1. Are the concepts “mutually exclusive events” and “independent events” the same? If not, what is the difference? Explain with examples.

Mutually exclusive event: Two events are mutually exclusive when they cannot occur at the same time. For example, if we flip a coin it can only show a head OR a tail, not both.

Independent event: The occurrence of one event does not affect the occurrence of the others. For example, if we flip a coin two times, the first time may show a head, but the next time we flip the coin, the outcome will be head. From this example, we can see that the first event does not affect the occurrence of the next event.

Difference between Mutually exclusive and independent events	
Mutually exclusive events	Independent events
The non-occurrence of an event will end up in the occurrence of an event.	There is no influence of an occurrence with another and they are independent of each other.
The mathematical formula for mutually exclusive events can be represented as P(X and Y) = 0	The mathematical formula for independent events can be defined as P(X and Y) = P(X) P(Y)
The sets will not overlap in the case of mutually exclusive events.	The sets will overlap in the case of independent events.

Reference,

https://cst.uwaterloo.ca/316/Tutorials/The%20difference%20between%20mutually%20exclusive%20and%20independent%20events.pdf

https://byjus.com/maths/difference-between-mutually-exclusive-and-independent-events/#:~:text=The%20difference%20between%20mutually%20exclusive,occurrence%20of%20the%20other%20event

2. For the same n and x values, which of the following is always true? $C(n,x) \geq P(n,x)$ or $P(n,x) \geq C(n,x)$? Explain.

According to formula of $P(n,x) = n! / x!$ and $C(n,x) = n! / x!(n-x)!$

We can say that the $P(n,x)$ always be greater and equal to $C(n,x)$, because in the combinations formula we have $(n-x)!$ more than the permutations formula.

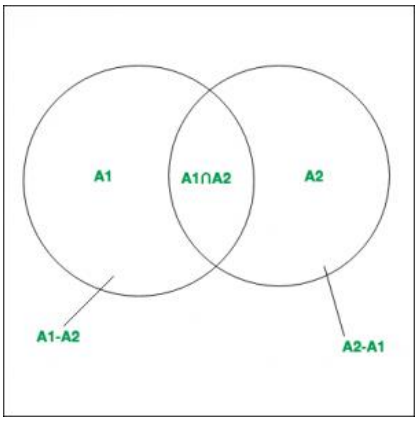
The reason for the difference it seems is hidden in the basic definition of each one and keeping order or not.

3. What is Inclusion-Exclusion Principle? Explain how it can be used in probability.

In the field of Combinatorics, it is a counting method used to compute the cardinality of the union set.

It is worked for,

1- Computes the total number of elements that satisfy at least one of several properties.
2- It prevents the problem of double counting.



$$|A \cup B| = |A| + |B| - |A \cap B|$$

In probability we can take it from both sides of equality,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

And when we talk about independence probability this formula can change to

$P(A \cup B) = P(A) + P(B) - P(A).P(B)$

Reference,

http://www.cs.cornell.edu/courses/cs2800/2014fa/lnotes/07_incl_excl.pdf

https://en.wikipedia.org/wiki/Inclusion%E2%80%93exclusion_principle

https://www.geeksforgeeks.org/inclusion-exclusion-various-applications/