MEASUREMENT

Science and engineering are based on measurements and comparisons. Thus, we need rules about how things are measured and compared. And for these purposes, we need to establish the units.

For example, physicists strive to develop clocks of extreme accuracy, so that any time or time interval can be precisely determined and compared.

Without clocks of extreme accuracy, GPS would be useless.

Quantities and How to Measure Them

Physics: length, time, mass, temperature, pressure, electric current, etc...

We measure each physical quantity in its own units, by comparison with a standard. The standard corresponds to exactly 1.0 unit of the quantity.

The standard for length, 1.0m is the distance travelled by light in a vacuum during a certain fraction of a second. The important thing is the scientists around the world will agree that our definitions are both sensible and practical.

Once we have set up a standard, we need to find ways to measure the quantity. For example, length → rulers but we can't use a ruler to measure the radius of an atom or the distance to a star.

There are so many physical quantities that is a problem to organize them. Fortunately, they are not all independent: e.g., speed is the ratio of a length to a time.

Thus we pick out a small number of -internationally agreed- physical quantities such as length & time, and assign standards to them alone. We then define all other physical quantities in terms of these base quantities and their standards: speed is defined in terms of the base quantities length & time.

The International System of units

In 1971, the 14th General Conference on weights and Measures picked seven quantities as Base quantities

→ SI (metric System)

length	meter	m
Time	second	s
Mass	kilogram	kg

Many SI derived units are defined in terms of these base units. SI unit for power, the Watt (W) is defined in terms of mass, length and time.

$$1 \text{ Watt} = 1 \underbrace{\text{kg m}^2/\text{s}^3}$$

↳ "kilogram meter squared per second cubed"

To express very large/small quantities, we use scientific notation, which employs powers of 10:

$$3\,560\,000\,000 \text{ m} = 3.56\times10^{9} \text{ m} \quad (3.56E9)$$

$$0.000\,000\,492 \text{ s} = 4.92\times10^{-7} \text{ s} \quad (4.92E-7)$$

↓ "exponent of ten"

PREFIXES

10^{24}	yotta-	Y	10^{-1}	deci-	d
10^{21}	zetta-	Z	10^{-2}	centi-	c
10^{18}	exa-	E	10^{-3}	mili-	m
10^{15}	peta-	P	10^{-6}	micro-	μ
10^{12}	tera-	T	10^{-9}	nano-	n
10^{9}	giga-	G	10^{-12}	pico-	p
10^{6}	mega-	M	10^{-15}	femto-	f
10^{3}	kilo-	k	10^{-18}	atto-	a
10^{2}	hecto-	h	10^{-21}	zepto-	z
10^{1}	deka-	da	10^{-24}	yocto-	y

("10^{100} = googol")

Changing units

$$\frac{1\,\text{min}}{60\,\text{s}} = 1 \longrightarrow \frac{60\,\text{s}}{1\,\text{min}} = 1$$

→ This is not the same as writing $\frac{1}{60} = 1$ or $60 = 1$

each number and its unit must be treated together

$$2\,\text{min} = (2\,\text{min})\,(1) = (2\,\cancel{\text{min}})\left(\frac{60\,\text{s}}{1\,\cancel{\text{min}}}\right) = 120\,\text{s}$$

In conversions, the units obey the same algebraic rules as variables & numbers.

LENGTH

1792 : French Republic (Revolution 1789 - 1799)

established a new system of weights and measures.

meter : $\frac{1}{10 \text{ million}}$ of the distance from the north pole to the equator. (!?)

→ meter : The Distance between two fine lines engraved near the ends of a platinum-iridium bar.

the Standard meter was kept at the International Bureau of Weights and Measures near Paris,

and accurate copies were sent to standardizing labs throughout the world.

Eventually, a better equipment was required. In 1960 a new standard for meter was based on the wavelength of light:

$\boxed{1650763.73}$ wavelengths of a particular orange-red light emitted by atoms of Krypton-86 in a gas discharge tube that can be set up anywhere in the world.

→ this awkward number was chosen so that it was close to the old standard.

1983 : $1/299792458$ of a second path travelled by light in a vacuum.

