Automotive Engineering, FIZ137 1st Midterm Exam | Instructor: Emre S. Taşcı | 13 / 11 / 2015

Choose 5 questions among the 7

1) Some of the physical laws belonging to alternative universes/computer games are listed below, among the wrong ones. Analyze each of them and mark the wrong ones, giving your reasons.

[E: Mechanical Energy; K: Kinetic Energy; U: Potential Energy; ρ: density;
μ_k: Kinetic friction coefficient; P: Power]

(4 points each)

a) $x=vt^2+at$

b) $\frac{dE}{dt}=mv\frac{dv}{dt}$

c) $v=\sqrt{(\frac{3mat}{\rho V})}$

d) $K=\frac{Pt}{3}\frac{mv^2}{Fx}$

e) $\mu_k=\frac{mv^2x}{U^2at^2}$

2) A particle's trajectory is given as: $y(x)=-\frac{1}{2}x^2+6x+5$.

a) If at t=0s, the object is located at (0,5)m and at t=5s located at (10,15); what are the components and magnitude of the average velocity for this interval? (7 Points)

b) If the x-component of its velocity is constant, then derive the y component of the velocity and acceleration with respect to time ($v_y(t)=?, a_y(t)=?$) (13 points)

3) A ball is dropped from rest from a height h. Each time it bounces off the ground, it loses half of its mechanical energy.

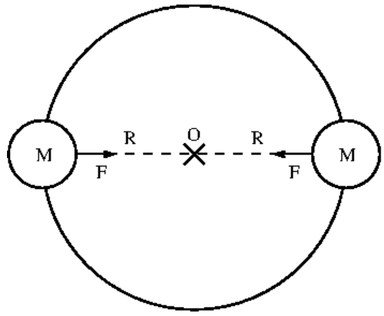
a) After n^{th} bounce, what is the ratio of its speed with respect to the speed it had just before the 1^{st} bounce? (10 Points)

b) Roughly draw the plots of x-t, v-t and a-t. (10 Points)

4) Two identical planets, each having a mass of M, orbit in a circle (radius R) around their mid point O. The force between them is attractive and related to their mass and the distance between them (2R) as :

$$F=\frac{GM^2}{4R^2}$$

where G is a constant.



a) Find the orbital speed of each planet. (6 Points)

b) Find the period of orbit. (4 Points)

c) Find the energy required to separate the two planet to infinity (in terms of G, M and R). (10 Points)

$$\int\frac{1}{x}dx=\ln|x|+C\,;\int\frac{1}{x^2}dx=-\frac{1}{x}+C$$

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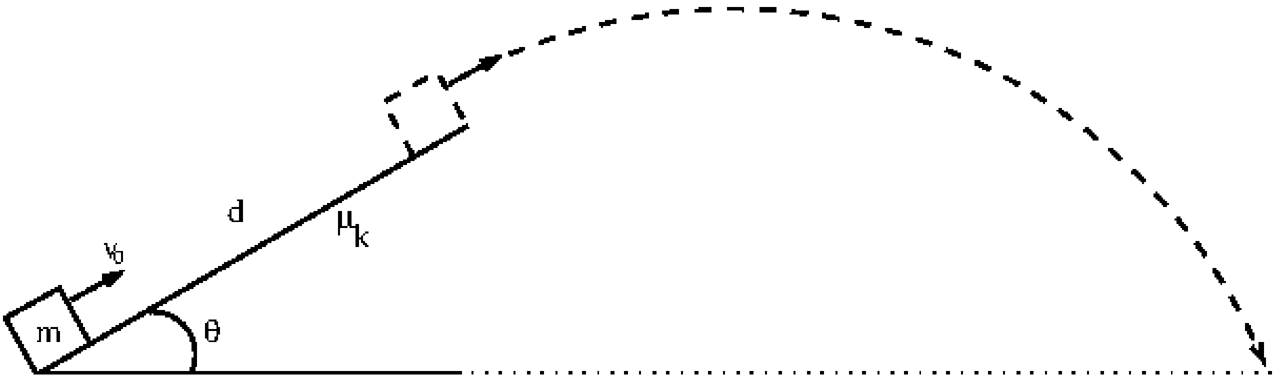
5) Two cars are coming toward each other, each with a velocity of magnitude v as observed from a person standing by the side of the road.

a) Calculate the energy according to the observer that will be released when the two cars crash. (5 Points)

b) Calculate the energy that will be released when the two cars crash, according to one of the drivers. (5 Points)

c) If you have calculated the results of (a) and (b) as same, does this point to the conservation of energy? If you have calculated different results for (a) and (b), then what is the difference between the two parts? If the event is recorded by two cameras, one by the observer standing still at the side of the road, another by a camera attached to one of the cars, which recording will appear as strange and why? (10 Points)

6) A block of mass m=3kg has been sent upwards on an inclined plane making an angle of $\theta=37^o$ by an initial velocity of v_0=16m/s. The coefficient of kinetic friction between the block and the plane, μ_k is 0.5 and the total length of the plane's side on which the block moves is d=1m.

