Games and Adversarial Search

BBM 405 – Fundamentals of Artificial Intelligence

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Slides are mostly adapted from AIMA, MIT Open Courseware and Svetlana Lazebnik (UIUC)


MATT
KASPAROV BEATS 'DEEP BLUE' IN ONE MOVE
OFF

Why study games?

- Games are a traditional hallmark of intelligence
- Games are easy to formalize
- Games can be a good model of real-world competitive or cooperative activities
  - Military confrontations, negotiation, auctions, etc.

Types of game environments

	Deterministic	Stochastic
Perfect information (fully observable)	Chess, checkers, go	Backgammon, monopoly
Imperfect information (partially observable)	Battleships	Scrabble, poker, bridge

Alternating two-player zero-sum games

- Players take turns
- Each game outcome or **terminal state** has a **utility** for each player (e.g., 1 for win, 0 for loss)
- The sum of both players' utilities is a constant

Games vs. single-agent search

- We don’t know how the opponent will act
  - The solution is not a fixed sequence of actions from start state to goal state, but a ***strategy*** or ***policy*** (a mapping from state to best move in that state)
- Efficiency is critical to playing well
  - The time to make a move is limited
  - The branching factor, search depth, and number of terminal configurations are huge
    - In chess, branching factor ≈ 35 and depth ≈ 100, giving a search tree of 10^{154} nodes
      - Number of atoms in the observable universe $\approx 10^{80}$
  - This rules out searching all the way to the end of the game

Games

- Multi agent environments : any given agent will need to consider the actions of other agents and how they affect its own welfare.
- The unpredictability of these other agents can introduce many possible contingencies
- There could be competitive or cooperative environments

- Competitive environments, in which the agent's goals are in conflict require adversarial search – these problems are called as games

Games

- In game theory (economics), any multiagent environment (either cooperative or competitive) is a game provided that the impact of each agent on the other is significant
- AI games are a specialized kind - deterministic, turn taking, two-player, zero sum games of perfect information
- In our terminology – deterministic, fully observable environments with two agents whose actions alternate and the utility values at the end of the game are always equal and opposite (+1 and –1)

Games – history of chess playing

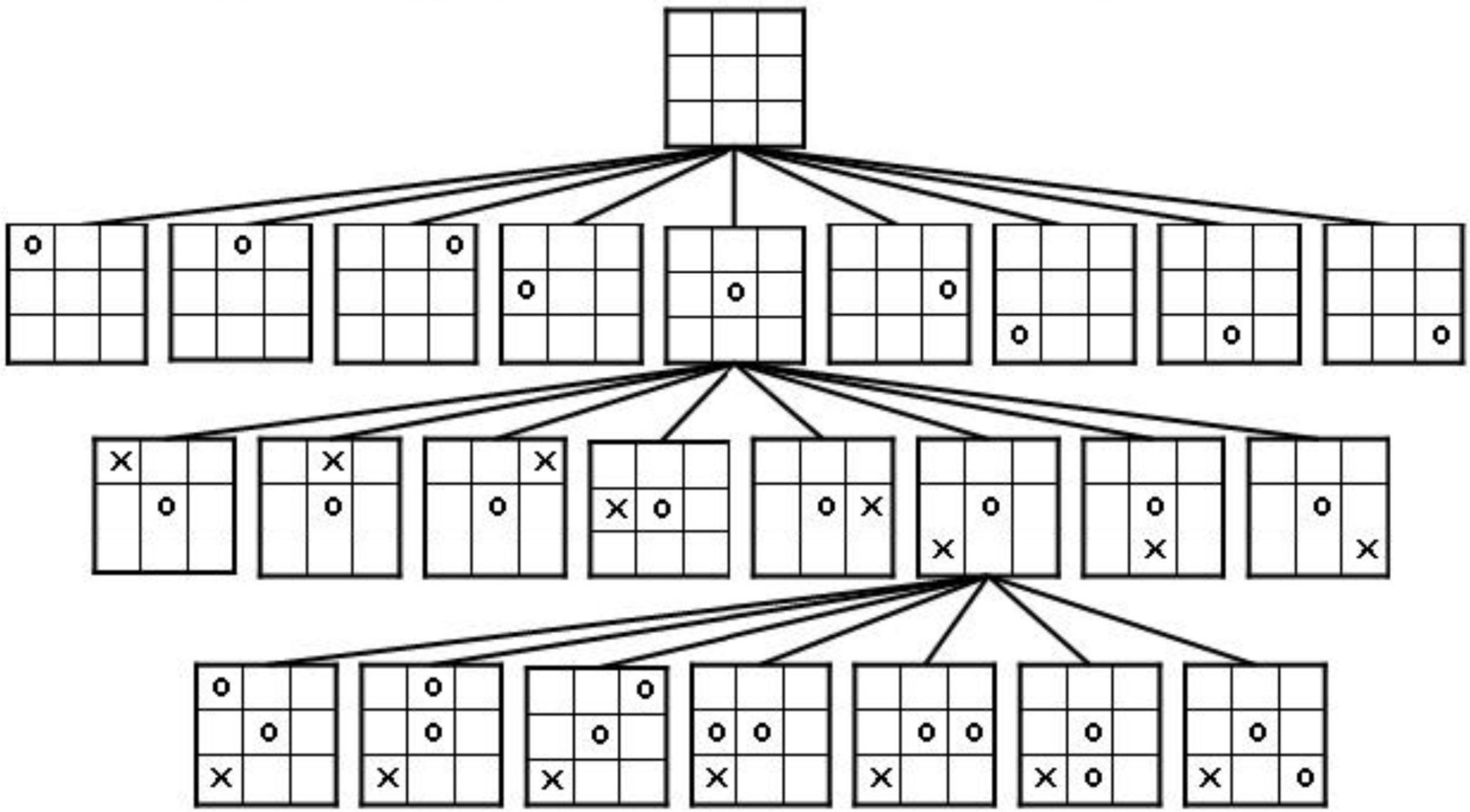
- 1949 – Shannon paper – originated the ideas
- 1951 – Turing paper – hand simulation
- 1958 – Bernstein program
- 1955-1960 – Simon-Newell program
- 1961 – Soviet program
- 1966 – 1967 – MacHack 6 – defeated a good player
- 1970s – NW chess 4.5
- 1980s – Cray Bitz
- 1990s – Belle, Hitech, Deep Thought,
- 1997 - Deep Blue - defeated Garry Kasparov

Game Tree search

- Initial state: initial board position and player
- Operators: one for each legal move
- Goal states: winning board positions
- Scoring function: assigns numeric value to states
- Game tree: encodes all possible games

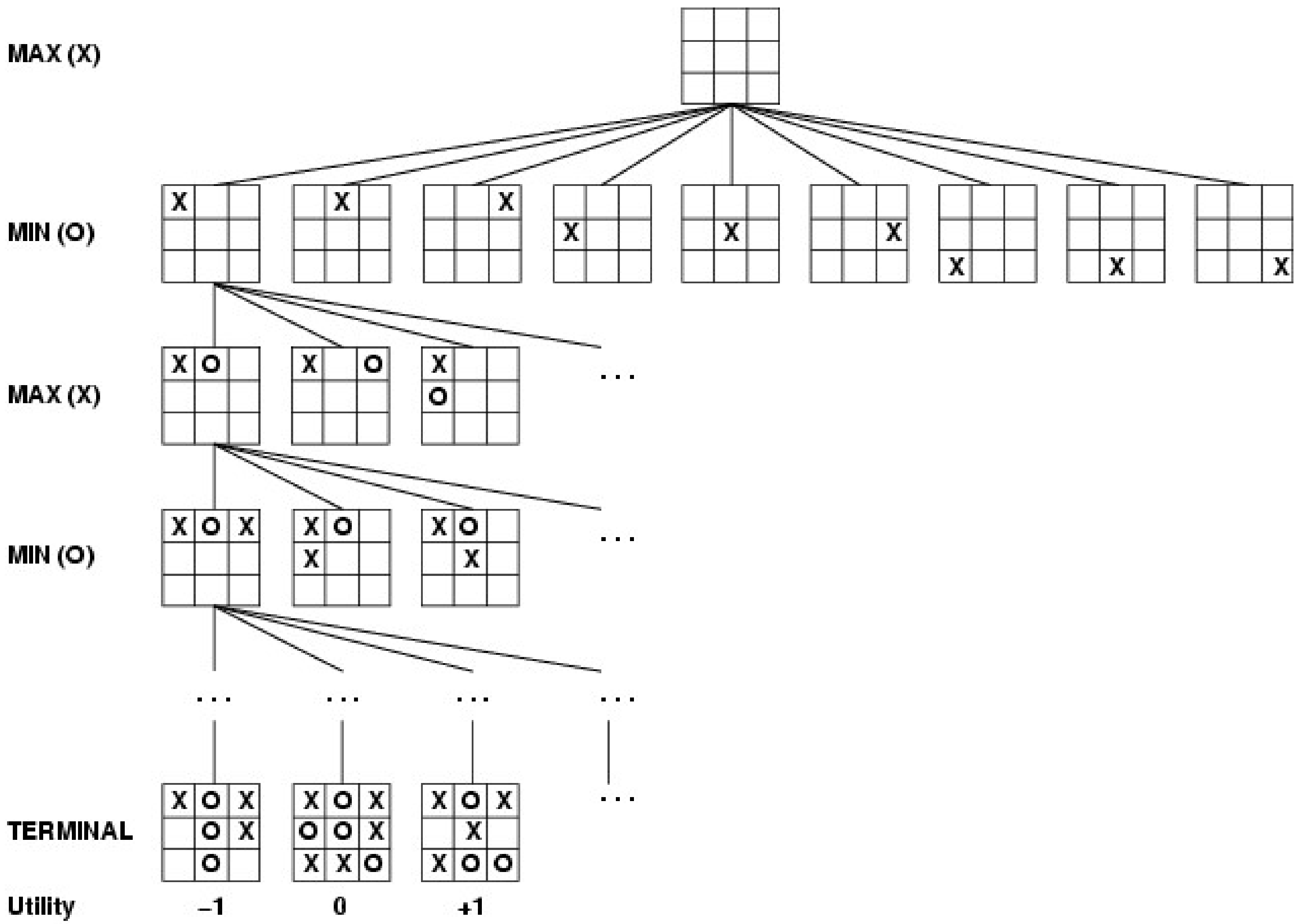
- We are not looking for a path, only the next move to make (that hopefully leads to a winning position)
- Our best move depends on what the other player does

Partial Game Tree for Tic-Tac-Toe



Game tree

- A game of tic-tac-toe between two players, “max” and “min”

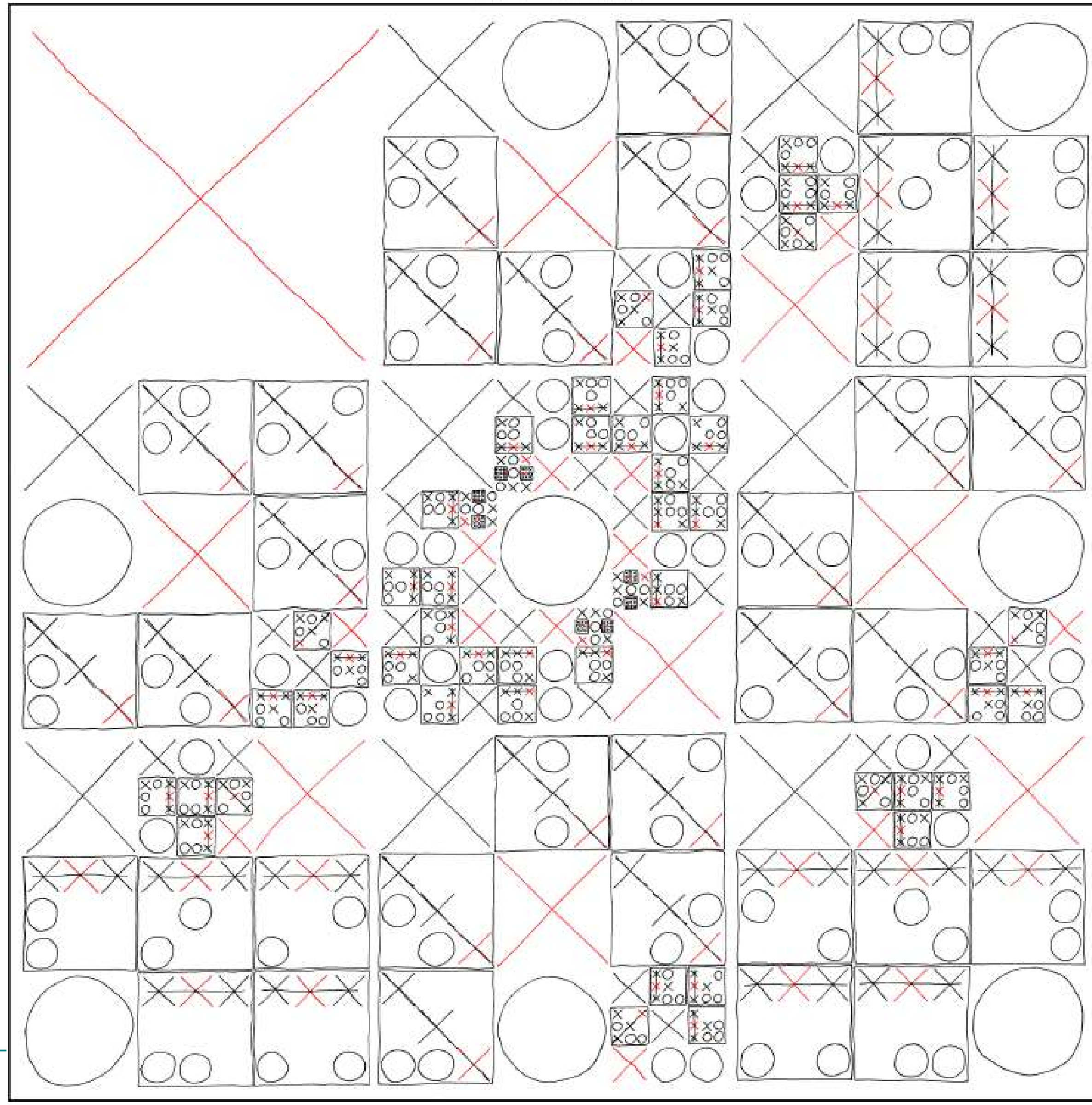


COMPLETE MAP OF OPTIMAL TIC-TAC-TOE MOVES

YOUR MOVE IS GIVEN BY THE POSITION OF THE LARGEST RED SYMBOL ON THE GRID. WHEN YOUR OPPONENT PICKS A MOVE, ZOOM IN ON THE REGION OF THE GRID WHERE THEY WENT. REPEAT.

http://xkcd.com/832

MAP FOR X:



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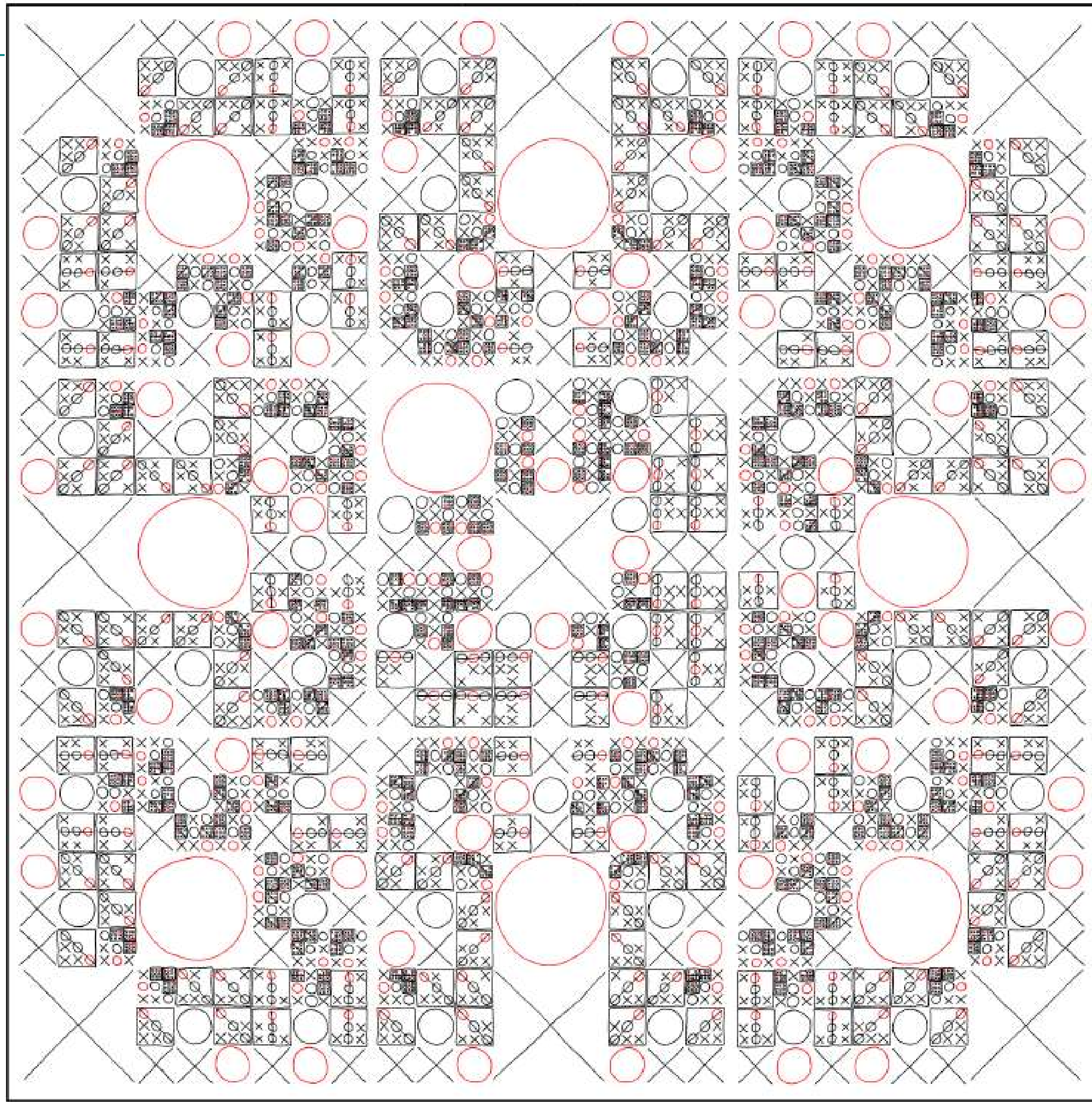

