FRICTION, DRAG FORCE & CENTRIPETAL FORCE

Friction

About 20% of the Gasoline used in an automobile is needed to counteract Friction.

→ if there was no friction - we couldn't drive a car, walk, hold a Pen, write, nails and screws would be useless, woven cloth would fall apart, knots would untie.

THOUGHT EXPERIMENT

1) Slide a book over the counter

→ it slows down, then stops

a ← opposite direction, hence F is in the opposite direction to movement also.

2) Push horizontally to make it travel at constant velocity.

There must be some countering force, otherwise it would accelerate

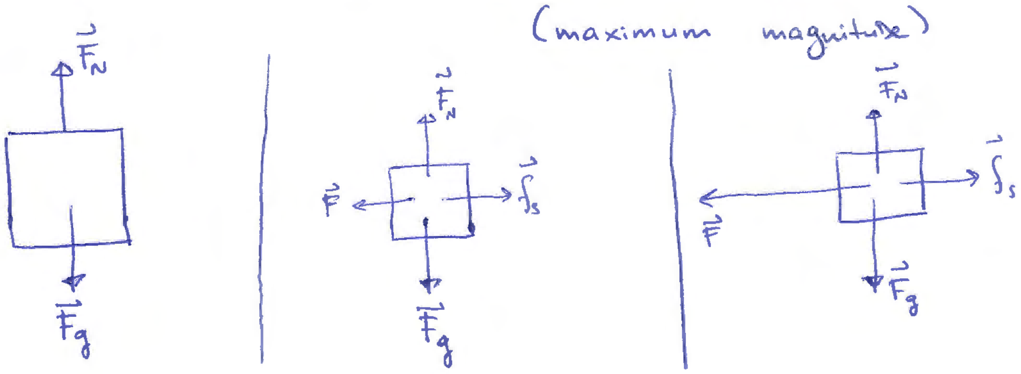
⇒ Friction force: same magnitude, opposite direction.

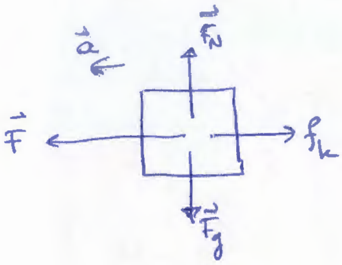
3) push a heavy crate and it won't move.

Push harder → still doesn't move, so the frictional can change in magnitude, such that two forces still Balance.

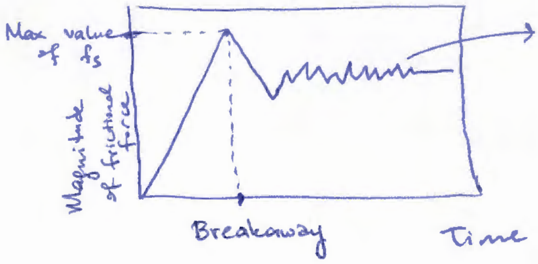
push with all your strength and it will start moving

→ so there must be an upper limit (maximum magnitude)





To Maintain the speed, weaken force $\vec{F}$ to match the weak frictional force.



$\longrightarrow f_k$ (approximately constant)

Static frictional force can only match growing applied force.

f_s : static frictional force

f_k : kinetic frictional force

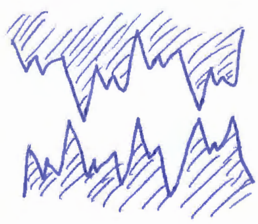
$f_s > f_k$ (usually)

Many factors behind Friction: If two highly polished, carefully cleaned metal surfaces are brought together in vacuum, they can not slide over each other. Because the surfaces are too smooth, many of the atoms on the surfaces bond with each other $\longrightarrow$ cold weld.

When two ordinary surfaces are placed together, only the high points touch each other.

The actual microscopic area of contact is much less than the apparent macroscopic contact area $\sim 10^4$

$\sim$ cold-weld



If the applied force is great enough to pull one surface across other, there is first a tearing of welds (at breakaway) and then a continous re-forming and tearing of welds as movement occurs $\longrightarrow f_k$

If the two surfaces are pressed together harder, many more points cold-weld $\longrightarrow f_s$

PROPERTIES of FRICTION

$\vec{F}$ attempts to slide the Body along the surface.

