

# STAT 216 Coursepack



Spring 2023  
Montana State University

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This resource was developed by Melinda Yager, Jade Schmidt, and Stacey Hancock in 2021 to accompany the online textbook: Carnegie, N., Hancock, S., Meyer, E., Schmidt, J., and Yager, M. (2021). *Montana State Introductory Statistics with R*. Montana State University. <https://mtstateintrostats.github.io/IntroStatTextbook/>.

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# Preface

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This coursepack accompanies the textbook for STAT 216: Introduction to Statistics at Montana State University, which can be found at <https://mtstateintrostats.github.io/IntroStatTextbook/>. The syllabus for the course (including the course calendar), data sets, and links to D2L Brightspace, Gradescope, and the MSU RStudio server can be found on the course webpage: <https://math.montana.edu/courses/s216/>. Videos assigned in the course calendar and other notes and review materials are linked in D2L.

Each of the activities in this workbook is designed to target specific learning outcomes of the course, giving you practice with important statistical concepts in a group setting with instructor guidance. In addition to the in-class activities for the course, the coursepack includes reading guides to aid in taking notes while you complete the required readings and videos. Bring this workbook with you to class each class period, and take notes in the workbook as you would your own notes. A well-written completed workbook will provide an optimal study guide for exams!

The activities and labs in this coursepack will be completed during class time. Parts of each lab will be turned in on Gradescope. To aid in your understanding, read through the introduction for each activity before attending class each day.

STAT 216 is a 3-credit in-person course. In our experience, it takes six to nine hours per week outside of class to achieve a good grade in this class. By “good” we mean at least a C because a grade of D or below does not count toward fulfilling degree requirements. Many of you set your goals higher than just getting a C, and we fully support that. You need roughly nine hours per week to review past activities, read feedback on previous assignments, complete current assignments, and prepare for the next day’s class. The following will give you an idea of what a typical week in the life of a STAT 216 student looks like.

- *Prior to class meeting:*
  - Read assigned sections of the textbook, using the provided reading guides to take notes on the material.
  - Watch assigned videos on that week’s content, pausing to take notes and answer video quiz questions.
  - Read through the introduction to the day’s in-class activity.
  - Read through the week’s homework assignment and note any questions you may have on the content.
- *During class meeting:*
  - Work through the in-class activity or weekly lab with your classmates and instructor, taking detailed notes on your answers to each question in the activity.
- *After class meeting:*
  - Complete any parts of the activity you did not complete in class.
  - Review the activity solutions in the Math and Stat Center, and take notes on key points.
  - Finish watching any remaining assigned videos or readings for the week.
  - Complete the week’s homework assignment.

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## Spring 2023 Calendar of In-Class Activities

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This calendar only lists the in-class activities, RStudio labs and exams each week. For required readings and videos as well as due dates for assignments, refer to the calendar at:

[https://mtstateintrostats.github.io/Syllabus/#Course\\_calendar](https://mtstateintrostats.github.io/Syllabus/#Course_calendar)

Week	Day	Date	Activity
1	W	1/18	Intro to Data
1	F	1/20	Week 1 Lab
2	M	1/23	American Indian Address Part A
2	W	1/25	American Indian Address Part B
2	F	1/27	Week 2 Lab
3	M	1/30	Myopia and Nightlights
3	W	2/1	IMDb Movie Reviews
3	F	2/3	Week 3 Lab
4	M	2/6	Movie Profits — Linear Regression
4	W	2/8	Movie Profits — Correlation
4	F	2/10	Week 4 Lab
5	M	2/13	Exam 1 Review
5	W	2/15	Group Midterm Exam 1
5	F	2/17	Midterm Exam 1
6	M	2/26	(No class) Helper Hinderer — Simulation HT
6	W	2/22	Helper Hinderer — Simulation HT continued
6	F	2/24	Week 6 Lab
7	M	2/27	Handedness of Male Boxers — Theory HT
7	W	3/1	Handedness of Male Boxers — Theory CI
7	F	3/3	Week 7 Lab
8	M	3/6	Good Samaritan — Simulation HT
8	W	3/8	Good Samaritan — Simulation CI
8	F	3/10	Week 8 Lab
Holiday	M–F	3/13–3/17	<b>No Class — Spring Break</b>
9	M	3/20	Helmet Use and Head Injuries — Theory HT
9	W	3/22	Helmet Use and Head Injuries — Theory CI
9	F	3/24	Week 9 Lab
10	M	3/27	Exam 2 Review
10	W	3/29	Group Midterm Exam 2
10	F	3/31	Midterm Exam 2
11	M	4/3	COVID and Air Pollution
11	W	4/5	Color Interference
11	F	4/7	(No class) Week 11 Lab
12	M	4/10	Does Behavior Impact Performance?
12	W	4/12	Week 12 Lab
12	F	4/14	
13	M	4/17	Diving Penguins
13	W	4/19	Golf Driving Distances
13	F	4/21	Week 13 Lab
14	M	4/24	What's the probability?
14	W	4/26	Relative Risk
14	F	4/28	Week 14 Lab

<b>Week</b>	<b>Day</b>	<b>Date</b>	<b>Activity</b>
15	M	5/1	Final Exam Review
15	W	5/3	Final Group Exam Part 1
15	F	5/5	Final Group Exam Part 2
Finals	T	5/9 6 - 7:50 pm	Common Final Exam See <a href="http://www.montana.edu/registrar/Schedules.html">www.montana.edu/registrar/Schedules.html</a>

## Basics of Data

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### 1.1 Week 1 Reading Guide: Basics of Data

Sections 1.1 (Case study) and 1.2 (Data basics)

#### Videos

- Stat 216 Course\_Tour
- Instructor bio
- 1.2.1and1.2.2
- 1.2.3to1.2.5

#### Vocabulary

Data:

Sample size:

Case/Observational unit:

Variable:

Quantitative variable:

Discrete variables:

Examples of discrete variables using the County data:

Continuous variables:

Examples of continuous variables using the County data:

Example of a number which is NOT a numerical (quantitative) variable:

Categorical variable:

Ordinal variable:

Example of an ordinal variable using the County data:

Nominal variable:

Examples of nominal variables using the County data:

**Note: Ordinal and nominal variables will be treated the same in this course. We recommend taking more statistics courses in the future to learn better methods of analysis for ordinal variables.**

Data frame:

Summary statistics:

Scatterplot:

Each point represents:

Positive association:

Negative association:

Associated or Dependent variables:

Independent variables:

Explanatory variable:

Response variable:

Observational study:

Randomized Experiment:

Placebo:



## Notes

Big Idea: Variability is inevitable! We would not expect to get *exactly* 50 heads in 100 coin flips. The statistical question then is whether any differences found in data are due to random variability, or if something else is going on.

The larger the difference, the **less we believe the difference was due to chance**.

In a data frame, rows correspond to \_\_\_\_\_

and columns correspond to \_\_\_\_\_.

How many types of variables are discussed? Explain the differences between them and give an example of each.

True or False: A pair of variables can be both associated AND independent.

True or False: Given a pair of variables, one will always be the explanatory variable and one the response variable.

True or False: If a study does have an explanatory and a response variable, that means changes in the explanatory variable must **cause** changes in the response variable.

True or False: Observational studies can show a naturally occurring association between variables.

### Example (Section 1.1 — Case study: Using stents to prevent strokes)

1. What is the principle question the researchers hope to answer? (We call this the **research question**.)
2. When creating two groups to compare, do the groups have to be the same size (same number of people in each)?
3. What are the cases or observational units in this study?
4. Is there a clear explanatory and response variable? If so, name the variable in each role and determine the type of variable (categorical or quantitative).
5. What is the purpose of the control group?
6. Is this an example of an observational study or a randomized experiment? How do you know?

7. Consider Tables 1.1 and 1.2. Which table is more helpful in answering the research question? Justify your answer.
8. Describe in words what is shown in Figure 1.2. Specifically, compare the proportion of patients who had a stroke between the treatment and control groups after 30 days as well as after 365 days.
9. Given the notion that the larger the difference between the two groups (for a given sample size), the less believable it is that the difference was due to chance, which measurement period (30 days or 365 days) provide stronger evidence that there is an association between stents and strokes, or that the differences are not due to random chance?
10. This study reported finding evidence that stents *increase* the risk of stroke. Does this conclusion apply to all patients and all stents?
11. This study reported finding evidence that stents *increase* the risk of stroke. This conclusion implies a causal link between stents and an increased risk of stroke. Is that conclusion valid? Justify your answer.

## Section 2.1 (Sampling principles and strategies)

### Videos

- 2.1

### Vocabulary

(Target) Population:

Sample:

Statistic:

Parameter:

Anecdotal evidence:

Bias:

Selection bias:

Non-response bias:

Response bias:

Convenience sample:

Simple Random Sample:

Non-response rate:

Representative:

## Notes

Ideally, how should we sample cases from our target population? What sampling method should be used?

## Notes on types of sampling bias

- Someone must first be *chosen* to be in a study and refuse to participate in order to have **non-response bias**.
- There must be a valid reason for someone to lie or be untruthful to justify saying **response bias** is present. Yes, anyone could lie at any time to any question. Response bias is when those lies are *predictable and systematic* based on outside influences.

