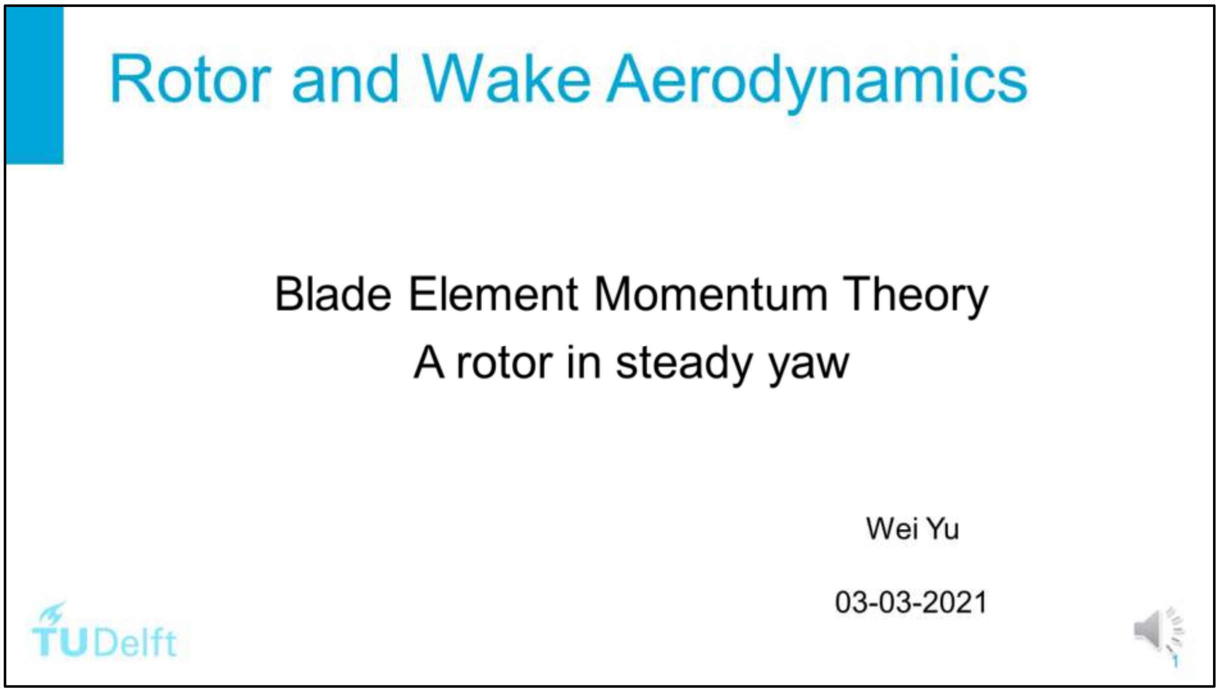
BEM for yawed rotor

Rotor / wake Aerodynamics (Technische Universiteit Delft)



Welcome to the lecture of rotor and wake aerodynamics. This module deals with BEM for a wind turbine rotor in steady yaw. My name is Wei Yu.

In the previous modules, you have:

- Derived BEM of an axial rotor;
- Defined an algorithm for setting up a BEM model;
- Discussed the main corrections for heavily loaded streamtubes and tip/root losses;

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By the end of this module, you should be able to

- Apply **Momentum Theory, Glauert Theory and vortex cylinder model** to a horizontal axis rotor in yaw or a helicopter in forward flight.

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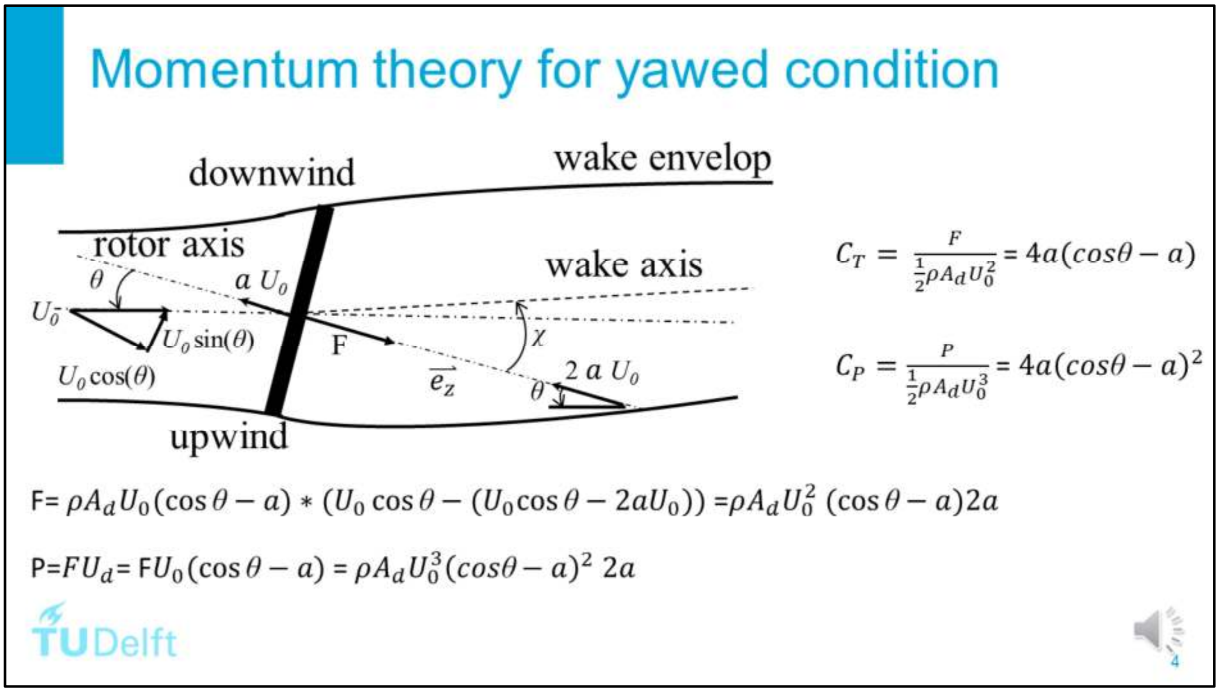


