KINETIC ENERGY & WORK

Any type of motion Requires Energy. Flying across the ocean requires it. Lifting material to the top Floor of a Building or to an orbiting space station Requires it. We spend a huge amount of money to acquire and use energy. Wars are going on Because of Energy resources. Wars have Been Ended Because of a sudden, overpowering use of energy by one side.

So, we know what it is about — but what does the term ENERGY really mean?

- × 6 Aug 1945, Hiroshima
  15 kilotons of TNT
  (63 TJ)
- × 9 Aug 1945, Nagasaki
  20 kilotons of TNT
  (84 TJ)

1 ton TNT $= 4.184 \times 10^{9}$ Joule

CERN : HIGGS BOSON
4 July 2012
14 TeV
$1\ eV = 1.602 \times 10^{-19}\ J$
$14\ TeV = 2.243 \times 10^{-6}\ J = 5.361 \times 10^{-16}$ ton TNT
(1 barn $= 10^{-23}\ m^2$)

WHAT IS ENERGY?

Technically, energy is a scalar quantity associated with the state (or condition) of one or more objects.

→ Energy is a number that we associate with a system of one or more objects. If a force changes one of the objects by, say, making it move, Then the energy number changes.

If the process/method by which we assign Energy is planned carefully, the numbers can Be used to predict the outcomes of experiments and to Build machines.

→ "Wonderful" property of the Universe

Energy can Be transformed from one type to another and transferred from one object to another. But the total amount is always the same → ENERGY IS CONSERVED

No exception to this principle of energy conservation has ever been found.

(It's a bit like currency)

We'll first focus on kinetic energy type and only on one way in which energy can be transformed (work).

Kinetic Energy

Kinetic Energy K is energy associated with the state of motion of an object. The faster the object moves, the greater is its kinetic energy. If the object is not moving, i.e., stationary $\longrightarrow$ its kinetic energy is zero.

For an object of mass m and speed v ($\ll c$):

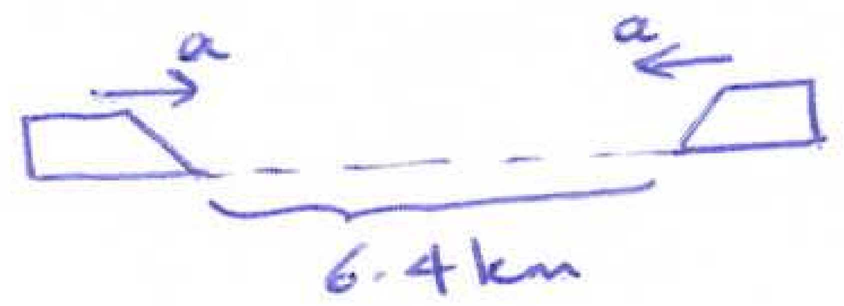
$$K = \frac{1}{2} m v^2$$

for example, a bird with mass $m = 3.0\,kg$, flying with $2.0\,m/s$ has a kinetic energy of $6.0\,kg\,m^2/s^2$ $\Rightarrow$ we associate that number with the duck's motion.

SI unit of K (and every other type of energy) is: Joule (J)

$$1 \text{ Joule} = 1\,J = 1\,kg\,m^2/s^2$$

Ex: Train Crash (1896, Texas, William Crash)



Assume: $w = 1.2 \times 10^6\,N$
$a = 0.26\,m/s^2$ (constant).

* What is the total kinetic energy before crash?

First, we need to calculate their speeds just before collision.

