$\begin{array}{c} {\rm BBM}\ 205 \\ {\rm Spring}\ 2015\ {\rm Final}\ {\rm Exam} \end{array}$

SHOW YOUR WORK TO RECEIVE FULL CREDIT. KEEP YOUR CELLPHONE TURNED OFF.

1. (3 points) Use pigeonhole principle to show that in any simple connected graph, there are two vertices that have the same degree.								
2. (2 points) Use a proof by contraposition to show that if $x + y \ge 2$								

where x and y are real numbers, then $x \ge 1$ or $y \ge 1$.

- 3. (7 points) (a) (1 point) How many license plates can be made using either three letters followed by three digits or four letters followed by two digits?
 - (b) (.5 points) How many different functions are there from a set with 10 elements to a set with 5 elements?
 - (c) (1.5 points) How many permutations of the letters ABCDEFG contain
 - a) the string BCD?
 - b) the strings ABC and CDE?
 - c) the strings CBA and BED?
 - (d) (1 point) Show that if n and k are integers with $1 \le k \le n$, then $\binom{n}{k} \le n^k/2^{k-1}$.
 - (e) (1 point) How many different ways are there to choose 6 donuts from the 21 varieties at a donut shop?
 - (f) (1 point) How many different strings can be made from the letters in ABRACADABRA, using all letters?
 - (g) (1 point) A bowl contains 10 red balls and 10 blue balls. A person selects balls at random without looking at them. How many balls must be selected to be sure of having at least three balls of the same color?

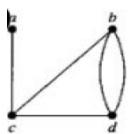
4.	(8 points)	(a)	(2 points)	Draw	these graphs:	K_4 ,	C_5 ,	$K_{2,3},$	Q_3 .
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- (b) (1.5 points) For which values of n are these graphs bipartite?
 - a) K_n b) C_n c) Q_n

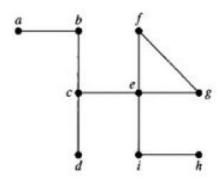
- (c) (2 points) How many vertices and how many edges do these graphs have?
- a) K_n b) C_n c) $K_{m,n}$ d) Q_n

- (d) (1.5 points) Find the degree sequence of each of the following graphs:
- a) K_4 b) C_5 c) $K_{2,3}$
- (e) (1 point) Determine whether each of these sequences is the degree sequence of a graph. For those that are, draw a graph having the given degree sequence.
 - a) 5,4,3,2,1,0
 - b) 1,1,1,1,1,1

- 5. (2 points) Represent the graph below using
 - a) an adjacency list,
 - b) an adjacency matrix.

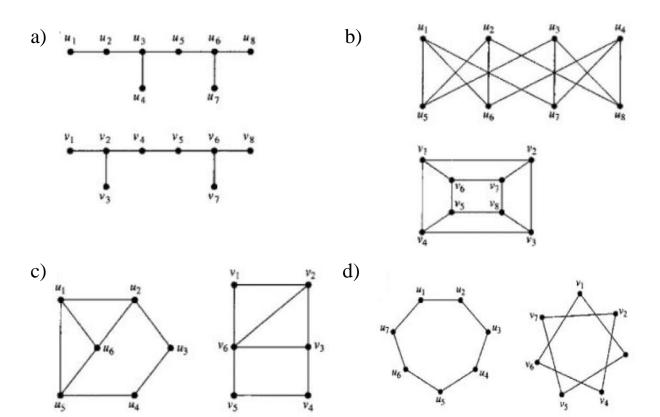


6. (1 point) Find all cut-vertices of the graph below.

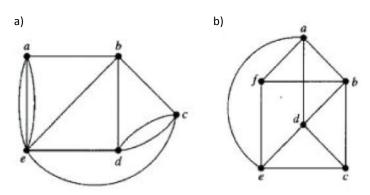


7. (1 point) A simple graph is called k-regular if every vertex has degree k. Show that if a bipartite graph G = (V, E) is k-regular for some positive integer k and (V_1, V_2) is a bipartition of V, then $|V_1| = |V_2|$.

8. (4 points) Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



9. (2 points) Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



- 10. (2 points) (a) (1 point) For which values of m and n does the complete bipartite graph $K_{m,n}$ have a Hamilton circuit?
 - (b) (1 point) Can you find a simple graph with n vertices (and $n \ge 3$) that does not have a Hamilton circuit, yet the degree of every vertex in the graph is at least (n-1)/2?

11. (3 points) (a) (1 point) Derive a recurrence relation for $C(n, k) = \binom{n}{k}$, the number of k-element subsets of an n-element subset. Specifically, write C(n+1,k) in terms of C(n,i) for appropriate i.

(b) (2 points) Solve the recurrence relation with the given initial condition below. $a_n = 7a_{n-1} - 10a_{n-2}$; $a_0 = 5$, $a_1 = 16$.

12. (2 points) Let f_i be the *i*th Fibonacci number. **Use induction** to prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ when n is a positive integer.

13. (3 points) (a) (1 point) Show that $x^2 + 4x + 17$ is $O(x^3)$.

(b) (2 points) Show that x^3 is **not** $O(x^2 + 4x + 17)$.

Extra Point Questions:

You can answer up to 3 questions to earn extra points.

Please **mark** which questions you choose to answer.

- 14. (3 points) Prove that at least one of the real numbers a_1, a_2, \ldots, a_n is greater than or equal to the average of these numbers.
- 15. (3 points) Use induction to prove that if n is a positive integer, then 133 divides $11^{n+1} + 12^{2n-1}$.
- 16. (3 points) Show that if G is a bipartite simple graph with n vertices and e edges, then $e \le n^2/4$.
- 17. (3 points) Show that a simple graph G with n vertices is connected if it has more than (n-1)(n-2)/2 edges.
- 18. (3 points) Show that every connnected graph with n vertices has at least n-1 edges.
- 19. (3 points) Suppose that v is an endpoint of a cut edge. Prove that v is a cut vertex if and only if this vertex is not pendant.
- 20. (3 points) Let S(n,k) denote the number of functions from $\{1,\ldots,n\}$ onto $\{1,\ldots,k\}$. Show that S(n,k) satisfies the recurrence relation

$$S(n,k) = k^n - \sum_{i=1}^{k-1} C(k,i)S(n,i).$$