Theory Walk Through for Cross-Correlating Redshift-Free Standard Candles

Mukherjee and Wandelt (2018) Paper

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1 Abstract:

LSST will supply up to 10^6 supernovae (SNe) to constrain dark energy through the distance - redshift (D$_L$ - z) test. Obtaining spectroscopic SN redshifts (spec-zs) is unfeasible; alternatives are suboptimal and may be biased. We propose a powerful multi-tracer generalization of the Alcock- Paczynski test that pairs redshift-free distance tracers and an overlapping galaxy redshift survey. Cross-correlating 5×10^4 redshift-free SNe with DESI or Euclid outperforms the classical D$_L$L - z test with spec-zs for all SN. Our method also applies to gravitational wave sirens or any redshift-free distance tracer.

2 Introduction:

- Accurate trace of the expansion history, e.g. through the D$_L$ - z relation, is one of the foremost goals of current and next-generation surveys such as SDSS-IV, DES, DESI, EUCLID, LSST, and WFIRST
- LSST going to generate large type Ia SN sample at a rate of $\geq 10^4 yr^{-1}$. Already now, the $\geq 10^3 yr^{-1}$ SNe being observed over a wide redshift range make it impossible to obtain time-consuming spectroscopic follow-up for every SN, (the traditional approach underlying the success of cosmology with standard candles over the last two decades).
- Alternative approach combines photometric types with spectroscopic redshift measurements of the presumed SN host galaxy, Errors may lead to biases and loss of information in the inferred cosmological parameters.
- Future galaxy redshift surveys (DESI, EUCLID) going to measure tens of millions of galaxy redshifts over large fractions of sky. It is realistic to expect a galaxy catalog with 10^7 spectroscopic redshifts overlapping SN data

sets over a wide redshift range on the time scale of LSST.

3 Idea:

Propose a new method to infer cosmological parameters accurately from distance tracers (e.g. SNe) without redshifts:

- Exploit fact that both distance tracers and galaxies are tracers of the matter density, and therefore spatially correlated through the underlying matter field
- Can tightly constrain cosmology by maximizing the spatial cross-correlation of overlapping distance catalog and redshift catalogs
- Approach shows a classical cosmological test in a new light as a limit of a multi-tracer generalization of the Alcock-Paczynski (A-P) test
- Particular feature of this cross-correlation approach is its robustness to both data systematics and modeling assumptions

4 Set-Up:

4.1 Apparent Magnitude:

Constrain Luminosity Distance, D_L through relationship between apparent, m, and absolute, M, magnitudes calibrated from light curves:

$$m = 5log_{10}(\frac{D_L(z)}{pc}) + M - 5 \tag{1}$$

- Get light-curves from photometric observations, then calibrate the relationship and get D_L. For some data might already have this, like for DES-Sn.

- Equation is a definition of D_L, which is the thing you want to get and carry forward.

- m is the apparent magnitude, this comes from observations, at what point in the light curve do you pick for m ???

- M is the absolute magnitude, this is gotten from fitting the light curve

4.2 Luminosity Distance:

The luminosity distance is related to the cosmological model and redshift through this equation:

$$D_L(z) = \frac{c}{H}(1+z)\int_0^z \frac{dz'}{\sqrt{\mathcal{E}(z)}} \tag{2}$$

$$\mathcal{E}(z) \equiv \Omega_m(1+z')^3 + \Omega_{de}\exp(3\int_0^z d\,ln(1+z')(1+\omega(z'))) \tag{3}$$

- Assume a flat universe ($\Omega_K = 0$, or $\Omega_{de} = 1 - \Omega_m$) with dark energy equation of state: $\omega(z) = \omega_0 + \omega_a(z/(1+z))$

4.3 Isotropic Two-Point Correlation Function:

At a seperation, r, between two tracers (x and y) of the density fluctuations, e.g. $1+\delta_x(\mathbf{s}) = \rho_{\mathbf{x}}(\mathbf{s})/\bar{\rho}$, with respect to the background density, $\bar{\rho}$, can be written as:

$$\xi^{iso}_{x-y}(r) = \frac{1}{2\pi^2}\int k^2\, dk\; b_x(z)\; b_y(z)\; P(k)\; j_0(kr)\; e^{-k^2/k^2_{max}} \tag{4}$$

- $P(k)$ is the non-linear power spectrum obtained from the ensemble average of the density fluctuations in the Fourier domain for wavenumber $\mathbf{k}$

- $j_0(kr)$ is the spherical Bessel function

- $b_x = \delta_x/\delta_{dm}$ is the bias of the tracer x WRT Dark Matter - Things Ross has talked about before

- The cutoff k_{max} is introduced for a numerical convergence at high k, to avoid the oscillatory behavior of $j_0(kr)$

