Quiz 11's Solutions
(Sample from a submission)

(1) In this question the sample space is chosing any 3 random people from 100 people and this is $\binom{100}{3}$.

And favourable case is Zeynep wins one of the prizes so Zeynep is one of 3 prize winners. So there are still 2 prizes to win and 99 people can win it and this is $\binom{99}{2}$. All of those cases Zeynep wins one of the prizes.

$$Pr = \frac{(\text{Favourable Case})}{(\text{Sample Space})} = \frac{\binom{99}{2}}{\binom{100}{3}} = \frac{\frac{99.98}{2}}{\frac{100.99.98}{3,2}}$$

$$= \frac{\cancel{99}.\cancel{98}}{\cancel{2}} \cdot \frac{3.\cancel{2}}{100.\cancel{99}.\cancel{98}} = \frac{3}{100}$$

$$\boxed{Pr = \frac{3}{100} = 0{,}03}$$

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(2) When die shows 1, 3 or 5, it does not show any even numbers so we can just consider showing odd numbers cases. When a fair die rolls sample space is 6, but favourable case is 3 (odd numbers). And it's probability is $\frac{3}{6} = \frac{1}{2}$. When we roll that die 6 times, all rolls are independent from each other. Because one of them doesn't effect other one. So the probability of a die never shows an even number when it is rolled six times is

$$Pr = \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} = \left(\frac{1}{2}\right)^6 = \frac{1}{64} \qquad \boxed{Pr = \frac{1}{64}}$$

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(3) In the question our sample space (possible outcomes) is 100 (Because there are 100 positive integer not exceeding 100). And favourable case is the number of numbers that are divisible by 5 or 7.

A = Positive integers equal or less than 100 are divisible by 5
B = Positive integers equal or less than 100 are divisible by 7.
$A \cap B$ = Positive integers equal or less than 100 are divisible by 5 and 7.

$A = \{5, 10, \ldots, 100\} \longrightarrow |A| = \frac{100-5}{5} + 1 = 19 + 1 = 20$

$B = \{7, 14, \ldots, 98\} \longrightarrow |B| = \frac{98-7}{7} + 1 = 13 + 1 = 14$

$A \cap B = \{35, 70\} \longrightarrow |A \cap B| = 2$

Favourable Case $\longrightarrow |A| + |B| - |A \cap B| = 20 + 14 - 2 = 32$

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$$Pr = \frac{\text{Favourable Case}}{\text{Sample Space}} = \frac{32}{100} = \frac{8}{25} \qquad \boxed{Pr = \frac{8}{25}}$$

$\{1, 2, \ldots, 100\} \leftarrow$ Sample Space

$S = \{TTT, TTH, THT, THH, HHH, HTT, HTH, HHT\}$

(4) a. $E_1 = \{TTT, TTH, THT, THH\}$

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$$P(E_1) = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{4}{8} = \frac{1}{2}$$

$E_2 = \{THT, THH, HHT, HHH\}$

$$P(E_2) = \frac{4}{8} = \frac{1}{2}$$

$E_1 \cap E_2 = \{THT, THH\}$ $\qquad P(E_1 \cap E_2) = \frac{2}{8} = \frac{1}{4}$ ✓

$P(E_1) \cdot P(E_2) = P(E_1 \cap E_2) \longrightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ ✓ E_1 and E_2 are independent

b. $E_1 = \{TTT, TTH, THT, THH\}$

$P(E_1) = \dfrac{\text{Number of favourable outcomes}}{\text{Number of passible outcomes}} = \dfrac{4}{8} = \dfrac{1}{2}$

$E_2 = \{HHT, THH\}$

$P(E_2) = \dfrac{2}{8} = \dfrac{1}{4}$

$P(E_1 \cap E_2) = \{THH\}$ $\quad P(E_1 \cap E_2) = \dfrac{1}{8}$

$P(E_1) \cdot P(E_2) = P(E_1 \cap E_2) \longrightarrow \dfrac{1}{2} \cdot \dfrac{1}{4} = \dfrac{1}{8}$ ✓ E_1 and E_2 are independent

c. $E_1 = \{TTT, TTH, HTT, HTH\}$

$P(E_1) = \dfrac{\text{Number of favourable outcomes}}{\text{Number of passible outcomes}} = \dfrac{4}{8} = \dfrac{1}{2}$

$E_2 = \{HHT, THH\}$

$P(E_2) = \dfrac{2}{8} = \dfrac{1}{4}$

$E_1 \cap E_2 = \dfrac{0}{8} \longrightarrow$ It is not passible two consecutive heads when second coin comes up tail.

$P(E_1 \cap E_2) = \dfrac{0}{8} = 0$

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$P(E_1) \cdot P(E_2) = P(E_1 \cap E_2) \longrightarrow \dfrac{1}{2} \cdot \dfrac{1}{4} \Rightarrow \dfrac{1}{8} \neq 0$ E_1 and E_2 are not independent

5 a. In any permutation, event of 1 precedes 2 or event of 2 precedes 1 have same probability. So they are equally likely.

$\boxed{Pr(1 \text{ precedes } 2) = Pr(2 \text{ precedes } 1) = \dfrac{1}{2}}$

(b) In any permutation, event of n precedes 1 or 1 precedes n have same probability. So they are equally likely.
Likewise, event of $(n-1)$ precedes 2 or 2 precedes $(n-1)$ have same probability. So they are equally likely, too.

$$Pr(n \text{ precedes } 1) = Pr(1 \text{ precedes } n) = \frac{1}{2}$$

$$Pr((n-1) \text{ precedes } 2) = Pr(2 \text{ precedes } (n-1)) = \frac{1}{2}$$

We have four different numbers. So one of them doesn't effect other one's probability in any permutation. So those two events are independent from each other.

$$Pr(n \text{ precedes } 1 \text{ and } (n-1) \text{ precedes } 2) = Pr(n \text{ precedes } 1) \times Pr((n-1) \text{ precedes } 2)$$

$$\boxed{Pr = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}}$$

(c) In any permutation, we can choose any 3 positions for n, 1 and 2. We must put n to the first place so we can put 1 and 2 to any place. We can permute 1 and 2 in $2!$ ways. The rest of the numbers in permutation can permute $(n-3)!$ ways.

$$Pr = \frac{\text{Favourable Case}}{\text{Sample space}} = \frac{2! \cdot \binom{n}{3} \cdot (n-3)!}{n!} = \frac{2 \cdot n \cdot (n-1) \cdot (n-2) \cdot (n-3)!}{3 \cdot 2 \cdot n!}$$

Possible Outcomes

$$\boxed{Pr = \frac{\cancel{2} \cdot \cancel{n!}}{3 \cdot \cancel{2} \cdot \cancel{n!}} = \frac{1}{3}}$$