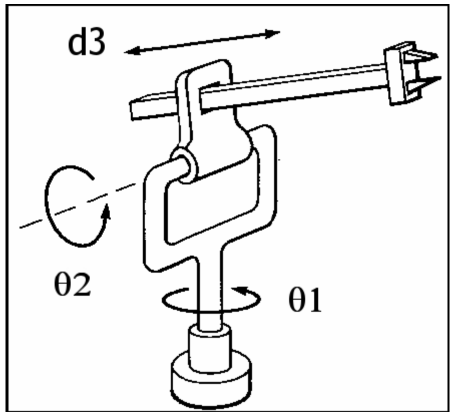
Week 6 – Question 3



(from J.J. Craig)

i	a_i	α_i	d_i	θ_i
1	0	90^o	a_1	θ_1*
2	a_2	-90^o	0	θ_2*
3	0	0	d_3*	0

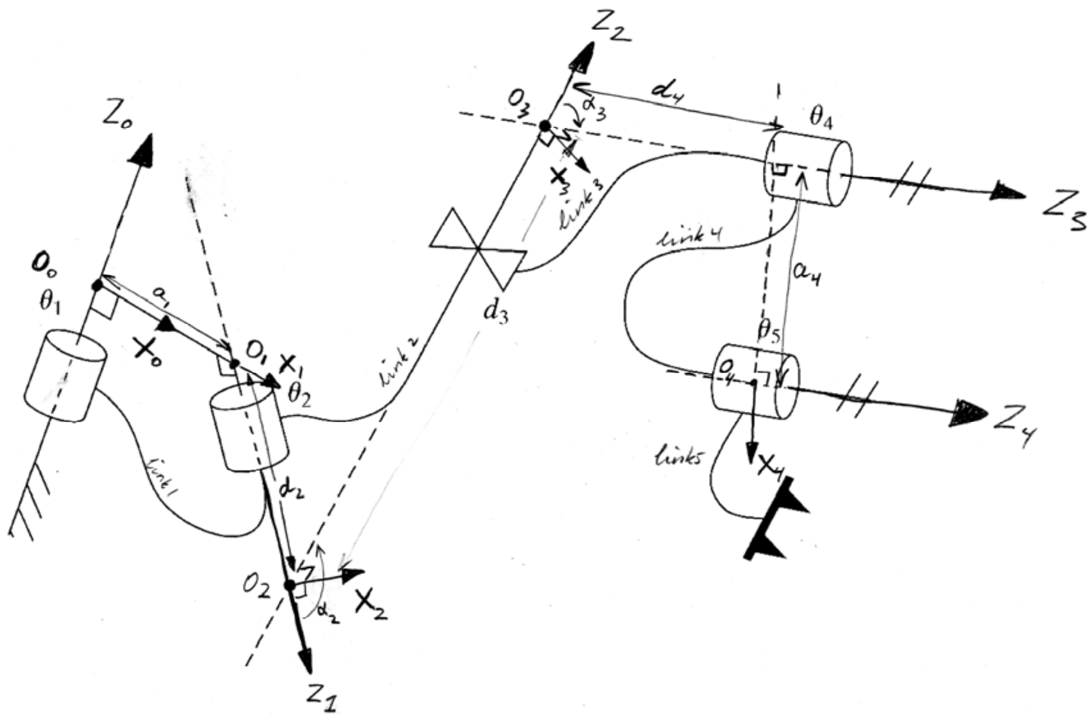
$$ {}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$
\begin{aligned}
{}^0_3T &= {}^0_1T\,{}^1_2T\,{}^2_3T \\
&= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & c_1c_2a_2 + c_1s_2d_3 \\ s_1c_2 & -c_1 & s_1s_2 & s_1c_2a_2 + s_1s_2d_3 \\ s_2 & 0 & -c_2 & s_2a_2 - c_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Week 6 – Question 4



	a_i	d_i	α_i	θ_i
1	a_1	0	α_1	θ_1^*
2	0	d_2	α_2	θ_2^*
3	0	d_3^*	α_3	θ_3
4	a_4	d_4	0	θ_4^*
5	0	0	0	θ_5^*

