AMS 562 Lecture Notes

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CHAPTER

ONE

WHY C++?

Table of Contents

- You Should Consider Using C++ If
- You Should NOT Consider Using C++ If
 - "By the end of the day, all I want is something..."
 - "My project/research is still at infant stage, I wanna test some ideas."
- "I'm confused..."
- How To Read The Lecture
- Acknowledgments

1.1 You Should Consider Using C++ If

The chances are that you may heard a lot of good things of C++, and finally you have decided to give it a shot. Great! There is not doubt with the power of C++. It is one of the most widely used programming languages, found in a large range of applications. Unsurprisingly, *scientific computing* communities use C++ quite often in their projects. For instance, if you are interested in solving *partial differential equations* (PDEs), then open-sourced frameworks such as OpenFOAM and SU2 might attract people who work in *computational fluid dynamics* (CFD), and FEniCS and deal. II for researchers who are interested in *finite element methods* (FEM) for solving, say, structural problems. Of course, the *computational geometry algorithms library* (CGAL) is very popular for computer scientists who develop geometry-based algorithms. What about for students who work in much lower level research areas, such as developing numerical algorithms. No problem, Eigen has been around for awhile that supports efficient representations of fundamental numerical computation objects, like vectors, matrices, and tensors. OK, I think I can stop here!

Concepts that you learn from C++ can be directly used in other programming languages such as Java, Python, MATLAB, C, Fortran, etc. Personally, I believe writing an algorithm in C++ helps you fully understand the idea and, potentially, can also improve the algorithm design. Let's say, you probably has already learned how to do matrix multiplications in your college linear algebra classes. Then, when your instructors asked you to implement this as homework assignment in MATLAB, what you did is probably something like the following

```
A = rand(3);
B = rand(3);
C = A*B;
```

or in Python

```
import numpy as np
A = np.random.random((3,3))
B = np.random.random((3,3))
C = A.dot(B)
```

Well, I am kidding! This is cheating as homework assignments. You probably implemented the triple for-loops. If you did a timing analysis, you would probably see that the triple for-loop version is more than 100 times slower. Languages like Python or MATLAB hide details from users, for instance, many students don't understand how matrices are stored in MATLAB and NumPY and the difference between accessing from column j to column j+1 in these two languages, i.e. speed is different when accessing A[i,j:j+2] and A(i,j:j+1). On the other side, C++ hides nothing from users.

1.2 You Should NOT Consider Using C++ If

Very well, so I should only code with C++!

Wait, wait... There are many things that can simply be done with in 1 or 2 lines in Python, but may take you hundreds of lines in C++. Let's still use the above matrix multiplication example. First, the concept of matrices does not exist in **native** C++, so you need define it. Once you have the matrix *class* (future), you, of course, cannot expect that C++ is smart enough to automatically generate those linear algebra operations for you, as a result, you need to implement them. Finally, debugging your implementations will take a fairly amount of time.

In short, there are several situations that I think you should not consider directly using C++

1.2.1 "By the end of the day, all I want is something..."

Well, with C++, "by the end of the day", you are still struggling with software designs. Typically, it's not easy to get something working in C++ as nicely as high level (future) programming languages in short terms.

1.2.2 "My project/research is still at infant stage, I wanna test some ideas."

C++ is not a friendly language for developing algorithms... MATLAB, for example, is a great framework for developing numerical algorithms. As a matter of fact, all SBU students can use it "freely" under its educational license.

Note: This statement works for most students who study in typical applied math fields. However, for students majoring in *high performance computing* (HPC), stories are different.

1.3 "I'm confused..."

"I am here to learn C++, but it seems like I should not use it..."

Learning C++ is always beneficial, even though you may not use it right away! Personally, I believe learning C++ can help you understand more and be more professional in programming. For instance, if you program for Python and Java, C++ can help you understand more about their *garbage collections* (GC). Also, *shadow copy* will no longer be an enemy of you, because you know what's going on under the hood.

If you code for MATLAB, C++ can help you be aware of *pass by value* (PBV), and how to avoid unnecessary data copying so that you can write more efficient MATLAB scripts.

1.4 How To Read The Lecture

By "I", I mean myself. By "We", I mean the Department of Allied Mathematics & Statistics (AMS) and/or Institute for Advanced Computational Science (IACS). Also, it's worth noting that this lecture note is mainly created for preparing the course, and sharing some experiences in programming C++. So I will use the informal word "You" to address the students. Finally, the term C++ implicitly refer to C++11 (see C++ Standard), for other versions, I will address them explicitly.

Sphinx is really a great tool for writing this note. Here, I leverage several nice directives to do highlighting.

Tips, used for giving some tips/tricks

Tip: Lecture Zero is great place to start.

Notes, used for emphasis on technical points

Note: My fundamental assumption is that you all are new to C++ programming but with decent experiences in at least one of Python, MATLAB/Octave, or Java.

Warnings, danger areas!

Warning: Generally speaking, C++ is not for someone who has no programming background.

1.5 Acknowledgments

I would like first to thank AMS and IACS for giving me this great opportunity for teaching this course. Also, my supervisor, Prof. Jim Jiao, helps a lot in sharing his experiences in teaching and setting up the Docker images.

CHAPTER

TWO

LECTURE ZERO

Table of Contents

- Comment! Please Comment...
 - Making comments make others life better.
 - "I'm so glad that I commented on this file nicely!"
- Naming Conventions
 - Rule I: Make the variable names meaningful.
 - Rule II: Make the variable names compact.
 - Rule III: Most functions should start with verbs.
- Format Your Files
- C++ Standard

2.1 Comment! Please Comment...

All programming languages have their way to comment the source codes. In Python, people use number sign to start a line of comment

```
# This is a comment
# This is another comment
```

In MATLAB, people use percentage sign, i.e.

```
% This is a comment
%{
This
is
a block of
comments
%}
```

In C++, we use double forward slashes // to comment, i.e.

```
// This is a c++ comment
// Compiler will not read me!
```

Of course, the old C style comment is also valid, i.e.

```
/*
compiler will
not read
anything
in this
block
*/
```

Tip: For C++, it's better to stick with // even though you have multiple lines of comments.

