BBM 205
Problem Set 2:
Proof Techniques

1. Use a direct proof to show that the sum of two even integers is even.

2. Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

3. Use a direct proof to show that the product of two odd numbers is odd.

4. Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

5. Show that these statements are equivalent, where a and b are real numbers: (i) a is less than b, (ii) the average of a and b is greater than a, and (iii) the average of a and b is less than b.

6. Find a counterexample to the statement that every positive integer can be written as the sum of the squares of three integers.

7. Prove the triangle inequality, which states that if x and y are real numbers, then $|x| + |y| \geq |x + y|$ (where $|x|$ represents the absolute value of x, which equals x if $x \geq 0$ and equals $-x$ if $x < 0$.

8. Prove or disprove that if a and b are rational numbers, then a^b is also rational.

9. (Spring 2014) Let $n_1, n_2, \ldots, n_t$ be positive integers. Show that if $n_1 + n_2 + \cdots + n_t - t + 1$ objects are placed into t boxes, then for some i $(1 \leq i \leq t)$, the ith box contains at least n_i objects.

10. (Spring 2015) Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

11. (Spring 2015) Prove that if x is rational and $x \neq 0$, then $1/x$ is rational.

12. (Spring 2015) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then either $x \geq 1$ or $y \geq 1$.

13. (Spring 2015) Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers.
14. (Spring 2015) Suppose that there are nine students in a discrete mathematics class at a small college.
    (a) Show that the class must have at least five male students or at least five female students.
    (b) Show that the class must have at least three male students or at least seven female students.
15. (Fall 2016) Prove the inequality, which states if x and y are real numbers, then $|x| + |y| \geq |x + y|$.
16. (Fall 2016) Use a proof by contradiction to prove that the product of an irrational number and a rational number is irrational.