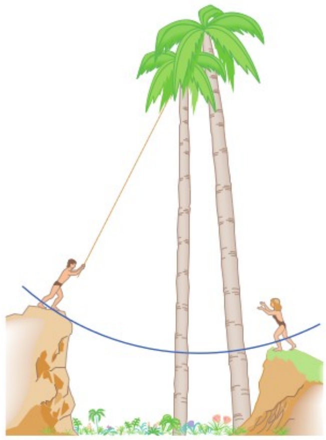


a) Show that after being launched from the end of the plane, the object will travel a duration of t in the air before hitting the ground where t satisfies the equation:

$5t^2-1.2t-0.6=0$ (having a root at t $\approx$ 0.5s). (15 Points)

b) How much further along the x-direction will the block have moved at the end of its flight with respect to its starting position? (5 Points)

7) Tarzan, of weight of mg, swings from the vine. The vine's length is r and the lowest point of the vine's arc is h=r/2 from Tarzan's starting point. Until the lowest part is covered (i.e., half distance of the ideal swing) everything is fine but upon passing, the vine starts to weaken and the moment a force of half of Tarzan's weight is applied, it breaks. What is the angle with the vertical when it breaks? (20 Points)



1) In terms of units:

a.) $m \stackrel{?}{=} \frac{m}{\cancel{s}} \cdot s^{2} + \frac{m}{s^{2}} \cdot \cancel{s}$ ✗

b.) $\frac{kg\, m/s^{2} \cdot m}{s} \stackrel{?}{=} kg\, m/s\; m/s^{2}$ ✓

c.) $\frac{m}{s} \stackrel{?}{=} \sqrt{\frac{kg\, m/s^{2} \cdot s}{kg/m^{3}\; m^{3}}} = \sqrt{\frac{m}{s}}$ ✗

d.) $kg\frac{m^{2}}{s^{2}} \stackrel{?}{=} \frac{J\, \cancel{s}}{\cancel{s}} \quad J \to kg\frac{m}{s^{2}}m \quad \frac{kg\, m^{2}/s^{2}}{kg\, m/s^{2}\, m}$ ✓

e.) $1 \stackrel{?}{=} \frac{kg\, m^{2}/s^{2}\, m}{kg\, m/s^{2}\, m\; kg\, m/s^{2}\, m\; m/s^{2}\, s^{2}} = \frac{s^{2}}{kg\, m^{2}}$ ✗

2) a.) $\Delta x = 10m \quad \Delta y = 10m \quad \Delta t = 5s$

$$v_{avg,x} = \frac{\Delta x}{\Delta t} = \frac{10m}{5s} = 2m/s$$

$$v_{avg,y} = \frac{\Delta y}{\Delta t} = \frac{10m}{5s} = 2m/s$$

$$\boxed{\vec{v}_{avg} = 2\,m/s\,\hat{i} + 2\,m/s\,\hat{j} \qquad v_{avg} = |\vec{v}_{avg}| = \sqrt{2^2+2^2}\,m/s = 2\sqrt{2}\,m/s}$$

b) $y(x) = -\frac{1}{2}x^2 + 6x + 5$ (units are adjusted accordingly)

$v_x(t) = \text{const} \Rightarrow v_x = v_{avg,x} = 2m/s$

$$v_y(t) = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = -\frac{1}{2}\cdot 2x\frac{dx}{dt} + 6\frac{dx}{dt} = -x v_x + 6 v_x$$

$$\left.\begin{array}{l} v_x = 2m/s \\ x = v_x t \end{array}\right\} \rightarrow v_y(t) = -v_x^2 t + 12$$

$$= -4t + 12 \quad m/s$$

$$a_y(t) = -4\,m/s^2$$

$$y(t) = -4\frac{t^2}{2} + 12t + y_0$$

$$y(0) = y_0 = 5m \Rightarrow \boxed{y(t) = -2t^2 + 12t + 5\ m}$$

3) a.) $U_0 = mgh$, just before hitting the ground the first time:

