# BBM 205
## Quiz 10
## Solutions

a) $A(n) = A(n-1) + 1$, where $A(0) = 0$.

(Proof skipped, induction would be helpful.)

$$A(n) = A(n-1)+1 = A(n-2)+2 = \dots = \underset{i \in \{1,2,\dots n\}}{A(n-i)+i} = \dots$$

$$\dots = A(n-n) + n = A(0) + n = n$$

b) $$B(n) = \begin{cases} 0 & \text{if } n<5 \\ B(n-5)+2 & \text{otherwise} \end{cases}$$

$$B(n) = B(n-5)+2 = B(n-10)+4 = \dots = B(n-5i)+2i = \dots$$

$$= \dots = B(n-5k)+2k, \text{ where } k = \lfloor \tfrac{n}{5} \rfloor$$

$$= 0 + 2k = 2\lfloor \tfrac{n}{5} \rfloor$$

c) $C(n) = C(n-1) + 2n - 1$, where $C(0) = 0$.

$$C(n) = C(n-1) + 2n - 1 = C(n-2) + 2(n+(n-1)) - 2 =$$

$$= \dots = C(n-i) + \left[\sum_{k=0}^{i-1} 2(n-k)\right] - 2i = \dots$$

$$= C(n-n) + \left[\sum_{k=0}^{n-1} 2(n-k)\right] - 2n = C(0) + 2n^2 - \left(\sum_{k=0}^{n-1} 2k\right) - 2n =$$

$$= 0 + 2n^2 - \frac{\cancel{2}n \cdot (n-1)}{\cancel{2}} - 2n = n^2 - n$$

d) $D(n) = D(n-1) + \binom{n}{2}$, where $D(0) = 0$.

$$D(n) = D(n-1) + \binom{n}{2} = D(n-2) + \binom{n}{2} + \binom{n-1}{2} = \dots$$

$$\dots = D(n-i) + \binom{n}{2} + \binom{n-1}{2} + \dots + \binom{n-i+1}{2} = \dots$$

$$\dots = \underbrace{D(\cancel{n}-n)}_{0} + \binom{n}{2} + \binom{n-1}{2} + \dots + \underbrace{\cancel{\binom{1}{2}}}_{0} =$$

$$= \frac{1}{2} \sum_{i=1}^{n-1} i \cdot (i+1) = \frac{1}{2} \sum_{i=1}^{n-1} i^2 + \frac{1}{2} \sum_{i=1}^{n-1} i =$$

$$= \frac{1}{2} \left( \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} \right) = \frac{n^3 - n}{6}.$$

e) $E(n) = E(n-1) + 2^n$, where $E(0) = 0$.

$$E(n) = E(n-1) + 2^n = E(n-2) + 2^n + 2^{n-1} = \dots =$$

$$= E(n-i) + \sum_{k=n-i+1}^{n} 2^k = \dots = \underbrace{E(\cancel{n}-n)}_{0} + \sum_{k=1}^{n} 2^k =$$

$$= (2^{n+1} - 1) - 2^0 = 2^{n+1} - 2$$

f) $F(n) = 3 \cdot F(n-1)$, where $F(0) = 1$.

$$F(n) = 3 \cdot F(n-1) = 3^2 \cdot F(n-2) = \dots = 3^i F(n-i) = \dots = 3^n \cdot \underbrace{F(\cancel{n}-n)}_{1}$$

$$F(n) = 3^n$$