QUESTIONS - II

1) Find the inverse of $A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ by computing A^6.

2) Find A when $A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

3) If A and B are invertible symmetric matrices such that $AB = BA$, show that A^{-1} and AB are also invertible and symmetric.

4) Find the determinant of the following matrices by using properties.

a) $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{bmatrix}$

5) By using the properties of the determinant, show that

$$\det \begin{bmatrix} a & b & b & a & b \\ b & b & a & b & a \\ b & a & b & a & b \\ a & b & a & b & b \\ b & a & b & b & a \end{bmatrix} = (2a+3b)(a-b)^4.$$

6) Find the determinant of 3×3 Vandermonde matrix.

7) If $A^{-1} = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 3 \\ 3 & 1 & -1 \end{bmatrix}$, then find $\operatorname{adj} A$.

8) For $A = \begin{bmatrix} -1 & 1 & 0 \\ 5 & -3 & 4 \\ 2 & 1 & -1 \end{bmatrix}$ find a row echelon matrix B and a product P of elementary matrices such that $B = PA$. Find P^{-1}.

9) Let $\begin{aligned} x + y + z &= 5 \\ x - 2y - 3z &= -1 \\ 2x + y - z &= 3. \end{aligned}$ Solve the system by Cramer's rule.

10) Let a, b, c be any numbers. Are the matrices

$A = \begin{bmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{bmatrix}$ row equivalent?