Automotive Engineering, FIZ138 1st Midterm Exam | Instructor: Emre S. Taşcı | 1 / 4 / 2016

Name: Number: Signature:

{ SOLUTIONS }

Mark the 5 questions you want to be evaluated from (each question is worth 20 points):

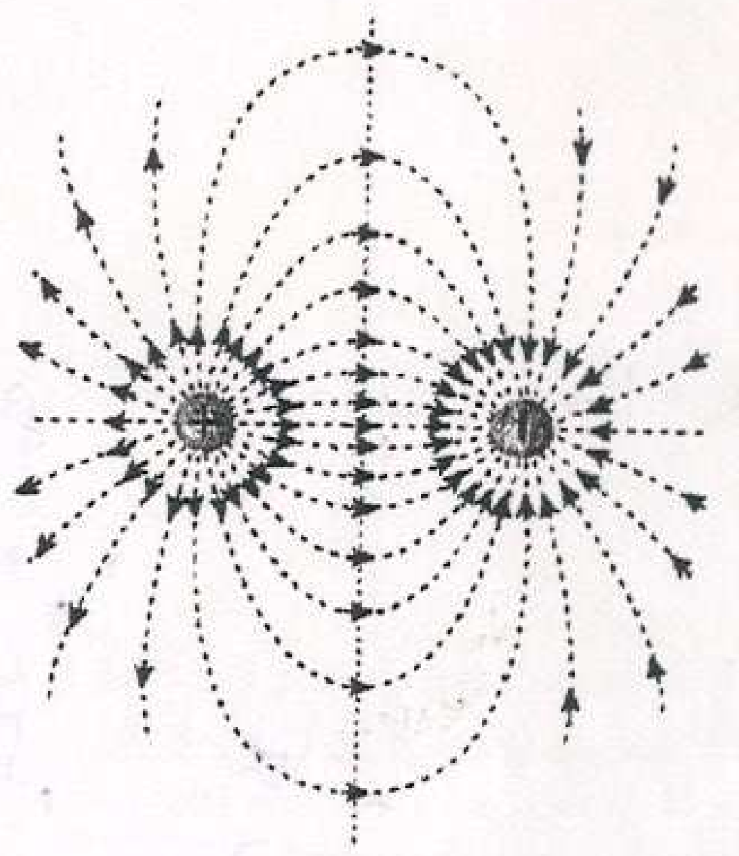
Mark:	Q1	Q2	Q3	Q4	Q5	Q6	Q7
Grade:							

Q1) a) **New Coulomb's Law:** Suppose that we invent a new unit of charge, "newC" which is defined as $1\,newC = \frac{1}{\sqrt{4\pi\epsilon_0}}C$ in order to get rid of the Coulomb's constant *k*. Discuss the problem with this approach.

b) Show that $R \times C$ multiplication has a unit of seconds.

c) An electric dipole is enclosed in a cubic box of side length *a*. If the electric flux for this cubic box is Φ_0, what will it be for another cubic box with side length *2a*? (The boxes' centers coincide)

d) An electric dipole is centered at the origin (with the charges placed at $x = \pm d/2$. What is the ratio of the magnitudes of electric field at a distance *x* and 6*x* from the origin of the dipole where $x >> d$?



a.) If we are to take only the value of ϵ_0 (i.e. without the units), then the result would be in units of $\frac{C^2}{m^2}$ which is not equal to Newton, hence we would have found something other than force, and still would need a constant $k' = 1 \frac{Nm^2}{C^2}$ to fix it.
If, on the other hand we had included the unit of ϵ_0 in our "new C", then we would be using something other than charge, which would be incorrect.

In summary: with such an approach, the units do not match any longer on the two sides of the equation!

b) $[|V|]$, $V = -\int E\,dx$

$$\Omega = \frac{V}{A} = \left(\frac{N}{C}m\right)\left(\frac{s}{C}\right) = \frac{N\,ms}{C^2} = \frac{kg\,m/s^2\,ms}{C^2} = \frac{kg\,m^2}{C^2 s}$$

$[|I|]$, $\frac{s}{C} \rightarrow \frac{1}{A}$

$$F = \left(\frac{C^2}{Nm^2}\right)\frac{m^2}{m} = \frac{C^2}{kg\,m/s^2\,m^2}\frac{m^2}{m} = \frac{C^2 s^2}{kg\,m^2}$$

$[|\epsilon_0|]$, $[|A|]$, $[|d|]$

$$\rightarrow \Omega \times F = \frac{\cancel{kg}\,\cancel{m^2}}{\cancel{C^2}\cancel{s}} \times \frac{\cancel{C^2}s^{\cancel{2}}}{\cancel{kg}\,\cancel{m^2}} = \underline{\underline{s}}$$

b.) Alternative: $\Omega . F = \frac{V}{A} \cdot \frac{C}{V} = \frac{C}{A} = \frac{C}{C/t} = \underline{\underline{s}}$

c) Since the net charge inside is equal to 0:

$$\Phi_0(a) = \Phi_0(2a) = 0$$

d.) $\vec{E}_{dipole} = \frac{1}{2\pi\epsilon_0}\frac{p}{x^3}$ $(x >> d)$

$$\rightarrow \frac{E(a)}{E(6a)} = \frac{1/a^3}{1/(6a)^3} = 6^3 = \underline{216}$$

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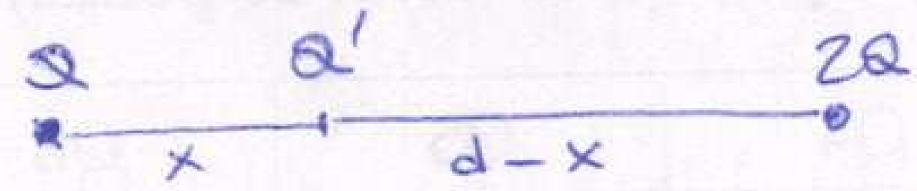
Q2) Two charges Q and 2Q are separated by a distance of *d*.

