Modeling and Control of Mechatronic Systems

Exercise 4: Design of Control Systems - Direct Analytical Method

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Exercise 4

Design of Control Systems - Direct Analytical Method

A system plant transfer function is given as,

$$G_p(s) = \frac{1}{s(s+1)}$$

A controller needs to be designed to satisfy the following conditions;

1. Time to peak t_p = 0.2s.
2. Damping ration ζ=0.8.
3. Zero steady state error to a step input.
4. Zero steady state error to a ramp input.

Do the following:

a). Determine a suitable sampling interval T.

b). Obtain $G_p(z)$ assuming the plant is proceeded by a ZOH.

c). Propose $F(z)$ ignoring the ringing effects.

d). Give the controller that corresponds to the $F(z)$ above.

e). Build a Simulink block diagram using your controller, ZOH and the continuous plant. Obtain a plot of step response and ramp response.

f). Propose an $F(z)$ to eliminate the ringing effects.

g). Give a controller that corresponds to $F(z)$ in f).

h). Replace the controller in item e). above with the controller obtained in item g). Obtain a plot of step response and ramp response

i). Discuss the effect of the presence and absence of ringing in the two plots you generated.

Solution

Part a). If the time to peak is t_p, then the period of damped oscillations can be approximated by $2t_p$. Hence the damped natural frequency is $\omega_d = 2\pi/2t_p = 15.70$ rad/sec. Since the period of damped oscillations is $2t_p = 0.4$, and assuming that we sample 10 times faster, a suitable sampling interval is,

$$T = 0.04s.$$

□

Part b). The $G_p(z)$ with a ZOH and using table reference is,

$$G_p(z) = \frac{0.0007894(z + 0.9868)}{(z - 1)(z - 0.9608)}$$

□

Part c). Given that $\zeta = 0.8$, the real part of the pole locations is $\omega_d/\tan\psi = -20.944$ where $\zeta = \cos\psi$=0.8. Therefore the desired pole locations on the s-plane are,

$$s_1, s_2 = -20.9440 \pm \text{j}15.70.$$

This corresponds to z-plane pole locations of,

$$z_1, z_2 = e^{(-20.9440 \pm \text{j}15.70)0.04} = 0.3501 \pm \text{j}0.2542.$$

Therefore the desired characteristic equation,

$$z^2 - 0.7003z + 0.1872 = 0$$

and hence the denominator of $F(z)$ is,

$$z^2 - 0.7003z + 0.1872$$

Taking into account the two requirements of zero steady state error to a step input and the velocity error requirement,

$$F(z) = \frac{(b_0 z + b_1)}{z^2 - 0.7003z + 0.1872}$$

The pole-zero deficiency is 1 and hence no additional causality constraint satisfaction is necessary. Note that there is a plant zero that will cause ringing. However, in the first part we ignore the ringing.

Using the zero steady state condition,

$$F(z)|_{z=1} = 1 = \frac{(b_0 + b_1)}{1 - 0.7003 + 0.1872}$$

This gives us,

$$b_0 + b_1 = 0.4870 \tag{4.1}$$

Using the velocity error condition,

$$\frac{1}{TK_v} = -\left.\frac{dF(z)}{dz}\right|_{z=1}$$

and given that zero velocity error is required, $1/K_v = 0$. Hence,

$$0 = \frac{(z^2 - 0.7003z + 0.1872)b_0 - (b_0 z + b_1)(2z - 0.7003)}{(z^2 - 0.7003z + 0.1872)^2}$$

Evaluating at $z = 1$,

$$-0.8127b_0 - 1.2997b_1 = 0 \tag{4.2}$$

Solving eq.(4.1) and eq.(4.2),

$$\begin{aligned} b_0 &= 1.2997 \\ b_1 &= -0.8127 \end{aligned}$$

This gives us the required $F(z)$ of,

$$F(z) = \frac{1.2997(z - 0.6253)}{z^2 - 0.7003z + 0.1872}$$

□

Part d). The controller can now be found as,

$$\begin{aligned} G_c(z) &= \frac{(z-1)(z-0.9608)}{0.0007894(z+0.9868)} \frac{1.2997(z-0.6253)}{z^2 - 0.7003z + 0.1872 - 1.2997(z-0.6253)} \\ &= \frac{(z-1)(z-0.9608)}{0.0007894(z+0.9868)} \frac{1.2997(z-0.6253)}{(z-1)^2} \\ &= 1646.4\frac{(z-0.9608)(z-0.6253)}{(z+0.9868)(z-1)} \end{aligned}$$