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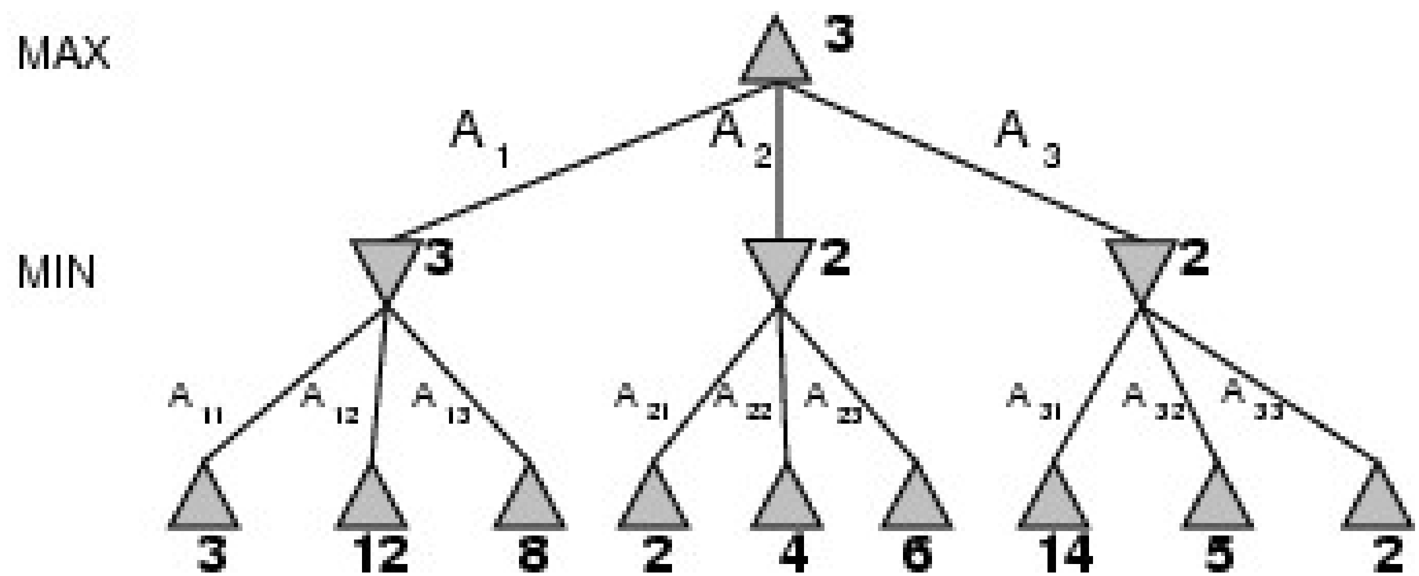
Optimal strategies

- In a normal search problem, the optimal solution would be a sequence of moves leading to a goal state - a terminal state that is a win
- In a game, MIN has something to say about it and therefore MAX must find a contingent strategy, which specifies
  - MAX's move in the initial state,
  - then MAX's moves in the states resulting from every possible response by MIN,
  - then MAX's moves in the states resulting from every possible response by MIN to those moves
  - …
- An optimal strategy leads to outcomes at least as good as any other strategy when one is playing an infallible opponent

Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-ply game:



Minimax value

- Given a game tree, the optimal strategy can be determined by examining the minimax value of each node (MINIMAX-VALUE(n))
- The minimax value of a node is the utility of being in the corresponding state, assuming that both players play optimally from there to the end of the game
- Given a choice, MAX prefer to move to a state of maximum value, whereas MIN prefers a state of minimum value

Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
    v ← MAX-VALUE(state)
    return the action in SUCCESSORS(state) with value v
```

```
function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← −∞
    for a, s in SUCCESSORS(state) do
        v ← MAX(v, MIN-VALUE(s))
    return v
```

