A Two-Stage Bayesian Approach for CP Response Estimation of Sensing Vehicles for Indirect Bridge Monitoring

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Abstract. Indirect bridge health monitoring using vehicle-based measurements offers a cost-effective alternative to traditional methods that require sensors mounted directly on the bridge. However, extracting the bridge's dynamic responses from vehicle vibrations is challenging due to road roughness and the complexity of the vehicle's own mechanics. This paper presents a novel two-stage approach to estimate on-bridge contact point (CP) responses without relying on prior knowledge of vehicle properties. In the first stage, before the vehicle enters the bridge, a Kalman Filter (KF) combined with an optimization framework uses measured vehicle's body accelerations to identify the vehicle's mechanical parameters, creating a surrogate model without direct input measurements. In the second stage, as the vehicle crosses the bridge, this surrogate model and a second KF are used to estimate the CP responses. To evaluate the method's robustness, simulations were performed using a half-car test vehicle driving on a Class B road profile that transitions onto a two-span bridge. Results show that while our method can identify the residual CP responses with high accuracy, the surrogate model itself is not unique, implying multiple configurations may yield similar CP estimates. Unlike existing techniques, this approach does not assume known vehicle properties, making it highly adaptable to real-world scenarios, where such data is often unavailable and particularly valuable for crowdsensing-based bridge monitoring, where diverse, uncalibrated vehicles are used.

Keywords: Indirect Bridge Health Monitoring · Residual Contact-Point Responses · Vehicle Scanning Method · Augmented Kalman Filter · Drive-by SHM.

1 Introduction

Indirect drive-by bridge health monitoring, in which moving sensing vehicles measure bridge vibrations, offers a cost-effective alternative to traditional methods that rely on fixed sensors [4]. Conventional bridge health monitoring typically places high-resolution sensors at a single location on the structure, whereas vehicle-mounted sensors can collect data from multiple points along the bridge. This approach provides much higher spatial resolution of the bridge's response, although each segment is scanned for only a very short time frame.

Yang et al. [16] first extracted bridge modal frequencies from vehicle acceleration data, establishing the groundwork for subsequent advances. Later studies focused on detecting higher bridge modes [8, 13, 14], estimating bridge damping and mode shapes [2, 5], mitigating road roughness through axle response subtraction [3, 15], and identifying bridge damage [9, 21, 20]. Despite these developments, isolating the bridge's dynamic response from vehicle measurements remains challenging due to road roghness impacts and the vehicle's mechanical complexity.

Recently, Yang et al. [17] introduced the concept of CP responses, defined at the vehicle-bridge interface, as a more effective alternative to raw vehicle acceleration data. CP responses filter out vehicle frequencies and highlight higher bridge modes in the Fourier spectrum. Experimental work using simplified vehicle models confirmed that CP responses can improve bridge modal identification and damage detection [17, 18]. However, those studies relied on undamped single-degree-of-freedom (SDoF) models, limiting their applicability to more complex vehicles. Nayek et al. [7] broadened the framework by formulating CP response extraction as a stochastic input-estimation problem and incorporating damping through a quarter-car model, thus accommodating model uncertainties and measurement noise.

Even with these advancements, most CP-based approaches assume known vehicle mechanical properties, treating only the unknown CP responses as inputs to be identified. To address this limitation, we propose a novel method that does not depend on prior knowledge of vehicle properties or bridge geometry. Our approach combines off-bridge and on-bridge data within an adaptable physics-based model, enabling us to identify vehicle dynamics and use them for accurate CP response estimation.

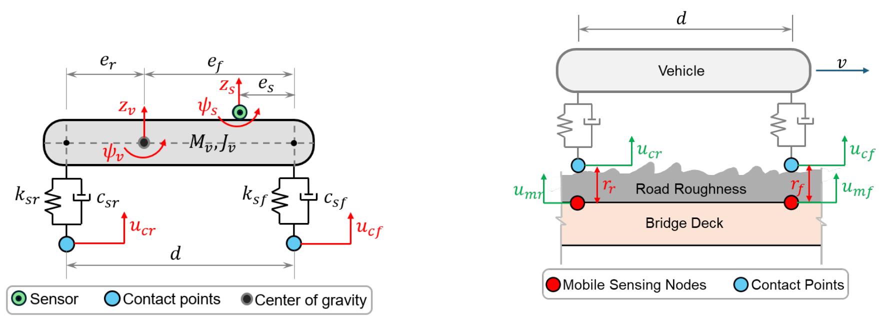
The remainder of this paper is organized as follows. Section 2 introduces the proposed methodology. Section 3 presents numerical simulations, results, and key findings. Section 4 concludes with overarching insights and directions for future work.

