Name-Last name :
Student No :
Section :

FİZ 138 PHYSICS II
MIDTERM EXAM I
March 28, 2017
13:00 – 14:30 (90 min)

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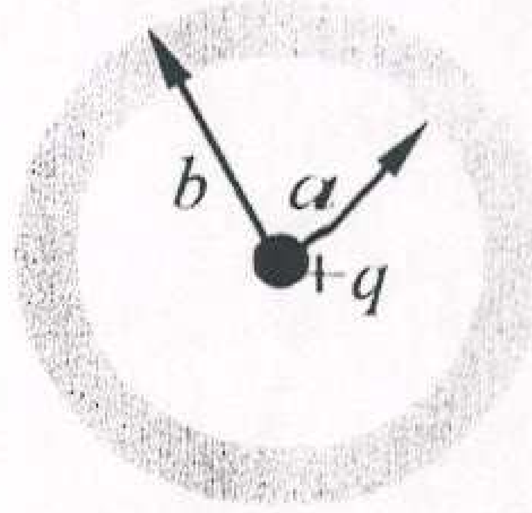
Questions

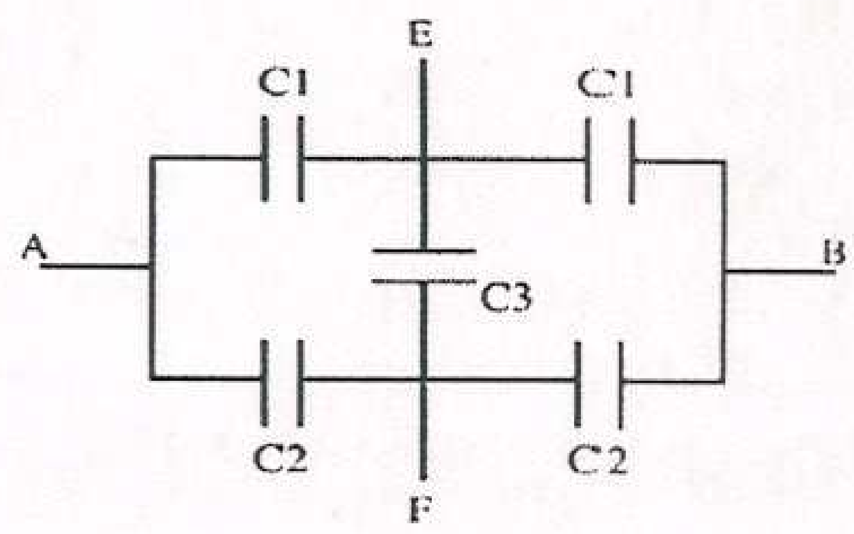
1. Figure shows a nonconducting spherical shell, of inner radius a and outer radius b, has a positive volume charge density $\rho = A/r$ (within its thickness), where A is a constant and r is the distance from the center of the shell. In addition, a positive charge q is located at the center. What value should A have if the electric field in the shell ($a \le r \le b$) is to be uniform?



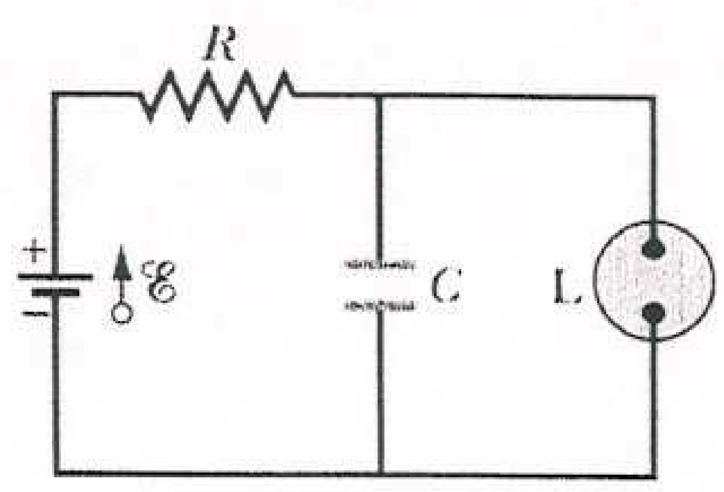
2. In a region where the electric potential is given by $V(x,y,z) = 2x^2 + yz$, find the electric field at the point with coordinates $x = 2$, $y = 1$, and $z = 2$. Everything is in SI units.