```
function MIN-VALUE(state) returns a utility value
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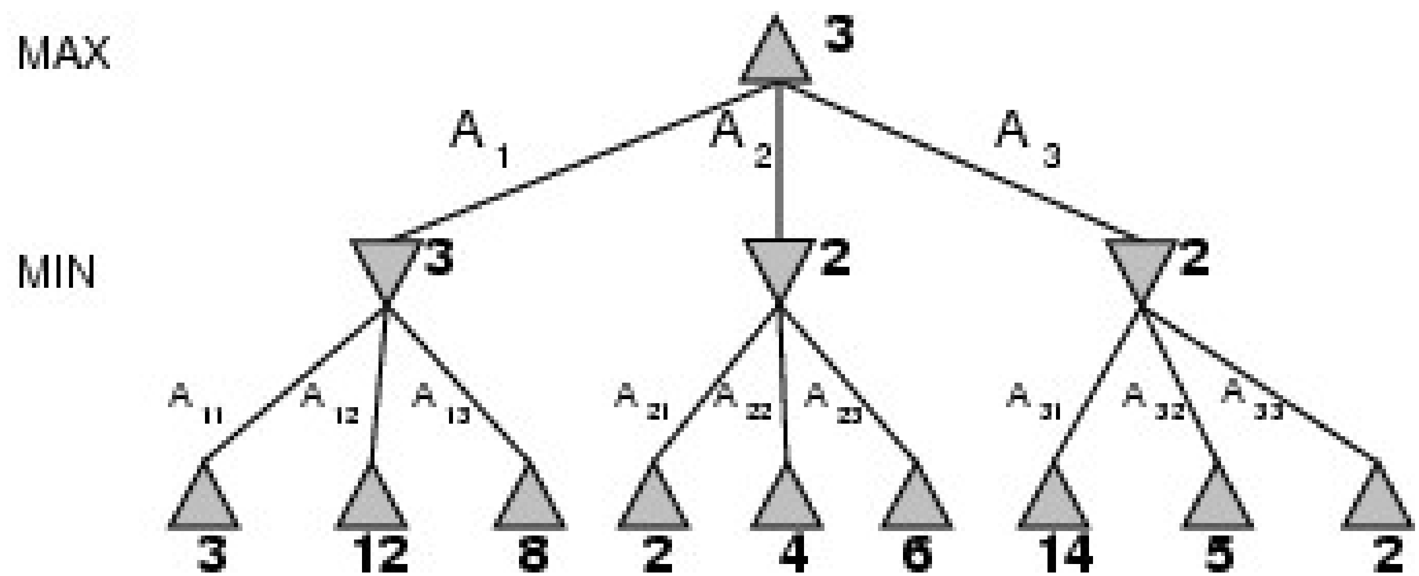
    v ← ∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s))
    return v
```

Minimax

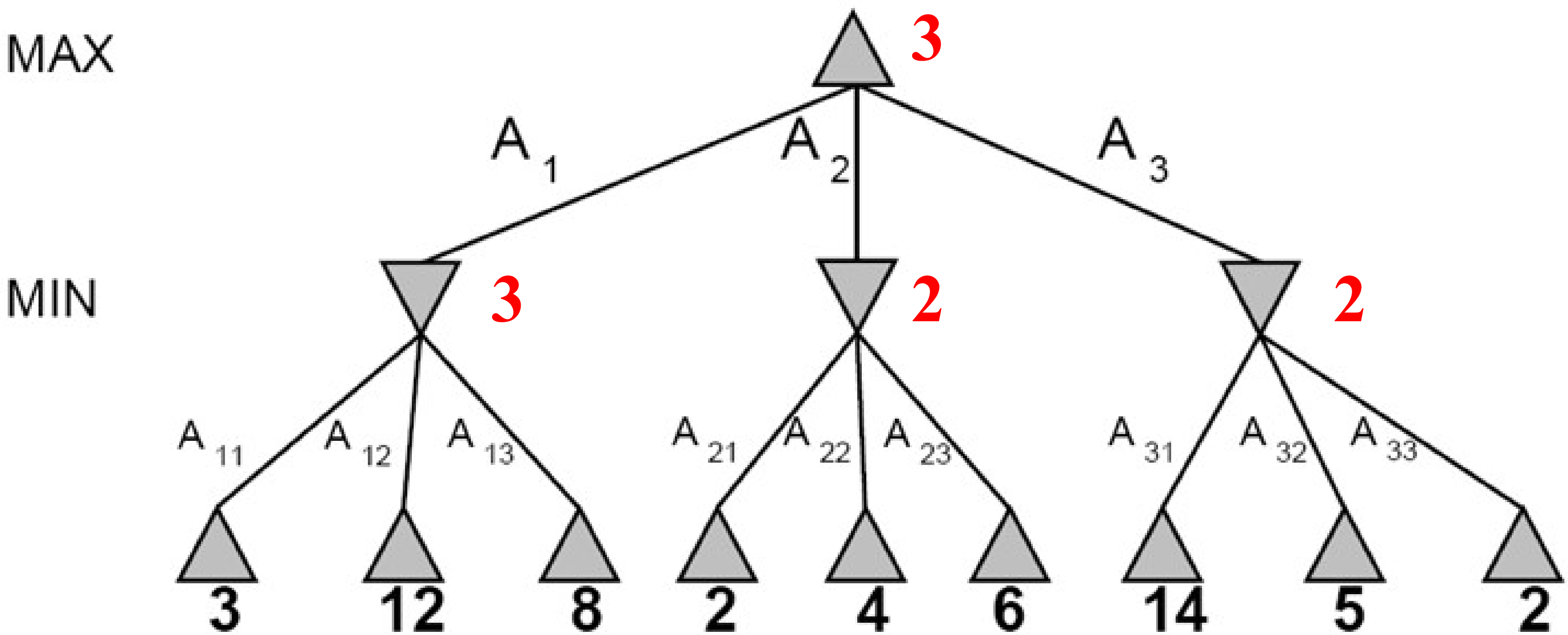
MINIMAX-VALUE(root) = max(min(3,12,8), min(2,4,6), min(14,5,2))

= max(3,2,2)

= 3

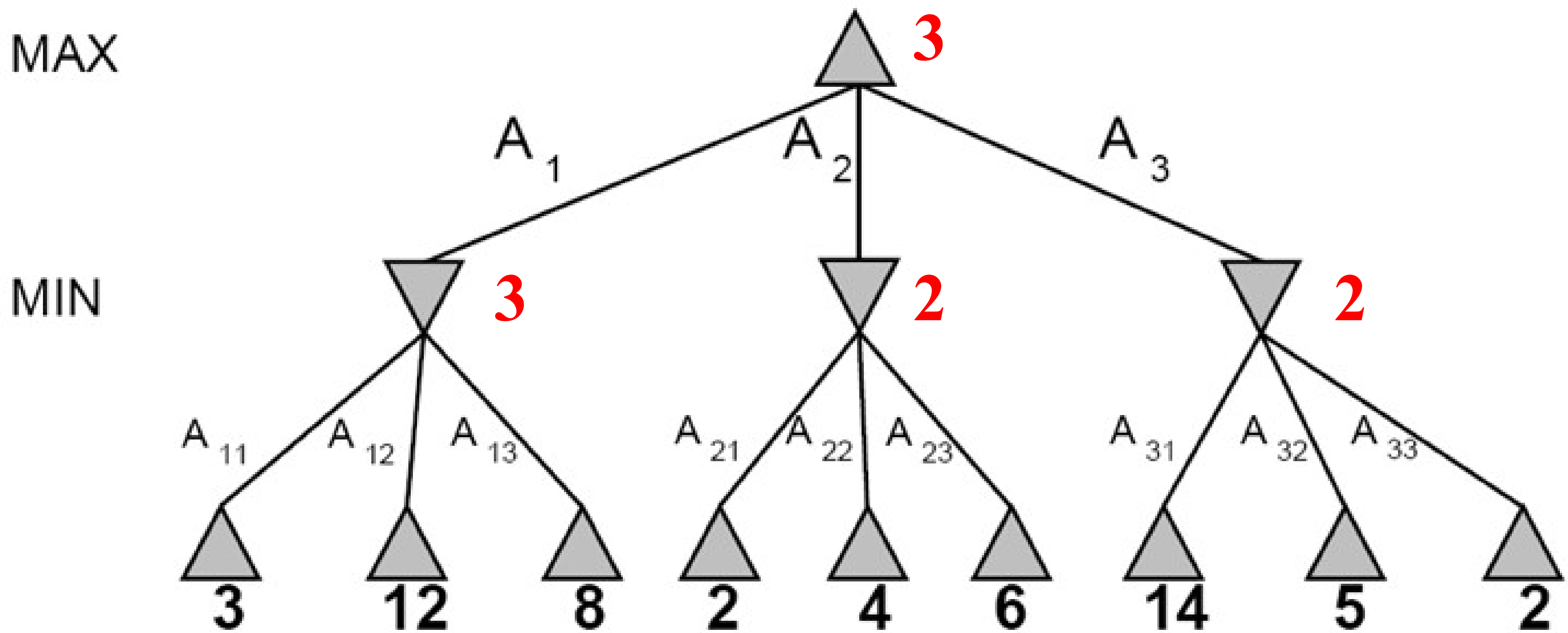


Game tree search



- **Minimax value of a node**:.the utility (for MAX) of being in the corresponding state, assuming perfect play on both sides
- **Minimax strategy:** Choose the move that gives the best worst-case payoff

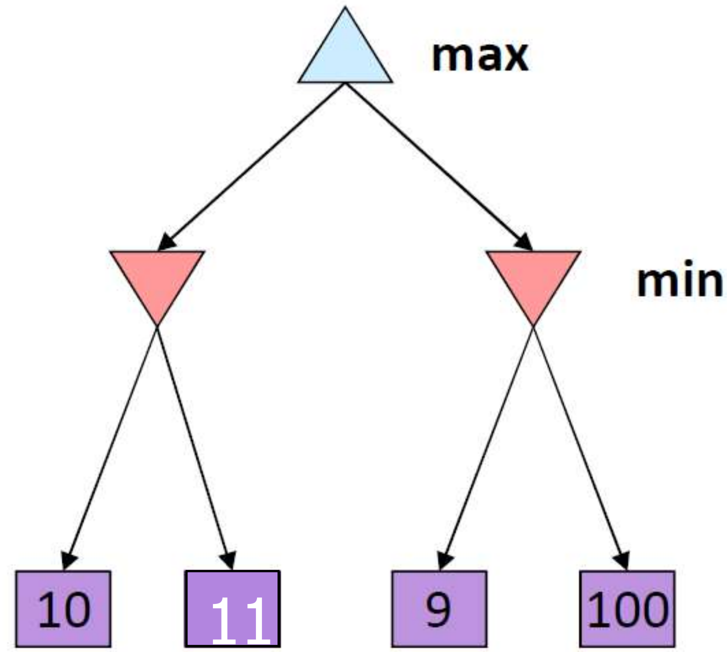
Computing the minimax value of a node



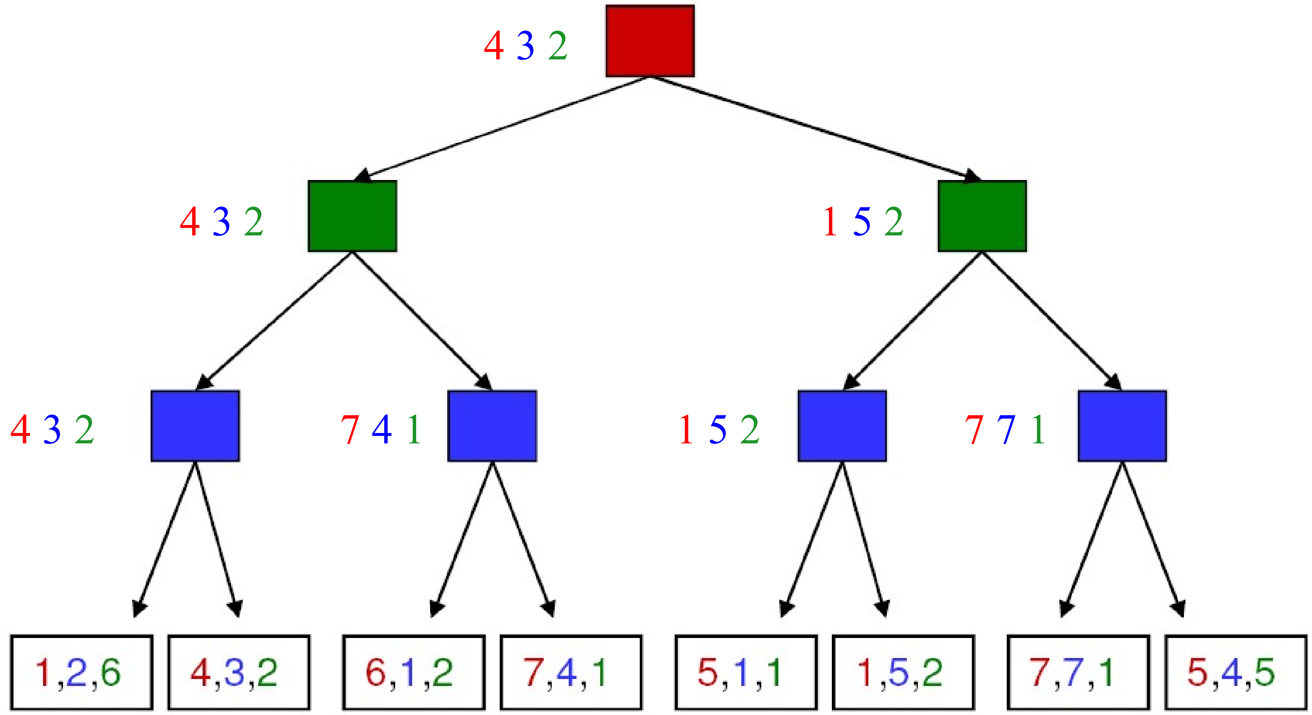
- **Minimax**(*node*) =
  - Utility(*node*) if *node* is terminal
  - $\max_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MAX
  - $\min_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MIN

Optimality of minimax

- The minimax strategy is optimal against an optimal opponent
- What if your opponent is suboptimal?
  - Your utility can only be higher than if you were playing an optimal opponent!
  - A different strategy may work better for a sub-optimal opponent, but it will necessarily be worse against an optimal opponent

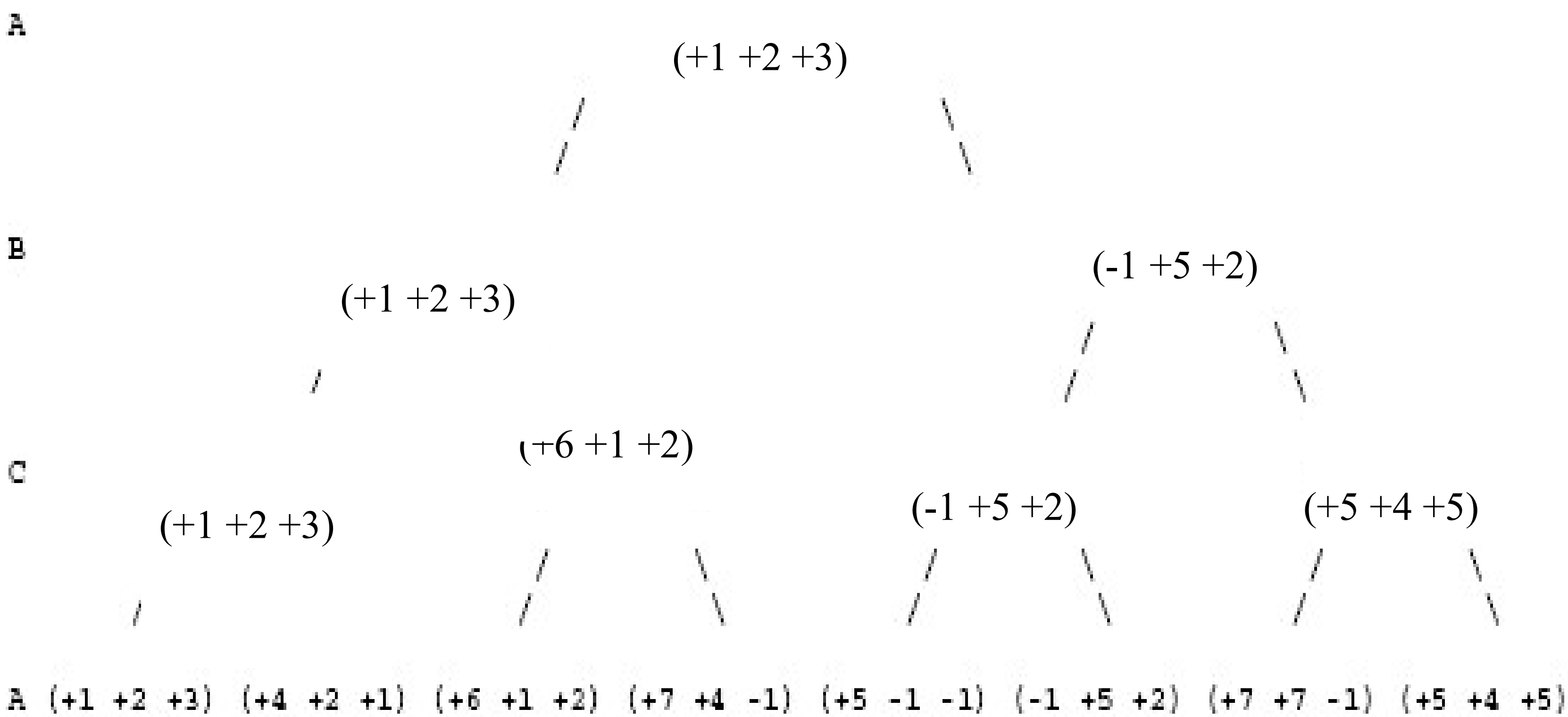


More general games



- More than two players, non-zero-sum
- Utilities are now tuples
- Each player maximizes their own utility at their node
- Utilities get propagated (*backed up*) from children to parents

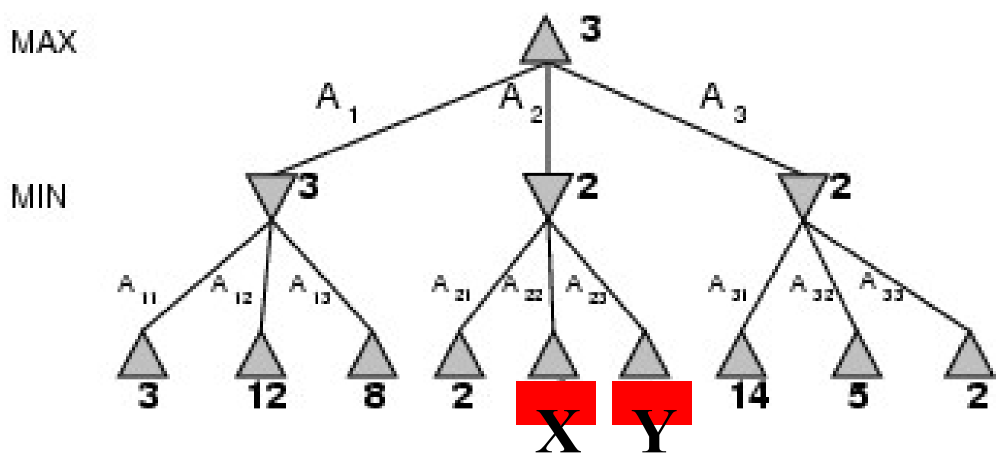
Tree Player and Non-zero sum games



α-β pruning

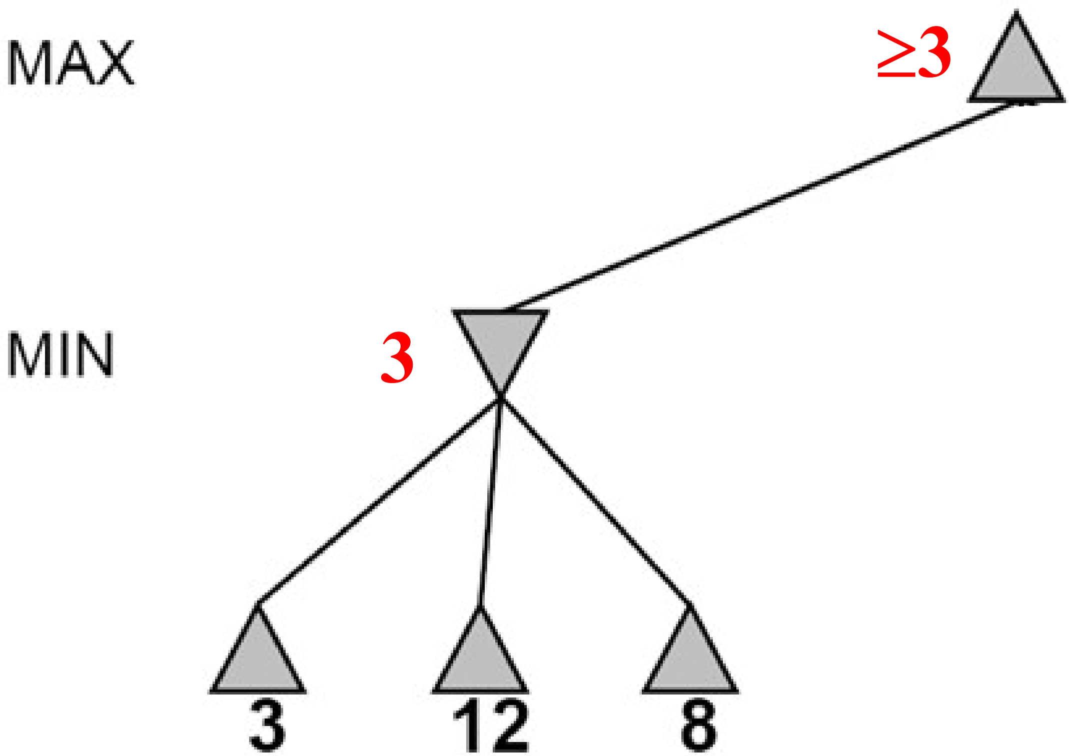
- It is possible to compute the correct minimax decision without looking at every node in the game tree

$$\text{MINIMAX-VALUE}(root) = \max(\min(3,12,8), \min(2,x,y), \min(14,5,2))$$
$$= \max(3,\min(2,x,y),2)$$
$$= \max(3,z,2) \quad \text{where } z <= 2$$
$$= 3$$



Alpha-beta pruning

- It is possible to compute the exact minimax decision without expanding every node in the game tree



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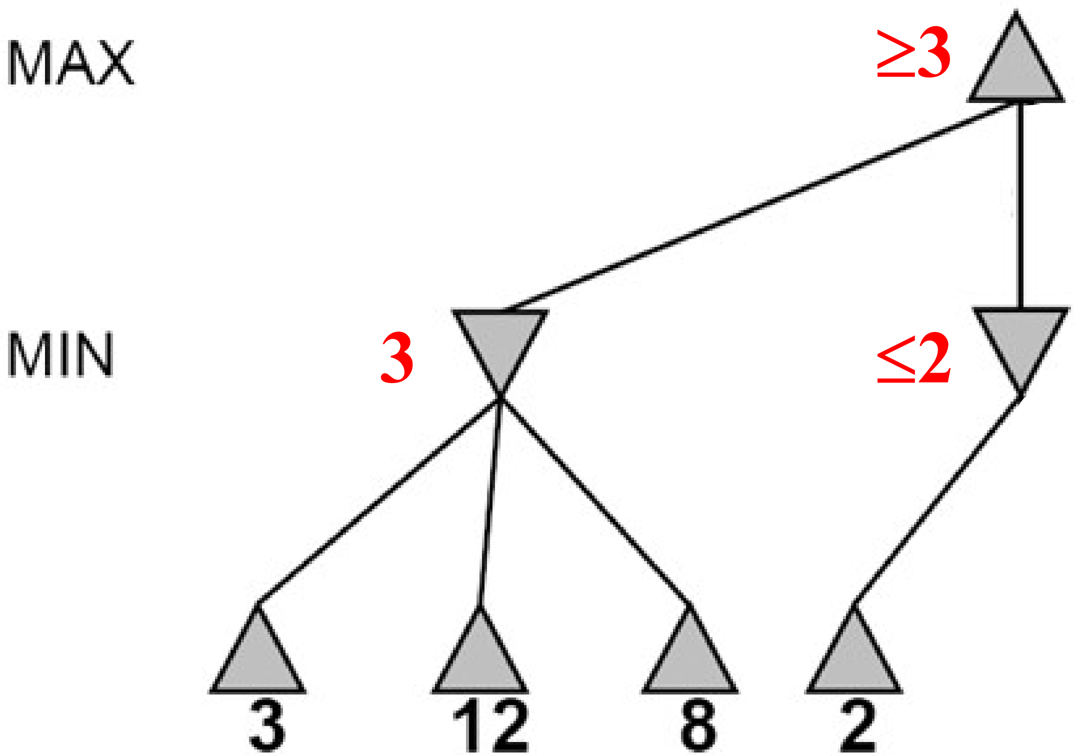

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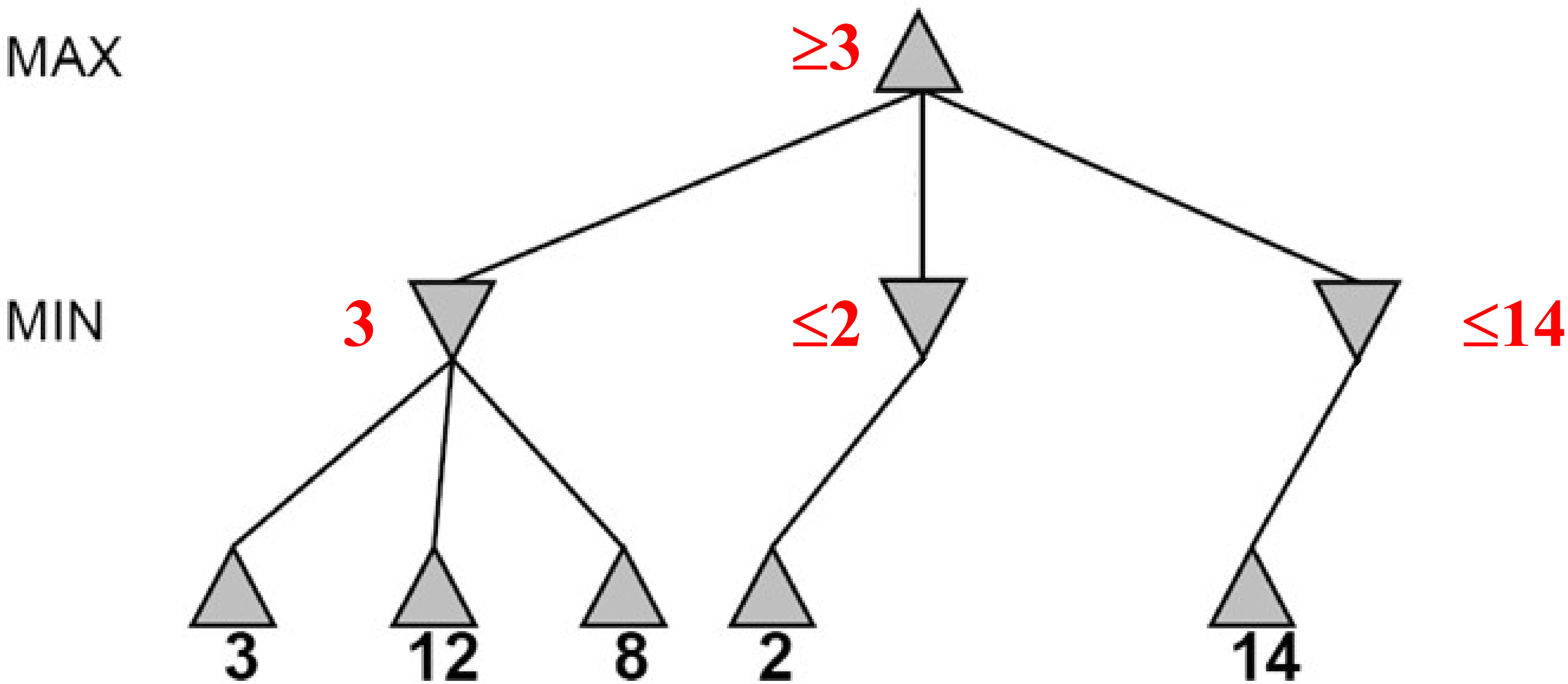

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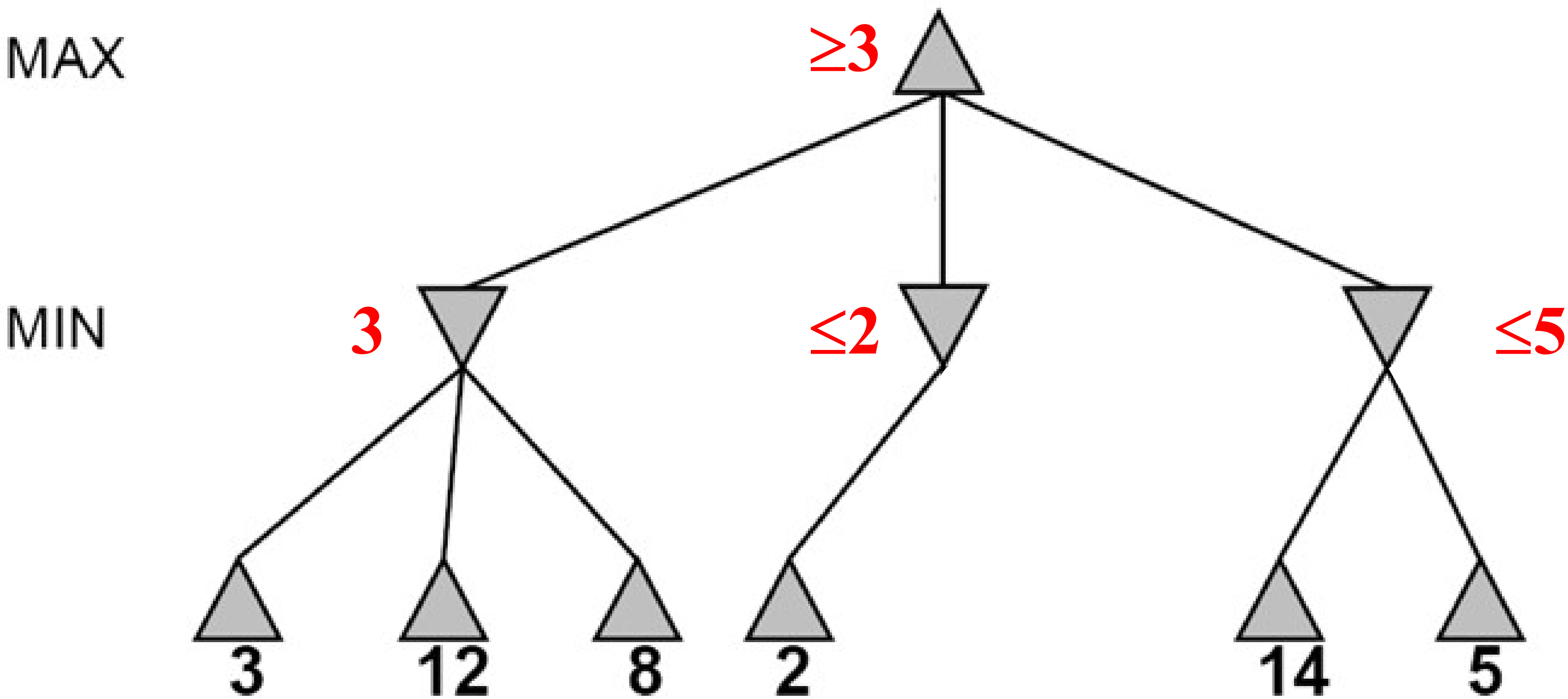

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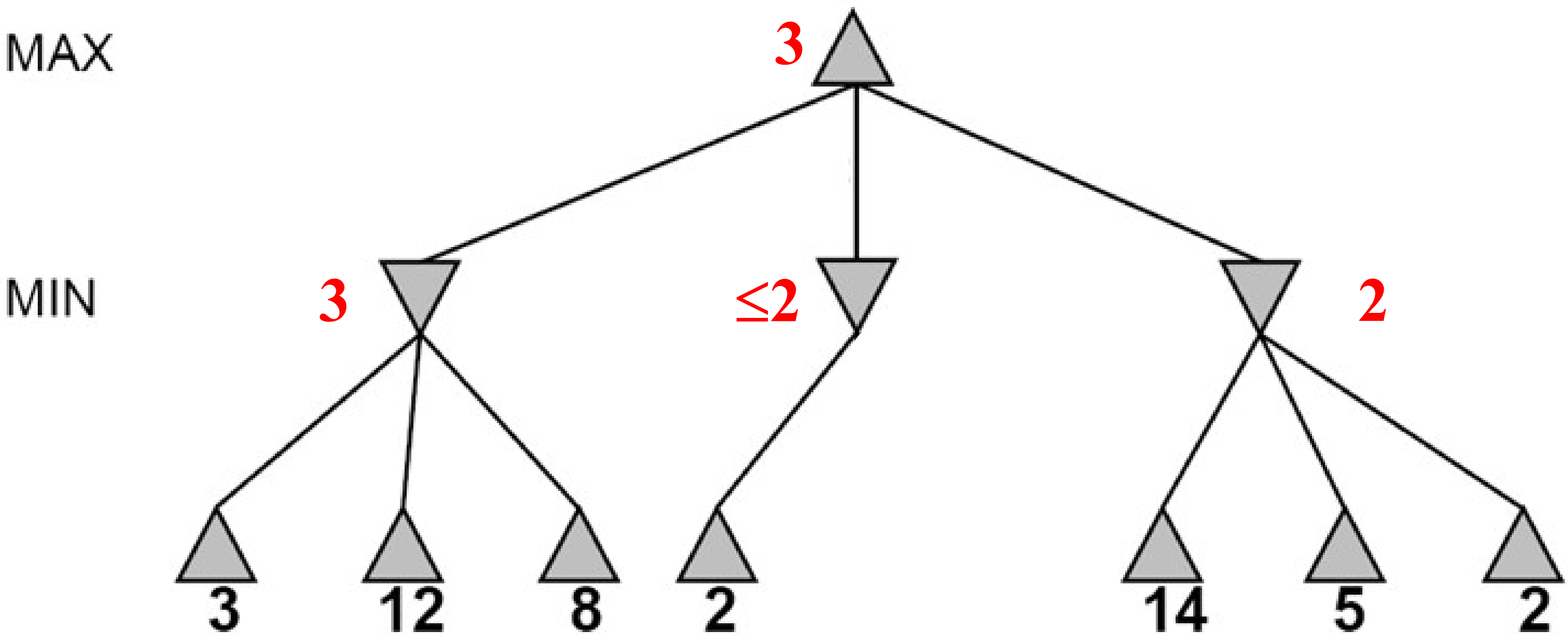
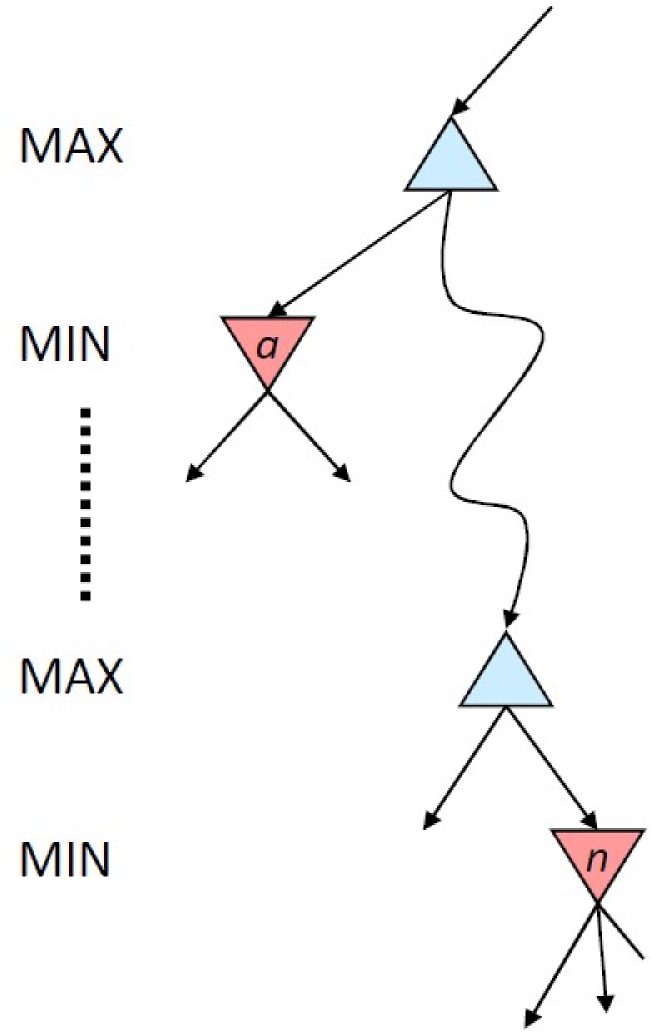


Properties of α-β

- Pruning does not affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Alpha-beta pruning

- **α** is the value of the best choice for the MAX player found so far at any choice point above node *n*
- We want to compute the MIN-value at *n*
- As we loop over *n*'s children, the MIN-value decreases
- If it drops below **α**, MAX will never choose *n*, so we can ignore *n*'s remaining children
- Analogously, **β** is the value of the lowest-utility choice found so far for the MIN player



The α-β algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  inputs: *state*, current state in game

  $v \leftarrow$ MAX-VALUE(*state*, $-\infty, +\infty$)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  inputs: *state*, current state in game
    α, the value of the best alternative for MAX along the path to *state*
    β, the value of the best alternative for MIN along the path to *state*

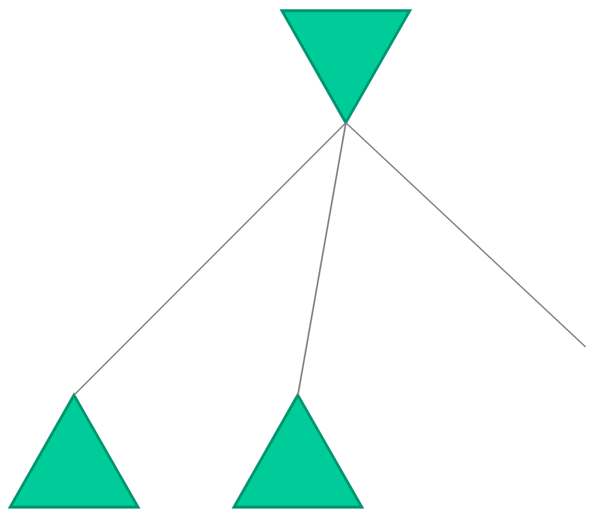
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s, α, β))
    if $v \geq \beta$ **then return** v
    $\alpha \leftarrow$ MAX(α, v)
  return v

Alpha-beta pruning

Function *action* = **Alpha-Beta-Search**(*node*)
$\quad v$ = **Min-Value**(*node*, $-\infty$, ∞)
$\quad$return the *action* from *node* with value v

α: best alternative available to the Max player
β: best alternative available to the Min player

Function v = **Min-Value**(*node*, α, β)
$\quad$if Terminal(*node*) return Utility(*node*)
$\quad v = +\infty$
$\quad$for each *action* from *node*
$\quad\quad v$ = Min(v, **Max-Value**(Succ(*node*, *action*), α, β))
$\quad\quad$if $v \leq \alpha$ return v
$\quad\quad \beta$ = Min(β, v)
$\quad$end for
$\quad$return v



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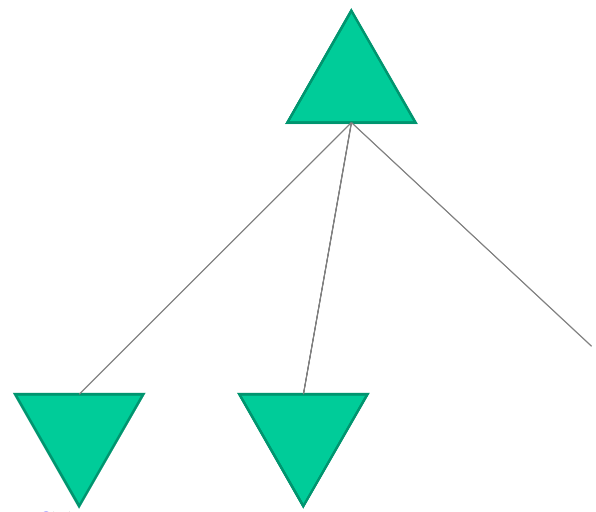
        if $v \geq \beta$ return v
        α = Max(α, v)
    end for
    return v



α-β pruning example

α - β

