Analysis of selected joint-life contingency models for males and females in Austria, Germany and Switzerland

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ABSTRACT:

The study made use of the 2022 mortality data provided by the respective statistics bureaus of the three selected countries in Western Europe: Austria, Germany and Switzerland. The researchers took into account the number of survivors per age from 0 to 99 between males and females to calculate the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the first death, expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year starting on the first death, expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the first death, expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death, expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year starting on the last death and expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death. The researchers also conducted comparative analysis to highlight potential differences amongst the models across the three countries in addition to the behavior of the three actuarial functions to bring to light the implications of such differences that eventually provide a strong foundation when it comes to understanding the nature of how the three countries calculate life insurance.

Keywords: mortality data, life contingencies, joint-life contingency models

1. INTRODUCTION

This section revolves around the profile of the actuarial societies in Austria, Germany and Switzerland in addition to the scope and limitations of the study.

1.1. Background of the study

Life insurance is a contract between an insurance company and a policyholder that offers financial protection to the named beneficiaries in the case of the policyholder's death. It acts as a key safety net, ensuring that families are not left financially vulnerable during times of loss. Beyond protection, life insurance holds a significant value as a fundamental element of financial planning that helps individuals to invest for the future and prepare for unexpected financial difficulties.

Austria, Germany, and Switzerland are three pivotal countries located in Western Europe, collectively known as the Germanosphere. As of 2024, the population of Austria is approximately 9.12 million, as reported by Worldometer. Germany has a significantly larger population, with approximately 84.55 million residents, also according to Worldometer. In Switzerland, the population stands at around 8.88 million, as indicated by Statista 2024.

Life expectancy in these countries reflects their advanced healthcare systems and quality of life. According to the source in the United Nations, Department of Economic and Social Affairs, Population Division (2024), In Austria, life expectancy was estimated to be about 82.12 years, largely due to the robust health services and lifestyle factors given. While Germany's average life expectancy is approximately 81.54 years, indicating a high standard of health among its populaces.

Meanwhile, Switzerland boasts the highest life expectancy among the three at around 84.09 years, demonstrating the effectiveness of public health initiatives in the region. The life expectancy figures represented by each of the three countries underline the ongoing investments in health promotion that positively affect the ongoing rate at the end of 2024.

The leading causes of death in Austria, Germany, and Switzerland are primarily attributed to non-communicable diseases. According to World Health Organization data (deaths per 100,000 population, 2021), the predominant cause of mortality in these three countries is ischemic heart disease. In Austria, the mortality rate for heart disease stands at 188.8. In Germany, the rate is slightly higher at 204.8 deaths. Meanwhile, in Switzerland, the rate is comparatively lower at 110.5 deaths, reflecting some differences in health outcomes across these countries.

Assessing the case profile of the leading three countries in the Germanosphere is ideal to conduct the analysis as they have a long-life expectancy, strong economy and stable social security systems. Therefore, conducting such study is essential to conduct given their stable demographic and economic conditions that aids in insurance firms to create an optimal package for their customers and boost their growth in the market.

1.2. Actuarial societies in Austria, Germany and Switzerland

This section revolves around the background of the actuarial societies in Austria, Germany and Switzerland.

1.2.1. Actuarial Association of Austria

Aktuarvereinigung Österreichs (AVÖ), anglicized as Actuarial Association of Austria, is the actuarial society of Austria. It was founded in 1971. The society is a member of the International Actuarial Association and Actuarial Association of Europe. This ensures that they are aligned with the global practices and standards of practicing actuary. Currently it has over 500 members, where 380 of those members are fully qualified actuaries (Advanced Solutions International, Inc., n.d.). Furthermore, in 2005 they established the European Actuarial Academy in collaboration with the following countries: Germany, Switzerland and Netherlands. This academy focuses on being a knowledge hub for actuarial education in Europe (*European Actuarial Academy*, n.d.).

1.2.2. German Actuarial Society

Deutsche Aktuarvereinigung (DAV), anglicized as either German Actuarial Society or German Association of Actuaries, is the actuarial society of Germany. It was established on April 14, 1993. As of March 2024, it currently comprises 7,100 total members, 5,500 of which are fully qualified actuaries and 1,600 are students who are in the process of becoming fully qualified members. The DAV also maintains active membership in international organizations like International Actuarial Association and Actuarial Association of Europe (Deutsche Aktuarvereinigung e.V. (DAV), n.d.). Moreover, the society is heavily involved in the development of actuarial education in Germany as it often partners with universities for instance, with Ulm University (*German Academic Exchange Service*, n.d.).

1.2.3. Swiss Association of Actuaries

Schweizerische Aktuarvereinigung, anglicized as Swiss Association of Actuaries, is the actuarial society of Switzerland. It was established in 1905 and is the professional organization that represents actuaries in Switzerland. As of 2017, there are approximately 1,364 members of this organization where 778 of them are qualified actuaries. The Swiss Association of Actuaries are actively involved in the International Actuarial Association and Actuarial Association of Europe. Their involvement in these organizations are deemed to further shape global actuarial practices and standards. (*Schweizerische Aktuarvereinigung*, n.d.).

1.3. Scope and limitations of the study

The study solely revolves around the 2022 male and female life tables of Austria, Germany and Switzerland, which were retrieved from Statistics Austria, Statistiches Bundesamt and Bundesamt für Statistik, respectively. The ages of both genders are from 0 to 99 to compensate for the missing data of those aged 100 in Switzerland. Lastly, the variables used in the study include the columns revolving around the number of survivors per age, expected value of whole life annuity, where benefit is paid at the insured's end of year of death and whole life insurance premium.

The study did not dwell into the life tables of the aforementioned three countries in the other years, let alone other countries. It did not also include external factors that influence the overall mortality of the countries such as environmental and social factors.

2. METHODOLOGY

This section revolves around the methodology used in analyzing the mortality data for males and females in Austria, Germany and Switzerland. The interest rates used for the three countries all assumed an interest rate of 6% for consistency.

2.1. Actuarial functions

This section contains the essential actuarial functions in the study.

Mortality rate. Mortality rate revolves around the statistical measure that measures the likelihood of death in the population given one timeframe. Furthermore, it denotes the probability of an individual dying between age x and x + n where x is any integer greater than or equal to 0 whereas n is any integer greater than 0. It is used in this study as it is useful to assess the life expectancy of a population and helps actuaries assess both population health and make informed decisions in insurance policy making. With the formula represented as q_x, the sole essential representation is ${}_nq_z$, which represents the probability an individual aged x survives to age x + n.

$$q_x = 1 - (1 - {}_nq_x)^{1/n}$$

Fig. 2.1. Formula of mortality rate

Number of survivors per age. The number of survivors per age is defined as the number of individuals in a population that are expected to survive until a particular age and further denotes the number of individuals that remain alive. It is used in this study as it is useful in tagging the associated health risks tied to a certain age, projecting future benefits and calculating insurance premiums optimally with the help of survival probability. Denoted as l_x, the sole essential representation is q_z, which represents the mortality rate.

$$l_x = nl_x[((1 - q_x)^{n-x})]$$

Fig. 2.2. Formula of number of survivors per age

Expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the first death. Taking account of the mortality rate and interest rate, the expected present value of a whole life annuity of a joint life status issued to individuals (x) and (y) payable at the end of each year until the first death calculates the present value of the annuity payouts to a male aged x and a female aged y that continues to make payments at the end of each year that halts making end-of-the-year payments until either one of the two dies within the year. It is used in the study as it is useful to assist insurance policymakers to create optimal annuity products and determine premiums for two individuals at the same time rather than treating the individuals independently that give rise to complexities and tediousness in calculations. It is noted that the age of a male denoted by x is equal to the age of a female denoted by in this context. Denoted by $ä_{xy}$, essential parameters include the discount function and the probability the two individuals aged x survive until age x +k, respectively.

$$ä_{xy} = \sum_{k=0}^{n-1} v^k {}_kp_{xy}$$

Fig. 2.3. Formula of expected present value of a whole life annuity of a joint life status issued to an individual ages x and y payable at the end of each year until the first death

Expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year starting on the first death. Taking account of the mortality rate and interest rate, the expected value of the whole life insurance of a joint life status issued to an individual ages x and y payable at the end of each year starting on the first death calculates the present value of the benefit to be paid at the end of the year of the first death amongst the male insurer aged x and the female insurer aged y where both insurers are of the same age. It is used in the study as it is useful in studying the optimality of the payout in addition to assessing the effect of the interest rate and survival probabilities given the deferral of the two insured individuals. It is noted that the age of a male denoted by x is equal to the age of a female denoted by in this context. Denoted as A_{xy}, essential parameters include v^k, ${}_kp_{xy}$ and $q_{x+k:y+k}$, which are the discount function, probability the two individuals aged x survive until age x +k and the probability the two individuals aged x+k die before age x + k + 1.

$$A_{xy} = \sum_{k=0}^{n} v^k {}_kp_{xy}\ q_{x+k:y+k}$$

Fig. 2.4. Formula of expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year starting on the first death.

Expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the first death. Taking account of the mortality rate and interest rate, the expected present value of a whole life annuity of a joint life status issued to individuals (x) and (y + 5) payable at the end of each year until the first death calculates the present value of the annuity payouts to a male aged x and a female aged y +5 that continues to make payments at the end of each year that halts making end-of-the-year payments until either one of the two dies within the year. It is used in the study as it is useful to assist insurance policymakers to create optimal annuity products and determine premiums for two individuals at the same time rather than treating the individuals independently that give rise to complexities and tediousness in calculations. It is noted that the age of a male denoted by x that files for a joint insurance is five years younger than the female.

$$ä_{x:y+5} = \sum_{k=0}^{\infty} v^k {}_kp_{x:y+5}$$

Fig. 2.5. Formula of expected present value of a whole life annuity of a joint life status issued to an individual ages x and y + 5 payable at the end of each year until the first death

Expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death. Taking account of the mortality rate and interest rate, the expected present value of a whole life annuity of a joint life status issued to individuals (x) and (y) payable at the end of each year until the last death calculates the present value of the annuity payouts to a male aged x and a female aged y that continues to make payments at the end of each year that halts making end-of-the-year payments until both of them died. It is used in the study as it is useful to assist insurance policymakers to create optimal annuity products and determine premiums for two individuals at the same time rather than treating the individuals independently that give rise to complexities and tediousness in calculations. It is noted that the age of a male denoted by x is equal to the age of a female denoted by in this context. Denoted by $\ddot{a}_{\overline{x:y}}$, essential parameters include the discount function and the probability the two individuals aged x survive until age x +k, respectively.

$$\ddot{a}_{\overline{x:y}} = \sum_{k=0}^{\infty} v^{k} {}_{k}p_{\overline{x:y}}$$

Fig. 2.6. Formula of expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death

Expected present value of a whole life insurance of a joint life (x) and (y) paid at the end of the year of the last death. The expected value of the whole life insurance of a joint life is a type of life insurance where the benefit is paid at the insured individuals' end of the year of last death that takes into account the present value of an annuity that pays the benefit at the end of the year of the death as opposed to paying it straightaway at the time of the death of the two insured individuals. It is used in the study as it is useful in studying the optimality of the payout in addition to assessing the effect of the interest rate and survival probabilities given the deferral of the two insured individuals. Denoted as A_x, essential parameters include v^k, ${}_kp_{xy}$ and $q_{x+k:y+k}$, which are the discount function, probability the two individuals aged x survive until age x +k and the probability the two individuals aged x+k die before age x + k + 1.

$$A_{\overline{x:y}} = \sum_{k=0}^{n} v^{k} {}_{k}p_{\overline{x:y}}\ q_{\overline{x+k:y+k}}$$

Fig. 2.7. Formula of expected present value of a whole life insurance of a joint life (x) and (y)

Expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death. The expected value of a whole life annuity of a joint life is the present value of the annuity payouts given the expected lifetime of two individuals that were taken into account alongside both mortality rate and interest rate. It is used in the study as it is useful to assist insurance policymakers to create optimal annuity products and determine premiums for two individuals at the same time rather than treating the individuals independently that give rise to complexities and tediousness in calculations. Denoted as $\ddot{a}_{xx}$, essential parameters include the discount function and the probability the two individuals aged x survive until age x +k, respectively.

$$A_{\overline{x:y+5}} = \sum_{k=0}^{\infty} v^{k} {}_{k}p_{\overline{x:y+5}}$$

Fig. 2.8. Expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death.

Overall, these actuarial functions are essential in the study as they help insurance firms calculate the most optimal insurance and premium packages for their clients using survival probability and mortality rates whilst prioritizing profitability and risk mitigation in joint cases. Furthermore, it helps them secure financial stability in managing payouts, insurance packages and premiums in achieving optimality in their allocation to clients when they apply for joint insurance. Lastly, it helps insurance firms boost their profile in customer value and market competitiveness by capitalizing on their optimal calculations in their products and services in hopes of attracting customers to avail them.

2.2. Data

The 2022 life tables used in the study to assess the mortality data of Austria, Germany and Switzerland were obtained from Statistics Austria, Statistiches Bundesamt und Bundesamt für Statistik, respectively. The data gives comprehensive information about the survival patterns and life expectancy trends of Austria, Germany and Switzerland. The essential variables were l_x and ${}_nq_z$ and represent the number of survivors given the age that was represented by x and the probability of dying between ages x and x + n, respectively. These variables will be utilized in the analysis to understand dynamics of the population aging and life expectancy when comparing the mortality between the countries.

Each life table assumes an initial cohort of 100,000 live births. This gives a standardised baseline in comparing the mortality rate across the populations of each country.

3. RESULTS

This section contains the results of the statistical analysis: overall tabular results of the actuarial functions and comparative analysis between ages 20, 40, 60, 80 and 99. The former was aided by a curve graph whereas the latter was aided by a bar graph and a table consisting of the five different ages.

	lx		End-of-the-year payments until the end of the year of first death			End-of-the-year payments until the end of the year of last death		
Age	Male	Female	$\ddot{a}_{x:y}$	$A_{x:y}$	$\ddot{a}_{x:y+5}$	$\ddot{a}_{x:y}$	$A_{x:y}$	$\ddot{a}_{x:y+5}$
0	100000	100000	17.5552	0.0062	17.5364	17.2238	0.0251	17.2261
1	99756	99755	17.5493	0.0065	17.5290	17.2816	0.0218	17.2440
2	99738	99734	17.5423	0.0069	17.5207	17.2652	0.0227	17.2223
3	99715	99724	17.5349	0.0073	17.5120	17.2468	0.0238	17.2015
4	99711	99708	17.5270	0.0077	17.5028	17.2251	0.0250	17.1755
5	99700	99701	17.5187	0.0082	17.4930	17.2017	0.0263	17.1491
6	99696	99688	17.5098	0.0087	17.4826	17.1768	0.0277	17.1208
7	99680	99685	17.5004	0.0092	17.4716	17.1506	0.0292	17.0925
8	99672	99674	17.4905	0.0097	17.4599	17.1229	0.0308	17.0598
9	99667	99667	17.4800	0.0103	17.4475	17.0924	0.0325	17.0271
10	99663	99660	17.4688	0.0109	17.4344	17.0598	0.0343	16.9906
11	99652	99648	17.4570	0.0116	17.4205	17.0273	0.0362	16.9551
12	99634	99638	17.4444	0.0123	17.4059	16.9937	0.0381	16.9191
13	99620	99636	17.4311	0.0130	17.3903	16.9561	0.0402	16.8820
14	99614	99619	17.4171	0.0138	17.3738	16.9173	0.0424	16.8395
15	99605	99612	17.4021	0.0146	17.3563	16.8751	0.0448	16.7953
16	99575	99593	17.3864	0.0155	17.3379	16.8359	0.0470	16.7539
17	99544	99572	17.3697	0.0164	17.3183	16.7948	0.0493	16.7072
18	99501	99540	17.3521	0.0174	17.2977	16.7551	0.0516	16.6617
19	99457	99519	17.3334	0.0184	17.2758	16.7113	0.0541	16.6131
20	99411	99496	17.3136	0.0195	17.2525	16.6655	0.0567	16.5597
21	99368	99461	17.2927	0.0206	17.2279	16.6185	0.0593	16.5034
22	99319	99444	17.2704	0.0219	17.2018	16.5666	0.0623	16.4447
23	99265	99415	17.2470	0.0232	17.1742	16.5144	0.0652	16.3857
24	99202	99389	17.2221	0.0245	17.1451	16.4600	0.0683	16.3248
25	99150	99376	17.1956	0.0260	17.1141	16.3984	0.0718	16.2563
26	99094	99358	17.1676	0.0275	17.0813	16.3344	0.0754	16.1859
27	99066	99340	17.1378	0.0292	17.0464	16.2621	0.0795	16.1045
28	99005	99307	17.1065	0.0309	17.0097	16.1931	0.0834	16.0268
29	98937	99273	17.0733	0.0327	16.9708	16.1213	0.0875	15.9435
30	98864	99252	17.0381	0.0347	16.9297	16.0438	0.0919	15.8595

31	98806	99221	17.0008	0.0367	16.8860	15.9608	0.0965	15.7659
32	98750	99203	16.9612	0.0389	16.8398	15.8703	0.1017	15.6686
33	98686	99165	16.9194	0.0412	16.7909	15.7788	0.1068	15.5668
34	98623	99139	16.8750	0.0437	16.7389	15.6796	0.1125	15.4563
35	98542	99091	16.8283	0.0462	16.6842	15.5807	0.1181	15.3436
36	98465	99056	16.7786	0.0490	16.6261	15.4731	0.1241	15.2241
37	98363	99006	16.7263	0.0519	16.5648	15.3652	0.1303	15.1002
38	98269	98955	16.6708	0.0549	16.4999	15.2495	0.1368	14.9697
39	98148	98919	16.6120	0.0582	16.4315	15.1286	0.1436	14.8381
40	98040	98872	16.5498	0.0616	16.3590	15.0000	0.1509	14.6973
41	97905	98820	16.4841	0.0652	16.2826	14.8682	0.1584	14.5538
42	97772	98774	16.4144	0.0690	16.2019	14.7272	0.1664	14.4034
43	97629	98713	16.3409	0.0731	16.1164	14.5811	0.1746	14.2439
44	97481	98634	16.2633	0.0774	16.0261	14.4294	0.1832	14.0769
45	97343	98548	16.1811	0.0819	15.9305	14.2678	0.1924	13.9004
46	97159	98449	16.0946	0.0867	15.8300	14.1047	0.2016	13.7211
47	96985	98334	16.0031	0.0917	15.7236	13.9321	0.2114	13.5310
48	96778	98227	15.9064	0.0970	15.6115	13.7524	0.2215	13.3353
49	96585	98108	15.8042	0.1026	15.4929	13.5609	0.2324	13.1266
50	96342	97972	15.6966	0.1086	15.3682	13.3667	0.2434	12.9142
51	96080	97824	15.5830	0.1148	15.2369	13.1643	0.2548	12.6939
52	95794	97662	15.4633	0.1214	15.0991	12.9542	0.2667	12.4699
53	95476	97486	15.3372	0.1283	14.9536	12.7366	0.2790	12.2337
54	95099	97300	15.2044	0.1356	14.8011	12.5140	0.2916	11.9941
55	94696	97092	15.0647	0.1433	14.6407	12.2830	0.3047	11.7461
56	94269	96859	14.9176	0.1514	14.4721	12.0430	0.3183	11.4887
57	93753	96569	14.7641	0.1598	14.2974	11.8054	0.3317	11.2400
58	93142	96292	14.6026	0.1686	14.1147	11.5620	0.3455	10.9877
59	92558	95977	14.4323	0.1779	13.9220	11.3034	0.3601	10.7205
60	91903	95628	14.2539	0.1877	13.7194	11.0395	0.3751	10.4407
61	91137	95243	14.0675	0.1979	13.5090	10.7747	0.3900	10.1633
62	90268	94728	13.8749	0.2084	13.2910	10.5178	0.4046	9.8905
63	89350	94193	13.6724	0.2195	13.0610	10.2505	0.4197	9.5983
64	88321	93606	13.4613	0.2310	12.8237	9.9819	0.4349	9.3143
65	87185	93037	13.2393	0.2430	12.5757	9.7039	0.4506	9.0208
66	85952	92377	13.0085	0.2556	12.3176	9.4253	0.4664	8.7220
67	84653	91594	12.7690	0.2686	12.0483	9.1454	0.4822	8.4138
68	83175	90854	12.5177	0.2822	11.7708	8.8591	0.4984	8.1094
69	81605	89937	12.2587	0.2962	11.4840	8.5775	0.5144	7.8055
70	80027	89012	11.9856	0.3109	11.1855	8.2757	0.5314	7.4933
71	78371	88012	11.7008	0.3262	10.8766	7.9647	0.5490	7.1783