a) Find the equilibrium point for a third charge of -Q placed between the positive charges.

b) Find the equilibrium point for a third charge of -3Q placed between the positive charges.

c) Analytically (mathematically) show that it is not possible to find an equilibrium point for a third charge lying outside the line passing through the two charges.

a, b)



$$\vec F_{Q'} = \vec F_{QQ'} + \vec F_{2QQ'} = 0 \rightarrow \vec E_{Q'} = 0 \leftarrow \text{independent of } Q'$$

$$\Rightarrow (a)\equiv(b)$$

$$k\frac{Q}{x^2} = k\frac{(2Q)}{(d-x)^2}$$

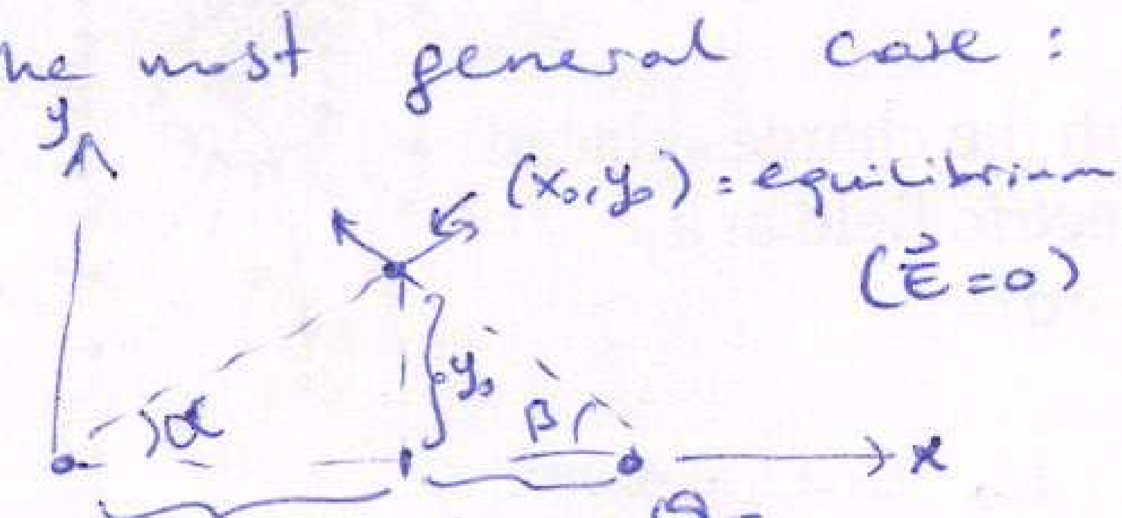
$$(d-x)^2 = 2x^2$$

$$d^2-2dx+x^2 = 2x^2 \rightarrow x^2+2dx-d^2=0$$

$$x = \frac{-2d\pm\sqrt{4d^2+4d^2}}{2} = -d\pm d\sqrt{2} = -d(1\mp\sqrt{2})$$

$$d(\sqrt{2}-1)$$

c) In the most general case:



$\vec E$ components

$$\vec E_A = \frac{kQ_A}{(x_0^2+y_0^2)^{3/2}}(x_0\hat i + y_0\hat j)$$

$$|\vec E_A| = \frac{kQ_A}{x_0^2+y_0^2}$$

$$\cos\alpha = \frac{x_0}{(x_0^2+y_0^2)^{1/2}}$$

$$\sin\alpha = \frac{y_0}{(x_0^2+y_0^2)^{1/2}}$$

$$\vec E_B = \frac{kQ_B}{[(d-x_0)^2+y_0^2]^{3/2}}(-(d-x_0)\hat i + y_0\hat j)$$

$$|\vec E_B| = \frac{kQ_B}{(d-x_0)^2+y_0^2}$$

$$\cos\beta = \frac{(d-x_0)}{[(d-x_0)^2+y_0^2]^{1/2}}$$

$$\sin\beta = \frac{y_0}{[(d-x_0)^2+y_0^2]^{1/2}}$$

At the equilibrium point components of $\vec E$ must sum to 0:

x-components:

$$\frac{A}{(x_0^2+y_0^2)^{3/2}}x_0 = \frac{B}{[(d-x_0)^2+y_0^2]^{3/2}}(d-x_0) \rightarrow \frac{A}{B} = \left[\frac{x_0^2+y_0^2}{(d-x_0)^2+y_0^2}\right]^{3/2}\frac{(d-x_0)}{x_0} \quad (1)$$

x_0 can't be zero!

y-components:

$$\frac{A}{(x_0^2+y_0^2)^{3/2}}y_0 = -\frac{B}{[(d-x_0)^2+y_0^2]^{3/2}}y_0 \rightarrow \frac{A}{B} = -\left[\frac{x_0^2+y_0^2}{(d-x_0)^2+y_0^2}\right]^{3/2} \quad (2)$$

Substitute (2) in (1):

$$\frac{A}{B} = \left(-\frac{A}{B}\right)\frac{d-x_0}{x_0} \rightarrow x_0-d = x_0$$