The blade element momentum theory has been derived under the assumption that the rotor plane is perpendicular to the wind direction. This is only true on a time-averaged basis. In reality, the wind direction fluctuates around the mean wind direction implies the wind turbine to be in a continuous yawed situation.

As we understand from previous modules, the momentum theory is only capable of determining the averaged induction on the rotor disc. By assuming radial independence, it is extended to annuli for aligned flow. However, in yawed case, the blade circulation also varies with azimuthal position. Therefore, applying MT to a yawed actuator disc is somewhat problematical. However, we can still use it for the averaged performance prediction.

Let's denote the yaw angle as θ (which is the angle formed by the wind direction and the rotor rotating axis). The wake follows a main direction, referred to as the wake axis, which forms an angle χ with the rotor axis referred to as the wake skew angle. This angle is higher than the yaw angle. Different models exist to determine the wake skew angle.

By assuming that the force on the disc is a pressure force and so normal to the disc. Thus, this force is responsible for the rate of change of momentum of the flow. Then the averaged induced velocity must also be perpendicular to the disc plane. Consequently, the wake is deflected to one side because a component of the induced velocity is at the right angles of the wind direction. Similar to the non-yawed case, the averaged induced velocity in the wake is twice that at the disc.

The same as the non-yawed condition, the force F should be balanced by the momentum fluxes that enter and leave the control volume in the axial direction of the rotor, which equals the mass flow rate times the change of velocity normal to the rotor plane.

F= $\rho A_d U_0(\cos\theta - a) * (U_0 \cos\theta - (U_0 \cos\theta - 2aU_0))$ =$\rho A_d U_0^2 \ (\cos\theta - a)2a$

The power equals the force times the velocity normal to the disc.

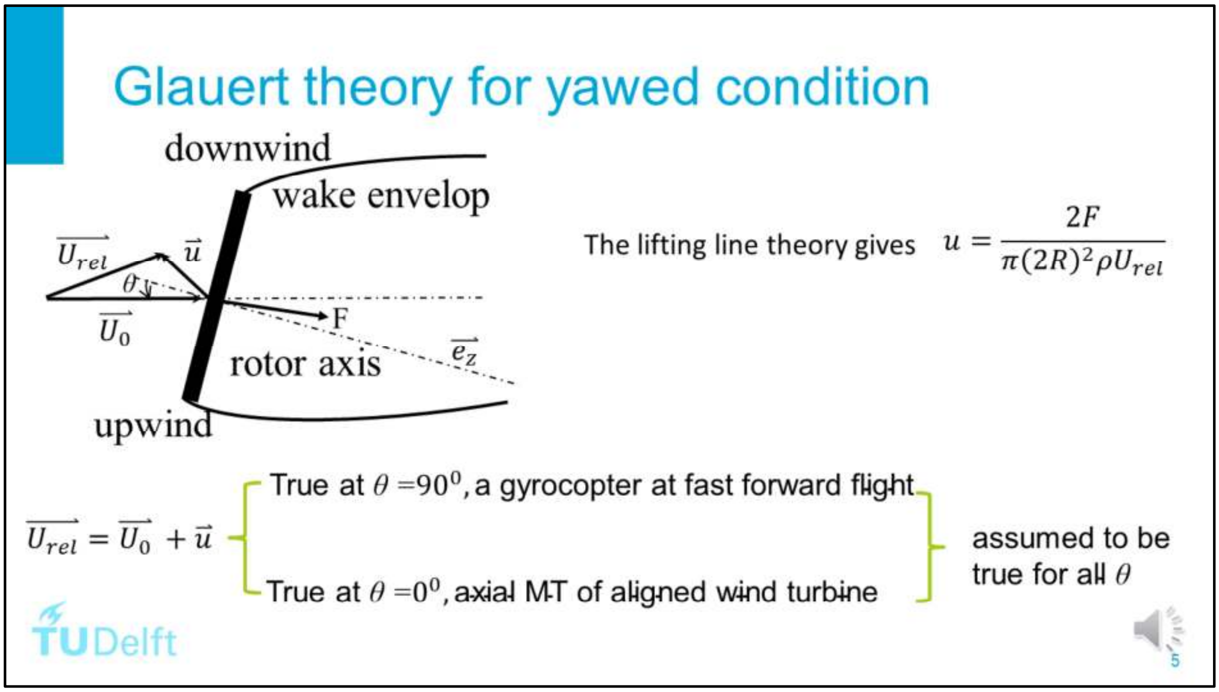
P=FU_d= F$U_0(\cos\gamma - a)$ = $\rho A_d U_0^3 (cos\gamma - a)^2 \ 2a$

