# Questionably Bayesian Psychology

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# Overview

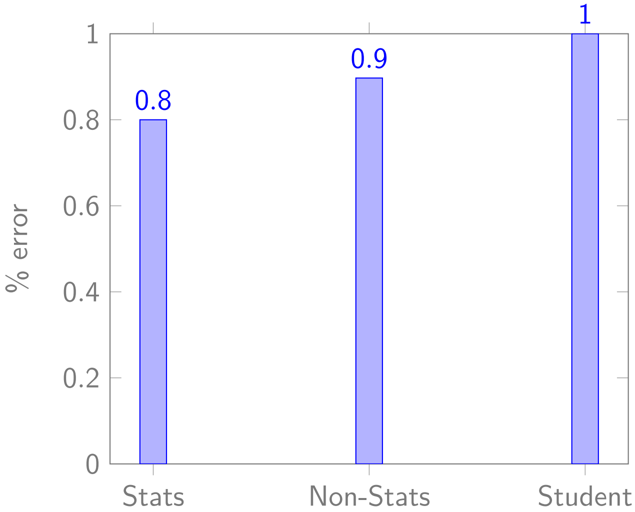
Section 1

# The Need for Bayesian Methods

# Haller & Krauss, 2002

An independent means t-test results in $(t = 2.7, p = 0.01)$. Which of the following are true?

- You have absolutely disproved the null hypothesis.
- You have found the probability of the null hypothesis being true.
- You have absolutely proved your experimental hypothesis.
- You can deduce the probability of the experimental hypothesis being true.
- You know, if you decide to reject the null hypothesis, the probability that you are making the wrong decision.
- You have a reliable experimental finding in the sense that if, hypothetically, the experiment were repeated a great number of times, you would obtain a significant result on 99% of occasions.


1
0.9
0.8
0.8
0.6
0.4
0.2
0
% error
Stats
Non-Stats
Student
0.8
0.9
1

Similar:

**Oakes (1986)** 97% Error rate

**Falk and Greenbaum (1995)** 87% Error rate

# Hypothesis Testing

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$$\bar{X} \xrightarrow{d} \mathrm{N}\left(\mu, \frac{\sigma^2}{n}\right)$$

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- So, if the true mean $\mu$ is zero, then $\bar{X}$ is (approximately) normally distributed with mean 0 and standard deviation $\frac{\sigma}{\sqrt{n}}$.
- If this is unlikely, then "reject the null".

# The p-value is uninteresting

Setup:

- Police force in a city of 1,000,000 people with 100 criminals
- Lie detector with 99% accuracy
- $H_0$: Person is innocent

Experiment:

- Test one person and find guilty.
- p-value?

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$$
\begin{aligned}
P(H_1|D) &= \frac{P(D|H_1)P(H_1)}{P(D)} \\
&= \frac{0.99 \cdot 0.0001}{0.99 \cdot 0.0001 + 0.01 \cdot 0.9999} \\
&\approx 0.001
\end{aligned}
$$

# The Case for Bayes

- Gives us what we want: The probability that a hypothesis is true.
- Allows us to specify more complex/realistic models.
- Allows us to incorporate prior information/research.

Section 2

# Questionably Bayesian Factors

# Popularity

- Google scholar reports 64,500 articles in psychology journals using or discussing Bayes Factors.
    - 10,900 since 2014
- Two of the most downloaded articles in the Journal of Mathematical Psychology are on Bayes Factors.

# The Essence of Bayes

Want to estimate a parameter $\theta$ (say, the parameters of a regression model)

1. Start with prior information
2. Update with the data (the likelihood function)
3. Posterior summarizes our new belief

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)}$$

# Questionably Bayesian Factors

Want to compare two models: $M_0$ and $M_1$.

- Using Bayes theorem, we can compute the posterior odds of $M_0$ over $M_1$.
- Written as the product of prior odds and the Bayes factor, which updates the prior according to evidence contained in the data.

$$\frac{P(M_0|D)}{P(M_1|D)} = \frac{P(D|M_0)}{P(D|M_1)} \cdot \frac{P(M_0)}{P(M_1)}$$

# BF Guidelines

Jeffreys (1961)

| **Bayes Factor** | **Evidence for $M_0$** |
|---|---|
| $< 1$ | Negative |
| $(1, 10^{1/2})$ | Barely worth mentioning |
| $(10^{1/2}, 10)$ | Substantial |
| $(10, 10^{3/2})$ | Strong |
| $(10^{3/2}, 10^2)$ | Very strong |
| $> 10^2$ | Decisive |

# Conceptual problems

- Bayes factor *is not the posterior odds*
  - Is not what we want to know: "Which model is more probable?"
- Reflects the evidence contained in the data alone
- Posterior odds are meaningless without an informative prior

Section 3

# The Problem of (Weak) Priors