2.1.1 Making comments make others life better.

Imagine you join a research group and continue on some former member's work. The only thing you get is his/her codes with no comments. Then, let's just hope the codes are very robust and have no errors (99.99% chance no!). Otherwise, it will waste you lots of time to figure out what he/she has done.

Also notice that, eventually, you will become someone's "former" member, so do not make your academic litter brothers/sisters hate you.

2.1.2 "I'm so glad that I commented on this file nicely!"

Actually, you are the one who benefit the most from making comments. While doing research, an important thing that everybody needs to keep in mind is to make the work reproducible. One needs to make sure that his/her results can be reproduced in the future. Therefore, make comments for yourselves in the future.

Of course, there are better tools, e.g. Git see *here*, to help you manage your work. But making comments are the most fundamental requirement that you need to organize yourselves nicely.

Let's take a look at the following examples:

```
// This function does blah blah ...
// This is the core component in the algorithm of step 1 in my paper...
// Essentially, this function is an extension of blah blah ...
//
// The following inputs arguments are needed:
// ...
// This function returns an integer flag that indicates ...
//
// See Also
// compute_next, compute_final
int compute_core(...) {
    // The first step is to ...
    ...
    // The second step is to ...
    // WARNING! The following codes assume ...
    // Finally clean up everything with ...
}
```

vs.

```
// compute core function
int compute_core(...) {
    ...
}
```

2.2 Naming Conventions

Defining variable names is not a trivial task. There are some standard rules, but for this class, I will briefly share some of my **personal** styles.

Warning: The following information is based on personal experience thus it can be subjective.

2.2.1 Rule I: Make the variable names meaningful.

Historically, people like to use foo, bar, or spam to demonstrate ideas. Usually, they don't assume or know the backgrounds of the readers, so they choose some *placeholder* names that can be replaced in real codes. Hence, avoid using these names in your projects; no one wants to see placeholders in the final products.

Another commonly used word is temp or tmp that stands for temporary variables. Personally, I use this word only for some variables that have short lifetime.

A rule of thumb is to ask your workmates and check whether they can understand the meaning of your variables.

2.2.2 Rule II: Make the variable names compact.

Making things meaningful doesn't mean you have to be verbose. Also, don't forget we have just learned how and why to make comments! Commonly, people use abbreviations for their variable names, and this is what we shall follow.

Tip: Choose an abbreviation that will appear in your project documentation or your papers/reports.

2.2.3 Rule III: Most functions should start with verbs.

This idea is commonly agreed by most of people, especially in scientific programming.

Tip: Some commonly used verbs: set, get, fetch, extract, is, assign, compute, determine, swap, move, init, destroy, create, remove, reset, reserve, put, resize, etc.

Note: C++ encourages people to hide *member variables* (future) and access them through *member functions* (future), e.g. size, length, data, capacity, etc.

2.3 Format Your Files

Besides comment your codes, another important aspect is to have a nice layout of your codes, so that it makes others who read your work enjoyable.

```
for (int a=0; a <10; ++a) {
    foo[a] =1.0; bar[a]=2.0;
    spam[a] = 3.0; }</pre>
```

vs.

```
for (int a = 0; a < 10; ++a) {
    foo[a] = 1.0;
    bar[a] = 2.0;
    spam[a] = 3.0;
}</pre>
```

Well, I vote for the second one!

However, manually doing this is a pain. We shall leverage the automatic tool such as clang-format, which can be used through our editor (vscode) through the Docker image for this course.

2.4 C++ Standard

The first ISO C++ standard was ratified in 1998, so that version is referred as C++98. Later on, we got C++03. **The game changer is C++11**, which has introduced many unique features. Personally, I think C++11 is totally a different language compared to its previous versions. For this course, all materials are based on C++11, and the term C++ is implicitly referred to version 11, for instance, the following statement is correct under this assumption: In C++, you can use lambdas instead of functors.

The current standard is C++17. However, in scientific computing, the truth is that there is usually a lag between current standard in practice and current ISO standard. The good news is that most of open-sourced projects have moved/ are moving to C++11, it's safe to assume C++11 and teach it in a course with title *scientific programming*. As a matter of fact, the first feature-complete GCC was version 4.8.1¹, which was released on May 31, 2013². Also, our Seawulf cluster's system GCC has version 4.8.5, which means it fully supports C++11.

¹ see GCC C++ standard supports

² see GCC release

CHAPTER

THREE

SIMPLE C++

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- The main functions
- Compile the one-line program
- "Hello World!"

3.1 The main functions

Each C++ program is written in a main function in C++, where a main function must return an integer to indicate your system the exit status of the program.

Note: 0 is used for indicating exiting successfully.

Here is the simplest C++ program, int main() { return 0; }, which does nothing but just returns EXIT_SUCCESS to your system.

3.2 Compile the one-line program

Unlike Python and MATLAB, where you can directly run programs, all C++ programs must be first **compiled** into executable binaries.

Note: The compilation stage is one of the key reason why static languages are faster than dynamic languages.

Now, copy the one line program into a file with name simple.cpp, inside a terminal, invoke:

```
$ g++ simple.cpp
```

This by default will compile the program into an executable file called a .out that lies in the same directory. To run the program, type:

```
$ ./a.out
```

Tip: ./ in front of the executable means the file is under *current working directory* (cwd). You can type pwd in the terminal to check *cwd*. In general, . means current, . . means previous, and / is the path separator on Linux. To navigate toi a directory through terminal, you need to use the built-in command cd, which stands for *change directory*. For instance, if I want to go to previous directory, I can simply type cd . . . cd ./foo will bring me to the foo folder that locates at *cwd* (you can omit ./ in this case). Absolute path can also be used. For example, cd /path/to/my-homework will navigate to /path/to/my-homework, and the leading / on Linux means the root directory.

Of course, this program seemingly does nothing. To check the returned code, type echo \$?, which will give you the exit-code of most recent program. You should see 0 on the screen.

3.3 "Hello World!"