1) If the body does not move, then the static frictional force $\vec{f}_s$ and the component of $\vec{F}$ that is parallel to the surface balance each other.

   Equal in magnitude, $\vec{f}_s$ is directed opposite of that component of $\vec{F}$.

2) f_s has a max value $f_{s,max}$

   $$f_{s,max} = \mu_s F_N$$

   μ_s : Coefficient of static friction

   F_N is the magnitude of the normal force on the body.

3) If the body begins to slide

   $$f_k = \mu_k F_N$$

   μ_k : Coefficient of kinetic friction

2 & 3 : If the Body presses harder

Newton's 3rd Law : $\vec{F}_N$ is greater

Direction of f_s & f_k always parallel to surface and opposite of the direction of the attempted sliding.

μ_k, μ_s : dimensionless (determined experimentally)

Ex: A car's wheels are locked in an Emergency Breaking.

Record: 1960, Jaguar on the M1 highway in England

290 m long!

Assume $\mu_k = 0.60$, $a =$ constant during Breaking

→ How fast was the car going?

$-f_k = ma$ → due only to a kinetic frictional Force.

$f_k = \mu_k F_N$, $F_N = mg$

$$a = -\frac{f_k}{m} = -\frac{\mu_k mg}{m} = -\mu_k g$$

$$\underset{0}{v^2} = v_0^2 + 2a\underbrace{(x - x_0)}_{290\,m}$$

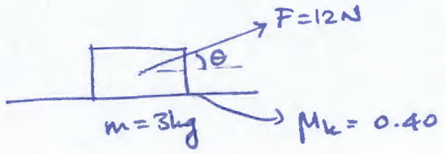
$$v_0 = \sqrt{2\mu_k g(x-x_0)}$$

$$= \sqrt{2(0.60)(9.8\,m/s^2)(290\,m)}$$

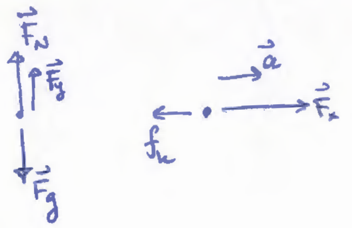
$$= 58\,m/s = 210\,km/h$$

(Actually, the marks ended only because Jaguar left the Road after 290 m → so v_0 was "at least" 210 km/h!)

Ex:



What θ gives maximum value of the Block's acceleration magnitude a?



$$\underbrace{F_N + F\sin\theta}_{F_y} - mg = m(0)$$

$$F_N = mg - F\sin\theta$$

$$\underbrace{F\cos\theta}_{F_x} - \mu_k F_N = ma$$

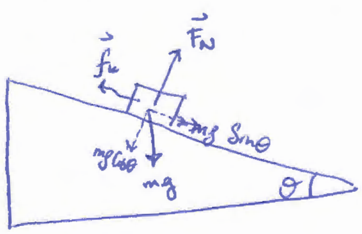
$$a = \frac{F}{m}\cos\theta - \mu_k\left(g - \frac{F}{m}\sin\theta\right)$$

Find max: $\frac{da}{d\theta} = 0 \Rightarrow -\frac{F}{m}\sin\theta + \mu_k \frac{F}{m}\cos\theta = 0$

$$\tan\theta = \mu_k$$

$$\theta = \tan^{-1}\mu_k = 21.8^\circ \approx \underline{22^\circ}$$

Ex: A block slides down a certain 35° slide in twice the time it would take it to slide down if there was no friction. What is the Coeff. of kinetic friction μ_k between the block & slide?



$$mg\sin\theta - f_k = ma$$

$$F_N - mg\cos\theta = 0$$

$$f_k = \mu_k F_N$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$l = v_0 t + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2l}{a}}$$

$$\mu_k = 0 \Rightarrow \frac{t}{t'} = \frac{\sqrt{2l/a}}{\sqrt{2l/a'}} = \sqrt{\frac{a'}{a}}$$

$$\frac{t}{t'} = 2 \rightarrow \frac{a'}{a} = 4$$

$$g\sin\theta = 4g(\sin\theta - \mu_k\cos\theta)$$

$$\theta = 35^\circ \rightarrow \underline{\mu_k = 0.53}$$

DRAG FORCE & TERMINAL SPEED

A fluid is anything that can flow - generally a gas or a liquid. When there is a relative velocity between a fluid and a body (either because the body moves through the fluid or because the fluid moves past the body (a submarine moving in the sea vs. water passing around the rocks in a river), the body experiences a <u>drag force</u> $\vec{D}$ that opposes the relative motion and points in the direction in which the fluid flows relative to the body.