```
// α = best score for MAX,  β = best score for MIN
// initial call is MAX-VALUE(state,-∞, ∞,MAX-DEPTH)

function MAX-VALUE (state, α, β, depth)
    if (depth == 0) then return EVAL (state)
    for each s in SUCCESSORS (state) do
        α = MAX (α, MIN-VALUE (s, α, β,depth-1))
        if α ≥ β then return α // cutoff
    end
    return α

function MIN-VALUE (state, α, β, depth)
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        if β ≤ α then return β // cutoff
    end
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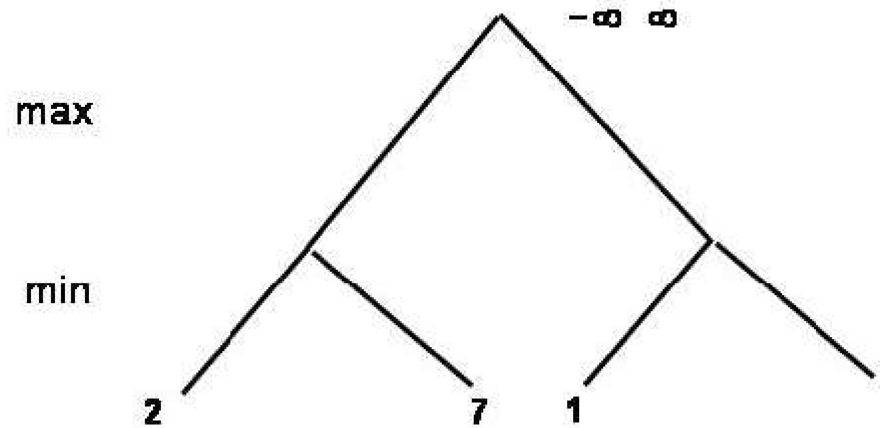

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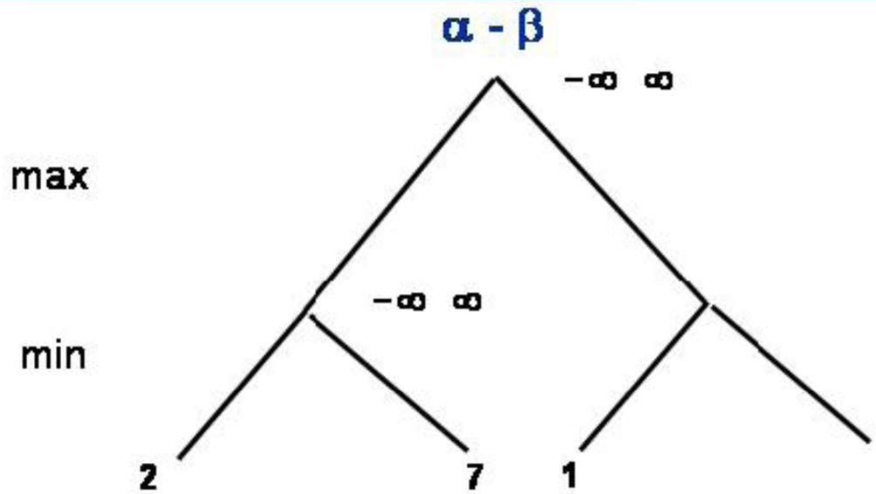

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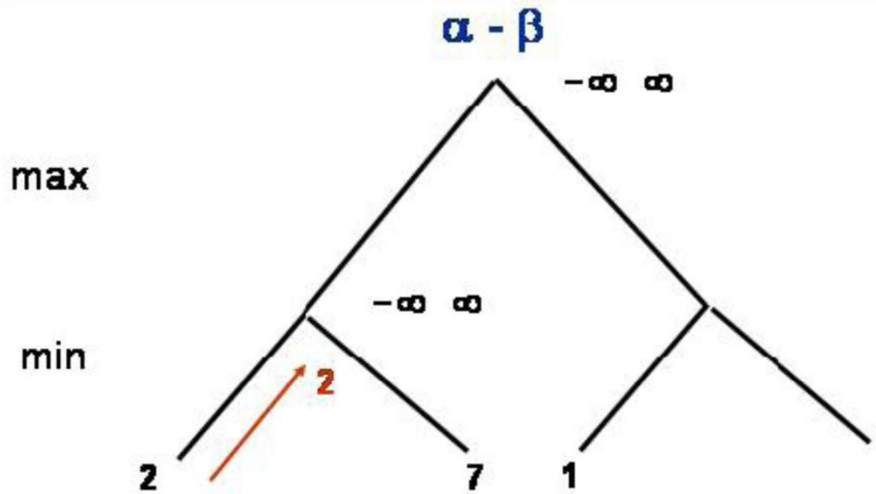

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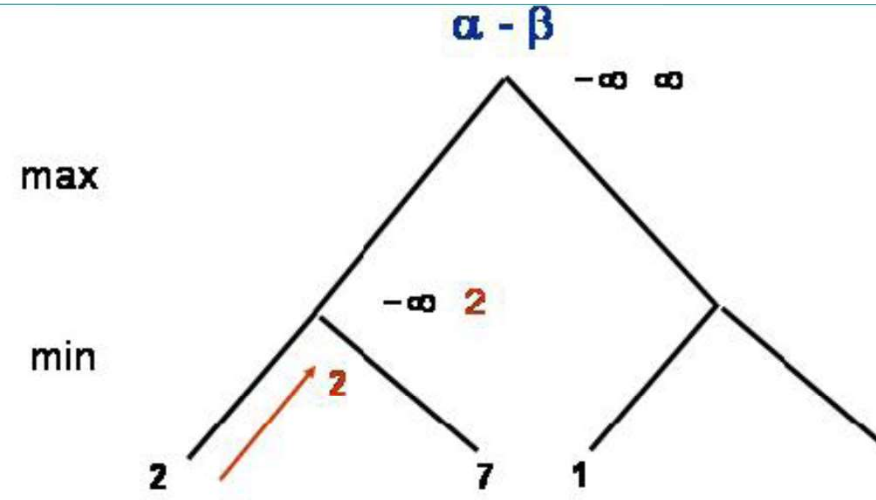

