Modeling and Control of Mechatronic Systems

Exercise 3: Mathematical Modeling and Simulation of an Anti-lock Braking System

Dr Jayantha Katupitiya

Exercise 3

Modeling and Control of an Anti-lock Braking System

The aim of this exercise is to model an anti-lock braking system. The system components include,

1. A pilot valve that needs to be actuated by the controller you design. The pilot valuve will receive a voltage input and will generate a brake pressure rate. Its dynamics can be described by the transfer function,
$$G_1(s) = \frac{15000}{(0.01s+1)} \tag{3.1}$$
The break pressure can be obtained by integrating the brake pressure rate. The brake pressure has un upper limit of 32400 N/m^2. Its lower limit is 0. The brake pressure can be converted to brake force by a constant 0.0028863. The brake force can be converted to brake torque by a constant 80, ending up with brake torque in Nm.

2. The coefficient of friction between the tyre and the road surface is a non-linear function of slip. Slip is defined as
$$s = 1 - \frac{\omega_w}{\omega_v} \tag{3.2}$$
in which ω_w is the wheel speed in rad/sec, and ω_v is the wheel speed calculated, in rad/sec, using the actual speed of the vehicle. The coefficient of friction values are given in the look up table below in Table. 3.1.

3. Weight of the vehicle is 2000 kg and has 4 wheels.

4. The wheel radius is 0.28 m.

5. The moment of inertia of each wheel about its axis is 2 kg m^2.

6. The maximum vehicle speed is 100 m/s.

The controller is required to adjust the pilot valve position to ensure the maximum coefficient of friction at the road-tyre surface. Do the following:

a). Develop a control strategy. That is, what will you measure?, what will you specify as the control input to achieve the control objective of maximum coefficient of friction.

Table 3.1: μ-slip data

Slip	μ
0	0
0.0500	0.4000
0.1000	0.8000
0.1500	0.9700
0.2000	1.0000
0.2500	0.9800
0.3000	0.9600
0.3500	0.9400
0.4000	0.9200
0.4500	0.9000
0.5000	0.8800
0.5500	0.8550
0.6000	0.8300
0.6500	0.8100
0.7000	0.7900
0.7500	0.7700
0.8000	0.7500
0.8500	0.7300
0.9000	0.7200
0.9500	0.7100
1.0000	0.7000

b). Derive the equilibrium equation for the rotational dynamics of a single wheel.

c). Devise a method to determine the vehicle speed using its inertial data, i.e. considering its acceleration.

d). Device a method to determine the wheel speed using its inertial data.

e). Complete a block diagram to achieve your control objectives.

f). Design a P controller and a PI controller to achieve the control objective.

g). Plot the stopping distance when the controller is in place and when the controller is not in place and do a comparison.