Here we assume that air is the fluid, body is blunt (~baseball) rather than slender (~javelin), and the relative motion is fast enough so that the air becomes turbulent (breaks up into swirls) behind the body.

In such cases, the magnitude of the drag force $\vec{D}$ is related to the relative speed v by an experimentally determined <u>drag coefficient</u> C according to

$$D = \frac{1}{2} C \rho A v^2$$

A → effective cross-sectional area of the body (area of a cross-section taken perpendicular to the velocity v)

ρ → air density (density of the medium)

C → typically between 0.4 - 1.0 (is not actually a constant as it may change if v changes significantly but we'll ignore this fact)

When a blunt body falls from rest through air, $\vec{D}$ is directed upwards; its magnitude gradually increases from zero as the speed of the body increases. $\vec{D}$ opposes $\vec{F}_g$

$$F_{net,y} = ma_y \longrightarrow D - F_g = ma$$

if the Body falls long enough, D eventually equals F_g

$\Rightarrow a = 0 \rightarrow$ body's speed no longer increases,

the Body now falls at constant speed

called terminal Speed v_t.

$a = 0$:

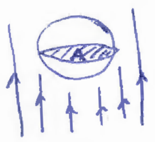
$$\frac{1}{2} C \rho A v_t^2 - F_g = 0 \rightarrow v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

Spread Eagle free-fall skydiving

Ex: A raindrop with radius $R = 1.5\,mm$ falls from a cloud $h = 1200\,m$

$C_{drop} = 0.60$, spherical shape. $\rho_w = 1000\,kg/m^3$, $\rho_a = 1.2\,kg/m^3$

a) Terminal speed?



$$A = \pi R^2 \qquad F_g = mg \qquad V = \frac{4}{3}\pi R^3 \qquad \rho_w = \frac{m}{V}$$

$$F_g = \underbrace{V \rho_w}_{m} g = \underbrace{\frac{4}{3}\pi R^3 \rho_w}_{m} g$$

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} = \sqrt{\frac{8\pi R^3 \rho_w g}{3C\rho_a \pi R^2}} = \sqrt{\frac{8R\rho_w g}{3C\rho_a}}$$

$$= \sqrt{\frac{(8)(1.5\times10^{-3}\,m)(1000\,kg/m^3)(9.8\,m/s^2)}{(3)(0.60)(1.2\,kg/m^3)}}$$

$= 7.4\,m/s \simeq 27\,km/h$ ← note that the height of the cloud does not enter into calculation.

b) What would be the drop's speed just before impact if there was no drag force?

$$v = \sqrt{2gh} = \sqrt{(2)(9.8\,m/s^2)(1200\,m)}$$

$= 153\,m/s \simeq \underline{550\,km/h}$

↳ bullet from a high caliber hand gun!

UNIFORM CIRCULAR MOTION

$a = \frac{v^2}{R}$, centripetal acceleration, directed toward the center of the circle

Two examples of uniform Circular Motion:

1) Rounding a curve in a car:

While the car moves in the circular arc, it has an accelaration directed toward the center of the circle. Newton's 2^{nd} Law says that a force must cause this acceleration. Moreover, the force must also be directed toward the center of the circle, hence "centripetal force". In this example, the centripetal force is a frictional force on the Tires from the Road; it Makes the turn possible. If you are to move in a uniform circular motion along with the car, there must also be a centripetal force on you. However, apparently the frictional force on you from the seat was not great enough to make you go in a circle with the car. Thus, the seat slid beneath you, until the Right wall of the car is jammed into you. Then its push on you provided the needed centripetal force on you and you joined the car's uniform circular motion.

2) Orbiting Earth:

As a Passanger in a space shuttle orbiting Earth, you float through your cabin. The centripetal forces are gravitational pulls exerted by earth and directed Radially inward.

∴ In both car and Shuttle cases, it is the centripetal force yet the sensation is different — in the car, jammed up against the wall you are aware of Being compressed by the wall; in the orbiting shuttle you do not feel any sensation. What is the difference?