Normalize them in the same way as in the non-yawed condition yields

$C_T = \frac{F}{\frac{1}{2}\rho A_d U_0^2}$ = $4a(cos\theta - a)$

$C_P = \frac{P}{\frac{1}{2}\rho A_d U_0^3}$ = $4a(cos\theta - a)^2$

The application of MT to yawed condition might be acceptable for disc-averaged induction estimation, but it is even less justification to apply it to annuli than that in the non-yawed conditions.



Here I will introduce to you the widely used Glauert theory for yawed condition.

Glauert theory is applicable at a high tip speed ratio and at a large yaw angle, where the rotor disc is analogous to an elliptic wing. The wing is flat, hence untwisted and with constant profiles along the span. The loading of the circular wing is thus elliptical which in turn implies that the induced velocity is constant along the span under lifting-line assumptions. Glauert further assumes that the induced velocity is constant along the wing chord. This lifting-line analysis allows us to link the mean induced velocity over the rotor to the rotor loading.

The lifting line theory gives the uniform (averaged) induced velocity as

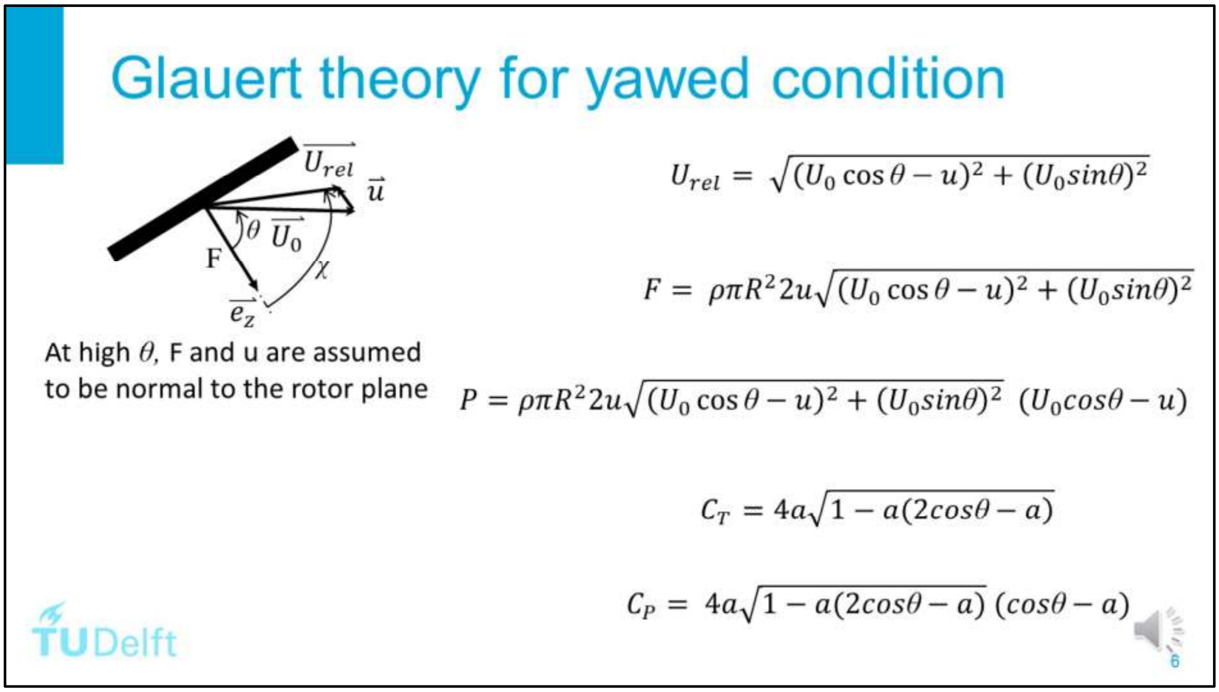
$$u = \frac{2F}{\pi(2R)^2\rho U_{rel}}$$

Where the relative velocity U_{rel} to the lifting line can be obtained by adding the incoming flow velocity and the averaged induced velocity vectorially.

$$\overrightarrow{U_{rel}} = \overrightarrow{U_0} + \vec{u}$$

Glauert's model is based on the fact that the above expression is the correct

expression for a gyrocopter at fast forward flight, which is similar to a yaw angle of 90 degrees of a wind turbine.
At a yaw angle of 0 degree, the $U_{rel} = U_0 - u$, which is also true from the MT of the aligned flow of wind turbine rotor.
Hence, the equation is valid for 90 degrees and 0 degrees yaw misalignment, and it is assumed to be true for in-between values.



