Modeling and Control of Mechatronic Systems

Exercise 3: Mathematical Modeling and Simulation of an Anti-lock Braking System

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Exercise 3

Modeling and Control of an Anti-lock Braking System

The aim of this exercise is to model an anti-lock braking system. The system components include,

1. A pilot valve that needs to be actuated by the controller you design. The pilot valuve will receive a voltage input and will generate a brake pressure rate. Its dynamics can be described by the transfer function,

$$G_1(s) = \frac{15000}{(0.01s + 1)} \tag{3.1}$$

   The break pressure can be obtained by integrating the brake pressure rate. The brake pressure has un upper limit of 32400 N/m^2. Its lower limit is 0. The brake pressure can be converted to brake force by a constant 0.0028863. The brake force can be converted to brake torque by a constant 80, ending up with brake torque in Nm.

2. The coefficient of friction between the tyre and the road surface is a non-linear function of slip. Slip is defined as

$$\mathbf{s} = 1 - \frac{\omega_w}{\omega_v} \tag{3.2}$$

   in which ω_w is the wheel speed in rad/sec, and ω_v is the wheel speed calculated, in rad/sec, using the actual speed of the vehicle. The coefficient of friction values are given in the look up table below in Table. 3.1.

3. Weight of the vehicle is 2000 kg and has 4 wheels.

4. The wheel radius is 0.28 m.

5. The moment of inertia of each wheel about its axis is 2 kg m^2.

6. The maximum vehicle speed is 100 m/s.

The controller is required to adjust the pilot valve position to ensure the maximum coefficient of friction at the road-tyre surface. Do the following:

a). Develop a control strategy. That is, what will you measure?, what will you specify as the control input to achieve the control objective of maximum coefficient of friction.

Table 3.1: μ-slip data

Slip	μ
0	0
0.0500	0.4000
0.1000	0.8000
0.1500	0.9700
0.2000	1.0000
0.2500	0.9800
0.3000	0.9600
0.3500	0.9400
0.4000	0.9200
0.4500	0.9000
0.5000	0.8800
0.5500	0.8550
0.6000	0.8300
0.6500	0.8100
0.7000	0.7900
0.7500	0.7700
0.8000	0.7500
0.8500	0.7300
0.9000	0.7200
0.9500	0.7100
1.0000	0.7000

b). Derive the equilibrium equation for the rotational dynamics of a single wheel.

c). Devise a method to determine the vehicle speed using its inertial data, i.e. considering its acceleration.

d). Device a method to determine the wheel speed using its inertial data.

e). Complete a block diagram to achieve your control objectives.

f). Design a P controller and a PI controller to achieve the control objective.

g). Plot the stopping distance when the controller is in place and when the controller is not in place and do a comparison.

Solution

Part a). The aim of the control system development is to ensure the shortest stopping distance for a motor vehicle that has activated its brakes. This can only be ensured by maintaining the highest coefficient of friction at the road wheels. It has been found that when the wheels are locked (i.e. slip $\mathbf{s}$=1), the coefficient of friction is lower than when the wheels maintain a slip $0 < \mathbf{s} < 1$ and hence the term anti-lock braking systems. Therefore our control strategy must be to find out off-line, the slip value that gives us the maximum coefficient of friction and then to make sure that the controller maintains the slip at that optimum value. In this exercise, the maximum

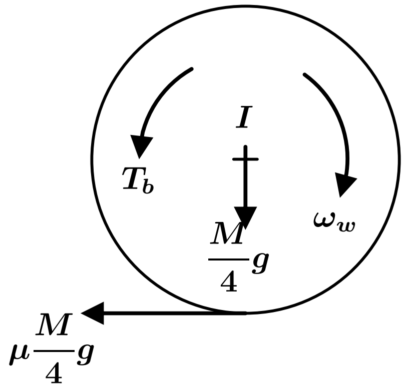


Figure 3.1: The rotational dynamics of the road wheel.

coefficient of friction occurs at a slip value of **s**=0.2. Therefore, the controlled quantity is **s** and the measured quantity is also **s**.

An extension of this is known as anti-skid braking systems, in which the aim is to maintain the braking forces at the four wheels to be identical and thereby avoiding a turning moment around a vertical axis that passes through the centre of gravity of the vehicle.

Part b). Let the control effort be $m(k)$ and the braking torque be T_b. Then,

$$\frac{T_b(s)}{M(s)} = \frac{15000}{(0.01s+1)} \times \frac{1}{s} \times 0.0028863 \times 80$$

Given that the aim is to control the slip **s** and if the slip error is $e(t)$, then

$$\frac{M(s)}{E(s)} = G_c(s)$$

in which $G_c(s)$ is the controller to be designed. Let $r(t)$=0.2 be the reference input and let $c(t)$ be the controlled slip. Then,

$$E(s) = R(s) - C(s)$$

while $R(s)$ is a constant, $c(t)$ is given as below,

$$c(t) = \mathbf{s} = 1 - \frac{\omega_w}{\omega_v}$$

Determining ω_w is as follows.

The angular speed ω_w refers to the actual angular speed of the wheel. Hence we must consider the rotational dynamics of the wheel. This is shown in Fig. 3.1. For a rotational inertia of I,

$$I\dot{\omega}_w = \mu\frac{Mg}{4}R - T_b$$

where R is the wheel radius. The coefficient of friction is a function of slip **s** as given in Table. 2.

$$\mu = f(\mathbf{s})$$

A block diagram that shows the implementation of the rotational dynamics is shown in Fig. 3.2.