Unfortunately, you cannot write one line code for "Hello World!" in C++. In Python, you can write a hello_world.py script with:

```
print('Hello World!')
```

And simply type:

```
$ python3 hello_world.py
```

You should see "Hello World!" on your screen. Or even something like:

```
$ python3 -c "print('Hello World!')"
```

However, there is no built-in print method in C++, we have to include the standard input and output library, i.e. iostream.

```
#include <iostream>

int main() {
    std::cout << "Hello World!" << std::endl;
    return 0;
}</pre>
```

Once we include the IO library (line 1), we can use the standard output streamer, i.e. std::out, to write out messages (line 4).

Now, copy the program into hello_world.cpp, and compile and run it.

Note: This section is basically to demonstrate some simple codes. There will be a specific section talking about IO.

3.3. "Hello World!"

CHAPTER

FOUR

SOFTWARE REQUIREMENTS

Table of Contents

- Using Our Docker Container
- Using Git with SSH Keys
- Code with Visual Studio Code

Note: In this section, we will briefly introduce the software skill suggestions for taking this course. We will provide useful links for you to further train yourself.

4.1 Using Our Docker Container

Why not take the advantage of cloud computing for teaching? The answer we come up with is Docker contains, which allow you run a collection of software packages (including OS) regardless your host machine systems. This class is cluster-free, i.e. you don't need to worry about dealing with our clusters and running everything through command lines.

First, read the description of our Docker container, i.e. AMS 562 container. Once you have installed Docker and Python, download these two scripts that will ease the process for using our container.

- Desktop driver: ams562_desktop.py
- Jupyter driver: ams562_jupyter,py

For the first time, open a terminal/console/powershell session:

```
$ python ams562_desktop
```

This will automatically pull the container and create the desktop environment in your default web browser. To see all options:

(continues on next page)

(continued from previous page)

```
open up the URL in the default web browser. It also sets up port forwarding
for ssh.
optional arguments:
-h, --help
                     show this help message and exit
-i IMAGE, --image IMAGE
                     The Docker image to use. The default is
                     ams562/desktop.
-t TAG, --tag TAG
                    Tag of the image. The default is latest. If the image
                     already has a tag, its tag prevails.
-v VOLUME, --volume VOLUME
                     A data volume to be mounted at \sim/" + projdir + ". The
                     default is ams562_project.
-w WORKDIR, --workdir WORKDIR
                     The starting work directory in container. The default
                     is ~/project.
-p, --pull
                     Pull the latest Docker image. The default is not to
                    pull.
                    Reset configurations to default.
-r, --reset
-c, --clear
                    Clear the project data volume (please use with
                     caution).
-d, --detach
                     Run in background and print container id
-s SIZE, --size SIZE Size of the screen. The default is to use the current
                     screen size.
-n, --no-browser
                    Do not start web browser
-N, --nvidia
                    Mount the Nvidia card for GPU computation. (Linux
                    only, experimental, sudo required).
-V, --verbose
                   Enable verbose mode and print debug info to stderr.
-q, --quiet
                    Disable screen output (some Docker output cannot be
                     disabled).
-A ARGS, --args ARGS Additional arguments for the "docker run" command.
                     Useful for specifying additional resources or
                     environment variables.
```

Tip: Always run with \$ python ams562_desktop -p to get the newest container since our Docker image is automatically rebuilt weekly.

The following directories are mirrored to your local machine:

Docker directories	Host directories	
\$DOCKER_HOME/shared	Current working directory	
\$DOCKER_HOME/project	Data volume	
\$DOCKER_HOME/.ssh	\$HOME/.ssh	
\$DOCKER_HOME/.config	\$HOME/.config	

The most commonly used is the shared directory, which allows you rapidly exchange data between the container and your host machine.

Warning: Except the above four directories, any changes to the container will not be persistent.

4.2 Using Git with SSH Keys

- What is Git?
- · Set up your private repository on Bitbucket
- Set up an SSH Key

4.2.1 What is Git?

Version control system definitely helps your work. I found this online material is interesting and helpful, please take a look at it.

SmartGit is a nice GUI system for Git.

Tip: You may also find using Git in Visual Studio Code is handy, see *here*.

4.2.2 Set up your private repository on Bitbucket

We will use one of the popular online git network, Bitbucket, to collect your homework assignments. Register an account with your **SBU** email address, then create a **private** repository following this description.

Name your repository with ams 562_<your name>. And initialize your repository with a README. md that at least includes your SBU ID and name, something like the following is fine:

```
# Welcome to my repository for AMS 562

* name: <your name>
     * SBU ID: <your id>
```

4.2.3 Set up an SSH Key

Using Secure Shell is the preferred way for using Git. Follow this description to setup an SSH key for your Bitbucket account.

Note: Keep your private key and **passphrase** secure!

4.3 Code with Visual Studio Code

- Using git Inside VScode
- Using Terminal Inside VScode

Using a decent editor is necessary, and develop with IDE-like environment is extremely helpful. Among all existing popular editors, we have decided to provide the Visual Studio Code that is developed by Microsoft. This editor has been installed and properly configured in our container.

4.3.1 Using git Inside VScode

Using Git trough terminal might be confusing for people who first work. To make things more consistent with our Docker setting, we find that using Git through Visual Studio Code is extremely convenient. See this description.

Notice that if you use Git through *SSH*, then you need to run the following command inside the terminal of our container:

```
$ ssh-add
Enter passphrase for /path/to/.ssh/<your private key>:
```

Then enter your passphrase that your created for the key. This is done once only.

4.3.2 Using Terminal Inside VScode

Using the integrated terminal of vscode is recommended, you can create/return to a terminal session by type CTRL-`. By default, it will put you at the *current working directory*, then you can invoke any commands inside this integrated terminal.

