QUESTION 3

3. Consider the basis $S = \{(1,0,-1),(-1,1,0),(0,1,1)\}$ of $\mathbb{R}^3$
Apply Gram-Schmidt orthogonalization process to S and find and orthonormal basis for $\mathbb{R}^3$

$w_1 = v_1$

$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1$

$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2$

$w_1 = v_1 = (1,0,-1)$

$w_2 = v_2 - \frac{v_2 \cdot w_1}{w_1 \cdot w_1} \cdot w_1 = (-1,1,0) - \frac{(-1,1,0) \cdot (1,0,-1)}{(1,0,-1)(1,0,-1)} \cdot (1,0,-1)$

$(-1,1,0) - \left[\frac{-1}{2} \cdot (1,0,-1)\right.$

$(-1,1,0) - (-1/2, 0, 1/2) = (-1/2, 1, -1/2)$

$w_3 = (0,1,1)$

$w_3 = v_3 - \frac{v_3 \cdot v_1}{v_1 \cdot v_1} \cdot v_1 - \frac{v_3 \cdot w_2}{w_2 \cdot w_2} \cdot w_2$

$(0,1,1) - \frac{(0,1,1) \cdot (1,0,-1)}{(1,0,-1)(1,0,-1)} \cdot (1,0,-1) - \frac{(0,1,1) \cdot (-1/2,1,-1/2)}{(-1/2,1,-1/2)(1/2,1,-1/2)} \cdot (-1/2, 1, 1/2)$

$(0,1,1) - \left(\frac{-1}{2} \cdot (1,0,-1)\right) - \frac{(0,1,1) \cdot (-1/2,1,-1/2)}{(-1/2,1,-1/2)(-1/2,1,-1/2)} \cdot (-1/2,1,-1/2)$

$\frac{+1/4}{3/4} \cdot (-1/2, 1, -1/2)$

$(0,1,1) - (-1/2, 0, 1/2) - (-1/6, 1/3, -1/6) = (2/3, 2/3, 2/3)$

$w_3 = (2/3, 2/3, 2/3)$

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$$u_1 = \frac{w_1}{\|w_1\|} = \frac{(1,0,-1)}{\sqrt{2}} = (1/\sqrt{2}, 0, -1/\sqrt{2})$$

$$u_2 = \frac{w_2}{\|w_2\|} = \frac{(-1/2, 1, -1/2)}{\sqrt{3/2}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot (-1/2, 1, -1/2) = \frac{-\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{-\sqrt{2}}{2\sqrt{3}}$$

$$u_3 = \frac{w_3}{\|w_3\|} = \frac{(2/3, 2/3, 2/3)}{2/\sqrt{3}} = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$$

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orthonormal $S''' = \left\{(1/\sqrt{2}, 0, -1/\sqrt{2}), \left(-\frac{\sqrt{2}}{2\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}, -\frac{\sqrt{2}}{2\sqrt{3}}\right), \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)\right\}$

QUESTION-4

4-) Find the eigen values and the eigen vectors corresponding to the eigen values of the matrix

$$A = \begin{bmatrix} -4 & 0 & 3 \\ 0 & -1 & 0 \\ -6 & 0 & 5 \end{bmatrix}$$

step1 of eigenvalues

subtracking diagonal entries of matrix by λ

$$\begin{bmatrix} -\lambda-4 & 0 & 3 \\ 0 & -\lambda-1 & 0 \\ -6 & 0 & 5-\lambda \end{bmatrix}$$

Step 2

find the determinant

$$\begin{vmatrix} -\lambda-4 & 0 & 3 \\ 0 & -\lambda-1 & 0 \\ -6 & 0 & 5-\lambda \end{vmatrix} = \cancel{0 \cdot (-1)^{2+1} \cdot \begin{vmatrix} 0 & 3 \\ 0 & 5-\lambda \end{vmatrix}} + (-\lambda-1) \cdot (-1)^{2+2} \cdot \begin{vmatrix} -\lambda-4 & 3 \\ -6 & 5-\lambda \end{vmatrix}$$

$$\cancel{+ 0 \cdot (-1)^{2+3} \cdot \begin{vmatrix} -\lambda-4 & 0 \\ -6 & 0 \end{vmatrix}}$$

$(-\lambda-4)(5-\lambda) - (+3)\cdot(-6)$

$-5\lambda + \lambda^2 - 20 + 4\lambda + 18$

$= \lambda^2 - \lambda - 2$

$(-\lambda-1)(\lambda^2-\lambda-2)$

$-\lambda^3 + \cancel{\lambda^2} + 2\lambda - \cancel{\lambda^2} + \lambda + 2$

$-\lambda^3 + 3\lambda + 2$

$\cancel{-\lambda^3 + 3\lambda + 2}$

step 3

find the roots

$-\lambda^3 + 3\lambda + 2$

$-(\lambda^3 - 3\lambda - 2) \cdot \dfrac{(\lambda+1)}{\lambda+1}$

$\dfrac{-(\lambda^3 - \lambda - 2)}{\lambda+1} = x^2 + \dfrac{-x^2-3x-2}{x+1}$

$\dfrac{-x^2-3x-2}{x+1} = -x + \dfrac{-2x-2}{x+1} \rightarrow -2$

result: $x^2 - x - 2$

-2

$+1$

QUESTION 4

eigenvectors

* $\lambda = 2$

$$\begin{bmatrix} -\lambda-4 & 0 & 3 \\ 0 & -\lambda-1 & 0 \\ -6 & 0 & 5-\lambda \end{bmatrix} = \begin{bmatrix} -6 & 0 & 3 \\ 0 & -3 & 0 \\ -6 & 0 & 3 \end{bmatrix}$$

<u>step 1</u>

find the reduced row echelon

$$\begin{bmatrix} -6 & 0 & 3 \\ 0 & -3 & 0 \\ -6 & 0 & 3 \end{bmatrix} \begin{array}{c} \\ \\ \\ +6 \quad 0 \quad -3 \end{array} \xrightarrow{R_3 - R_1 \to R_3} \begin{bmatrix} -6 & 0 & 3 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

<u>step 2</u>

solve

$$\begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 - \frac{v_3}{2} = 0$ $\quad v_1 = \frac{v_3}{2}$

$v_2 = 0$

if we take $v_3 = t$ then $v_1 = t/2$ $\quad v_2 = 0$

$$v = \begin{bmatrix} t/2 \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix} t$$

* $\lambda = -1$

$$\begin{bmatrix} -\lambda-4 & 0 & 3 \\ 0 & -\lambda-1 & 0 \\ -6 & 0 & 5-\lambda \end{bmatrix} = \begin{bmatrix} -3 & 0 & 3 \\ 0 & 0 & 0 \\ -6 & 0 & 6 \end{bmatrix}$$

reduced row echelon

$$\xrightarrow{R_3 - 2R_1 \to R_3} \begin{bmatrix} -3 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$v_1 - v_3 = 0$

$v_1 = v_3$

if we take $v_2 = s$ $\quad v_3 = t$

then $v_1 = t$

$$v = \begin{bmatrix} t \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} t$$