True or False: Convenience sampling tends to result in non-response bias.

True or False: Volunteer sampling tends to result in response bias.

True or False: Random sampling helps to resolve selection bias, but has no impact on non-response or response bias.

## 1.2 Activity 1: Intro to Data

### 1.2.1 Learning outcomes

- Identify observational units, variables, and variable types in a statistical study.
- Identify biased sampling methods.

### 1.2.2 Terminology review

Statistics is the study of how best to collect, analyze, and draw conclusions from data. Today in class you will be introduced to the following terms:

- Observational units or cases
- Variables: categorical or quantitative
- Types of sampling bias

For more on these concepts, read Chapter 1 and Section 2.1 in the textbook.

### 1.2.3 General information on the Coursepack

Information is provided throughout each activity and lab to guide students through that day's activity or lab. Be sure to read ALL the material provided at the beginning of the activity and between each question. At the end of each activity is a section called *Take-home messages* that contains key points from the day's activity. Use these to review the day's activity and make sure you have a full understanding of that material.

#### Steps of the statistical investigation process

As we move through the semester we will work through the six steps of the statistical investigation process.

1. Ask a research question.
2. Design a study and collect data.
3. Summarize and visualize the data. *Weeks 3–4*
4. Use statistical analysis methods to draw inferences from the data. *Weeks 6–13*
5. Communicate the results and answer the research question. *Weeks 6–13*
6. Revisit and look forward.

Today we will focus on the first two steps.

**Step 1:** The first step of any statistical investigation is to *ask a research question*. As stated in the textbook, “with the rise of data science, however, we might not start with a research question, and instead start with a data set.” Today we will create a data set by collecting responses on students in class.

**Step 2:** To answer any research question, we must *design a study and collect data*. Our study will consist of answers from each student. Your responses will become our observed data that we will explore.

**Observational units or cases** are the subjects data are collected on. In a spreadsheet of the data set, each row will represent a single observational unit.

1. What are the observational units or cases for today's study?

2. How many students are in class today? This is the **sample size**.

A **variable** is information collected or measured on each observational unit or case. Each column in a data set will represent a different variable. The rows in a data set represent the observational units.

We will look at two types of variables: **quantitative** and **categorical** (see Figure 1.1).

Quantitative variables are numerical measurements that can be discrete (whole, non-negative numbers) or continuous (any value within an interval). The number of pets one owns would be a discrete variable as you can not have a partial pet. GPA would be a continuous variable ranging from 0 to 4.0.

The outcome of a categorical variable is a group or category such as eye color, state of residency, or whether or not a student lives on campus. Categorical variables with a natural ordering are considered ordinal variables while those without a natural ordering are considered nominal variables. All categorical variables will be treated as nominal for analysis in this course.

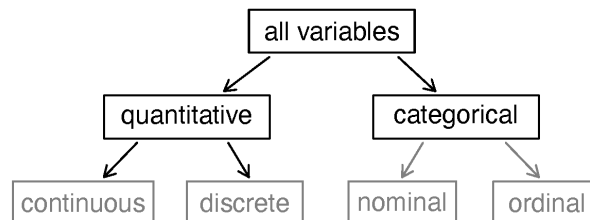


Figure 1.1: Types of variables.

3. One person from each group open the Google sheet linked in D2L and fill in the responses for the following questions for each group member. When creating a data set for use in R it is important to use single words or an underscore between words. Each outcome must be written the same way each time. Make sure to use all lowercase letters to create this data set to have consistency between responses. Do not give units of measure with the numerical values for the length of forearm. For **Residency** use in\_state or out\_state as the two outcomes.

- Major: what is your declared major?
- Residency: do you have in-state or out-of-state residency?
- Forearm\_Length: what is the length of your forearm in inches from the end of your elbow to the end of your index finger?
- Num\_Credits: how many credits are you taking this semester?

4. For each column of data, fill in the following table to write out the variable we are collecting on each observational unit in this study and the type of each variable.

Column	Variable	Type of Variable
Major		
Residency		
Forearm Length		
Num Credits		

5. Write down some issues found with the created class data set.

#### **1.2.4 Take-home messages**

1. When creating a data set, each row will represent a single observational unit or case. Each column represents a variable collected. It is important to write each variable as a single word or use an underscore between words.
2. There are two types of variables: categorical (groups) and quantitative (numerical measures).
3. Make sure to be consistent with writing each outcome in the data set.

#### **1.2.5 Additional notes**

Use this space to summarize your thoughts and take additional notes on today's activity and material covered, and to write down the names and contact information of your teammates.

## 1.3 Week 1 Lab - Sampling Methods

### 1.3.1 Learning outcomes

- Identify biased sampling methods.

### 1.3.2 Terminology review

Statistics is the study of how best to collect, analyze, and draw conclusions from data. Today in class you will be introduced to the following terms:

- Types of sampling bias
  - Selection bias
  - Response bias
  - Non-response bias

For more on these concepts, read Chapter 1 and Section 2.1 in the textbook.

### 1.3.3 General information on labs

On Friday of each week you will complete a lab. Questions are selected from each lab to be turned in on Gradescope. The questions to be submitted on Gradescope are bolded in the lab. As you work through the lab have the Gradescope lab assignment open so that you can answer those questions as you go.

### Types of bias

In the next few weeks we will look at how to summarize data both numerically and graphically. For now we will focus on sampling methods and the type of sampling bias that may be present.

- Selection bias: a part of the target population is not included or underrepresented in the sample
- Non-response or non-participation bias: part of the already selected sample does not respond or chooses not to participate
- Response bias: survey participant gives an untruthful or misleading response

To help determine the type of bias present, it is helpful to think about the observational units, the sample, and the target population represented by the problem. The **target population** is the group of cases that makes up the population the researcher is interested in. If sampling bias is present, then the sample taken will not be representative of the actual target population. In these next questions, identify the target population, the sample selected, the variable collected and its type (categorical or quantitative), and the type of bias present.

1. To determine if the proportion of out-of-state undergraduate students at Montana State University has increased in the last 10 years, a statistics instructor sent an email survey to 500 randomly selected current undergraduate students. One of the questions on the survey asked whether they had in-state or out-of-state residency. She only received 378 responses.

Sample size:

Sample taken:

Target population:

Variable:

Type of Variable:   categorical      quantitative

Justify why there is non-response bias in this study.

2. A television station is interested in predicting whether or not a local referendum to legalize marijuana for adult use will pass. It asks its viewers to phone in and indicate whether they are in favor or opposed to the referendum. Of the 2241 viewers who phoned in, forty-five percent were opposed to legalizing marijuana.

Sample size:

Sample taken:

Target population:

Variable:

Type of Variable:   categorical      quantitative

Justify why there is selection bias in this study.



3. To gauge the interest in a new swimming pool, a local organization stood outside of the Bogart Pool in Bozeman, MT, during open hours. One of the questions they asked was, “Since the Bogart Pool is in such bad repair, don’t you agree that the city should fund a new pool?”

Sample size:

Sample taken:

Target population:

Variable:

Type of Variable:   categorical      quantitative

Justify why there is response bias in this study.

Justify why there is selection bias in this study.

4. **The Bozeman school district was interested in surveying parents of students about their opinions on returning to in-person classes following the COVID-19 pandemic. They divided the school district into 10 divisions based on location and randomly surveyed 20 households within each division. Explain why selection bias would be present in this study design.**

## Gas prices

In this part of the lab we will explore two different websites to explore the cost of gas. Open both the Gas Buddy Website ([www.gasbuddy.com](http://www.gasbuddy.com)) and a government website (<https://www.eia.gov/petroleum/>). Spend some time exploring each site.

5. Choose a city listed on both sites. Write down three gas prices found on Gas Buddy for this city and the reported gas price from the government website for the same city.
6. Compare the two websites.
  - Why are gas stations selected to appear in each data set?
  - Do we know if gas stations were left out for any given time period?
  - Can we make claims about what the mean price is for all gas stations in a region?
7. **Which of the following questions are best answered with the government data, and which with Gas Buddy?**
  - How do average gas prices compare across regions of the country?
  - Where should I go to buy gas right now?
  - What will prices be like in one week? One year?

### 1.3.4 Take-home messages

1. There are three types of bias to be aware of when designing a sampling method: selection bias, non-response bias, and response bias.

### 1.3.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered, and to write down the names and contact information of your teammates.

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## Study Design

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### 2.1 Week 2 Reading Guide: Sampling, Experimental Design, and Scope of Inference

Sections 2.2 (Observational studies), 2.3 (Experiments), and 2.4 (Scope of inference)

Videos

- 2.2to2.4

#### Reminders from Section 1.2

**Explanatory variable:** The variable researchers think *may be* affecting the other variable. What the researchers control/assign in an experiment. If comparing groups, the explanatory variable puts the observational units into groups.

**Response variable:** The variable researchers think *may be* influenced by the other variable. This variable is always observed, never controlled or assigned.