72	76451	86936	11.4070	0.3419	10.5623	7.6616	0.5662	6.8769
73	74428	85745	11.1025	0.3582	10.2363	7.3540	0.5836	6.5674
74	72271	84377	10.7890	0.3749	9.9003	7.0487	0.6008	6.2548
75	69969	82831	10.4666	0.3919	9.5538	6.7462	0.6179	5.9361
76	67607	81101	10.1336	0.4094	9.1959	6.4383	0.6353	5.6092
77	64926	79174	9.7952	0.4270	8.8357	6.1487	0.6517	5.2952
78	62080	77141	9.4457	0.4451	8.4732	5.8582	0.6681	4.9988
79	59241	74956	9.0825	0.4637	8.0956	5.5538	0.6853	4.6826
80	56334	72648	8.7060	0.4828	7.7133	5.2374	0.7031	4.3750
81	53133	70133	8.3243	0.5018	7.3328	4.9330	0.7203	4.0830
82	49945	67383	7.9312	0.5211	6.9481	4.6161	0.7381	3.8024
83	46682	64186	7.5360	0.5399	6.5596	4.3053	0.7556	3.5269
84	43222	60886	7.1328	0.5585	6.1734	3.9892	0.7733	3.2602
85	39409	57128	6.7400	0.5755	5.7996	3.7037	0.7893	3.0138
86	35492	53071	6.3492	0.5911	5.4335	3.4254	0.8048	2.7835
87	31367	48523	5.9755	0.6038	5.0859	3.1817	0.8182	2.5846
88	27263	43726	5.6093	0.6137	4.7463	2.9527	0.8305	2.4052
89	23151	38800	5.2546	0.6195	4.4141	2.7469	0.8412	2.2400
90	19358	33828	4.9009	0.6206	4.0664	2.5400	0.8514	2.0564
91	15668	28805	4.5682	0.6132	3.7270	2.3686	0.8585	1.8747
92	12479	23841	4.2377	0.5955	3.3712	2.2007	0.8634	1.6827
93	9609	19155	3.9171	0.5616	3.0071	2.0572	0.8630	1.4407
94	7107	15000	3.5921	0.5050	2.6542	1.9348	0.8529	
95	5109	11466	3.2329	0.4202	2.4392	1.8032	0.8254	
96	3560	8462	2.8273	0.2971	2.1893	1.6557	0.7569	
97	2385	5854	2.3643	0.1308	1.8818	1.4997	0.5734	
98	1434	3819	1.8002		1.5546	1.3503		
99	843	2412						

Fig. 3.1. Joint life table of Austria

It can be observed that the number of survivors per age in Austria when comparing the number from men and women is that the number of women that the number of survivorship in women is higher than the number in men with the notable exception of ages 9, 10 and 11. By age 42, the difference between the two genders crosses 1,000. By age 64, the difference between the two genders crossed 5,000 and saw the number of survivors in men diverted from the number of survivors in women greatly shortly thereafter. However, by their early 90s, the number of survivors in women started to converge with the number of survivors in men after the difference started to cool down by that time.

The expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers amongst (x) and (y) in Austria begins to collapse at age 49 as there is a 0.1 difference in the aforementioned age when comparing it to age 48. It is noteworthy to state that the magnitude of the decrease starts at age 49 as the difference in the values with respect to the preceding age before age 49 is not that wide (i.e., less than 0.1). Furthermore, the aforementioned age is the time when the values see a growing dramatic decline of the value in the aforementioned actuarial function in the ages that follow. When comparing the aforementioned actuarial function to the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y) in Austria, the latter generated smaller values than that of the former. However, the noteworthy decline starts earlier at age 36 as there is a 0.1 difference in the aforementioned age when comparing it to age 35. Furthermore, age 36 marks the start of the continuous dramatic decline of the aforementioned actuarial function in the ages that follow as one sees the gap between the preceding age grow wider as time passes. It is also noteworthy that the values under the two aforementioned actuarial functions decrease when comparing the value under the succeeding age to the preceding age.

The expected present value of a whole life insurance paid at the end of every year of starting at the first death of the two insurers amongst (x) and (y) in Austria begins to widen at age 61 as there is a 0.1 increase when comparing the value at age 60. It is noteworthy to state that the magnitude of the increase starts at age 61 as the difference in the values with respect to the preceding age before age 61 is not that wide (i.e., less than 0.1). Furthermore, the aforementioned age is the time when the values see a growing sharper increase of the value in the aforementioned actuarial function in the ages that follow. However, the increase gets interrupted at age 91 as the value in the aforementioned actuarial function starts dipping as the ages progress until it reaches a halts at age 97. When comparing the aforementioned actuarial function to the expected present value of a whole life insurance paid at the end of every year before the death of the last of the two insurers amongst (x) and (y) in Austria, the latter generated bigger values than that of the former. However, the noteworthy increase starts earlier at age 48 as there is a 0.1 difference in the aforementioned age when comparing it to age 47. Furthermore, age 48 marks the start of the continuous sharper increase of the value in the aforementioned actuarial function in the ages that follow as one sees the gap between the preceding age grow wider as time passes. However, the increase gets interrupted at age 93 as the value in the aforementioned actuarial function starts dipping as the ages progress until it halts at age 97, experiencing a delay when compared to its counterpart concerning the expected present value of an annuity paid at the end of every year before the death of the first insurer. It is also noteworthy that the values under the two aforementioned actuarial functions increase when comparing the value under the succeeding age to the preceding age.

The expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers amongst (x) and (y + 5) in Austria begins to collapse at age 46 as there is a 0.1 difference in the aforementioned age when comparing it to age 45. It is noteworthy to state that the magnitude of the decrease starts at age 46 as the difference in the values with respect to the preceding age before age 49 is not that wide (i.e., less than 0.1). Furthermore, the aforementioned age is the time when the values see a growing dramatic decline of the value in the aforementioned actuarial function in the ages that follow. When comparing the aforementioned actuarial function to the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y + 5) in Austria, the latter generated smaller values than that of the former. However, the noteworthy decline starts earlier at age 33 as there is a 0.1 difference in the aforementioned age when comparing it to age 32. Furthermore, age 33 marks the start of the continuous dramatic decline of the aforementioned actuarial function in the ages that follow as one sees the gap between the preceding age grow wider as time passes. However, the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y + 5) in Austria has no value as there is no survivorship record for those ages 100 to 104. It is also noteworthy that the values under the two aforementioned actuarial functions decrease when comparing the value under the succeeding age to the preceding age.

Overall, it is worth noting that the values under the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y) is cheaper than the expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers amongst (x) and (y). However, when one looks at the age of the male insurer, the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y + 5) is cheaper than the when one looks at the age of the male insurer expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers amongst (x) and (y + 5). Lastly, the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y + 5) is cheaper than the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers amongst (x) and (y). It is also observed that the expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers amongst (x) and (y + 5) is cheaper than the expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers amongst (x) and (y).

	lx	Payment at the moment of first death	Payment at the moment of last death

Age	Male	Female	$\ddot{a}_{x:y}$	$A_{x:y}$	$\ddot{a}_{x:y+5}$	$\ddot{a}_{x:y}$	$A_{x:y}$	$\ddot{a}_{x:y+5}$
0	100000	100000	17.5511	0.0064	17.5317	17.2013	0.0263	17.2101
1	99675	99713	17.5452	0.0067	17.5241	17.2790	0.0219	17.2397
2	99653	99692	17.5379	0.0071	17.5156	17.2632	0.0228	17.2190
3	99642	99680	17.5302	0.0075	17.5065	17.2429	0.0240	17.1950
4	99630	99670	17.5221	0.0080	17.4969	17.2213	0.0252	17.1700
5	99619	99660	17.5135	0.0085	17.4868	17.1982	0.0265	17.1431
6	99610	99654	17.5043	0.0090	17.4760	17.1727	0.0280	17.1141
7	99601	99648	17.4946	0.0095	17.4646	17.1456	0.0295	17.0835
8	99593	99642	17.4843	0.0101	17.4525	17.1168	0.0311	17.0509
9	99586	99635	17.4734	0.0107	17.4397	17.0862	0.0329	17.0166
10	99578	99629	17.4618	0.0113	17.4261	17.0537	0.0347	16.9808
11	99572	99624	17.4495	0.0120	17.4117	17.0188	0.0367	16.9427
12	99565	99618	17.4365	0.0127	17.3965	16.9822	0.0387	16.9030
13	99558	99612	17.4227	0.0135	17.3803	16.9433	0.0409	16.8609
14	99548	99604	17.4081	0.0143	17.3632	16.9030	0.0432	16.8170
15	99536	99593	17.3927	0.0151	17.3451	16.8611	0.0456	16.7710
16	99521	99581	17.3763	0.0160	17.3259	16.8173	0.0481	16.7228
17	99498	99566	17.3590	0.0170	17.3056	16.7727	0.0506	16.6732
18	99472	99551	17.3407	0.0180	17.2841	16.7260	0.0532	16.6208
19	99435	99535	17.3212	0.0191	17.2613	16.6784	0.0559	16.5671
20	99390	99517	17.3007	0.0202	17.2372	16.6297	0.0587	16.5112
21	99350	99499	17.2789	0.0214	17.2117	16.5771	0.0617	16.4516
22	99308	99480	17.2559	0.0227	17.1846	16.5219	0.0648	16.3888
23	99266	99463	17.2314	0.0240	17.1559	16.4630	0.0681	16.3222
24	99223	99446	17.2055	0.0255	17.1256	16.4006	0.0717	16.2523
25	99180	99430	17.1780	0.0270	17.0934	16.3344	0.0754	16.1782
26	99138	99411	17.1490	0.0286	17.0593	16.2644	0.0794	16.0999
27	99092	99392	17.1182	0.0303	17.0233	16.1909	0.0835	16.0182
28	99046	99373	17.0855	0.0321	16.9851	16.1129	0.0879	15.9318
29	99000	99350	17.0510	0.0340	16.9447	16.0309	0.0926	15.8408
30	98950	99327	17.0144	0.0360	16.9019	15.9445	0.0975	15.7457
31	98896	99301	16.9757	0.0381	16.8566	15.8539	0.1026	15.6461
32	98840	99271	16.9347	0.0404	16.8086	15.7588	0.1080	15.5407
33	98780	99239	16.8913	0.0428	16.7579	15.6589	0.1136	15.4306
34	98717	99203	16.8453	0.0453	16.7043	15.5540	0.1196	15.3153
35	98642	99162	16.7968	0.0480	16.6475	15.4454	0.1257	15.1956
36	98562	99117	16.7454	0.0509	16.5875	15.3315	0.1322	15.0694
37	98472	99072	16.6910	0.0539	16.5241	15.2121	0.1389	14.9389
38	98367	99020	16.6336	0.0570	16.4571	15.0889	0.1459	14.8031
39	98258	98961	16.5728	0.0604	16.3863	14.9597	0.1532	14.6614

40	98141	98897	16.5086	0.0639	16.3115	14.8245	0.1609	14.5129
41	98012	98832	16.4407	0.0677	16.2324	14.6829	0.1689	14.3578
42	97875	98755	16.3689	0.0716	16.1489	14.5355	0.1772	14.1963
43	97725	98674	16.2931	0.0758	16.0608	14.3815	0.1859	14.0277
44	97556	98581	16.2131	0.0802	15.9679	14.2223	0.1949	13.8551
45	97374	98482	16.1286	0.0849	15.8700	14.0560	0.2044	13.6751
46	97183	98377	16.0393	0.0898	15.7666	13.8813	0.2142	13.4871
47	96974	98258	15.9451	0.0950	15.6576	13.7002	0.2245	13.2918
48	96742	98131	15.8458	0.1004	15.5427	13.5120	0.2351	13.0891
49	96483	97976	15.7413	0.1062	15.4218	13.3193	0.2460	12.8804
50	96188	97810	15.6312	0.1122	15.2949	13.1208	0.2573	12.6671
51	95861	97628	15.5153	0.1186	15.1615	12.9159	0.2689	12.4472
52	95503	97430	15.3933	0.1253	15.0212	12.7039	0.2809	12.2203
53	95108	97217	15.2649	0.1324	14.8740	12.4850	0.2933	11.9875
54	94681	96979	15.1299	0.1398	14.7195	12.2590	0.3061	11.7477
55	94206	96709	14.9883	0.1476	14.5576	12.0282	0.3191	11.5031
56	93664	96415	14.8398	0.1557	14.3885	11.7934	0.3324	11.2563
57	93068	96093	14.6840	0.1643	14.2115	11.5529	0.3460	11.0032
58	92416	95736	14.5208	0.1732	14.0262	11.3070	0.3599	10.7446
59	91688	95341	14.3501	0.1826	13.8331	11.0578	0.3740	10.4834
60	90889	94901	14.1717	0.1923	13.6316	10.8048	0.3883	10.2183
61	90001	94407	13.9856	0.2025	13.4217	10.5506	0.4027	9.9510
62	89023	93876	13.7912	0.2131	13.2032	10.2927	0.4173	9.6805
63	87959	93296	13.5882	0.2241	12.9753	10.0314	0.4321	9.4043
64	86802	92654	13.3768	0.2356	12.7386	9.7681	0.4470	9.1260
65	85543	91951	13.1567	0.2476	12.4928	9.5031	0.4620	8.8459
66	84178	91183	12.9277	0.2600	12.2376	9.2366	0.4771	8.5627
67	82711	90352	12.6892	0.2729	11.9728	8.9673	0.4923	8.2767
68	81143	89471	12.4405	0.2864	11.6982	8.6933	0.5078	7.9875
69	79469	88503	12.1823	0.3003	11.4140	8.4178	0.5234	7.6964
70	77695	87447	11.9141	0.3147	11.1191	8.1395	0.5391	7.3995
71	75842	86312	11.6348	0.3296	10.8141	7.8547	0.5552	7.0996
72	73869	85081	11.3451	0.3451	10.4987	7.5680	0.5714	6.7953
73	71820	83748	11.0442	0.3611	10.1729	7.2747	0.5880	6.4872
74	69620	82293	10.7335	0.3776	9.8383	6.9827	0.6045	6.1791
75	67348	80745	10.4103	0.3946	9.4922	6.6813	0.6216	5.8627
76	64909	79035	10.0782	0.4120	9.1398	6.3836	0.6384	5.5520
77	62408	77208	9.7334	0.4300	8.7769	6.0758	0.6558	5.2352
78	59741	75198	9.3800	0.4482	8.4089	5.7707	0.6730	4.9246
79	56951	73021	9.0165	0.4667	8.0352	5.4629	0.6904	4.6175
80	54070	70683	8.6420	0.4856	7.6557	5.1475	0.7082	4.3125

81	50964	68088	8.2632	0.5043	7.2788	4.8421	0.7254	4.0258
82	47721	65270	7.8776	0.5230	6.9016	4.5372	0.7426	3.7497
83	44325	62157	7.4892	0.5413	6.5278	4.2388	0.7593	3.4905
84	40784	58742	7.0997	0.5588	6.1555	3.9481	0.7756	3.2389
85	37110	55015	6.7118	0.5752	5.7896	3.6671	0.7913	3.0030
86	33288	50908	6.3321	0.5897	5.4348	3.4060	0.8058	2.7894
87	29406	46504	5.9599	0.6020	5.0877	3.1604	0.8192	2.5907
88	25480	41808	5.6009	0.6108	4.7514	2.9397	0.8310	2.4122
89	21710	37035	5.2457	0.6159	4.4138	2.7242	0.8422	2.2381
90	18044	32165	4.9054	0.6150	4.0812	2.5319	0.8513	2.0754
91	14635	27294	4.5778	0.6058	3.7416	2.3593	0.8582	1.9081
92	11575	22621	4.2562	0.5856	3.3876	2.1981	0.8622	1.7226
93	8834	18249	3.9443	0.5485	3.0207	2.0626	0.8603	1.4612
94	6559	14325	3.6221	0.4894	2.6565	1.9327	0.8488	
95	4697	10899	3.2821	0.3983	2.4520	1.8145	0.8160	
96	3247	8028	2.8975	0.2686	2.2264	1.6956	0.7348	
97	2143	5672	2.4390	0.1050	1.9697	1.5812	0.5258	
98	1381	3896	1.8244		1.5950	1.3917		
99	871	2565						

Fig. 3.2. Life table for a joint life in Germany

It can be observed from the survival statistics in Germany, it’s evident that women tend to outlive men at every age. The only exception is during the early years of life, where the differences are quite small. However, by the time individuals reach 42, the number of male survivors falls behind female survivors by over 1,000. This gap grows even larger, exceeding 5,000 by age 64. After this point, the number of men surviving starts to diverge significantly from that of women. Interestingly, as people reach their mid-80s, this gap begins to close again, suggesting that both men and women face similar survival rates as they near the end of their lives.

The expected present value of a whole life insurance paid at the end of every year before the death of the first of the two insurers among (x) and (y) in Germany begins to decline sharply at age 49, as evidenced by a decrease greater than 0.10 in the value compared to age 48. Prior to this, the annual differences were relatively small and further suggests the start of a more rapid decrease in value from this point onwards. This trend continues as age progresses. When comparing this function to the expected present value of a whole life annuity paid at the end of every year before the death of the last of the two insurers among (x) and (y), it is observed that the latter generates smaller values. Furthermore, the decline in the last-death annuity due begins earlier, with a 0.1 drop already observed by age 34. This age marks the beginning of a sustained and dramatic decrease in value, with larger differences between successive ages becoming more apparent as time goes on. It is also notable that for both annuity types, the expected present values decrease with each succeeding age, reflecting the increasing probability of death.

On the other hand, the expected present value of a whole life insurance paid at the end of every year before the death of the first of the two insurers among (x) and (y) in Germany begins to increase more markedly at age 61, with a 0.01 increase from the preceding age After age 61, the value grows more steadily but begins to decrease at age 90, eventually halting at age 97. When compared to the expected present value of a whole life insurance paid at the end of every year before the death of the last of the two insurers among (x) and (y), the latter consistently yields higher values. However, its more notable increase starts earlier at age 34, where the 0.01 increment compared to age 51 signals the beginning of a sharper rise, fluctuating between 0.01 or 0.02. This rise continues until age 92, after which the values begin to decline and halt at age 97. This decline occurs slightly later than in the first-death insurance case. It is also noteworthy that the values under both life insurance functions increase with each succeeding age until they begin to decline.