3. A spherical capacitor has radii a and b. What is the radius r for which the energy stored within, i.e., the spherical shell from radius a to radius r, is one third (1/3) of the total energy stored in the capacitor?

4. Calculate the equivalent capacitance (in terms of C_1, C_2 and C_3)
   a) C_{AB} between the points A and B,
   b) C_{EF} between the points E and F.



5. The fluorescent lamp L only functions when the potential difference across it reaches V_L— below that value, no current passes through it; then the capacitor discharges completely through the lamp and the lamp flashes briefly.
   a) With C, $\mathcal{E}$ (ideal *emf* device) and V_L given, calculate the necessary R in order to achieve n flashes per second from the lamp.
   b) If the circuit is turned on at $t = 0$, plot the voltage across the lamp's terminals with respect to time.



Charge on a charging capacitor: $q(t) = \mathcal{E}C[1 - \exp(-t/RC)]$
Charge on a discharging capacitor: $q(t) = q_o \exp(-t/RC)$

1) $\epsilon_0 E(a) 4\pi a^2 = q$ ← central charge ①

← charge in the shell (a to r)

$$\epsilon_0 E(r) 4\pi r^2 = q + \int_a^r dr' 4\pi r'^2 \frac{A}{r'} = q + 4\pi A \frac{1}{2}(r^2 - a^2) \quad ②$$

E : uniform $\longrightarrow E(a) = E(r) = E$

$$E = \underbrace{\frac{1}{4\pi\epsilon_0}\frac{q}{a^2}}_{①} = \underbrace{\frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\left[q + 2\pi A(r^2 - a^2)\right]}_{②}$$

$$\rightarrow \frac{qr^2}{a^2} = q + 2\pi A(r^2 - a^2)$$

$$q\left(\frac{r^2}{a^2} - 1\right) = 2\pi A(r^2 - a^2)$$

$$q\frac{\cancel{(r^2 - a^2)}}{a^2}\frac{1}{2\pi\cancel{(r^2 - a^2)}} = A \Rightarrow \underline{A = \frac{q}{2\pi a^2}}$$

2) $E_i = -\frac{dV}{dx_i} \rightarrow$ $E_x = -4x$, $E_y = -z$, $E_z = -y$

$$\vec{E}(x,y,z) = -4x\hat{i} - z\hat{j} - y\hat{k}$$

$$\underline{\vec{E}(2,1,2) = (-8\hat{i} - 2\hat{j} - \hat{k})\,N/C}$$

3) 1st Way

$u = \frac{1}{2}\epsilon_0 E^2$: Energy Density

$$E(r) = \frac{Q}{4\pi\epsilon_0 r^2} \rightarrow u = \frac{1}{2}\epsilon_0 \frac{Q^2}{16\pi^2\epsilon_0^2 r^4}$$

$$dU = u.dV = \frac{1}{2}\frac{Q^2}{16\pi^2\epsilon_0 r^4}4\pi r^2 dr$$

$$dU = \frac{Q^2}{8\pi\epsilon_0 r^2}dr$$

$$\frac{1}{3} = \frac{\int_a^r \frac{Q^2}{8\pi\epsilon_0 r'^2}dr'}{\int_a^b \frac{Q^2}{8\pi\epsilon_0 r^2}dr} = \frac{-\left[\frac{1}{r} - \frac{1}{a}\right]}{-\left[\frac{1}{b} - \frac{1}{a}\right]} = \frac{a-r}{ar}\cdot\frac{ab}{a-b}$$

2nd Way

$$U = \frac{1}{2}QV$$

$$E(r)4\pi r^2\epsilon_0 = Q \rightarrow E(r) = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$V(r) = -\int_a^r E(r')dr' = -\frac{Q}{4\pi\epsilon_0}\int_a^r \frac{dr'}{r'^2}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{a}\right] = \frac{Q}{4\pi\epsilon_0}\frac{(a-r)}{ar}$$

