BBM 205

Problem Set 3:

Proof by Induction

1. Use induction to show that $5^n - 1$ is divisible by 4 for $n \geq 1$.
2. Use induction to show that $2^n \geq n^2$ for $n \geq 4$.
3. Use induction to show that if $S(n)$ is the sum of integers $1, \ldots, n$, then $S(n) = n(n+1)/2$.
4. Use induction to show that $n! \geq 2^{n-1}$ for $n \geq 1$.
5. Use induction to show that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$;
6. Use induction to show that $\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$;
7. Use induction to show that n straight lines in the plane divide the plane into $(n^2+n+2)/2$ regions. Assume that no two lines are parallel and no three lines have a common point.
8. Use induction to show that the regions in the question above can be colored red and green so that no two regions that share an edge have the same color.
9. (Spring 2014) Use induction to show that postage of 24 kuruş or more can be achieved by using only 5-kuruş and 7-kuruş stamps.
10. (Spring 2014) Use induction to show that $7^n - 1$ is divisible by 6.
11. (Spring 2014) Use induction to show that if $r \neq 1$, then
$$a + ar^1 + ar^2 + \cdots + ar^n = \frac{a(r^{n+1} - 1)}{r - 1} \qquad \text{for } n \geq 1.$$
12. (Spring 2014) Let f_i be the ith Fibonacci number. Use induction to prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.
13. (Spring 2014) Use induction to prove that if n is a positive integer, then 133 divides $11^{n+1} + 12^{2n-1}$.

14. (Spring 2015) Let $P(n)$ be the statement that

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

where n is an integer greater than 1. Show that $P(n)$ is true for all $n \geq 2$ using induction by following the steps below.

(a) Show that $P(2)$ is true.

(b) What is the inductive hypothesis?

(c) Complete the inductive step.

15. (Spring 2015) Prove by using induction that 2 divides $n^2 + n$ whenever n is a positive integer.

16. (Spring 2015) Let f_i be the ith Fibonacci number. Use induction to prove that $f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$ when n is a positive integer.

17. (Fall 2106) Use induction to show that $2^n \geq n^3$ for $n \geq 10$.