National University
of Computer & Emerging Sciences
Islamabad

MT1003 – Calculus and Analytical Geometry

Assignment No: 01 **Individual Assignment**

Section: All BSCS Sections **Semester:** Fall 2022

Due date: Thursday, 15 September 2022, before 03:30pm **Marks:** 22*10=220

Instructions:

1. Plagiarized work will result in zero marks.
2. No retake or late submission will be accepted.
3. Attach complete code, results, and screenshot for questions that require programming solution. Programs/codes should not be handwritten.
4. Questions with  icon require partial or complete solution using the approved programming tool.
5. **The assignment is to be submitted in softcopy as well as in hardcopy. Submit the hardcopy before the deadline through CR, and softcopy on GCR by Thursday 15 September 2022 before 03:30pm.**
6. The softcopy should be a single PDF file of your complete assignment including programming and non-programming questions.
7. The PDF file should be according to the following format: id_section_A1 e.g., i22-123456_A_A1. A1 in the end denotes Assignment 1. **The first page must include complete student information, including name, section, id, course name, and assignment number.**
8. The images of by-hand solution should be properly scanned. You can use any mobile application such as CamScanner or Adobe Scan for scanning. Each of these applications allow you to export pdf or image files which you can use to combine with your programming solutions. Do not attach direct images from the camera application of your mobile phone, or screenshots.
9. MATLAB is the only approved programming tool. Octave-Online is also allowed.

Question 1

A lab technician controls the temperature T inside a kiln. From an initial temperature of 0 degrees Celsius, he allows the temperature to increase by 2 degrees Celsius per minute for the next 60 minutes. After the 60th minute, he allows the temperature to cool by 3 degrees Celsius per minute. If t is the number of minutes, write the function for temperature T and find continuity of the function T at $t = 60 minutes$.

Question 2

In New York City, taxicabs charge passengers $2.50 for entering a cab and then $0.50 for each one-fifth of a mile (or fraction thereof) traveled. There are additional charges for slow traffic and idle times, but these are not considered in this problem. If x represents the distance traveled in miles, then write a function $C(x)$ that represent the cost of the taxi fare.

Sketch the graph of C using MATLAB. Using this graph of the fare function, find each of the following limits, if it exists.

i)	a) $\lim_{x\to 0.25^-} C(x)$	b) $\lim_{x\to 0.25^+} C(x)$	c) $\lim_{x\to 0.25} C(x)$
ii)	a) $\lim_{x\to 0.20^-} C(x)$	b) $\lim_{x\to 0.20^+} C(x)$	c) $\lim_{x\to 0.20} C(x)$
iii)	a) $\lim_{x\to 0.6^-} C(x)$	b) $\lim_{x\to 0.6^+} C(x)$	c) $\lim_{x\to 0.6} C(x)$

Question 3

If the air temperature on a given day is 80^0F, the heat index $I(h)$ (also in ^{0}F) can be approximated by the following function, where h is the relative humidity as a percentage:

$$I(h) = \begin{cases} 80 & \text{if } 0 \le h \le 40 \\ 80 + 0.1(h - 40) & \text{if } 40 < h \le 80 \\ 0.005h^2 - 0.65h + 104 & \text{if } 80 < h \le 100 \end{cases}$$

a) What is the heat index if the relative humidity is 30%? What if it is 90%?
b) What relative humidity produces a heat index of 83^0F?
c) Is the heat index function $I(h)$ continuous at $h = 40$? What about at $h = 80$?

Question 4

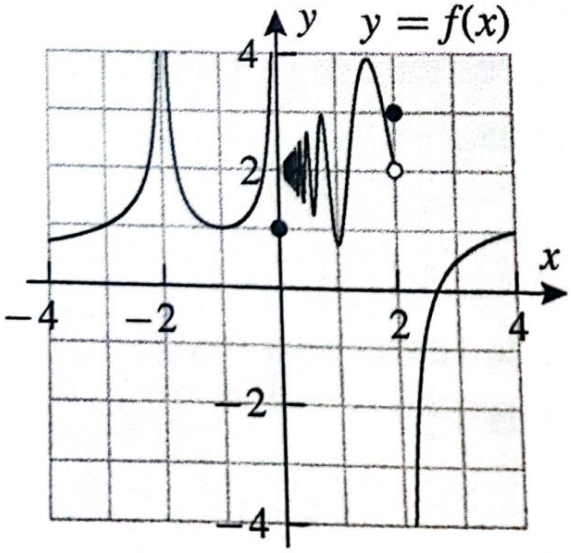
Find the value(s) of the constant A so that the function $f(x)$ will be continuous for all x.

$$f(x) = \begin{cases} Ax - 3 & \text{if } x < 2 \\ 3 - x + 2x^2 & \text{if } x \ge 2 \end{cases}$$

Question 5

For the function f graphed below, find

a) $\lim_{x\to -2} f(x)$
b) $\lim_{x\to 0^-} f(x)$
c) $\lim_{x\to 0^+} f(x)$
d) $\lim_{x\to 2^-} f(x)$
e) $\lim_{x\to 2^+} f(x)$
f) the vertical asymptotes of the graph of f.



Question 6

Classify the locations and types of discontinuities of the following functions. Write the corresponding limits.

a) $f(x) = -\frac{1}{(x-1)^2}$

b) $h(x) = \frac{3-\sqrt{x}}{9-x}$

c) $g(x) = \begin{cases} x^2 - 2 & if\, x \neq -3 \\ 5 & if\ x = -3 \end{cases}$

d) $f(x) = \begin{cases} e^x & x < 0 \\ x^2 & x \geq 0 \end{cases}$

Question 7

Evaluate $\lim_{x\to 1} \arccos\left(\frac{1-\sqrt{x}}{1-x}\right)$.

