Modeling and Control of Mechatronic Systems

Exercise 5: Design of Control Systems - Indirect Design in Frequency Domain

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Exercise 5

Design of Control Systems - Indirect Design in Frequency Domain

Note: This question is extracted from "Continuous and Discrete Control Systems" by John Dorsey.

A system plant transfer function is given as,

$$G_p(s) = \frac{100}{(s+4)(s+50)}$$

A controller needs to be designed to satisfy the following conditions;

1. Sample rate of 50 Hz or less.
2. Velocity error constant $K_v > 100$.
3. $\dfrac{|E(j\omega)|}{|R(j\omega)|} \leqslant 0.02 \quad \text{for} \quad \omega < 1 \quad \text{rad/s.}$
4. $\dfrac{|C(j\omega)|}{|R(j\omega)|} \leqslant 0.1 \quad \text{for} \quad \omega > 100 \quad \text{rad/s.}$
5. Crossover frequency ω_c of at least 15 rad/s.
6. Phase margin of at least $50°$.
7. The controller $G_c(s) = K \dfrac{\prod_i (1+\tau_i s)}{\prod_j (1+\tau_j s)}$ with the order of the controller as low as possible.

Do the following:

a). Change $G_p(s)$ to its time constant form.

b). Determine the controller terms you would use to satisfy the velocity error requirement K_v and determine the controller gain K.

c). Generate the Bode magnitude plot (bodemag) of $G'_p(s)$, which is the adjusted version of $G_p(s)$ that has taken into account the controller gain and any controller terms you may have already finalized. Use the grid command to show grid on the Bode magnitude plot.

d). Using plot(x,y,'+'); command plot the disturbance rejection and noise rejection conditions given above in the problem statement. Use the same command to plot a cross "(+)" at the crossover frequency point.

e). Re-shape the Bode plot to ensure maximum possible phase margin.

f). Determine the phase lag introduced by the ZOH, by using the formula $\frac{\omega_c T}{2}$.

g). Having fulfilled the gain margin requirement, determine the complete controller.

h). Using matched pole-zero method, calculate the complete discrete time controller $G_c(z)$.

i). Build a Simulink model and obtain the step response and ramp response.

j). Investigate the disturbance rejection and noise rejection capabilities of the system, by injecting a sinusoidal signal of varying frequency at the output of the control system to emulate disturbances and at the input of the control system to emulate the noise.

Solution

Part a). The time constant form of the transfer function is,

$$G_p(s) = \frac{1/2}{(1 + s/4)(1 + s/50)}$$

Part b). To fulfil the velocity error requirement the controller must incorporate a pure integrator. As can be seen the plant transfer function does not have an integrator, hence the controller must have the integrator. The controller must have the following form.

$$G_c(s) = K\frac{\prod_i(1 + \tau_i s)}{s\prod_j(1 + \tau_j s)}$$

The velocity error constant is defined as,

$$K_v = \text{Lim}_{s\to 0} sG_c(s)G_p(s) = K \times \frac{1}{2} = 100.$$

This gives a controller gain of $K = 200$.

Part c&d). Taking into account the gain K, the gain adjusted transfer function $G'_p(s)$ is,

$$G'_p(s) = \frac{100}{(1 + s/4)(1 + s/50)}$$

The Bode magnitude plot is shown in Fig. 5.1. The disturbance rejection condition, the noise rejection condition and the cross over frequency point are plotted in this figure. It can clearly be seen that none of the specified requirements are satisfied.