$$\Rightarrow d = 0\ ???$$

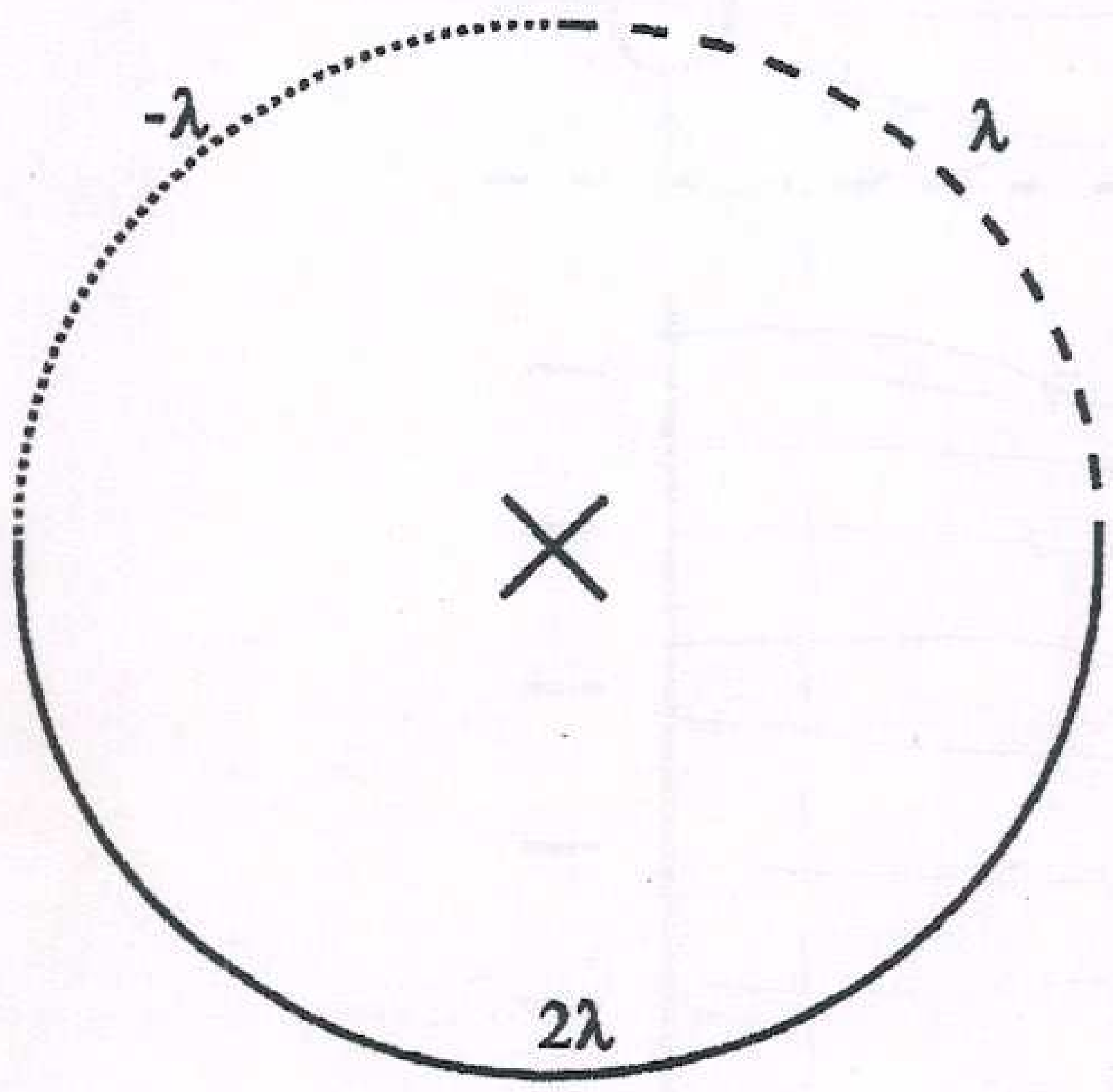
→ There is a consistent solution only when y is taken to be zero, i.e.:

$$y=0 \Rightarrow (1): \quad \frac{A}{B} = \left[\frac{x_0^2}{(d-x_0)^2}\right]^{3/2}\frac{d-x_0}{x_0} = \frac{x_0^2}{(d-x_0)^2}\ ✓$$

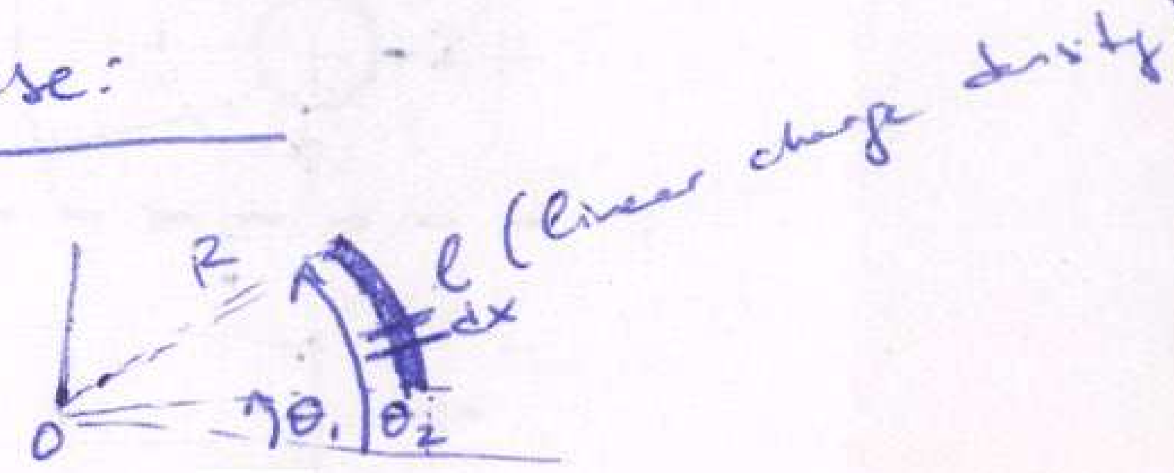
$$\left((2): \quad \frac{A}{(\ )^{3/2}}\cdot 0 = -\frac{B}{(\dots)^{3/2}}\cdot 0 \Rightarrow 0=0\ ✓\right)$$

Name: Number: Signature:

Q3) 3 wires with charge densities λ $(\theta=0\ldots\frac{\pi}{2})$, $-\lambda$ $(\theta=\frac{\pi}{2}\ldots\pi)$ and 2λ $(\theta=\pi\ldots2\pi)$ are arranged into a loop of radius R as shown in the figure. Calculate the electric field at the center in vector form.