2 Methodology

The proposed methodology begins with collecting acceleration data from the sensing vehicle before it enters the bridge. At this stage, a normalized surrogate model of the vehicle dynamics is used to identify model parameters. In the second stage, once the vehicle enters the bridge, the identified model parameters are applied in a Kalman filter to estimate the vehicle's on-bridge residual contact-point (CP) responses. These responses are free from road roughness effects and vehicle dynamics, offering valuable insights into the bridge's modal characteristics, which are critical for indirect monitoring applications.

2.1 A Surrogate Model for HC Vehicles

The equation of motion for a half-car (HC) model is formulated in the global coordinate system, utilizing the vehicle's response at the sensor location. The CP responses are treated as base excitations affecting the vehicle dynamics, as illustrated in Fig. 1a and the equation of motion is expressed as:



(a) Vehicle Parameters

(b) CP Responses, Road Roughness Inputs, and Bridge Responses at Axles

Fig. 1: Half-Car Model Parameters and Road Roughness Input to Vehicles.

$$\mathbf{M}\mathbf{T}\ddot{\mathbf{z}}_s(t) + \mathbf{C}\mathbf{T}\dot{\mathbf{z}}_s(t) + \mathbf{K}\mathbf{T}\mathbf{z}_s(t) = \mathbf{E}_k\mathbf{u}_c(t) + \mathbf{E}_c\dot{\mathbf{u}}_c(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the mass, damping, and stiffness matrices of the vehicle, respectively. The matrices $\mathbf{E}_k$ and $\mathbf{E}_c$ represent the load effect matrices. The response vector $\mathbf{z}(t) = \left[z_s(t)\ \psi_s(t)\right]^\top$ comprises the vertical displacement and pitch rotation at the sensor location, and $\mathbf{T}$ is a transformation matrix. The specific forms of these matrices for the HC model are [12]:

$$\mathbf{M} = \begin{bmatrix} M_v & 0 \\ 0 & J_v \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} c_{sr} + c_{sf} & e_f c_{sf} - e_r c_{sr} \\ e_f c_{sf} - e_r c_{sr} & e_r^2 c_{sr} + e_f^2 c_{sf} \end{bmatrix},$$
$$\mathbf{K} = \begin{bmatrix} k_{sr} + k_{sf} & e_f k_{sf} - e_r k_{sr} \\ e_f k_{sf} - e_r k_{sr} & e_r^2 k_{sr} + e_f^2 k_{sf} \end{bmatrix}. \tag{2}$$

$$\mathbf{E}_k = \begin{bmatrix} k_{sr} & k_{sf} \\ -e_r k_{sr} & e_f k_{sf} \end{bmatrix}, \quad \mathbf{E}_c = \begin{bmatrix} c_{sr} & c_{sf} \\ -e_r c_{sr} & e_f c_{sf} \end{bmatrix},$$
$$\mathbf{T} = \begin{bmatrix} 1 & -(e_f - e_s) \\ 0 & 1 \end{bmatrix}. \tag{3}$$

In these expressions, M_v and J_v represent the vehicle's vertical mass and mass moment of inertia, respectively. The vector $\mathbf{u}_c(t) = \left[u_{cr}(t)\ u_{cf}(t)\right]^\top$ includes the CP responses at the rear and front axle locations, respectively. The parameters

k_{sf} and c_{sf} correspond to the suspension stiffness and damping of the front axle, while k_{sr} and c_{sr} represent those of the rear axle.

