Modeling and Control of Mechatronic Systems

Exercise 5: Design of Control Systems - Indirect Design in Frequency Domain

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Exercise 5

Design of Control Systems - Indirect Design in Frequency Domain

Note: This question is extracted from "Continuous and Discrete Control Systems" by John Dorsey.

A system plant transfer function is given as,

$$G_p(s) = \frac{100}{(s+4)(s+50)}$$

A controller needs to be designed to satisfy the following conditions;

1. Sample rate of 50 Hz or less.
2. Velocity error constant $K_v > 100$.
3. $\dfrac{|E(j\omega)|}{|R(j\omega)|} \leqslant 0.02$ for $\omega < 1$ rad/s.
4. $\dfrac{|C(j\omega)|}{|R(j\omega)|} \leqslant 0.1$ for $\omega > 100$ rad/s.
5. Crossover frequency ω_c of at least 15 rad/s.
6. Phase margin of at least $50°$.
7. The controller $G_c(s) = K\dfrac{\prod_i (1+\tau_i s)}{\prod_j (1+\tau_j s)}$ with the order of the controller as low as possible.

Do the following:

a). Change $G_p(s)$ to its time constant form.

b). Determine the controller terms you would use to satisfy the velocity error requirement K_v and determine the controller gain K.

c). Generate the Bode magnitude plot (`bodemag`) of $G'_p(s)$, which is the adjusted version of $G_p(s)$ that has taken into account the controller gain and any controller terms you may have already finalized. Use the `grid` command to show grid on the Bode magnitude plot.

d). Using `plot(x,y,'+');` command plot the disturbance rejection and noise rejection conditions given above in the problem statement. Use the same command to plot a cross "(+)" at the crossover frequency point.

e). Re-shape the Bode plot to ensure maximum possible phase margin.

f). Determine the phase lag introduced by the ZOH, by using the formula $\dfrac{\omega_c T}{2}$.

g). Having fulfilled the gain margin requirement, determine the complete controller.

h). Using matched pole-zero method, calculate the complete discrete time controller $G_c(z)$.

i). Build a Simulink model and obtain the step response and ramp response.

j). Investigate the disturbance rejection and noise rejection capabilities of the system, by injecting a sinusoidal signal of varying frequency at the output of the control system to emulate disturbances and at the input of the control system to emulate the noise.