BBM 205. Spring 2015 £xam 2

(4 points)

1. Let P(n) be the statement that $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for the positive in teger n.

- a) What is the stetement P(1)?
- b) Show that P(1) is true, completing the basis step of the proof.
- c) What is the inductive hypothesis?
- d) Complete the inductive step.

(2 points)
2. Prove that 2 divides n²+n whenever n is a positive integer. (using induction)

(1 paint)
3. What is the cardinality of each of these sets?

a) Ø b) {Ø} c) {Ø, {Ø}} d) {Ø, {Ø}, {Ø}, {Ø}, {Ø}}

(2 points)
4. Determine whether each of these statements is true or false.
a) $x \in \{x\}$ b) $\{x\} \subseteq \{x\}$ c) $\{x\} \in \{x\}$ 1) $\{x\} \in \{\{x\}\}$ e) $\emptyset \subseteq \{x\}$ f) $\emptyset \in \{x\}$

(2 points)
5. Use the Euclidean algorithm to find gcd (1529, 14039).

(3 points)

6. Solve the recurrence relation with the given initial conditions: $a_n = 2a_{n-1} + 8a_{n-2}$, $a_0 = 4$, $a_1 = 10$

- (3 points)
- ta) Find a recurrence relation and initial conditions for cn, the minimum number of moves in which the n-disk Tower of Hanoi puzzle can be solved.
 - b) Solve this recurrence relation.

(3 points)

8. Let f_n be the nth Fibonacci number. Show that $f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ when n is a positive integer.