

Q1 Academic Honesty

0 Points

It is a violation of the Academic Integrity Code to look at any reference material other than your textbook and lecture notes, or to give inappropriate help to someone or to receive unauthorized aid by someone in person or electronically via messaging apps such as WhatsApp. Academic Integrity is expected of all students of Hacettepe University at all times, whether in the presence or absence of members of the faculty. Do NOT sign nor take this exam if you do not agree with the honor code.

Understanding this, I declare I shall not give, use or receive unauthorized aid in this examination.

Signature (Specify your name and surname as your signature)

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While answering the following questions, please consider the implementations that we discussed in our lectures unless stated otherwise.

Q2 Minimum Spanning Trees

12 Points

Simple graphs refers graphs which do not have parallel edges between any two vertices and no self-loops (edges starting and ending at the same vertex).

Suppose that you are given a simple graph containing n vertices where the weight of every edge is set to w .

(a) (5 pts) How does the weight of the minimum spanning tree change if the weights of two edges of the original simple graph is changed from w to $w/2$? *While answering this question, you should give either an exact expression or a specific range of values.*

2 edge weight decrease W to $W/2$. Total decrease $2W$ to W
($W/2 * 2$). $2W - W = W$

(b) (7 pts) How does the weight of the minimum spanning tree change if the weights of three edges of the original simple graph is changed from w to $w/2$? *While answering this question, you should give either an exact expression or a specific range of values.*

3 edge weight decrease W to $W/2$. Total decrease $3W$ to $3W/2$
($w/2 * 3$). $3W - 3W/2 = 3W/2$

Q3 True/False Questions

12 Points

For each part, determine whether the statement is true or false. *To get full credits, you need to provide a correct reasoning.

(a) (4 pts) Mergesort need to perform $\sim N \log N$ comparisons to sort a reverse-sorted array containing N distinct elements.

☒ True

☐ False

Your reasoning:

reverse-sorted array is a worst case array for Mergesort. So Mergesort need to perform $\sim N \log N$ comparisons to sort a reverse-sorted array

(b) (4 pts) Suppose that you are inserting a key into a red-black binary search tree (BST). The height of the red-black BST can decrease after this key insertion.

☒ True

☐ False

Your reasoning:

Red-Black tree is height-balanced.

(c) (4 pts) Suppose that you are given a directed simple graph having at least 3 vertices with distinct positive edge weights, and two of the vertices are selected as the source and the target, respectively. Assuming that every vertex is reachable from the source vertex, any shortest path from the source to the target should contain the edge having the lowest weight.

☐ True

☒ False

Your reasoning:

the shortest paths do not have to have the lowest weight. As the complexity increases, the number of roads that can occur increases. The path formed may not contain the lowest weight in the options.

exp:

$$3-3-3-3-3-3 = 18$$

$$1-1-1-1-1-8 = 13$$

Q4 String Sorting

12 Points

Suppose that you want to sort the following array of strings in ascending order by using MSD radix sort (with a radix of 10):

105 340 114 240 458 673 124 456 231 111

Q4.1

6 Points

What is the array immediately after performing key-indexed counting for the third time? Your answer should be a sequence of strings of 3-digit numbers separated by whitespace, e.g. 105 340 114

105 111 114 124 240 231 340 458 456 673

Q4.2

6 Points

What is the array immediately after performing key-indexed counting for the fourth time? Your answer should be a sequence of strings of 3-digit numbers separated by whitespace, e.g. 105 340 114

105 111 114 124 231 240 340 458 456 673

Q5 Algorithmic Complexity

22 Points

Suppose A and B are given arrays of the same length n . A is sorted in increasing order, and B is not sorted. Consider the following function:

```
public static int what(int A[], int B[])
{
    for (int j = 0; j < A.length; j++)
        for (int i = 0; i < B.length-1; i++)
            if (B[j] == A[i] + A[i+1])
                return 1;
    return 0;
}
```

Q5.1

2 Points

What is the worst-case time complexity of the function?

- ☐ $O(n)$
- ☐ $\Theta(n)$
- ☒ $O(n^2)$
- ☐ $\Theta(n^2)$

Q5.2

4 Points

What task does the function perform? Explain briefly what the function does in general terms, not how it executes the task.

function looks for sum of two consecutive data($a[i]$ ve $a[i+1]$) in b.

If the value in $b[j]$ is equal to the sum of $a[i]$ and $a[i + 1]$, it returns 1, if not 0.

Q5.3

16 Points

Write a function in JAVA that performs the same task efficiently. The efficient solution has an order-of-magnitude (not a constant factor) improvement in time complexity. A function with the same or a greater (time or space) complexity will not get full credit. You may define supplementary functions for ease.