For the case of a male insured aged (x) and a female insured aged (y+5), the expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insurers begins to collapse with a 0.1 decrease from age 45 then at age 57, the decrease it began to fluctuate between 0.2 and 0.3. When comparing this to the expected present value of a whole life annuity paid before the death of the last of the two insurers among (x) and (y + 5), the latter produces smaller values. Moreover, the decline begins earlier, at age 33 with a 0.1 decrease but it begins to fluctuate more with the values between 0.2 and 0.3 at age 43. This trend continues more sharply in the following ages. Notably, has got no record for ages 95 to 99 thanks to the absence of survivorship records for those advanced ages. As in the previous comparisons, both annuity types experience decreasing values as the insured ages increase.

Overall, the expected present value of a whole life annuity that pays out at the end of each year until the last of two insurers, (x) and (y), passes away, we find that it's generally lower than the value of an annuity that pays out before the first death. This difference is due to the longer duration and the added uncertainty that comes with the last-death annuity, leading to a reduced present value. Interestingly, if the female partner is five years younger (i.e. (y+5)), the expected present value of the whole life annuity paid before the last death drops even further compared to the one paid before the first death. Additionally, in the scenario where (x) is paired with (y+5), both types of annuities, first-death and last-death, show lower present values than those in the (x,y) scenario.

	lx		Payment at the moment of first death			Payment at the moment of last death		
f.Age	f.lx	g.lx	$\ddot{a}_{x:y}$	$A_{x:y}$	$\ddot{a}_{x:y+5}$	$\ddot{a}_{x:y}$	$A_{x:y}$	$\ddot{a}_{x:y+5}$
0	100000	100000	17.5669	0.0054	17.5510	17.2375	0.0243	17.2623
1	99575	99672	17.5619	0.0056	17.5446	17.3421	0.0184	17.3132
2	99553	99645	17.5557	0.0060	17.5373	17.3312	0.0190	17.2970
3	99538	99626	17.5491	0.0063	17.5296	17.3169	0.0198	17.2781
4	99525	99610	17.5421	0.0067	17.5214	17.3010	0.0207	17.2578
5	99515	99597	17.5346	0.0071	17.5127	17.2831	0.0217	17.2353
6	99507	99588	17.5267	0.0075	17.5034	17.2630	0.0228	17.2113
7	99502	99581	17.5184	0.0080	17.4936	17.2408	0.0241	17.1852
8	99498	99577	17.5095	0.0085	17.4833	17.2167	0.0255	17.1577
9	99495	99573	17.5001	0.0090	17.4723	17.1909	0.0269	17.1284
10	99493	99571	17.4901	0.0095	17.4606	17.1630	0.0285	17.0978
11	99491	99568	17.4795	0.0101	17.4483	17.1337	0.0302	17.0661
12	99489	99566	17.4683	0.0107	17.4352	17.1024	0.0319	17.0331
13	99486	99562	17.4564	0.0113	17.4214	17.0697	0.0338	16.9989
14	99480	99558	17.4438	0.0120	17.4067	17.0356	0.0357	16.9630
15	99471	99550	17.4304	0.0127	17.3912	17.0007	0.0377	16.9256
16	99454	99538	17.4163	0.0135	17.3748	16.9656	0.0397	16.8872
17	99429	99522	17.4014	0.0143	17.3574	16.9306	0.0417	16.8474
18	99397	99503	17.3856	0.0152	17.3389	16.8951	0.0437	16.8064
19	99360	99484	17.3689	0.0161	17.3194	16.8582	0.0457	16.7635
20	99321	99465	17.3511	0.0170	17.2987	16.8196	0.0479	16.7184
21	99282	99446	17.3324	0.0180	17.2768	16.7785	0.0503	16.6704
22	99242	99430	17.3124	0.0191	17.2536	16.7347	0.0527	16.6199
23	99203	99414	17.2913	0.0202	17.2290	16.6880	0.0554	16.5662
24	99164	99399	17.2690	0.0214	17.2030	16.6383	0.0582	16.5095
25	99126	99384	17.2453	0.0227	17.1753	16.5855	0.0612	16.4494

26	99086	99370	17.2201	0.0241	17.1461	16.5296	0.0643	16.3862
27	99046	99355	17.1935	0.0255	17.1151	16.4705	0.0677	16.3198
28	99005	99339	17.1653	0.0270	17.0823	16.4082	0.0712	16.2496
29	98963	99322	17.1355	0.0286	17.0476	16.3424	0.0749	16.1760
30	98919	99304	17.1038	0.0303	17.0107	16.2731	0.0789	16.0984
31	98873	99284	17.0703	0.0321	16.9718	16.2003	0.0830	16.0171
32	98824	99261	17.0348	0.0340	16.9305	16.1240	0.0873	15.9317
33	98773	99237	16.9972	0.0361	16.8868	16.0436	0.0918	15.8418
34	98720	99209	16.9574	0.0382	16.8406	15.9593	0.0966	15.7473
35	98663	99180	16.9153	0.0405	16.7916	15.8707	0.1016	15.6481
36	98603	99147	16.8707	0.0429	16.7397	15.7778	0.1069	15.5440
37	98540	99112	16.8235	0.0454	16.6849	15.6800	0.1124	15.4346
38	98474	99074	16.7735	0.0481	16.6268	15.5772	0.1182	15.3194
39	98403	99033	16.7205	0.0510	16.5653	15.4694	0.1243	15.1989
40	98328	98989	16.6645	0.0540	16.5002	15.3561	0.1307	15.0724
41	98247	98941	16.6052	0.0572	16.4314	15.2374	0.1375	14.9400
42	98161	98889	16.5424	0.0605	16.3586	15.1128	0.1445	14.8012
43	98069	98834	16.4760	0.0641	16.2816	14.9819	0.1519	14.6559
44	97969	98773	16.4058	0.0679	16.2003	14.8451	0.1596	14.5043
45	97860	98707	16.3315	0.0719	16.1142	14.7020	0.1677	14.3460
46	97741	98635	16.2529	0.0761	16.0233	14.5524	0.1762	14.1810
47	97611	98556	16.1699	0.0806	15.9273	14.3962	0.1850	14.0090
48	97468	98469	16.0821	0.0853	15.8259	14.2334	0.1943	13.8302
49	97309	98372	15.9894	0.0903	15.7189	14.0641	0.2038	13.6446
50	97134	98266	15.8915	0.0955	15.6060	13.8879	0.2138	13.4518
51	96940	98148	15.7881	0.1011	15.4868	13.7050	0.2241	13.2518
52	96724	98018	15.6790	0.1069	15.3613	13.5152	0.2349	13.0448
53	96485	97873	15.5639	0.1131	15.2289	13.3187	0.2460	12.8306
54	96219	97713	15.4426	0.1196	15.0895	13.1154	0.2575	12.6093
55	95923	97536	15.3146	0.1264	14.9428	12.9053	0.2694	12.3810
56	95595	97341	15.1799	0.1337	14.7884	12.6883	0.2817	12.1455
57	95232	97125	15.0379	0.1412	14.6261	12.4645	0.2943	11.9028
58	94829	96887	14.8885	0.1492	14.4554	12.2340	0.3074	11.6532
59	94384	96624	14.7314	0.1576	14.2762	11.9967	0.3208	11.3965
60	93892	96334	14.5661	0.1664	14.0881	11.7529	0.3346	11.1328
61	93350	96015	14.3924	0.1757	13.8908	11.5023	0.3487	10.8624
62	92754	95664	14.2100	0.1854	13.6839	11.2451	0.3633	10.5851
63	92099	95277	14.0185	0.1955	13.4673	10.9815	0.3782	10.3011
64	91382	94852	13.8176	0.2062	13.2405	10.7111	0.3935	10.0105
65	90599	94385	13.6069	0.2173	13.0032	10.4341	0.4091	9.7133
66	89745	93870	13.3861	0.2290	12.7553	10.1507	0.4252	9.4097

67	88816	93305	13.1550	0.2412	12.4964	9.8606	0.4416	9.0996
68	87807	92683	12.9131	0.2539	12.2264	9.5639	0.4583	8.7834
69	86711	91998	12.6602	0.2672	11.9449	9.2609	0.4754	8.4614
70	85522	91244	12.3960	0.2810	11.6520	8.9517	0.4929	8.1339
71	84231	90413	12.1203	0.2953	11.3476	8.6366	0.5107	7.8015
72	82830	89498	11.8330	0.3103	11.0317	8.3159	0.5288	7.4647
73	81309	88487	11.5339	0.3257	10.7045	7.9902	0.5472	7.1243
74	79655	87371	11.2231	0.3417	10.3663	7.6601	0.5659	6.7813
75	77855	86137	10.9006	0.3581	10.0175	7.3264	0.5847	6.4367
76	75895	84772	10.5666	0.3751	9.6588	6.9899	0.6037	6.0920
77	73759	83260	10.2214	0.3924	9.2909	6.6518	0.6228	5.7487
78	71433	81583	9.8655	0.4101	8.9150	6.3132	0.6418	5.4084
79	68903	79721	9.4995	0.4280	8.5323	5.9751	0.6609	5.0729
80	66153	77654	9.1242	0.4461	8.1444	5.6391	0.6798	4.7448
81	63173	75356	8.7407	0.4642	7.7529	5.3064	0.6985	4.4260
82	59954	72802	8.3502	0.4821	7.3597	4.9786	0.7168	4.1188
83	56490	69964	7.9544	0.4996	6.9667	4.6575	0.7348	3.8253
84	52784	66813	7.5550	0.5164	6.5757	4.3448	0.7522	3.5470
85	48844	63321	7.1542	0.5321	6.1885	4.0428	0.7689	3.2854
86	44692	59468	6.7545	0.5462	5.8063	3.7534	0.7847	3.0414
87	40364	55248	6.3580	0.5580	5.4296	3.4784	0.7995	2.8148
88	35910	50675	5.9670	0.5668	5.0579	3.2195	0.8131	2.6046
89	31399	45790	5.5835	0.5713	4.6889	2.9777	0.8252	2.4075
90	26915	40664	5.2084	0.5701	4.3185	2.7538	0.8354	2.2174
91	22553	35399	4.8422	0.5608	3.9408	2.5486	0.8431	2.0223
92	18415	30131	4.4833	0.5405	3.5495	2.3619	0.8471	1.7995
93	14603	25015	4.1277	0.5047	3.1442	2.1928	0.8449	1.4983
94	11205	20211	3.7674	0.4474	2.7533	2.0394	0.8316	
95	8287	15865	3.3881	0.3614	2.5129	1.8979	0.7959	
96	5883	12081	2.9665	0.2409	2.2591	1.7606	0.7109	
97	3995	8910	2.4655	0.0954	1.9653	1.6097	0.5078	
98	2586	6355	1.8304		1.5808	1.3998		
99	1592	4375						

Fig. 3.3. Life table for a joint life in Switzerland

It can be observed that the number of survivors per age in Switzerland when comparing the number from men and women is that the number of women remains higher than men across all ages. By age 48, the difference between the two genders crosses 1,000. By age 69, the difference between the two genders crossed 5,000 and saw the number of survivors in men diverted from the number of survivors in women greatly. However, by their early 80s, the number of survivors in women started to converge with the number of survivors in men after the difference started to cool down and closed in at the tail-end.

It can be observed from the survival statistics in Switzerland that women tend to outlive men at every age. The only exception is during the early years of life, where the differences between the number of male and female survivors are quite minimal. However, by the time individuals reach age 42, the number of male survivors falls behind female survivors by over 1,000. This gap continues to widen, exceeding 5,000 by age 64. After this point, the number of men surviving begins to diverge more significantly from that of women. Interestingly, in the later years, particularly in their mid- to late-80s, the gap begins to narrow once again, indicating that both genders face increasingly similar survival prospects toward the end of life.

The expected present value of a whole life annuity paid at the end of every year before the death of the first of the two insured persons (x and y) in Switzerland begins to decline noticeably at age 48, with a drop of about 0.1. Prior to this, the decrease in value between successive ages remains fairly steady and below 0.1, indicating a more gradual reduction. From age 59 onwards, this decline accelerates and fluctuates between 0.2 and 0.3. In contrast, the expected present value of a whole life annuity paid before the death of the last of the two insured individuals consistently yields smaller values. Interestingly, its more pronounced decrease begins slightly earlier, around age 23, with a 0.1 reduction that marks the start of a more rapid decline. This downward trend continues with increasingly larger at age 42 in which it began to fluctuate more between 0.2 and 0.3. As with other countries, both annuity types steadily decline with age, in line with rising mortality risks.

On the other hand, the expected present value of a whole life insurance benefit paid at the end of the year before the first of the two individuals dies starts to increase more noticeably at age 63, showing a 0.01 increase from the prior age. Before this age, the value changes between years are smaller and less regular. After age 66, the increase becomes steadier or more likely to fluctuate between 0.02 and 0.03, then it begins to decline by age 89 and displaying no record at ages 98 and 99, similar to Germany and Austria for the same reason. When compared to the corresponding last-death insurance, the latter produces consistently higher values. What's notable is that the more significant growth for the last-death insurance begins earlier, at around age 34, where a 0.01 increment from the prior value can be observed and it steadily started at age 44. From age 66, values grow incrementally, often increasing by 0.01 to 0.02 between ages. The peak is reached around age 92, after which the values gradually decline and have no record by ages 98 and 99, though the peak is slightly later than in the first-death insurance scenario. As expected, both insurance values follow an upward trend until late life, reflecting the cumulative mortality risk, before they eventually decline.

For the case where the male is aged (x) and the female is (y+5), the expected present value of a whole life annuity paid before the first death begins to fall more noticeably at age 48, with a 0.1 drop. This decline intensifies at age 57, where differences between successive years fluctuate more widely between 0.2 and 0.3. The same pattern is observed when looking at the last-death annuity for this age setup, however, this one shows an earlier drop starting around age 33, with a 0.1 reduction. The fluctuations between 0.2 and 0.3 become more consistent by age 46 and continue more sharply thereafter. The life table is blank between ages 95 and 99, reflecting the absence of survivors at those advanced ages. As in previous comparisons, both types of annuities for the (x, y+5) case decline with each successive age.

Overall, when comparing the expected present value of a whole life annuity that pays out annually until the last of two insured individuals dies to one that pays out until the first death, the last-death annuity is generally lower in value. This is due to the longer payout period but also higher uncertainty, which reduces its present value. When the female is five years younger (i.e., (y+5)), the expected value of the last-death annuity drops even further compared to the first-death version. Moreover, both annuities, first-death and last-death, show lower present values in the (x, y+5) scenario than in the (x, y) scenario, due to the longer joint survivorship period and the compounded discounting effect.

	Austria		Germany		Switzerland	
Age	Male	Female	Male	Female	Male	Female
0	100000	100000	100000	100000	100000	100000
1	99756	99755	99675	99713	99575	99672
2	99738	99734	99653	99692	99553	99645
3	99715	99724	99642	99680	99538	99626
4	99711	99708	99630	99670	99525	99610
5	99700	99701	99619	99660	99515	99597
6	99696	99688	99610	99654	99507	99588

7	99680	99685	99601	99648	99502	99581
8	99672	99674	99593	99642	99498	99577
9	99667	99667	99586	99635	99495	99573
10	99663	99660	99578	99629	99493	99571
11	99652	99648	99572	99624	99491	99568
12	99634	99638	99565	99618	99489	99566
13	99620	99636	99558	99612	99486	99562
14	99614	99619	99548	99604	99480	99558
15	99605	99612	99536	99593	99471	99550
16	99575	99593	99521	99581	99454	99538
17	99544	99572	99498	99566	99429	99522
18	99501	99540	99472	99551	99397	99503
19	99457	99519	99435	99535	99360	99484
20	99411	99496	99390	99517	99321	99465
21	99368	99461	99350	99499	99282	99446
22	99319	99444	99308	99480	99242	99430
23	99265	99415	99266	99463	99203	99414
24	99202	99389	99223	99446	99164	99399
25	99150	99376	99180	99430	99126	99384
26	99094	99358	99138	99411	99086	99370
27	99066	99340	99092	99392	99046	99355
28	99005	99307	99046	99373	99005	99339
29	98937	99273	99000	99350	98963	99322
30	98864	99252	98950	99327	98919	99304
31	98806	99221	98896	99301	98873	99284
32	98750	99203	98840	99271	98824	99261
33	98686	99165	98780	99239	98773	99237
34	98623	99139	98717	99203	98720	99209
35	98542	99091	98642	99162	98663	99180
36	98465	99056	98562	99117	98603	99147
37	98363	99006	98472	99072	98540	99112
38	98269	98955	98367	99020	98474	99074
39	98148	98919	98258	98961	98403	99033
40	98040	98872	98141	98897	98328	98989
41	97905	98820	98012	98832	98247	98941
42	97772	98774	97875	98755	98161	98889
43	97629	98713	97725	98674	98069	98834
44	97481	98634	97556	98581	97969	98773
45	97343	98548	97374	98482	97860	98707
46	97159	98449	97183	98377	97741	98635
47	96985	98334	96974	98258	97611	98556

48	96778	98227	96742	98131	97468	98469
49	96585	98108	96483	97976	97309	98372
50	96342	97972	96188	97810	97134	98266
51	96080	97824	95861	97628	96940	98148
52	95794	97662	95503	97430	96724	98018
53	95476	97486	95108	97217	96485	97873
54	95099	97300	94681	96979	96219	97713
55	94696	97092	94206	96709	95923	97536
56	94269	96859	93664	96415	95595	97341
57	93753	96569	93068	96093	95232	97125
58	93142	96292	92416	95736	94829	96887
59	92558	95977	91688	95341	94384	96624
60	91903	95628	90889	94901	93892	96334
61	91137	95243	90001	94407	93350	96015
62	90268	94728	89023	93876	92754	95664
63	89350	94193	87959	93296	92099	95277
64	88321	93606	86802	92654	91382	94852
65	87185	93037	85543	91951	90599	94385
66	85952	92377	84178	91183	89745	93870
67	84653	91594	82711	90352	88816	93305
68	83175	90854	81143	89471	87807	92683
69	81605	89937	79469	88503	86711	91998
70	80027	89012	77695	87447	85522	91244
71	78371	88012	75842	86312	84231	90413
72	76451	86936	73869	85081	82830	89498
73	74428	85745	71820	83748	81309	88487
74	72271	84377	69620	82293	79655	87371
75	69969	82831	67348	80745	77855	86137
76	67607	81101	64909	79035	75895	84772
77	64926	79174	62408	77208	73759	83260
78	62080	77141	59741	75198	71433	81583
79	59241	74956	56951	73021	68903	79721
80	56334	72648	54070	70683	66153	77654
81	53133	70133	50964	68088	63173	75356
82	49945	67383	47721	65270	59954	72802
83	46682	64186	44325	62157	56490	69964
84	43222	60886	40784	58742	52784	66813
85	39409	57128	37110	55015	48844	63321
86	35492	53071	33288	50908	44692	59468
87	31367	48523	29406	46504	40364	55248
88	27263	43726	25480	41808	35910	50675

89	23151	38800	21710	37035	31399	45790
90	19358	33828	18044	32165	26915	40664
91	15668	28805	14635	27294	22553	35399
92	12479	23841	11575	22621	18415	30131
93	9609	19155	8834	18249	14603	25015
94	7107	15000	6559	14325	11205	20211
95	5109	11466	4697	10899	8287	15865
96	3560	8462	3247	8028	5883	12081
97	2385	5854	2143	5672	3995	8910
98	1434	3819	1381	3896	2586	6355
99	843	2412	871	2565	1592	4375

Fig. 3.4. Expected number of survivors per age in Austria, Germany and Switzerland

From ages 1 to 27, Switzerland has the lowest number of male survivors. From ages 29 to 46, Austria had the lowest number of male survivors. From ages 47 to 98, Germany had the lowest number of male survivors. Finally, at age 99, Austria finished as the country with the lowest number of survivors. However, from ages 1 to 22, Austria had the highest number of male survivors. From ages 23 to 33, Germany held the highest number of male survivors. Finally, from ages 34 to 99, Switzerland holds the highest number of male survivors.