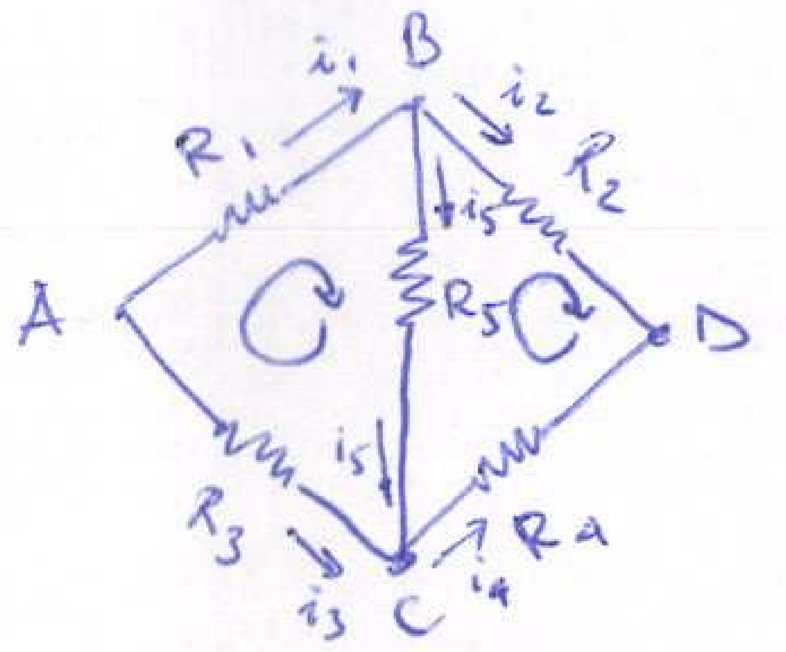
$$\frac{1}{3} = \frac{\frac{1}{2}QV(r)}{\frac{1}{2}QV(b)} = \frac{V(r)}{V(b)} = \frac{(a-r)}{ar}\cdot\frac{ab}{(a-b)}$$

$$\frac{1}{3} = \frac{(a-r)b}{(a-b)r} \longrightarrow 3ab - 3br = ar - br$$

$$3ab = r(a - b + 3b)$$

$$\boxed{r = \frac{3ab}{a+2b}}$$

Wheatstone Bridge Revisited



5 unknowns: i_1, i_2, i_3, i_4, i_5

$$\left.\begin{aligned} &B: \quad i_1 - i_2 - i_5 = 0 \\ &C: \quad i_3 + i_5 - i_4 = 0 \\ &ABC: \quad -i_1R_1 - i_5R_5 + i_3R_3 = 0 \\ &BDC: \quad -i_2R_2 + i_4R_4 + i_5R_5 = 0 \end{aligned}\right\} \text{4 equations}$$

(ABCD is just ABC + BDC, so nothing new)

5 unknowns − 4 equations = Not sufficient

Let's assume $i_5 = 0$. Then: $B: \; i_1 - i_2 = 0 \rightarrow i_1 = i_2 \equiv i_A$

$C: \; i_3 - i_4 = 0 \rightarrow i_3 = i_4 \equiv i_B$

$$\rightarrow \begin{aligned} &ABC: \; -i_AR_1 + i_BR_3 = 0 \rightarrow i_AR_1 = i_BR_3 \\ &BDC: \; -i_AR_2 + i_BR_4 = 0 \rightarrow i_AR_2 = i_BR_4 \end{aligned} \Big\} \rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\Rightarrow \boxed{i_5 = 0 \longleftrightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

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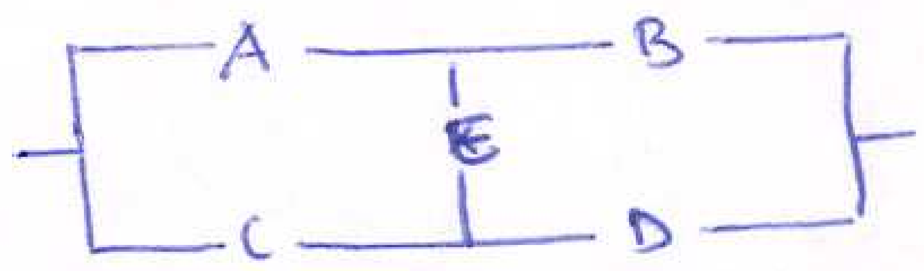
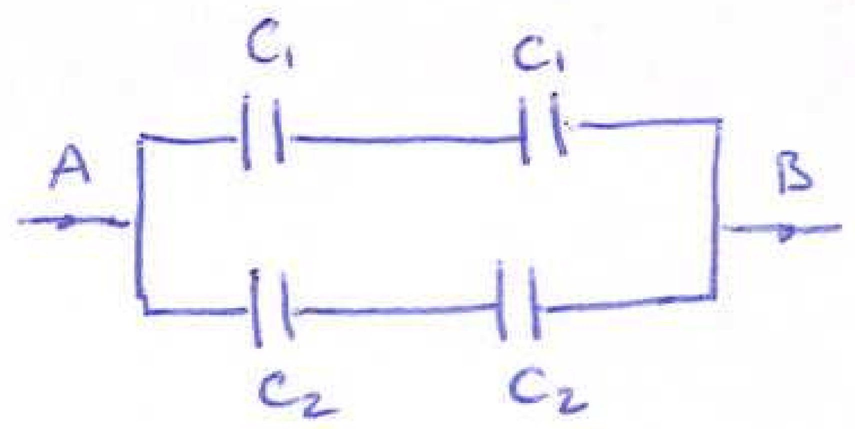
4) a) AB (Wheatstone Bridge)



