BBM 205. Quiz 9

18.12.2018

1. How many bitstrings of length ten both begin and end with a 1?

1 . . . . . 1 2^8 choices for the remaining 8 entries

2. How many positive integers less than 1000 are divisible by 7 but not by 11? (Use one of the cardinality rules in your answer.)

$\lfloor \frac{999}{7} \rfloor = 142 = |A|$ where A = set of all numbers $x < 1000$ such that $7 | x$

$\lfloor \frac{999}{11} \rfloor = 90 = |B|$ B = set of all numbers $x < 1000$ such that $11 | x$.

$\lfloor \frac{999}{77} \rfloor = 12 = |A \cap B|$

We need to calculate $|A - B| = |A| - |A \cap B| = 142 - 12$.

3. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

a) How many balls must she select to be sure of having at least three balls of the same color?

$\lceil \frac{N}{2} \rceil \geq 3$. So, $N = 5$.

By using pigeonhole principle, we need to find smallest N so that

b) How many balls must she select to be sure of having at least three blue balls?

In the worst case, we draw all of the red balls until seeing three blue balls, that makes $10 + 3 = 13$ balls.