From ages 1 to 22, Switzerland had the lowest number of female survivors. From ages 23 to 41, Austria had the lowest number of female survivors. Finally, from ages 42 to 99, Germany had the lowest number of female survivors. From ages 1 to 17, Austria has the highest number of female survivors. From 18 to 33, Germany had the highest number of female survivors. Finally, from ages 34 to 99, Switzerland had the highest number of female survivors.

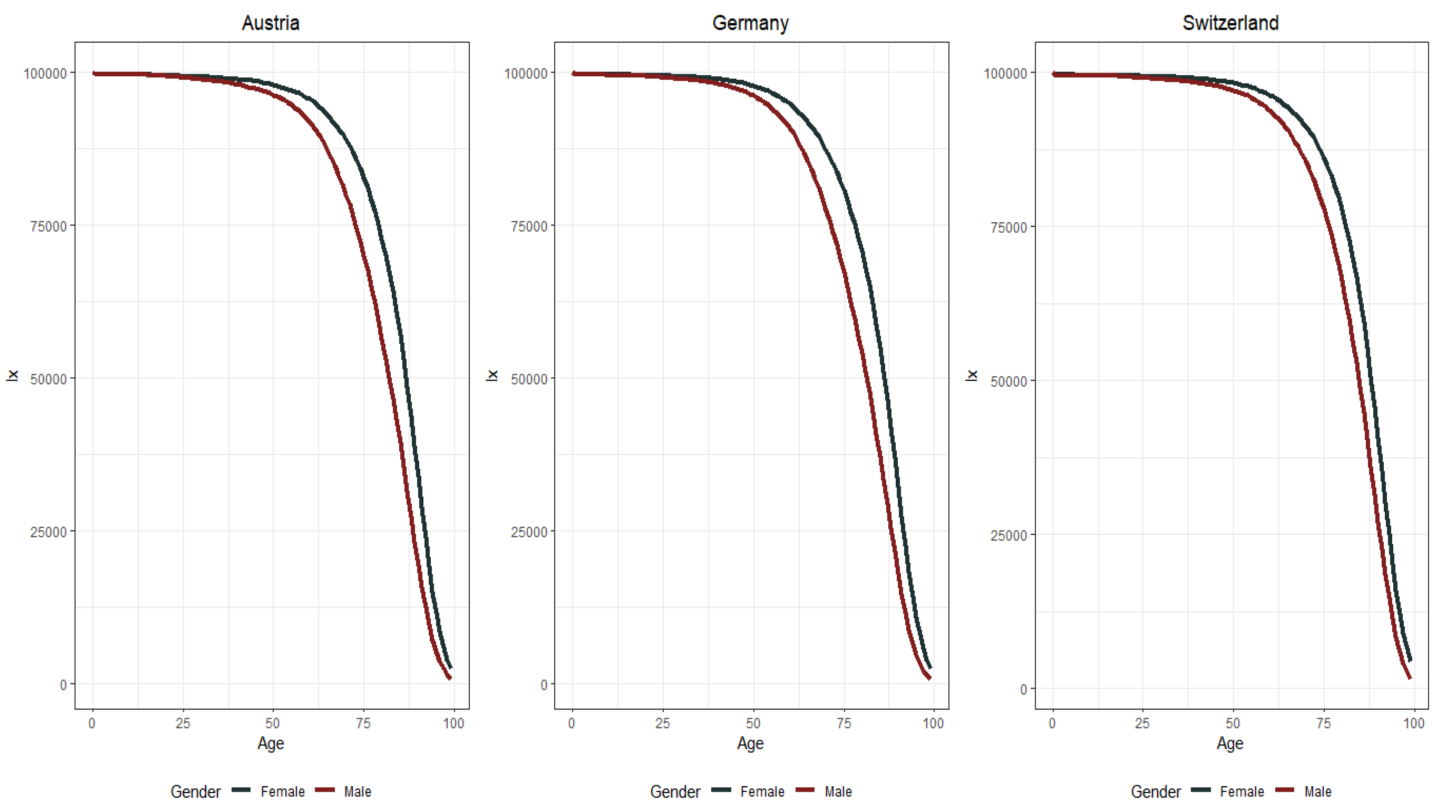

Fig. 3.5. Graphical comparison of the expected number of survivors for each gender in Austria, Germany and Switzerland

The graphs of the number of survivors for each gender in Austria, Germany and Switzerland behave similarly. It follows the downwards exponential behavior of the graph.

For the case of Austria, the graph begins to diverge at age 32. It then peaked at age 60, indicating the greatest difference in survival between genders. It began to converge at age 85 (which the difference is 17,719 and at age 86 becomes 17,579 in which it started narrowing down) as the number of survivors for both genders decreased and their survival trajectories became more similar.

For the case of Germany, the graph begins to diverge at age 32, following a pattern similar to Austria. The peak divergence occurs at age 60, where the survival gap between males and females is the most pronounced. It began to converge at age 84 (which the difference is 17,958 and at age 85 becomes 17,905 in which it started narrowing down), as both genders approach similar survival rates in older age.

For the case of Switzerland, the graph begins to diverge at age 30, showing that males start to experience a slightly lower survival rate than females. The peak divergence happens at age 59, consistent with Austria and Germany. It began to converge at age 87 (which the difference is 14,884 and at age 88 becomes 14,765 in which it started narrowing down), where both survival curves approach each other again.

Age	Austria	Germany	Switzerland
0	17.5552	17.5511	17.5669
1	17.5493	17.5452	17.5619
2	17.5423	17.5379	17.5557
3	17.5349	17.5302	17.5491
4	17.5270	17.5221	17.5421
5	17.5187	17.5135	17.5346
6	17.5098	17.5043	17.5267
7	17.5004	17.4946	17.5184
8	17.4905	17.4843	17.5095
9	17.4800	17.4734	17.5001
10	17.4688	17.4618	17.4901
11	17.4570	17.4495	17.4795
12	17.4444	17.4365	17.4683
13	17.4311	17.4227	17.4564
14	17.4171	17.4081	17.4438
15	17.4021	17.3927	17.4304
16	17.3864	17.3763	17.4163
17	17.3697	17.3590	17.4014
18	17.3521	17.3407	17.3856
19	17.3334	17.3212	17.3689
20	17.3136	17.3007	17.3511
21	17.2927	17.2789	17.3324
22	17.2704	17.2559	17.3124
23	17.2470	17.2314	17.2913
24	17.2221	17.2055	17.2690
25	17.1956	17.1780	17.2453

26	17.1676	17.1490	17.2201
27	17.1378	17.1182	17.1935
28	17.1065	17.0855	17.1653
29	17.0733	17.0510	17.1355
30	17.0381	17.0144	17.1038
31	17.0008	16.9757	17.0703
32	16.9612	16.9347	17.0348
33	16.9194	16.8913	16.9972
34	16.8750	16.8453	16.9574
35	16.8283	16.7968	16.9153
36	16.7786	16.7454	16.8707
37	16.7263	16.6910	16.8235
38	16.6708	16.6336	16.7735
39	16.6120	16.5728	16.7205
40	16.5498	16.5086	16.6645
41	16.4841	16.4407	16.6052
42	16.4144	16.3689	16.5424
43	16.3409	16.2931	16.4760
44	16.2633	16.2131	16.4058
45	16.1811	16.1286	16.3315
46	16.0946	16.0393	16.2529
47	16.0031	15.9451	16.1699
48	15.9064	15.8458	16.0821
49	15.8042	15.7413	15.9894
50	15.6966	15.6312	15.8915
51	15.5830	15.5153	15.7881
52	15.4633	15.3933	15.6790
53	15.3372	15.2649	15.5639
54	15.2044	15.1299	15.4426
55	15.0647	14.9883	15.3146
56	14.9176	14.8398	15.1799
57	14.7641	14.6840	15.0379
58	14.6026	14.5208	14.8885
59	14.4323	14.3501	14.7314
60	14.2539	14.1717	14.5661
61	14.0675	13.9856	14.3924
62	13.8749	13.7912	14.2100
63	13.6724	13.5882	14.0185
64	13.4613	13.3768	13.8176
65	13.2393	13.1567	13.6069
66	13.0085	12.9277	13.3861

67	12.7690	12.6892	13.1550
68	12.5177	12.4405	12.9131
69	12.2587	12.1823	12.6602
70	11.9856	11.9141	12.3960
71	11.7008	11.6348	12.1203
72	11.4070	11.3451	11.8330
73	11.1025	11.0442	11.5339
74	10.7890	10.7335	11.2231
75	10.4666	10.4103	10.9006
76	10.1336	10.0782	10.5666
77	9.7952	9.7334	10.2214
78	9.4457	9.3800	9.8655
79	9.0825	9.0165	9.4995
80	8.7060	8.6420	9.1242
81	8.3243	8.2632	8.7407
82	7.9312	7.8776	8.3502
83	7.5360	7.4892	7.9544
84	7.1328	7.0997	7.5550
85	6.7400	6.7118	7.1542
86	6.3492	6.3321	6.7545
87	5.9755	5.9599	6.3580
88	5.6093	5.6009	5.9670
89	5.2546	5.2457	5.5835
90	4.9009	4.9054	5.2084
91	4.5682	4.5778	4.8422
92	4.2377	4.2562	4.4833
93	3.9171	3.9443	4.1277
94	3.5921	3.6221	3.7674
95	3.2329	3.2821	3.3881
96	2.8273	2.8975	2.9665
97	2.3643	2.4390	2.4655
98	1.8002	1.8244	1.8304
99			

Fig. 3.6. Expected value of $\ddot{\mathbf{a}}_{xy}$ in Austria, Germany and Switzerland

From ages 1 to 89, Germany exhibited the cheapest expected value of a joint whole life annuity issued at the end of every year until the first death of the two insurers amongst (x) and (y). However, for the ages 90 to 98, Austria exhibited the cheapest expected value of the aforementioned actuarial function.

For all ages from 1 to 98, Switzerland exhibits the highest expected value of a joint whole life annuity issued at the end of every year until the first death of the two insurers amongst (x) and (y).

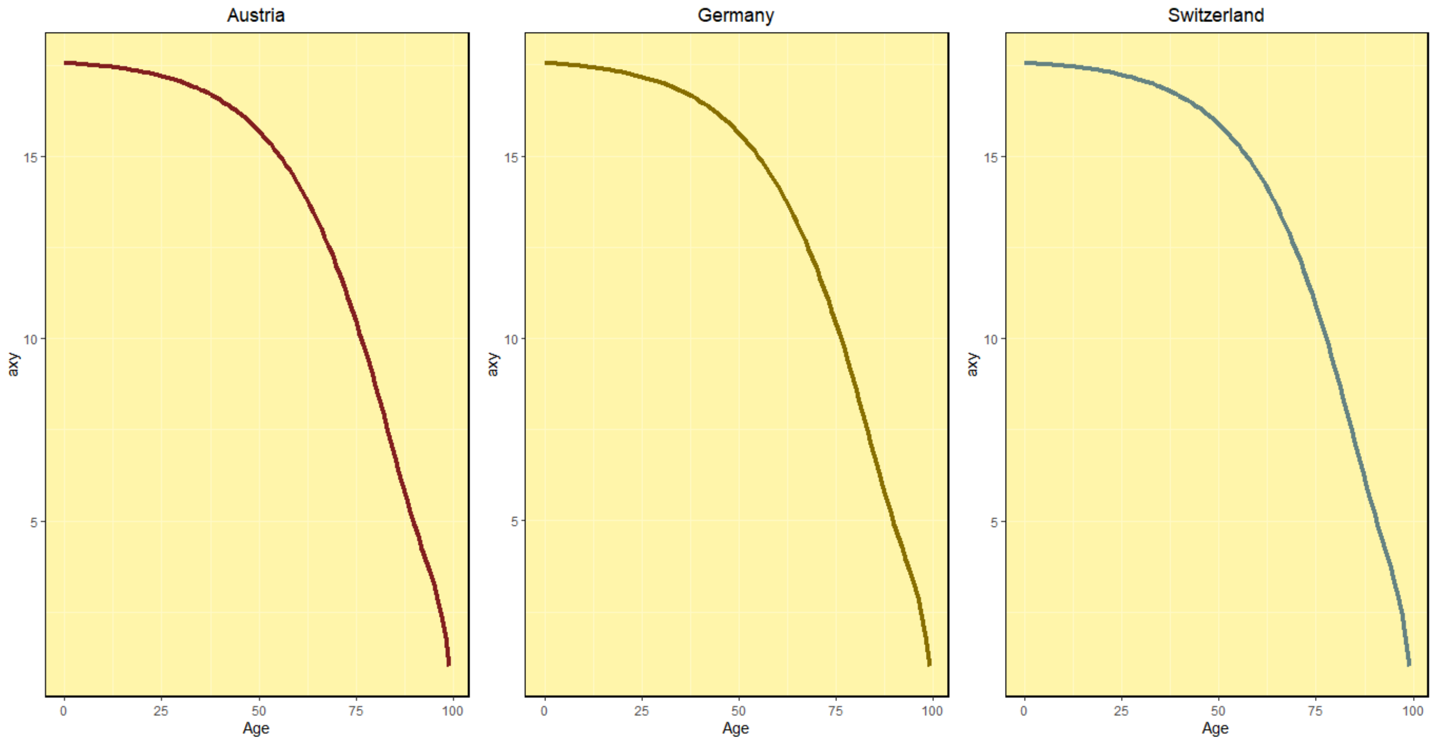

Fig. 3.7. Graphical comparison of expected value of whole life annuity of Austria, Germany and Switzerland

Austria, Germany, and Switzerland display a trend that resembles a downward parabolic graph, with values gradually declining as age increases. These values, which can be interpreted as average remaining life years or expected life durations, show a smooth but non-linear descent over time. All three countries follow a similar pattern, but the curvature, or the part of the graph where the decline becomes more rounded and less steep, is especially noteworthy during middle adulthood. This curvature reflects a temporary moderation in the rate of decline before it accelerates ageing in older ages.

In Austria the curve becomes noticeable for people between the ages of 31 and 55. During this time, the decline in values shifts from a sharp drop in the earlier years to a more gradual slope. For example, from ages 30 to 50, Austria's values decrease from about 17.0008 to 15.0647, which is a slower decline compared to the steeper drop seen from age 0 to 30. Germany shows a similar pattern between ages 30 and 54, with values dropping from around 17.0144 at age 29 to 15.1299 at age 54, again reflecting a gentler decline. On the other hand, Switzerland displays a comparable but slightly delayed curve from ages 32 to 57, with values falling from 17.0348 to 15.0379.

Age	Austria	Germany	Switzerland
0	0.0062	0.0064	0.0054
1	0.0065	0.0067	0.0056
2	0.0069	0.0071	0.0060
3	0.0073	0.0075	0.0063
4	0.0077	0.0080	0.0067
5	0.0082	0.0085	0.0071
6	0.0087	0.0090	0.0075
7	0.0092	0.0095	0.0080
8	0.0097	0.0101	0.0085
9	0.0103	0.0107	0.0090
10	0.0109	0.0113	0.0095

11	0.0116	0.0120	0.0101
12	0.0123	0.0127	0.0107
13	0.0130	0.0135	0.0113
14	0.0138	0.0143	0.0120
15	0.0146	0.0151	0.0127
16	0.0155	0.0160	0.0135
17	0.0164	0.0170	0.0143
18	0.0174	0.0180	0.0152
19	0.0184	0.0191	0.0161
20	0.0195	0.0202	0.0170
21	0.0206	0.0214	0.0180
22	0.0219	0.0227	0.0191
23	0.0232	0.0240	0.0202
24	0.0245	0.0255	0.0214
25	0.0260	0.0270	0.0227
26	0.0275	0.0286	0.0241
27	0.0292	0.0303	0.0255
28	0.0309	0.0321	0.0270
29	0.0327	0.0340	0.0286
30	0.0347	0.0360	0.0303
31	0.0367	0.0381	0.0321
32	0.0389	0.0404	0.0340
33	0.0412	0.0428	0.0361
34	0.0437	0.0453	0.0382
35	0.0462	0.0480	0.0405
36	0.0490	0.0509	0.0429
37	0.0519	0.0539	0.0454
38	0.0549	0.0570	0.0481
39	0.0582	0.0604	0.0510
40	0.0616	0.0639	0.0540
41	0.0652	0.0677	0.0572
42	0.0690	0.0716	0.0605
43	0.0731	0.0758	0.0641
44	0.0774	0.0802	0.0679
45	0.0819	0.0849	0.0719
46	0.0867	0.0898	0.0761
47	0.0917	0.0950	0.0806
48	0.0970	0.1004	0.0853
49	0.1026	0.1062	0.0903
50	0.1086	0.1122	0.0955
51	0.1148	0.1186	0.1011

52	0.1214	0.1253	0.1069
53	0.1283	0.1324	0.1131
54	0.1356	0.1398	0.1196
55	0.1433	0.1476	0.1264
56	0.1514	0.1557	0.1337
57	0.1598	0.1643	0.1412
58	0.1686	0.1732	0.1492
59	0.1779	0.1826	0.1576
60	0.1877	0.1923	0.1664
61	0.1979	0.2025	0.1757
62	0.2084	0.2131	0.1854
63	0.2195	0.2241	0.1955
64	0.2310	0.2356	0.2062
65	0.2430	0.2476	0.2173
66	0.2556	0.2600	0.2290
67	0.2686	0.2729	0.2412
68	0.2822	0.2864	0.2539
69	0.2962	0.3003	0.2672
70	0.3109	0.3147	0.2810
71	0.3262	0.3296	0.2953
72	0.3419	0.3451	0.3103
73	0.3582	0.3611	0.3257
74	0.3749	0.3776	0.3417
75	0.3919	0.3946	0.3581
76	0.4094	0.4120	0.3751
77	0.4270	0.4300	0.3924
78	0.4451	0.4482	0.4101
79	0.4637	0.4667	0.4280
80	0.4828	0.4856	0.4461
81	0.5018	0.5043	0.4642
82	0.5211	0.5230	0.4821
83	0.5399	0.5413	0.4996
84	0.5585	0.5588	0.5164
85	0.5755	0.5752	0.5321
86	0.5911	0.5897	0.5462
87	0.6038	0.6020	0.5580
88	0.6137	0.6108	0.5668
89	0.6195	0.6159	0.5713
90	0.6206	0.6150	0.5701
91	0.6132	0.6058	0.5608
92	0.5955	0.5856	0.5405

93	0.5616	0.5485	0.5047
94	0.5050	0.4894	0.4474
95	0.4202	0.3983	0.3614
96	0.2971	0.2686	0.2409
97	0.1308	0.1050	0.0954
98			
99			

Fig. 3.8. Tabular comparison of the $\mathbf{A_{xy}}$ in Austria, Germany and Switzerland

From ages 1 to 97, Switzerland exhibited the cheapest expected value of a joint whole life insurance issued at the end of every year until the first death of the two insurers amongst (x) and (y).