* denotes that it is a joint variable

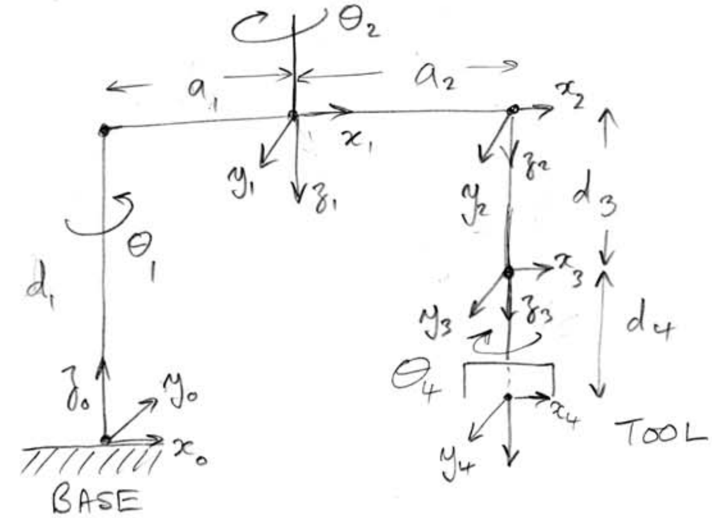
Week 6 – Question 5

i	a_i	α_i	d_i	θ_i
1	l	-90^o	0	θ_1*
2	l	90^o	0	θ_2*
3	l	-90^o	0	θ_3*

$$
\begin{aligned}
{}^0_1T &= \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^1_2T &= \begin{bmatrix} c_2 & -s_2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\Rightarrow {}^0_2T &= {}^0_1T\,{}^1_2T = \begin{bmatrix} c_1c_2 & -c_1s_2 & -s_1 & c_1 \\ s_1c_2 & -s_1s_2 & c_1 & s_1 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^2_3T &= \begin{bmatrix} c_3 & -s_3 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\Rightarrow {}^0_3T &= {}^0_2T\,{}^2_3T = \begin{bmatrix} c_1c_2c_3 - s_1s_3 & -c_1c_2s_3 - s_1c_3 & c_1s_2 & c_1c_2 + c_1 \\ s_1c_2c_3 + c_1s_3 & -s_1c_2s_3 + c_1c_3 & s_1s_2 & s_1c_2 + s_1 \\ -s_2c_3 & s_2s_3 & c_2 & -s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^3_4T &= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\Rightarrow {}^0_4T &= {}^0_3T\,{}^3_4T = \boxed{\begin{bmatrix} c_1c_2c_3 - s_1s_3 & -c_1s_2 & -c_1c_2s_3 - s_1c_3 & c_1c_2c_3 - s_1s_3 + c_1c_2 + c_1 \\ s_1c_2c_3 + c_1s_3 & -s_1s_2 & -s_1c_2s_3 + c_1c_3 & s_1c_2c_3 + c_1s_3 + s_1c_2 + s_1 \\ -s_2c_3 & -c_2 & s_2s_3 & -s_2c_3 - s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
\end{aligned}
$$

Week 6 – Questions 6 and 7

Consider the SCARA robot:



There are four transformations needed to get from BASE to TOOL.

$$\therefore \quad {}^{0}p = {}^{0}_{1}T \; {}^{1}_{2}T \; {}^{2}_{3}T \; {}^{3}_{4}T \cdot {}^{4}p$$

using previous notation
NOT the same as SCHILLING

This time we have to take into account movement in the origins and allow for each set of axes to be rearranged. So for the ${}^{0}_{1}T$, we move a_1C1 in the x direction, a_1S1 in the y direction and z. However the new set of axes are upside down so that $x \to x$, $y \to -y$ and $z \to -z$

So $\begin{bmatrix} C1 & -S1 & 0 \\ S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ becomes $\begin{bmatrix} C1 & S1 & 0 \\ S1 & -C1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and adding

the translation gives

$${}^{0}_{1}T = \begin{bmatrix} C1 & S1 & 0 & a_1C1 \\ S1 & -C1 & 0 & a_1S1 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

likewise
$$ {}^{1}_{2}T = \begin{bmatrix} C2 & -S2 & 0 & a_2C2 \\ S2 & C2 & 0 & a_2S2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
with $x \rightarrow x$
$y \rightarrow y$
$z \rightarrow z$

and with d_3 becoming a variable so that the gripper can be moved in the vertical direction.

$$ {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Finally
$$ {}^{3}_{4}T = \begin{bmatrix} C4 & -S4 & 0 & 0 \\ S4 & C4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
with the Tool point remaining on the axis of z_4

Multiplying together to get ${}^{0}_{4}T$ gives
$$ \begin{bmatrix} C(1-2-4) & S(1-2-4) & 0 & a_1C1 + a_2C(1-2) \\ S(1-2-4) & -C(1-2-4) & 0 & a_1S1 + a_2S(1-2) \\ 0 & 0 & -1 & d_1 - d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$
to give the full forward kinematic solution.

[SCHILLING P 71]

SCARA EXAMPLE 1

$$
{}^{0}_{4}T = \begin{bmatrix} \cos(45-45-90) & \sin(45-45-90) & 0 & \cos 45 + \cos 0 \\ \sin(45-45-90) & -\cos(45-45-90) & 0 & \sin 45 + \sin 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\therefore \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0.707 \\ -1 & 0 & 0 & 0.707 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.707 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

SCARA EXAMPLE 2

$$
{}^{0}_{4}T = \begin{bmatrix} \cos(-50) & \sin(-50) & 0 & 1.5\cos 30 + 1.7\cos(-50) \\ \sin(-50) & -\cos(-50 & 0 & 1.5\sin 30 + 1.7\cos(-50) \\ 0 & 0 & -1 & 3.4 - 2.6 - 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\therefore \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.643 & -0.766 & 0 & 2.39 \\ -0.766 & -0.643 & 0 & 1.843 \\ 0 & 0 & -1 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.777 \\ 1.328 \\ 0.7 \\ 1 \end{bmatrix}
$$

EXAMPLE 2 HARDER. SCARA.

FORWARD

$d_1 = 3.4,\ a_1 = 1.5,\ a_2 = 1.7,\ d_3 = 2.6,\ d_4 = 0.1$

$\theta_1 = 30^\circ,\ \theta_2 = 70^\circ,\ \theta_4 = 0.$

find the position of the EE wrt base $(0,0,0)$ if the EE reaches out 0.8 in the y direction.

$$ {}^0_4T = \begin{bmatrix} \cos(-50) & \sin(-50) & 0 & 1.5\cos 30 + 1.7\cos(-50) \\ \sin(-50) & -\cancel{\cos}(-50) & 0 & 1.5\sin 30 + 1.7\cancel{\cos}(-50) \\ 0 & 0 & -1 & 3.4 - 2.6 - 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.643 & -0.766 & 0 & 2.39 \\ -0.766 & -0.643 & 0 & 1.843 \\ 0 & 0 & -1 & 0.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8 \\ 0 \\ 1 \end{bmatrix} $$

$$ = \begin{bmatrix} 1.777 \\ 1.328 \\ 0.7 \\ 1 \end{bmatrix} $$