Q1 Academic Honesty

1 Point

It is a violation of the Academic Integrity Code to look at any reference material other than your textbook and lecture notes, or to give inappropriate help to someone or to receive unauthorized aid by someone in person or electronically via messaging apps such as WhatsApp. Academic Integrity is expected of all students of Hacettepe University at all times, whether in the presence or absence of members of the faculty. Do NOT sign nor take this exam if you do not agree with the honor code.

Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination.

Signature (Specify your name and surname as your signature)

MEHMET TAHA USTA MTUSTA

While answering the following questions, please consider the implementations that we discussed in our lectures unless stated otherwise.

Q2 True/False Questions

30 Points

For each part, determine whether the statement is true or false.

Q2.1

5 Points

In a DAG G, we are interested in finding the longest path between any two vertices. We can find that by running the Dijkstra's shortest path algorithm from every vertex in G after negating all the edge weights.

- [x] True
- [ ] False

Q2.2
5 Points

We can always use Bellman-Ford's algorithm to determine the shortest paths from a source vertex to any other vertex even if some edge weights are 0.

- [x] True
- [ ] False

Q2.3
5 Points

Consider a weighted directed graph $G = (V, E, w)$ and let P be a shortest path from some vertex s to some other vertex t for $s, t \in V$. If we increase the weight of every edge in the graph by 1 by setting $w'(e) = w(e) + 1$ for each edge $e \in E$, then P will still be a shortest path from s to t in $G' = (V, E, w')$

- [x] True
- [ ] False

Q2.4
5 Points

Suppose G is a weighted directed graph that is known to have no shortest paths longer than k edges. Then, in order to solve the single-source shortest paths problem on G, it is sufficient to run the Bellman-Ford algorithm for only $k - 1$ passes.

- [ ] True
- [x] False

Q2.5
5 Points

Suppose G is a connected graph. Then, the number of edges in BFS, DFS, MST and shortest path trees will always the same.

- ○ True
- ◉ False

Q2.6
5 Points

Let T denote a shortest path tree for some graph G and x be a node in T. Then, for any vertex y, the shortest path from x to y in G is always the same as the path from x to y in T.

- ○ True
- ◉ False

Q3
30 Points

Given a weighted directed graph

Q3.1
10 Points

If all edges have the same weight, say $w(e) = 1, e \in E$, what is the minimum time complexity of computing the single-source shortest path problem?

E V

Q3.2
10 Points

If the edges have only two distinct weights, say $w(e) = 1$ or $w(e) = 2$, $e \in E$, what is the minimum time complexity of

computing the single-source shortest path problem?

E log V

Q3.3

4 Points

Is it always possible to find the shortest path in between any source s and destination d vertices where $s, d \in V$?

- (●) True
- ( ) False

Q3.4

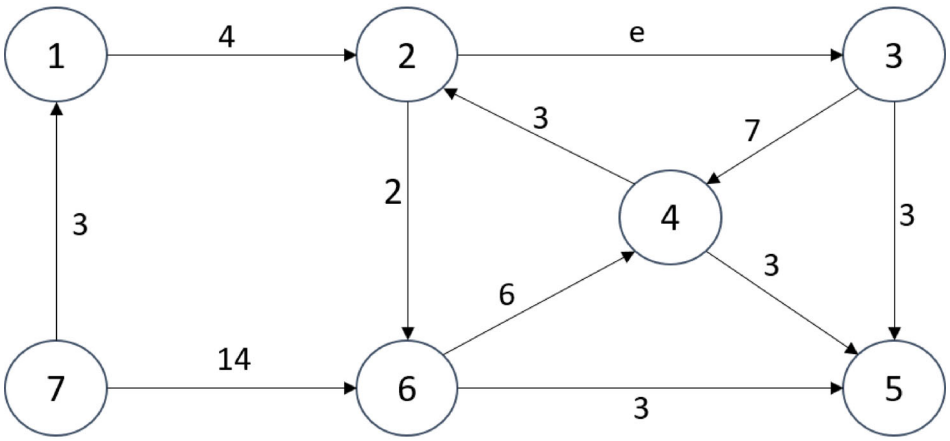
6 Points

Explain the rationale for Q3.4. Please explicitly state all possible causes.

Q4

20 Points

Answer the following questions for the graph given below. Assume that we are trying to calculate the shortest paths originating from vertex 1.



Q4.1
10 Points

For which value(s) of e it is not possible to use Dijkstra's algorithm?

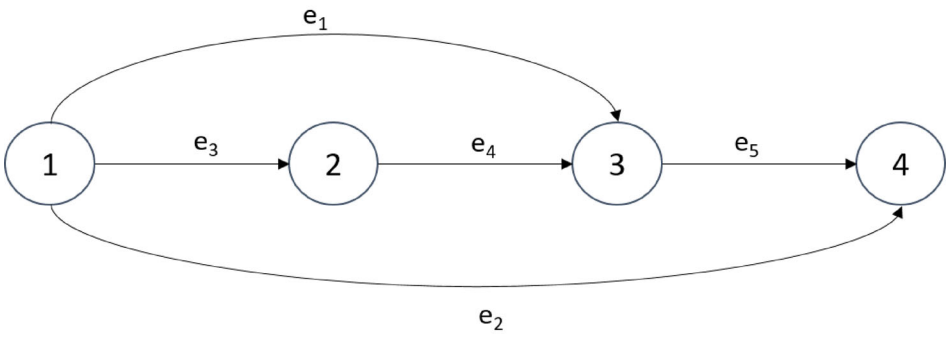
- [ ] 1
- [ ] 10
- [x] -2
- [x] -4
- [ ] None

Q4.2
10 Points

For which values(s) of e it is not possible to use Bellman-Ford's algorithm?

- [ ] -1
- [ ] -10
- [x] -12
- [ ] -4
- [ ] None

Q5
20 Points



Consider the graph given above, consider Bellman-Ford's algorithm with a FIFO queue, where vertex 1 is the source. Edges are processed in the order of their indices. Answer the following questions, considering all possible edge weights.

Q5.1

10 Points

For the graph in the picture (not in general), minimum number of calls to relax function in the best case?

E

Q5.2

10 Points

For the graph in the picture (not in general), maximum number of calls to relax function in the worst case?

E V

Quiz 4 - Shortest Paths

GRADED

STUDENT

Mehmet Taha Usta

TOTAL POINTS

37 / 101 pts

QUESTION 1

Academic Honesty **1** / 1 pt

QUESTION 2

True/False Questions **10** / 30 pts

2.1 (no title) **0** / 5 pts

2.2 (no title) **0** / 5 pts

2.3 (no title) **0** / 5 pts

2.4 (no title) **5** / 5 pts

2.5 (no title) **0** / 5 pts

2.6 (no title) **5** / 5 pts

QUESTION 3

(no title) **6** / 30 pts

3.1 (no title) **0** / 10 pts

3.2 (no title) **0** / 10 pts

3.3 (no title) **0** / 4 pts

3.4 (no title) **6** / 6 pts

QUESTION 4

(no title) **20** / 20 pts

4.1 (no title) **10** / 10 pts

4.2 (no title) **10** / 10 pts

QUESTION 5

(no title) **0** / 20 pts

5.1 (no title) **0** / 10 pts

5.2 (no title) **0** / 10 pts