- For galaxies: $x = g$, with galaxy bias $b_g \approx 1.6$,

- For SNe: $x = sn$ and $b_{sn} = \delta_{sn}/\delta_{sdm}$. While b_{sn} is uncertain there are studies which indicate that the b_{sn} may exceed b_g by around 60%. We will conservatively take $b_{sn} \approx 1.6$ as a fiducial value

4.4 Red-Shift Space Distortions:

RSD's will affect galaxy positions in redshift space (not SNe because no red-shift label for us). Need to add a single Kaiser factor to the galaxy-SN cross-power spectrum:

$$P_{gs}(k) = (1 + f\mu_k^2/b_g)P_k \tag{5}$$

- $f \equiv d\,lnG/d\,lna$ is dimensionless growth rate - G is growth factor, μ_k is the cosine of the angle between the Fourier modes and the line of sight

Keith Suggestions:

- Kaiser 1987 - this seems to be the seminar work in the field: *Clustering in real space and in red shift space - Kaiser, 1987*
- *Hamilton 1992 - fairly technical, similar in spirit to the Kaiser 1987:* Measuring Omega and the Real Correlation Function from the Redshift Correlation Function - Hamilton, 1991
- Percival et al. 2011 - more accessible review article: *Redshift-Space Distortions - Percival et al., 2011*

4.5 Anisotropic Cross Correlation Function:

$$\xi_{g-sn}(\boldsymbol{r}) = (1 + \frac{f}{3b_g})\ \xi^{iso}_{g-sn}(r)\ \mathcal{P}_0(\mu_r) + \frac{2f}{3b_g}\ (\xi^{iso}_{g-sn}(r) - \bar{\xi}_{gs}(r))\ \mathcal{P}_2(\mu_r) \tag{6}$$

- $\bar{\xi}_{gs}(r) = \frac{3}{r^3}\int_0^r \xi^{iso}_{g-sn}(s)s^2\ ds$
- μ_r is the cosine of the angle between the line of sight and $\mathbf{r}$
- $\mathcal{P}_\ell(\mu_r)$ is the ℓth order Legendre Polynomial.
- Anisotropic $g-g$ and $sn-sn$ autocorrelations take on analogous forms.

4.6 Giant Equation:

$$\xi_{g-sn}(\mathbf{r}) = (1 + \frac{f}{3b_g})\ \xi^{iso}_{g-sn}(r)\ \mathcal{P}_0(\mu_r) + \frac{2f}{3b_g}\ (\xi^{iso}_{g-sn}(r) - \frac{3}{r^3}\int_0^r \xi^{iso}_{g-sn}(s)s^2\ ds)\ \mathcal{P}_2(\mu_r) \tag{7}$$

$$\xi^{iso}_{x-y}(r) = \frac{1}{2\pi^2}\int k^2\ dk\ b_x(z)\ b_y(z)\ P(k)\ j_0(kr)\ e^{-k^2/k^2_{max}} \tag{8}$$

5 Method:

Consider both galaxies and SNe as tracers of δ_{dm} in comoving coordinates. Galaxies observed in redshift space and SNe in D_L space.

Model these observed galaxy and SNe over-densities, $\boldsymbol{\delta}_{g,sn}$ as correlated Gaussian random fields. Then log-likelihood for: $\boldsymbol{\theta} \equiv \{\Omega_m, \omega_0, \omega_a, H_0\}$ becomes:

$$-2\ \mathcal{L}_{full}(\boldsymbol{\delta}_g, \boldsymbol{\delta}_{sn}|\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{\delta}_g \\ \boldsymbol{\delta}_{sn} \end{pmatrix}^T \boldsymbol{\Xi}^{-1} \begin{pmatrix} \boldsymbol{\delta}_g \\ \boldsymbol{\delta}_{sn} \end{pmatrix}\ +\ \ln|\boldsymbol{\Xi}| \tag{9}$$

- The Covariance matrix, $\boldsymbol{\Xi}(\boldsymbol{\theta})$ is a block matrix of form:

$$\boldsymbol{\Xi}(\boldsymbol{\theta}) = \begin{pmatrix} \boldsymbol{Z}^T(\boldsymbol{\theta})\boldsymbol{\xi}_{g-g}\boldsymbol{Z}(\boldsymbol{\theta}) & \boldsymbol{Z}^T(\boldsymbol{\theta})\boldsymbol{\xi}_{g-sn}\boldsymbol{D}(\boldsymbol{\theta}) \\ \boldsymbol{D}^T(\boldsymbol{\theta})\boldsymbol{\xi}^T_{g-sn}\boldsymbol{Z}(\boldsymbol{\theta}) & \boldsymbol{D}^T(\boldsymbol{\theta})\boldsymbol{\xi}_{sn-sn}\boldsymbol{D}(\boldsymbol{\theta}) \end{pmatrix} \tag{10}$$

- The $\boldsymbol{\xi}$ are computed as outlines above. - Parameter dependence in $\boldsymbol{\Xi}$