Therefore the ringing controller is,

$$\boxed{G_c(z) = 1646.4\frac{(z-0.9608)(z-0.6253)}{(z+0.9868)(z-1)}}$$

□

Part e). A Simulink block diagram is shown in Fig. 4.1. When simulating, choose a variable step with a min step size of 0.001 and max step size of 0.002, which are both integer factors of the sampling interval 0.04 s. To obtain the discrete response, choose fixed step solver. The responses are plotted in Fig. 4.2. Note the ringing response. It dies down very slowly as the closed loop pole is very close to the unit circle, hence very close to the origin of the s-plane. A zoomed in view showing the ringing in response to a unit ramp input is shown in Fig. 4.3. □

Part f). To eliminate ringing, we will embed the ringing zero of the plant in the numerator of $F(z)$. This will give us,

$$F(z) = \frac{(b_0 z + b_1)(z + 0.9868)}{z(z^2 - 0.7003z + 0.1872)}$$

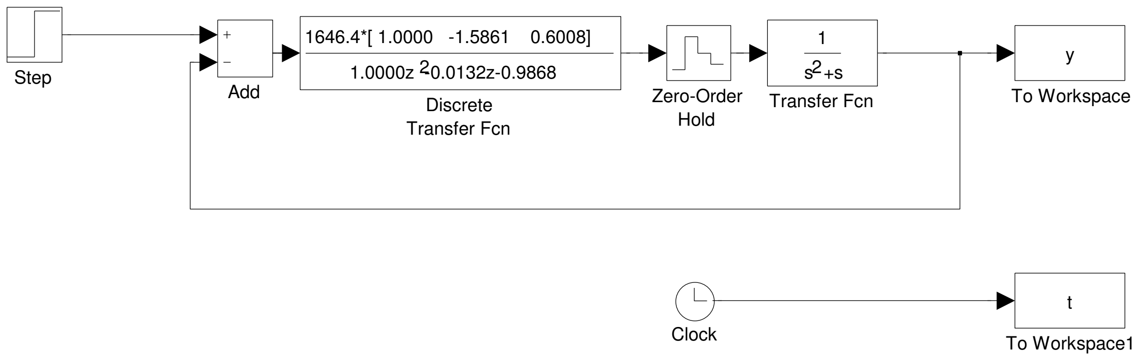

Figure 4.1: Simulink block diagram

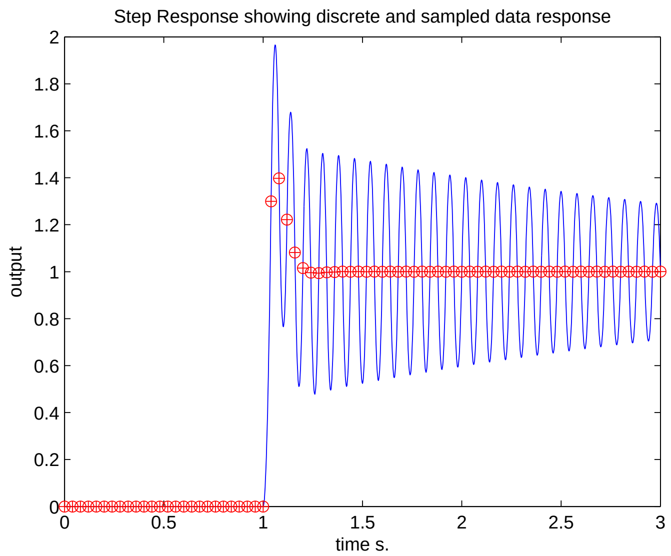

Figure 4.2: Discrete response shown with discrete red circles at T=0.04 s apart and the continuous time response shown in blue.

Note the addition of a z in the denominator to satisfy the causality constraint. Applying $F(1)=1$ we get,

$$b_0 + b_1 = 0.245067 \tag{4.3}$$

Applying velocity error condition we get,

$$\begin{aligned} &b_0(-0.374093 + z)z^2(2.34769 + z) \\ &\qquad -2b_1(1.59851 + z)(0.0577816 + (-0.46846 + z)z)|_{z=1} = 0 \end{aligned}$$