#### Vocabulary

Observational study:

Observational data:

Prospective study:

Retrospective study:

Confounding variable:

Experiment:

Randomized experiment:

Blocking:

Treatment group:

Control group:

Blinding:

Placebo:

Placebo effect:

Scope of inference:

Generalizability:

Causation:

## Notes

What are the four principles of a well-designed randomized experiment?

Fill in the appropriate scope of inference for each study design.

Selection of Cases	Study Type	
	Randomized experiment	Observational study
Random sample (and no other sampling bias)		
Non-random sample (or other sampling bias)		

True or False: Observational studies can show an association between two variables, but cannot determine a causal relationship.

True or False: In order for an experiment to be valid, a placebo must be used.

True or False: If random sampling of the target population is used, and no other types of bias are suspected, results from the sample can be generalized to the entire target population.

True or False: If random sampling of the target population is used, and no other types of bias are suspected, results from the sample can be inferred as a causal relationship between the explanatory and response variables.

## 2.2 Activity 2A: American Indian Address

### 2.2.1 Learning outcomes

- Explain why a sampling method is unbiased or biased.
- Identify biased sampling methods.
- Explain the purpose of random selection and its effect on scope of inference.

### 2.2.2 Terminology review

In today's activity, we will examine unbiased and biased methods of sampling. Some terms covered in this activity are:

- Random sample
- Unbiased vs biased methods of selection
- Generalization

To review these concepts, see Section 2.1 in the textbook.

### 2.2.3 American Indian Address

For this activity, you will read a speech given by Jim Becenti, a member of the Navajo American Indian tribe, who spoke about the employment problems his people faced at an Office of Indian Affairs meeting in Phoenix, Arizona, on January 30, 1947 (Moquin and Van Doren 1973). His speech is below:

**It is hard for us to go outside the reservation where we meet strangers. I have been off the reservation ever since I was sixteen. Today I am sorry I quit the Santa Fe [Railroad]. I worked for them in 1912-13. You are enjoying life, liberty, and happiness on the soil the American Indian had, so it is your responsibility to give us a hand, brother. Take us out of distress. I have never been to vocational school. I have very little education. I look at the white man who is a skilled laborer. When I was a young man I worked for a man in Gallup as a carpenter's helper. He treated me as his own brother. I used his tools. Then he took his tools and gave me a list of tools I should buy and I started carpentering just from what I had seen. We have no alphabetical language.**

**We see things with our eyes and can always remember it. I urge that we help my people to progress in skilled labor as well as common labor. The hope of my people is to change our ways and means in certain directions, so they can help you someday as taxpayers. If not, as you are going now, you will be burdened the rest of your life. The hope of my people is that you will continue to help so that we will be all over the United States and have a hand with you, and give us a brotherly hand so we will be happy as you are. Our reservation is awful small. We did not know the capacity of the range until the white man come and say "you raise too much sheep, got to go somewhere else," resulting in reduction to a skeleton where the Indians can't make a living on it. For eighty years we have been confused by the general public, and what is the condition of the Navajo today? Starvation! We are starving for education. Education is the main thing and the only thing that is going to make us able to compete with you great men here talking to us.**

**By eye selection**

1. Circle ten words in Jim Becenti's speech which are a representative sample of the length of words in the entire text. Describe your method for selecting this sample.
2. Fill in the table below with the length of each word (number of letters/digits in the word) selected in question 1:

Observation	Length
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

3. Calculate the mean (average) word length in your selected sample. Is this value a parameter or a statistic?
4. Report your mean word length to your instructor. Your instructor will guide the class in creating a visualization of the distribution of results generated by your class. Draw a picture of the plot here. Include a descriptive  $x$ -axis label.
5. Based on the plot of sample mean word lengths in question 4, what is your best guess for the average word length of the population of all 359 words in the speech?

6. The true mean word length of the population of all 359 words in the speech is 3.95 letters. Is this value a parameter or a statistic?

Where does the value of 3.95 fall in the plot created in question 4? Near the center of the distribution? In the tails of the distribution?

7. If the class samples were truly representative of the population of words, what proportion of sample means would you expect to be below 3.95?
8. Using the graph created in question 4, estimate the proportion of students' computed sample means that were lower than the true mean of 3.95 letters?
9. Based on your answers to questions 7 and 8, would you say the sampling method used by the class is biased or unbiased? Justify your answer.
10. If the sampling method is biased, what type of sampling bias (selection, response, non-response) is present? What is the direction of the bias, i.e., does the method tend to overestimate or underestimate the population mean word length?
11. Should we use results from our by eye samples to make a statement about the word length in the population of words in Becenti's address? Why or why not?

#### 2.2.4 Take-home messages

1. When we use a biased method of sample selection, the method will tend to overestimate or underestimate the parameter.
2. To see if a method is biased, we compare the distribution of the estimates to the true value. We want our estimate to be on target or unbiased. When using unbiased methods of selection, the mean of the distribution matches or is very similar to our true parameter.
3. If the sampling method is biased, inferences made about the population based on a sample estimate will not be valid.



### **2.2.5 Additional notes**

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## 2.3 Activity 2B: American Indian Address (continued)

### 2.3.1 Learning outcomes

- Explain the purpose of random selection and its effect on scope of inference.
- Select a simple random sample from a finite population using a random number generator.
- Explain why a sampling method is unbiased or biased.
- Explain the effect of sample size on sampling variability.

### 2.3.2 Terminology review

In today's activity, we will examine unbiased and biased methods of sampling. Some terms covered in this activity are:

- Random sample
- Unbiased vs biased methods of selection
- Generalization

To review these concepts, see Section 2.1 in the textbook.

#### Random selection

Today we will return to the American Indian Address introduced in Activity 2A. Suppose instead of attempting to select a representative sample by eye (which did not work), each student used a random number generator to select a simple random sample of 10 words. A **simple random sample** relies on a random mechanism to choose a sample, without replacement, from the population, such that every sample of size 10 is equally likely to be chosen.

To use a random number generator to select a simple random sample, you first need a numbered list of all the words in the population, called a **sampling frame**. You can then generate 10 random numbers from the numbers 1 to 359 (the number of words in the population), and the chosen random numbers correspond to the chosen words in your sample.

1. Use the random number generator at <https://istats.shinyapps.io/RandomNumbers/> to select a simple random sample from the population of all 359 words in the speech.
- Set “Choose Minimum” to 1 and “Choose Maximum” to 359 to represent the 359 words in the population (the sampling frame).
  - Set “How many numbers do you want to generate?” to 10 and ensure the “No” option is selected under “Sample with Replacement?”
  - Click “Generate”.

Fill in the table below with the random numbers selected and use the **Becenti.csv data file** found on D2L to determine each number's corresponding word and word length (number of letters/digits in the word):

Observation	Number	Length
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

- Calculate the mean word length in your selected sample in question 1. Is this value a parameter or a statistic?
- Report your mean word length to your instructor. Your instructor will guide the class in creating a visualization of the distribution of results generated by your class. Draw a picture of the plot here. Include a descriptive  $x$ -axis label.
- Where does the value 3.95, the true mean word length, fall in the distribution created in question 3? Near the center of the distribution? In the tails of the distribution?

One set of randomly generated sample mean word lengths from a single class may not be large enough to visualize the distribution results. Let's have a computer generate 1,000 sample mean word lengths for us.

- Navigate to the “One Variable with Sampling” Rossman/Chance web applet: <http://www.rossmanchance.com/applets/2021/sampling/OneSample.html?population=gettysburg>.
  - Click “Clear” below the text box containing data from the Gettysburg address to delete that data set.
  - Download the Becenti.csv file from D2L and open the spreadsheet on your computer.
  - Copy and paste the population of word lengths (column C) into the applet from the data set provided making sure to include the header. Click “Use Data”. Verify that the mean for the data set is 3.953 with a sample size of 359. If these are not the values you got, check with your instructor for help with copying in the data set correctly.
  - Click the check-box for “Show Sampling Options”
  - Select 1000 for “Number of samples” and select 10 for the “Sample size”.
  - Click “Draw Samples”.
5. The plot labeled “Statistics” displays the 1,000 randomly generated sample mean word lengths. Sketch this plot below. Include a descriptive  $x$ -axis label and be sure to write down the provided mean and SD (standard deviation) of the distribution.

6. What is the center value of the distribution created in question 5?
7. Explain why the sampling method of using a random number generator to generate a sample is a “better” method than choosing 10 words “by eye”.
8. Is random selection an unbiased method of selection? Explain your answer. Be sure to reference your plot from question 5.

## Effect of sample size

We will now consider the impact of sample size.

9. First, consider if each student had selected 20 words, instead of 10, by eye. Do you think this would make the plot from question 4 in Activity 2A centered on 3.95 (the true mean word length)? Explain your answer.

10. Now we will select 20 words instead of 10 words at random.

- In the “One Variable with Sampling” Rossman/Chance web applet([http://www.rossmanchance.com/applets/2021/sampling/OneSample.html?population=gettysburg.](http://www.rossmanchance.com/applets/2021/sampling/OneSample.html?population=gettysburg)), change the Sample size to 20.
- Click “Draw Samples”.

The plot labeled “Statistics” displays the 1,000 randomly generated sample mean word lengths. Sketch this plot below. Include a descriptive  $x$ -axis label and be sure to write down the provided mean and SD (standard deviation) of the distribution.