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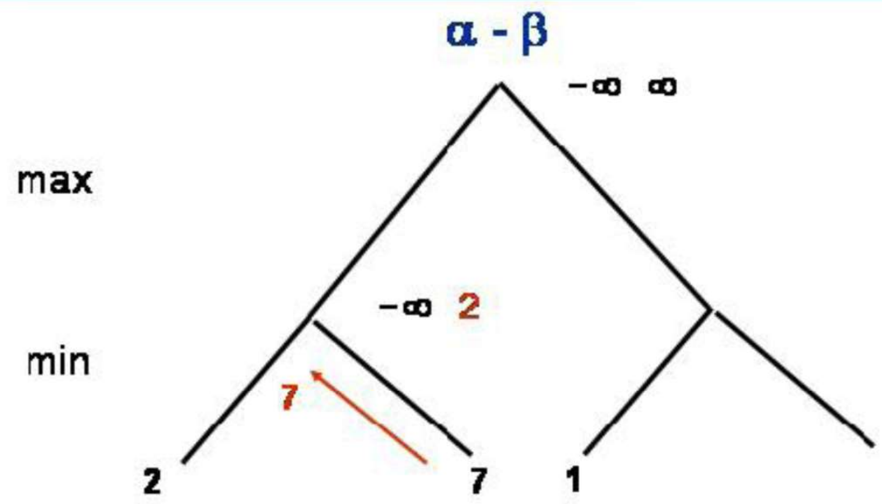

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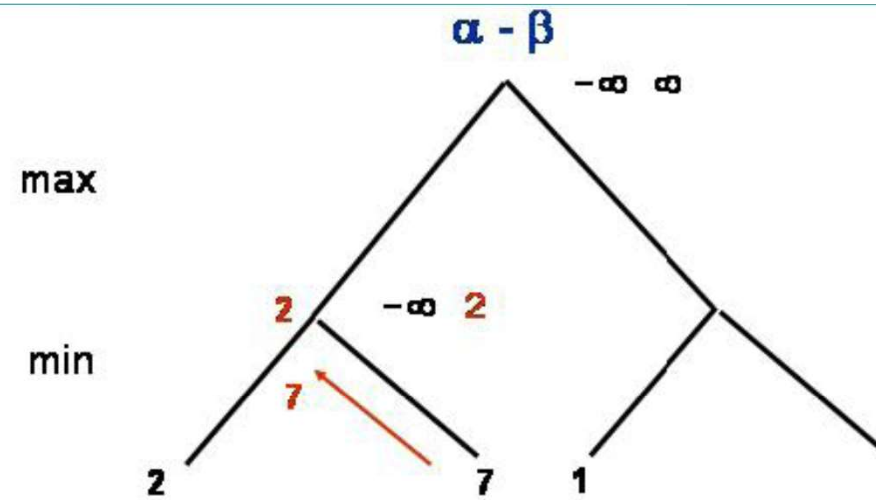

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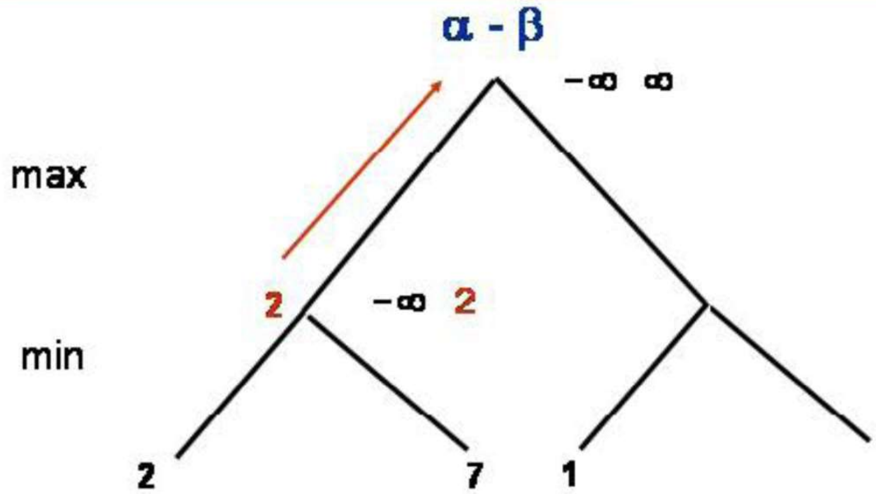

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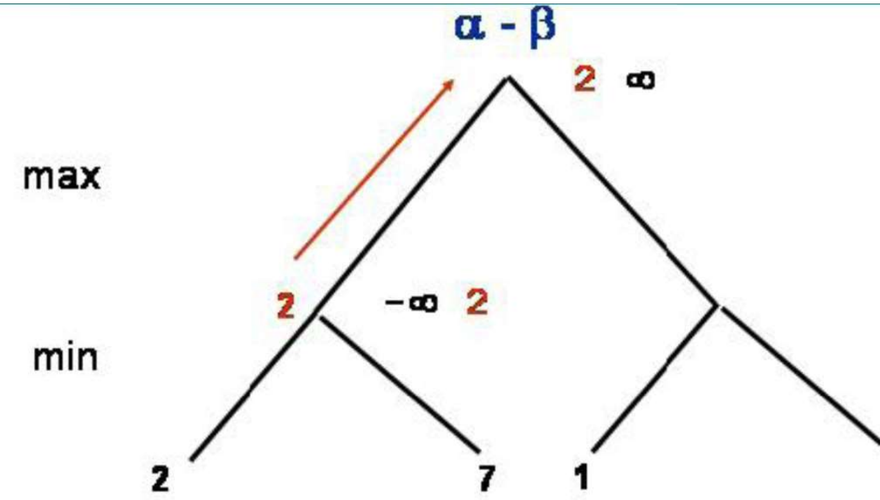

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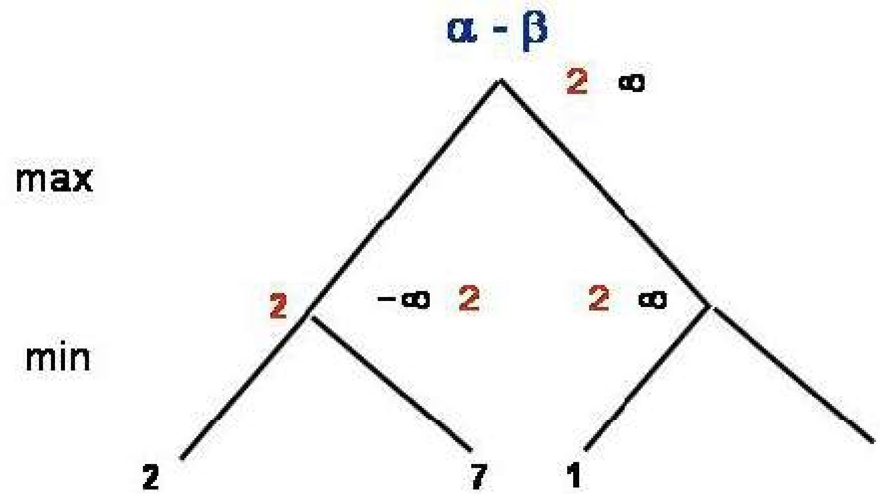

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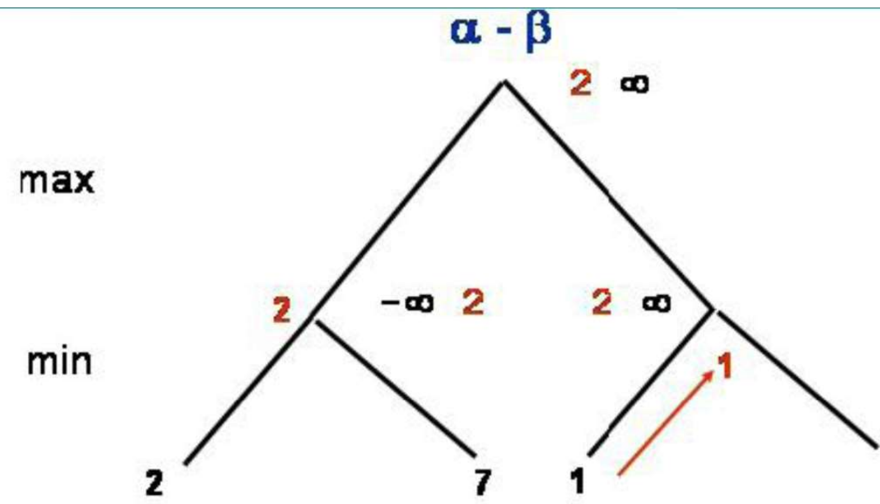
    for each s in SUCCESSORS (state) do
        α = MAX (α, MIN-VALUE (s, α, β,depth-1))
        if α ≥ β then return α // cutoff
    end
    return α

function MIN-VALUE (state, α, β, depth)
    if (depth == 0) then return EVAL (state)
    for each s in SUCCESSORS (state) do
        β = MIN (β, MAX-VALUE (s, α, β,depth-1))
        if β ≤ α then return β // cutoff
    end
    return β
```



α-β pruning example

α - β

// α = best score for MAX, β = best score for MIN
// initial call is MAX-VALUE(state,-∞, ∞,MAX-DEPTH)

```
function MAX-VALUE (state, α, β, depth)
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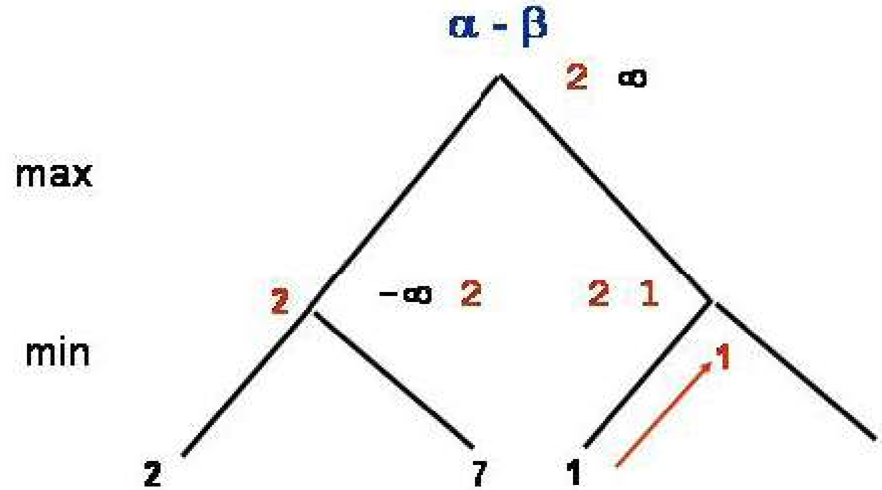
```



α-β pruning example

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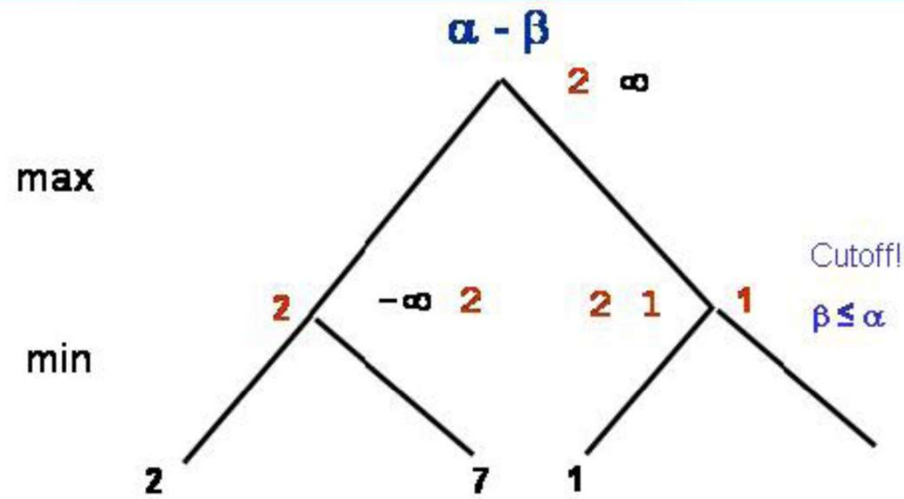
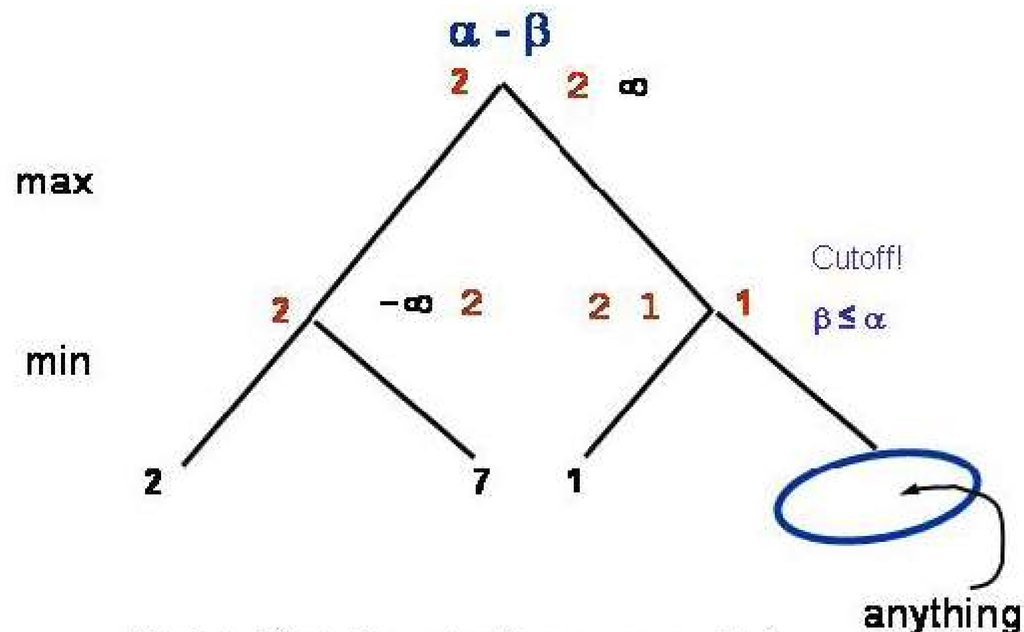
```