A General Case:



$$dq = \ell\, dx = \ell . R\, d\theta$$

$$|d\vec{E}_0| = k\frac{dq}{R^2} = k\,\ell\frac{R\,d\theta}{R^2} = \frac{k\ell}{R}\,d\theta$$

$$d\vec{E}_0 = \frac{k\ell}{R}\,d\theta\left[-\cos\theta\,\hat{i} - \sin\theta\,\hat{j}\right]$$

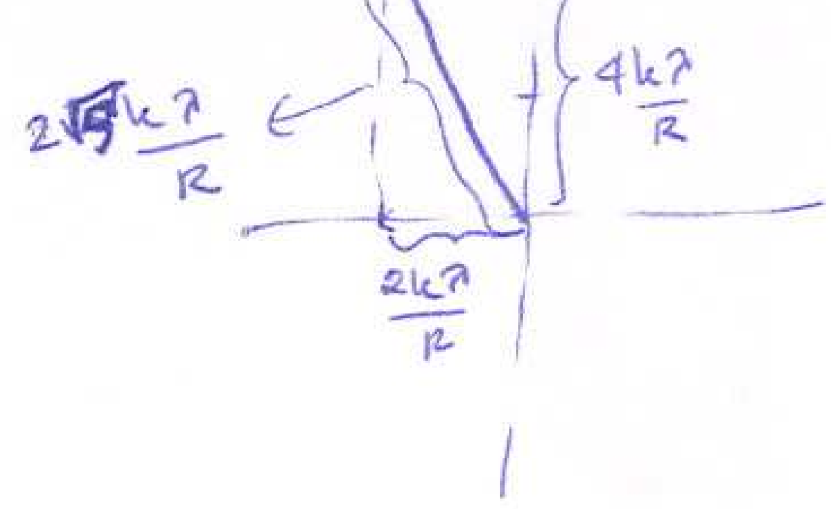
$$\rightarrow \vec{E}_0 = -\frac{k\ell}{R}\int_{\theta_1}^{\theta_2}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\,d\theta = -\frac{k\ell}{R}\begin{bmatrix}(\sin\theta_2 - \sin\theta_1)\,\hat{i} \\ -(\cos\theta_2 - \cos\theta_1)\,\hat{j}\end{bmatrix}$$

θ_1	θ_2	ℓ	$-\frac{k\ell}{R}\left[(\sin\theta_2 - \sin\theta_1)\hat{i} - (\cos\theta_2 - \cos\theta_1)\hat{j}\right]$
0	$\pi/2$	λ	$-\frac{k\lambda}{R}\left[(1-0)\hat{i} - (0-1)\hat{j}\right] = -\frac{k\lambda}{R}(\hat{i}+\hat{j})$
$\pi/2$	π	$-\lambda$	$-\frac{k(-\lambda)}{R}\left[(0-1)\hat{i} - (-1-0)\hat{j}\right] = \frac{k\lambda}{R}(-\hat{i}+\hat{j})$
π	2π	2λ	$-\frac{k(2\lambda)}{R}\left[(0-0)\hat{i} - (1-(-1))\hat{j}\right] = \frac{-2k\lambda}{R}(-2\hat{j}) = \frac{4k\lambda}{R}\hat{j}$

+

$$\Rightarrow \vec{E}_0 = \frac{k\lambda}{R}\left[-\hat{i}-\hat{j}-\hat{i}+\hat{j}+4\hat{j}\right] = \frac{k\lambda}{R}(-2\hat{i}+4\hat{j})$$