To perform state estimation using the Kalman Filter approach, Equation (1) must first be converted into a state-space (SS) representation. Unlike traditional methods, This study introduces a novel approach that uses the first derivative of Equation (1) to construct the SS model. This approach is motivated by the fact that, in vehicle scanning techniques, acceleration is typically the primary measured quantity. By directly incorporating recorded accelerations into the state vector, the measurement equation becomes independent of vehicle-specific parameters. The state vector is defined as:

$$\mathbf{x}(t) = \left[\dot{z}_s(t)\ \dot{\psi}_s(t)\ \ddot{z}_s(t)\ \ddot{\psi}_s(t)\right]^\top, \tag{4}$$

where $\dot{z}_s(t)$ and $\dot{\psi}_s(t)$ represent the vertical and pitch velocities, and $\ddot{z}_s(t)$ and $\ddot{\psi}_s(t)$ represent the corresponding accelerations of the vehicle body at the sensor location. Similarly, the input vector is defined as:

$$\mathbf{u}(t) = \left[\dot{u}_{cr}(t)\ \dot{u}_{cf}(t)\ \ddot{u}_{cr}(t)\ \ddot{u}_{cf}(t)\right]^\top, \tag{5}$$

where $\dot{u}_{cr}(t)$ and $\dot{u}_{cf}(t)$ are the velocity inputs, and $\ddot{u}_{cr}(t)$ and $\ddot{u}_{cf}(t)$ are the acceleration inputs at the rear and front contact points, respectively.

With these definitions, the continuous form of state-space representation is expressed as:

$$\dot{\mathbf{x}}(t) = \mathbf{F}\mathbf{x}(t) + \mathbf{G}\mathbf{u}(t), \tag{6}$$

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t). \tag{7}$$

Here, $\mathbf{F}$ is the system matrix, $\mathbf{G}$ is the input matrix, $\mathbf{y}(t) = \left[\ddot{z}_s(t)\ \ddot{\psi}_s(t)\right]^\top$ is the measurement vector, and $\mathbf{H}$ is the output matrix. The feedthrough matrix $\mathbf{D}$ can be set to zero, as the measurement vector is already included in the state vector. The matrices are defined as follows:

$$\begin{aligned} &\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{E}_k & \mathbf{M}^{-1}\mathbf{E}_c \end{bmatrix}, \\ &\mathbf{H} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{8}$$

By assuming $e_f k_{sf} \approx e_r k_{sr}$ in Equations (2) and (3), and considering that the measured data is recorded at the vehicle's center of gravity, the vertical and rotational mode shapes of the vehicle can be decoupled. Rearranging these equations allows the system matrix and input matrix to be simplified as follows. It should be noted that while this serves as a surrogate model with relatively few parameters, it is not the most accurate one. However, it incorporates the physics of the problem into the parameter identification process:

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\omega_z^2 \begin{bmatrix} 1 & 0 \\ 0 & \left(\frac{\omega_\psi}{\omega_z}\right)^2 \end{bmatrix} & -2\xi_z\omega_z \begin{bmatrix} 1 & 0 \\ 0 & \left(\frac{\omega_\psi}{\omega_z}\right)^2 \end{bmatrix} \end{bmatrix}, \tag{9}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \omega_z^2 \begin{bmatrix} \widetilde{e}_f & (1-\widetilde{e}_f) \\ -\frac{1}{d}\left(\frac{\omega_\psi}{\omega_z}\right)^2 & \frac{1}{d}\left(\frac{\omega_\psi}{\omega_z}\right)^2 \end{bmatrix} & 2\xi_z\omega_z \begin{bmatrix} \widetilde{e}_f & (1-\widetilde{e}_f) \\ -\frac{1}{d}\left(\frac{\omega_\psi}{\omega_z}\right)^2 & \frac{1}{d}\left(\frac{\omega_\psi}{\omega_z}\right)^2 \end{bmatrix} \end{bmatrix}. \tag{10}$$

where ω_z and ω_ψ are the vertical and rotational natural frequencies of the vehicle, respectively. ξ_z is the damping ratio for the vertical mode, d is the wheelbase, and $\widetilde{e}_f$ represents the normalized distance of the vehicle's center of gravity from the front axle with respect to d.