For all ages from 1 to 84, Germany exhibits the highest expected value of a joint whole life insurance issued at the end of every year until the first death of the two insurers amongst (x) and (y). However, for the ages 85 to 97, Austria exhibited the highest expected value of the aforementioned actuarial function.

It is also worth noting that the values of the aforementioned actuarial function increases from ages 1 to 90 in Austria whereas the value of the aforementioned actuarial function increases from ages 1 to 89 in Germany and Switzerland as the countries faces a swift decline in the value from the respective ages to age 97.

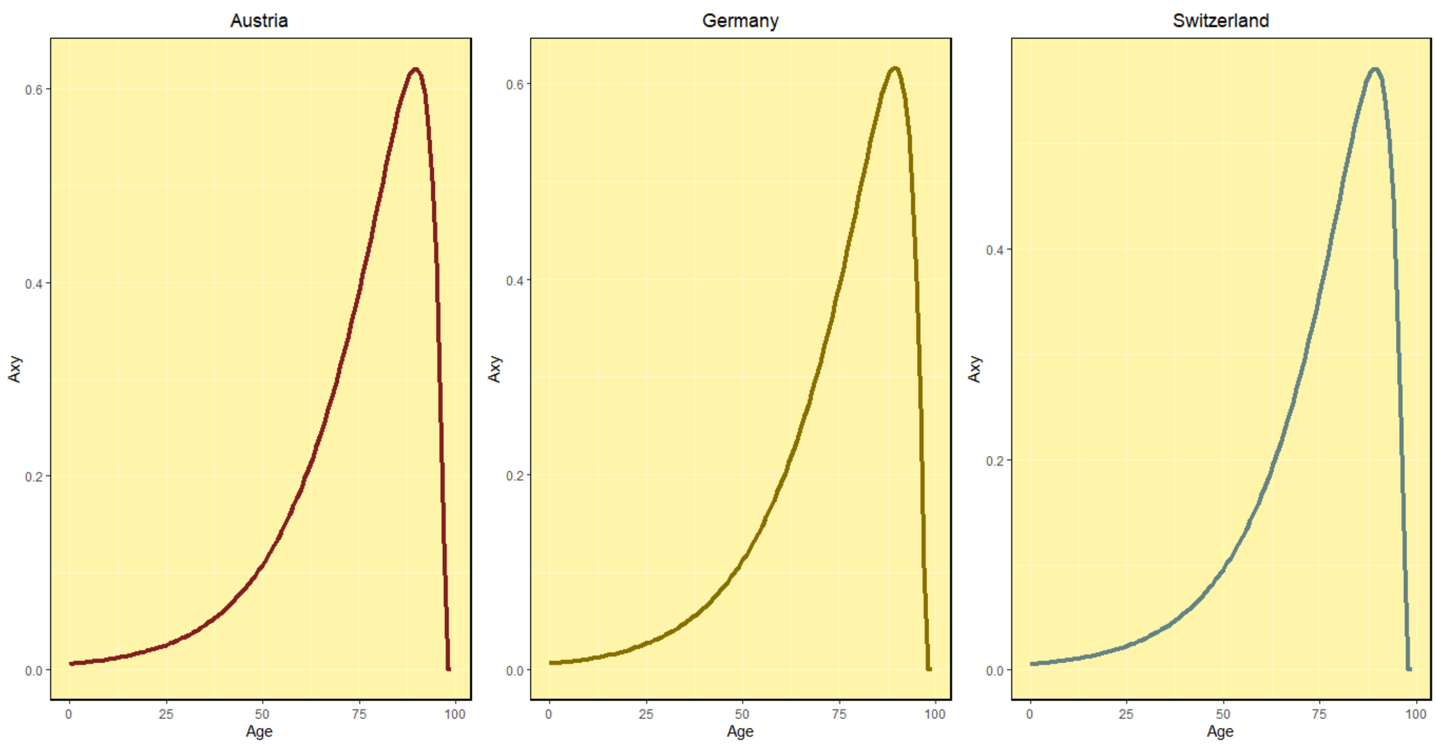

Fig. 3.9. Graphical comparison of expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year starting on the first death of Austria, Germany and Switzerland

The values for Austria, Germany, and Switzerland show a clear upward trend across all ages, but especially from early childhood to late adulthood. This upward trend suggests that the values are more likely to be linked to life insurance or annuity factors, which rise steadily as people get older, either because the risk of death is higher or the expected payouts are higher. But this trend starts to change in the very late stages of life, around age 90, and the values start to go down. This change could mean that annuity values are going down or that the chances of survival are going down at very old ages. All of the countries show this general pattern, but Switzerland always has values that are a little lower than Austria and Germany. This could be because of differences in demographic or actuarial assumptions between the two countries.

In Austria, the most stable rate of increase appears between ages 28 and 55, where the values climb gradually from 0.0309 to 0.1433, marking a smoother and less steep rise compared to the sharper growth before age 28 or the faster escalation after age 60. In Germany, values rise from 0.0303 to 0.1476 between the ages of 27 and 55, which shows the same gentler curve. The slower rate of growth for Switzerland starts a little later, between the ages of 30 and 58, when values go from 0.0303 to 0.1492. After a sharper growth, it picks up again as it reaches to the highest point around ages 88 to 90. It was shown that Austria had a maximum value of 0.6206 at age 90, Germany had a maximum value of 0.6159 at age 89, and Switzerland had a maximum value of 0.5713 at age 88. Following this, the values begin to drop significantly at older ages. This change shows that the limited life expectancy left and lower expected payouts at older ages.

Age	Austria	Germany	Switzerland
0	17.5364	17.5317	17.5510
1	17.5290	17.5241	17.5446
2	17.5207	17.5156	17.5373
3	17.5120	17.5065	17.5296
4	17.5028	17.4969	17.5214
5	17.4930	17.4868	17.5127
6	17.4826	17.4760	17.5034
7	17.4716	17.4646	17.4936
8	17.4599	17.4525	17.4833
9	17.4475	17.4397	17.4723
10	17.4344	17.4261	17.4606
11	17.4205	17.4117	17.4483
12	17.4059	17.3965	17.4352
13	17.3903	17.3803	17.4214
14	17.3738	17.3632	17.4067
15	17.3563	17.3451	17.3912
16	17.3379	17.3259	17.3748
17	17.3183	17.3056	17.3574
18	17.2977	17.2841	17.3389
19	17.2758	17.2613	17.3194
20	17.2525	17.2372	17.2987
21	17.2279	17.2117	17.2768
22	17.2018	17.1846	17.2536
23	17.1742	17.1559	17.2290
24	17.1451	17.1256	17.2030
25	17.1141	17.0934	17.1753
26	17.0813	17.0593	17.1461
27	17.0464	17.0233	17.1151
28	17.0097	16.9851	17.0823
29	16.9708	16.9447	17.0476
30	16.9297	16.9019	17.0107
31	16.8860	16.8566	16.9718
32	16.8398	16.8086	16.9305

33	16.7909	16.7579	16.8868
34	16.7389	16.7043	16.8406
35	16.6842	16.6475	16.7916
36	16.6261	16.5875	16.7397
37	16.5648	16.5241	16.6849
38	16.4999	16.4571	16.6268
39	16.4315	16.3863	16.5653
40	16.3590	16.3115	16.5002
41	16.2826	16.2324	16.4314
42	16.2019	16.1489	16.3586
43	16.1164	16.0608	16.2816
44	16.0261	15.9679	16.2003
45	15.9305	15.8700	16.1142
46	15.8300	15.7666	16.0233
47	15.7236	15.6576	15.9273
48	15.6115	15.5427	15.8259
49	15.4929	15.4218	15.7189
50	15.3682	15.2949	15.6060
51	15.2369	15.1615	15.4868
52	15.0991	15.0212	15.3613
53	14.9536	14.8740	15.2289
54	14.8011	14.7195	15.0895
55	14.6407	14.5576	14.9428
56	14.4721	14.3885	14.7884
57	14.2974	14.2115	14.6261
58	14.1147	14.0262	14.4554
59	13.9220	13.8331	14.2762
60	13.7194	13.6316	14.0881
61	13.5090	13.4217	13.8908
62	13.2910	13.2032	13.6839
63	13.0610	12.9753	13.4673
64	12.8237	12.7386	13.2405
65	12.5757	12.4928	13.0032
66	12.3176	12.2376	12.7553
67	12.0483	11.9728	12.4964
68	11.7708	11.6982	12.2264
69	11.4840	11.4140	11.9449
70	11.1855	11.1191	11.6520
71	10.8766	10.8141	11.3476
72	10.5623	10.4987	11.0317
73	10.2363	10.1729	10.7045

74	9.9003	9.8383	10.3663
75	9.5538	9.4922	10.0175
76	9.1959	9.1398	9.6588
77	8.8357	8.7769	9.2909
78	8.4732	8.4089	8.9150
79	8.0956	8.0352	8.5323
80	7.7133	7.6557	8.1444
81	7.3328	7.2788	7.7529
82	6.9481	6.9016	7.3597
83	6.5596	6.5278	6.9667
84	6.1734	6.1555	6.5757
85	5.7996	5.7896	6.1885
86	5.4335	5.4348	5.8063
87	5.0859	5.0877	5.4296
88	4.7463	4.7514	5.0579
89	4.4141	4.4138	4.6889
90	4.0664	4.0812	4.3185
91	3.7270	3.7416	3.9408
92	3.3712	3.3876	3.5495
93	3.0071	3.0207	3.1442
94	2.6542	2.6565	2.7533
95	2.4392	2.4520	2.5129
96	2.1893	2.2264	2.2591
97	1.8818	1.9697	1.9653
98	1.5546	1.5950	1.5808
99			

Fig. 3.10. Tabular comparison of the expected present value of a whole life annuity of a joint life status issued to an individual ages x and y + 5 payable at the end of each year until the first death in Austria, Germany and Switzerland

From ages 1 to 76, Germany exhibited the lowest expected present value of a joint whole life annuity-due issued at the end of every year until the first death of the two lives aged (x) and (y+5). However, for ages 77 to 96, Austria exhibited the lowest expected value of the aforementioned actuarial function.

For all ages from 1 to 96, Switzerland consistently exhibited the highest expected value of a joint whole life annuity-due issued at the end of every year until the first death of the two lives aged (x) and (y+5).

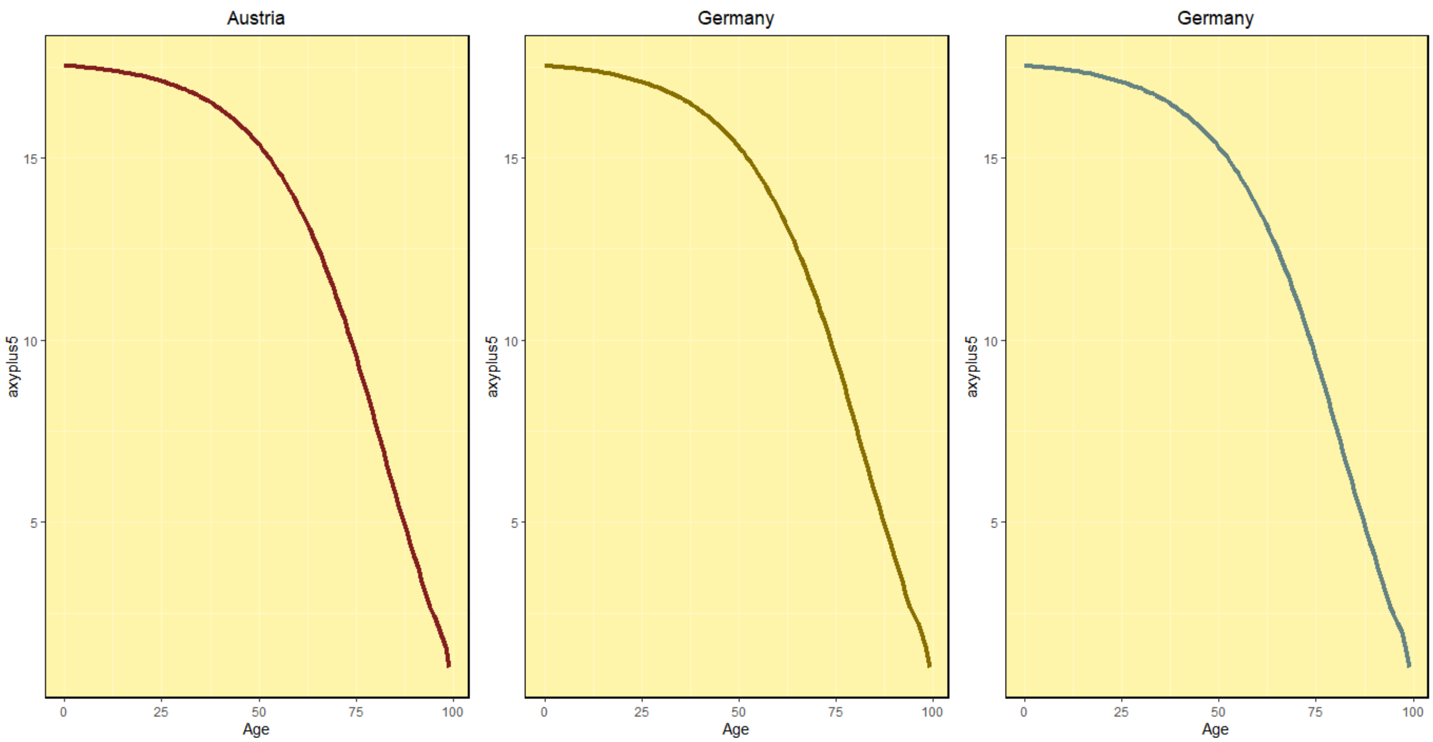

Fig. 3.11. Graphical comparison of the expected present value of a whole life annuity of a joint life status issued to an individual ages x and y + 5 payable at the end of each year until the first death in Austria, Germany and Switzerland

A downward parabolic trend is shown in Austria, Germany, and Switzerland, where values progressively decrease as age continuously increases. Over time, these values shown demonstrate a smooth but non-linear decline. As this pattern is consistent across all three countries; however, in middle adulthood, the curvature, where the decline becomes less steep and more rounded thus becomes particularly noticeable.

In Austria, the curvature shown becomes more distinct between the ages of 29 to 58. During this age period, the decline in values shifts from a sharper drop in earlier years to a more gradual slope. For instance, the expected value drops from 16.9708 at age 29 to 14.1147 at age 58, a slower decline compared to the steeper drop from 17.5364 at age 0 to 16.9297 at age 30. Germany shows a similar curvature between ages 28 to 58, with values falling from 16.9851 to 14.0262, reflecting a comparable slowdown in the rate of decline during this mid-age range.

On the other hand, Switzerland displays a slightly delayed yet consistent curvature line between the ages of 31 to 60, where values decrease from 17.0107 to 14.0881. This pattern emphasizes the same gradual decline in the middle years before a steeper drop resumes in older ages.

Age	Austria	Germany	Switzerland
0	17.2238	17.2013	17.2375
1	17.2816	17.279	17.3421
2	17.2652	17.2632	17.3312
3	17.2468	17.2429	17.3169
4	17.2251	17.2213	17.301
5	17.2017	17.1982	17.2831
6	17.1768	17.1727	17.263

7	17.1506	17.1456	17.2408
8	17.1229	17.1168	17.2167
9	17.0924	17.0862	17.1909
10	17.0598	17.0537	17.163
11	17.0273	17.0188	17.1337
12	16.9937	16.9822	17.1024
13	16.9561	16.9433	17.0697
14	16.9173	16.903	17.0356
15	16.8751	16.8611	17.0007
16	16.8359	16.8173	16.9656
17	16.7948	16.7727	16.9306
18	16.7551	16.726	16.8951
19	16.7113	16.6784	16.8582
20	16.6655	16.6297	16.8196
21	16.6185	16.5771	16.7785
22	16.5666	16.5219	16.7347
23	16.5144	16.463	16.688
24	16.46	16.4006	16.6383
25	16.3984	16.3344	16.5855
26	16.3344	16.2644	16.5296
27	16.2621	16.1909	16.4705
28	16.1931	16.1129	16.4082
29	16.1213	16.0309	16.3424
30	16.0438	15.9445	16.2731
31	15.9608	15.8539	16.2003
32	15.8703	15.7588	16.124
33	15.7788	15.6589	16.0436
34	15.6796	15.554	15.9593
35	15.5807	15.4454	15.8707
36	15.4731	15.3315	15.7778
37	15.3652	15.2121	15.68
38	15.2495	15.0889	15.5772
39	15.1286	14.9597	15.4694
40	15	14.8245	15.3561
41	14.8682	14.6829	15.2374
42	14.7272	14.5355	15.1128
43	14.5811	14.3815	14.9819
44	14.4294	14.2223	14.8451
45	14.2678	14.056	14.702
46	14.1047	13.8813	14.5524
47	13.9321	13.7002	14.3962

48	13.7524	13.512	14.2334
49	13.5609	13.3193	14.0641
50	13.3667	13.1208	13.8879
51	13.1643	12.9159	13.705
52	12.9542	12.7039	13.5152
53	12.7366	12.485	13.3187
54	12.514	12.259	13.1154
55	12.283	12.0282	12.9053
56	12.043	11.7934	12.6883
57	11.8054	11.5529	12.4645
58	11.562	11.307	12.234
59	11.3034	11.0578	11.9967
60	11.0395	10.8048	11.7529
61	10.7747	10.5506	11.5023
62	10.5178	10.2927	11.2451
63	10.2505	10.0314	10.9815
64	9.9819	9.7681	10.7111
65	9.7039	9.5031	10.4341
66	9.4253	9.2366	10.1507
67	9.1454	8.9673	9.8606
68	8.8591	8.6933	9.5639
69	8.5775	8.4178	9.2609
70	8.2757	8.1395	8.9517
71	7.9647	7.8547	8.6366
72	7.6616	7.568	8.3159
73	7.354	7.2747	7.9902
74	7.0487	6.9827	7.6601
75	6.7462	6.6813	7.3264
76	6.4383	6.3836	6.9899
77	6.1487	6.0758	6.6518
78	5.8582	5.7707	6.3132
79	5.5538	5.4629	5.9751
80	5.2374	5.1475	5.6391
81	4.933	4.8421	5.3064
82	4.6161	4.5372	4.9786
83	4.3053	4.2388	4.6575
84	3.9892	3.9481	4.3448
85	3.7037	3.6671	4.0428
86	3.4254	3.406	3.7534
87	3.1817	3.1604	3.4784
88	2.9527	2.9397	3.2195

89	2.7469	2.7242	2.9777
90	2.54	2.5319	2.7538
91	2.3686	2.3593	2.5486
92	2.2007	2.1981	2.3619
93	2.0572	2.0626	2.1928
94	1.9348	1.9327	2.0394
95	1.8032	1.8145	1.8979
96	1.6557	1.6956	1.7606
97	1.4997	1.5812	1.6097
98	1.3503	1.3917	1.3998
99			

Fig. 3.12. Tabular comparison of the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death in Austria, Germany and Switzerland

From ages 1 to 92, Germany exhibited the cheapest expected value of a joint whole-life annuity issued at the end of every year until the last death of the two insurers, (x) and (y). However, from ages 93 to 98, Austria exhibited the lowest expected value for this actuarial function.