At a low angle of attack of the lifting line (which means a high angle of yaw of the rotor), then both F and u can be assumed to be normal to the rotor plane. Then we can get the relative velocity as this expression

$$U_{rel} = \sqrt{(U_0 \cos\theta - u)^2 + (U_0 sin\theta)^2}$$

Substitute U_{rel} into the lifting line equation which links the force and the induced velocity, and rewrite the equation we get

$$F = \rho\pi R^2 2u\sqrt{(U_0 \cos\theta - u)^2 + (U_0 sin\theta)^2}$$

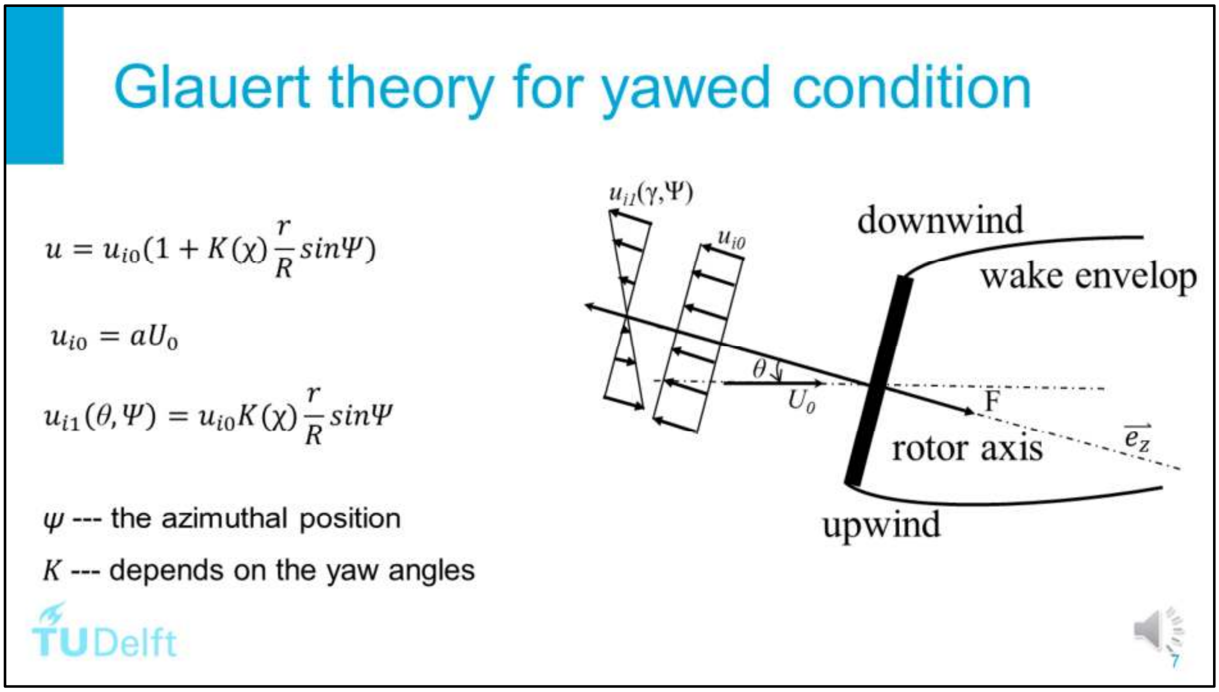
Similarly, the power equals the force times the velocity normal to the disc.

$$P = \rho\pi R^2 2u\sqrt{(U_0 \cos\theta - u)^2 + (U_0 sin\theta)^2}\ (U_0 cos\theta - u)$$

