Laboratory Exercise 5: Diffusion equation

OBJECTIVES: To experiment with a forward in time, centred in space scheme for integrating the linear diffusion equation

$$\frac{\partial \phi}{\partial t} = K \frac{\partial^2 \phi}{\partial x^2}$$

on a periodic domain $0 \leq x \leq 1$. The initial condition is given as

$$\phi(x,0) = \begin{cases} 1 & \text{if } 1/4 \leq x \leq 3/4 \\ 0 & \text{otherwise} \end{cases}$$

Browse the procedure `diffuse.py` (supplied). Make sure you understand how it works. Note how the solution at a new time level overwrites that at an old time level, so that data is stored at only two time levels. Note also how the periodic boundary conditions are implemented.

Run the procedure. Is the output what you expect? Experiment with the initial conditions. What happens to a sine wave?

Modify the procedure to use double the number of grid points in the x direction and run it again. Is the solution more accurate? What happens?

Use the theory given in lectures to calculate the maximum stable timestep at this new resolution. Modify the procedure to use a stable time step and compile it and run it again. Now is it more accurate than before?

Optional

Create a new procedure, based on `diffuse.py`, that uses a backward Euler time step instead of a forward Euler time step. You will have to formulate the tridiagonal system of simultaneous equations for the $\phi_j^{(n+1)}$, then use `numpy` to solve that system.

Run your new procedure for the original Δx and Δt specified in `diffuse.py`. Compare the accuracy of the two schemes.

Now double the number of grid points in the x direction and recompile and run your new procedure. Does it remain stable? How big can you make the time step without it blowing up?