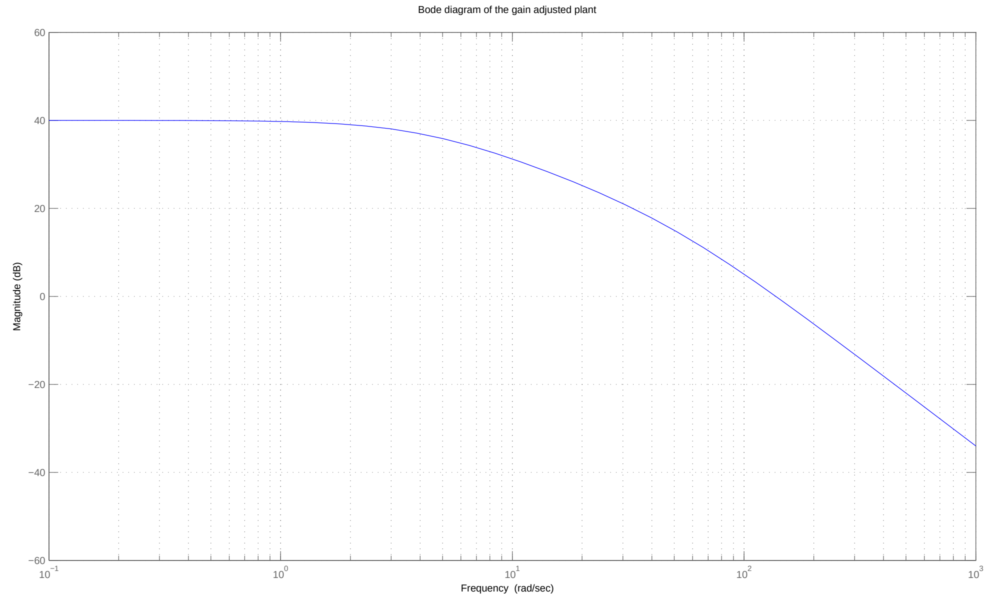


Figure 5.1: The Bode magnitude plot of the original system with new gain.

