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# Laboratory Experiment II : Design of a Speed Controller

MTRN3020 Modelling and Control of Mechatronic Systems

## Abstract

This laboratory exercise is to design a speed controller to be implemented in a motor generator system. The direct analytical design method or the so-called Ragazzini's method is to be used to design the controller. The controller design involves only a small amount of calculations leading to a difference equation. This document only describes the pre-experiment calculations that needs to be done. Follow the step by step procedures to obtain the parameters you need to bring to the experiment. You must have these parameters calculated as each student will generate his/her own data files.

## 1 Experimental Setup

The experimental set up consists of a motor that is connected to a generator. In practice, the generators are driven by some form of a prime mover powered by a gasoline or steam engine/turbine. For the purpose of clean experimentation, in this case, the generator is driven by an electric motor.

## 2 Controller Design

The block diagram to be used is shown in Fig. 1.

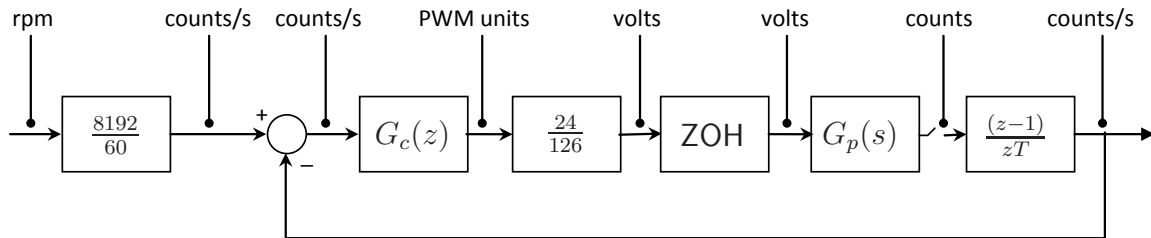


Figure 1: Speed control block diagram

### 2.1 Step 1

Download the file `noLoad.m` from Blackboard. This file shows the response of the motor when a voltage of 24 volts is applied. Plot a graph of time (column 1) as  $x$ -axis and motor speed (column 3) as  $y$ -axis. Time is in milliseconds and hence need to be converted to seconds. The speed is in counts/second. As shown in Fig. 1, the controller  $G_c(z)$  relates the error in counts/second to PWM Units. Hence the motor speed need not be converted to any other units. Through first order response

fitting to the motor response data, obtain a transfer function that relates the applied voltage to the speed in counts/second. Let this be,

$$G_{p1}(s) = \frac{A}{1 + \tau s}. \quad (1)$$

Note that by this time you should have the numerical values for  $A$  and  $\tau$ .

In this experiment, we cannot sample the speed. We only sample the counts using an encoder. The speeds are calculated. Hence the experimental situation is such that we should form the transfer function that relates voltage to counts. This can be obtained by including an integrator and is,

$$G_p(s) = \frac{A}{s(1 + \tau s)}. \quad (2)$$

## 2.2 Step 2

By combining all blocks in the block diagram, except  $G_c(z)$ , the plant transfer function can be obtained as follows.

$$G_p(z) = \mathcal{Z} \left[ \frac{24}{126} \frac{(1 - e^{-sT})}{s} \frac{A}{s(1 + \tau s)} \frac{(z - 1)}{zT} \right]. \quad (3)$$

Break this into two parts,

$$G_p(z) = A(z) \frac{(z - 1)}{zT}. \quad (4)$$

Use the following commands in Matlab to find the  $A(z)$  part.  $T$  to be used right throughout can be found against your name in a file in Blackboard.

```
>> num = [24*A];
>> den = 126*[tau 1 0];
>> [numd, dend] = c2dm(num,den,T,'zoh');
>> roots(numd)
>> roots(dend)
```

The `c2dm` function automatically takes into account the ZOH as we have mentioned so with 'zoh' in the `c2dm` function. Now, using the above found roots,  $A(z)$  part can be written in the form,

$$A(z) = \frac{C(z - z_1)}{(z - 1)(z - p_1)}, \quad (5)$$

where the constant  $C$  is also known. Substituting (5) in (4),  $G_p(z)$  is now,

$$G_p(z) = \frac{C(z - z_1)}{Tz(z - p_1)} \quad (6)$$

Note that all numerical values of the parameters of  $G_p(z)$  are now known.

## 2.3 Step 3

We now need to form our  $F(z)$ . To do this we need the design specifications. First of these, the design time constant, that is the desired time constant of  $F(z)$  denoted here as  $\tau_d$ , can be found against your

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name in a file in Blackboard. The other is the fact that we need zero steady state error - that is unity DC gain which also means  $F(z) = 1$ .

It is possible that  $(z - z_1)$  in (6) could cause ringing. If so it must be absorbed by the numerator of  $F(z)$ , else it need not be absorbed by the numerator of  $F(z)$ . A time constant of  $\tau_d$  corresponds to  $s = -1/\tau_d$  in  $s$ -domain. Its location in  $z$ -domain is  $e^{-T/\tau_d}$ . Hence,  $F(z)$  takes the form,

$$F(z) = \frac{b_0(z - z_1)}{(z - e^{-T/\tau_d})} \quad \text{if ringing or} \quad F(z) = \frac{b_0}{(z - e^{-T/\tau_d})} \quad \text{if not ringing.} \quad (7)$$

Note that, in the case of ringing, the causality constraint (the pole zero deficiency must at least be that of  $G_p(z)$ ) is not satisfied. Therefore, the acceptable forms of  $F(z)$  are

$$F(z) = \frac{b_0(z - z_1)}{z(z - e^{-T/\tau_d})} \quad \text{if ringing or} \quad F(z) = \frac{b_0}{(z - e^{-T/\tau_d})} \quad \text{if not ringing.} \quad (8)$$

Note: if your  $G_p(z)$  has a ringing zero, it MUST be eliminated!

Only unknown in (8) is  $b_0$ . This can be obtained using  $F(z) = 1$  condition. So,

$$b_0 = \frac{(1 - e^{-T/\tau_d})}{(1 - z_1)} \quad \text{if ringing or} \quad b_0 = (1 - e^{-T/\tau_d}) \quad \text{if not ringing} \quad (9)$$

At this stage, all numerical values of  $F(z)$  are completely known.

## 2.4 Step 4

Use

$$G_c(z) = \frac{1}{G_p(z)} \frac{F(z)}{(1 - F(z))} \quad (10)$$

to obtain the controller transfer function. Bring this transfer function to the experiment. The demonstrator will explain how it forms the final difference equation to be used as the controller.

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