Normalize them the same way as in aligned condition

$$C_T = 4a\sqrt{1 - a(2cos\theta - a)}$$

$$C_P = 4a\sqrt{1 - a(2cos\theta - a)}\ (cos\theta - a)$$



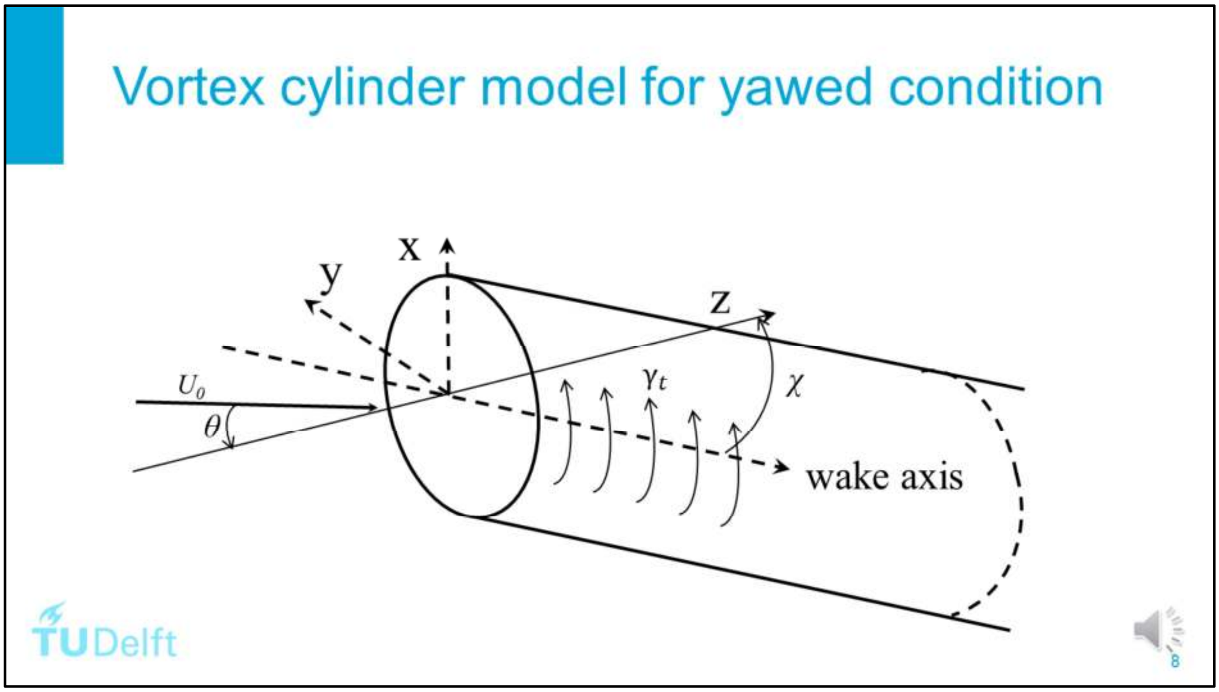
Yet, the normal velocity should also vary with azimuthal position, so Glauert yaw-model assumes that the induced velocity normal to the rotor, u , can be decomposed into a constant component u_{i0} and a component u_{i1} that varies linearly with the radius and sinusoidally with the azimuth angle:

$$u_{i1}(\theta, \Psi) = u_{i0}K(\chi)\frac{r}{R}sin\Psi$$

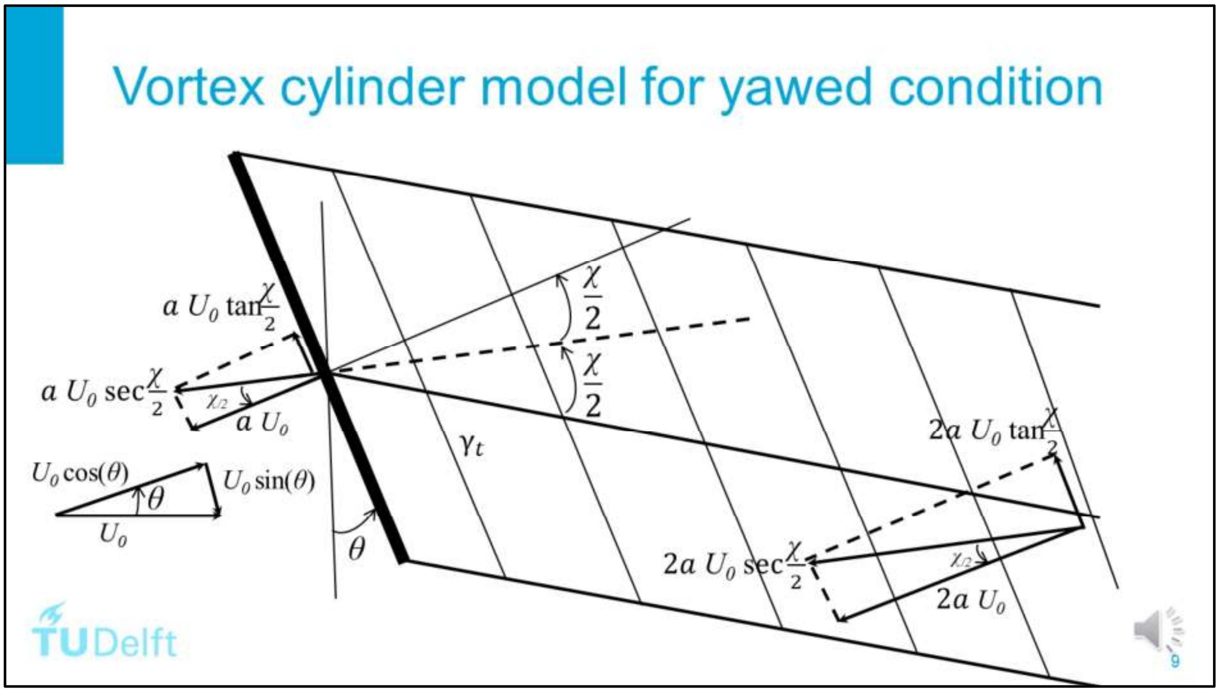
where $u_{i0} = aU_0$ is the induction value in the non-yawed case or the azimuthally averaged value.
ψ is the azimuthal position, K is a parameter that depends on the yaw angles.
The induced velocity can be written as

$$u = u_{i0}(1 + K(\chi)\frac{r}{R}sin\Psi)$$

In many engineering yaw-models, the radial dependency and the dependency on yaw angles (K) are calibrated from measurements, vortex models and high-fidelity CFD results.



