

Hacettepe Univ. MAT 124-02-05										Midterm Exam		26.04.2019	
Surname :					Instructor :					Signature			
Name :					Time :								
ID :					Duration : 90 <i>min.</i>								
4 questions, 2 pages										100 points			
1	2	3	4										

**Q1**(20 p.) Find a formula to find the area of the region that lies inside both of the circles  $r = 2(\sin \theta + \cos \theta)$  and  $r = 2 \sin \theta$ .

**Q2 (a)**(10 p.) Using the Implicit Function Theorem show that the equations  $F(x, y, z) = \cos(xz) - \sin(y + z) + 1 = 0$  and  $G(x, y, z) = x + y + z - 1 - \pi/2 = 0$  can be simultaneously solved for  $y$  and  $z$  as functions of  $x$  near  $P = (1, 0, \pi/2)$ .

**(b)**(10 p.) Evaluate the partial derivatives  $\left. \frac{\partial y}{\partial x} \right|_P$  and  $\left. \frac{\partial z}{\partial x} \right|_P$  of the functions  $y$  and  $z$  of the item **(a)** above, where  $P = (1, 0, \pi/2)$ .

**(c)** (20 p.) Find the tangent line to the curve that is the intersection of the surfaces  $F(x, y, z) = 0$  and  $G(x, y, z) = 0$ , where  $F$  and  $G$  are as in **(a)**, at the point  $P = (1, 0, \pi/2)$ .

**Q3**(20 p.) Find the critical point of the function

$$f(x, y) = xy + 2x - \ln x^2 y$$

in the open first quadrant ( $x > 0$ ,  $y > 0$ ) and show that  $f$  takes on a minimum value there.

**Q4**(20 p.) Find absolute maximum and minimum values of the function  $f(x, y) = (4x - x^2) \cos y$  on the rectangular plate  $0 \leq x \leq 5$ ,  $-\pi/2 \leq y \leq \pi$ .