In contrast, Switzerland consistently displayed the highest expected value of a joint whole life annuity to the last death of the two lives insured, across all ages from 1 to 98. This indicates a systematically longer combined life expectancy in Switzerland relative to Austria and Germany.

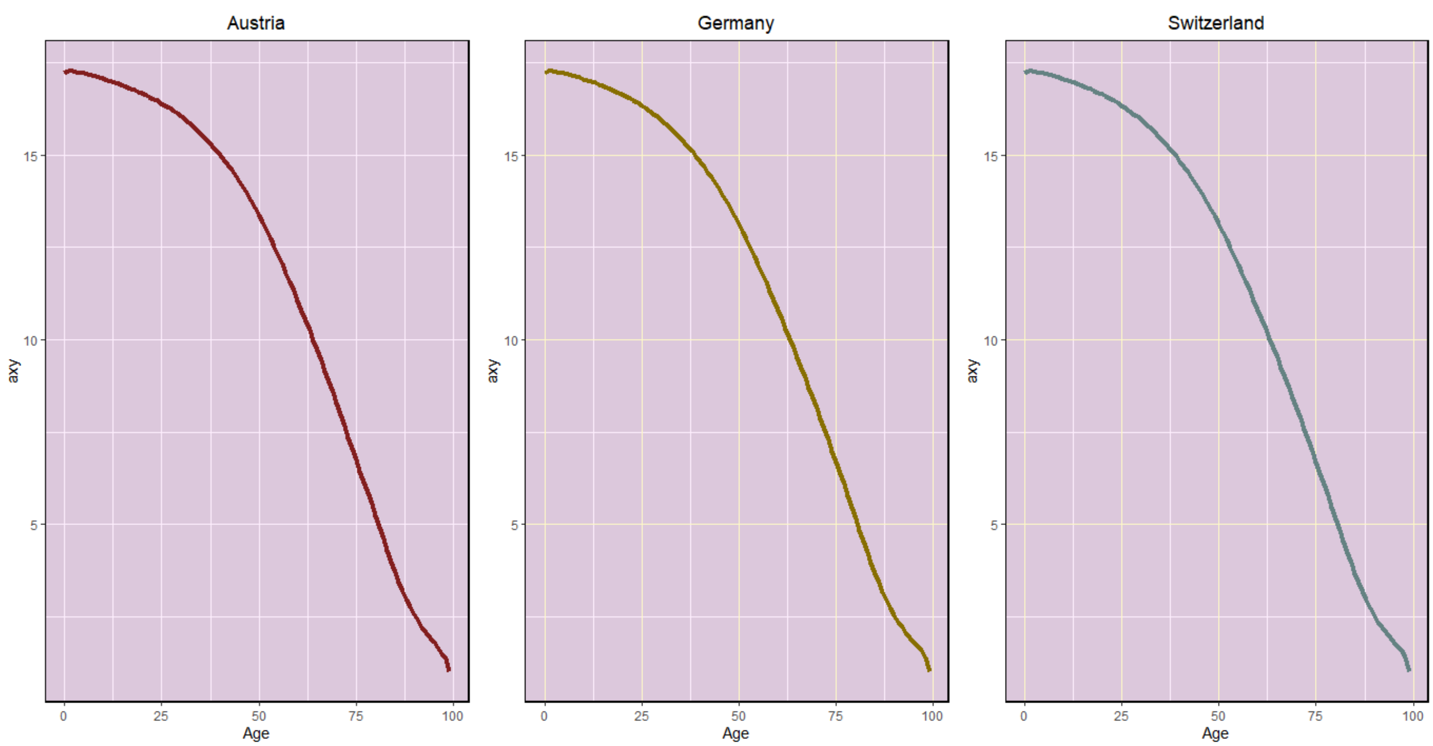

Fig. 3.13. Graphical comparison of the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death in Austria, Germany and Switzerland

Austria, Germany, and Switzerland exhibit a downward trend in the last-death annuity values that closely mirrors the shape observed in the first-death counterpart. These values follow a downward parabolic trend across ages, starting at high early ages, declining steeply in youth, and then leveling out more gradually in middle adulthood. However, the key similarity lies in the pattern's transformation only up to age 87. Just as in the first-death graph, the curve ceases to behave like a parabola until the point at age 87. Instead, the values begin to rapidly decline more linearly and steadily at age 88, signaling that the changes in expected annuity duration become more consistent and less curved in the oldest ages.

In Austria, the parabolic curvature is most noticeable between ages 31 to 64, with the values dropping from 15.9608 to 9.9819 causing it to have a smoother and more gradual decline compared to earlier years. Germany shows a similar curve from ages 30 to 64, where the values move from 15.9445 to 9.7681. On the other hand, Switzerland also follows this pattern albeit slightly later than other countries mentioned. It becomes more evident from ages 34 to 67, with the values decreasing from 15.9593 to 9.8606. These ranges highlight the shared mid-life curvature among the three countries, reflecting a moderation in the decline of expected annuity values during those years.

After age 87, the downward slope in all three countries no longer curves significantly and instead begins to descend almost linearly. This shift marks the end of the parabolic shape and indicates that as age increases, expected values to the last death drop at a steadier and more uniform pace.

Age	Austria	Germany	Switzerland
0	0.0251	0.0263	0.0243
1	0.0218	0.0219	0.0184
2	0.0227	0.0228	0.019
3	0.0238	0.024	0.0198
4	0.025	0.0252	0.0207
5	0.0263	0.0265	0.0217
6	0.0277	0.028	0.0228
7	0.0292	0.0295	0.0241
8	0.0308	0.0311	0.0255
9	0.0325	0.0329	0.0269
10	0.0343	0.0347	0.0285
11	0.0362	0.0367	0.0302
12	0.0381	0.0387	0.0319
13	0.0402	0.0409	0.0338
14	0.0424	0.0432	0.0357
15	0.0448	0.0456	0.0377
16	0.047	0.0481	0.0397
17	0.0493	0.0506	0.0417
18	0.0516	0.0532	0.0437
19	0.0541	0.0559	0.0457
20	0.0567	0.0587	0.0479
21	0.0593	0.0617	0.0503
22	0.0623	0.0648	0.0527
23	0.0652	0.0681	0.0554
24	0.0683	0.0717	0.0582
25	0.0718	0.0754	0.0612
26	0.0754	0.0794	0.0643
27	0.0795	0.0835	0.0677
28	0.0834	0.0879	0.0712

29	0.0875	0.0926	0.0749
30	0.0919	0.0975	0.0789
31	0.0965	0.1026	0.083
32	0.1017	0.108	0.0873
33	0.1068	0.1136	0.0918
34	0.1125	0.1196	0.0966
35	0.1181	0.1257	0.1016
36	0.1241	0.1322	0.1069
37	0.1303	0.1389	0.1124
38	0.1368	0.1459	0.1182
39	0.1436	0.1532	0.1243
40	0.1509	0.1609	0.1307
41	0.1584	0.1689	0.1375
42	0.1664	0.1772	0.1445
43	0.1746	0.1859	0.1519
44	0.1832	0.1949	0.1596
45	0.1924	0.2044	0.1677
46	0.2016	0.2142	0.1762
47	0.2114	0.2245	0.185
48	0.2215	0.2351	0.1943
49	0.2324	0.246	0.2038
50	0.2434	0.2573	0.2138
51	0.2548	0.2689	0.2241
52	0.2667	0.2809	0.2349
53	0.279	0.2933	0.246
54	0.2916	0.3061	0.2575
55	0.3047	0.3191	0.2694
56	0.3183	0.3324	0.2817
57	0.3317	0.346	0.2943
58	0.3455	0.3599	0.3074
59	0.3601	0.374	0.3208
60	0.3751	0.3883	0.3346
61	0.39	0.4027	0.3487
62	0.4046	0.4173	0.3633
63	0.4197	0.4321	0.3782
64	0.4349	0.447	0.3935
65	0.4506	0.462	0.4091
66	0.4664	0.4771	0.4252
67	0.4822	0.4923	0.4416
68	0.4984	0.5078	0.4583
69	0.5144	0.5234	0.4754

70	0.5314	0.5391	0.4929
71	0.549	0.5552	0.5107
72	0.5662	0.5714	0.5288
73	0.5836	0.588	0.5472
74	0.6008	0.6045	0.5659
75	0.6179	0.6216	0.5847
76	0.6353	0.6384	0.6037
77	0.6517	0.6558	0.6228
78	0.6681	0.673	0.6418
79	0.6853	0.6904	0.6609
80	0.7031	0.7082	0.6798
81	0.7203	0.7254	0.6985
82	0.7381	0.7426	0.7168
83	0.7556	0.7593	0.7348
84	0.7733	0.7756	0.7522
85	0.7893	0.7913	0.7689
86	0.8048	0.8058	0.7847
87	0.8182	0.8192	0.7995
88	0.8305	0.831	0.8131
89	0.8412	0.8422	0.8252
90	0.8514	0.8513	0.8354
91	0.8585	0.8582	0.8431
92	0.8634	0.8622	0.8471
93	0.863	0.8603	0.8449
94	0.8529	0.8488	0.8316
95	0.8254	0.816	0.7959
96	0.7569	0.7348	0.7109
97	0.5734	0.5258	0.5078
98			
99			

Fig. 3.14. Tabular comparison of the expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death in Austria, Germany and Switzerland

From ages 1 to 97, Switzerland consistently exhibited the lowest expected value of joint whole life insurance payable at the moment of last death between lives (x) and (y). This reflects that relative to Austria and Germany, Switzerland estimates lower expected payouts for this benefit structure, possibly due to longer joint survival assumptions or differing demographic or interest rate bases.

For all ages from 1 to 89, Germany displayed the highest expected values for this last-death insurance function, suggesting higher anticipated payouts or earlier last-death occurrences within that age span. However, beginning at age 90 and continuing through age 97, Austria overtakes Germany, showing the highest expected values. This late-age shift implies that Austrian mortality assumptions yield longer joint life spans or increased present value accumulation at advanced ages.

When comparing this to the first-death insurance counterpart, both share an upward trend in values from early ages. However, a key distinction lies in the age of peak value and the nature of the decline. For last-death insurance, values increase until age 92 in Austria, Germany, and Switzerland, after which the values begin a sharp descent. This is later than the first-death function, which typically peaks slightly earlier, around ages 87–88, before starting its parabolic downturn.

Lastly, for all three countries, the expected values of this actuarial function collapse to 0 at ages 98 and 99, indicating that by this point, death is virtually certain for both lives involved, and thus, the insurance benefit has already been paid or lost all time value.

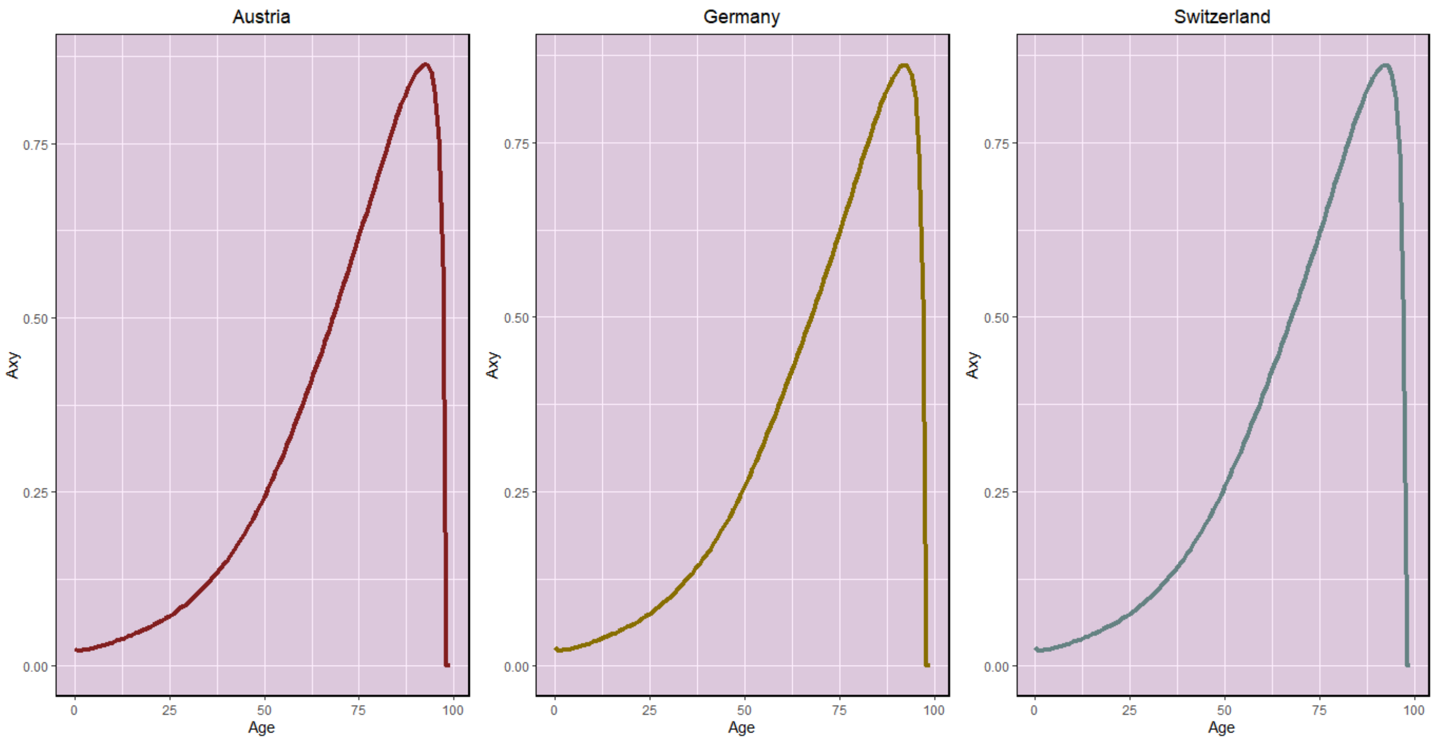

Fig. 3.15. Graphical comparison of the expected present value of a whole life insurance of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death in Austria, Germany and Switzerland

The values for Austria, Germany, and Switzerland follow an upward trend from early childhood through adulthood, which mirrors the general shape seen in the counterpart actuarial function based on the first death. However, unlike the first-death function, which rises quickly and starts to flatten or decrease as early as the late 80s, the values for the last-death continue to increase well into the early 90s. This is because a payout at the moment of the last death involves both lives surviving longer, resulting in a delayed and extended increase in expected present values.

In both the first-death and last-death functions, the shape of the curve shown begins with a slow rise in childhood, becomes steeper during adulthood, and eventually peaks in old age. However, while the first-death values peak earlier and decline due to the high probability that at least one death occurs, the last-death function shows a prolonged increase, since it requires both individuals to die, extending the peak into the early 90s.

Austria shows a smooth rise from ages 28 to 55, increasing from 0.0834 to 0.3047. Similarly, the slope of Germany increased from 0.0835 at age 27 to 0.3061 at age 54. In parallel to both countries, the slopes of Switzerland increased from 0.0712 at age 28 to 0.3074 at age 58. These periods of steady growth are common in both first-death and last-death functions, though the last-death version peaks at a later age.

The highest values for each of the 3 countries occurred around the age of 92: Austria at 0.8634, Germany at 0.8622, and Switzerland at 0.8471. This is shown noticeably later than the first-death function, where peak values were often seen in the high age of 80s and began a parabolic decline afterward. For the last-death values, the decline is delayed and more gradual, starting around age 93.

Finally, just like in the first-death function, values collapse to 0 between ages 98 and 99, signaling that by this age range, death is practically certain for both lives and no future benefit remains to be discounted.

Age	Austria	Germany	Switzerland
0	17.2261	17.2101	17.2623
1	17.244	17.2397	17.3132
2	17.2223	17.219	17.297
3	17.2015	17.195	17.2781
4	17.1755	17.17	17.2578
5	17.1491	17.1431	17.2353
6	17.1208	17.1141	17.2113
7	17.0925	17.0835	17.1852
8	17.0598	17.0509	17.1577
9	17.0271	17.0166	17.1284
10	16.9906	16.9808	17.0978
11	16.9551	16.9427	17.0661
12	16.9191	16.903	17.0331
13	16.882	16.8609	16.9989
14	16.8395	16.817	16.963
15	16.7953	16.771	16.9256
16	16.7539	16.7228	16.8872
17	16.7072	16.6732	16.8474
18	16.6617	16.6208	16.8064
19	16.6131	16.5671	16.7635
20	16.5597	16.5112	16.7184
21	16.5034	16.4516	16.6704
22	16.4447	16.3888	16.6199
23	16.3857	16.3222	16.5662
24	16.3248	16.2523	16.5095
25	16.2563	16.1782	16.4494
26	16.1859	16.0999	16.3862
27	16.1045	16.0182	16.3198
28	16.0268	15.9318	16.2496
29	15.9435	15.8408	16.176
30	15.8595	15.7457	16.0984
31	15.7659	15.6461	16.0171
32	15.6686	15.5407	15.9317
33	15.5668	15.4306	15.8418
34	15.4563	15.3153	15.7473
35	15.3436	15.1956	15.6481
36	15.2241	15.0694	15.544
37	15.1002	14.9389	15.4346
38	14.9697	14.8031	15.3194

39	14.8381	14.6614	15.1989
40	14.6973	14.5129	15.0724
41	14.5538	14.3578	14.94
42	14.4034	14.1963	14.8012
43	14.2439	14.0277	14.6559
44	14.0769	13.8551	14.5043
45	13.9004	13.6751	14.346
46	13.7211	13.4871	14.181
47	13.531	13.2918	14.009
48	13.3353	13.0891	13.8302
49	13.1266	12.8804	13.6446
50	12.9142	12.6671	13.4518
51	12.6939	12.4472	13.2518
52	12.4699	12.2203	13.0448
53	12.2337	11.9875	12.8306
54	11.9941	11.7477	12.6093
55	11.7461	11.5031	12.381
56	11.4887	11.2563	12.1455
57	11.24	11.0032	11.9028
58	10.9877	10.7446	11.6532
59	10.7205	10.4834	11.3965
60	10.4407	10.2183	11.1328
61	10.1633	9.951	10.8624
62	9.8905	9.6805	10.5851
63	9.5983	9.4043	10.3011
64	9.3143	9.126	10.0105
65	9.0208	8.8459	9.7133
66	8.722	8.5627	9.4097
67	8.4138	8.2767	9.0996
68	8.1094	7.9875	8.7834
69	7.8055	7.6964	8.4614
70	7.4933	7.3995	8.1339
71	7.1783	7.0996	7.8015
72	6.8769	6.7953	7.4647
73	6.5674	6.4872	7.1243
74	6.2548	6.1791	6.7813
75	5.9361	5.8627	6.4367
76	5.6092	5.552	6.092
77	5.2952	5.2352	5.7487
78	4.9988	4.9246	5.4084
79	4.6826	4.6175	5.0729

80	4.375	4.3125	4.7448
81	4.083	4.0258	4.426
82	3.8024	3.7497	4.1188
83	3.5269	3.4905	3.8253
84	3.2602	3.2389	3.547
85	3.0138	3.003	3.2854
86	2.7835	2.7894	3.0414
87	2.5846	2.5907	2.8148
88	2.4052	2.4122	2.6046
89	2.24	2.2381	2.4075
90	2.0564	2.0754	2.2174
91	1.8747	1.9081	2.0223
92	1.6827	1.7226	1.7995
93	1.4407	1.4612	1.4983
94			
95			
96			
97			
98			
99			

Fig. 3.16. Tabular comparison of the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y +5) payable at the end of each year until the last death in Austria, Germany and Switzerland

From ages 1 to 93, Switzerland exhibited the highest expected present value of a joint whole life annuity-due issued at the end of every year until the last death of the two lives aged (x) and (y+5). This behavior contrasts with its first-death counterpart, in which Switzerland held the highest expected values across all ages from 1 to 96, indicating that Switzerland's advantage in early annuity value persists longer under the first-death model than in the last-death setting.

