# Practice Problems

__/__/____

① $f(n) = n-10$ $\quad g(n) = n+10$

$f(n) = \Theta(g(n))$

② $f(n) = n$ $\quad g(n) = n$

$f(n) = \Theta(g(n))$

③ $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

④ $\frac{4^n}{2^n} = O(2^n)$

⑤ $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

① $f(n) = n-10$ $\quad g(n) = n+10$

$f(n) = \Theta(g(n))$

Note :- $f(n) \le c.g(n)$ & $f(n) \ge c.g(n)$ i.e Both Big O and Big Omega

Big O

$f(n) = c.g(n)$

$n-10 <= c.n+10$

$n-10 <= n+10$ when $c=1$

Since L.H.S is less than R.H.S

Big O is true

Omega

$f(n) >= c.\cancel{n+10}\, g(n)$

$n-10 >= n+10$

$n-10 >= \frac{n+10}{2}$

when

$c = 1/2$

Omega

$\therefore f(n) = \Theta(g(n)) \rightarrow$ True

② $f(n) = n$ $\quad g(n) = n$

$f(n) = \Theta(g(n))$

Big O

$n <= c.n$

$n <= n$ when $c=1$

Big O $\longrightarrow$ True

Omega

$n >= c.n$

$n >= n$ when $c=1$

Omega $\rightarrow$ True

Hence $f(n) = \Theta(g(n)) \rightarrow$ True.

__/__/__

③ $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

$n^{\log_2 64} \cdot n^{\log_2 32} = O(n^5)$

$n^{\log_2 2^6} \cdot n^{\log_2 2^5} = O(n^5)$

$n^6 \times n^5 = O(n^5)$

$n^{11} = O(n^5)$

Big O

$f(n) <= c.g(n)$

$n^{11} <= c.n^5$

$n^{11} <= n^6 n^5$ when $c = n^6 \rightarrow$ where n is not

$= n^{11} <= n^{11}$ a constant,

$\therefore$ It is false. [Big O $\rightarrow$ False]

④ $\frac{4^n}{2^n} = O(2^n)$

$= \frac{\cancel{2}^n \times 2^n}{\cancel{2^n}} = O(2^n)$

Big O

$2^n <= c.2^n$

$2^n <= 2^n$ when $c = 1$

Big O $\rightarrow$ True.

⑤ $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

~~$n^{\log_2 2^7} <= c.n^2$~~

$n^{\log_2 2^7} n^2 <= c.n^9$

$n^7 n^2 <= n^9$

Big O is

$128^{\log_2 n} \cdot n^2 >= c.n^9$

$n^{\log_2 2^7} \cdot n^2 >= c.n^9$

$n^7 n^2 >= n^9$

$n^9 >= n^9$

Omega is

$128^{\log_2 n} \cdot n^2 = \Theta(n^9) \rightarrow$ True