It is due to the nature of the two forces. In the car the centripetal force is the push on the part of your Body touching the car wall. In the shuttle, the centripetal force is earth's gravitational pull on every atom of your body. Thus, there is no compression (pull) on any part of your Body.

Another example is a puck moving around in a circle at a constant speed v while tied to a string. This time, the centripetal force is the radially inward pull on the puck from the string — without that force, the puck would slide off in a straight line, instead of moving in a circle.

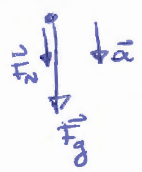
→ It is not a new force – the name just indicates the direction of the force – it can be a frictional force, gravitational force, the force from a car wall or a string.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed.

$$a = \frac{v^2}{R} \rightarrow F = m\frac{v^2}{R} \text{ (magnitude of centripetal force)}$$

Ex: Bicycle

$R = 2.7\,m$, $v = ?$ in order to not to fall at the top



$-F_N - F_g = m(-a)$

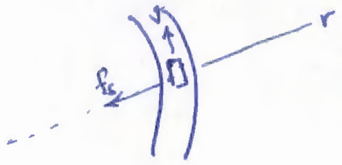
$-F_N - mg = m\left(-\frac{v^2}{R}\right)$

lowest speed = verge of losing contact → $F_N = 0$

$\Rightarrow v = \sqrt{gR} = \sqrt{(9.8\,m/s^2)(2.7\,m)} = 5.1\,m/s$

↑ independent of mass

Ex: Race car, negative lift.

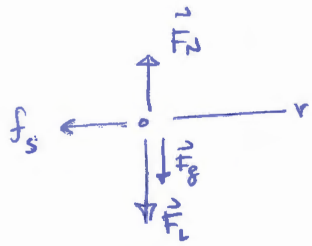


$m = 600\,kg$

$R = 100\,m$

$\mu_s = 0.75$

a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of the negative lift $\vec{F}_L$ acting downward on the car?



Radial

$F_{net,r} = m a_r$

$-f_s = m\left(-\frac{v^2}{r}\right)$

$f_{s,max} = \mu_s \cdot F_N \longrightarrow \boxed{\mu_s F_N = m\left(\frac{v^2}{R}\right)}$

Vertical

$\vec{F}_g = m\vec{g}$

$F_{net,y} = m a_y$

$F_N - mg - F_L = 0$

$\boxed{F_N = mg + F_L}$

$\longrightarrow F_L = m\left(\frac{v^2}{\mu_s R} - g\right)$

$= (600\,kg)\left(\frac{(28.6\,m/s)^2}{(0.75)(100\,m)} - 9.8\,m/s^2\right)$

$= 663.7\,N \approx \underline{660\,N}$

b) The magnitude F_L of the negative lift on a car depends on the square of the car speed v^2 just as the Drag force.

What is the magnitude of the negative lift for $v = 90\,m/s$?

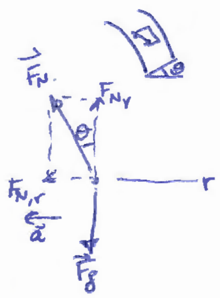
$$\frac{F_{L,90}}{F_L} = \frac{(90\,m/s)^2}{(28.6\,m/s)^2}$$

$F_L \rightarrow 663.7\,N$

$\hookrightarrow F_{L,90} = 6572\,N \approx 6600\,N$

$F_g = mg = (600\,kg)(9.8\,m/s^2) = 5880\,N \rightarrow$ The car could run on a long ceiling provided that it moves at about 90 m/s (= 324 km/h)

Ex: Car in a Banked Turn.



m, $v = 20\,m/s$

$R = 190\,m$

What θ prevents sliding?

Radial: $F_{N,r} = F_N \cdot \sin\theta$

$F_{net,r} = m a_r \longrightarrow \boxed{-\underbrace{F_N}_{unknown} \sin\theta = \underbrace{m}\left(-\frac{v^2}{R}\right)}$

Vertical: $F_{net,y} = m a_y$

$F_N \cos\theta - mg = m(0) \longrightarrow \boxed{F_N \cos\theta = mg}$

$\rightarrow \theta = \tan^{-1}\frac{v^2}{gR}$

$= \tan^{-1}\frac{(20\,m/s)^2}{(9.8\,m/s^2)(190\,m)}$

$= 12°$