Tip: You can jump to a specific location of a file if the file is shown with absolute path. Therefore, it's convenient to pass in abs path of a file inside the integrated terminal. One easy way to do so is to add `pwd` / in front of your file.

```
$ g++ `pwd`/main.cpp
```

CHAPTER

FIVE

LECTURE 1: TYPES & I/O

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- C++ is all about TYPES!
- Standard Input & Output

5.1 C++ is all about TYPES!

- The Built-in Types
 - The Integral Types
 - * Integers
 - * Characters
 - * Boolean
 - Floating Numbers
 - * Precision
- The string Type
- Literals
 - Integer Literals
 - Floating Point Literals
 - Character Literals
 - Boolean Literals
 - String Literals
 - Escape Sequences
- Define & Initialize Variables
 - Define Variables
 - Initialize Variable Values

```
Type Conversions
Type Conversions Between Integers
Converting Floating Point Numbers to Integers
The const Specifier
Array
Accessing Array Elements
Multidimensional Array
The char[]
Scope & Lifetime of Variables
```

Unlike Python (or any other dynamic languages), all C++ variables must be initialized with their types explicitly given. And the variable names cannot be reused within the same *cope*. Consider the following Python code

```
In [1]: a = 1
In [2]: type(a)
Out[2]: int
In [3]: a = 1.0
In [4]: type(a)
Out[4]: float
In [5]: a = 'a'
In [6]: type(a)
Out[6]: str
```

In the program, variable a first is initialized as an integer, but later on it switches to floating point number and string. This, however, is not allowed in C++.

5.1.1 The Built-in Types

A built-in is a component that comes with the programming language; using built-in components does not require you import any external interfaces (even including official ones). In C++, we have built-in data types that define the foundation of the language (or even other languages). The built-in types can be mainly divided into three groups:

- 1. integral types,
- 2. floating number types, and
- 3. the valueless type, i.e. void.

The Integral Types

Let's put item 3 apart now. The integral types can be further categorized into:

- · integers
- characters
- boolean

Integers

Integers types are used to store the whole numbers in programming. The most commonly used one is probably int. For integers, both **signed** and **unsigned** versions are provided, where the former allows negative values.

Table 1: Integer Types Table

Types	Size (bytes)	Range	
short	2	-32,768 to 32,767	
unsigned short	2	0 to 65,535	
int	4	-2,147,483,648 to 2,147,483,647	
unsigned int	4	0 to 4,294,967,295	
long	8	-2^63 to 2^63-1	
unsigned long	8	0 to 2^64-1	

Warning: In general, the sizes of integer types are platforms and compilers depended. The above information is for GCC with 64bit machines (as our docker image). On some old machines, you may find that the sizes of int and long are 2 and 4 byte, respectively. In order to have 64bit (8-byte) integer, you need long long.

Note: signed is also a keyword in C++, but using it is optional, i.e. signed int is identical to int.

Characters

The keyword to store character data is char, whose size must be 1 byte. char ranges from [-128, 127] and [0, 255] for unsigned char.

Boolean

A bool is used to store logical values, i.e. either true or false. Typically, the size of bool is 1 byte (in theory, we only need 1 bit).

Warning: signed and unsigned are not applicable to bool

Floating Numbers

Clearly, integers are not enough! Especially in scientific computing, we need real numbers that can store data from our models. In C++, the concept of *floating numbers* is used to represent real numbers. Like the *integer* table, the following is the table of floating numbers

ID

ID

Table 2: Floating Numbers Table

where ID stands for implementation depended.

long double

Note: The size and range of long double is implementation depended. The size may be 8, 12, or 16 bytes depending on different compilers. In general, long double is not a commonly used type.

Precision

The fact is that floating numbers, in general, cannot represent real numbers **exactly**. This is particularly true for irrational numbers, i.e. $\sqrt{2}$, π , etc. We refer float as *single-precision format*¹ while *double-precision format*² for double.

Table 3: Floating Numbers Precision Table

Precision	Significant digits ³
single-precision	typically 7
double-precision	typically 15

For instance, given two real numbers 1.1 and 1.100000004, which are, of course, different numbers in the exact arithmetic setting. However, under single-precision format, they are equal. What about double-precision? Checkout notebook precision.

Double-precision format is about twice accurate than single-precision, and has a much wider range. **Question:** what are the points of using single-precision numbers?

5.1.2 The string Type

string is also an important type in all programming language. With standard C++, string is not a built-in type, it's defined in the standard library <string>. Therefore, including the interface is needed for using strings.

```
#include <string>
std::string name = "John";
std::string age = "32";
```

¹ please read Wikipedia page for more

² please read Wikipedia page for more

³ please read Wikipedia page for more

Note: If you are familiar with C, there is so-called C-string type, which is an *array* of characters, i.e. char.

5.1.3 Literals

Literals are constant values of any programs. In C++ (almost all other languages), there are five types:

- 1. integer literals,
- 2. floating point literals,
- 3. character literals,
- 4. boolean literals, and
- 5. string literals.

Each literal has its own **form** and **type**. Notice that literals are commonly used in *initializing* variables.

Integer Literals

Examples of integer literals are:

```
32, 1, -2, -100, 30001, ...
```

Their **types** are int. Then, how to specify literals of other integer types? You need suffix u and 1 (ell), where the former represents unsigned and the latter is for long types:

```
321 // long literal
32u // unsigned int literal
32ul // unsigned long literal
```

Note: There does not exist numeric literals for short and unsigned short.

Floating Point Literals

The following forms:

```
1.0, 2.0, 3.0, 4.0, ...
```

are all double-precision floating number literals. Suffix f is used to denote single-precision, i.e. float.

```
1.0f // float 1
-2.0f // float -1
-5.21f // float -5.21
```

You can also use scientific notations:

```
1.0e0 // double, 1x10^0, i.e. 1
2.0e-3 // double, 2x10^-3, i.e. 0.002, or
2e-3
1.e10f // float, 1x10^10
5.32001e3f // float, 5320.01
```

Character Literals

For character literals, use the single quotations:

```
'a' // character a
'A' // character A
'7' // character 7
```

Boolean Literals

C++ uses true and false for Logical literals.

String Literals

For strings, C++ uses double quotations, for instance:

```
"Hello World!" // string value of Hello World!
"AMS 562" // string of AMS 562
```

A string is a sequence of characters.