Coleman is the first person to do so using a cylinder wake model. By ignoring the wake expansion and assuming an infinite number of blades and assuming that the bound circulation on the rotor disc is radially and azimuthally uniform. The wake of a yawed rotor can be represented by a skewed semi-infinite cylinder with tangential vorticity which is parallel to the disc.

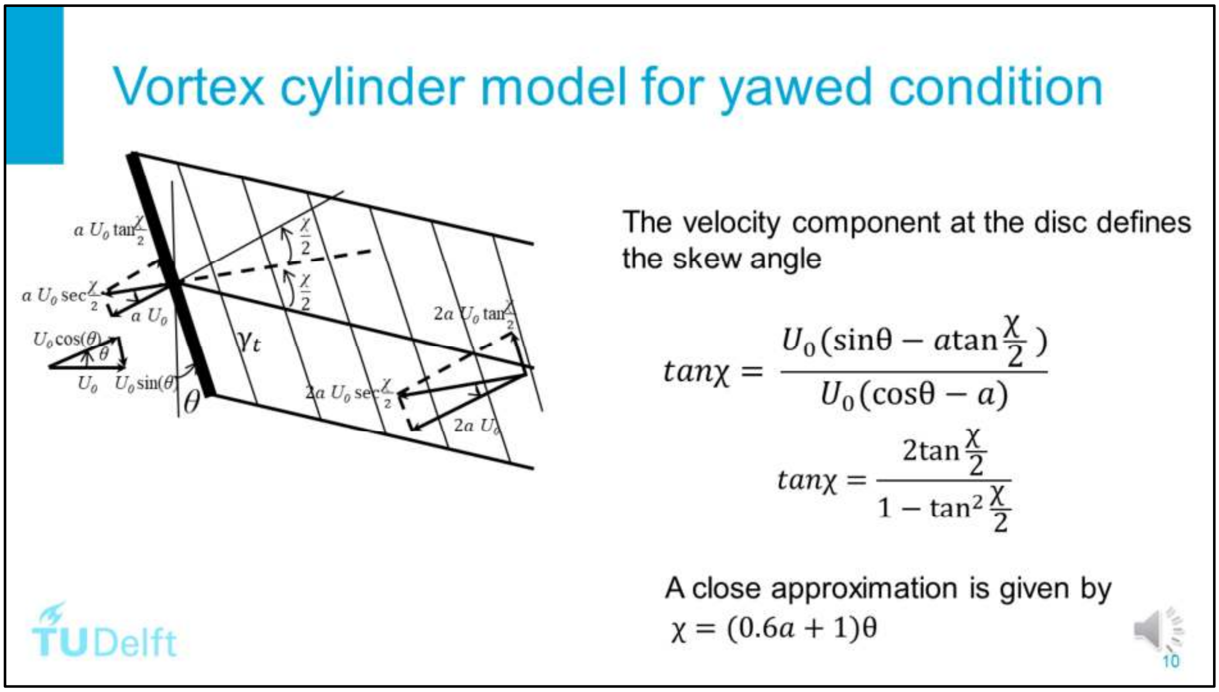


Using Biot-Savart law, an averaged induced velocity at the disc can be derived as $aU_0 \sec\frac{\chi}{2}$, which is in the direction that bisects the skew angle.

The induced velocity has a component of aU_0 at the direction normal to the disc and a component of $aU_0 \tan\frac{\chi}{2}$ parallel to the disc .

The incoming wind velocity has a component of $U_0 cos\theta$ normal to the disc and a component of $U_0 \sin\theta$ parallel to the disc.