Derivation $v^2 = v_0^2 + 2a(x - x_0) \longrightarrow v_0 = 0, \quad x - x_0 = 3.2 \times 10^3\,m$

$$v^2 = 0 + 2\,(0.26\,m/s^2)\,(3.2 \times 10^3\,m) \Rightarrow v = 40.8\,m/s$$
$\sim 150\,km/h$

$$v^2 = 1664\,m^2/s^2$$

$$m = \frac{1.2 \times 10^6\,N}{9.8\,m/s^2} = 1.22 \times 10^5\,kg$$

$$\rightarrow K = 2\left(\frac{1}{2} m v^2\right) = (1.22 \times 10^5\,kg)\,(40.8\,m/s)^2 = 2.0 \times 10^8\,J$$

$\sim$ a bomb

Because, there are two trains

a: const

$\rightarrow a = a_{avg} = \frac{v - v_0}{t - 0} \rightarrow v = v_0 + at$ (1)

$x = x_0 + v_{avg} t$

$v_{avg} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(v_0 + v_0 + at)$
$= v_0 + \frac{1}{2} at$

$x = x_0 + (v_0 + \frac{1}{2} at) t$

$x - x_0 = v_0 t + \frac{1}{2} a t^2$ (2)

eliminate t:

(1): $\frac{v - v_0}{a} = t$

$\Rightarrow$ (2): $x - x_0 = v_0 \left(\frac{v - v_0}{a}\right) + \frac{1}{2} a \left(\frac{v - v_0}{a}\right)^2 \Rightarrow 2a(x - x_0) = v^2 - v_0^2 \Rightarrow \boxed{v^2 = v_0^2 + 2a(x - x_0)}$ (3)

WORK

If you accelerate an object to a higher speed by applying a force, you increase its kinetic energy. Similarly, if you decelerate the object to a lesser speed by applying a force, you decrease its kinetic energy.

We account these changes by saying that your force has transferred energy to the object from yourself or from the object to yourself. In such an energy transfer via a force, work W is said to be done on the object by the force.

> Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Work $\Rightarrow$ Transferred Energy $\Rightarrow$ same type as energy
$\Rightarrow$ same unit & scalar quantity.

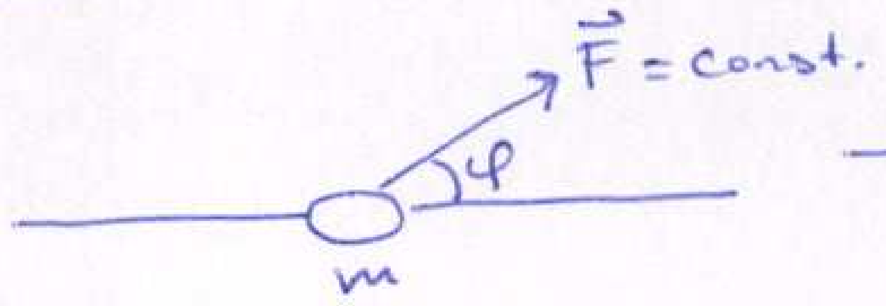
It's not like a transfer of an object/material like a flow of water, but more like an electronic transfer of money between two bank accounts $\rightarrow$ nothing material goes across the accounts.

Even though we are using the word "work", it mustn't be understood in the common way. For instance, pushing a wall - from physical point of view - doesn't contribute a change in the kinetic energy of the wall (i.e., it doesn't start moving/gain speed) therefore the net work done on the wall by us is zero (even though our body tells the opposite!).

WORK & KINETIC ENERGY

Finding an expression FOR WORK

Consider a Bead on a frictionless wire:



$\rightarrow \vec{a}$ along the wire

$F_x = m . a_x$

As the bead moves through a displacement $\vec{d}$, the Force changes the bead's velocity from an initial V_0 to some other value $\vec{v}$.

$\vec{F} =$ const $\rightarrow \vec{a}$ is also constant.

$$\Rightarrow v^2 = V_0^2 + 2a_x d \rightarrow a_x d = \frac{v^2 - V_0^2}{2}$$

$$\frac{F_x}{m} . d = \frac{1}{2}(v^2 - V_0^2) \Rightarrow \boxed{F_x . d = \underbrace{\frac{1}{2}mv^2}_{K_f} - \underbrace{\frac{1}{2}mV_0^2}_{K_i}}$$

$$\Rightarrow W = F_x d$$

$$W = F . \cos\varphi . d = Fd\cos\varphi = \vec{F} \cdot \vec{d}$$

CAUTION! 1) F must be a constant* Force. (no change in magnitude or direction)

*Later, we'll deal with those kind of situations where the force changes as well.