In terms of the lowest values, Germany held the lowest expected present value of the last-death annuity from ages 1 to 85. From age 86 onward, Austria's values dropped below Germany's. This again differs from the first-death case, where Germany already had the lowest values from age 1 to 76, with Austria becoming the lowest afterward. The delayed decline in the last-death values suggests a more prolonged annuity payout period in Austria's case when both lives are considered.

Finally, a striking distinction between the two types of annuities is seen in the last 5 years (ages 95 to 99): the last-death annuity values completely collapse to 0 for all three countries, which is not observed in the first-death annuity, where values remain positive and gradually taper off. This reflects the nature of the last-death annuity, which ceases only when both individuals have died, and at such extreme ages, joint survival is virtually impossible.

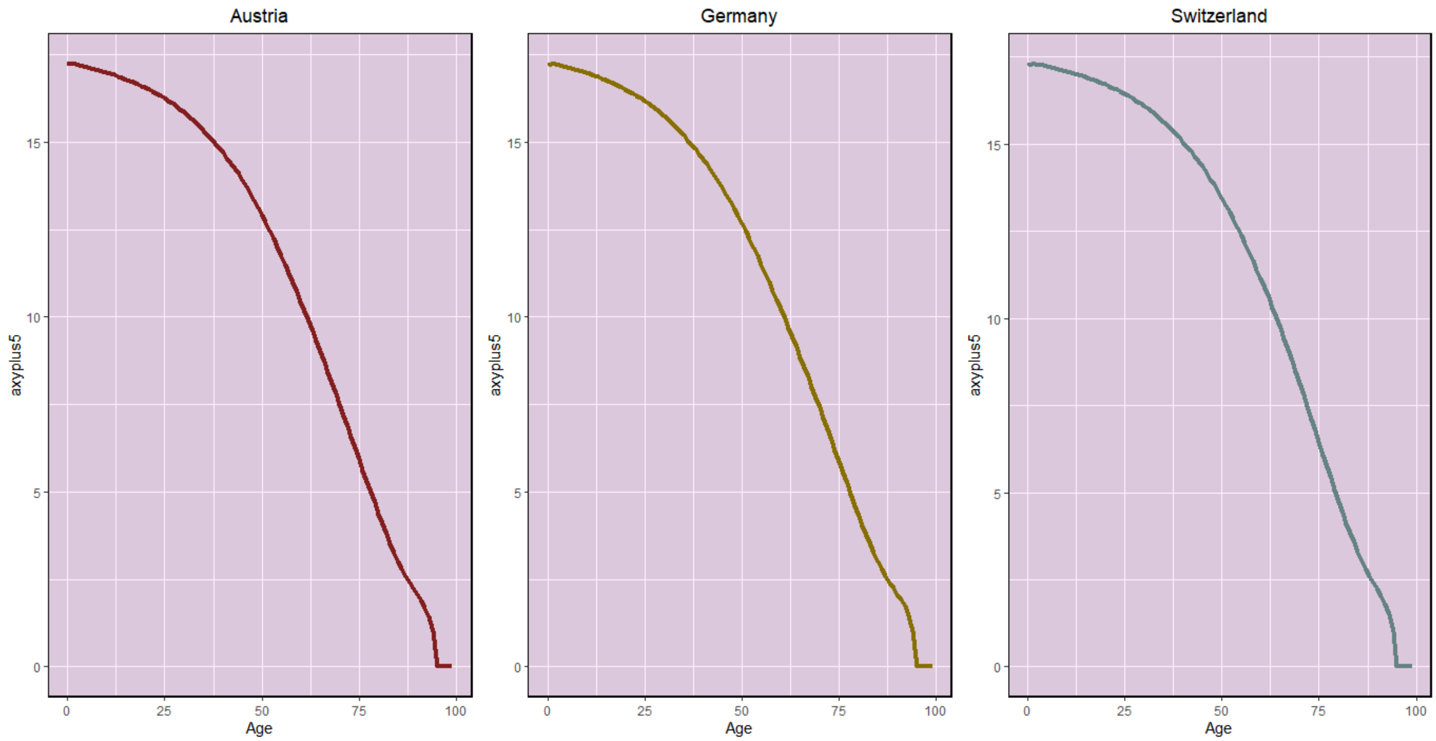

Fig. 3.17. Graphical comparison of the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y +5) payable at the end of each year until the last death in Austria, Germany and Switzerland

The graph shows a downward parabolic trend in Austria, Germany, and Switzerland. As age continues to increase from 0 to 99, the values decrease consistently, highlighting a very smooth but non-linear decline. Early in life, the decrease is much sharper, however as individuals move into youth or mid-adulthood, the decline becomes more evident and gradual. The patterns shown are consistent across all three countries with each display's slight variations in the point where the curvature flattens and begins to descend more gently.

In Austria, the curve shown in the graph becomes noticeably more rounded between ages 29 to 61. During the given age period, the annuity values dropped from 15.9435 to 10.1633 and showed a gradual decline compared to the sharper fall from 17.2261 at age 0 to 16.0268 at age 28. This suggests that in mid-adulthood, the likelihood of both individuals surviving longer contributes to a slower reduction in the expected value of the annuity. Germany shows a similar pattern as well between ages 28 to 60, with a decrease from 15.9318 to 10.2183. Like Austria, the early years show a steep drop, but this all flattens during mid-adulthood before resuming a sharper decline in later years.

Switzerland, in contrast, shows a slightly delayed but smoother curvature from ages 31 to 60. During this interval, values decline from 16.0171 to 11.1328. This later and more stretched curvature suggests that, among the three countries, Switzerland sustains joint survival for a longer time, delaying the sharper drop-off seen in Austria and Germany. The prolonged plateau during midlife reinforces the idea of enhanced longevity, particularly in at least one of the two insured lives.

When compared to their first-death annuity counterparts, the values of the last-death annuities are consistently higher across all ages and countries. This is expected since payments continue until both individuals have passed, resulting in a longer expected payout period. While first-death annuities tend to decline more rapidly as soon as one individual becomes more susceptible to mortality, the last-death annuity functions maintain a higher value for a longer period.

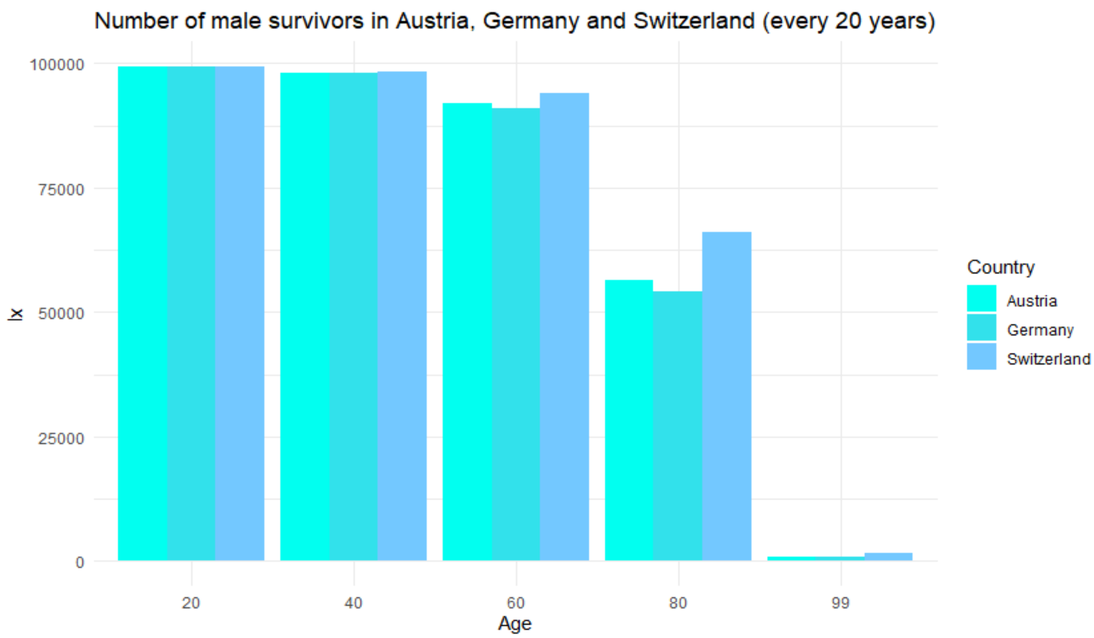

Fig. 3.18. Number of male survivors in Austria, Germany and Switzerland (every 20 years)

From the graph there is an evident declining trend on the number of male survivors in Austria, Germany and Switzerland as age increases. This graph follows typical mortality patterns. At the younger age groups (20 and 40), the number of survivors between the three countries are high and quite similar with each of the countries. Though the disparity becomes larger from the older age groups (50, 80, and 99). This suggests that there is a difference in life expectancy in Austria, Germany and Switzerland.

Furthermore, Switzerland notably has higher survival numbers in the older age groups. This indicates that the population in Switzerland has longer life longevity compared to Austria and Germany. Some possible factors that influence this is Switzerland has better quality of life, healthcare and socioeconomic conditions compared to the two other countries.

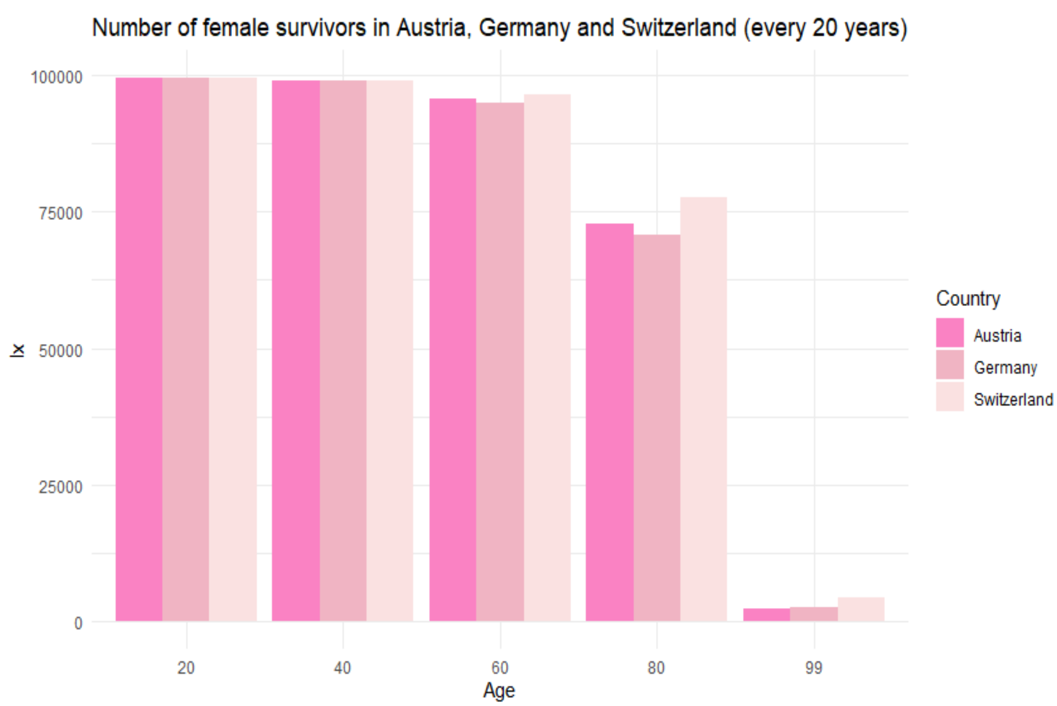

Fig. 3.19. Number of female survivors in Austria, Germany and Switzerland (every 20 years)

It is shown in the graph that there is an evident declining trend on the number of female survivors in Austria, Germany and Switzerland as age increases. Furthermore the graph follows typical mortality patterns. From ages 20-40 years, the number of female survivors between the three countries are quite similar. Though there is a larger disparity from the older age groups (50, 80, and 99). This suggests that there is a difference in life expectancy in Austria, Germany and Switzerland.

It is also important to note that Switzerland notably has higher survival numbers in the older age groups. This indicates that the population in Switzerland has longer life longevity compared to Austria and Germany. Some possible factors that influence this is Switzerland has better quality of life, healthcare and socioeconomic conditions compared to the two other countries.

Furthermore, when comparing the graph of the number of male and female survivors in Austria, Germany and Switzerland it is observed that female survivors are greater than male survivors in the older age groups (50, 80 and 99) which aligns with global observed demographic trends that females typically have longer life expectancies than males.

4. SUMMARY

This section contains a summary of the findings under the morality data of Austria, Germany and Switzerland. This section also contains tabular comparison and graphical comparison concerning the single life and joint life contingency models.

4.1. Mortality data

Looking through the graphical comparisons of life insurance metrics from Austria, Germany, and Switzerland, the researchers uncover some intriguing trends and a few similarities. These analyses shed light on how survival rates, expected whole life annuities, and whole life benefit premiums evolve over time, illustrating the differences in their divergence, peak, and convergence points. Typically, Austria and Germany are in sync, whereas Switzerland often shows a delay in divergence and converges later in several cases.

The survival trends for men and women in Austria, Germany, and Switzerland show a downwards exponential pattern. This divergence starts in the early 30s and reaches its peak around age 60. Interestingly, by the mid-80s, the survival rates for both genders began to converge, indicating that the differences in survival based on gender are consistent across these three countries but tend to lessen as people get older.

4.2. Joint-life contingency models

The expected present values of whole life annuities and insurance under joint life status. Whether for first death or last death coverage shows consistent age related patterns across Austria, Germany and Switzerland. Values typically decrease for annuities and increase for insurances until late life. First death annuities decline sharply from ages 45-49, while last death annuities drop earlier, around ages 33-35 with 0.1+ year over year differences. Both collapse toward zero by ages 98-99.

Insurance values under first death rise significantly from ages 47-54 until peaking at 87-90 then decline to zero. While for last death insurance it increases more gradually from age 34 to about 92, peaking higher due to longer payout horizons then drops sharply. Switzerland holds the highest values for most intervals, with Germany and Austria trading ranks by age and function.

With a five-year age gap (x, y+5), values are generally lower than in (x,y), especially for annuities due to deferred payouts. Last death annuities show earlier and sharper midlife declines. Annuities follow a downward parabolic pattern, flattening in mid-adulthood before steep late-age drops, especially in last death setups. Insurance values steadily rise until plateauing, it is earlier for first death and later for last death. Peak last-death insurance values occur later and higher than first death, this reflects extended risk horizons. The sharp late age collapse in last death annuities goes to zero by 99 is unique to this function. Overall, trends align with actuarial expectations: annuities decrease and insurances increase with age, more extreme and delayed under last-death with cross-country differences reflecting mortality and interest assumptions.

5. CONCLUSIONS

This section contains conclusions with regards to the findings gathered from the mortality data, single life contingency models and joint life contingency models.

5.1. Mortality data

For all three countries, as time passes, there are less male survivors than female survivors. It is because more men in Austria, Germany and Switzerland partake in risky health behaviors such as alcohol consumption, poor dietary habits and ignorance in seeking preventive healthcare. Youth mortality rates are higher in Switzerland as the Swiss youth are more inclined to partake in risky behaviors that cut one's life short in addition to the expensive healthcare costs creating disparities for the Swiss youth in contrast to Austria and Germany. However, elderly mortality rates in Switzerland are lower in Switzerland as their advanced healthcare is often geared towards the elderly in addition to the high standard of living and strong social support systems that are less prevalent in Austria and Germany.

5.2. Joint life contingency models where benefit is paid after the end of the year of first death

When assuming a 6% interest rate for the three countries then comparing the actuarial functions with the three countries, the count of the male survivors and female survivors play a role in the pricing strategies of the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the first death. The higher survival probability translates into more delayed payments that inflates the present value of future payments as it shall be paid at a future date. It also translates into more risk taken into account for the living insurer. As the insurer lives on, insurance companies take into account administrative and liability costs to cover the services offered to the insurers. Furthermore, insurance companies also apply effective external management risk management strategies to consider longevity risk brought by the variability and risk that were associated with the duration concerning how long the payments were made. Lastly, higher survivorship rates also translate into the inflation of the required reserves to fund the payout of the insurers and to account for future liabilities for the insurance company. Economic conditions also play a role in the pricing of the benefits as the cost of living of a country plays a role in determining the capacity of an individual in paying off loans with regards to insurance payments in addition to making up for the living costs in a particular country. Since Switzerland had the highest cost of living amongst the three countries, the aforementioned country had the highest expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the first death. However, it is also worth noting that Switzerland also had a high survivorship rate. With Austria and Germany alternating as the country that follows Switzerland is dependent on the number of survivors between the two as a result of Austria having the same cost of living as Germany due to the former's economic dependence on the latter.

The same findings are applied for the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the first death when it comes to the implications of survivorship rates and economic conditions. It is worth noting that expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the first death is pricier than the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the first death as the former has a lower survival probability than the latter that ultimately paved the way for a more delayed termination of the package and the payments being extended for a longer period of time. In turn, insurance companies conclude that less risk and volatility are involved in the former when compared to the latter as administrative costs are less when accounting for the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the first death.

5.3. Joint life contingency models where insurance is paid after the end of the year of first death

For the case of the pricing strategies of the whole life insurance at a 6% interest rate, higher survivorship translates into a cheaper policy as payouts occur in the future due to the longer wait associated with the payout. Insurance companies also use this to their advantage as the chance of first death translates into the low chance of a claim occurring early on. Therefore, this notion prompted insurance companies to price their insurance at a lower price when faced with a high survivorship rate that leads to the deferral of expected payouts as investing premiums for a longer time lead to lower liabilities and cheaper premiums. A high survivorship rate also leads to payments occurring further out that results in heavy discounting in addition to a smaller claim frequency. Furthermore, a higher survivorship rate prompts less risk to account for volatility in risk margins and reduced internal administrative costs. It is worth noting that the value starts to dip at age 91 in Austria, age 90 in Germany and age 89 in Switzerland as those are the ages where life expectancy drops rapidly that gives rise to the devaluation of insurance leverage as a result of the weakening of pooling benefits and less time for investment growth. Furthermore, it is due to the control that insurance companies enforce on the aforementioned actuarial function as viewing the aforementioned ages as guaranteed payout rather than a long-term protection to account for the likelihood of the first death of the two insurers that further prompted insurance companies to control the allocated reserves to be handed out to the elderly. It is also due to the smaller opportunity for the allotted reserves to generate greater investment returns.