$c = 29979245 \, m/s$

	(m)
Distance to: the first galaxies formed	2×10^{26}
Andromeda Galaxy	2×10^{22}
nearby star Proxima Centauri	4×10^{16}
Pluto	6×10^{12}
Radius of Earth	6×10^{6}
Height of Mt. Everest	9×10^{3}
Thickness of a Page	1×10^{-4}
Length of a typical virus	1×10^{-8}
Radius of the atom	5×10^{-9}
proton	1×10^{-15}

TIME

Two aspects: i) time of the day
ii) How long an event lasts (duration)

	s
Lifetime of the proton (predicted)	3×10^{40}
Age of the universe	5×10^{17}
Age of the Pyramid Cheops	1×10^{11}
Human life expectancy	2×10^{9}
Length of a day	9×10^{4}
Time between two human-heartbeats	8×10^{-1}
Lifetime of the muon	2×10^{-6}
Shortest lab light pulse	1×10^{-16}
Lifetime of the most unstable particle	1×10^{-23}
The Planck time	1×10^{-43}

↳ time it took for the laws of physics as we know after the big Bang.

Atomic clock at the National Institute of Standards and Tech. (NIST)

1967: 9192631770 oscillations of the light (of a specified wavelength) emitted by a Cesium-133 atom.

MASS

Standard kilogram

The SI standard of mass is a platinum-iridium cylinder kept at the International Bureau of weights and measures.

1 kg

Second mass standard

Masses of atoms can be compared with one another standard more precisely than kg.

Carbon-12 atom is assigned a mass of 12 atomic mass units (u)

$$1u = 1.66053886 \times 10^{-27}\ kg$$

	kg
Known Universe	1×10^{53}
Our Galaxy	2×10^{41}
Sun	2×10^{30}
Moon	7×10^{22}
Small Mountain	1×10^{12}
Ocean Liner	7×10^{7}
Elephant	5×10^{3}
Grape	3×10^{-3}
Dust	7×10^{-10}
penicilin Molecule	3×10^{-17}
Uranium Atom	4×10^{-25}
Proton	2×10^{-29}
Electron	9×10^{-31}

DENSITY

$\rho = \frac{m}{V}$ (density of water: 1 gram per cm^3)

VECTORS

Physics deals with a great many quantities that have both size and direction and the language required for this is vectors.

Navigation (e.g., "go 300m down this street, then turn left") of any sorts is based on vectors, but physicists and Engineers also need vectors in special ways to explain phenomena involving rotation and magnetic forces (later courses).

Vectors and Scalars

A particle moving along a straight line can only move in two directions (1 Dimensional). For this, we can mark its moving in one direction as positive and the other as negative and the problem is solved.

However, in 3D, a plus or minus sign is no longer sufficient and therefore we must use a vector.

A vector has magnitude as well as direction and they follow certain rules of combination.

Some physical quantities that are vector:

displacement, velocity and acceleration.

Not all physical quantities involve a direction:

Temperature, pressure, energy, mass and time do not "point" in the spatial sense.

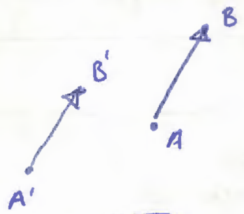
⇒ We call such quantities as <u>SCALARs</u>.

A single value, with a sign (e.g. −23°C) specifies a scalar.

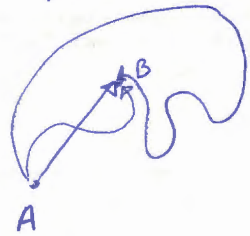
* Dimensions of a scalar?

The Simplest vector quantity is displacement (change of position)

$\longrightarrow$ <u>displacement Vector</u>



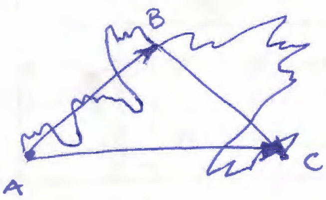
They specify identical displacement vectors and represent the same <u>change of position</u> for the particle. A vector can be shifted without changing its value.



The displacement vector tells us nothing about the path the particle takes. All of the three paths connecting A and B correspond to the same displacement vector.

<u>Displacement vectors Represent only the overall effect of the motion, not the motion itself.</u>

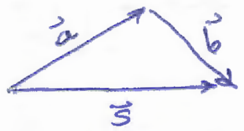
<u>Adding Vectors Geometrically</u>



Suppose that a particle moves from A to B, then from B to C. We can represent its overall displacement vectors as AB and BC.

The net Displacement is a single displacement from A to C. We call AC the <u>vector sum</u> (or <u>resultant</u>) of the vectors AB and BC.

This sum is not the algebraic sum.