11. Compare the distribution created in question 10 to the one created in question 5.

Which features are similar?

Which features differ?

12. Compare the spreads of the plots in question 10 and in question 5. You should see that in one plot all sample means are closer to the population mean than in the other. Which plot shows this?

13. Using the evidence from your simulations, answer the following questions:

Does changing the sample size impact whether the sample estimates are unbiased? Explain your answer.

Does changing the sample size impact the variability (spread) of sample estimates? Explain your answer

14. What is the purpose of random selection of a sample from the population?

### 2.3.3 Take-home messages

1. Random selection is an unbiased method of selection.
2. To determine if a sampling method is biased or unbiased, we compare the distribution of the estimates to the true value. We want our estimate to be on target or unbiased. When using unbiased methods of selection, the mean of the distribution matches or is very similar to our true parameter.
3. Random selection eliminates selection bias. However, random selection will not eliminate response or non-response bias.
4. The larger the sample size, the more similar (less variable) the statistics will be from different samples.
5. Sample size has no impact on whether a *sampling method* is biased or not. Taking a larger sample using a biased method will still result in a sample that is not representative of the population.

### 2.3.4 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## 2.4 Week 2 Lab: Study Design

### 2.4.1 Learning outcomes

- Explain the purpose of random assignment and its effect on scope of inference.
- Identify whether a study design is observational or an experiment.
- Identify confounding variables in observational studies and explain why they are confounding.

### 2.4.2 Terminology review

In this activity, we will examine different study designs, confounding variables, and how to determine the scope of inference for a study. Some terms covered in this activity are:

- Scope of inference
- Explanatory variable
- Response variable
- Confounding variable
- Experiment
- Observational study

To review these concepts, see Sections 2.2 through 2.5 in the textbook.

### 2.4.3 General information labs

Remember that each Friday you will complete a lab. Questions are selected from each lab to be turned in on Gradescope. The questions to be submitted on Gradescope are bolded in the lab. As you work through the lab have the Gradescope lab assignment open so that you can answer those questions as you go.

#### **Atrial fibrillation**

Atrial fibrillation is an irregular and often elevated heart rate. In some people, atrial fibrillation will come and go on its own, but others will experience this condition on a permanent basis. When atrial fibrillation is constant, medications are required to stabilize the patient's heart rate and to help prevent blood clots from forming. Pharmaceutical scientists at a large pharmaceutical company believe they have developed a new medication that effectively stabilizes heart rates in people with permanent atrial fibrillation. They set out to conduct a trial study to investigate the new drug. The scientists will need to compare the proportion of patients whose heart rate is stabilized between two groups of subjects, one of whom is given a placebo and the other given the new medication.

1. Identify the explanatory and response variable in this trial study.

Explanatory variable:

Response variable:

Suppose 24 subjects with permanent atrial fibrillation have volunteered to participate in this study. There are 16 subjects that self-identified as male and 8 subjects that self-identified as female.

2. One way to separate into two groups would be give all the males the placebo and all the females the new drug. Explain why this is not a reasonable strategy.
  
3. Could the scientists fix the problem with the strategy presented in question 2 by creating equal sized groups by putting 4 males and 8 females into the drug group and the remaining 12 males in the placebo group? Explain your answer.
  
4. A third strategy would be to **block** on sex. In this type of study, the scientists would assign 4 females and 8 males to each group. Using this strategy, what **proportion** of males out of the 12 individuals would be in each group?
  
5. **Assume the scientists used the strategy in question 4, but they put the four tallest females and eight tallest males into the placebo group and the remaining subjects into the control group. They found that the proportion of patients whose heart rate stabilized is higher in the drug group than the placebo group.**

Could that difference be due to the sex of the subjects? Explain your answer.

Could it be due to other variables? Explain your answer.

While the strategy presented in question 5 controlled for the sex of the subject, there are more potential **confounding variables** in the study. A confounding variable is a variable that is *both*

1. associated with the explanatory variable, *and*
2. associated with the response variable.

When both these conditions are met, if we observe an association between the explanatory variable and the response variable in the data, we cannot be sure if this association is due to the explanatory variable or the confounding variable—the explanatory and confounding variables are “confounded.”

**Random assignment** means that subjects in a study have an equally likely chance of receiving any of the available treatments.



6. You will now investigate how randomly assigning subjects impacts a study's scope of inference.

- Navigate to the “Randomizing Subjects” applet under the “Other Applets” heading at: <http://www.rossmanchance.com/ISIapplets.html>. This applet lists the sex and height of each of the 24 subjects. Click “Show Graphs” to see a bar chart showing the sex of each subject. Currently, the applet is showing the strategy outlined in question 7.
- Click “Randomize”.

In this random assignment, what proportion of males are in group 1 (the placebo group)?

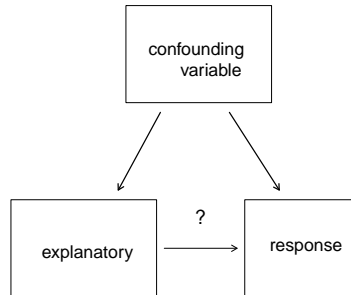
What proportion of males are in group 2 (the drug group)?

What is the difference in proportion of males between the two groups (placebo - drug)?

7. Notice the difference in the two proportions is shown as a dot in the plot at the bottom of the web page. Un-check the box for Animate above “Randomize” and click “Randomize” again. Did you get the same difference in proportion of males between the placebo and drug groups?
8. Change “Replications” to 998 (for 1000 total). Click “Randomize” again. Sketch the plot of the distribution of difference in proportions from each of the 1000 random assignments here. Be sure to include a descriptive  $x$ -axis label.

9. Does random assignment *always* balance the placebo and drug groups based on the sex of the participants? Does random assignment *tend* to make the placebo and drug groups *roughly* the same with respect to the distribution of sex? Use your plot from question 8 to justify your answers.
10. Change the drop-down menu below Group 2 from “sex” to “height”. The applet now calculates the average height in the placebo and drug groups for each of the 1000 random assignments. The dot plot displays the distribution of the difference in mean heights (placebo - drug) for each random assignment. Based on this dot plot, is height distributed equally, on average, between the two groups? Explain how you know.


The diagram below summarizes these ideas about confounding variables and random assignment. When a confounding variable is present (such as sex or height), and an association is found in a study, it is impossible to discern what caused the change in the response variable. Is the change the result of the explanatory variable or the confounding variable? However, if all confounding variables are *balanced* across the treatment groups, then only the explanatory variable differs between the groups and thus *must have caused* the change seen in the response variable.




11. **What is the purpose of random assignment of the subjects in a study to the explanatory variable groups?**
  
12. Suppose in this study on atrial fibrillation, the scientists did randomly assign groups and found that the drug group has a higher proportion of subjects whose heart rates stabilized than the placebo group. Can the scientists conclude the new drug *caused* the increased chance of stabilization? Explain your answer.
  
13. Is the sample of subjects a simple random sample or a convenience sample?
  
14. **Both the sampling method (which we covered last week) and the study design will help to determine the *scope of inference* for a study: To *whom* can we generalize, and can we conclude *causation or only association*? Use the table below to determine the scope of inference of this trial study described in question 12.**

*Scope of Inference:* If evidence of an association is found in our sample, what can be concluded?

	Study Type		
Selection of cases	Randomized experiment	Observational study	
Random sample (and no other sampling bias)	Causal relationship, and can generalize results to population.	Cannot conclude causal relationship, but can generalize results to population.	→ Inferences to population can be made
No random sample (or other sampling bias)	Causal relationship, but cannot generalize results to a population.	Cannot conclude causal relationship, and cannot generalize results to a population.	→ Can only generalize to those similar to the sample due to potential sampling bias



Can draw cause-and-effect conclusions



Can only discuss association  
due to potential confounding  
variables

## Study design

The two main study designs we will cover are **observational studies** and **experiments**. In observational studies, researchers have no influence over which subjects are in each group being compared (though they can control other variables in the study). An experiment is defined by assignment of the treatment groups of the *explanatory variable*, typically via random assignment. In today's activity we will discover the purpose behind random assignment.

For the next exercises identify the study design (observational study or experiment), the sampling method, and the scope of inference.

15. The pharmaceutical company Moderna Therapeutics, working in conjunction with the National Institutes of Health, conducted Phase 3 clinical trials of a vaccine for COVID-19 last fall. US clinical research sites enrolled 30,000 volunteers without COVID-19 to participate. Participants were randomly assigned to receive either the candidate vaccine or a saline placebo. They were then followed to assess whether or not they developed COVID-19. The trial was double-blind, so neither the investigators nor the participants knew who was assigned to which group.

Study design:

Sampling method:

Scope of inference:

16. In another study, a local health department randomly selected 1000 US adults without COVID-19 to participate in a health survey. Each participant was assessed at the beginning of the study and then followed for one year. They were interested to see which participants elected to receive a vaccination for COVID-19 and whether any participants developed COVID-19.

