Mapping $L: V \to V \quad L(A) = A^T$

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in R \right\}$$

(A) Is L a linear transformation?

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$$L(a+b) = \ell(a) + \ell(b) \quad \Big\} \quad L(c.a) = c\,\ell(a)$$

$$a = \begin{bmatrix} j & k \\ L & m \end{bmatrix} \quad b = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$$

$$L(a+b) = \begin{bmatrix} j+x & k+y \\ L+z & m+t \end{bmatrix}^T = \begin{bmatrix} J+x & L+z \\ k+y & m+t \end{bmatrix}$$

$$L(a)+L(b) = \begin{bmatrix} j & k \\ L & m \end{bmatrix}^T + \begin{bmatrix} x & y \\ z & t \end{bmatrix}^T$$

$$= \begin{bmatrix} j & L \\ k & m \end{bmatrix} + \begin{bmatrix} x & z \\ y & t \end{bmatrix} = \begin{bmatrix} j+x & L+z \\ k+y & m+t \end{bmatrix}$$

$$L(c.a) = \begin{bmatrix} cj & ck \\ cL & cm \end{bmatrix}^T = \begin{bmatrix} cj & cL \\ ck & cm \end{bmatrix} = c \begin{bmatrix} j & L \\ k & m \end{bmatrix}$$

$$c\,L(a) = c \begin{bmatrix} j & k \\ L & m \end{bmatrix}^T = c \begin{bmatrix} j & L \\ k & m \end{bmatrix}$$

L is a lineer Transformation

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(B) find the matrix ${}_B[L]_B$

$$L\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right)=\begin{bmatrix}1&0\\0&0\end{bmatrix}=1a+0b+0c+0d$$

$$L\left(\begin{bmatrix}0&1\\0&0\end{bmatrix}\right)=\begin{bmatrix}0&0\\1&0\end{bmatrix}=0a+0b+1c+0d$$

$$L\left(\begin{bmatrix}0&0\\1&0\end{bmatrix}\right)=\begin{bmatrix}0&1\\0&0\end{bmatrix}=0a+1b+0c+0d$$

$$L\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right)=\begin{bmatrix}0&0\\0&1\end{bmatrix}=0a+0b+0c+1d$$

$${}_B[L]_B=\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$$