Warning: In Python, single quotations can be used for strings, i.e. see the *Hello World* example. However, this rule cannot be applied to C++, i.e. 'abc' is referred as multi-character literal that has type of int instead of char and the value is ID.

Escape Sequences

Questions: How to use literals to represent string "A" and character ' with the quotation marks?

Such special characters are so-called *escape sequences* and start with backslash. Commonly used ones are:

Table 4: Commonly Used Escape Sequences (ES) ⁴				
	ES	Description		

ES	Description	
\ '	single quote	
\"	double quote	
	backslash	
\n	new line	
\t	horizontal tab	
\v	vertical tab	

Now let's consider the following string literals with escape sequences:

```
"Hello\nWorld!" // Hello<new line>World!
"Hello\tWorld!" // Hello<tab>World!
"\"Hello World\"" // "Hello World!"
```

5.1.4 Define & Initialize Variables

At the beginning of this lecture, we have showed an *Python program* to demonstrate one of the major differences between C++ and dynamic language. In C++, you must to explicitly construct variables with their types given. The format is [type] var; where [type] is legal types, e.g. int, double, std::string, etc.

Define Variables

Here are some examples of defining variables:

Warning: Once a variable name is been occupied, you cannot reuse it for anything else (within the same *scope*).

Initialize Variable Values

It's good practice to initialize a variable while defining it.

Checkout notebook types and run it.

Note: One should not expect any default behaviors of uninitialized variables, e.g. when you write int a;, a might be zero but you should not assume this!

Type Conversions

Let's take a look at the following seemingly trivial code:

```
double two = 2;
```

It defines a double-precision floating point number two and initializes it to 2. However, recall that each literal has its own type, which means the above code assigns a double precision number with an integer. This is called type conversion in C++.

⁴ Check cppreference page for more.

Type Conversions Between Integers

In general, type conversions between integers are simply copying the values. However, keep in mind that all integer types have their ranges. Converting from larger size types to smaller ones may potentially cause troubles, i.e. integer *overflow* and *underflow*.

```
unsigned int wha = -1; // What the value of wha??
```

Typically, the issues come from converting between signed and unsigned integers. Let's take a look at the *code* above. It tries to convert int value -1 to unsigned int variable wha.

Note: You can consider each integer type form a cyclic list that MAX+1 is its MIN and MIN-1 is the MAX.

As a result, the actual value of wha is 4,294,967,295. Checkout and run notebook conv.

Warning: Unless you 100% know what you are doing, converting between signed and unsigned integers should be avoid!

Converting Floating Point Numbers to Integers

The rule for converting floating numbers to integers is to truncate them into whole numbers.

```
int a = 12.03;  // a is 12
int b = -1.234e2; // b is -123
```

5.1.5 The const Specifier

const is a keyword in C++ that indicates an variable is immutable. Once a variable is defined as constant, you cannot modify its value, so initialization is must for defining constant variables!

```
const int a = 4;  // define a constant integer of value 4
// a = 2;  // ERROR! you cannot modify constant vars
// const double b;  // ERROR! const var must be initialized
```

Tip: Use const whenever possible!

5.1.6 Array

Array is one of the most basic data structures in programming. As a matter of facet, it is also the most commonly used data structure in scientific computing. An array is a sequence of objects that have the same size and type. In C++, an array can be constructed with square bracket [N], where N is the size of array.

```
double arr[3];  // create an array of 3 doubles
int pos[5];  // an array of 5 integers
```

To initialize an array, the curly brackets { } are needed, e.g.

```
double tols[2] = {1e-4, 2e-5};  // array of two with values 1e-4 and 2e-5
int mappings[] = {2, 0, 1};  // array of three integers, size 3

// short a[];  // ERROR! size must be provided
// const int b[2];  // ERROR! b must be initialized
int z[3] = {1};  // partial initialization is ok
// int m[2] = {1,2,3};  // ERROR! exceeded the size
```

It is allowed to implicitly provide the size if you initialize the array, as shown in line 2. Also, partially initialize an array is allowed, but the right-hand side must be no larger than the actual array size. Checkout and run notebook array.

Accessing Array Elements

To access an specific element of an array, we need to use operator [index].

Warning: Un like Fortran and MATLAB, C++ is zero-based indexing, i.e. the first element index starts from 0 instead of 1.

Multidimensional Array

Multidimensional arrays are useful, for instance, a matrix can be represented as a 2D array, where the first dimension is the row size and column size for the second dimension. The concept of multidimensional array can be interpreted as *an array of arrays*.

```
// array of two arrays of size 2
  double mat[2][2];
  mat[0][0] = 1;
2
  mat[0][1] = 2;
  mat[1][0] = 3;
  mat[1][1] = 4;
5
6
       a matrix of 2x2
7
8
       1121
9
       1 3 4 1
10
```

What is the type of mat? It is an array but its elements also have type of array, i.e. double[2]. In line 2, mat[0][0] first accesses to the first element of mat, which is an array, say mat0, the accesses the first element of mat0, i.e. mat0[0].

The char[]

As we already learned in section *string*, a string is just a sequence of char, i.e. char[]. Therefore, char[] is also called *C string*, or the native string type of C++. You can initialize a C string either using the array fashion or the *string literal* way.

```
// The follow two are identical
char str1[] = "AMS 562";
char str2[] = {'A', 'M', 'S', '', '5', '6', '2'};
```

5.1.7 Scope & Lifetime of Variables

We have already learned that reusing a variable name is not allowed in C++, but this rule applies to the variables with the same scope. Scope operators must appear as a pair of scope opener and scope closer, where are { and }, respectively.

```
int a = 1;  // define integer a
// double a; // ERROR! reusing a within the same scope

// start a new scope
{
    double a; // OK!
}
// scope ends
```

Note: The *main* function or any other functions all have local scope.

The lifetime of a variable is associated with its scope. When it reaches the end of scope, it will become inaccessible and be popped out from the program stack.

```
// start with a child scope
2
       int a;
3
   } // a becomes invalid!
   // a = 1; // ERROR! a does not exist!!
6
   int b = 1; // define b, and its life begins
   {
10
       double b = 2.0; // define local b
11
       b = 3;
12
       // which b?
13
14
15
  b = 2;
16
   // which b?
```

Note: Child scope overwrites its parent scopes.