Study design:

Sampling method:

Scope of inference:

#### **2.4.4 Take-home messages**

1. The study design (observational study vs, experiment) determines if we can draw causal inferences or not. If an association is detected, a randomized experiment allows us to conclude that there is a causal (cause-and-effect) relationship between the explanatory and response variable. Observational studies have potential confounding variables within the study that prevent us from inferring a causal relationship between the variables studied.
2. Confounding variables are variables not included in the study that are related to both the explanatory and the response variables. When there are potential confounding variables in the study we cannot draw causal inferences.
3. Random assignment balances confounding variables across treatment groups. This eliminates any possible confounding variables by breaking the connections between the explanatory variable and the potential confounding variables.
4. Observational studies will always carry the possibility of confounding variables. Randomized experiments, which use random assignment, will have no confounding variables.

### **2.4.5 Additional notes**

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## Exploring Categorical and Quantitative Data

---

### 3.1 Week 3 Reading Guide: Introduction to R, Categorical Variables, and a Single Quantitative Variable

#### Chapter 3 (Applications: Data)

##### Videos

- [Starting\\_with\\_R](#)

##### Notes

R is case sensitive, meaning it reads `data` differently from `Data`. If you get an error message, check that your capitalization is correct.

R does not like spaces or special characters. This means the column and row headers in the data set should not have spaces, periods, commas, etc. Instead of titling the variable `column header`, use `column_header` or `ColumnHeader`.

**Tidy data:** Data frames should have

1 row per \_\_\_\_\_,

1 column per \_\_\_\_\_.

We highly recommend completing the R/RStudio tutorials in section 3 to help understand R better.

We will not expect you to be able to write full code independently for this course. For Stat 216, you will need to understand types (categorical or quantitative) and roles (explanatory or response) of variables, as well as the structure of data, in order to fill in a few blanks in provided code to graph or analyze data.

##### Functions

State what these introductory functions do in R:

```
glimpse(data_set_name)
```

```
head(data_set_name)
```

```
data_set_name$variable_name
```

```
<-
```

```
%>%
```

## Chapter 4 (Exploring categorical data)

### Videos

- 4.1
- 4.2
- 4.4

### Vocabulary

Frequency table:

Relative frequency table:

Contingency or two-way table:

Association (between two variables):

Unconditional proportion:

Conditional proportion:

Row proportions:

Column proportions:

Statistic:

Sample proportion:

Notation:

Parameter:

Population proportion:

Notation:

Bar plot:

Segmented bar plot:

Mosaic plot:

Simpson's Paradox:

## Notes

In a contingency table, which variable (explanatory or response) generally will make the columns of the table? Which variable will make the rows of the table?

In a segmented bar plot, the bars represent the levels of which variable? The segments represent the levels of which variable?

What type of plot(s) are appropriate to display a single categorical variable?

What type of plot(s) are appropriate to display two categorical variables?

What is the difference between a standardized segmented bar plot and a mosaic plot?

True or false: Pie charts are generally highly recommended ways to graphically display categorical data.

True or false: Two categorical variables are associated if the conditional proportions of a particular outcome (typically of the response variable) differ across levels of the other variable (typically the explanatory variable).

True or false: When a segmented bar plot has segments that sum to 1 (or 100%), the segment heights correspond to the proportions conditioned on the **segment**.

## Review of Simpson's Paradox

Based on the segmented bar plot in Figure 4.6, which race of defendant was more likely to have the death penalty invoked?

Based on the segmented bar plot in Figure 4.7 and Table 4.9, which race of defendant was more likely to have the death penalty invoked when the victim was Caucasian?

Based on the segmented bar plot in Figure 4.7 and Table 4.9, which race of defendant was more likely to have the death penalty invoked when the victim was African American?

The direction of the relationship between the \_\_\_\_\_ and \_\_\_\_\_ variables is **reversed** when accounting for a \_\_\_\_\_ variable.



## Chapter 5 (Exploring quantitative data)

### Videos

- 5.2to5.4
- 5.5to5.6
- 5.7

### Type of Plots

Scatterplot:

Dot plot:

Histogram:

Density plot:

Box plot:

### Vocabulary

Four characteristics of a scatterplot:

Form:

Strength:

Direction:

Unusual observations or outliers:

Distribution (of a variable):

Four characteristics of the distribution of one quantitative variable:

Center:

Variability:

Shape:

Outliers:

Point estimate:

Histogram:

Data density:

Tail:

Skew:

Symmetric:

Modality:

Density plot:

Deviation:

Variance:

Standard deviation:

Boxplot:

Five number summary:

Median:

$X^{th}$  percentile:

Interquartile range (IQR):

Robust statistics:

## Notes

What type of plot(s) are appropriate for displaying one quantitative variable?

What type of plot(s) are appropriate for displaying two quantitative variables?

What type of plot(s) are appropriate for displaying one quantitative variable and one categorical variable?

What are the two ways to measure the ‘center’ of a distribution? Which one is considered robust to skew/outliers?

What are the three ways to measure the ‘variability’ of a distribution? Which one is considered robust to skew/outliers?

How are variance and standard deviation related?

Fill in the following table with the appropriate notation.

Summary Measure	Parameter	Statistic
Mean		
Variance		
Standard deviation		

How are outliers denoted on a box plot? How can you mathematically determine if a data set has outliers?

### 3.1.1 Summarizing Chapters 4 and 5

Look at the table of vocabulary terms in the final section of each chapter. If there are any you do not know, be sure to review the appropriate section of your text.

#### Notes

Statistics summarize \_\_\_\_\_ .

Parameters summarize \_\_\_\_\_.

Fill in the following table with the appropriate notation for each summary measure.

Summary measure	Statistic	Parameter
Sample size		
Proportion (used to summarize one categorical variable)		
Mean (used to summarize one quantitative variable)		
Correlation (used to summarize two quantitative variables)		
Regression line slope (used to summarize two quantitative variables)		

### Data visualization summary

Fill in the following table to help associate type of plot for each of several scenarios.

	Appropriate plot(s)
One categorical variable (categorical response, no explanatory)	
One quantitative variable (quantitative response, no explanatory)	
Two categorical variables (categorical response, categorical explanatory)	
One of each (quantitative response, categorical explanatory)	
Two quantitative variables (quantitative response, quantitative explanatory)	

## 3.2 Activity 3A: Graphing Categorical Variables

### 3.2.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set or research question involving categorical variables.
- Plots for a single categorical variable: bar plot.
- Plots for association between two categorical variables: segmented bar plot, mosaic plot.

### 3.2.2 Terminology review

In today's activity, we will review summary measures and plots for categorical variables. Some terms covered in this activity are:

- Proportions
- Bar plots
- Segmented bar plots
- Mosaic plots

To review these concepts, see Chapter 4 in the textbook.

### 3.2.3 Graphing categorical variables

For today's activity we will begin to use the statistical package R to analyze data through the IDE (integrated development environment) RStudio. For almost all activities and labs it will be necessary to upload the provided R script file from D2L for that day. Follow these steps to upload the necessary R script file for today's activity:

- Download the Myopia Activity R script file from D2L.
- Click "Upload" in the "Files" tab in the bottom right window of RStudio. In the pop-up window, click "Choose File", and navigate to the folder where the Myopia Activity R script file is saved (most likely in your downloads folder). Click "Open"; then click "Ok".
- You should see the uploaded file appear in the list of files in the bottom right window. Click on the file name to open the file in the Editor window (upper left window).

Notice that the first three lines of code contain a prompt called, `library`. Packages needed to run functions in R are stored in directories called libraries. When using the MSU RStudio server, all the packages needed for the class are already installed. We simply must tell R which packages we need for each R script file. We use the prompt `library` to load each **package** (or library) needed for each activity. Note, these `library` lines **MUST** be run each time you open a R script file in order for the functions in R to work. Before class today you should have worked through an R tutorial to prepare for class and to make sure you can login to the RStudio server. This tutorial will be a great resource as you begin to use R.

Highlight and run lines 1–3 to load the packages needed for today's activity. Notice the use of the `#` symbol in the R script file. The `#` sign is not part of the R code. It is used by these authors to add comments to the R code and explain what each call is telling the program to do. R will ignore everything after a `#` sign when executing the code. Refer to the instructions following the `#` sign to understand what you need to enter in the code.

## Nightlight use and myopia

In a study reported in Nature (Quinn et al. 1999), a survey of 479 children found that those who had slept with a nightlight or in a fully lit room before the age of two had a higher incidence of nearsightedness (myopia) later in childhood.

In this study, there are two variables studied: **Light**: level of light in room at night (no light, nightlight, full light) and **Sight**: level of myopia developed later in childhood (high myopia, myopia, no myopia).

1. Which variable is the explanatory variable? Which is the response variable?

An important part of understanding data is to create visual pictures of what the data represent. In this activity, we will create graphical representations of categorical data.