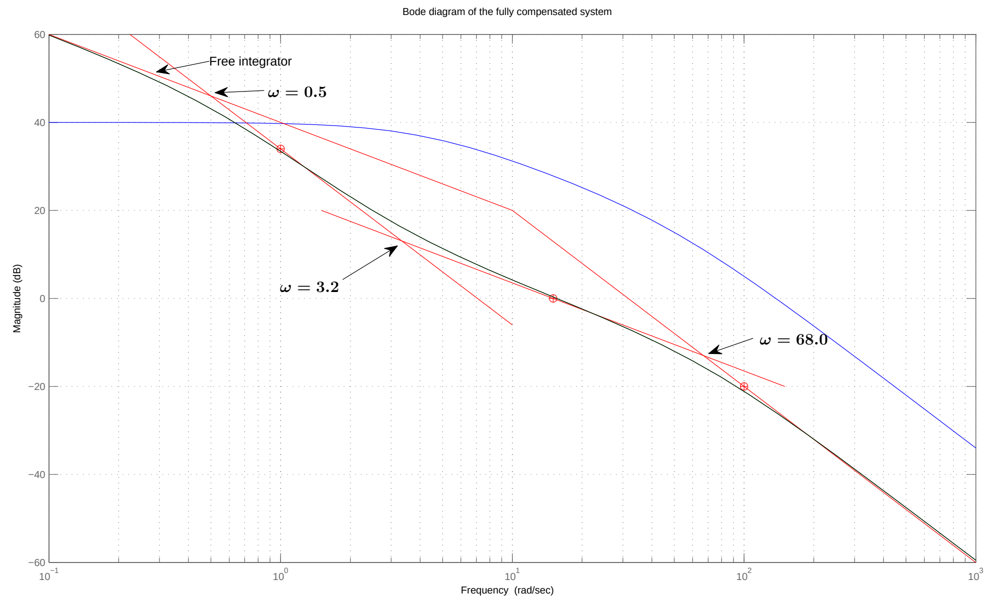


Figure 5.2: The Bode diagram of the fully compensated system

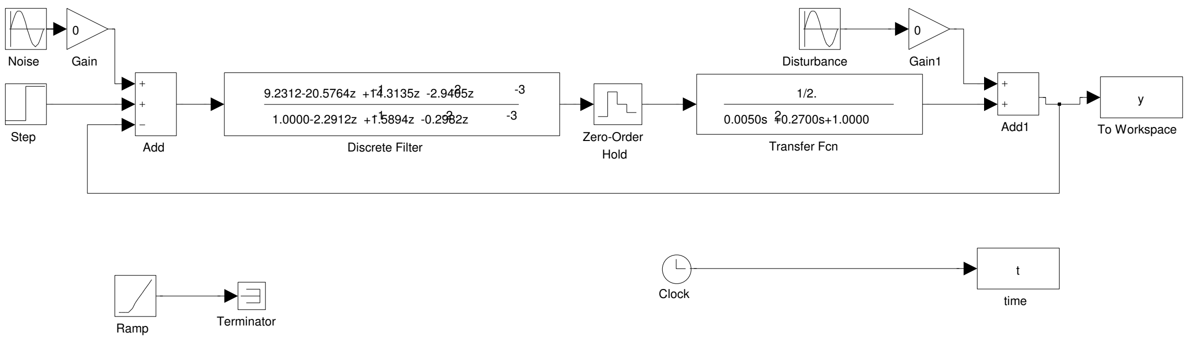


Figure 5.3: The Simulink block diagram

Part e). A reshaped Bode plot is shown in Fig. 5.2. Note the break frequencies shown on the plot. The transfer function that corresponds to the reshaped plot is,

$$G(s) = \frac{100(1 + s/3.2)}{s(1 + s/0.5)(1 + s/68)}$$

Part f). The crossover frequency ω_c of the exact plot (use commands: `conv`, `tf`, `margin`) shown in Fig. 5.2 is 15.5435 rad/s. For a sampling time $T = 0.02$ seconds, the phase lag introduced by the ZOH is,

$$\angle ZOH(j\omega) = -\frac{\omega_c T}{2} \times \frac{180}{\pi} = 8.9053$$

Part g). The phase margin that corresponds to the reshaped Bode plot is 67.3338°. This figure can be obtained using `margin` function of Matlab. After taking into account the phase lag introduced by the ZOH, the final phase margin is, $\phi_m = 67.3338 - 8.9053 = 58.4280$ which is acceptable.

The complete controller can be obtained using $G_c(s)G_p(s) = G(s)$. Therefore,

$$G_c(s)\frac{1/2}{(1 + s/4)(1 + s/50)} = \frac{100(1 + s/3.2)}{s(1 + s/0.5)(1 + s/68)}$$

Hence,

$$G_c(s) = \frac{200(1 + s/3.2)(1 + s/4)(1 + s/50)}{s(1 + s/0.5)(1 + s/68)}$$

Part h). Using matched pole-zero mapping method, the discrete equivalent of $G_c(s)$ can be obtained as

$$G_c(z) = \frac{9.231z^3 - 20.58z^2 + 14.31z - 2.941}{z^3 - 2.291z^2 + 1.589z - 0.2982}$$

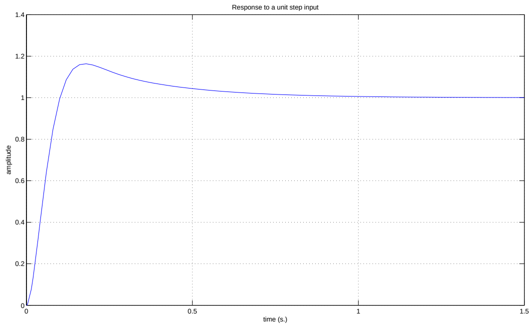
which results in the difference equation,

$$\begin{aligned} m(k) = 2.291m(k-1) - 1.589m(k-2) - 0.2982m(k-3) \\ + 9.231e(k) - 20.58e(k-1) + 14.31e(k-2) - 2.941e(k-3). \end{aligned}$$

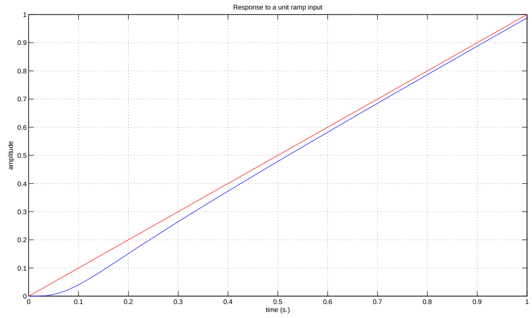
□

Part i). The Simulink block diagram is shown in Fig. 5.3. The step response and the ramp response are shown in Fig. 5.4.

Part j). The Simulink block diagram shown in Fig. 5.3 shows the noise inputs and the disturbance inputs. The disturbance rejection is shown in Fig. 5.6 and noise rejection is shown in Fig. 5.5.