$$\frac{X_A}{X_B} = \frac{X_C}{X_D} \Rightarrow i_E = 0$$

$$\frac{C_1}{C_1} = \frac{C_2}{C_2} \rightarrow i_3 = 0$$

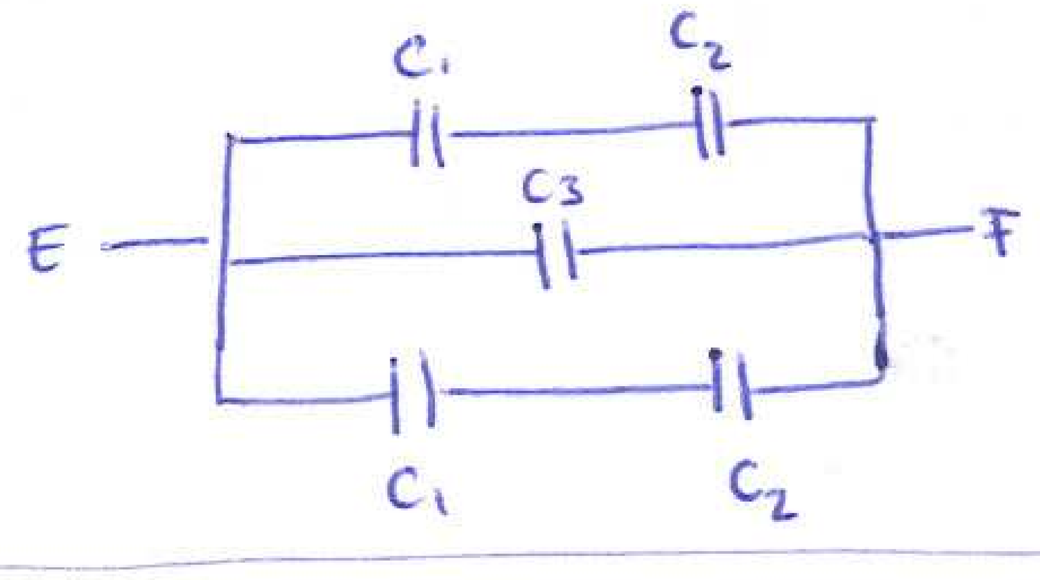
$\rightarrow$



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_1}\right)^{-1} + \left(\frac{1}{C_2} + \frac{1}{C_2}\right)^{-1}$$

$$= \frac{C_1}{2} + \frac{C_2}{2} = \underline{\frac{C_1 + C_2}{2}}$$

b) EF



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} + C_3 + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

$$= 2\,\frac{C_1C_2}{C_1 + C_2} + C_3$$

$$= \frac{2C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$$

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5) a) $V = \frac{Q}{C} \rightarrow V(t) = \frac{\mathcal{E}\cancel{C}[1 - \exp(-t/RC)]}{\cancel{C}}$

$V = V_L \leftrightarrow t = \frac{1}{n} \Rightarrow V_L = \mathcal{E}\left[1 - \exp\left(-\frac{1}{nRC}\right)\right]$

$$1 - \frac{V_L}{\mathcal{E}} = \exp\left(-\frac{1}{nRC}\right)$$

$$\ln\left(1 - \frac{V_L}{\mathcal{E}}\right) = -\frac{1}{nRC}$$

$$R = -\frac{1}{nC}\,\frac{1}{\ln\left(1 - \frac{V_L}{\mathcal{E}}\right)} = -\frac{1}{nC}\,\frac{1}{\ln\left(\frac{\mathcal{E} - V_L}{\mathcal{E}}\right)} = \frac{1}{nC\ln\left(\frac{\mathcal{E}}{\mathcal{E} - V_L}\right)}$$

b)