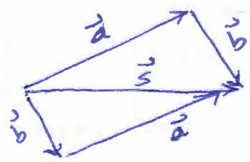
arrow over symbol: vector ($\vec{a}$)

magnitude (without sign or direction: symbol (a)

$\vec{s} = \vec{a} + \vec{b}$ Vector Equation

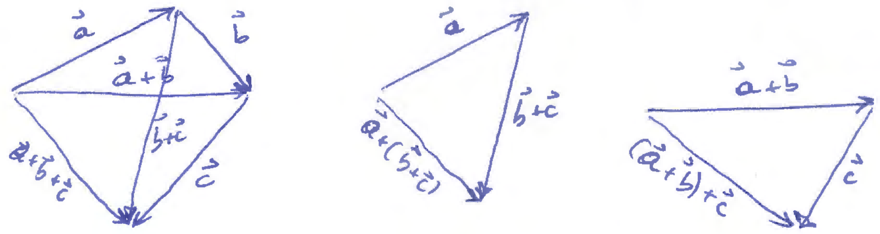
The symbol "+" and the words "sum" and "add" have different meanings for vectors, because they involve both magnitude <u>and</u> direction.

* $(\vec{a} + \vec{b}) = (\vec{b} + \vec{a})$ (Commutative Law)

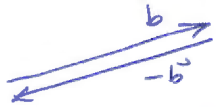


* Secondly, we can group and add more than two vectors in any order:

$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c}) \quad \text{(associate Law)}$$



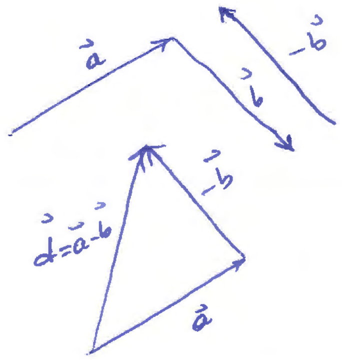
* The vector $-\vec{b}$ is a vector with the same magnitude as $\vec{b}$ but in the opposite direction.



$$\vec{b}+(-\vec{b})=0$$

$\Rightarrow$ difference between two vectors: $\vec{d}=\vec{a}-\vec{b}$

$\vec{d}=\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$ (vector subtraction)



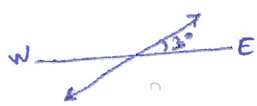
$\vec{d}=\vec{a}-\vec{b}$

$\rightarrow \vec{d}+\vec{b}=\vec{a}$ or $\vec{a}=\vec{d}+\vec{b}$

Ex: You can go (in each direction +/-)

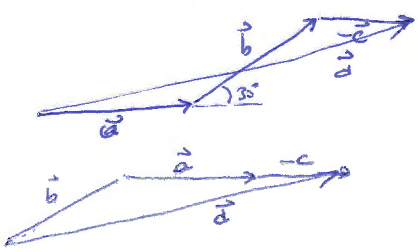
a) $\vec{a}$: 2km due east (/west)

b) $\vec{b}$: 2km 30° north of east (/30° south of west)



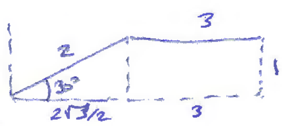
c) $\vec{c}$: 1km west (/east)

What is the greatest distance you can move?



$\vec{d}=\vec{a}+\vec{b}-\vec{c}$

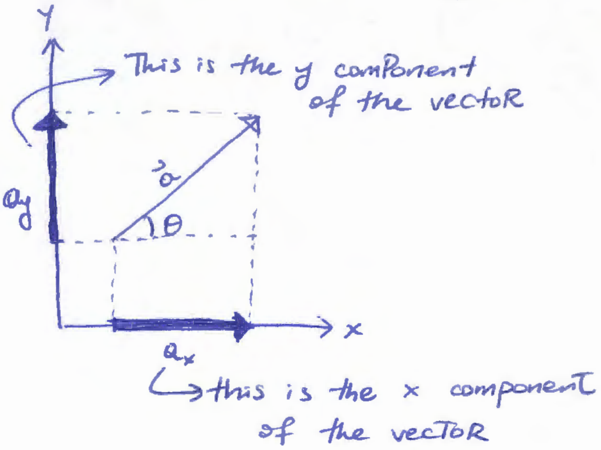
$d=4.8$ km (measured by a Ruler)

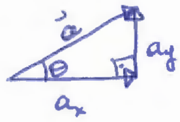


$\sqrt{1^2+(\underbrace{3+\sqrt{3}}_{4.7321})^2}=\sqrt{23.3923}=4.8366$ km

Components of Vectors

Adding vectors geometrically can be difficult. An easier technique is to use a Rectangular coordinate system and placing vectors on it.