### R code

Throughout these activities, we will often include the R code you would use in order to produce output or plots. These “code chunks” appear in gray. In the code chunk below, we demonstrate how to read the data set into R using the `read.csv()` function. The line of code shown below (line 6 in the R script file) reads in the data set and names the data set `myopia`. Highlight and run line 6 in the R script file to load the data from the Stat 216 webpage.

```
# This will read in the data set
myopia <- read.csv("https://math.montana.edu/courses/s216/data/ChildrenLightSight.csv")
```

2. Click on the data set name (`myopia`) in the Environment tab (upper right window). This will open the data set in a 2nd tab in the Editor window (upper left window). R is case sensitive, which means that you must always type the name of a variable EXACTLY as it is written in the data set including upper and lower case letters and without misspellings! Write down the name of each variable (column names) as it is written in the data set.

### Displaying a single categorical variable

If we wanted to know how many children in our data set were in each level of myopia, we could create a frequency bar plot of the variable **Sight**. In the R script file, enter the variable name, **Sight** (*note the capital S*), for **variable** into the `ggplot` code at line 10. Highlight and run lines 9–15 to create the plot. Note: this is a **frequency** bar plot plotting counts (the number of children in each level of sight is displayed on the *y*-axis).

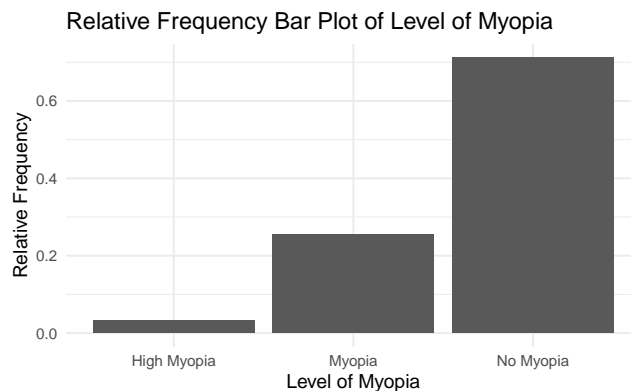
```
myopia %>% # Data set piped into...
ggplot(aes(x = variable)) + # This specifies the variable
  geom_bar(stat = "count") + # Tell it to make a bar plot
  labs(title = "Frequency Bar Plot of Level of Myopia", # Give your plot a title
       x = "Level of Myopia", # Label the x axis
       y = "Frequency") # Label the y axis
```

3. Sketch the bar chart created below. Be sure to label the axes.

4. Using the bar chart created, estimate how many children have some level of myopia.

We could also choose to display the data as a proportion in a **relative frequency** bar plot. To find the relative frequency, divide the count in each level of myopia by the sample size. These are sample proportions. Notice that in this code we told R to create a bar plot with proportions.

```
myopia %>% # Data set piped into...
  ggplot(aes(x = Sight)) + # This specifies the variable
  geom_bar(aes(y = ..prop.., group = 1)) + # Tell it to make a bar plot with proportions
  labs(title = "Relative Frequency Bar Plot of Level of Myopia", # Give your plot a title
       x = "Level of Myopia", # Label the x axis
       y = "Relative Frequency") # Label the y axis
```



5. Which features in the relative frequency bar plot are the same as the frequency bar plot? Which are different?

## Displaying two categorical variables

Is there an association between the level of light in a room and the development of myopia? To examine the differences in level of myopia for the level of light, we would create a segmented bar plot of **Light** segmented by **Sight**. To create the segmented bar plot enter the variable name, **Light** for **explanatory** and the variable name, **Sight** for **response** in the R script file in line 27. Highlight and run lines 26–33.

```
myopia %>% # Data set piped into...
ggplot(aes(x = explanatory, fill = response)) + # This specifies the variables
  geom_bar(stat = "count", position = "fill") + # Tell it to make a stacked bar plot
  labs(title = "Segmented Bar Plot of Night Light Use by Level of Myopia",
        # Make sure to title your plot
        x = "Level of Light", # Label the x axis
        y = "") + # Remove y axis label
  scale_fill_grey() # Make figure black and white
```

6. Sketch the segmented bar plot created here. Be sure to label the axes.

7. From the segmented bar plot, estimate the proportion of no myopia for those that used a nightlight.

8. Which level of light has the highest proportion of No Myopia?

We could also plot the data using a mosaic plot. Fill in the variable name, **Light** for **explanatory** and the variable name, **Sight** for **response** in line 38 in the R script file. Highlight and run lines 36–43.

```
myopia %>% # Data set piped into...
ggplot() + # This specifies the variables
geom_mosaic(aes(x=product(explanatory), fill = response)) + # Tell it to make a mosaic plot
  labs(title = "Mosaic Plot of Night Light Use by Level of Myopia",
        # Make sure to title your plot
        x = "Level of Light", # Label the x axis
        y = "") + # Remove y axis label
  scale_fill_grey() # Make figure black and white
```



9. What is similar and what is different between the segmented bar chart and the mosaic bar chart?

10. Explain why the bar for `Nightlight` is the widest in the mosaic plot.

Fill in the name of the explanatory variable and the response variable in line 46 in the R script file, highlight and run line 46 to get the counts for each combination of levels of variables.

```
myopia %>% group_by(explanatory) %>% count(response)
```

11. Fill in the following table with the values from the R output.

	Light Level			
Myopia Level	Full Light	Nightlight	No Light	Total
High Myopia				
Myopia				
No Myopia				
Total				

In the following questions, use the table to calculate the described proportions. Notation is important for each calculation. Since this is sample data it is appropriate to use statistic notation for the proportion,  $\hat{p}$ . When calculating a proportion dependent on a single level of a variable subscripts are needed when reporting the notation.

12. Calculate the proportion of children with high myopia. Use appropriate notation.

13. Calculate the proportion of children with high myopia among those that slept with full light. Use appropriate notation.

14. Calculate the proportion of children with high myopia among those that slept with no light. Use appropriate notation.

15. Calculate the difference in proportion of children with high myopia for those that slept with full light minus those who slept with no light. Give the appropriate notation. Use full light minus no light as the order of subtraction.

### **3.2.4 Take-home messages**

1. Bar charts can be used to graphically display a single categorical variable either as counts or proportions. Segmented bar charts and mosaic plots are used to display two categorical variables.
2. Segmented bar charts always have a scale from 0 - 100%. The bars represent the outcomes of the explanatory variable. Each bar is segmented by the response variable. If the heights of each segment are the same for each bar there is no association between variables.
3. Mosaic plots are similar to segmented bar charts but the widths of the bars also show the number of observations within each outcome.

### **3.2.5 Additional notes**

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## 3.3 Activity 3B: IMDb Movie Reviews — Displaying Quantitative Variables

### 3.3.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set or research question for quantitative data.
- Interpret the following summary statistics in context: median, lower quartile, upper quartile, standard deviation, interquartile range.

### 3.3.2 Terminology review

In today’s activity, we will review summary measures and plots for quantitative variables. Some terms covered in this activity are:

- Two measures of center: mean, median
- Two measures of spread (variability): standard deviation, interquartile range (IQR)
- Types of graphs: box plots, dot plots, histograms
- Identify and create appropriate summary statistics and plots given a data set or research question for a single categorical and a single quantitative variable.
- Interpret the following summary statistics in context: median, lower quartile, upper quartile, standard deviation, interquartile range.
- Given a plot or set of plots, describe and compare the distribution(s) of a single quantitative variable (center, variability, shape, outliers).

To review these concepts, see Section 2.3 in the textbook.

### 3.3.3 Movies released in 2016

A data set was collected on movies released in 2016 (“IMDb Movies Extensive Dataset” 2016). Here is a list of some of the variables collected on the observational units, movies released in 2016.

Variable	Description
budget_mil	Amount of money (in US \$ millions) budgeted for the production of the movie
revenue_mil	Amount of money (in US \$ millions) the movie made after release
duration	Length of the movie (in minutes)
content_rating	Rating of the movie (G, PG, PG-13, R, Not Rated)
imdb_score	IMDb user rating score from 1 to 10
genres	Categories the movie falls into (e.g., Action, Drama, etc.)
facebook_likes	Number of likes a movie receives on Facebook

## Summarizing a single quantitative variable

The `favstats()` function from the `mosaic` package gives the summary statistics for a quantitative variable. Here we have the summary statistics for the variable `imdb_score`. The summary statistics give the two measures of center and two measures of spread for IMDb score. Highlight and run lines 1 – 8 in the provided R script file to load the data set. Check that the summary statistics match that printed in the coursepack.

```
# Read in data set
movies <- read.csv("https://math.montana.edu/courses/s216/data/Movies2016.csv")
movies %>% # Data set piped into...
  summarise(favstats(imdb_score)) # Apply favstats function to imdb_score
```

```
#>   min    Q1 median   Q3 max    mean      sd  n missing
#> 1  3.4  5.65    6.4  7.1  8.2  6.309783 1.086689 92      0
```

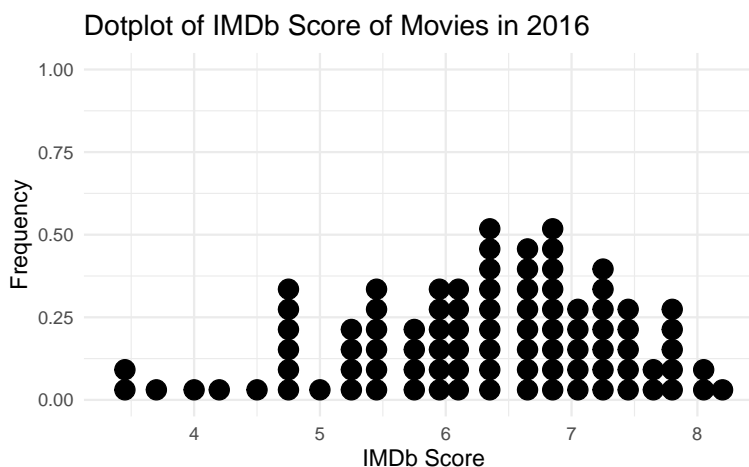
1. Give the values for the two measures of center (mean and median).
2. Report the value for quartile 1 and interpret this value in context of the problem.
3. Calculate the interquartile range ( $IQR = Q3 - Q1$ ).
4. Report the value of the standard deviation and interpret this value in context of the problem.