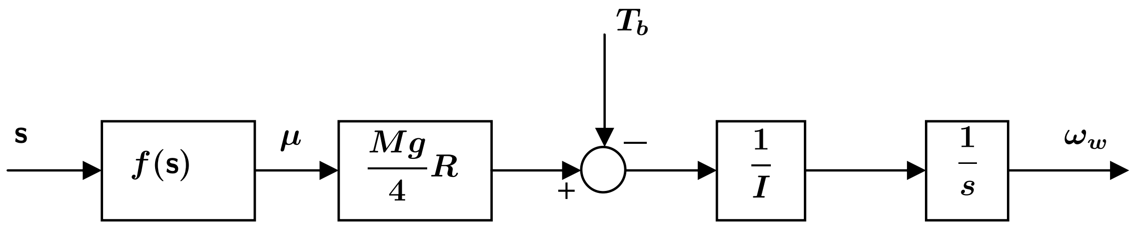


Figure 3.2: The block diagram for rotational dynamics

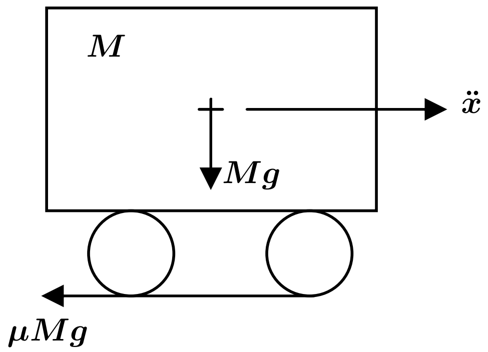


Figure 3.3: The vehicle dynamics to determine ω_v.

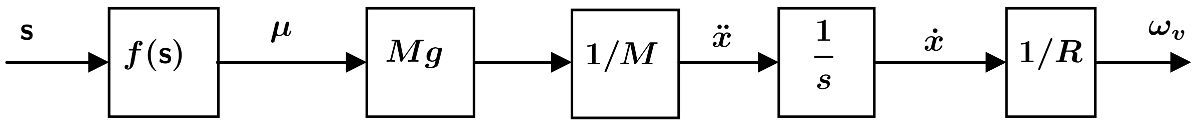


Figure 3.4: The block diagram showing the vehicle dynamics.

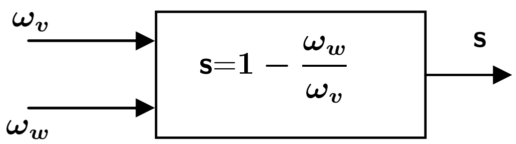


Figure 3.5: Slip calculation

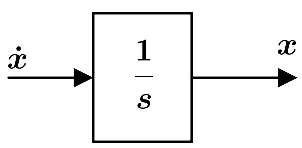
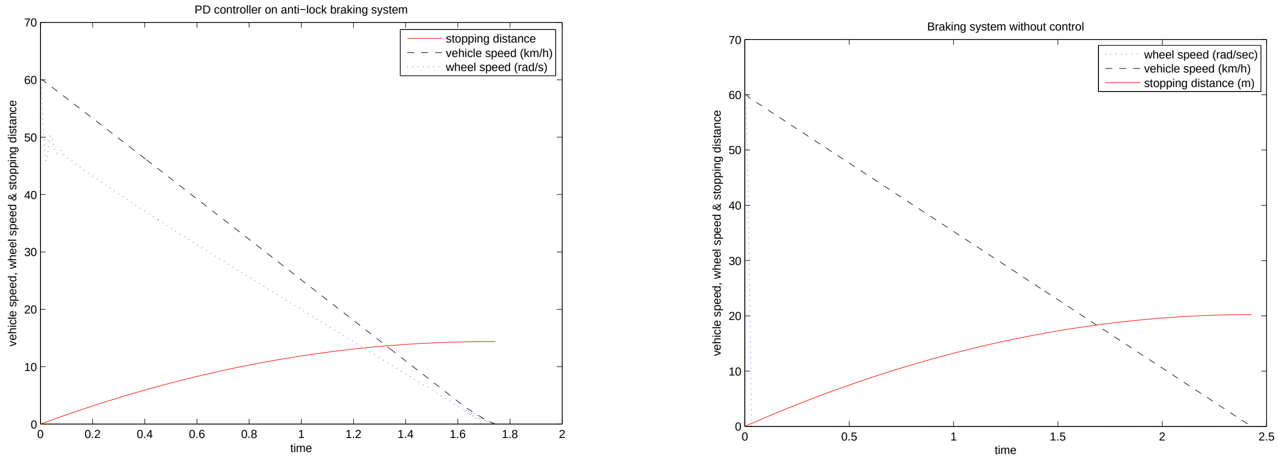


Figure 3.6: Stopping distance determination.



(a) (b)

Figure 3.7: (a). With ABS and (b) Without ABS

Part c) & d). Determining wheel speed ω_v using vehicle dynamics is done as follows.

This quantity is the apparent wheel velocity determined based on the vehicle motion. Hence, we must consider vehicle dynamics. This is shown in Fig. 3.3. The equation of motion is,

$$M\ddot{x} = -\mu M g = -f(\mathsf{s})Mg$$

having determined $\dot{x}$ from the above equation, the apparent wheel speed ω_v can be determined using,

$$\omega_v = \frac{\dot{x}}{R}$$

A block diagram showing vehicle dynamics leading up to the apparent wheel speed ω_v is shown in Fig. 3.4.

The slip **s** can now be determined as follows. Having determined ω_w and ω_v, the slip **s** can be calculated using a block diagram as shown in Fig. 3.5. Note that in a Simulink model, precautions must be taken to avoid the divide by zero at the time the vehicle stops. The stopping distance x can be calculated by integrating the vehicle's linear velocity $\dot{x}$. This is shown in Fig. 3.6.

Part e). For you to do!

Part f). The two plots In Fig. 3.7 shows the difference between the two cases.