2) The object must be particle like (– "rigid")

$0 \leq \varphi < \frac{\pi}{2}$: $\vec{F}$ has an active component along $\vec{d} \Rightarrow W > 0$

$\pi/2 < \varphi < \pi$ $\quad \vec{F}$ " " " " opposite of $\vec{d} \Rightarrow W < 0$

Units of Work

$[W] = [K] = \text{Joule}$

Also: $[W] = [F \cdot d] = Nm$

$\Rightarrow 1J = 1 kg\, m^2/s^2 = 1 Nm$

Net Work done by Several Forces:

$$W = \left(\sum_i \vec{F}_i\right) \cdot \vec{d} = \vec{F}_{net} \cdot \vec{d}$$

Work – Kinetic Energy Theorem:

$$\Delta K = K_f - K_i = W$$

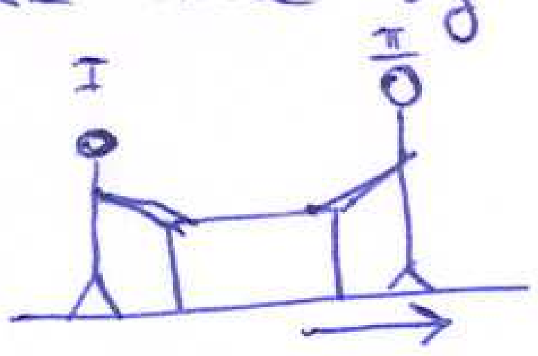
(Change in the Kinetic Energy of the particle) = (Net work done on the particle)

We can also write:

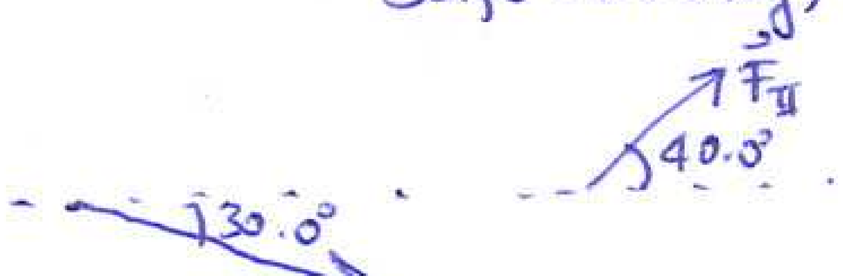
$$K_f = K_i + W$$

(Kinetic Energy after the Work is done) = (Kinetic energy before the net work) + (the net work done)

Ex: Work done by two constant Forces



Safe : 225 kg, initially static, $\vec{d} = 8.50m$ displacement



The magnitudes and directions of the forces do not change as the safe moves $\Rightarrow$ const. $\vec{a}$ and the floor & safe make frictionless contact.

a) What is the net work done on the safe by forces $\vec{F}_1$ and $\vec{F}_2$ during the displacement $\vec{d}$?

$W = \vec{F} \cdot \vec{d} \Rightarrow W = Fd\cos\varphi$

$W_1 = F_1 d \cos\varphi_1 = (12.0N)(8.50m)(\cos 30.0°)$
$= 88.33J$

$W_2 = F_2 d \cos\varphi_2 = (10.0N)(8.50m)(\cos 40.0°)$
$= 65.11J$

$\Rightarrow$ Net work: $W = W_I + W_{II} = 88.33J + 65.11J$
$= 153.4J \simeq 153J$

(Example, cont'd)

b) During the displacement, what is the work W_g done on the safe by the gravitational force $\vec{F}_g$ and what is the work W_N done on the safe by the normal force $\vec{F}_N$ from the floor?

$$W_g = mgd\cos 90^\circ = mgd(0) = 0$$

$$W_N = F_N d\cos 90^\circ = F_N d(0) = 0$$

c) Speed v_f at the end of the 8.50m displacement?