$$K_0 = \frac{1}{2}mv_0^2 = mgh = U_0$$

$$\rightarrow v_0^2 = 2U_0$$

After 1^{st} bounce: $E = \frac{mgh}{2} = \frac{U_0}{2}$

After 2^{nd} bounce: $E = \frac{U_0}{2^2}$

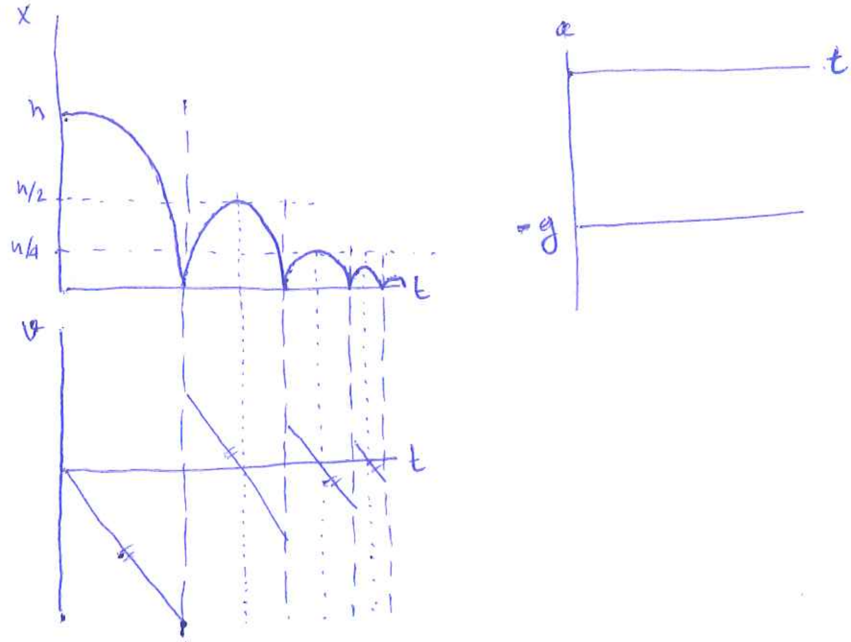
⋮

After n^{th} bounce: $E = \frac{U_0}{2^n} \Rightarrow \frac{U_0}{2^n} = \frac{1}{2}mv_n^2 \rightarrow v_n^2 = \frac{2U_0}{2^n}$

$$\Rightarrow \left(\frac{v_n}{v_0}\right)^2 = \frac{2U_0/2^n}{2U_0} = \frac{1}{2^n}$$

$$\boxed{\frac{v_n}{v_0} = 2^{-\frac{n}{2}}}$$

3) b)



4) a.) $F = \frac{GM^2}{4R^2} = ma_{rad} = \frac{Mv^2}{R} \rightarrow v = \sqrt{\frac{GM}{4R}} = \frac{1}{2}\sqrt{\frac{GM}{R}}$

b.) $T = \frac{2\pi R}{v} = 4\pi\sqrt{\frac{R^3}{GM}}$

c) $W + K_i + U_i = K_f + U_F \qquad K_f = U_f = 0 \qquad U_i = 2\left(-\int_R^\infty \frac{GM^2}{4x^2}dx\right) = 2\left(-\frac{GM^2}{4}\int_R^\infty \frac{dx}{x^2}\right)$

$= 2\left(+\frac{GM^2}{4}\frac{1}{x}\Big|_R^\infty\right) = 2\left(0 - \frac{GM^2}{4R}\right)$

$= -\frac{GM^2}{2R}$

$W + \underbrace{\frac{1}{2}Mv^2 + \frac{1}{2}Mv^2}, - \frac{GM^2}{2R} = 0$

$\Rightarrow W = -\frac{GM^2}{4R} + \frac{GM^2}{2R} = \frac{GM^2}{4R}$

5) In the event of a crash, the energy that will be releaseD is the kinetic energy.

a.) $K = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

b.) $K' = \frac{1}{2}M(2v)^2 = 2Mv^2$

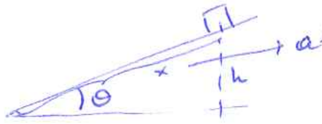
c.) Inertial Reference Frame is the one in which the person "standing" by the side of the RoaD is stationary. Therefore, the Recording by the camera on the side of the Road will appEAR to Be normal, while the one attached to the car will appear strange (the particles will Be appearing to fly off backwords from the car that was stationary with respect to the camera).

