Mehmet Taha
USTA
21527472
MTUSTA

Ⓐ Let $V = R^3$ and $W = \langle (1,0,-1),(0,1,-1) \rangle$ be a subspace of V. find orthonormal basis for W

$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \rightarrow$ Gram schmit process

$$\vec{u_k} = \vec{v_k} - \sum_{j=1}^{k-1} proj_{\vec{u_j}}(\vec{v_k})$$

$$proj_{\vec{u_j}} = (\vec{v_k}) = \frac{\vec{u_j} \cdot \vec{v_k}}{|\vec{u_j}|} \vec{u_j}$$

$$\vec{e_k} = \frac{\vec{u_k}}{|\vec{u_k}|}$$

$$\vec{u_1} = \vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \rightarrow e_1 = \frac{\vec{u_1}}{|\vec{u_1}|} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$u_2 = \vec{v_2} - proj_{u_1}(\vec{v_2}) = \begin{bmatrix} -1/2 \\ 1 \\ -1/2 \end{bmatrix} \quad \vec{e_2} = \frac{\vec{u_2}}{|\vec{u_2}|} = \begin{bmatrix} -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \\ -\frac{\sqrt{6}}{6} \end{bmatrix}$$

$$\left\{ \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix}, \begin{bmatrix} -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \\ \frac{-\sqrt{6}}{6} \end{bmatrix} \right\}$$

B

Mehmet Taha
USTA
21527472
MTUSTA

homogenius to matrix

$$\begin{array}{c} \begin{array}{cccc} x & y & z & t \end{array} \\ \begin{bmatrix} -1 & 2 & 1 & -2 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \rightarrow \text{I need to find null space}$$

Tranform reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \text{rref}$$

$$\left.\begin{array}{l} x + t = 0 \Rightarrow x = -t \\ y - t = 0 \Rightarrow y = +t \\ z + t = 0 = z = -t \\ \qquad t \Rightarrow \text{arbitrary} \end{array}\right\} \Rightarrow \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \underbrace{\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}}_{\text{null space}} \cdot t$$