## Displaying a single quantitative variable

5. What are the three types of plots used to plot a single quantitative variable?

A dotplot will plot a dot for each value in the data set. The following code will create a dotplot of IMDb scores. Notice that we put in the variable name `imdb_score` for `x =` in the `ggplot` function.

```
movies %>% # Data set piped into...
ggplot(aes(x = imdb_score)) + # Name variable to plot
  geom_dotplot() + # Create dotplot
  labs(title = "Dotplot of IMDb Score of Movies in 2016", # Title for plot
       x = "IMDb Score", # Label for x axis
       y = "Frequency") # Label for y axis
```



6. What is the shape of the distribution of IMDb scores?

To create a histogram of the IMDb scores, enter the variable name, `imdb_score` in the provided R script file for `variable` at line 20, highlight and run lines 19–24. Visually, this shows us the range of IMDb scores for Movies released in 2016.

Notice that the **bin width** is 0.5. For example the first bin consists of the number of movies in the data set with an IMDb score of 3.25 to 3.75. It is important to note that a movie with a IMDb score on the boundary of a bin will fall into the bin above it; for example, 4.75 would be counted in the bin 4.75–5.25.

```
movies %>% # Data set piped into...
ggplot(aes(x = variable)) + # Name variable to plot
  geom_histogram(binwidth = 0.5) + # Create histogram with specified binwidth
  labs(title = "Histogram of IMDb Score of Movies in 2016", # Title for plot
       x = "IMDb Score", # Label for x axis
       y = "Frequency") # Label for y axis
```

7. Sketch the histogram created here.

8. Which range of IMDb scores have the highest frequency?

To create a boxplot of the IMDb scores, enter the variable name, `imdb_score` in the provided R script file for `variable` at line 28, highlight and run lines 27–32. Visually, this shows us the range of IMDb scores for Movies released in 2016.

```
movieS %>% # Data set piped into...
ggplot(aes(x = variable)) + # Name variable to plot
  geom_boxplot() + # Create boxplot
  labs(title = "Boxplot of IMDb Score of Movies in 2016", # Title for plot
        x = "IMDb Score", # Label for x axis
        y = "Frequency") # Label for y axis
```

9. Sketch the boxplot created and identify the values of the 5-number summary (minimum value, first quartile ( $Q_1$ ), median, third quartile ( $Q_3$ ), maximum value) on the plot. Use the following formulas to find the invisible fence on both ends of the distribution. Draw a dotted line at the invisible fence to show how the outliers were found.

$$\text{Lower Fence: values} \leq Q_1 - 1.5 \times IQR$$

$$\text{Upper Fence: values} \geq Q_3 + 1.5 \times IQR$$

10. Compare the three graphs of IMDb scores created above.

Which graph is best used to show the shape of the distribution?

Which graph is best used to show the outliers of the distribution?

## Summary statistics for a single categorical and single quantitative Variable

Is there an association between content rating and budget for movies in 2016? To use the `favstats()` function in the `mosaic` package with two variables, we will enter the variables as a formula, response-explanatory. This function will give the summary statistics for budget for each content rating. Highlight and run lines 35–37 in the provided R script file and check that the summary statistics match those provided in the coursepack.

```
movies %>% # Data set piped into...
  filter(content_rating != "Not Rated") %>% # Remove Not Rated movies
  summarise(favstats(budget_mil~content_rating)) # Find the summary measures for each content rating
```

```
#>   content_rating min      Q1 median      Q3 max      mean      sd  n missing
#> 1             PG 0.5 11.00   74.0 151.250 175 86.54167 71.52795 12        0
#> 2          PG-13 0.0 17.25   33.5 138.750 250 74.17500 74.15190 46        0
#> 3              R 0.0  7.75   19.5  29.625  60 21.09375 16.99926 32        0
```

11. Which content rating has the largest IQR?
12. Report the mean budget amount for the PG rating. Use appropriate notation.
13. Report the mean budget amount for the R rating. Use appropriate notation.
14. Calculate the difference in mean budget amount for movies in 2016 with a PG rating minus those with a R rating. Use appropriate notation with informative subscripts.

## Displaying a single categorical and single quantitative variable

The boxplot of movie budgets (in millions) by content rating is plotted using the code below. Enter the variable `budget_mil` for `response` and the variable `content_rating` for explanatory at line 42, highlight and run code lines 40–46. This plot compares the budget for different levels of content rating.

```
movies %>% # Data set piped into...
  filter(content_rating != "Not Rated") %>% # Remove Not Rated movies
  ggplot(aes(y = response, x = explanatory))+ # Identify variables
  geom_boxplot()+ # Tell it to make a box plot
  labs(title = "Side by side box plot of budget by content rating", # Title
       x = "Content Rating", # x-axis label
       y = "Budget (in Millions)") # y-axis label
```

15. Sketch the box plots created using the R code.

16. Answer the following questions about the box plots created.

- Which content rating has the highest center?
- Which content rating has the largest spread?
- Which content rating has the most skewed distribution?
- Fifty percent of movies in 2016 with a PG-13 content rating fall below what value? What is the name of this value?

17. Which variable is the explanatory variable? Response variable?



### 3.3.4 Take-home messages

1. Histograms, box plots, and dot plots can all be used to graphically display a single quantitative variable.
2. The box plot is created using the five number summary: minimum value, quartile 1, median, quartile 3, and maximum value. Values in the data set that are less than  $Q_1 - 1.5 \times IQR$  and greater than  $Q_3 + 1.5 \times IQR$  are considered outliers and are graphically represented by a dot outside of the whiskers on the box plot.
3. Data should be summarized numerically and displayed graphically to give us information about the study.
4. When comparing distributions of quantitative variables we look at the shape, center, spread, and for outliers. There are two measures of center: mean and the median and two measures of spread: standard deviation and the interquartile range,  $IQR = Q_3 - Q_1$ .

### 3.3.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## 3.4 Week 3 Lab: IPEDs

### 3.4.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set or research question for quantitative data.
- Interpret the following summary statistics in context: median, lower quartile, upper quartile, standard deviation, interquartile range.

### 3.4.2 Terminology review

In today's lab, we will review summary measures and plots for quantitative variables. Some terms covered in this activity are:

- Two measures of center: mean, median
- Two measures of spread (variability): standard deviation, interquartile range (*IQR*)
- Types of graphs: box plots, dot plots, histograms
- Identify and create appropriate summary statistics and plots given a data set or research question for a single categorical and a single quantitative variable.
- Interpret the following summary statistics in context: median, lower quartile, upper quartile, standard deviation, interquartile range.
- Given a plot or set of plots, describe and compare the distribution(s) of a single quantitative variable (center, variability, shape, outliers).

To review these concepts, see Chapter 5 in the textbook.

### 3.4.3 The Integrated Postsecondary Education Data System (IPEDS)

Upload and open the provided R script file for the week 3 lab to answer the following questions. **Remember bolded questions will be answered on Gradescope for your group.**

These data are on a subset of institutions that met the following selection criteria (Education Statistics 2018):

- Degree granting
- United States only
- Title IV participating
- Not for profit
- 2-year or 4-year or above
- Has full-time first-time undergraduates
- Note that several variables have missing values for some institutions (denoted by “NA”).

Variable Name	Description
UnitID	Unique institution identifier
Name	Institution name
State	State abbreviation
Control	<ul style="list-style-type: none"> <li>Public</li> <li>Private</li> </ul>
Sector	<ul style="list-style-type: none"> <li>Public 2-year</li> <li>Private 2-year</li> <li>Public 4-year or higher</li> <li>Private 4-year or higher</li> </ul>
LandGrant	Is this a land-grant institution? (Yes/No)
Size	Institution size category based on total students enrolled for credit, Fall 2018: <ul style="list-style-type: none"> <li>Under 1,000</li> <li>1,000 - 4,999</li> <li>5,000 - 9,999</li> <li>10,000 - 19,999</li> <li>20,000 and above</li> </ul>
Cost_OutofState	Cost of attendance for full-time, first-time degree/certificate seeking out-of-state undergraduate students living on campus for academic year 2018-19. It includes in-out-of-state tuition and fees, books and supplies, on campus room and board, and other on campus expenses.
Cost_InState	Cost of attendance for full-time, first-time degree/certificate seeking in-state undergraduate students living on campus for academic year 2018-19. It includes in-state tuition and fees, books and supplies, on campus room and board, and other on campus expenses.
Retention	The full-time retention rate is the percent of the (fall full-time cohort from the prior year minus exclusions from the fall full-time cohort) that re-enrolled at the institution as either full- or part-time in the current year
Percent_InState	Percent of first-time degree/certificate seeking undergraduate students who reside in the same state of the institution.
Enrollment	Total number of people enrolled for credit in the fall of the academic year.
Graduation_Rate	Graduation rate of first-time, full-time degree or certificate-seeking students - 2012 cohort (4-year institutions) and 2015 cohort (less-than-4-year institutions). This rate is calculated as the total number of completers within 150% of normal time divided by the revised cohort minus any allowable exclusions.
Percent_FinancialAid	Percentage of all full-time, first-time degree/certificate-seeking undergraduate students who were awarded any financial aid.