5.2 Standard Input & Output

```
The std::cout Stream
The std::cin Stream
Use Input Operator >>
Read An Entire Line
The std::cerr Stream
std::cout vs. std::cerr
```

5.2.1 The std::cout Stream

By default, most program environments' standard output is screen. In C++, the global object std::cout is defined and guaranteed to be initialized at the beginning of any programs. The object itself is defined in the standard I/O library <iostream>, which must be included in order to perform I/O tasks.

std::cout stands for standard C output, which is a FILE * object in C (sys.stdout in Python). C++ uses the abstraction called *streams* to perform I/O operations.

Output operator << (bitwise left shift, or double less-than signs) is used to indicate "write to a streamer".

```
#include <iostream> // bring in std::cout

std::cout << "Hello World!" << std::endl;
std::cout << "1+1=" << 2 << std::endl;
std::cout << "size of double is: " << sizeof(double) << std::endl;</pre>
```

Notice that std::endl is *manipulator* to produce a newline. Line 4 and 5 show that you can recursively write to the cout streamer and the output contents can be different types, e.g. in line 4, "1+1=" is string but 2 is integer.

Note: In stead of using manipulator std::endl, you can also use the newline *escape sequence*—\n.Therefore, the outputs are identical between std::cout << "Hello World!" << std::endl; and std::cout << "Hello World!\n";.

Checkout and run notebook cout.

5.2.2 The std::cin Stream

Use Input Operator >>

The default standard input for most program environments is through keyboard. In C, the FILE * object is stdin (sys.stdin in Python). This input streamer is able to read the user inputs from keyboard. std::cin stands for standard C input.

Similar to output operator, the input operator is bitwise shift right (or double greater-than signs) >>. The basic syntax is std::cin>>var; and the user inputs will be stored in var.

```
#include <iostream> // cout and cin

// best practice, always indicate the user what to enter
std::cout << "Please enter your first name: ";
std::string name;

// the program will hang here til receive the user input
std::cin >> name;
std::cout << "Hello! " << name << std::endl;</pre>
```

std::cin read the user inputs into its buffer, and the input operator >> searches the user keyboard inputs and treated them as a sequence of white space separated arguments. Therefore, if you enter your full name, e.g. "Qiao Chen", only the first value will be printed because the program only asks for one input arguments, i.e. name.

It's also possible to handle multiple input arguments:

```
#include <iostream> // cout and cin

// best practice, always indicate the user what to enter
std::cout << "Please enter your first and last names: ";
std::string fname, lname;

// the program will hang here til receive the user input
std::cin >> fname >> lname;
std::cout << "Hello! " << fname << ' ' ' << lname << std::endl;</pre>
```

Compile and run program cin_demo.

Read An Entire Line

It's also convenient to read an entire line at once. To do this, you need to use the std::getline function. Like input operator >>, *getline* treats the user inputs as a sequence of \n separated arguments. The syntax is: std::getline([input streamer], string).

Warning: Special care must be taken into consideration if you want to mix the use of input operator >> and std::getline. When the user types "Hello" and press ENTER, the actual input value is "Hello\n".

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```
std::getline(std::cin, sent);
std::cout << "The sentence you entered is:\n" << sent << std::endl;</pre>
```

The above code will not work as what you expect, because after reading a word, there is still a **newline**, \n , character left, and this confuses the following *getline* operation. In order to skip the character \n , you need to call . The correct program is:

5.2.3 The std::cerr Stream

std::cerr stands for standard C error output. Its associated C FILE * object is stderr (sys. stderr in Python). It, like *cout*, is an output streamer. It also writes outputs on the screen.

```
#include <iostream>
std::cerr << "This is an error message!\n";
std::err << "WARNING! Converting from signed to unsigned is dangerous!\n";</pre>
```

5.2.4 std::cout VS. std::cerr

Conceptually, we understand that std::cout should be used for writing normal messages, e.g. logging information; std::cerr, on the other hand, should be used for indicating error messages. However, in terms of programming, what is the difference between these two streamers, since they seemingly both just output messages on the screen.

Before we dig into this question, we need to understand the concept *file descriptor* (FD). FD is a handle (usually non-negative integer) that uniquely indicates an open **file** object. On Linux, a file object can also be other input/output resources such as pipe and network sockets. Recall that both *cout* and *cerr* stand for *standard outputs*, the latter is specifically for error output. All these two are the default output FDs, to which a program can write outputs. On Linus, there should be three standard FDs:

Table 5:	Standard	File	Descriptors

Streams	File Types	FD handles
std::cin	standard input	0
std::cout	standard output	1
std::cerr	standard error	2

Consider the following program:

```
#include <iostream>
```

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```
int main() {
    // write a message to stdout
    std::cout << "This is from cout\n";
    // write something to stderr
    std::cerr << "This is from cerr\n";
    return 0;
}</pre>
```

and you can download it program cout_vs_cerr.

Inside a terminal, compile the program:

```
$ g++ cout_vs_cerr.cpp
```

Let's first run the program normally

```
$ ./a.out
This is from cout
This is from cerr
```

Both of "This is from cout" and "This is from cerr" are printed on the screen. Bash allows you to *redirect* standard outputs by using > when you run any programs. Now rerun the program:

```
$ ./a.out >cout.txt
This is from cerr
```

Only "This is from cerr" is shown on the screen, the output that was written to cout had been redirected to the file "cout.txt". Invoke the built-in commands 1s and cat to list *cwd* and print out the content of a file.

```
$ ls
a.out cout.txt cout_vs_cerr.cpp
$ cat cout.txt
This is from cout
```

In addition, you redirect a specific file descriptor by adding the FD handle in front of >. Finally rerun the program:

```
$ ./a.out 1>cout.txt 2>cerr.txt
$ ls
a.out cerr.txt cout.txt cout_vs_cerr.cpp
$ cat cout.txt cerr.txt
This is from cout
This is from cerr
```

This time, both *cout* and *cerr* wrote to files "cout.txt" and "cerr.txt", respectively. In practice, this allows you easily to group the program outputs into normal progress logging information and error/warning information. For instance:

```
$ ./my_prog 1>prog.log 2>error.log
```

If something goes wrong, the user can always trace back in "error.log" given the assumption that your program writes error/warning messages to std::cerr.