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}\underbrace{mv_i^2}_{0}$$

1st way: $$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(153.4\,J)}{225\,kg}} = 1.17\ m/s$$

2nd way: $$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = \frac{F}{m} = \frac{(12\cos 30 + 10\cos 40)N}{225\,kg} = \frac{18.053\,N}{225\,kg} \cong 0.080\ m/s^2$$

$$v^2 = 2a \cdot 8.5m = 1.364\ m^2/s^2$$

$$\Rightarrow v = 1.168\ m/s$$

Ex: $\vec{d} = (-3.0m)\hat{i}$

$\vec{F}_{wind} = (2.0N)\hat{i} + (-6.0N)\hat{j}$

$\overleftarrow{\vec{d}}$

The crate is initially sliding in the negative x-direction and a wind is blowing in the given vectorial form.

a) How much work does this force do on the crate during the displacement?

$$W = \vec{F}\cdot\vec{d} = \left[(2.0N)\hat{i} + (-6.0N)\hat{j}\right]\cdot\left[(-3.0m)\hat{i}\right]$$

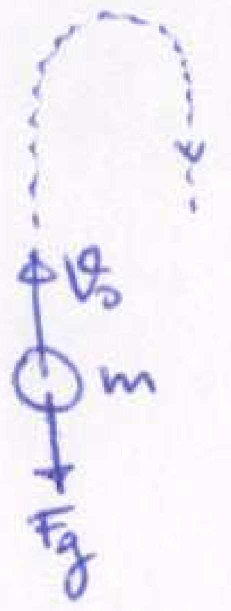
$$= (2.0N)(-3.0m)\underbrace{\hat{i}\cdot\hat{i}}_{1} + (-6.0N)(-3.0m)\underbrace{\hat{j}\cdot\hat{i}}_{0}$$

$= -6.0J \Rightarrow$ The force does a negative 6.0J of work on the crate, transferring 6.0J of energy from the kinetic energy of the crate. (IT __SLOWS__ THE CRATE)

b) If the crate has a kinetic energy of 10J at the beginning of displacement $\vec{d}$, what is its kinetic energy at the end of $\vec{d}$?

$K_f = K_i + W = 10J + (-6.0)J = 4.0J \rightarrow 10J > 4J \Rightarrow$ Less kinetic energy means that the crate has been slowed.

Work done by the Gravitational Force



When an object of mass m is thrown upwards, with an initial speed $v_0 \longrightarrow K_i = \frac{1}{2} m v_0^2$. As it Rises, it slows down due to the Gravitational Force $\vec{F}_g$.

$$W = Fd\cos\varphi$$

$$W_g = mgd\cos\varphi \quad \to \varphi = 180^\circ$$

$\to W_g = mgd(-1) = -mgd < 0 \Rightarrow$ During the object's Rise, The gravitational Force acting on the object transfers energy in the amount mgd From the Kinetic energy of the object.

After the object Reaches to the maximum height, and is falling down $\longrightarrow \varphi = 0$

$$\Rightarrow W_g = mgd\cos(0) = mgd(+1) = mgd > 0$$

WORK DONE BY LIFTING AND LOWERING AN OBJECT

Our applied Force does positive work W_a on The object while gravitational Force does negative work W_g.

$$\Delta K = K_f - K_i = W_a + W_g$$ (Also valid if we lower the object)

Suppose that the object is stationery Before & After: (e.g., Lifting a Book from the floor to shelf)

$\to K_f = K_i = 0$

$\Rightarrow W_a + W_g = 0 \Rightarrow W_a = -W_g$

$\longrightarrow$ The applied force transfers the same amount of energy to the object as the Gravitational force transfers from the object.