The main feature of this model is its ability to provide a reasonable range for most vehicle parameters. Within this range, the best parameters can be identified. This advantage was not possible with the original equations, as parameters such as stiffness and damping coefficients lacked a priori reference values. This is a surrogate model, meaning the goal is not to determine the exact values of these parameters but to find any combination that simulates the same dynamic responses. This simplification is sufficient for the intended purpose of estimating the CP responses of the vehicles.

For discrete-time implementation, a small time step Δt is considered. To reflect real-world conditions, process noise and measurement noise are incorporated into the model. The process noise vector $\mathbf{w}_k$ and the measurement noise vector $\mathbf{v}_k$ at time step k ($t = k\Delta t$) are typically assumed to be uncorrelated white noise processes with zero mean and known covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$, respectively [10]. Consequently, the discrete state-space equations at time step k are expressed as follows, allowing the state of the system at time k to be predicted based on the state at time $k-1$:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_k, \tag{11}$$

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{D}\mathbf{u}_k + \mathbf{v}_k. \tag{12}$$

For linear systems, the discrete system matrix $\mathbf{A}$ is computed using the matrix exponential $e^{\mathbf{F}\Delta t}$. Similarly, the discrete input matrix $\mathbf{B}$ is derived as $\mathbf{F}^{-1}(\mathbf{A}-\mathbf{I})\mathbf{G}$.

2.2 Input Estimation using Augmented Kalman Filter

To estimate the vehicle input vector $\mathbf{u}(t)$, the Augmented Kalman Filter (AKF) is employed. The AKF refines the traditional SS model by incorporating the input vector directly into the state vector. This augmentation allows for the simultaneous estimation of both system states and unknown inputs [11].

The augmented state vector is defined as:

$$\mathbf{x}_a(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{bmatrix} = \begin{bmatrix} \dot{z}_s(t) \; \dot{\psi}_s(t) \; \ddot{z}_s(t) \; \ddot{\psi}_s(t) \; \dot{u}_{cr}(t) \; \dot{u}_{cf}(t) \; \ddot{u}_{cr}(t) \; \ddot{u}_{cf}(t) \end{bmatrix}^\top$$

Conventionally, it is assumed that the input CP accelerations remain constant over short intervals with added noise error. Accordingly, the discrete augmented SS representation of the vehicle can be expressed as:

$$\mathbf{x}_{a,k} = \mathbf{A}_a \mathbf{x}_{a,k-1} + \mathbf{w}_{a,k}, \tag{13}$$

$$\mathbf{y}_k = \mathbf{H}_a \mathbf{x}_{a,k} + \mathbf{v}_k. \tag{14}$$

In this configuration, $\mathbf{A}_a$ and $\mathbf{H}_a$ serve as the process and output matrices for the augmented system, respectively.

$$\mathbf{A}_a = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{0} & \mathbf{\Gamma} \end{bmatrix}, \quad \mathbf{H}_a = \begin{bmatrix} \mathbf{H} & \mathbf{D} \end{bmatrix}. \tag{15}$$

The process noise $\mathbf{w}_{a,k}$ accounts for uncertainties associated with system states and input estimations. The matrix $\mathbf{\Gamma}$ is designed to specifically manage the transition of inputs from time step $k-1$ to k, under the assumption of constant acceleration motion, defined as follows:

$$\mathbf{u}_k = \mathbf{\Gamma} \mathbf{u}_{k-1} + \mathbf{w}_{u,k}, \quad \mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{16}$$

The AKF operates similarly to the standard Kalman Filter but utilizes the augmented state vector. The prediction and update steps are adapted to accommodate the additional states corresponding to the input forces.

Prediction Phase:

$$\hat{\mathbf{x}}^-_{a,k} = \mathbf{A}_a \hat{\mathbf{x}}^+_{a,k-1}, \tag{17}$$

$$\mathbf{P}^-_{a,k} = \mathbf{A}_a \mathbf{P}^+_{a,k-1} \mathbf{A}_a^\top + \mathbf{Q}_{a,k}. \tag{18}$$