This gives,

$$8.83844b_0 + 12.919b_1 = 0 \tag{4.4}$$

Solving eq. (4.3) and eq. (4.4) gives us,

$$\begin{aligned} b_0 &= 0.775887 \\ b_1 &= -0.53082 \end{aligned}$$

and hence,

$$F(z) = \frac{0.775887(z - 0.6841)(z + 0.9868)}{z(z^2 - 0.7003z + 0.1872)}$$

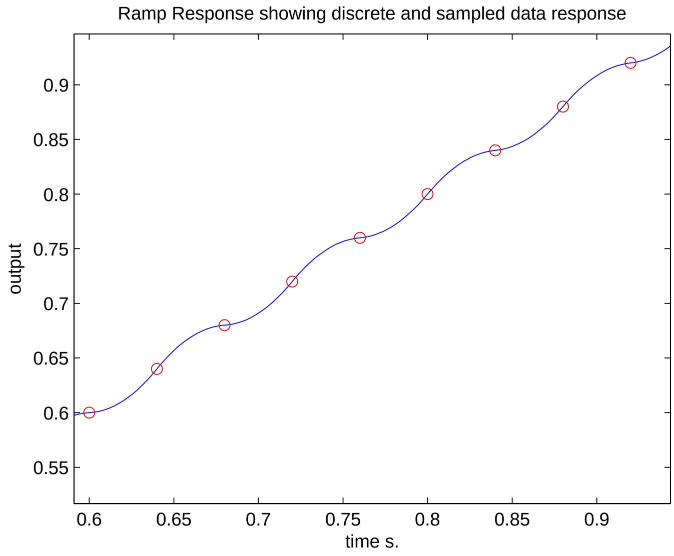

Figure 4.3: Zoomed view of discrete ramp response shown with discrete red circles at T=0.04 s apart and the continuous time response shown in blue. Note the ringing effect.

□

Part g). The controller can now be found by,

$$G_c(z) = \frac{(z-1)(z-0.9608)}{0.0007894(z+0.9868)} \frac{0.775887(z-0.6841)(z+0.9868)}{z(z^2-0.7003z+0.1872)-0.775887(z-0.6841)(z+0.9868)}$$

Factorizing,

$$G_c(z) = \frac{(z-1)(z-0.9608)}{0.0007894(z+0.9868)} \frac{0.775887(z-0.6841)(z+0.9868)}{(z-1)^2(z+0.5238)}$$

After simplification,

$$\boxed{G_c(z) = 982.8819\frac{(z-0.9608)(z-0.6841)}{(z+0.5238)(z-1)}}$$

Note the absence of the pole in the controller that could have caused ringing. When this controller is placed in series with the plant, there is no pole in the controller to cancel the ringing zero of the plant. As a result, no ringing takes place. □

Part h). The performance of the non-ringing controller is shown in Fig. 4.4 and Fig. 4.5. □

Part i). Obvious! □

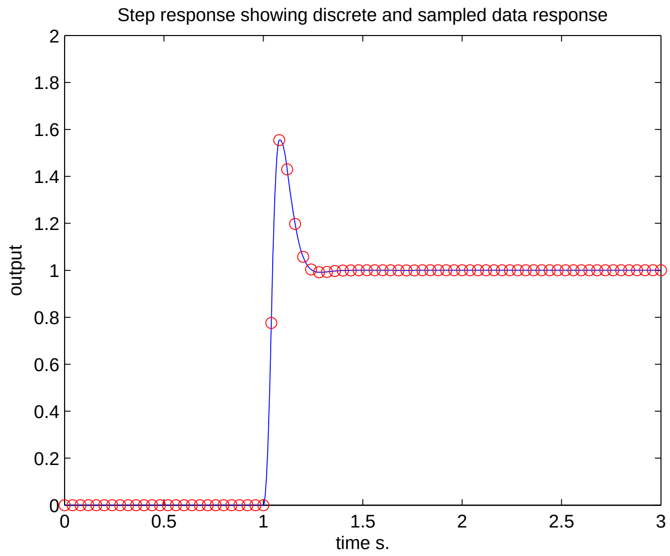

Figure 4.4: Discrete response shown with discrete red circles at T=0.04 s apart and the continuous time response shown in blue.

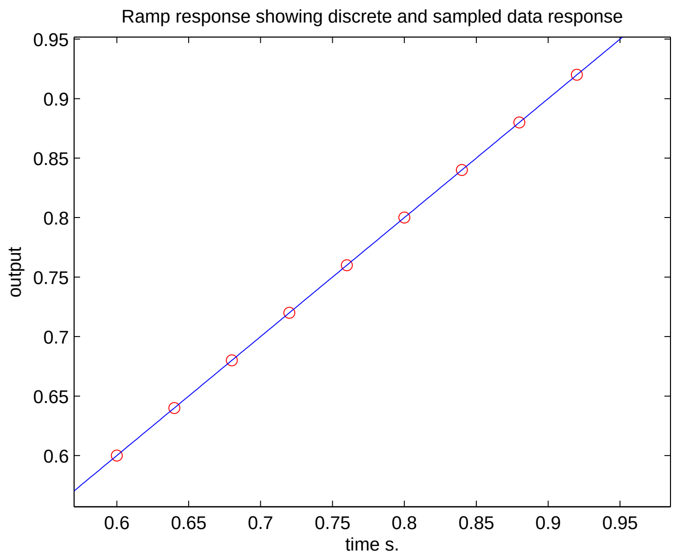

Figure 4.5: Zoomed view of discrete ramp response shown with discrete red circles at T=0.04 s apart and the continuous time response shown in blue. Note the absence of ringing effect.