CHAPTER

SIX

LECTURE 2: REFERENCES, POINTERS & DYNAMIC ARRAY

Table of Contents

- Understanding References in C++
- *Understanding Pointers in C++*
- Dynamic Memory Allocation/Deallocation
- Defining Multiple Variables

6.1 Understanding References in C++

- Initializing Variables vs. Initializing References
- References with const-Qualifier

A reference is an **alternative name** of another object in C++. The syntax is adding symbol & after the type identifier (declarator), i.e. [type] &var.

6.1.1 Initializing Variables vs. Initializing References

In lecture 1 Define & Initialize Variables, we have learned how to initialize a variable, e.g.

```
int a = 1; // define and initialize integer a with value 1
```

When we initialize a variable, the initializer (right-hand side) will be copied to the variable. The above code is to copy the right-hand side, which is 1, to variable a. Intuitively, recopying values is allowed for variables, so that you can write:

```
int a; // define a
a = 2; // recopy value 2
```

However, for references, we bind, instead of copy, them to their initializers. Each reference is bound to its initial object, and rebinding it is not allowed.

Warning: All references must be defined with initializers!

Note: A reference is just an alias of an object!

For the code above, if we modify the value of obj, say obj=2;, what will be the value of ref?

Checkout notebook ref and run it.

Note: Modifying a reference will affect the object that it bind to. Essentially, the behavior of an object and its references is synchronized.

Due to the fact that a reference can be used to modify the values of its original object, **binding a (normal)** reference to a constant object is not allowed!

```
const double tol = 1e-2;
double &tol_ref = tol; // error!
// similarly, a literal is considered to be constant object
char &A = 'A'; // error!
```

6.1.2 References with const-Qualifier

You can bind a constant object with **constant reference**. Because a reference itself already has the property of const-ness, i.e. you cannot rebind a reference, **constant reference** can also be used to bind temporary variables (future).

```
const float alpha = 1.0f;
const float &alpha_ref = alpha; // ok!
const std::string &str_ref = "ams562"; // ok!
```

Note: You can bind constant references to normal objects.

Checkout and run notebook const_ref.

6.2 Understanding Pointers in C++

- The address-of Operator
- The dereference Operator
- Initialize Pointers
- References vs. Pointers

• Pointers with const-Qualifier

Pointer is probably one of the most difficult concept to understand in C and C++. Before we jump into pointers, we need to first understand the memory addresses in C++.

An **object** has its unique memory address. A pointer is a special type of **objects** that can hold memory addresses as its values.

To define a pointer, we need to add symbol * after the type identifier, i.e. [type] *ptr.

```
int *ptr; // define a pointer
```

6.2.1 The address-of Operator

To extract the memory address of an object, we need to use the address-of operator, i.e. &.

Note: Do not get confused with the **symbol** & used in defining *references*, since it appears after the **type** identifier. The address-of operator is used in front of **variables**.

```
int a = 1;  // define an integer
int *a_ptr = &a;  // define a pointer that points to a's address
```

In line 2, the address-of operator is used in order to extract the memory address of a, and the value (of the memory address) is assigned to the pointer a_ptr. Moreover, a_ptr is defined as a pointer that points an integer object, typically, you cannot mix the pointer type and object type.

```
Warning: double *ptr = &a is not allowed with the code above!
```

6.2.2 The dereference Operator

Accessing objects is a typical usage of pointers. To do so, we need the dereference operator—*.

Note: Similarly, do not confuse with dereference operator and the symbol * for defining pointers.

Checkout and run notebook ptr.

6.2.3 Initialize Pointers

A **null pointer** points to a special memory address that indicates empty object. Typically, you should initialize a pointer with **null** if you don't know what addresses it needs to take. In C++, we can use (and

should use) nullptr for **null pointer**. Some programs prefer to use 0 and the traditional NULL (from C, requiring <cstdlib> interface).

```
double *ptr1 = nullptr; // C++ preferred
double *ptr2 = 0; // equiv way
double *ptr3 = NULL; // require <cstdlib>
```

Warning: It is legal to **initialize** a pointer with value 0 (null). However, you cannot reset a pointer with integer object with value zero.

```
double *ptr = 0;  // fine!
ptr = 0;  // ok!
int zero = 0;
ptr = zero;  // ERROR! cannot assign double * with int
```

Tip: Always use nullptr in C++.

Warning: Uninitialized pointers are extremely dangerous and using them is one of the typical error sources.

Danger: You cannot deference nullptr, since it will cause segmentation fault, which is a critical memory bug that will immediately abort your programs.

```
int *a = nullptr;
*a = 1; // Seg fault! program crushes!!
```

6.2.4 References vs. Pointers

There are some similarities between *references* and pointers. An obvious one is that you can use both to access and modify an object.

```
double tol = 1e-6;
double &tol_ref = tol;
double *tol_ptr = &tol;

tol_ref = 1e-2;
std::cout << tol << std::endl; // what is the output?
std::cout << *tol_ptr << std::endl; // what about this?
*tol_ptr = 0.0;
std::cout << tol_ref << std::endl; // this?</pre>
```

However, there is one fundamental difference: **references are not an object!** This means that you cannot get the memory addresses of references (well, technically the memory address of a reference is that of the object it refers to, since a reference is just an alias of an object.)

A pointer, on the other hand, is an object in C++ thus having its own memory address. You can also refer to pointers through the reference semantic.

Try the notebook ptr_ref.

6.2.5 Pointers with const-Qualifier

Unlike *references*, pointers are normal objects, so there is a difference between *pointers to constants* and *constant pointers*.

