# Questions

1) In $\mathbb{R}_2$, define $(x,y)\oplus(x',y')=(x+x', y+y')$ and $c\odot(x,y)=(0,0)$. Is $\mathbb{R}_2$ a vector space with $\oplus$ and $\odot$? $(c\in\mathbb{R})$

2) Let V be the set of all positive real numbers. Define
$u\oplus v=u.v$ , $c\odot v=v^c$ for any $u,v\in V$ and $c\in\mathbb{R}$.
Prove that V is a vector space.

3) Determine whether or not the following set W is a subspace of V.
a) $W=\{(2t,-3t)\mid t\in\mathbb{R}\}$ , $V=\mathbb{R}_2$.
b) $W=\{(2t+3,-4t)\mid t\in\mathbb{R}\}$ , $V=\mathbb{R}_2$
c) $W=\{(a,b,c)\mid a,b,c\in\mathbb{R},\ 3a+b-2c=0\}$ , $V=\mathbb{R}_3$.
d) $W=\{a_0+a_1t+a_2t^2\mid a_i\in\mathbb{R},\ a_1=3a_0\}$ , $V=P_2$
e) $W=\left\{\begin{bmatrix} a & b & c\\ d & e & f\end{bmatrix}\mid a,b,c,d,e,f\in\mathbb{R},\ a+c=0,\ b+d+f=0\right\}$ ; $V=M_{23}$
f) $V=C(-\infty,\infty)=\{f\mid f:\mathbb{R}\to\mathbb{R}$ is continuous$\}$,
$W_1=\{$all constent functions$\}$ , $W_2=\{$All functions such that $f(0)=0\}$
$W_3=\{$all functions s.t. $f(0)=3\}$ $W_4=\{$All differentiable functions$\}$

4) Determine the elements of the following subspaces in $\mathbb{R}_3$.
a) $\langle(4\ 2\ -6),(-2\ -1\ 3)\rangle$ b) $\langle(-1\ -3\ 2),(1\ 2\ -1),(1\ 1\ -1)\rangle$

5) Let V be a vector space and $\{\alpha_1,\alpha_2,\alpha_3\}$ is a linearly independent subset of V. Determine whether the following subsets are lin. independe
a) $\{\alpha_1,\alpha_2,d\alpha_3\}$ $(d\in\mathbb{R}, d\neq 0)$ b) $\{\alpha_1+2\alpha_3,\ 3\alpha_2-\alpha_3\}$ c) $\{\alpha_1,\ \alpha_2+d\alpha_3,\ \alpha_3\}$ $(d\in\mathbb{R}, d\neq 0)$

6) Show that $\{1-t,\ 3-t^2,\ t+4t^2\}$ is a basis for $P_2$.

7) Let $H=\left\{\begin{bmatrix} a & c & a+2c\\ 3b & a-b & c+a\end{bmatrix}\mid a,b,c\in\mathbb{R}\right\}$. Show that H is a subspace of $M_{23}$ and find a basis for H.

8) Find a basis for the subspaces
a) $\langle(1\ 2\ 3)\rangle$ in $\mathbb{R}_3$; b) $\langle(4\ 1\ 1),(-1\ 1\ 2)\rangle$ in $\mathbb{R}_3$;
c) $\langle(1\ 0\ 3\ 2),(1\ -4\ -5\ -2),(-2\ 1\ -4\ -3),(3\ -2\ 5\ 4)\rangle$ in $\mathbb{R}_4$.
d) $\left\{\begin{bmatrix} a\\ b\\ c\end{bmatrix}\mid 2a+b-c=0\right\}$ in $\mathbb{R}^3$.
e) $\{at^3+bt^2+ct+d\mid c=a-2d,\ b=5a+3d\}$ in $P_3$.

9) Find a basis and the dimension of the solution space of

$$\begin{aligned} x_1 + 2x_2 - x_3 + 3x_4 &= 0 \\ 2x_1 + 2x_2 - x_3 + 2x_4 &= 0 \\ x_1 \qquad + 3x_3 + 3x_4 &= 0 \end{aligned}$$

10) Find a basis for $\mathbb{R}^3$ that includes the vectors $\begin{bmatrix}1\\0\\2\end{bmatrix}$ and $\begin{bmatrix}0\\1\\3\end{bmatrix}$.

11) Let $L: M_{22} \to \mathbb{R}$ be defined by $L\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\right) = a+d$. Is $L$ a linear transformation?

12) Let $L: P_1 \to P_1$ be a linear trans. for which we know that $L(t+1) = 2t+3$, $L(t-1) = 3t-2$. a) Find $L(6t-4)$ b) Find $L(at+b)$.

13) Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear trans. represented by the matrix $\begin{bmatrix}1 & 3 & 1\\ 1 & 2 & 0\\ 0 & 1 & 1\end{bmatrix}$ with respect to the natural basis for $\mathbb{R}^3$. Find $L\left(\begin{bmatrix}0\\1\\2\end{bmatrix}\right)$.

14) Let $S = \{(1,-1),(-2,3)\}$ and $T = \{(1,0,1),(2,-1,3),(-2,0,-3)\}$ be ordered bases for $\mathbb{R}_2$ and $\mathbb{R}_3$, respectively. If $A = \begin{bmatrix}-3 & 7\\ -1 & 1\\ -3 & 6\end{bmatrix}$ is the representation of $L$ with respect to $S$ and $T$, then find $L((0,1))$.

15) Let the basis $S = \{\alpha_1, \alpha_2, \alpha_3\}$ of $\mathbb{R}_3$ where $\alpha_1 = (-1,2,1)$, $\alpha_2 = (2,0,1)$, $\alpha_3 = (0,1,1)$.

a) If $L(\alpha_1) = -4$, $L(\alpha_2) = 1$, $L(\alpha_3) = -2$ for $L: \mathbb{R}_3 \to \mathbb{R}$, find the general definition of $L$.

b) If $T = \{2\}$, then find $[L]_S^T$.

c) Find the matrix of $L$ w.r.t. the natural bases for $\mathbb{R}_3$ and $\mathbb{R}$.

16) Find all eigenvalues and asscociated eigenvectors of a) $A = \begin{bmatrix}4 & 0 & -6\\ 0 & 1 & 0\\ 3 & 0 & -5\end{bmatrix}$

b) $A = \begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & -1 & 1\end{bmatrix}$ c) $L: \mathbb{R}^{2\times1} \to \mathbb{R}^{2\times1}$, $L(u_1,u_2) = (3u_1 - 5u_2, 2u_1 - 3u_2)$.

17) Let $H$ be the set of all skew symmetric matrices in $M_{33}$.

a) Show that $H$ is a subspace of $M_{33}$.

b) Show that $H$ is isomorphic to $\mathbb{R}^3$.

18) Find a basis for the row space of $A = \begin{bmatrix}-1 & 2 & 0 & 3\\ 0 & 1 & 2 & 0\\ -1 & 3 & 2 & 3\end{bmatrix}$. What is rank $A$?