$\begin{array}{c} {\rm BBM~205} \\ {\rm Spring~2015~Butunleme~Exam} \end{array}$

SHOW YOUR WORK TO RECEIVE FULL CREDIT. KEEP YOUR CELLPHONE TURNED OFF.

Maraa.			
Name.			
1 1 WIII C			

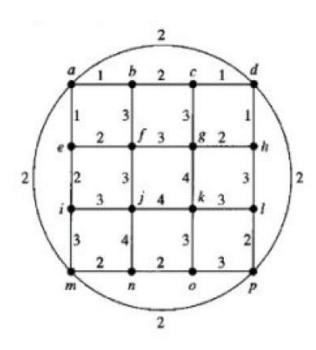
1. (3 points) Solve the recurrence relation with the given initial condition below. $a_n = 2a_{n-1} + 8a_{n-2}$; $a_0 = 4$, $a_1 = 10$.

2.	(3 points) (a) (1 point) How many bit strings of length seven either begin with two 0's or end with three 1's?
	(b) (1 point) How many subsets with more than two elements does a set with 100 elements have?
	(c) (1 point) How many ways are there to select three unordered elements from a set with five (different) elements when repetition is allowed ?
3.	 (3 points) Suppose that there are nine students in a discrete mathematics class at a small college. (a) (1.5 points) Show that the class must have at least five male students or at least five female students.
	(b) (1.5 points) Show that the class must have at least three male students or at least seven female students.

4. (4 points) Use

- (a) (2 points) Kruskal's algorithm
- (b) (2 points) Prim's algorithm

to find a minimum spanning tree for the weighted graph below.

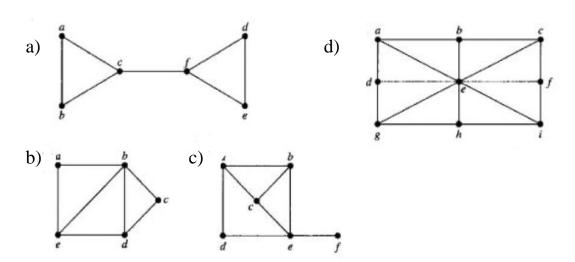


- 5. (3 points) (a) (1.5 points) Write the chromatic number of the graphs below depending on the values of m and n.
 - a) K_n b) C_n
- c) $K_{m,n}$

- (b) (1.5 points) For which values of n do these graphs have an Euler circuit?
 - a) K_n b) C_n c) Q_n

6. (2 points) Show that in a simple connected graph with at least two vertices there must be two vertices that have the same degree.

- 7. (4 points) (a) (2 points) Determine whether the given graph has a Hamilton cycle. Construct such a cycle when one exists.
 - (b) (2 points) If no Hamilton cycle exists, determine whether the graph has an Hamilton path and construct such a path if one exists.



8. (3 points) Let P(n) be the statement that

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

where n is an integer greater than 1. Show that P(n) is true for all $n \geq 2$ using induction by following the steps below.

- (a) (1 point) Show that P(2) is true.
- (b) (1 point) What is the inductive hypothesis?
- (c) (1 point) Complete the inductive step.

9. (3 points) (a) (1 point) Show that $x^2 + 4x + 17$ is $O(x^3)$.

(b) (2 points) Show that x^3 is **not** $O(x^2 + 4x + 17)$.

10. (3 points) Prove that at least one of the real numbers a_1, a_2, \ldots, a_n is greater than or equal to the average of these numbers.

11. (3 points) Show that if G is a bipartite simple graph with n vertices and e edges, then $e \le n^2/4$.

12. (3 points) Suppose that v is an endpoint of a cut edge. Prove that v is a cut vertex if and only if this vertex is not pendant.

13. (3 points) Let S(n,k) denote the number of functions from $\{1,\ldots,n\}$ onto $\{1,\ldots,k\}$. Show that S(n,k) satisfies the recurrence relation

$$S(n,k) = k^n - \sum_{i=1}^{k-1} C(k,i)S(n,i).$$