BBM 205 - Discrete Structures: Quiz 3 - Solutions
Date: 31.10.2018

Name:
Student ID:

1. (10 points) **Use proof by induction** to show that the following statement is true for all $n \geq 0$.

$$1 + 2 + \cdots + 2^n = 2^{n+1} - 1.$$

Solution: P(n) is the statement $1 + 2 + \cdots + 2^n = 2^{n+1} - 1$.
Base case: P(0) is true, since $1+2+\cdots+2^0 = 2^0 = 1$, which is equal to $2^{0+1} - 1 = 1$.
Inductive Step: We can assume that P(n) is true (called inductive hypothesis, I.H.) to show that P(n+1) is true.
$1 + 2 + \cdots + 2^n + 2^{n+1}$ can be separated into two parts:
Let the first part be $1 + 2 + \cdots + 2^n$, which is equal to $2^{n+1} - 1$ by I.H.
Now, adding the second part 2^{n+1} to this gives $(2^{n+1} - 1) + 2^{n+1}$. This equals $2 \cdot 2^{n+1} - 1 = 2^{n+2} - 1$, we are done.

2. (10 points) **Use proof by induction** to show that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for all $n \geq 0$.

Solution: P(n) is the statement that $7^{n+2} + 8^{2n+1}$ is divisible by 57.
Base case: P(0) is true, since $7^{0+2} + 8^{2 \cdot 0+1} = 7^2 + 8 = 57$, divisible by 57.
Inductive Step: We can assume that P(n) is true (called inductive hypothesis, I.H.) to show that P(n+1) is true.
Observe that $7^{(n+1)+2} + 8^{2(n+1)+1}$ can be rewritten as $7 \cdot 7^{n+2} + 7 \cdot 8^{2n+1} + 57 \cdot 8^{2n+1}$.
The first part $7 \cdot 7^{n+2} + 7 \cdot 8^{2n+1} = 7 \cdot (7^{n+2} + 8^{2n+1})$ and this is divisible by 57 by I.H.
The second part $57 \cdot 8^{2n+1}$ is also divisible by 57, done.