Question 8 </>

Use the Squeeze Theorem to show that $\lim_{x\to 0}(x^2 cos 20\pi x) = 0$. Illustrate by graphing the functions $f(x) = -x^2, g(x) = x^2 cos 20\pi x, and\ h(x) = x^2$ on the same plot using MATLAB.

Question 9

An old window has the shape of a rectangle surmounted by a semicircle. It is desired to express the area A of the window as function of its width x. The perimeter of the window is $30ft$.

Question 10

Given an initial quantity P_0 of a radioactive isotope strontium 90, find its half-life (the time, in years, it takes for the quantity P_0 to decay to half of its original mass) if the decay constant is $\lambda = 0.0244$.

Question 11

A farmer wants to build a closed cylindrical reservoir with a capacity of 11,000 cubic feet. The farmer has decided to use two different materials: one for the top, and the other for the bottom and sides. The cost of the first material (used for the top) is \$3 per square foot; the cost for the second material is \$5 used for sides and bottom. The farmer wants you to express the cost of constructing the reservoir in terms of its radius.

Question 12

Identify the domain of the following functions

a) $f(x) = \sqrt{2x+9} - \frac{1}{\sqrt{2-x}}$

b) $f(x) = \cos^{-1}(4x-3)$

c) $f(x) = [\log(\cos^{-1}(\sqrt{x^2+3x+2})]$

d) $f(x) = \ln|x^2-9| - 5$

Question 13

Consider the graph of $y = f(x)$ given below. Match each equation with its graph.

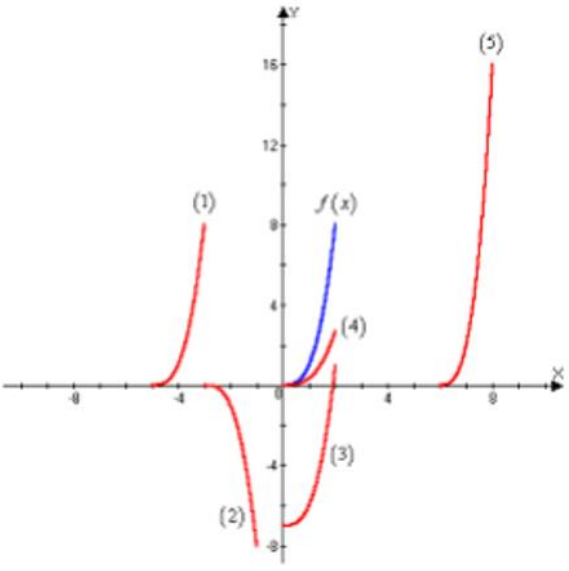
a) $y = \frac{1}{3}f(x)$

b) $y = f(x+5)$

c) $y = f(x) - 7$

d) $y = 2f(x-6)$

e) $y = -f(x+2)$



Question 14

Sketch the graph of $y = 2\cos\left(3x - \frac{3\pi}{4}\right) - 1$ for the interval $-2\pi \leq x \leq 2\pi$. State the amplitude, period, and principal axis of the function. Also sketch the graph using MATLAB.

Question 15

Sketch the graph of the function using the transformations, and determine the domain of the following function

$$f(x) = \log_3(x + 4) - 1.$$

Also sketch the graph using MATLAB.

Question 16

An internet service provider has given you the following data to consider:

Year	1999	2000	2001	2002	2003	2004	2005	2006
Customers (in millions)	76.4	78.6	81.5	87.8	88.4	92.4	94.0	95.0

Use MATLAB to plot the data and observe its trend. Find a linear function for the data and use it to predict the number of customers the company will have in 2014.

Question 17

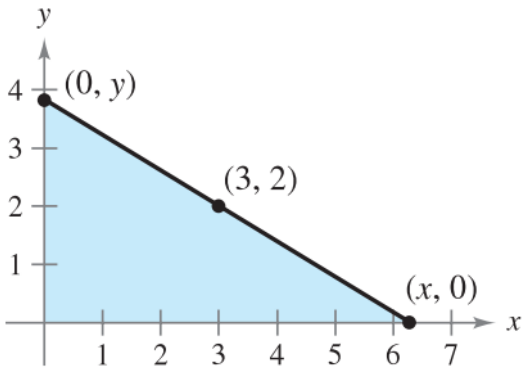
To extend the mission of a rover on Saturn's moon Titan, the engineers have calculated the minimum required power to be equal to 7 watts. The rover is nuclear powered, and the output of the power system is given by the equation

$$Y = 75e^{-\frac{t}{125}}$$

where t is the time in days that the supply is used. Calculate how long the rover can keep operating.

Question 18

A right triangle is formed in the first quadrant by the x- and y-axes and a line through the point (3, 2). Write the length L of the hypotenuse as a function of x.



Question 19

Describe the transformation of $f(x) = e^x$ represented by $g(x) = e^{x+3} + 2$. Then graph each function.

Question 20 </>

Give surface plot of $f(x) = \sin(x)^2$. Use surf command of MATLAB.

Question 21 </>

Plot $y = (x^2 - \sin(x^4))^{\frac{1}{2}}$ over the range [0,2] for both x- and y-axes using MATLAB.

Question 22 </>

Plot (using MATLAB) $f(x) = x(e^x - x - 1)$. Take [-2, 2] and [-1. 5] as range for x- and y-axes, respectively.