Problem Set 1

This first few problems on this homework are meant to exercise your innate problem solving skills. Some of you may find them harder than homework questions in subsequent assignments, which will be more closely tied to actual material covered in lecture.

1. **Cases**
   Prudence has a special (cubic) die. The values on its faces are the integers from 1 to 6, but they are not arranged as in a normal die. When Prudence first tosses the die, the sum of the values on the four side faces is 15. In her second toss, the sum of these values is 12. Find what value appears in the face opposite 6 on Prudence's special die. (Hint: what are possible values for the top and bottom face when the sum of the side faces is 12).

2. **Even and odd**
   25 families live in the squares of a 5×5 chessboard, each occupying a square. As it often happens, each family thinks that their neighbors (i.e. those families living in the squares with which their square shares a side) have better living units. At a town hall meeting it is resolved to make everyone happier by moving the 25 families around so that each ends up in a square of one of their former neighbors. Unfortunately try as they might, the families cannot figure out any relocation scheme to carry out this resolution. Give a very simple reason explaining why they were doomed to fail. (Hint: 25 is odd).

3. **Trying Small Examples**
   A and B play the following game: starting with a pile of n stones, A and B take turns, each removing 1, 2, 3 or 4 stones from the pile. The player who removes the last stone loses the game. A goes first. For which values of n is this game a forced win by B? By A? (Hint: Try small values of n starting with $n = 1$. Do you see a pattern?)

4. **Portia's Caskets**
   In Shakespeare's Merchant of Venice, Portia had three caskets, gold, silver and lead, inside one of which was her portrait. Her suitor was asked to choose one of the caskets, and if he chose the portrait, he could claim Portia as his bride.

   Here is Portia's casket test again with a twist. Suppose there are two casket makers, Bellini and Cellini. Bellini always writes true statements on his caskets, while Cellini always writes false ones.

   (a) Suppose one of the caskets contains, not a portrait, but a dagger, and the suitor's job is to avoid choosing that casket. The caskets are inscribed as follows:
   Gold: The dagger is in this casket
   Silver: This casket is empty.
   Lead: At most one of these three caskets was made by Bellini.
   Which casket should the suitor choose? Explain your answer.

   (b) Suppose Portia placed her portrait in one of the caskets and the suitor's job is to select the casket with the portrait and to determine the maker of each of the three caskets. The caskets are inscribed as follows:
   Gold: The portrait is in here.
   Silver: The portrait is in here.
   Lead: At least two of these caskets were made by Cellini.
   How should the suitor answer to pass the test? Explain.

5. **Everyday logic**

Let's try to apply the language of Logic and Propositions that we learned in class to the rag "Everybody loves my baby, but my baby don't love nobody but me" (by Jack Palmer, ca. the 1920s). Suppose that our universe has one relationship, LOVES, and two distinguished individuals, ME and MYBABY. For example, LOVES(ME, MYBABY) means "I love my baby" and $\exists x \neg$ LOVES(x, ME) means "not everybody loves me."

- Write a proposition, using this vocabulary and the Boolean connectives $\wedge$, $\vee$, $\neg$, $\Rightarrow$, as well as quantifiers $\forall$ and $\exists$, to transcribe this verse into the language of Logic.

- Look now at what you wrote, and argue that this proposition implies ME = MYBABY !

Moral: Translating everyday life into Logic is a tricky business...