$\Rightarrow W_a = -mgd\cos\varphi$

$\vec{F}_a \uparrow \varphi = 180^\circ \Rightarrow W_a = mgd$ (positive work done on the object)

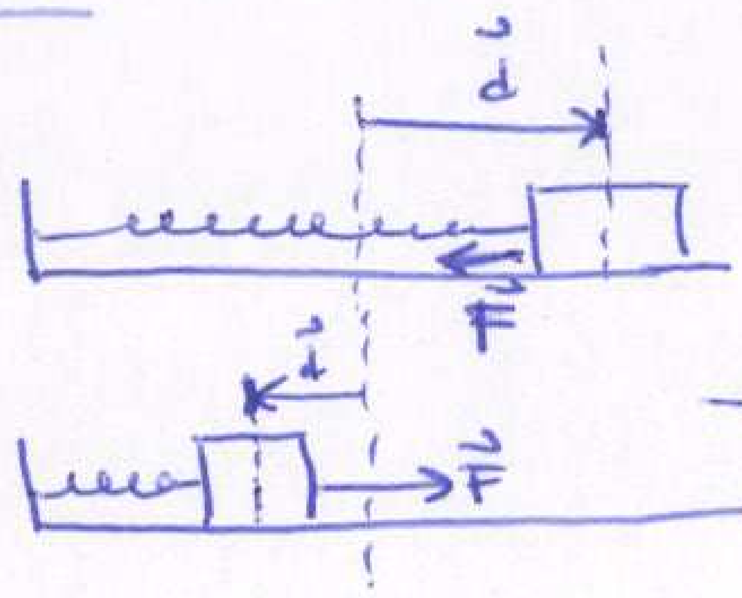
$\vec{F}_a \downarrow \varphi = 0^\circ \Rightarrow W_a = -mgd$ (negative work done on The object)

Work Done by the Spring Force

$\vec{F}_s = -k \cdot \vec{d}$

$\vec{d}$ → displacement

k → Spring Constant.



x positive

F_x negative

x negative

F_x positive

$F_x = -kx$ (Hooke's Law)

Spring: Massless & ideal (Assumption)

F_x does work but we can't write $W = Fd\cos\varphi$,

Because the Force is not constant!

$\varphi = 180°$

$$W_s = \sum_i -F_{x_i}\Delta x_i \ , \ \Delta x_i \to 0 : W_s = \int_{x_i}^{x_f} -F_x\,dx = -k\int_{x_i}^{x_f} x\,dx$$

$$= \left(-\frac{1}{2}k\right)x^2\Big|_{x_i}^{x_f} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$x_i = 0 : W_s = -\frac{1}{2}kx^2$$

$$\boxed{W = \int_{x_i}^{x_f} F(x)\,dx}$$

3-Dimensional Forces – General Case:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$dW = \vec{F}\cdot d\vec{r} = F_x\,dx + F_y\,dy + F_z\,dz$$

$$W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

1-D

$$W = \int_{x_i}^{x_f} F(x)\,dx = \int_{x_i}^{x_f} ma\,dx$$

$$ma\,dx = m\frac{dv}{dt}dx = m\frac{dv}{dx}v\,dx = mv\,dv \Rightarrow W = \int_{v_i}^{v_f} mv\,dv = m\int_{v_i}^{v_f} v\,dv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}v$$

$$\Rightarrow \boxed{W = K_f - K_i = \Delta K}$$

Power

$$P_{avg} = \frac{W}{\Delta t} \rightarrow P = \frac{dW}{dt} = \frac{F\cos\varphi\,dx}{dt} = F\cos\varphi\left(\frac{dx}{dt}\right) = Fv\cos\varphi \Rightarrow P = \vec{F}\cdot\vec{v}$$

$[P] = 1\,\text{Watt} = 1\,W = 1\,J/s$;

1 horsepower = 1 hp = 746 W

1 kilowatt-hour = 1 kWh = $(10^3\,W)(3600\,s)$

$= 3.60\times10^6\,J = 3.60\,MJ$