Update Phase:

$$\mathbf{K}_{a,k} = \mathbf{P}^-_{a,k} \mathbf{H}_a^\top \left(\mathbf{H}_a \mathbf{P}^-_{a,k} \mathbf{H}_a^\top + \mathbf{R}_k \right)^{-1}, \tag{19}$$

$$\hat{\mathbf{x}}^+_{a,k} = \hat{\mathbf{x}}^-_{a,k} + \mathbf{K}_{a,k} \left(\mathbf{y}_k - \mathbf{H}_a \hat{\mathbf{x}}^-_{a,k} \right), \tag{20}$$

$$\mathbf{P}^+_{a,k} = \left(\mathbf{I} - \mathbf{K}_{a,k} \mathbf{H}_a \right) \mathbf{P}^-_{a,k}. \tag{21}$$

In the above equations, $\hat{\mathbf{x}}^-_{a,k}$ and $\hat{\mathbf{x}}^+_{a,k}$ represent the augmented state estimates before and after updates. Similarly, $\mathbf{P}^-_{a,k}$ and $\mathbf{P}^+_{a,k}$ indicate the corresponding error covariance matrices at these stages. The term $\mathbf{K}_{a,k}$ is the augmented Kalman Gain. Finally, $\mathbf{Q}_{a,k}$ defines the augmented process-input noise covariance matrix. Through this standard procedure, the inputs to the vehicle can be estimated based on known system parameters and measured vehicle body accelerations.

2.3 Model Parameter Identification Procedure

In the first stage of the proposed framework, particle swarm optimization (PSO) is employed to identify the unknown model parameters in Equations (9) and (10) using off-bridge vehicle body acceleration data. The PSO algorithm explores different combinations of these parameters, and the AKF is used to estimate the CP responses. Based on the fact that the rear and front axles of the vehicle scan the same road inputs with a time lag (see Figure 1b) [18], the residuals of the estimated CP responses should ideally approach zero. This serves as the optimization objective, defined as follows:

$$\min_{\theta_i} \frac{1}{2T_d} \int_0^{T_d} \left[\Delta\ddot{u}_{cr}(t)^2 + \Delta\dot{u}_{cr}(t)^2 \right] dt, \tag{22}$$

where T_d represents the measurement time window from the vehicle before it enters the bridge. The residual CP acceleration $\Delta\ddot{u}_{cr}(t)$ and residual CP velocity $\Delta\dot{u}_{cr}(t)$ are defined as:

$$\Delta\ddot{u}_{ucr}(t) = \ddot{u}_{cr}(t) - \ddot{u}_{cf}(t-\tau), \quad \Delta\dot{u}_{ucr}(t) = \dot{u}_{cr}(t) - \dot{u}_{cf}(t-\tau). \tag{23}$$

where τ is the time lag between the front and rear axles.

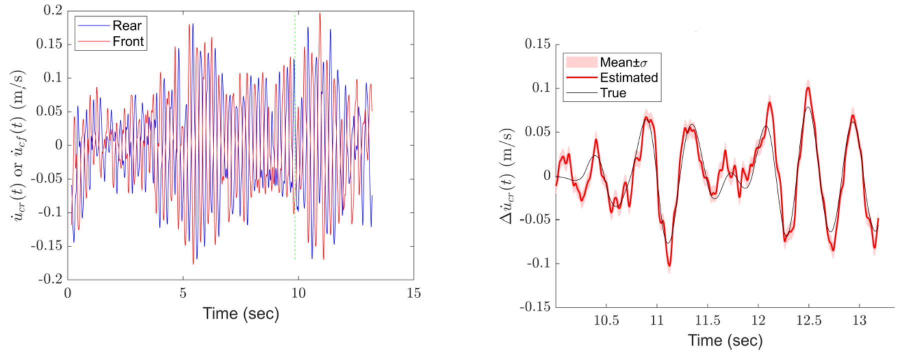
3 Results and Discussion

3.1 Numerical Setup

A two-span girder bridge model, spanning 70 meters, was simulated in OpenSees [6]. The bridge has a rectangular cross-section (3 m wide, 1.5 m high) and concrete material properties (2400 kg/m^3 density, 27.5 GPa elastic modulus). The model used an uncoupled iterative algorithm for dynamic vehicle-bridge interaction analysis. Eigenvalue analysis revealed natural frequencies of 2.13 Hz, 3.32 Hz, and 8.5 Hz for the first three modes. Vehicles were represented by the HC model with parameters from Yang et al. [19]: suspension stiffness of 230 kN/m (front) and 180 kN/m (rear), body mass of 2500 kg, pitch moment of inertia of 2300 kg·m^2, 3 m wheelbase, and speed of 76 km/h (20 m/s). The damping of the suspension system was assumed to be zero to simplify the analysis. A Class B road roughness profile, based on ISO8608 [1], was included to account for dynamic effects of road irregularities.