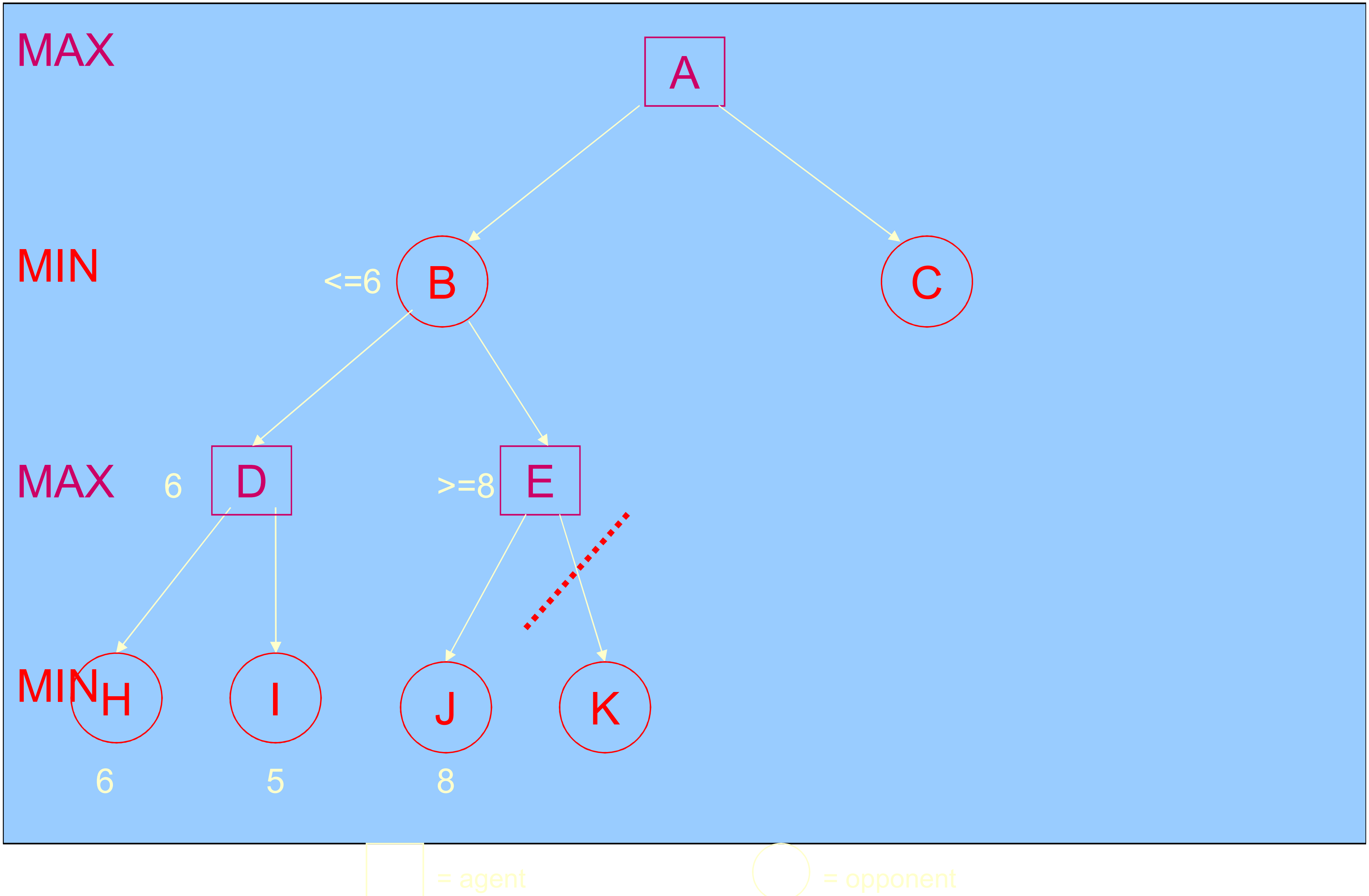
α-β pruning example

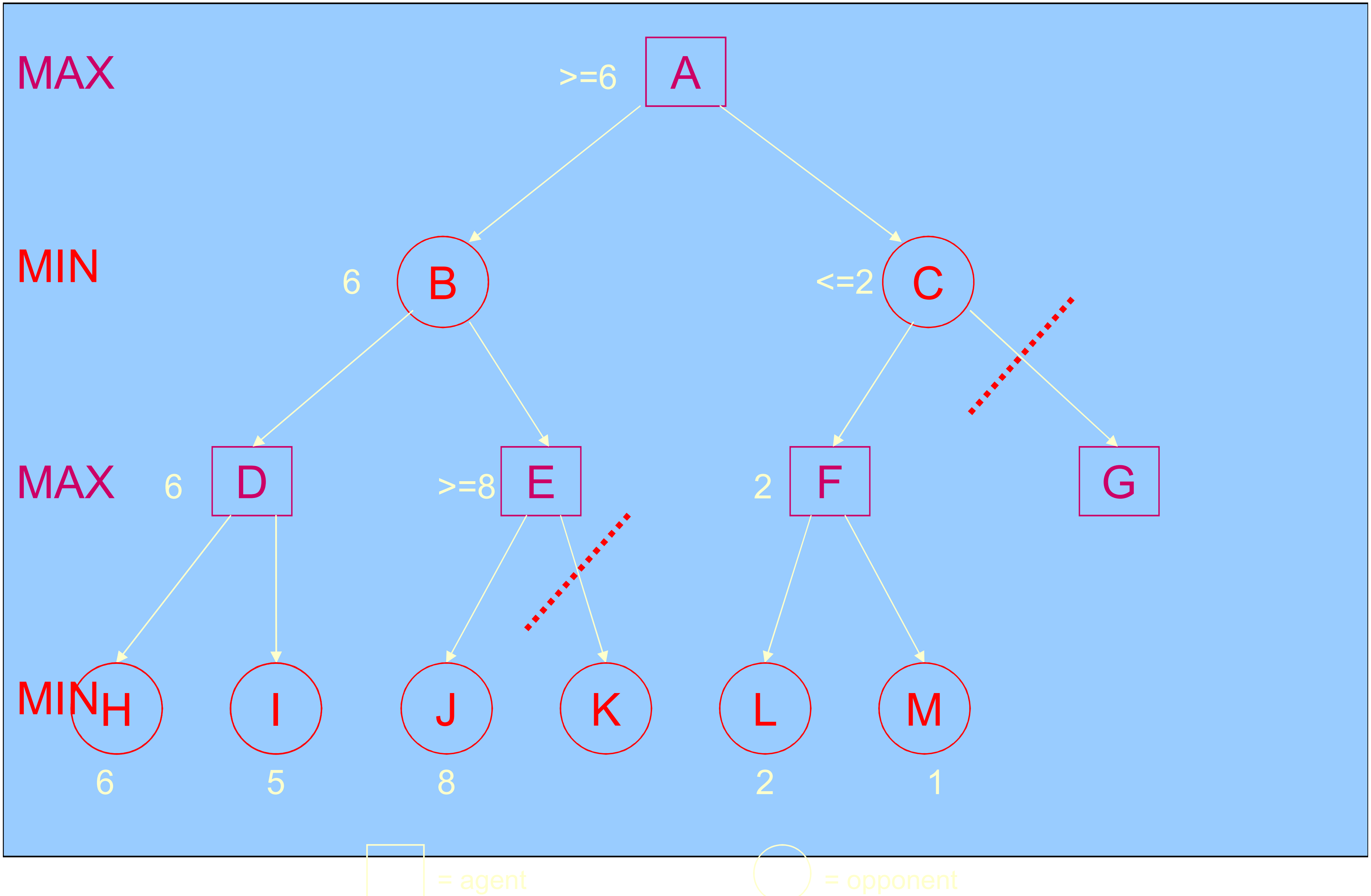


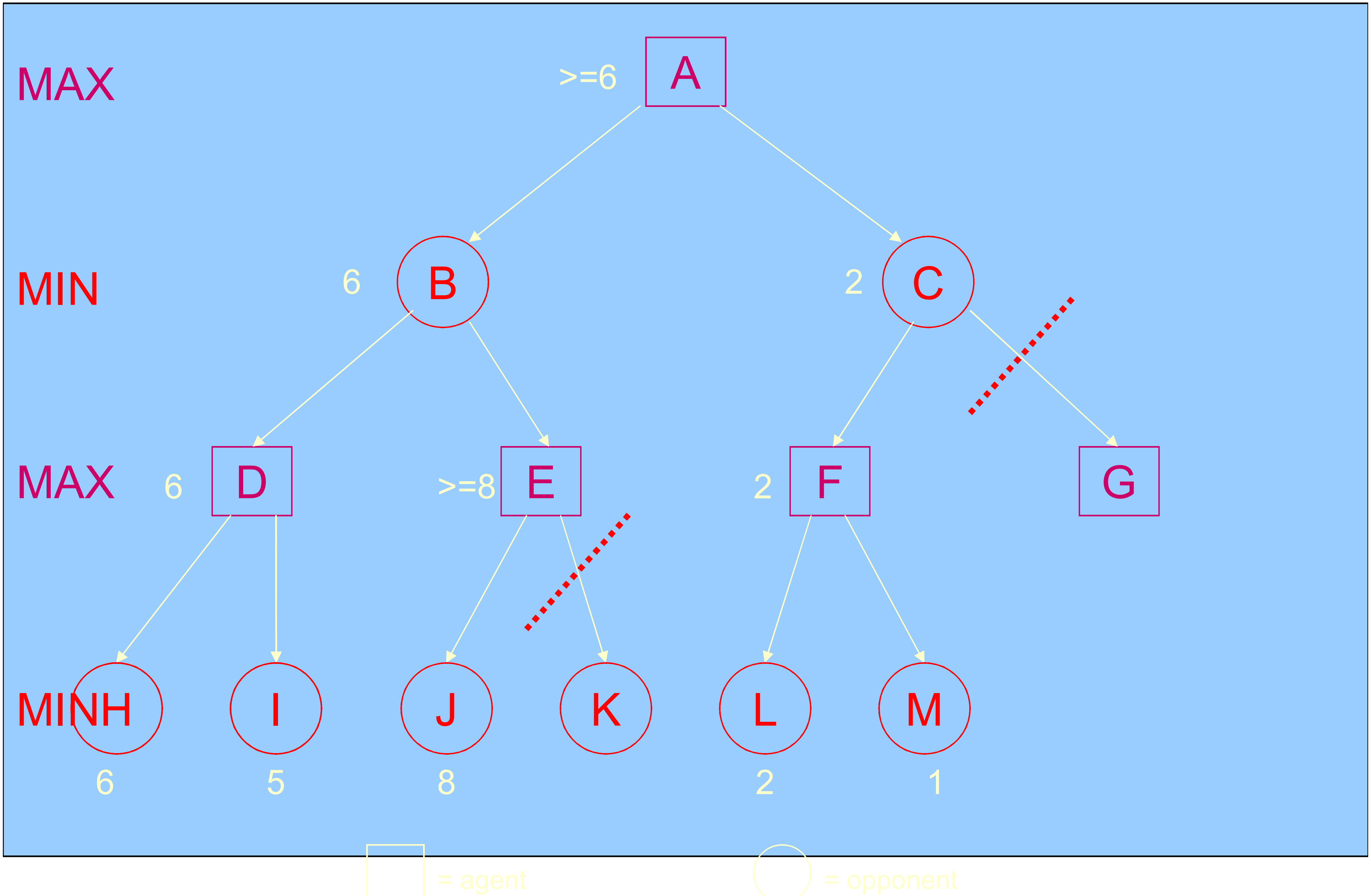
A total of 3 static evaluations were needed to obtain the value for the tree.

Alpha-beta pruning

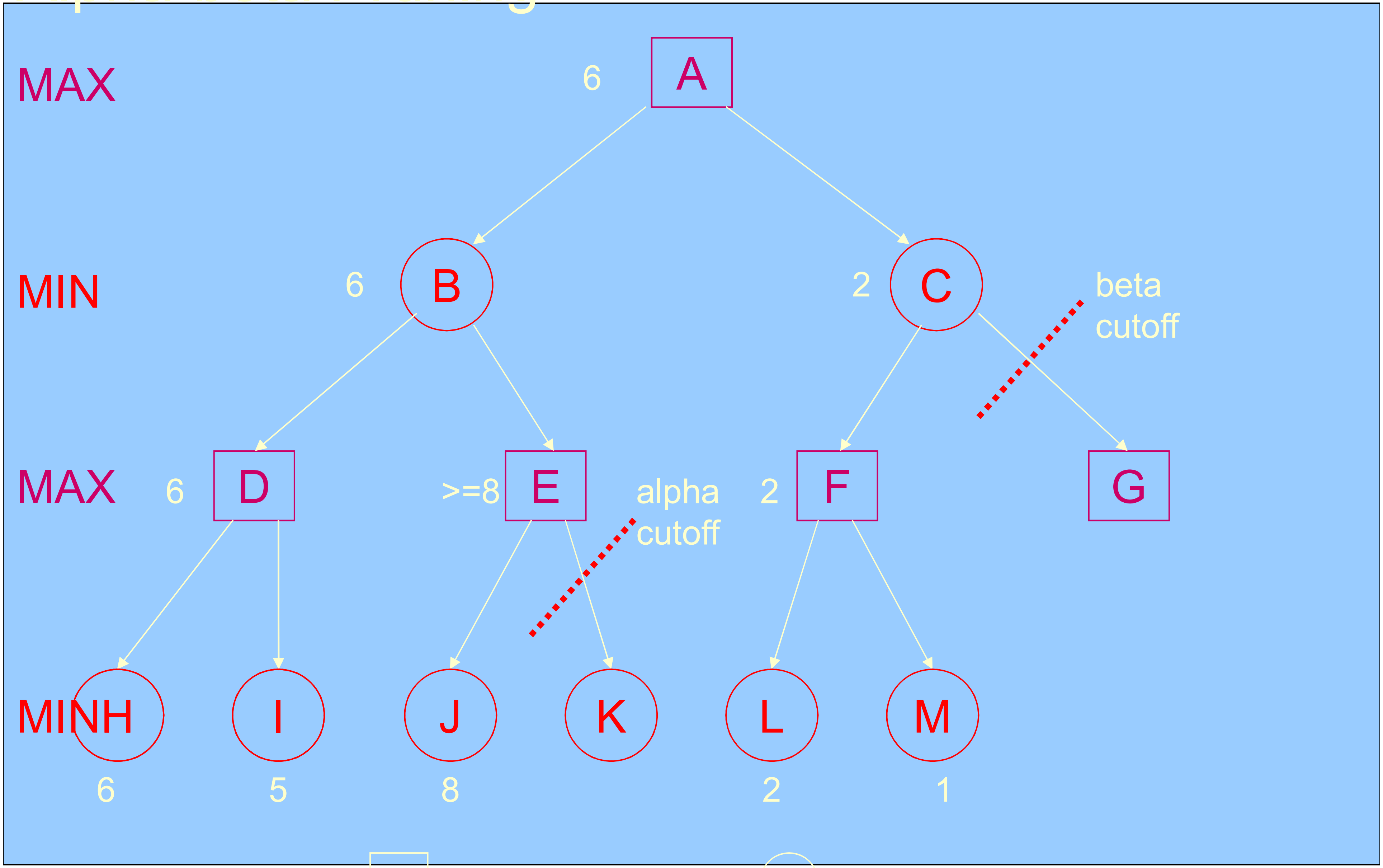
- Pruning does not affect final result
- Amount of pruning depends on move ordering
  - Should start with the “best” moves (highest-value for MAX or lowest-value for MIN)
  - For chess, can try captures first, then threats, then forward moves, then backward moves
  - Can also try to remember “killer moves” from other branches of the tree
- With perfect ordering, the time to find the best move is reduced to $O(b^{m/2})$ from $O(b^m)$
  - Depth of search is effectively doubled


MAX
A
MIN
<=6
B
C
MAX
6
D
>=8
E
MIN
H
I
J
K
6
5
8
= agent
= opponent


MAX
>=6
A
MIN
6
B
<=2
C
MAX
6
D
>=8
E
2
F
G
MIN
H
I
J
K
L
M
6
5
8
2
1
= agent
= opponent


MAX
>=6
A
MIN
6
B
2
C
MAX
6
D
>=8
E
2
F
G
MIN
H
I
J
K
L
M
6
5
8
2
1
= agent
= opponent

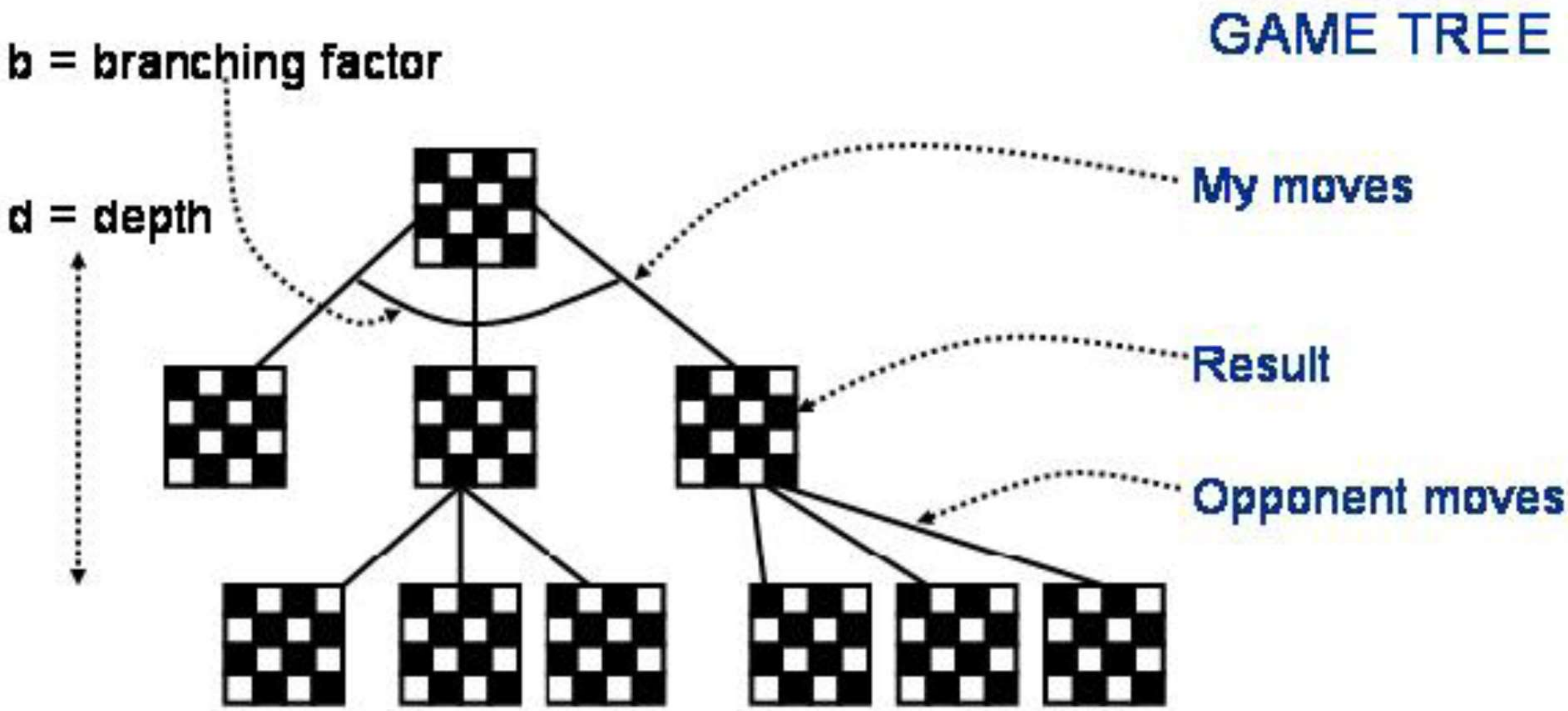
Alpha-beta Pruning



= agent

= opponent

Move generation



Chess

b = 36
d > 40

36^{40} is big!

