BBM 205 - Discrete Structures: Quiz 2 - Solutions
Date: 24.10.2018

Name:
Student ID:

1. (12 points) **Prove by cases** that

$$|r+s| \leq |r| + |s|$$

for all real numbers r, s.

Solution: We consider the following cases:

1. If $r, s \geq 0$, then
$$|r+s| = r+s \leq |r| + |s| = r+s.$$
2. If $r, s < 0$, then
$$|r+s| = -r-s \leq |r| + |s| = -r-s.$$
3. If $r < 0$ and $s \geq 0$, then first consider the case that $r+s \geq 0$. Then,
$$|r+s| = r+s \leq |r| + |s| = -r+s.$$
4. If $r < 0$ and $s \geq 0$, then secondly consider the case that $r+s < 0$. Then,
$$|r+s| = -r-s \leq |r| + |s| = -r+s.$$

The other two cases obtained by switching the role of r and s in the last two cases do not need to be considered, since the role of r and s are symmetric.

2. (8 points) **Prove by contradiction** that if $a \cdot b = n$, then either a or b must be less than or equal to $\sqrt{n}$, where a, b and n are nonnegative real numbers.

Solution: Let Q be the statement that either a or b must be less than or equal to $\sqrt{n}$, and assume that Q is false. This means that $a > \sqrt{n}$ and $b > \sqrt{n}$. In that case, $a \cdot b > (\sqrt{n})^2 = n$ which contradicts with the assumption that $a \cdot b = n$. Hence, Q must be true.