3.2 Interpretation of Results

Figure 2a illustrates the estimated rear and front CP velocity responses of the vehicle, derived from the optimized model parameters obtained through the proposed framework. As seen in the figure, the CP responses are inherently noisy due to road profile irregularities. However, the residual CP response, shown in Figure 2b, is smooth and free from these fluctuations. This demonstrates that the estimated CP responses successfully capture the residual patterns of the vehicle traversing the bridge. Importantly, the mean predicted results, within one standard deviation, fall within an acceptable range of accuracy. This is particularly notable given that the model parameters were not directly known and the CP responses were estimated solely from measured vehicle body responses.



(a) (b)

Fig. 2: Estimated on-bridge (a) rear and front axle CP responses and (b) residual CP response.

Figure 3a presents the optimization process and its convergence trajectory for the four model parameters. The figure demonstrates that the optimization achieves convergence efficiently, requiring minimal time and relatively few iterations. Additionally, the process shows stable performance across multiple runs. However, as depicted in Figure 3b, the error in the identified model parameters, when compared to the initial simulation model, can be relatively high, with some parameters showing errors of up to 20%. Despite this, such errors are acceptable within the context of the proposed multi-stage framework. The framework does not aim to identify a unique parameter set for the vehicle model; instead, it accommodates various parameter combinations within the gray-box model that can simulate the dynamic response accurately.

4 Concluding Remarks

In this paper, an innovative two-stage Bayesian approach was proposed for indirect bridge monitoring applications. The methodology utilizes acceleration data collected from the sensing vehicle prior to entering the bridge to identify a dynamic surrogate model for it through an optimization framework. Once the vehicle enters the bridge, the identified model parameters are employed in the Augmented Kalman Filter to estimate the residual CP responses of the vehicle. Numerical simulation results demonstrated the effectiveness of the proposed

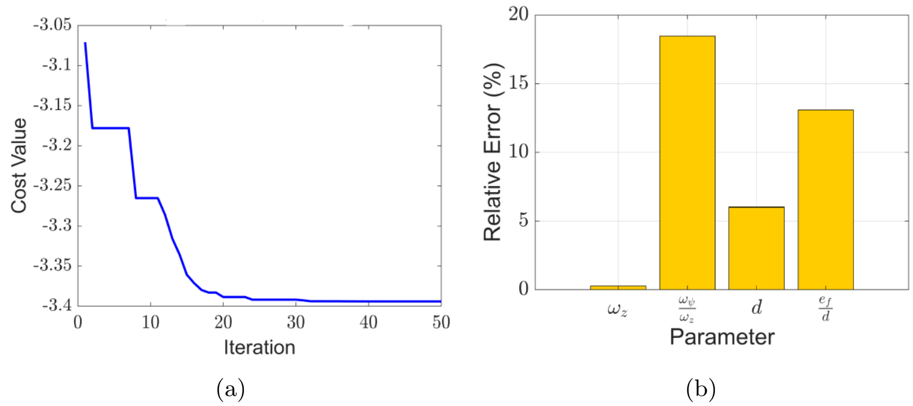


Fig. 3: (a) Optimization convergence process for the four model parameters. (b) Errors in the identified model parameters compared to the ground truth.

approach in accurately estimating the residual CP response with acceptable precision. Although the identified model parameters for the sensing vehicle are not necessarily unique, this is acceptable since the primary aim of the approach is to estimate the residual CP response with high accuracy, a key requirement for indirect bridge monitoring applications. However, the method has certain limitations that warrant further investigation. Future research could focus on validating the approach using results from field tests and exploring more comprehensive models that account for additional influencing factors, such as damping effects and more complex vehicle dynamics. Such advancements would enhance the robustness and applicability of the method in real-world scenarios.

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