Resource limits

Suppose we have 100 secs, explore 10^4 nodes/sec

→ 10^6 nodes per move

Standard approach:

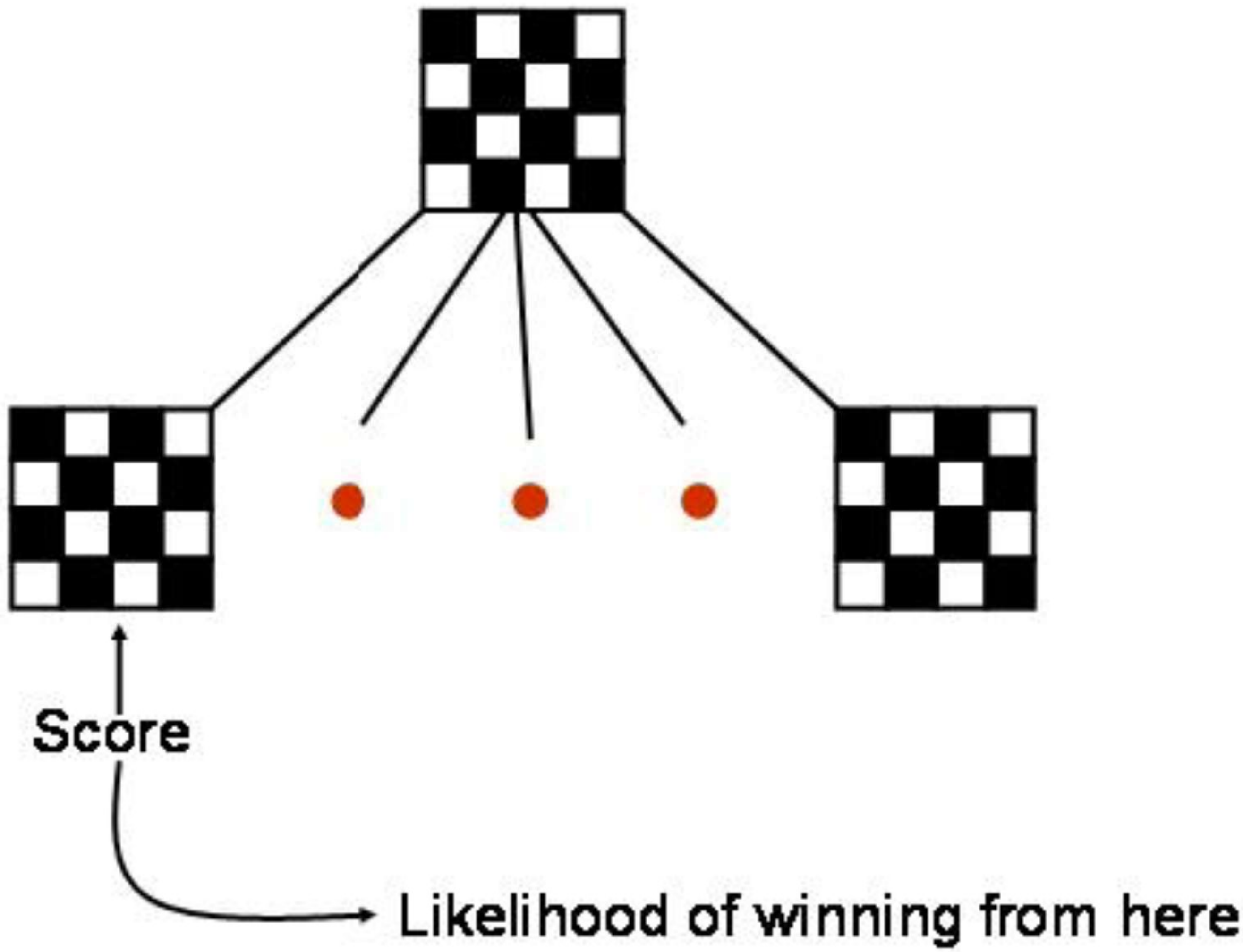
- cutoff test:

  e.g., depth limit (perhaps add quiescence search)

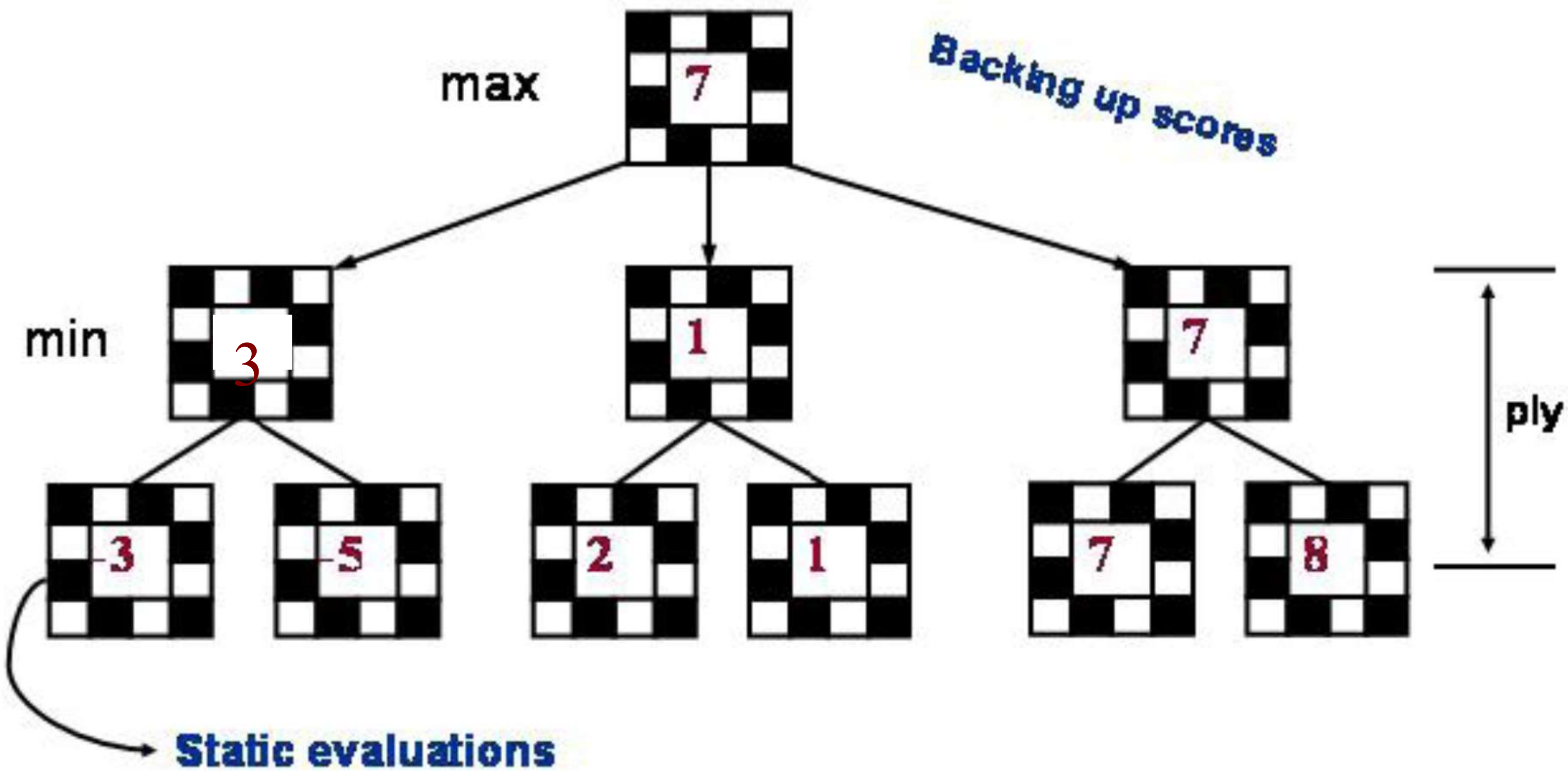
- evaluation function

  = estimated desirability of position

Evaluation function



Min-Max


max
7
Backing up scores
min
3
1
7
ply
-3
-5
2
1
7
8
Static evaluations

Evaluation functions

- A typical evaluation function is a linear function in which some set of coefficients is used to weight a number of "features" of the board position.

- For chess, typically linear weighted sum of features

$$Eval(s) = w_1\, f_1(s) + w_2\, f_2(s) + \ldots + w_n\, f_n(s)$$

- e.g., $w_1 = 9$ with

  $f_1(s)$ = (number of white queens) – (number of black queens), etc.

Evaluation function

$$
\begin{aligned}
S = \; & c_1 \times \text{material} \\
+ \; & c_2 \times \text{pawn structure} \\
+ \; & c_3 \times \text{mobility} \\
+ \; & c_4 \times \text{king safety} \\
+ \; & c_5 \times \text{center control} \\
+ \; & \ldots
\end{aligned}
$$

P	1
K	3
B	3.5
R	5
Q	9

- "material", : some measure of which pieces one has on the board.
- A typical weighting for each type of chess piece is shown
- Other types of features try to encode something about the distribution of the pieces on the board.

Cutting off search

MinimaxCutoff is identical to *MinimaxValue* except

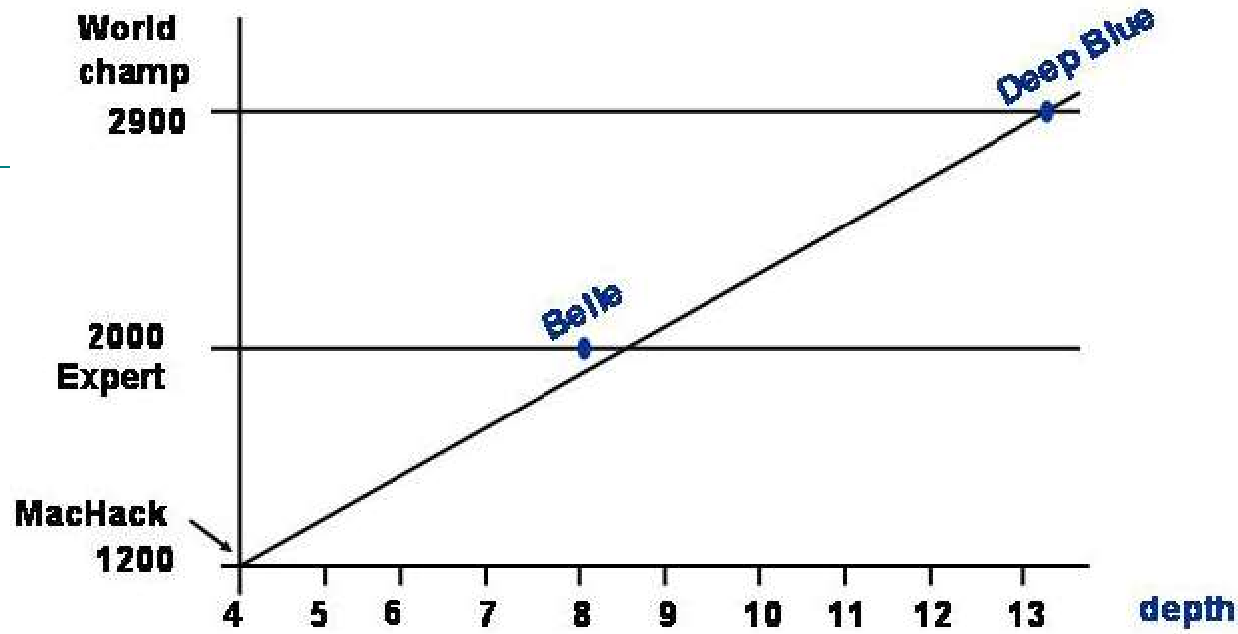
1. *Terminal?* is replaced by *Cutoff?*
2. *Utility* is replaced by *Eval*

Does it work in practice?

$b^m = 10^6$, b=35 → m=4

4-ply lookahead is a hopeless chess player!

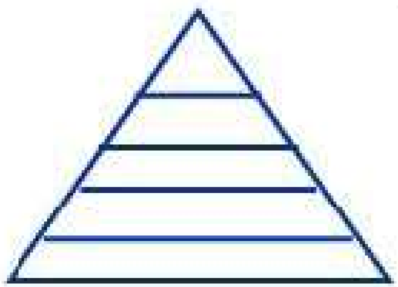
- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov



- The key idea is that the more lookahead we can do, that is, the deeper in the tree we can look, the better our evaluation of a position will be, even with a simple evaluation function. In some sense, if we could look all the way to the end of the game, all we would need is an evaluation function that was 1 when we won and -1 when the opponent won.

- it seems to suggest that brute-force search is all that matters.
- And Deep Blue is brute indeed... It had 256 specialized chess processors coupled into a 32 node supercomputer. It examined around 30 billion moves per minute. The typical search depth was 13ply, but in some dynamic situations it could go as deep as 30.

Practical issues



Variable branching

Iterative deepening

- order best move from last search first
- use previous backed up value to initialize [α, β]
- keep track of repeated positions (transposition tables)

Horizon effect

- quiescence
- Pushing the inevitable over search horizon

Parallelization

Evaluation function

- Cut off search at a certain depth and compute the value of an **evaluation function** for a state instead of its minimax value
  - The evaluation function may be thought of as the probability of winning from a given state or the *expected value* of that state
- A common evaluation function is a weighted sum of *features*:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

  - For chess, w_k may be the **material value** of a piece (pawn = 1, knight = 3, rook = 5, queen = 9) and $f_k(s)$ may be the advantage in terms of that piece
- Evaluation functions may be *learned* from game databases or by having the program play many games against itself

Cutting off search

- **Horizon effect:** you may incorrectly estimate the value of a state by overlooking an event that is just beyond the depth limit
  - For example, a damaging move by the opponent that can be delayed but not avoided
- Possible remedies
  - **Quiescence search:** do not cut off search at positions that are unstable – for example, are you about to lose an important piece?
  - **Singular extension:** a strong move that should be tried when the normal depth limit is reached

Advanced techniques

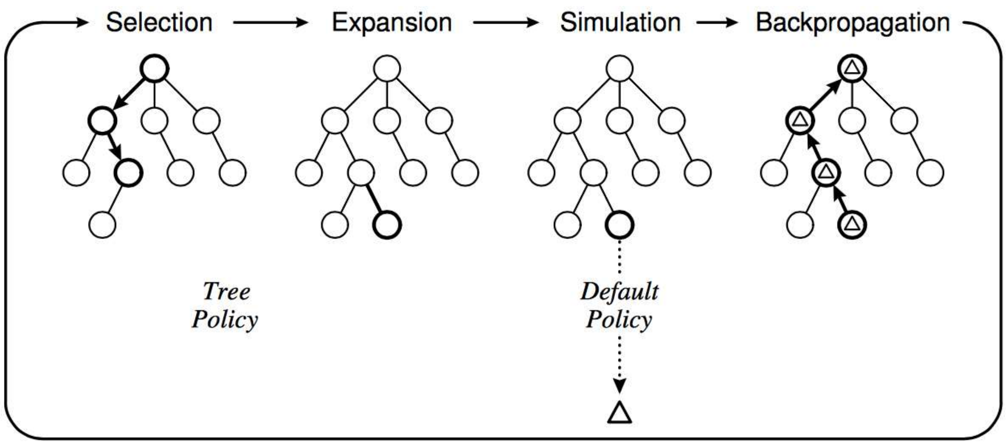
- **Transposition table** to store previously expanded states
- **Forward pruning** to avoid considering all possible moves
- **Lookup tables** for opening moves and endgames

Chess playing systems

- Baseline system: 200 million node evalutions per move (3 min), minimax with a decent evaluation function and quiescence search
  - 5-ply ≈ human novice
- Add alpha-beta pruning
  - 10-ply ≈ typical PC, experienced player
- Deep Blue: 30 billion evaluations per move, singular extensions, evaluation function with 8000 features, large databases of opening and endgame moves
  - 14-ply ≈ Garry Kasparov
- More recent state of the art (Hydra, ca. 2006): 36 billion evaluations per second, advanced pruning techniques
  - 18-ply ≈ better than any human alive?

Monte Carlo Tree Search

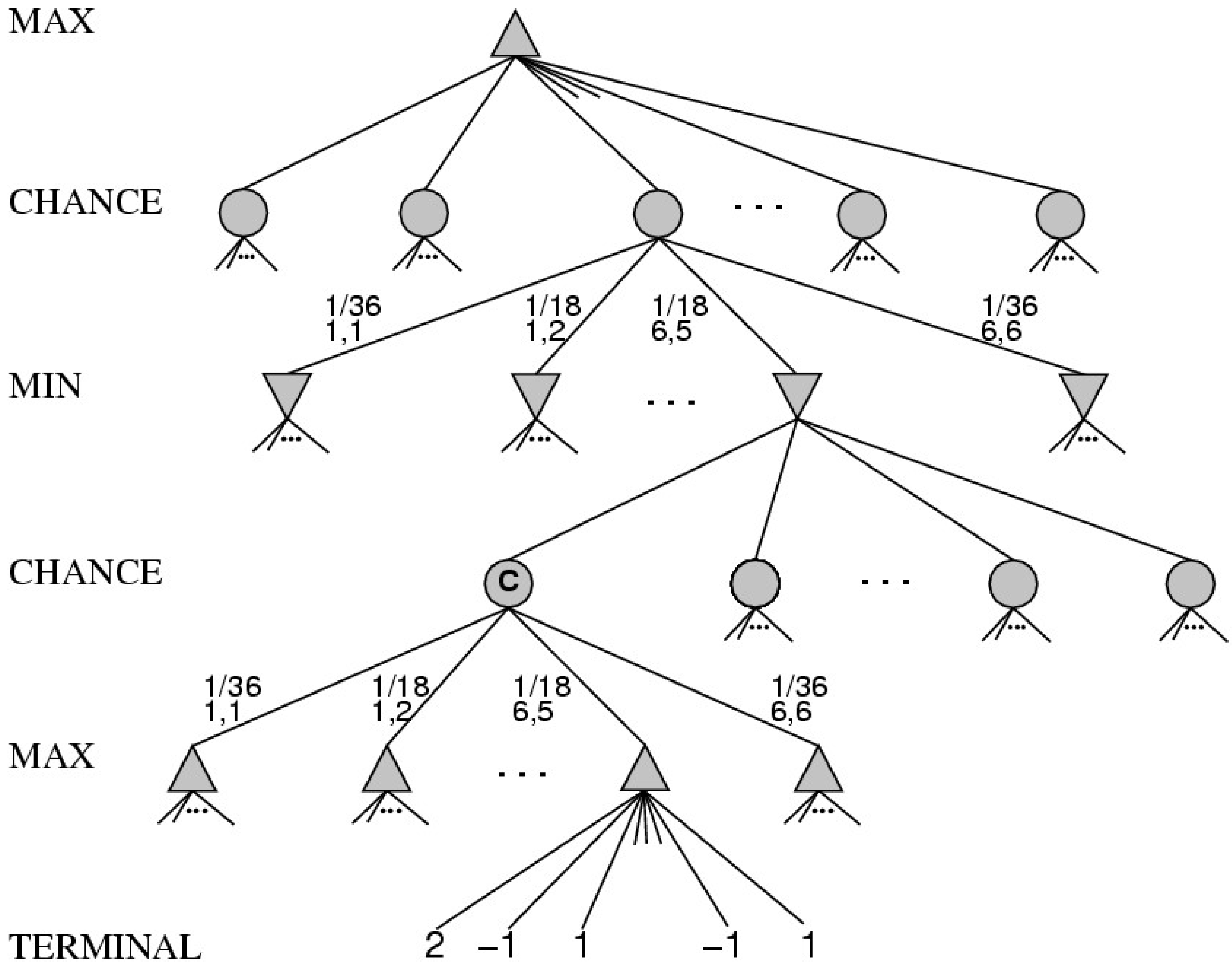
- What about games with deep trees, large branching factor, and no good heuristics – like Go?
- Instead of depth-limited search with an evaluation function, use randomized simulations
- Starting at the current state (root of search tree), iterate:
  - Select a leaf node for expansion using a *tree policy* (trading off *exploration* and *exploitation*)
  - Run a simulation using a *default policy* (e.g., random moves) until a terminal state is reached
  - Back-propagate the outcome to update the value estimates of internal tree nodes



Stochastic games

- How to incorporate dice throwing into the game tree?

