BBM 205
Problem Set 1: Logic

1. What is the negation of each of these propositions?
   (a) Today is Thursday.
   (b) There is no pollution in Istanbul.
   (c) $2 + 1 = 3$
   (d) The summer in Ankara is hot and sunny.

2. Let p and q be the propositions
   p: It is below freezing.
   q: It is snowing.
   Write these propositions using p and q and logical connectives.
   (a) It is below freezing and snowing.
   (b) It is below freezing but not snowing.
   (c) It is not below freezing and it is not snowing.
   (d) It is either snowing or below freezing (or both).
   (e) If it is below freezing, it is also snowing.
   (f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
   (g) That it is below freezing is necessary and sufficient for it to be snowing.

3. Show that $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent by using a truth table.

4. Determine the truth value of each of these statements if the domain consists of all real numbers.

   a) $\exists x(x^3 = -1)$ c) $\exists x(x^4 < x^2)$
   b) $\forall x((-x)^2 = x^2)$ d) $\forall x(2x > x)$

5. The notation $\exists! x P(x)$ denotes "There exists a unique x such that $P(x)$ is true.". What are the truth values of these statements?

   a) $\exists! x P(x) \rightarrow \exists x P(x)$
   b) $\forall x P(x) \rightarrow \exists! x P(x)$
   c) $\exists! x \neg P(x) \rightarrow \neg \forall x P(x)$

6. Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of
   (a) the positive real numbers.
   (b) the integers.
   (c) the nonzero real numbers.

7. (Spring 2014) Determine the truth value of each statement, assuming that the domain is the set of real numbers. Justify your answer.

(a) For every x, $x^2 > x$.

(b) For some x, $x^2 > x$.

(c) For every x, for every y, if $x < y$, then $x^2 < y^2$.

(d) For every x, for some y, $x^2 < y + 1$.

(e) For every x, for every y, $x^2 + y^2 = 9$.

8. (Spring 2015) The notation $\exists! x P(x)$ denotes "There exists a unique x such that $P(x)$ is true.". What are the truth values of these statements?

   (a) $\exists! x P(x) \to \exists x P(x)$

   (b) $\forall x P(x) \to \exists! x P(x)$

9. (Spring 2015) Let p, q and r be the propositions
   p: You get an A on the final exam.
   q: You do every exercise in this book. Write these propositions using p, q and r and
   r: You get an A in this class.
   logical connectives.

   (a) You get an A in this class, but you do *not* do every exercise in this book.

   (b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

   (c) To get an A in this class, it is *necessary* for you to get an A on the final.

   (d) You get an A on the final, *but* you do not do every exercise in this book; *nevertheless*, you get an A in this class.

   (e) Getting an A on the final and doing every exercise in this book is *sufficient* for getting an A in this class.

   (f) You will get an A in this class *if and only if* you either do every exercise in this book or you get an A on the final.

10. (Spring 2015) Show that this conditional statement is a tautology by using truth table: $(p \land q) \to (p \to q)$.

11. (Spring 2015) Determine the truth value of the statement $\forall x \exists y (xy = 1)$ if the domain for the variables consists of

    (a) the positive real numbers.

    (b) the nonzero integers.

    (c) the nonzero real numbers.

12. (Spring 2015) State the converse, contrapositive and inverse of each of these conditional statements.

    (a) *If* it snows tonight, *then* I will stay at home.

    (b) I go to the beach *whenever* it is a sunny summer day.

    (c) *When* I stay up late, it is *necessary* that I sleep until noon.

    (d) A positive integer is a prime only if it has no divisors other than 1 and itself.

13. (Spring 2015) Let $Q(x)$ be the statement "$x + 1 > 2x$". If the domain for x consists of all integers, what are the truth values below? Justify your answer.

$$\begin{array}{lll} Q(-1), & Q(1), & \forall x \neg Q(x), \\ \exists x Q(x), & \forall x Q(x), & \exists x \neg Q(x). \end{array}$$

14. (Spring 2015) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

$$\forall x(x^2 \neq x), \qquad \forall x(|x| > 0), \qquad \forall x(x^2 \neq 2).$$

15. (Fall 2016) Determine the truth value of each of these statements if the domain consists of real numbers. Give a short explanation for each answer to receive full credit.
    (a) $\exists x(x^3 = -1)$
    (b) $\exists x(x^4 < x^2)$
    (c) $\forall x((-x^2) = x^2)$
    (d) $\forall x(2x > x)$

16. (Fall 2016) Determine whether each of these arguments is valid. If it is valid, show the steps of your conclusion. If it is not valid, give a logical error.
    (a)

    If n is a real number such that $n > 2$, then $n^2 > 4$.
    $n \leq 2$

    $n^2 \leq 4$

    (b)

    If it snows today, the university will close.
    The university is not closed today.

    Therefore, it did not snow today.

17. (Fall 2016) Determine the truth value of the statement $\exists x \forall y(x \leq y^2)$ and if the domain for x and y consists of the following sets. Give a short explanation for your answer.
    (a) the positive real numbers
    (b) the integers
    (c) the nonzero real numbers