$$\vec{E}_0 = \frac{2k\lambda}{R}(-\hat{i}+2\hat{j})$$

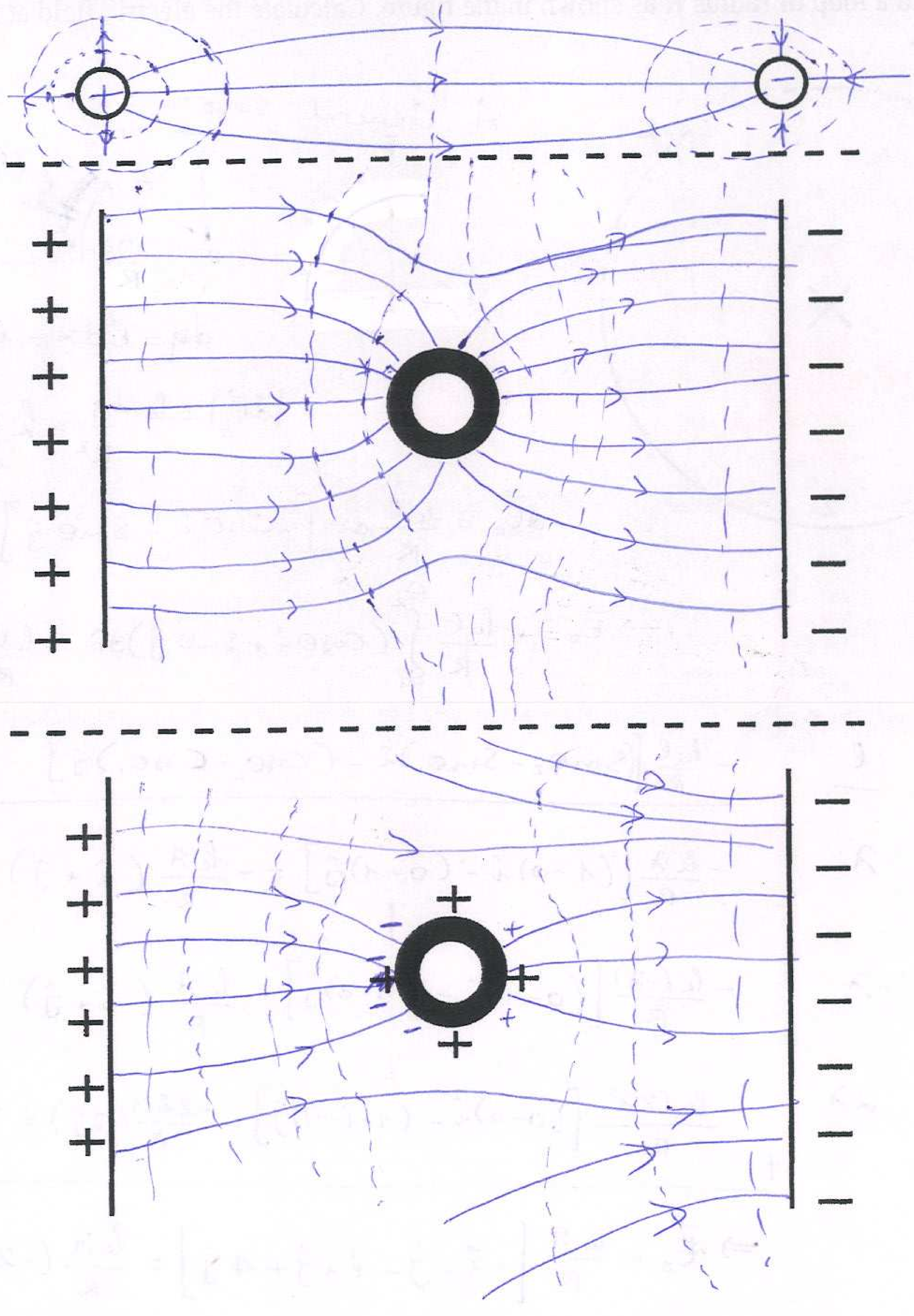


Q4) Draw the electric field lines and equipotential surfaces for the following systems:

a) Two oppositely charged point-like particles separated by a distance d

b) A conducting, spherical shell with no net charge placed in the middle of the distance between the parallel plates of a charged capacitor.

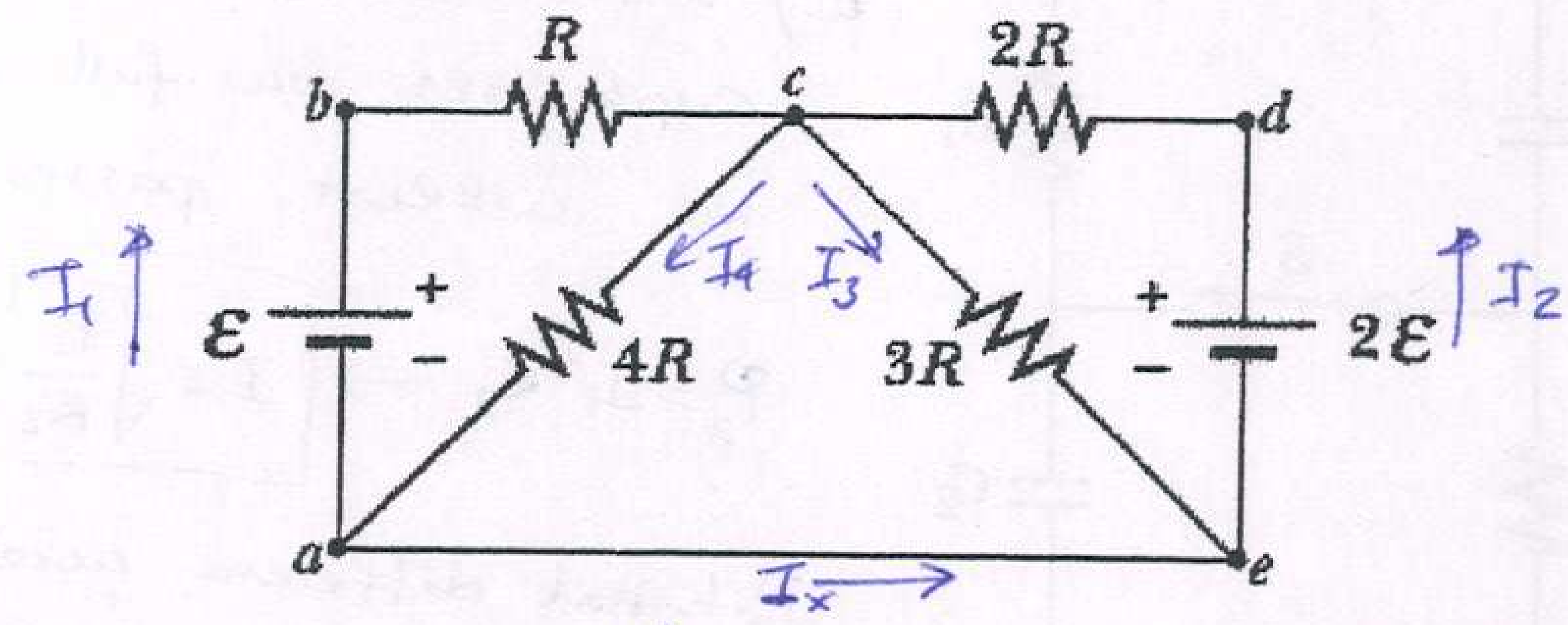
c) A conducting, positively charged (total charge Q_S) spherical shell placed in the middle of the distance between the charged parallel plates of Q_A and $-Q_B$, respectively ($Q_S < Q_A < |Q_B|$).



