Hacettepe University

MAT 124 MATHEMATICS II Final Examination	
Acad.Year: *2019-2020* Semester : *Spring* Date : *24.06.2020* Time : *10:00* Duration : *120 min.*	Name : Surname : Number : Total 100 points

1. (20)	2. (20)	3. (20)	4. (20)	5. (20)			

1. Let $F(x,y,z,u,v) = x^2+xy+yz^2+u^3+v-7 = 0$ and $G(x,y,z,u,v) = xyz+x+y+u^2-v^2-5 = 0$. Show that z and u can be solved as functions of x, y and v near the point $P_0 = (1,1,2,1,0)$. Compute $\frac{\partial z}{\partial x}|_{P_0}$.

2. Sketch the domain of the integral

$$\int_0^{\frac{\pi}{2}} dy \int_y^{\frac{\pi}{2}} \frac{\sin x}{x} dx$$

and evaluate it.

3. Convert

$$\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz dz dx dy$$

into an integral in cylindrical coordinates and evaluate it.

4. Let $\mathcal{C}$ be the path from $(0,0)$ to $(0,1)$ along the parabola $y = x^2$ from $(0,0)$ to $(1,1)$ and the line segment from $(1,1)$ to $(0,1)$. Then evaluate the integral

$$\int_{\mathcal{C}} F.dr$$

where $F(x,y,z) = 2xe^y i + 2x^2 e^{y^2} j$.

5. Let $F(x,y,z) = yzi + xzj + (xy+2z)k$.

(a) Show that the vector field F is conservative by finding a function f such that $F = \nabla f$.

(b) Use part (a) to evaluate

$$\int_{\mathcal{C}} F.dr$$

where $\mathcal{C}$ is any path from $(1,0,-2)$ to $(4,6,3)$.