## BBM 205 Spring 2015 Exam 3

## SHOW YOUR WORK TO RECEIVE FULL CREDIT. KEEP YOUR CELLPHONE TURNED OFF.

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1. (4 points) For each f(n) and g(n), fill in the blank f(n) = ...(g(n)) and g(n) = ...(f(n)) with one of the symbols  $O, \Omega$  or  $\Theta$ . There can be more than one correct symbol.

(a) (1 point) 
$$f(n) = 5 \log x, g(n) = x$$

(b) (1 point) 
$$f(n) = 2^n + n^2$$
,  $g(n) = n^3 + 3^n$ 

(c) (1 point) 
$$f(n) = (x^3 + 1)/(x + 1), g(n) = x^2$$

(d) (1 point) 
$$f(n) = n!, g(n) = 2^n$$

2. (3 points)

Show that, for any positive integer k,  $1^k + 2^k + 3^k + \cdots + n^k = \Theta(n^{k+1})$  by using the definition of  $\Theta$ -notation.

3. (3 points) Find the smallest integer n such that f(x) is  $O(x^n)$  for each of these functions.

(a) (1 point) 
$$f(x) = 2x^3 + x^2 \log x$$

(b) (1 point) 
$$f(x) = 3x^3 + (\log x)^4$$

(c) (1 point) 
$$f(x) = (x^4 + x^2 + 1)/(x^3 + 1)$$

4. (2 points) (a) (1 point) Let d be a positive integer. Use pigeonhole principle to show that among any group of d+1 integers there are two with exactly the same remainder when they are divided by d.

(b) (1 point) Let  $n_1, n_2, \ldots, n_t$  be positive integers. Use pigeonhole principle to show that if  $n_1 + n_2 + \cdots + n_t - t + 1$  objects are placed into t boxes, then for some i ( $1 \le i \le t$ ), the ith box contains at least  $n_i$  objects.

- 5. (2 points)
  - (a) (1 point) Find the coefficient of  $x^5$  in  $(4-3x)^{21}$ .
  - (b) (1 point) Find the coefficient of  $x^3y^2z^5$  in  $(x+y+z)^{10}$ .
- 6. (2 points) Use Pascal's identity to prove the following whenever n and r are positive integers.

$$\sum_{k=0}^{k=r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

7. (1 point) How many solutions are there to the inequality

$$x_1 + x_2 + x_3 \le 11$$
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where  $x_1$ ,  $x_2$ , and  $x_3$  are nonnegative integers?

- 8. (1 point) Seven women and nine men are on the faculty in the computer science department at a school.
  - (a) (.5 points) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
  - (b) (.5 points) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?
- 9. (2 points) How many ways are there to distribute five balls into seven boxes if each box must have at most one ball in it if
  - (a) (.5 points) both the balls and boxes are labelled?
  - (b) (.5 points) the balls are labelled, but the boxes are unlabelled?
  - (c) (.5 points) the balls are unlabelled, but the boxes are labelled?
  - (d) (.5 points) both the balls and boxes are unlabelled?