Mehmet Taha
USTA
21527472
MTUSTA

(A) $V_1 = 1+x_2$, $V_2 = x-x^3$, $V_3 = 1$, $V_4 = x$, $V_5 = x^2$

$V_6 = x^3$

$$\begin{matrix} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[-R_1+R_3]{R_2+R_4} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$\xrightarrow{-R_3}$ $\begin{bmatrix} \textcircled{1} & 0 & 1 & 0 & 0 & 0 \\ 0 & \textcircled{1} & 0 & 1 & 0 & 0 \\ 0 & 0 & \textcircled{1} & 0 & -1 & 0 \\ 0 & 0 & 0 & \textcircled{1} & 0 & 1 \end{bmatrix}$ $\Rightarrow$ leading entry

Basis $\{1+x^2, x-x^3, 1, x\}$

(B)

$u = x - x^2 + x^3 = w_2 + x^3 = w_2 + \frac{1}{2}(w_3 - w_1) =$

$= -\frac{1}{2} \cdot w_1 + 1 \cdot w_2 + \frac{1}{2} w_3$

$$[u]_C = \begin{bmatrix} -1/2 \\ 1 \\ 1/2 \end{bmatrix}$$