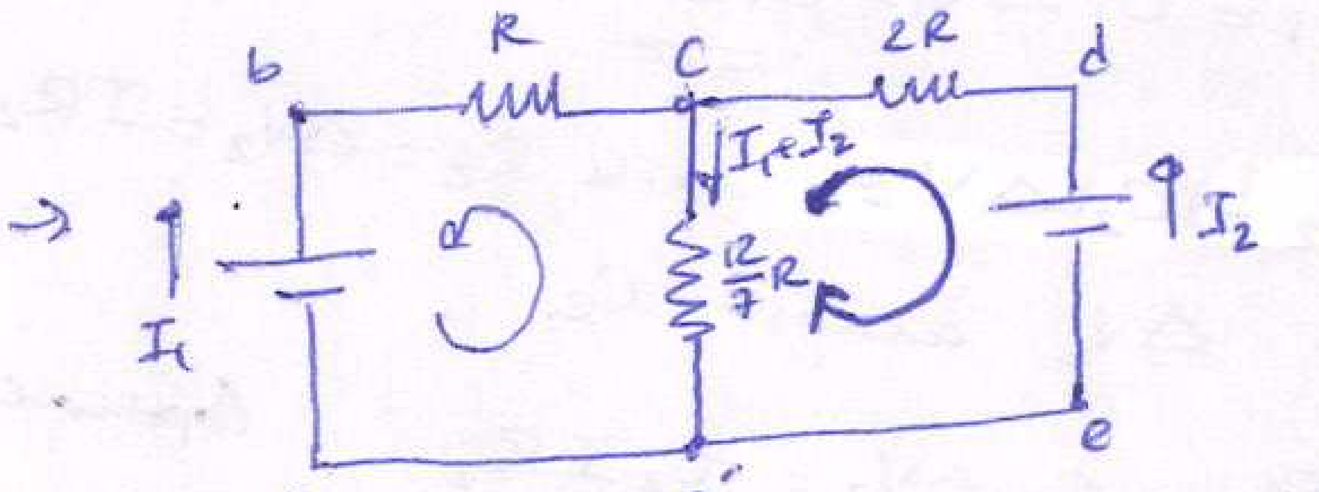
Automotive Engineering, FIZ138 1st Midterm Exam | Instructor: Emre S. Taşcı | 1 / 4 / 2016

Name: Number: Signature:

Q5) Calculate the direction and magnitude of the current in the wire between a and e in terms of R and ε.



$4R \parallel 3R \rightarrow R_{eq} = \left(\frac{1}{4R} + \frac{1}{3R}\right)^{-1} = \frac{12}{7}R$



bc aa': $I_1 R + (I_1+I_2)\left(\frac{12}{7}R\right) = \mathcal{E}$

cd aa: $(I_1+I_2)\left(\frac{12}{7}R\right) + I_2(2R) = 2\mathcal{E}$

$\frac{19}{7} I_1 + \frac{12}{7} I_2 = \frac{\mathcal{E}}{R}$

$\frac{\cancel{12}^{6}}{7} I_1 + \frac{\cancel{26}^{13}}{7} I_2 = \frac{\cancel{2\mathcal{E}}}{R} \quad \times\left(-\frac{12}{13}\right)$

$\frac{19}{7} I_1 + \frac{12}{7} I_2 = \frac{\mathcal{E}}{R}$

$-\frac{12\cdot 6}{13\cdot 7} I_1 - \frac{12}{13}\cdot\frac{13}{7} I_2 = -\frac{12}{13}\frac{\mathcal{E}}{R}$

$\frac{I_1}{7}\left(19 - \frac{12\cdot 6}{13}\right) = \frac{1}{13}\frac{\mathcal{E}}{R}$

$\frac{I_1}{7\cdot\cancel{13}}\left(19\cdot 13 - 12\cdot 6\right) = \frac{1}{\cancel{13}}\frac{\mathcal{E}}{R}$

$\frac{\cancel{175}^{25}}{\cancel{7}} I_1 = \frac{\mathcal{E}}{R}$

$\boxed{I_1 = \frac{1}{25}\frac{\mathcal{E}}{R}}$

$\frac{19}{7}\frac{1}{25}\frac{\mathcal{E}}{R} + \frac{12}{7} I_2 = \frac{\mathcal{E}}{R}$

$\frac{19}{25}\frac{\mathcal{E}}{R} + 12 I_2 = \frac{7\mathcal{E}}{R}$

$12 I_2 = \left(7 - \frac{19}{25}\right)\frac{\mathcal{E}}{R}$

$I_2 = \frac{175-19}{25\cdot 12}\frac{\mathcal{E}}{R}$

$I_2 = \frac{\cancel{156}^{13}}{25\cdot\cancel{12}}\frac{\mathcal{E}}{R}$

$\boxed{I_2 = \frac{13}{25}\frac{\mathcal{E}}{R}}$

$V_{ca'} = (I_1+I_2)\cdot\frac{12}{7}R$

$= \frac{\cancel{14}^{2}\mathcal{E}\,12}{25\cancel{R}\,\cancel{7}}\cancel{R} = \frac{24}{25}\mathcal{E}$

$I_4 = \frac{V_C - V_A}{4R} = \frac{\frac{24}{25}\mathcal{E}}{4R} = \frac{6}{25}\frac{\mathcal{E}}{R}$