A pointer to constant is that the pointer itself is a normal object but points to a constant object, i.e. you cannot modify the underlying object. However, you can modify the pointer, e.g. assign another memory address.

```
const int a = 1;
const int *p2c = &a;
*p2c = 2;  // ERROR! a is constant
p2c = nullptr; // fine
```

A constant pointer is that the pointer itself is constant object, i.e. you cannot modify the memory address. But you can still dereference it to modify the object that it points to.

```
int a;
int *const p = &a; // you must initialize p, why?
*p = 1; // fine
p = nullptr; // ERROR!
```

Of course, you can have constant pointers to constant, i.e.

```
int a;
const int *const p = &a;

*p = 2; // ERROR!
p = nullptr; // ERROR!
```

Tip: Read a declaration from right to left.

Note: Pointers to constant are widely used in practice.

6.3 Dynamic Memory Allocation/Deallocation

- Stack Memory vs. Heap Memory
- Dynamic Memory Allocation
- Dynamic Memory Deallocation
- Dynamic Array Allocation/Deallocation

6.3.1 Stack Memory vs. Heap Memory

The stack memory is fast but limited. In general, the stack is fixed. Ideally, we want to use the stack memory, because it is directly accessible to the CPU stack registers and the operating system can even directly allocate the stack in the cache. The fact is that all variables are constructed in the stack memory.

Note: All arrays are stored in the stack.

However, due to the limited size, we can easily get ourselves into overflow.

```
double huge_data[extremely_large_size]; // this will break!
```

An easier way to understand this limitation is to look at the following *recursive function*:

```
void never_call(int i) {
   int j = 1;
   int k = i;
   std::cout << k << std::endl;
   never_call(j+k); // recursively call "myself"
}</pre>
```

Each time, when the function never_call is invoked, three variables will be created—i (local copy), j and k. Due to the recursive mechanism, all the variables will live in the stack thus resulting segmentation fault eventually because of stack size overflow.

The **heap memory**, on the other side, is, loosely speaking, the left-over memory in the RAM after 1) your program is loaded and 2) the memory is allocated. Unlike the stack memory, where all variables are stored, the space we request in the heap is not directly appeared in the program, we need to create a *pointer* that points to the leading (usually) memory address of that chuck of memory space.

Note: We typically refer variables that created in the stack as the *meta data* that describes the actual data, which is typically large and stored in the heap.

The heap memory is used for *dynamic memory allocation* (a.k.a runtime memory allocation), which is usually used in the following two situations:

- 1. the data size is large, and
- 2. the data size is unknown and dynamically changing.

For the second one, a typical situation is that we you write a word processing program, you don't know how many characters the user may use, so a memory space that can grow dynamically is must.

Tip: For some applications, even though the data size cannot be determined beforehand, but the upper bound can be precisely estimated. In this case, if the upper bound is small, then you should use the stack memory! One example would be create a *C-string* to store the user input filename, i.e. char filename_buffer[200].

6.3.2 Dynamic Memory Allocation

Since we already learned that we need to use *pointers* to point to the memory locations in the head, you should not be surprised that dynamic memory allocations involve using pointers. The syntax is [type]*ptr = new [type], where the operator new is to allocate memory dynamically.

```
int *bad_ptr = 3; // ERROR!
int *ptr = new ptr; // request a valid place first
*ptr = 3; // dereferencing a valid pointer is fine
// or
int *ptr_init = new int (3);
```

6.3.3 Dynamic Memory Deallocation

As we already learned in the *scope*, all variables have the lifetime that is bounded by the scope, i.e.

```
{ // scope begins
int a; // push a into the stack
} // scope ends, a is popped
```

Does this rule applied for dynamic memory in the heap? Recall that a *Understanding Pointers in C++* is also a variable.

```
1 { // scope begins
2 int *ptr = new int; // allocate a dynamic chuck
3 } // scope ends, ptr is popped
```

In line 3, once the scope ends, ptr will be popped out and no long visible, what about the dynamic memory it used to point at?

```
Warning: Dynamic memory will not be automatically cleaned up!
```

Actually, the code above is extremely dangerous, because once ptr is popped out, it's impossible for you to access to the dynamic memory space that ptr used to point at. People refer this as *memory leaks*.

To relax a dynamic allocation, we need the deallocation operator delete, the syntax is delete [ptr];.

```
int * ptr = new int;
delete ptr; // freed here!
} // no leak!
```

Important: There must be exactly a delete that matches to a new.

```
double *p = new double; // allocate here
p = new double; // reallocate here, previous allocation leaked!
delete p; // the first double is leaked.
// delete p; // delete twice won't work and dangerous!
```

6.3.4 Dynamic Array Allocation/Deallocation

A common use of dynamic memory allocation is to request a large chunk of contiguous memory space, i.e. an array, during runtime. For this task, should use operator new[] and relax the dynamic array with operator delete[].

Note: A *pointer* can be accessed like an *array* using operator []. For instance, given the example above, you can do data[0] to access the first element in the dynamic array, and data[n] for the n-th one.

A common mistake is mixing new/new[] with delete/delete[].

Warning: new must be coupled with delete, and the same rule applies for new[] and delete[].

```
int *p1 = new int;
delete [] p1; // WRONG!
double *p2 = new double [2];
delete p2; // WRONG!
```

Try the notebook dyn.

Note: With modern C++, you really don't need to worry that much about managing dynamic memory. In the future lectures, we will learn using vector as well as the so-called *smart pointers*.

6.4 Defining Multiple Variables

Now, with the knowledge of compound types, i.e. *pointers* and *references*, I think it's a good time for you to understand a tricky part in C++—defining multiple variables.

You can define multiple variables like:

```
int i, j; // define two variables without initialization
int k=0, t; // define another two, initialize k
float x, y=1.0f; // only initialize the second one
std::string depart("ams"), course("562"); // initialize both
```

This is pretty intuitive. Now let's say I want create two points:

```
int *ptr1, ptr2;
ptr2 = nullptr; // ERROR!!
```

This is because compound types have the local property. Therefore, ptr2 is actually just an integer.

Now, try to figure out the types of the following variables.

```
int a=0,*b=0,&c=a,**d=&b,&d=c,**&e=d,f=d; // read from right to left
```

СНАРТЕ	3
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