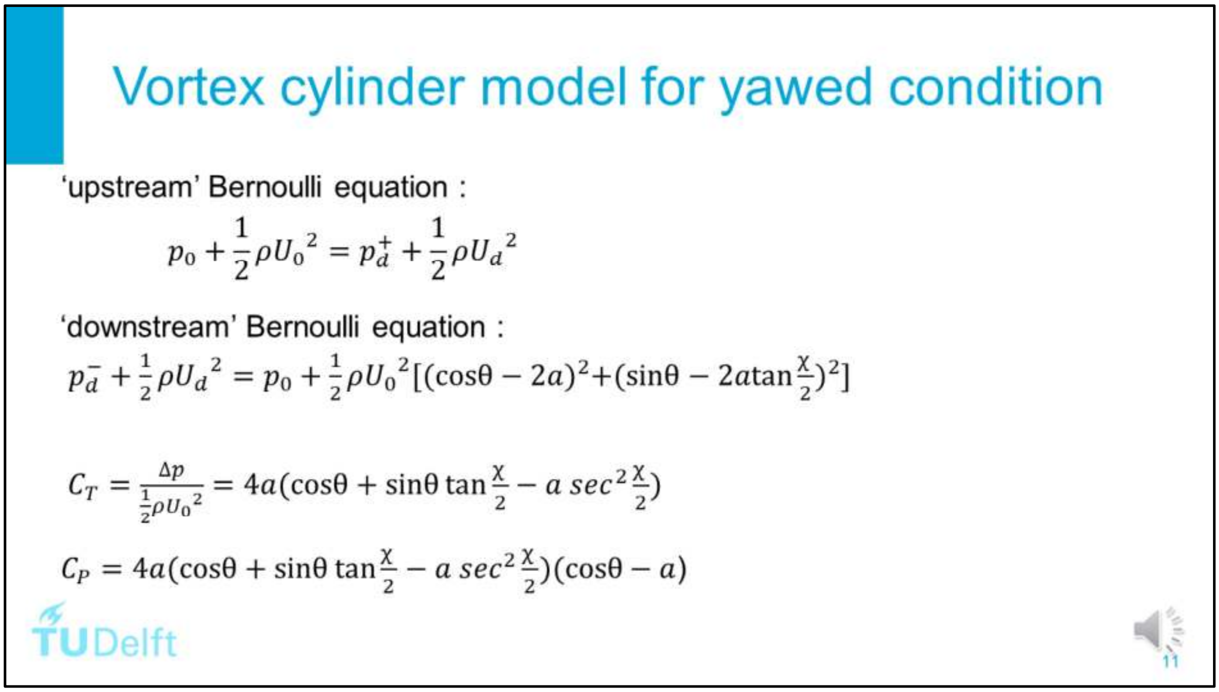
Similarly, the induction in the far wake is twice that at the rotor disc.



The velocity component at the disc defines the skew angle, which gives

$$tan\chi = \frac{U_0(\sin\theta - a\tan\frac{\chi}{2})}{U_0(\cos\theta \quad)} = \frac{2\tan\frac{\chi}{2}}{1 - \tan^2\frac{\chi}{2}}$$

A close approximation of the relationship is given by $\chi = (0.6a + 1)\theta$.



Now we can apply the Bernoulli equation for 'upstream' of the disc, we get

$$p_0 + \frac{1}{2}\rho {U_0}^2 = p_d^+ + \frac{1}{2}\rho {U_d}^2$$

For 'downstream' of the disc, we get

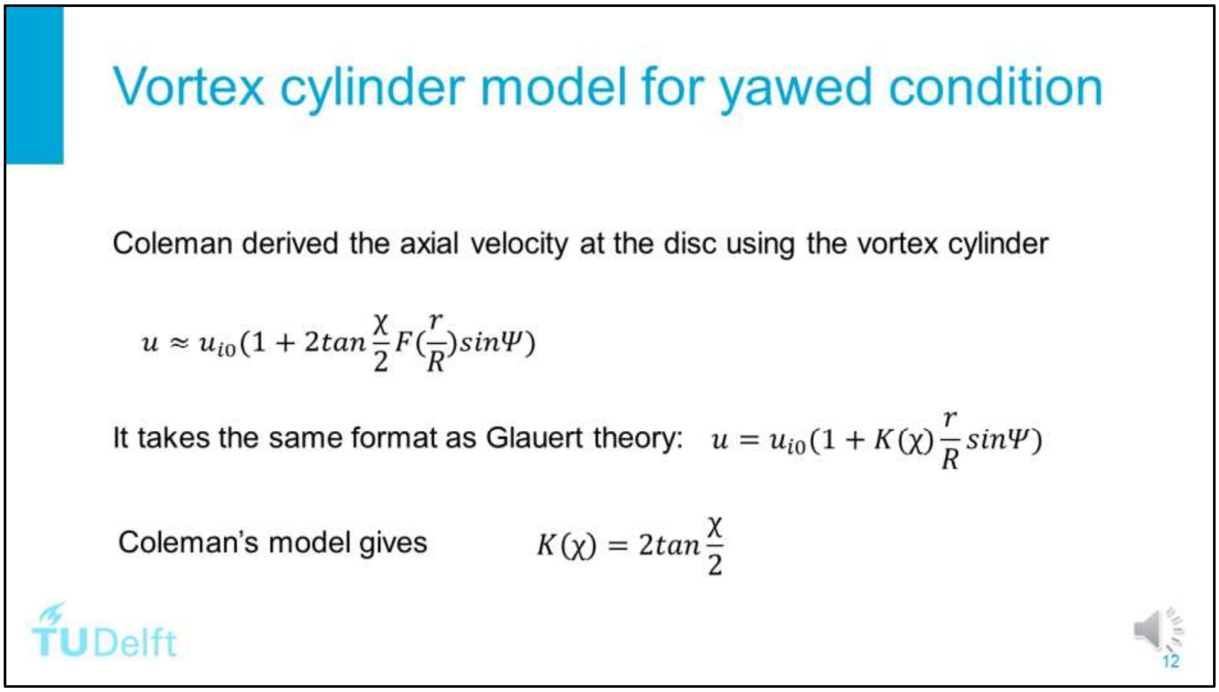
$$p_d^- + \frac{1}{2}\rho {U_d}^2 = p_0 + \frac{1}{2}\rho {U_0}^2[(\cos\theta - 2a)^2 + (\sin\theta - 2a\tan\frac{\chi}{2})^2]$$