Stochastic games



Stochastic games

- **Expectiminimax:** for chance nodes, sum values of successor states weighted by the probability of each successor

- **Value**(*node*) =
  - Utility(*node*) if *node* is terminal
  - $\max_{action}$ **Value**(Succ(*node, action*)) if *type* = MAX
  - $\min_{action}$ **Value**(Succ(*node, action*)) if *type* = MIN
  - sum_{action} P(Succ(*node, action*)) * **Value**(Succ(*node, action*)) if *type* = CHANCE

Stochastic games

- **Expectiminimax:** for chance nodes, sum values of successor states weighted by the probability of each successor
  - Nasty branching factor, defining evaluation functions and pruning algorithms more difficult
- **Monte Carlo simulation:** when you get to a chance node, simulate a large number of games with random dice rolls and use win percentage as evaluation function
  - Can work well for games like Backgammon

Stochastic games of imperfect information

States are grouped into *information sets* for each player

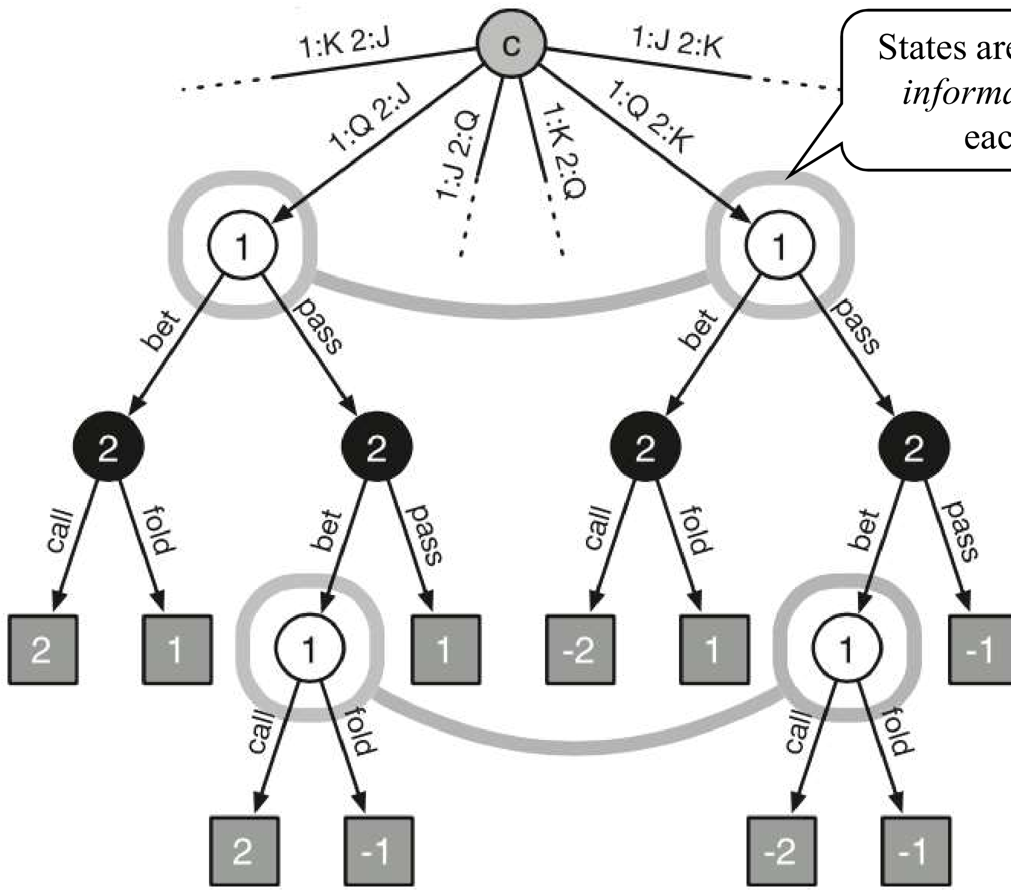


Fig. 1. Portion of the extensive-form game representation of three-card Kuhn poker (*16*). Player 1 is dealt a queen (Q), and the opponent is given either the jack (J) or king (K). Game states are circles labeled by the player acting at each state ("c" refers to chance, which randomly chooses the initial deal). The arrows show the events the acting player can choose from, labeled with their in-game meaning. The leaves are square vertices labeled with the associated utility for player 1 (player 2's utility is the negation of player 1's). The states connected by thick gray lines are part of the same information set; that is, player 1 cannot distinguish between the states in each pair because they each represent a different unobserved card being dealt to the opponent. Player 2's states are also in information sets, containing other states not pictured in this diagram.

Source

Stochastic games of imperfect information

- Simple Monte Carlo approach: run multiple simulations with random cards pretending the game is fully observable
  - "Averaging over clairvoyance"
  - Problem: this strategy does not account for bluffing, information gathering, etc.

Game AI: Origins

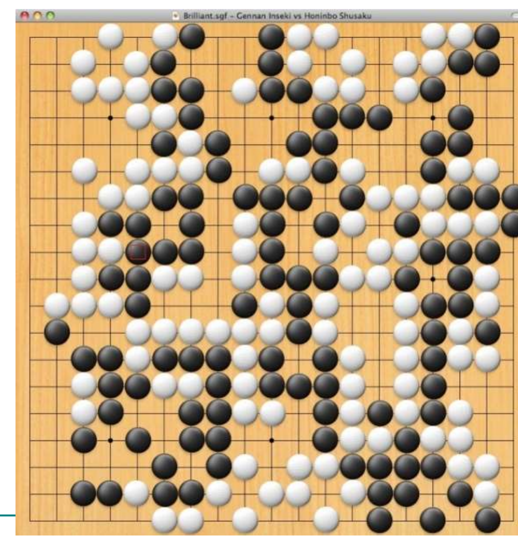
- Minimax algorithm: Ernst Zermelo, 1912
- Chess playing with evaluation function, quiescence search, selective search:
  Claude Shannon, 1949 (paper)
- Alpha-beta search: John McCarthy, 1956
- Checkers program that learns its own evaluation function by playing against itself: Arthur Samuel, 1956

Game AI: State of the art

- Computers are better than humans:
  - **Checkers:** solved in 2007
  - **Chess:**
    - State-of-the-art search-based systems now better than humans
    - Deep learning machine teaches itself chess in 72 hours, plays at International Master Level (arXiv, September 2015)
- Computers are competitive with top human players:
  - **Backgammon:** TD-Gammon system (1992) used *reinforcement learning* to learn a good evaluation function
  - **Bridge:** top systems use Monte Carlo simulation and alpha-beta search

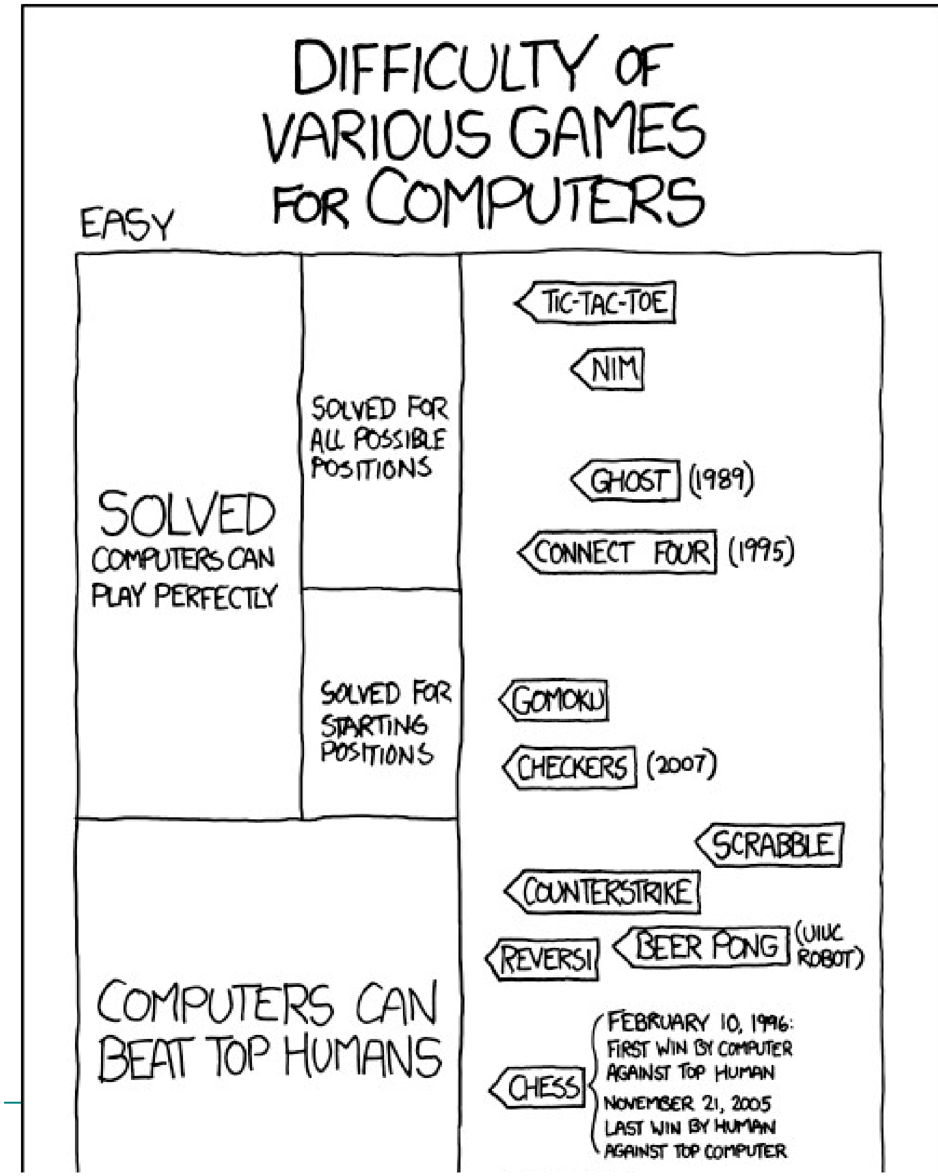
Game AI: State of the art

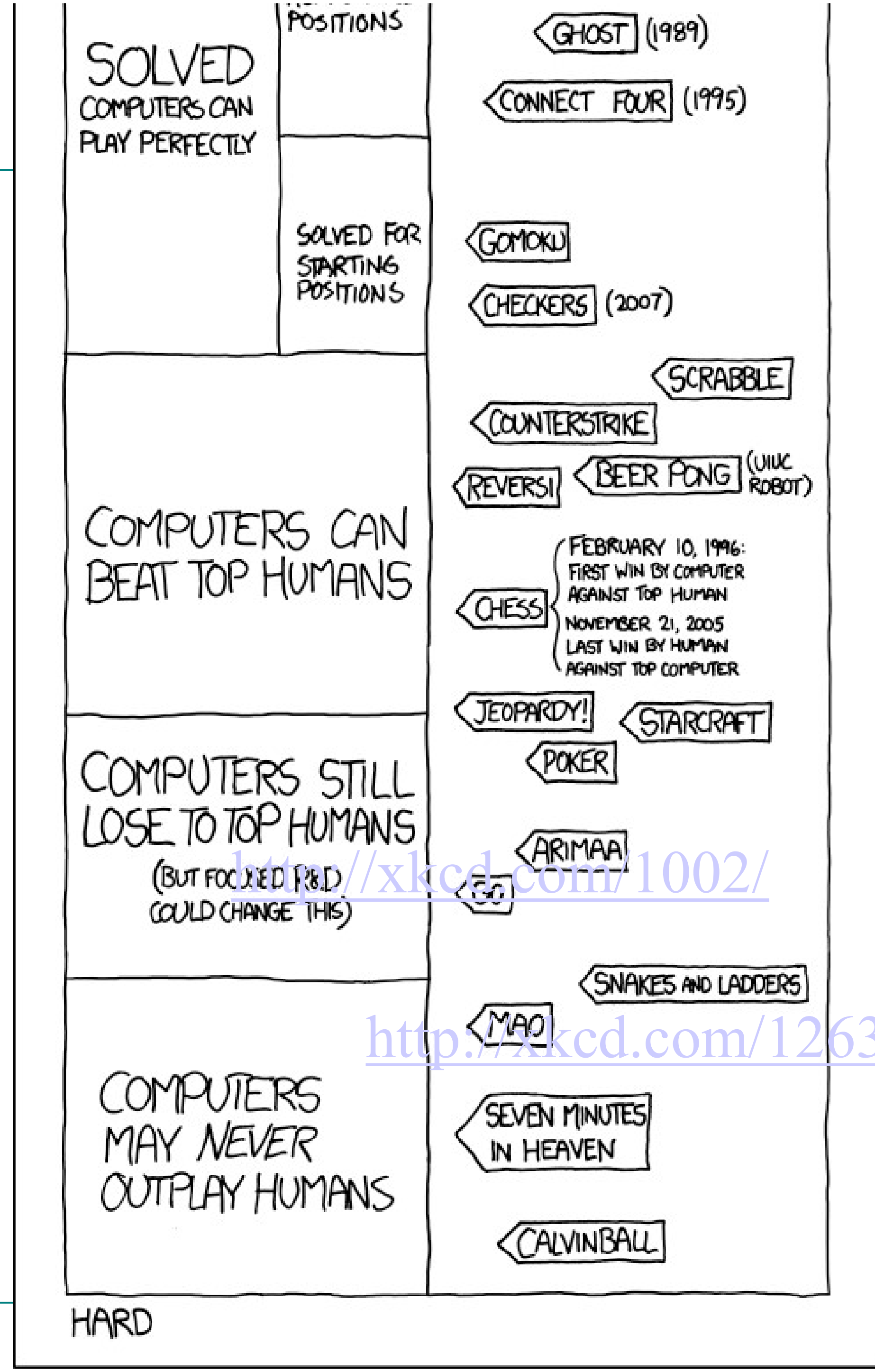
- Computers are not competitive with top human players:
  - **Poker**
    - Heads-up limit hold'em poker has been solved (Science, Jan. 2015)
      - Simplest variant played competitively by humans
      - Smaller number of states than checkers, but partial observability makes it difficult
      - *Essentially weakly solved* = cannot be beaten with statistical significance in a lifetime of playing
    - Huge increase in difficulty from limit to no-limit poker, but AI has made progress
  - **Go**
    - Branching factor 361, no good evaluation functions have been found
    - Best existing systems use Monte Carlo Tree Search and pattern databases
    - New approaches: deep learning (44% accuracy for move prediction, can win against other strong Go AI)



Review: Games

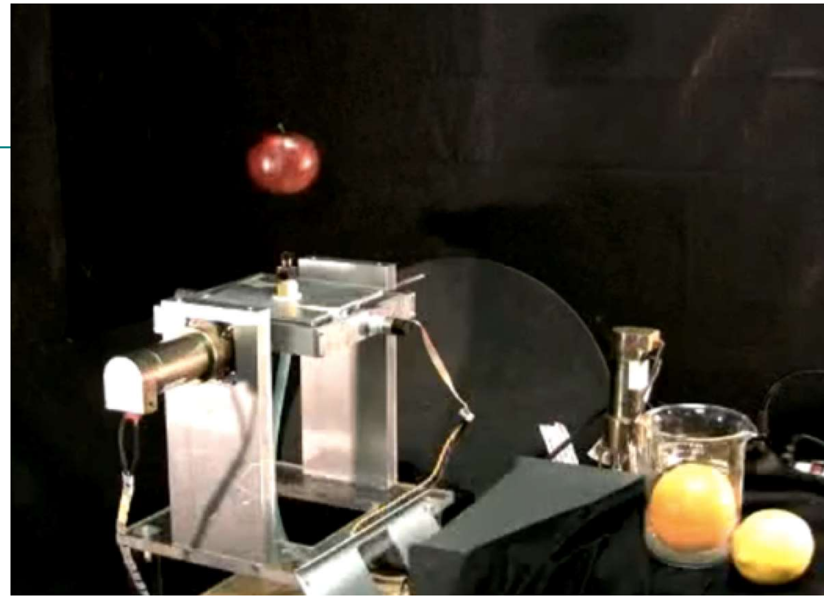
- Stochastic games
- State-of-the-art in AI





http://xkcd.com/1002/

http://xkcd.com/1263/



[UIUC robot (2009)]