BBM 205 - Discrete Structures: Midterm 1 Date: 24.10.2017, Time: 16:00 - 17:30

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Question:	1	2	3	4	5	6	7	8	9	Total
Points:	15	12	10	7	12	12	12	10	10	100
Score:										

1. (15 points) Determine by using a truth table whether the following statements are a tautology (true for all combinations of x and y), a contradiction (false for all combinations of x and y) or neither. (Asagidaki ifadelerin totoloji (her x ve y degeri icin dogru), celiski (her x ve y degeri icin yanlis) ya da bunlardan hicbiri oldugunu dogruluk tablosu kullanarak gosterin.

(a)
$$x \wedge (x \implies y) \wedge (\neg y)$$

(b)
$$x \implies (x \lor y)$$

(c)
$$(x \lor y) \land (\neg(x \land y))$$

- 2. (12 points) Let p, q and r be the propositions (p, q ve r ifadeleri asagidaki gibidir.)
 - p: You get an A in the final exam (Finalden A aliyorsun)
 - q: You do every exercise in the book (Kitaptaki her alistirmayi cozuyorsun)
 - r: You get an A in this class (Dersten A aliyorsun)

Write the following propositions using p, q, r and logical connectives. (Asagidaki onermeleri p, q, r ve mantiksal baglaclar kullanarak yazin.)

- (a) You get an A in this class, but you do not do every exercise in the book. (Dersten A aliyorsun, ama kitaptaki her alistirmayi cozmuyorsun.)
- (b) You get an A on the final exam, you do every exercise in the book, and you get an A in this class. (Finalden A aliyorsun, kitaptaki her alistirmayi cozuyorsun, ve dersten A aliyorsun.)
- (c) To get an A in this class, it is necessary for you to get an A on the final exam. (Dersten A alman icin finalden A alman gerekli.)
- (d) Getting an A on the final exam and doing every exercise in the book are sufficient for getting an A in this class. (Finalden A almak ve kitaptaki her alistirmayi cozmek, dersten A almak icin yeterli.)

3. (10 points) Show using proof by contrapositive for any number t that if t is irrational, then 5t is irrational. (Kontrapozitif ile ispat yontemini kullanarak her t sayisi icin eger t irrasyonel ise 5t'nin de irrasyonel olacagini gosterin.)

4. (7 points) Prove or disprove that for any integer n, if $n^2 \equiv 0 \pmod{4}$, then $n \equiv 0 \pmod{4}$. (Her n tamsayisi icin eger $n^2 \equiv 0 \pmod{4}$ ise $n \equiv 0 \pmod{4}$ oldugunun dogru ya da yanlis oldugunun gosterin.)

5. (12 points) Prove using induction that, for $n \ge 1$, $1 \cdot 1! + 2 \cdot 2! + \dots + n \cdot n! = (n+1)! - 1$. (Verilen ifadeyi her $n \ge 1$ icin tumevarim kullanarak gosterin.)

6. (12 points) Show using proof by contradiction the following statement: Let x and y be two positive integers. If xy < 36, then either x < 6 or y < 6. (Celiski ile ispat yontemini kullanarak verilen ifadenin dogrulugunu gosterin: Her x ve y pozitif tamsayisi icin, eger xy < 36 ise ya x < 6 ya da y < 6 dogrudur.)

7. (12 points) Use induction to prove that for all non-negative integers n,

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

(Yukaridaki ifadenin en az sifir degerinde olan her n tamsayisi icin dogrulugunu tumevarim kullanarak gosterin.)

8. (10 points) The Fibonacci number F_n is described as: $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Show by using induction that for all non-negative integers n, F_n is even if and only if n is divisible by 3. (Fibonacci sayisi yukaridaki soruda tanimlanmistir. Tumevarim kullanarak her sifirdan buyuk n tamsayisi icin, F_n 'in cift olmasinin ve n'in 3'e tam bolunebilmesinin denk kosullar oldugunu gosterin.)

9.	(10 points) Show by using induction that every non-negative integer can be written as
	the sum of <u>distinct</u> powers of 2. (Tumevarim kullanarak her sifirdan buyuk tamsayinin 2'nin <u>farkli</u> kuvvetlerinin toplami olarak yazilabilecegini gosterin.)
	2 inn <u>tarkii</u> kuvvetierinin topiann olarak yazhabhetegini gosterin.)

Reference Sheet for Logic and Program Proofs

Logical Equivalences

Definition of \land	Idempotent Laws	DeMorgan's Laws	Distributive Laws
$P \wedge \neg P \equiv False$	$p \lor p \equiv p$	$\neg(p \land q) \equiv \neg p \lor \neg q$	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
$P \wedge False \equiv False$	$p \wedge p \equiv p$	$\neg (p \lor q) \equiv \neg p \land \neg q$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
$P \wedge True \equiv P$			
Definition of ∨	Double Negation	Absorption Laws	Associative Laws
$P \lor \neg P \equiv True$	$\neg(\neg p) \equiv p$	$p \lor (p \land q) \equiv p$	$(p \lor q) \lor r \equiv p \lor (q \lor r)$
$P \vee False \equiv P$		$p \land (p \lor q) \equiv p$	$(p \land q) \land r \equiv p \land (q \land r)$
$P \lor True \equiv True$			
	Commutative Laws	Implication Laws	Biconditional Laws
	$p \vee q \equiv q \vee p$	$p \to q \equiv \neg p \vee q$	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$
	$p \wedge q \equiv q \wedge p$	$p \to q \equiv \neg q \to \neg p$	$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$

Inference Rules

Simplification	Modus Ponens	Modus Tollens	Hypothetical Syllogism	
$p \wedge q$	p	$\neg q$	p o q	
	$p \to q$	p o q	$q \rightarrow r$	
Therefore, p	Therefore, q	Therefore, $\neg p$	Therefore, $p \to r$	
Conjunction	Addition	Resolution	Disjunctive Syllogism	
p	p	$p \lor q$	$p \lor q$	
q		$\neg p \lor r$	$\neg p$	
Therefore, $p \wedge q$	Therefore, $p \lor q$	Therefore, $q \vee r$	Therefore, q	
Universal Instantiation $\forall x P(x)$	Universal Generalization $P(c)$	Existential Instantiation $\exists x P(x)$	Existential Generalization $P(c)$	
Therefore, $P(c)$	Therefore, $\forall x P(x)$	Therefore, $P(c)$	Therefore, $\exists x P(x)$	

Inference Rules For Program Proofs

Composition Rule	Conditional Rule	Conditional with Else Rule
$p\{S_1\}q$	$(p \land condition)\{S\}q$	$(p \land condition)\{S_1\}q$
$q\{S_2\}r$	$(p \land \neg condition) \rightarrow q$	$(p \land \neg condition)\{S_2\}q$
$p\{S_1; S_2\}r$	$p\{ \text{ if } condition } S\}q$	$p\{ \text{ if } condition } S_1 \text{ else } S_2 \} q$
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