Subtracting the upstream and downstream Bernoulli equations and normalize it with respect to $\frac{1}{2}\rho {U_0}^2$, we get

$$C_T = \frac{\Delta p}{\frac{1}{2}\rho {U_0}^2} = 4a(\cos\theta + \sin\theta\tan\frac{\chi}{2} - a\,sec^2\frac{\chi}{2})$$

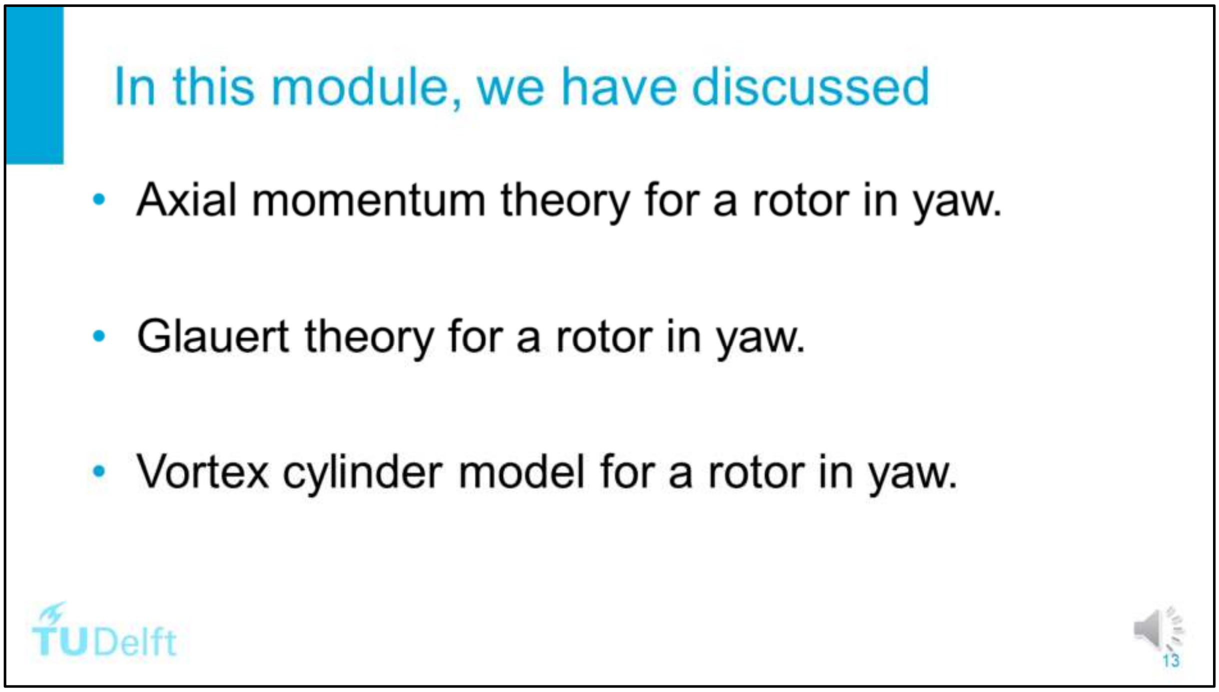
Similarly, the power equals to the force times the velocity normal to the disc, by normalizing it, we get the expression for C_P

$$C_P = 4a(\cos\theta + \sin\theta\tan\frac{\chi}{2} - a\,sec^2\frac{\chi}{2})(\cos\theta - a)$$



Coleman derived the axial velocity at the disc using the vortex cylinder model, which is given as $u \approx u_{i0}(1 + 2tan\frac{\chi}{2}F(\frac{r}{R})sin\Psi)$

We can see that it takes the same format as Glauert theory $u = u_{i0}(1 + K(\chi)\frac{r}{R}sin\Psi)$
By comparing the two equations, we know Coleman's model gives the yaw-angle dependent parameter K as
$K(\chi) = 2tan\frac{\chi}{2}$


In this module, we have discussed
• Axial momentum theory for a rotor in yaw.
• Glauert theory for a rotor in yaw.
• Vortex cylinder model for a rotor in yaw.
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Reference

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3. E. Branlard. Wind Turbine Aerodynamics and Vorticity-Based Methods, 2017.

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Here are some references.