$\theta = 37°$; $\sin\theta = 0.6$; $\cos\theta = 0.8$

b) (with $v_0 = 4\,m/s$)

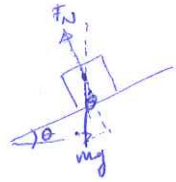
$$K_i = \frac{1}{2} m v_0^2 = 24\,J$$

How much does the object go along the inclined plane?



at a distance x, its potential energy is $U = mgh = mg\,x.\sin\theta$

Along this movement, the energy dissipated due to friction:



$$U_f = f\,x = \mu F_N x = \mu.(mg.\cos\theta).x$$

$$\Rightarrow \frac{1}{2} m v_0^2 = mg\,x \sin\theta + \mu\, mg \cos\theta\, x$$

$$8 = (9{,}8.0.6 + 9{,}8.0{,}5.0.8)\,x$$

$$\Rightarrow x = \frac{8}{9.8}\,m = 0.816\,m < 1 \Rightarrow$$ the object stops before reaching to the end!

(with $v_0 = 16\,m/s$)

a.) $\frac{1}{2}.3.16^2 = 3.9{,}8.0{,}6.x + 0.5.3.9{,}8.0{,}8.x \Rightarrow x = 13.061\,m > 1 \Rightarrow$ the block reaches to the end!

Let's substitute 1m for x and calculate the speed of the block as it reaches the edge:

$$\frac{1}{2}.3.16^2 = 3.9{,}8.0{,}6.1 + 0{,}5.3.9{,}8.0{,}8.1 + \frac{1}{2}\,3\,v_f^2$$

$$\Rightarrow v_f^2 = 236.4\,m^2/s^2 \Rightarrow v_f = 15.375\,m/s$$

$\rightarrow v_{fx} = v_f.\cos\theta = 12.30\,m/s$

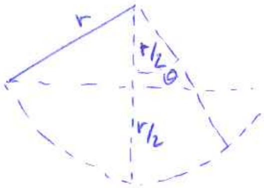
$\rightarrow v_{fy} = v_f \sin\theta = 9.23\,m/s$

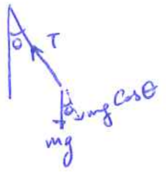
$$y - y_0 = v_{fy}t - \frac{1}{2}gt^2 \longrightarrow -0.6 = 9.23t - \frac{1}{2}9.8t^2$$

$$\Rightarrow \boxed{4.9t^2 - 9.23t - 0.6 = 0} \rightarrow t \begin{cases} -0.065\,s \\ \boxed{1.947\,s} \end{cases}$$

b.) $t = 1.947\,s \longrightarrow x = v_{fx}t = 12{,}3 \times 1{,}947 = \underline{23.95\,m}$

7)

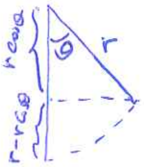




$\longrightarrow \quad F_{net} = \frac{mv^2}{r}$

$$T - mg\cos\theta = \frac{mv^2}{r}$$

$$\frac{mg}{2} - mg\cos\theta = \frac{mv^2}{r} \quad (1)$$



$\longrightarrow \quad mg\frac{r}{2} = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$

$$2mgr\left(\frac{1}{2} - (1-\cos\theta)\right) = mv^2$$

$$2mgr\left(\cos\theta - \frac{1}{2}\right) = mv^2 \quad (2)$$

$(2) \to (1):\quad \frac{mg}{2} - mg\cos\theta = 2mgr\left(\cos\theta - \frac{1}{2}\right)\cdot\frac{1}{r}$

$$\frac{1}{2} - \cos\theta = 2\cos\theta - 1$$

$$3\cos\theta = \frac{3}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \boxed{\theta = 60^\circ}$$

Question: Why we can't directly write:

$$T = \frac{mg}{2}, \quad T = mg\cos\theta$$

$$\to \frac{mg}{2} = mg\cos\theta \to \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \quad ?$$

Re-imagine the question, but this time let the vine break at $\frac{mg}{3}$ instead of $\frac{mg}{2}$. Then,

$(1):\quad \frac{mg}{3} - mg\cos\theta = \frac{mv^2}{r}$

but (2) would be the same : $2mgr\left(\cos\theta - \frac{1}{2}\right) = mv^2$

and joining (1) & (2) would yield:

$$\frac{mg}{3} - mg\cos\theta = 2mgr\left(\cos\theta - \frac{1}{2}\right)\frac{1}{r}$$

$$\frac{1}{3} - \cos\theta = 2\cos\theta - 1 \to 3\cos\theta = \frac{4}{3}$$

$$\boxed{\cos\theta = \frac{4}{9} \Rightarrow \theta = 63.61^\circ}$$

whereas, the incorrect approach would yield:

$$\frac{mg}{3} = mg\cos\theta \Rightarrow \cos\theta = \frac{1}{3}, \quad \theta = 70.53^\circ$$

So, a direct, yet incorrect approach/solution of $\frac{mg}{2} = mg\cos\theta \to \theta = 60^\circ$ is not acceptable.