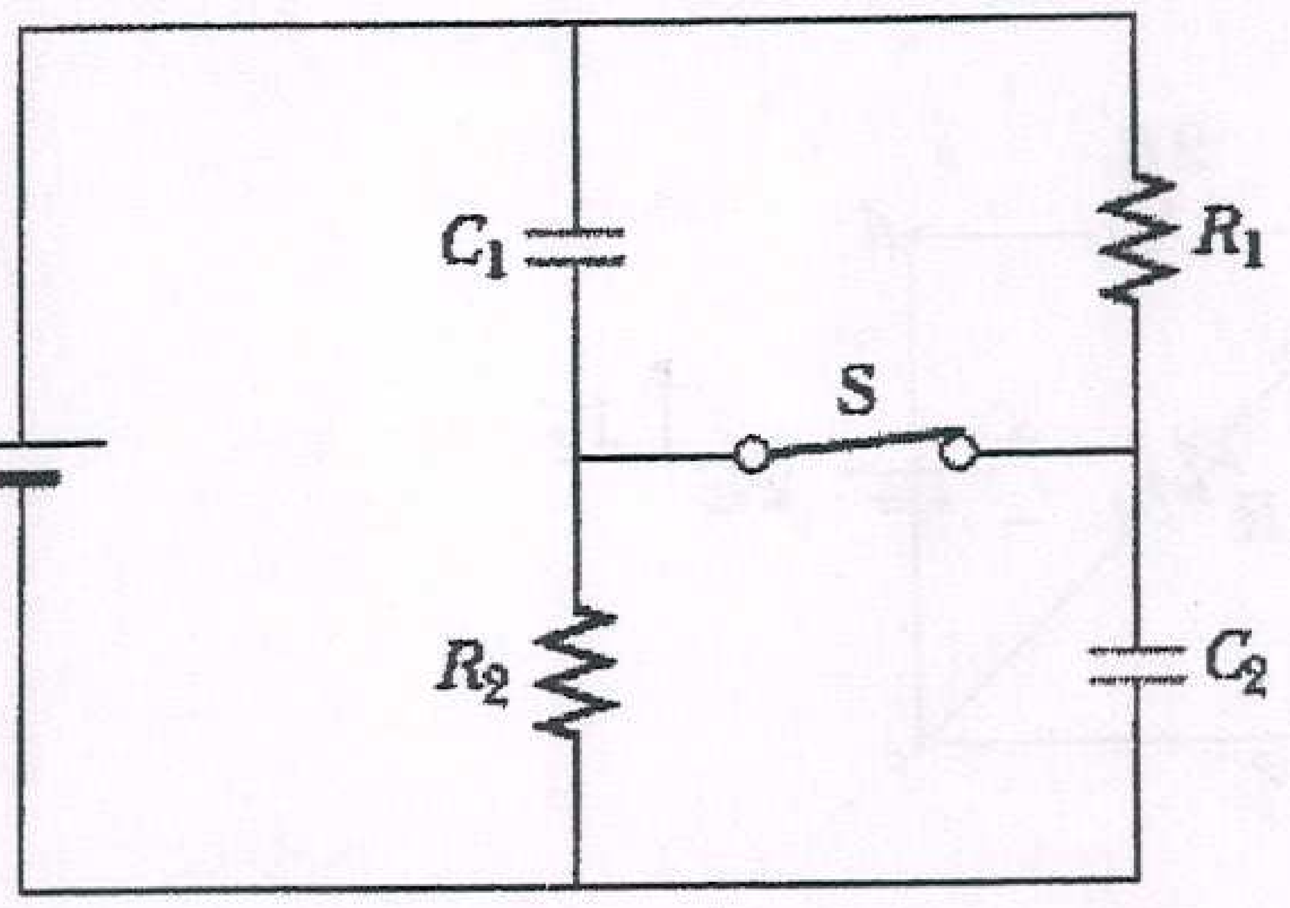
$I_4 = I_1 + I_x$

$I_x = I_4 - I_1 = \frac{6}{25}\frac{\mathcal{E}}{R} - \frac{1}{25}\frac{\mathcal{E}}{R} = \underline{\underline{\frac{1}{5}\frac{\mathcal{E}}{R}}}$

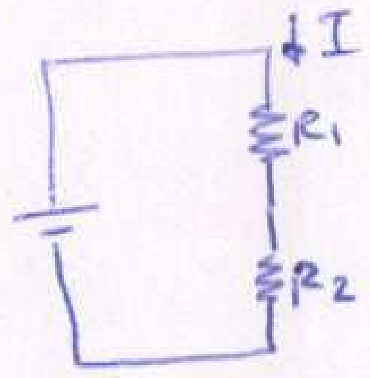
Q6) The circuit carries a constant current. The switch is closed for a long time.

a) If the power delivered to R_2 is P_2, calculate the charge on C_1.
b) After the switch is opened, and a long time has passed, calculate the charge on C_2.

(The source EMF is intentionally not given: you'll have to derive it.
Express all your results in terms of R_1, R_2, C_1, C_2 and P_2)



a) With the switch closed, the capacitors are full and thus no current passes them →.



$P_2 = I^2 R_2 \rightarrow \boxed{I = \sqrt{\frac{P_2}{R_2}}}$

Potential difference across R_1: $\Delta V_1 = I R_1$ is equal to ΔV_1 across C_1

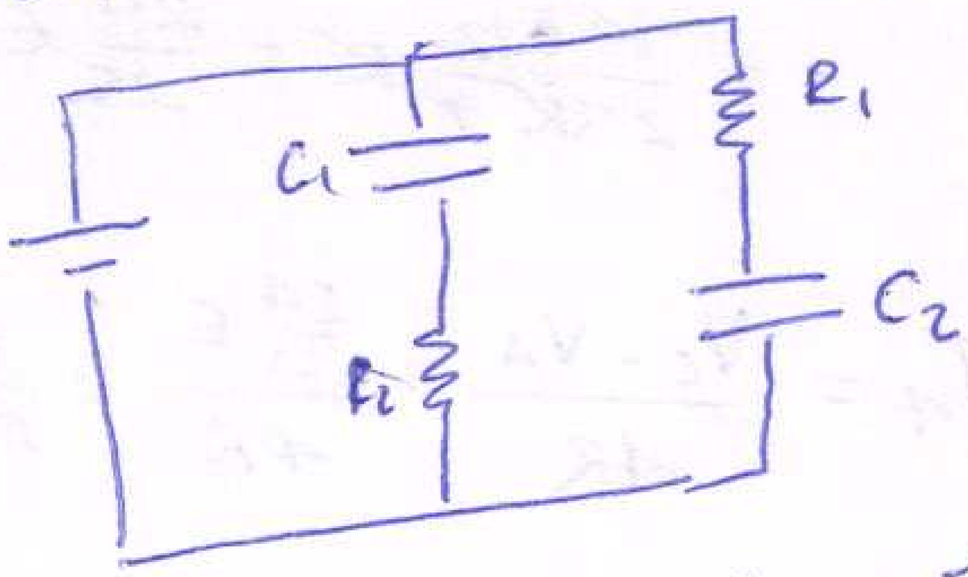
$\rightarrow Q_1 = C_1 \Delta V_1 = \underline{C_1 I R_1}$

