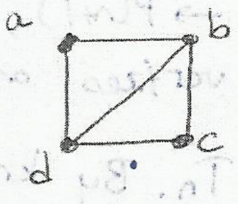
BBM 205 - Discrete Structures
Quiz 7 - 5. 12. 2018

Name:

Student ID:

SOLUTIONS

1. (6 points) Find the number of walks of length 3 between a and c in the following graph <u>by using the adjacency matrix</u> of the given graph.



$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad A_G^2 = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 3 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 3 \end{bmatrix}$$

$$A_G^3 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 2 & 5 & 2 & 5 \\ 5 & 4 & 5 & 5 \\ 2 & 5 & 2 & 5 \\ 5 & 5 & 5 & 4 \end{bmatrix} \end{array}$$

# of length-3 a,c-walks is 2, as seen as the marked entry in A_G^3.

2. (6points) Show by using induction on the number of vertices that the number of edges in a tree is one less than the number of vertices in a tree.

Let $P(n)$ be the statement that every tree on n vertices, say T_n, satisfies $|E(T_n)| = |V(T_n)| - 1$.

Base step: $P(1)$ is true, since there is no edge if only one vertex.

Induction step: To show $P(n) \rightarrow P(n+1)$, consider a tree, say T_{n+1}, with $n+1$ vertices and remove a leaf to obtain a new tree, say T_n. By knowing that $P(n)$ is true, we know $|V(T_n)| + 1 = |E(T_n)|$. Since removing the leaf reduced # edges and # vxs by 1, we have

$$|V(T_{n+1})| = |V(T_n)| + 1 \overset{I.H.}{=} |E(T_n)| = |E(T_{n+1})| - 1.$$

DONE.

3. (6points) Show that if a graph is bipartite, then it does not contain any odd cycle.

(There are many possible answers that count.)

Proof by contradiction: Consider a bipartite graph with parts V_1 and V_2. Say there is a cycle of length $2k+1$ with vertices $v_1, v_2, \dots, v_{2k+1}$. If v_1 is in one part, say V_1, then v_2 must be in V_2, by the same idea v_3 must be in V_1 and so on. At the end, we see that v_{2k+1} must be in V_1, but since $v_1 \in V_1$ as well, the edge $v_1 v_{2k+1}$ has both ends in V_1, contradiction.

Hence, for any $k \in \mathbb{Z}^+$, C_{2k+1} cannot be a subgraph.