5.4. Joint life contingency models where benefit is paid after the end of the year of last death

When assuming a 6% interest rate for the three countries and comparing the expected value of a whole life annuity under a joint life status payable annually at the end of each year until the last death of insurers (x) and (y), higher survival probabilities translates in the longer duration of payments that inflates the expected present value as more payments are being paid that further gives rise to insurance companies accounting for the increased administrative and liability expenditures as the annuity is continually being paid over a longer period of time. For this case, a higher survival probability rate heightens the uncertainty as to the duration it takes for the payments to be made; hence, risk management strategies are also incorporated in the pricing strategies to account for the corresponding variability that also affects reserve requirements and pricing strategies. Cost of living also plays a role in the pricing strategy of the aforementioned actuarial packages as it determines the capacity of insurers to pay the packages and accounting for the living standards of each country. It is worth noting that the the expected value of a whole life annuity under a joint life status payable annually at the end of each year until the last death of insurers (x) and (y) is cheaper than the the expected value of a whole life annuity under a joint life status payable annually at the end of each year until the first death of insurers (x) and (y) as last-death annuities have a longer payment delay that makes the former have a more drastic effect when discounting the value to the present. Furthermore, there is a lower probability of early payout in the eyes of the insurer because it happens much later.

The same findings are applied for the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death when it comes to the implications of survivorship rates and economic conditions. It is worth noting that expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death remains pricier than the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y) payable at the end of each year until the last death as the former has a lower survival probability than the latter that ultimately paved the way for a more delayed termination of the package and the payments being extended for a longer period of time. In turn, insurance companies conclude that less risk and volatility are involved in the former when compared to the latter as administrative costs are less when accounting for the expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death. The expected present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the last death remains cheaper than the present value of a whole life annuity of a joint life status issued to an individual (x) and (y + 5) payable at the end of each year until the first death as discounting is heavier on the former due to the longer duration.

5.5. Joint life contingency models where insurance is paid after the end of the year of last death

Assuming a 6% interest rate and under joint life contingency models where the insurance is paid after the end of the year of the last death, the expected present value of a whole life insurance for a joint life status for individuals (x) and (y) also remains dependent on the joint survivorship probability that translates on the duration of the payment periods. This policy remains in force until the last of the two insurers die that extends its duration that further increases the liability of the insurer and administrative costs thanks to the additional premium that must cover the risk over a more extended period of time. Another characteristic of this policy is that the payout is not made until the second of the two insurers passes away, which further delays the financial support the beneficiaries receive that also spikes the value of the premium due to inflation in addition to the associated risk. The extended uncertainty of predicting the mortality of two individuals also proves to be more unpredictable that prompts insurance companies to employ a conservative approach in pricing insurance packages in addition to dedicating more resources to assess risk more accurately thanks to a longer regulatory requirement in holding the policies that also translate to an increase in overhead costs. Lastly, there are only a few companies in Austria, Germany and Switzerland that offer last-death insurance policies and allows existing insurers to price their package at a higher price as opposed to a more intense competition in first-death insurance companies that leads to them pricing their packages at a lower price to keep up with the environment. Therefore, these notions translate into a higher price in the joint life contingency models where insurance is paid after the end of the year of last death when compared to the joint life contingency models where insurance is paid after the end of the year of first death.

6. APPENDICES

```
library(lifecontingencies)
setwd("C:/Users/Tommy/Downloads")

library(lifecontingencies)
setwd("C:/Users/Tommy/Downloads")

#Austria
a = read.csv("AA.csv", header=TRUE)
attach(a)
a.male.table  =  new("actuarialtable",x=a$x,
       lx=a$lx, interest=0.06)
b = read.csv("AB.csv", header=TRUE)
```

```
attach(b)
b.female.table =
      new("actuarialtable",x=b$x, lx=b$lx,
      interest=0.06)

ab.axy = numeric(length(a.male.table@x))
for(i in 1:length(a.male.table@x)) {
  a.male.table.age = a.male.table@x[i]
  b.female.table.age = b.female.table@x[i]
         tables = list(a.male.table,
      b.female.table)
          ab.axy[i] = axyzn(tables,
      x=c(b.female.table.age,
      b.female.table.age), i=0.06,
      status="last")
}
ab.Axy = numeric(length(a.male.table@x))
for(i in 1:length(a.male.table@x)) {
  a.male.table.age = a.male.table@x[i]
  b.female.table.age = b.female.table@x[i]
         tables = list(a.male.table,
      b.female.table)
          ab.Axy[i] = Axyzn(tables,
      x=c(a.male.table.age,
      b.female.table.age), i=0.06,
      status="last")
}
ab.axyplusfive =
      numeric(length(a.male.table@x))
for(i in 1:length(a.male.table@x)) {
  a.male.table.age = a.male.table@x[i]
  b.female.table.age = b.female.table@x[i]
         tables = list(a.male.table,
      b.female.table)
      ab.axyplusfive[i] = axyzn(tables,
      x=c(a.male.table.age,
      b.female.table.age + 5), i=0.06,
      status="last")
}
ab.aaxyy = numeric(length(a.male.table@x))
for(i in 1:length(a.male.table@x)) {
  a.male.table.age = a.male.table@x[i]
  b.female.table.age = b.female.table@x[i]
         tables = list(a.male.table,
      b.female.table)
         ab.aaxyy[i] = axyzn(tables,
      x=c(a.male.table.age,
      b.female.table.age), i=0.06,
      status="joint")
}
ab.AAxyy = numeric(length(a.male.table@x))
for(i in 1:length(a.male.table@x)) {
  a.male.table.age = a.male.table@x[i]
  b.female.table.age = b.female.table@x[i]
         tables = list(a.male.table,
      b.female.table)
         ab.AAxyy[i] = Axyzn(tables,
      x=c(a.male.table.age,
      b.female.table.age), i=0.06,
      status="joint")
}
ab.aaxyyplusfive =
      numeric(length(a.male.table@x))
for(i in 1:length(a.male.table@x)) {
  a.male.table.age = a.male.table@x[i]
  b.female.table.age = b.female.table@x[i]
         tables = list(a.male.table,
      b.female.table)
     ab.aaxyyplusfive[i] = axyzn(tables,
      x=c(a.male.table.age,
      b.female.table.age + 5), i=0.06,
      status="joint")
}
ab.finaltable = data.frame(a$x, a$lx, b$lx,
      ab.axy, ab.Axy, ab.axyplusfive,
      ab.aaxyy, ab.AAxyy,
      ab.aaxyyplusfive)
write.csv(ab.finaltable,
      "AustriaFinalTable.csv")

#Germany
c = read.csv("BA.csv", header=TRUE)
attach(c)
c.male.table =
new("actuarialtable",x=c$Age, lx=c$lx,
interest=0.06)
e = read.csv("BC.csv", header=TRUE)
attach(e)
e.female.table =
new("actuarialtable",x=e$Age, lx=e$lx,
interest=0.06)

ce.axy = numeric(length(c.male.table@x))
for(i in 1:length(c.male.table@x)) {
  c.male.table.age = c.male.table@x[i]
  e.female.table.age = e.female.table@x[i]
         tables = list(c.male.table,
e.female.table)
          ce.axy[i] = axyzn(tables,
x=c(c.male.table.age, e.female.table.age),
i=0.06, status="last")
}
ce.Axy = numeric(length(c.male.table@x))
for(i in 1:length(c.male.table@x)) {
  c.male.table.age = c.male.table@x[i]
  e.female.table.age = e.female.table@x[i]
         tables = list(c.male.table,
e.female.table)
          ce.Axy[i] = Axyzn(tables,
x=c(c.male.table.age, e.female.table.age),
i=0.06, status="last")
}
ce.axyplusfive =
numeric(length(c.male.table@x))
for(i in 1:length(c.male.table@x)) {
  c.male.table.age = c.male.table@x[i]
  e.female.table.age = e.female.table@x[i]
         tables = list(c.male.table,
e.female.table)
```

```
    ce.axyplusfive[i] = axyzn(tables, x=c(c.male.table.age, e.female.table.age + 5), i=0.06, status="last")
}
ce.aaxyy = numeric(length(c.male.table@x))
for(i in 1:length(c.male.table@x)) {
  c.male.table.age = c.male.table@x[i]
  e.female.table.age = e.female.table@x[i]
    tables = list(c.male.table, e.female.table)
    ce.aaxyy[i] = axyzn(tables, x=c(c.male.table.age, e.female.table.age), i=0.06, status="joint")
}
ce.AAxyy = numeric(length(c.male.table@x))
for(i in 1:length(c.male.table@x)) {
  c.male.table.age = c.male.table@x[i]
  e.female.table.age = e.female.table@x[i]
    tables = list(c.male.table, e.female.table)
    ce.AAxyy[i] = Axyzn(tables, x=c(c.male.table.age, e.female.table.age), i=0.06, status="joint")
}
ce.aaxyyplusfive = numeric(length(c.male.table@x))
for(i in 1:length(c.male.table@x)) {
  c.male.table.age = c.male.table@x[i]
  e.female.table.age = e.female.table@x[i]
    tables = list(c.male.table, e.female.table)
    ce.aaxyyplusfive[i] = axyzn(tables, x=c(c.male.table.age, e.female.table.age + 5), i=0.06, status="joint")
}
ce.finaltablet = data.frame(c$Age, c$lx, e$lx, ce.axy, ce.Axy, ce.axyplusfive, ce.aaxyy, ce.AAxyy, ce.aaxyyplusfive)
write.csv(ce.finaltablefirst, "GermanyFinalTable.csv")

#Switzerland
f = read.csv("CB.csv", header=TRUE)
attach(f)
f.male.table = new("actuarialtable",x=f$Age, lx=f$lx, interest=0.06)
f.male.Axn = Axn(f.male.table, 0:99)
f.male.axn = axn(f.male.table, 0:99)
f.male.Px = (Axn(f.male.table, 0:99))/(axn(f.male.table, 0:99))
f.male.axx = numeric(length(f.male.table@x))
for(i in 1:length(f.male.table@x)) {
  age = f.male.table@x[i]
  tables = list(f.male.table, f.male.table)
   f.male.axx[i] = axyzn(tables, x=c(age, age), i=0.06, status="joint")
}
f.male.Axx = numeric(length(f.male.table@x))
for(i in 1:length(f.male.table@x)) {
  age = f.male.table@x[i]
   f.male.Axx[i] = Axyzn(list(f.male.table, f.male.table), x=c(age, age), i=0.06, status="joint")
}
f.male.Pxx = f.male.Axx/f.male.axx
g = read.csv("CC.csv", header=TRUE)
g.female.table = new("actuarialtable",x=g$Age, lx=g$lx, interest=0.06)
g.female.Axn = Axn(g.female.table, 0:99)
g.female.axn = axn(g.female.table, 0:99)
g.female.Px = (Axn(g.female.table, 0:99))/(axn(g.female.table, 0:99))
g.female.axx = numeric(length(g.female.table@x))
for(i in 1:length(g.female.table@x)) {
  age = g.female.table@x[i]
    tables = list(g.female.table, g.female.table)
   g.female.axx[i] = axyzn(tables, x=c(age, age), i=0.06, status="joint")
}
g.female.Axx = numeric(length(g.female.table@x))
for(i in 1:length(g.female.table@x)) {
  age = g.female.table@x[i]
    g.female.Axx[i] = Axyzn(list(g.female.table,g.female.table), x=c(age, age), i=0.06, status="joint")
}
g.female.Pxx = g.female.Axx/g.female.axx

fg.finaljointtable = data.frame(f$Age, f$lx, g$lx, f.male.axx, g.female.axx, f.male.Axx, g.female.Axx, f.male.Pxx, g.female.Pxx)
fg.finaljointtable
write.csv(fg.finaljointtable, "SwitzerlandFinalJointTable.csv")

abcefg.survivors = data.frame(a$Age, a$lx, b$lx, c$lx, e$lx, f$lx, g$lx)
write.csv(abcefg.survivors, "lx.csv")
```

Fig. 6.1. Actuarial Table Codes

```
library(ggplot2)
library(gridExtra)
setwd("C:/Users/Tommy/Downloads")

#lx
h = read.csv("EC.csv", header=TRUE)
ha = ggplot(data=h, aes(x=Age, y=lx, group=Gender)) + theme_bw() + theme(plot.title = element_text(hjust=0.5))
```

```
+ geom_line(aes(color=Gender), size= 2) +
ggtitle("Austria") +
scale_color_manual(values=c("#E5F3F3",
"#FFE6EE")) +
theme(legend.position="bottom")

i = read.csv("EF.csv", header=TRUE)
ib = ggplot(data=i, aes(x=Age, y=lx,
group=Gender)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Gender), size= 1.7) +
ggtitle("Germany") +
scale_color_manual(values=c("#223332",
"#822222")) +
theme(legend.position="bottom")

j = read.csv("EG.csv", header=TRUE)
jb = ggplot(data=j, aes(x=Age, y=lx,
group=Gender)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Gender), size= 1.7) +
ggtitle("Switzerland") +
scale_color_manual(values=c("#223332",
"#822222")) +
theme(legend.position="bottom")

grid.arrange(ha, ib, jb, ncol=3)

#axy
k = read.csv("FA.csv", header=TRUE)
ka = ggplot(data=i, aes(x=Age, y=axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Austria") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#822222"))

l = read.csv("FE.csv", header=TRUE)
lb = ggplot(data=l, aes(x=Age, y=axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Germany") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#877000"))

n = read.csv("FH.csv", header=TRUE)
nb = ggplot(data=n, aes(x=Age, y=axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Switzerland") +
theme(plot.title = element_text(hjust=0.5),
legend.position = "none", panel.background
= element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#688686"))

grid.arrange(ka, lb, nb, ncol=3)

#Axy
o = read.csv("GA.csv", header=TRUE)
oa = ggplot(data=o, aes(x=Age, y=Axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Austria") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#822222"))

p = read.csv("GC.csv", header=TRUE)
pa = ggplot(data=p, aes(x=Age, y=Axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Germany") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#877000"))

q = read.csv("GE.csv", header=TRUE)
qa = ggplot(data=q, aes(x=Age, y=Axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Switzerland") +
theme(plot.title = element_text(hjust=0.5),
legend.position = "none", panel.background
```

```
= element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#688686"))

grid.arrange(oa, pa, qa, ncol=3)

#axy+5
r = read.csv("HB.csv", header=TRUE)
ra = ggplot(data=r, aes(x=Age, y=axyplus5,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Austria") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#822222"))

s = read.csv("HC.csv", header=TRUE)
sa = ggplot(data=s, aes(x=Age, y=axyplus5,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Germany") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#877000"))

t = read.csv("HG.csv", header=TRUE)
ta = ggplot(data=s, aes(x=Age, y=axyplus5,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Germany") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#688686"))

grid.arrange(ra, sa, ta, ncol=3)

#axxyy
library(ggplot2)
library(gridExtra)
setwd("C:/Users/Tommy/Downloads")

r = read.csv("HB.csv", header=TRUE)
ra = ggplot(data=r, aes(x=Age, y=axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Austria") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#822222"))

s = read.csv("HC.csv", header=TRUE)
sa = ggplot(data=s, aes(x=Age, y=axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Germany") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#877000"))

t = read.csv("HF.csv", header=TRUE)
ta = ggplot(data=s, aes(x=Age, y=axy,
group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Switzerland") +
theme(plot.title = element_text(hjust=0.5),
legend.position = "none", panel.background
= element_rect(fill = "#FFF7AA"),
panel.grid.major = element_line(color =
"#FFFCCC"), panel.grid.minor =
element_line(color = "#FFFCCC"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#688686"))

grid.arrange(ra, sa, ta, ncol=3)

#Axx
y = read.csv("KA.csv", header=TRUE)
ya = ggplot(data=y, aes(x=Age, y=Axx,
group=Gender)) + theme_bw() +
```

```
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Gender), size= 1.7) +
ggtitle("Austria") +
scale_color_manual(values=c("#223332",
"#822222")) +
theme(legend.position="bottom")

z = read.csv("KE.csv", header=TRUE)
za = ggplot(data=z, aes(x=Age, y=Axx,
group=Gender)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Gender), size= 1.7) +
ggtitle("Germany") +
scale_color_manual(values=c("#223332",
"#822222")) +
theme(legend.position="bottom")

aaaaa = read.csv("KG.csv", header=TRUE)
aaaaaa = ggplot(data=aaaaa, aes(x=Age,
y=Axx, group=Gender)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Gender), size= 1.7) +
ggtitle("Switzerland") +
scale_color_manual(values=c("#223332",
"#822222")) +
theme(legend.position="bottom")

grid.arrange(ya, za, aaaaaa, ncol=3)

#AAxyyplus5
bac = read.csv("LB.csv", header=TRUE)
bag = ggplot(data=bac, aes(x=Age,
y=axyplus5, group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Austria") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#E0CCE0"),
panel.grid.major = element_line(color =
"#FFF0FF"), panel.grid.minor =
element_line(color = "#FFF0FF"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#822222"))

cab = read.csv("LJ.csv", header=TRUE)
cac = ggplot(data=cab, aes(x=Age,
y=axyplus5, group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Germany") + theme(plot.title =
element_text(hjust=0.5), legend.position =
"none", panel.background =
element_rect(fill = "#E0CCE0"),
panel.grid.major = element_line(color =
"#FFF0FF"), panel.grid.minor =
element_line(color = "#FFF0FF"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#877000"))

fac = read.csv("LL.csv", header=TRUE)
fak = ggplot(data=fac, aes(x=Age,
y=axyplus5, group=Country)) + theme_bw() +
theme(plot.title = element_text(hjust=0.5))
+ geom_line(aes(color=Country), size= 1.7)
+ ggtitle("Switzerland") +
theme(plot.title = element_text(hjust=0.5),
legend.position = "none", panel.background
= element_rect(fill = "#E0CCE0"),
panel.grid.major = element_line(color =
"#FFF0FF"), panel.grid.minor =
element_line(color = "#FFF0FF"),
panel.border = element_rect(color =
"#000000", fill = NA, size = 1)) +
scale_color_manual(values=c("#688686"))

grid.arrange(bag, cac, fak, ncol=3)
```

Fig. 6.2. Graph Codes

```
library(ggplot2)
ua = data.frame(Country=rep(c("Austria",
    "Germany", "Switzerland"), each=5),
    Age=rep(c("20", "40", "60", "80",
    "99"), 3), lx=c(99411, 98040, 91903,
    56334, 843, 99390, 98141, 90889,
    54070, 871, 99321, 98328, 93892,
    66153, 1592))
ggplot(data=ua, aes(x=Age, y=lx,
    fill=Country)) +
    geom_bar(stat="identity",
    position=position_dodge())+
    ggtitle("Number of male survivors in
    Austria, Germany and Switzerland
    (every 20 years)") +
    theme(plot.title =
    element_text(hjust=0.5)) +
    scale_fill_manual(values=c("#01FFF1"
    , "#33E3EE", "#77CCFF")) +
    theme_minimal()
uc = data.frame(Country=rep(c("Austria",
    "Germany", "Switzerland"), each=5),
    Age=rep(c("20", "40", "60", "80",
    "99"), 3), lx=c(99496, 98872, 95628,
    72648, 2412, 99517, 98897, 94901,
    70683, 2565, 99465, 98989, 96334,
    77654, 4375))
ggplot(data=uc, aes(x=Age, y=lx,
    fill=Country)) +
    geom_bar(stat="identity",
    position=position_dodge())+
    ggtitle("Number of female survivors
    in Austria, Germany and Switzerland
    (every 20 years)") +
    theme(plot.title =
    element_text(hjust=0.5)) +
    scale_fill_manual(values=c("#FA86C3"
```

```
,     "#F2B8C6",     "#FEE5E5"))     +
theme_minimal()
```

Fig. 6.3. Comparison Codes

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