Potential difference across R_2: $\Delta V_2 = I R_2$ is equal to ΔV_2 across C_2

$\rightarrow Q_2 = C_2 \Delta V_2 = C_2 I R_2$ (optional)

The source emf : $\mathcal{E} = I R_{eq} = I(R_1 + R_2)$

b) After the switch is opened, eventually the capacitors will once again be filled in equilibrium and they will cut the currents:

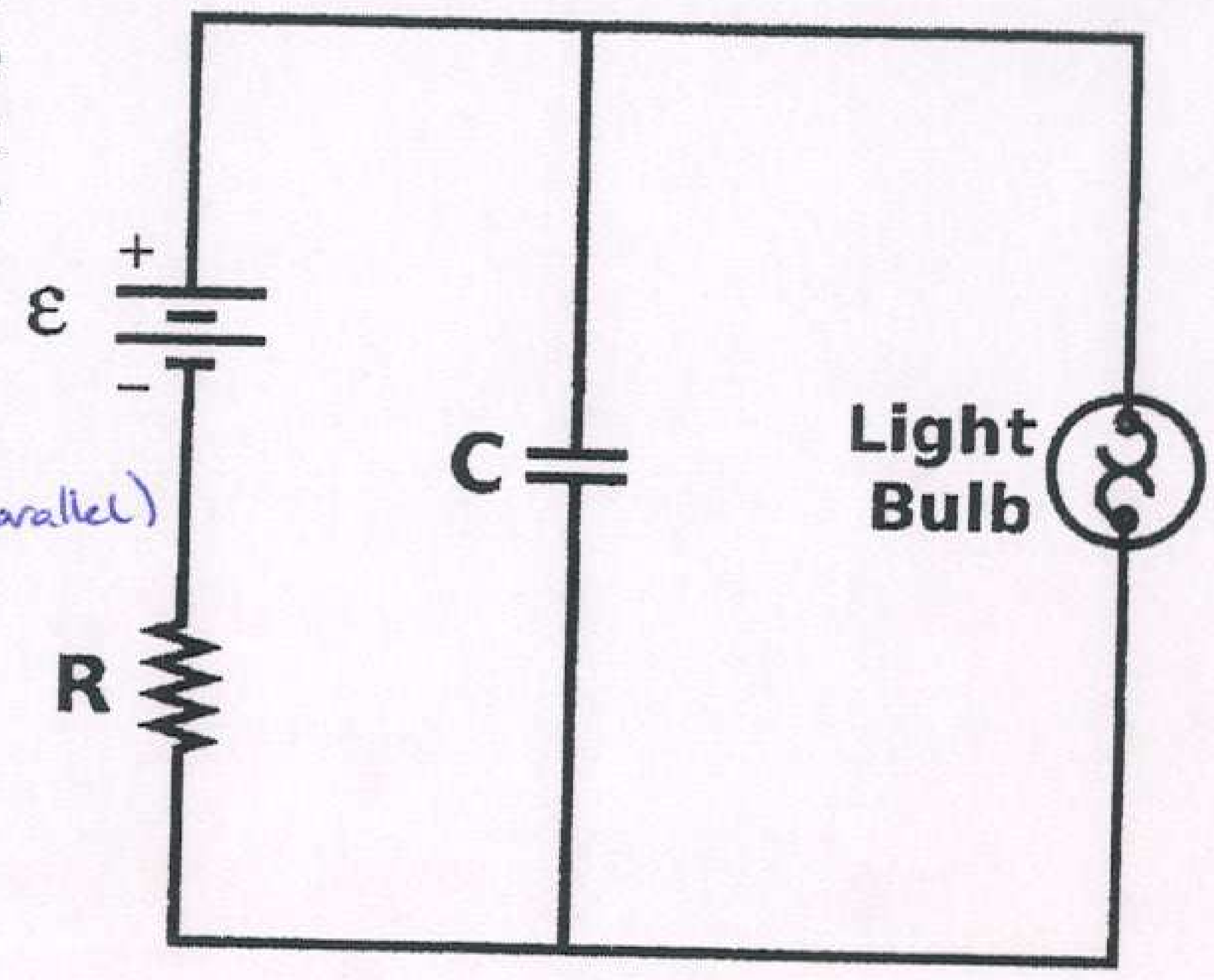


There is no current → the potential difference across each resistor is zero.
→ The potential difference across the capacitors is equal to that of the source (parallel)

$\rightarrow Q_2 = C_2 \Delta V_2' = C_2 \mathcal{E} = C_2 I (R_1 + R_2) = C_2 \sqrt{\frac{P_2}{R_2}} (R_1 + R_2)$

Name: Number: Signature:

Q7) A light bulb is connected to an RC circuit as shown in the figure. The light bulb has a voltage threshold V_L such that, below this voltage, it doesn't operate. In order to have the lamp flash n times per second, what should the value of R be in terms of n, C, ε and V_L? (Assume that the emf device is ideal, with no internal resistance)



Potential diff. across bulb = ΔV across C (parallel)

$$V_L = \varepsilon\left(1 - e^{-t/RC}\right)$$

$$\rightarrow \frac{V_L}{\varepsilon} = 1 - e^{-t/RC}$$

$$1 - \frac{V_L}{\varepsilon} = e^{-t/RC}$$

$$-t/RC = \ln\left(\frac{\varepsilon - V_L}{\varepsilon}\right)$$

$$t/RC = \ln\left(\frac{\varepsilon}{\varepsilon - V_L}\right)$$

f = n times per second

$$\rightarrow \underbrace{t = \frac{1}{f} = \frac{1}{n}}$$

$$\Rightarrow R = \frac{t}{C\ln\left(\frac{\varepsilon}{\varepsilon - V_L}\right)} = \frac{1}{nC\ln\left(\frac{\varepsilon}{\varepsilon - V_L}\right)}$$