B array can be sorted to facilitate the work done.
Binary search algorithm can be used after sorting.

```
public static int what(int A[], int B[]){
    // sorting B
    Collections.sort(B);
    for (int j = 0; j < A.length; j++){
        //if (B[j] == A[i] + A[i+1])
        if(BinarySearch(B, A[i] + A[i+1]) != -1){
            return 1;
        }
    }
    return 0;
}
```

Q6

14 Points

Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(|V|)$ time, independent of $|E|$.

```
public class Cycle {
    private boolean[] marked;
    private int[] edgeTo;
    private Stack<Integer> cycle;
    /**
     * Determines whether the undirected graph {@code G} has a
     cycle and,
     * if so, finds such a cycle.
     * @param G the undirected graph
     */

    public Cycle(Graph G) {
        // does this graph have a self loop? side effect: initialize
        cycle to be self loop
        if (hasSelfLoop(G)) return;
        // does this graph have two parallel edges? side
        effect: initialize cycle to be two parallel edges
        if (hasParallelEdges(G)) return;

        marked = new boolean[G.V()];
        edgeTo = new int[G.V()];
        // depth first search
        for (int v = 0; v < G.V(); v++)
            if (!marked[v])
                // run dfs
                dfs(G, -1, v);
    }

    // extra functions
}
```

Q7 Boyer Moore

8 Points

Execute Boyer-Moore's algorithm as discussed in the lectures for the pattern "BBM202" and the text "BBM101BBM102BBM201BBM202". Answer the following two questions

Q7.1

4 Points

How many text positions are checked for the pattern, i.e. how many times the outer loop of Boyer-Moore is executed?

5

Q7.2

4 Points

How many character comparisons are performed? Note that all comparisons should be considered even if it is not a match.

12

Q8 KMP

8 Points

Build the Deterministic Finite state automaton for the pattern "ABBA", answer the following questions assuming that alphabet only contains A and B characters.

Q8.1

4 Points

Write down the first row of the dfa array (corresponding to character A). Separate each transition with a single space character.

1 1 1 4

Q8.2

4 Points

Write down the second row of the dfa array (corresponding to character B). Separate each transition with a single space character.

0 2 3 0

Q9 Huffman Coding Compression

12 Points

A:10, B:2, C:1, D:5, E:12, F:4, G:6, H:3

Assume that for the characters between A-H the frequencies are given as above. Answer the following questions.

Q9.1

3 Points

After constructing the Huffman trie, which letter(s) is/are encoded with 2 bits

☒ A

☐ B

☐ C

☐ D

☒ E

☐ F

☐ G

☐ H

Q9.2

3 Points

After constructing the Huffman trie, which letter(s) is/are encoded with 4 bits

☐ A☐ B☐ C☐ D☐ E☐ F☐ G☒ H**Q9.3**

3 Points

Given a set of characters and distinct frequencies, is there a unique Huffman coding? Explain your answer?

☐ Yes, it is unique

☒ Not unique

Explanation

No, huffman coding is a set of frequencies.
trees can be built in the same structure with using different
frequencies and different characters.

Q9.4

3 Points

Given the characters A, B, C, D what should be their frequencies such that all characters are represented with equal number of bits in the Huffman coding.

BBACCDACABDD
A->3 B->3 C->3 D->3

all freq -> 3

Final Exam

GRADED

STUDENT
Mehmet Taha Usta

TOTAL POINTS
64 / 100 pts

QUESTION 1	
Academic Honesty	0 / 0 pts
QUESTION 2	
Minimum Spanning Trees	6 / 12 pts
QUESTION 3	
True/False Questions	2 / 12 pts
QUESTION 4	
String Sorting	12 / 12 pts
4.1 (no title)	6 / 6 pts
4.2 (no title)	6 / 6 pts
QUESTION 5	
Algorithmic Complexity	16 / 22 pts
5.1 (no title)	2 / 2 pts
5.2 (no title)	4 / 4 pts
5.3 (no title)	10 / 16 pts
QUESTION 6	
(no title)	6 / 14 pts

QUESTION 7

Boyer Moore	4 / 8 pts
7.1 (no title)	4 / 4 pts
7.2 (no title)	0 / 4 pts

QUESTION 8

KMP	8 / 8 pts
8.1 (no title)	4 / 4 pts
8.2 (no title)	4 / 4 pts

QUESTION 9

Huffman Coding Compression	10 / 12 pts
9.1 (no title)	3 / 3 pts
9.2 (no title)	3 / 3 pts
9.3 (no title)	1 / 3 pts
9.4 (no title)	3 / 3 pts