(a)



(b)

Figure 5.4: (a) Step response, (b) Ramp response

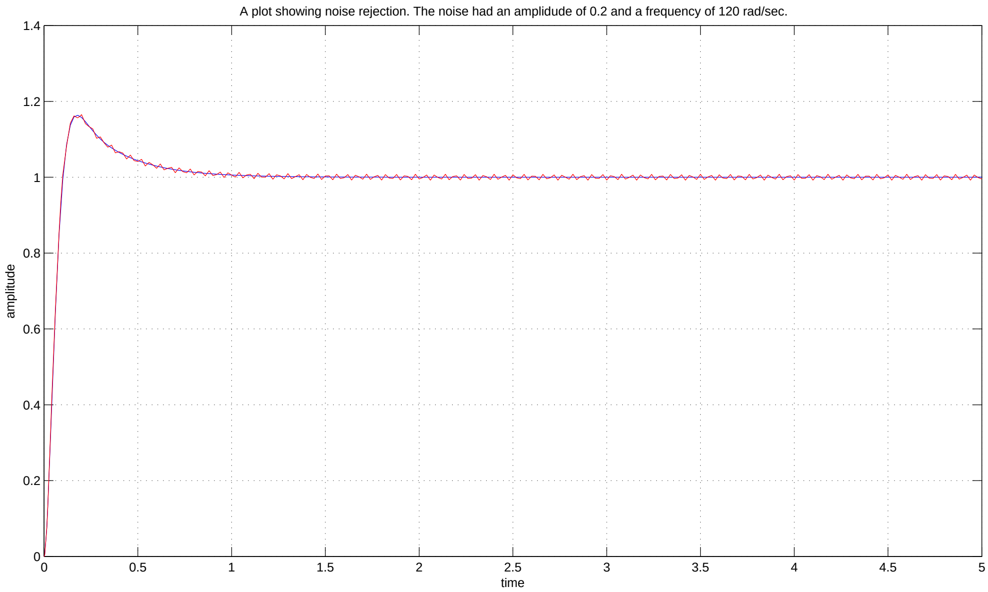


Figure 5.5: The noise rejection. Note that the noise signal has an amplitude of 0.2 and a frequency of 120 rad/sec. The effect of noise is suppressed as expected.

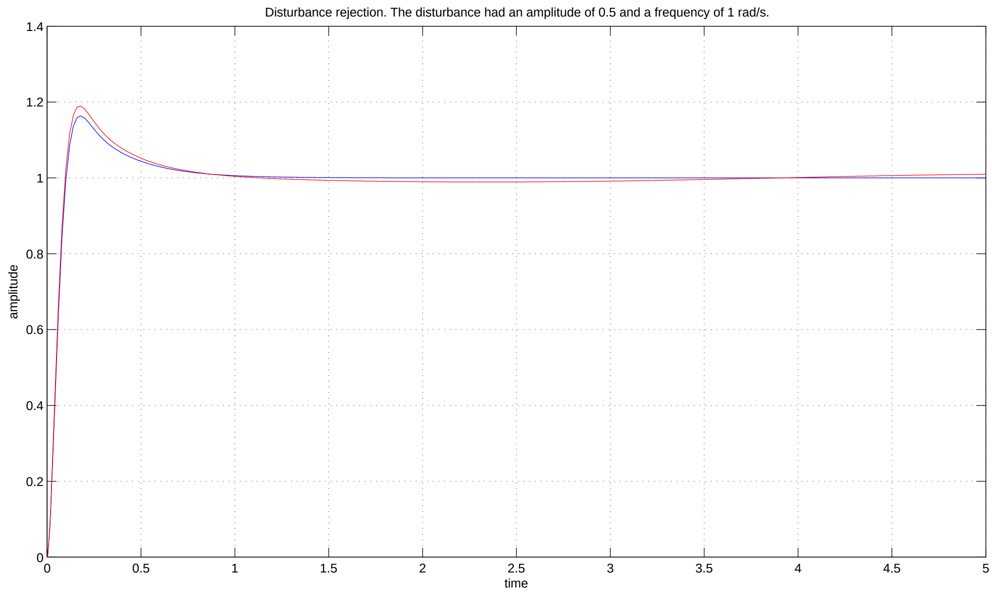


Figure 5.6: The disturbance rejection. Note that the disturbance signal has an amplitude of 0.5 and a frequency of 1 rad/sec. The effect of the disturbance is suppressed as expected.