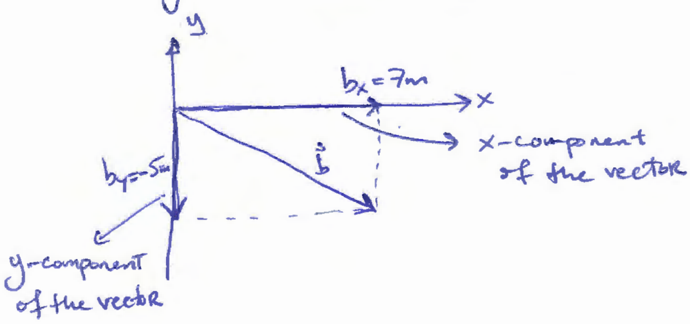
The components and the vector form a Right Triangle

A <u>component</u> of a vector is the projection of the vector on an axis.

(projector, light, shadow)

To find the projection of a vector along an axis, we draw perpendicular lines from the two ends of the vector to the axis. The projection of a vector on an x axis is its <u>x component</u>, and similarly the projection of a vector on a y axis is its y component. The process of finding the components of a vector is called <u>Resolving the Vector.</u>

A component of a vector has the same direction (along an axis) as the vector.



$a_x = a\cos\theta$

$a_y = a\sin\theta$

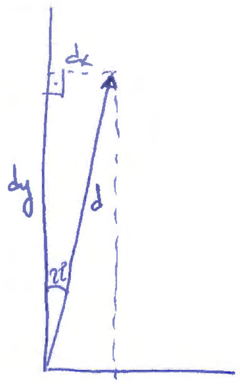
$a = \sqrt{a_x^2 + a_y^2}$

$\tan\theta = \dfrac{a_y}{a_x}$

where θ is the angle the vector makes with the positive direction of the x-axis, and a is the magnitude of $\vec{a}$.

Example : A small airplane leaves an airport and is later seen 215 km away, in a direction making an angle of 22° east of due north. How far east and how far north is the airplane from the airport when sighted?

($\sin 22° = 0.375 = \cos 68°$
$\cos 22 = 0.927 = \sin 68°$)



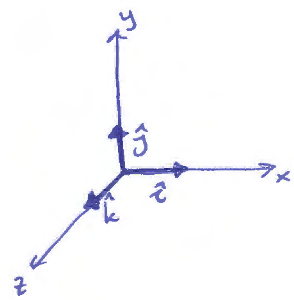
$$d_x = d\sin 22° = d\cos 68° = 215\,km \cdot 0.375 = 80.625\,km$$

$$d_y = d\cos 22 = d\sin 22° = 215\,km \cdot 0.927 = 199.31\,km$$

Unit vectors

A unit vector is a vector that has a magnitude of exactly $\underline{1}$ and points in a particular direction. It lacks both dimension and unit. Its only purpose is to point, that is to specify a direction

$x, y, z \longrightarrow \hat{i}, \hat{j}, \hat{k}$



right-handed coordinate system

Unit vectors are very useful for expressing other vectors

vector components of $\vec{a}$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

scalars $\rightarrow$ scalar components of $\vec{a}$

$$\vec{b} = b_x\hat{i} + b_y\hat{j}$$

ADDING VECTORS BY COMPONENTS

We can add vectors

i) geometrically

ii) algeabrically

iii) by combining their components axis by axis.

$\vec{r} = \vec{a} + \vec{b}$

thus: $r_x = a_x + b_x$

$r_y = a_y + b_y$

$r_z = a_z + b_z$

$\Leftrightarrow$ two vectors must be equal if their corresponding components are equal.

$\rightarrow$ 1) Resolve the vectors into their scalar components

2) Combine these scalar components, axis by axis to get the components of the sum $\vec{r}$

3) Combine the components of $\vec{r}$ to get $\vec{r}$ itself

This procedure for adding is also valid for substraction, since $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

$$\vec{d} = \vec{a} - \vec{b} \rightarrow \left.\begin{array}{l} d_x = a_x - b_x \\ d_y = a_y - b_y \\ d_z = a_z - b_z \end{array}\right\} \rightarrow \vec{d} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}$$