### Summary statistics for a single quantitative variable

Look through the provided chart above showing the description of variables measured. The UnitID and Name are identifiers for each observational unit, *US degree granting institutions in 2018*.

1. Identify in the chart above which variables collected on the US institutions are categorical (C) and which variables are quantitative (Q).

In Wednesday's activity, the code was provided to import the data set needed directly from the Stat 216 website. Follow these steps to upload and import the data set for today's lab.

- Download the provided data set `IPEDS_Data_2018` from D2L
- Upload the data set `IPEDS_Data_2018` to the RStudio server using the same steps to upload the R script file.
- Click on "Import Dataset" in the Environment tab in the upper right hand corner.
- Choose "From Text(base)" in the drop-down menu and select the correct csv file.
- Be sure that "Yes" is selected next to "Heading" in the pop-up screen. Click "Import".
- To view the data set, click on the data set name (`IPEDS_Data_2018`). Verify that that column names match the first column in the chart on the previous page. If the columns are named V1, V2, V3...etc, you did not select "Yes" for "Heading".

Enter the name of the data set (see the environment tab) for `datasetname` in the R script file in line 6. We will look at the retention rates for the 4-year institutions only. Enter the variable name `Retention` for `variable` in line 12. Highlight and run lines 1 – 12. **Note that the two lines of code (lines 8 and 10) are filtering to remove the 2-year institutions so we are only assessing Public 4-year and Private 4-year institutions.** The `favstats()` function from the `mosaic` package gives the summary statistics for a quantitative variable. The summary statistics give the two measures of center and two measures of spread for retention rate.

```
IPEDS <- datasetname #Creates the object IPEDS
IPEDS <- IPEDS %>%
  filter(Sector != "Public 2-year") #Filters the data set to remove Public 2-year
IPEDS <- IPEDS %>%
  filter(Sector != "Private 2-year") #Filters the data set to remove Private 2-year
IPEDS %>%
  summarise(favstats(variable)) #Gives the summary statistics
```

2. Report the value for quartile 3 and interpret this value in context of the study.

3. Calculate the interquartile range ( $IQR = Q_3 - Q_1$ ) for this study.

4. Report and interpret the value of the standard deviation.

5. How many missing values are there? What does this indicate?

### Displaying a single quantitative variable

There are three type of plots used to plot a single quantitative variable: a dotplot, a histogram or a boxplot. A dotplot of retention rates would plot a dot for the retention rate for each 4-year institution.

We will create both a histogram and a boxplot of the variable **Retention**. Enter the name of the variable in both line 16 and line 23 for **variable** in the R script file. **Give each plot a descriptive title**. Highlight and run lines 15 – 27 to give the histogram and boxplot. Notice that the **bin width** for the histogram is 10. For example, the first bin consists of the number of 4-year institutions in the data set with a retention rate of 0 to 10%. It is important to note that a 4-year institution with a retention rate on the boundary of a bin will fall into the bin above it; for example, 10 would be counted in the bin 10–20.

### Export and upload both plots to Gradescope for your group.

- To export the graphs: in the bottom right corner in the Plots tab, click on **Export**.
- Then choose **Save as Image**. Save the image as a png. This will save your graph to the server.
- In the Files tab, click on the box next to your saved image file, click **More** and choose **Export**. This will save your file to your downloads folder on your computer.

```
IPEDS %>% # Data set piped into...
ggplot(aes(x = variable)) + # Name variable to plot
  geom_histogram(binwidth = 10) + # Create histogram with specified binwidth
  labs(title = "Title", # Title for plot
        x = "Retention Rate", # Label for x axis
        y = "Frequency") # Label for y axis
```

```
IPEDS %>% # Data set piped into...
ggplot(aes(x = variable)) + # Name variable to plot
  geom_boxplot() + # Create boxplot
  labs(title = "Title", # Title for plot
        x = "Retention Rates", # Label for x axis
        y = "Frequency") # Label for y axis
```

6. What is the shape of the distribution of retention rates?

7. Identify any outliers in the data set.

## Robust Statistics

Let's examine how the presence of outliers affect the values of center and spread.

8. Report the two measures of center (mean and median) for retention given in the R output.
9. Report the two measures of spread (standard deviation and *IQR*) for retention given in the R output.

To show the effect of outliers on the measures of center and spread, the smallest values of retention rate in the data set were increased by 30%. Highlight and run lines 30–38.

```
IPEDS %>% # Data set piped into...  
  summarise(favstats(Retention_Inc))
```

```
IPEDS %>% # Data set piped into...  
  ggplot(aes(x = Retention_Inc)) + # Name variable to plot  
  geom_boxplot() + # Create histogram with specified binwidth  
  labs(title = "Boxplot of Retention Rates for US Higher Education Institutions", # Title for plot  
        x = "Retention Rate", # Label for x axis  
        y = "Frequency") # Label for y axis
```

10. Report the two measures of center for this new data set.
11. Report the two measures of spread for this new data set.
12. Which measure of center is robust to (not affected by) outliers? Explain your answer.
13. Which measure of spread is robust to outliers? Explain your answer.

## Summarizing a single categorical and single quantitative variable

Is there a difference in retention rates for public and private 4-year institutions? In the next part of the activity we will compare retention rates for public and private 4-year institutions. Note that this variable (public or private) is labelled **Control** in the data set.

14. Based on the research question, which variable will we treat as the explanatory variable? Response variable?

Enter the name of the explanatory variable and the name of the response variable in lines 42 and 45 of the R script file. Remember that the variable name must be typed in EXACTLY as it is written in the data set. Highlight and run lines 41 – 49 to find the summary statistics and create side by side boxplots of the data.

```
IPEDS %>% # Data set piped into...
  summarise(favstats(response~explanatory)) # Summary statistics for retention rates by sector
```

```
IPEDS %>% # Data set piped into...
  ggplot(aes(y = response, x = explanatory))+ # Identify variables
  geom_boxplot()+ # Create box plot
  labs(title = "Side by side box plot of retention rates by control", # Title
       x = "Control", # x-axis label
       y = "Retention Rates") # y-axis label
```

15. Compare the two boxplots.

Which type of university has the highest center?

Largest spread?

What is the shape of each distribution?

Does either distribution have outliers?

16. Report the difference in mean retention rates for private and public universities. Use private minus public as the order of subtraction. Use the appropriate notation.

17. Does there appear to be an association between retention rates and type of university? Explain your answer.

## Summarizing two categorical variables

Are private 4-year institutions smaller than public one? The following set of code will create a segmented bar plot of size of the institution by sector. Enter the variable **Sector** for explanatory and **Size** for response in line 64. Highlight and run lines 63–69 in the R script file.

```
IPEDS %>%  
  ggplot(aes(x=explanatory, fill = response)) + # Enter the explanatory and response variables  
  geom_bar(stat = "count", position = "fill") + # Create a segmented bar plot  
  labs(title = "Segmented Bar Plot of Sector by Size", # Title  
        x = "Sector", # x-axis label  
        y = "") # remove y-axis label
```

18. Does there appear to be an association between sector and size of 4-year institutions? Explain your answer using the plot.



## Exploring Multivariable Data

---

### 4.1 Week 4 Reading Guide: Two Quantitative Variables and Multivariable Concepts

#### Section 6.1 (Fitting a line, residuals, and correlation)

##### Videos

- 6.1

##### Reminders from Section 5.1

Scatterplot: displays two quantitative variables; one dot = two measurements  $(x, y)$  on one observational unit.

Four characteristics of a scatterplot:

- *Form*: pattern of the dots plotted. Is the trend generally linear (you can fit a straight line to the data) or non-linear?
- *Strength*: how closely do the points follow a trend? Very closely (strong)? No pattern (weak)?
- *Direction*: as the  $x$  values increase, do the  $y$ -values tend to increase (positive) or decrease (negative)?
- Unusual observations or *outliers*: points that do not fit the overall pattern of the data.

##### Vocabulary

Predictor:

Residual:

Formula:

Residual plot:

Correlation:

## Notes

General equation of a linear model for a *population*:  $y = \beta_0 + \beta_1 x + \epsilon$ , where

$x$  represents

$y$  represents

$\beta_0$  represents

$\beta_1$  represents

$\epsilon$  represents

General equation of a linear regression model from *sample* data:  $\hat{y} = b_0 + b_1 x$ , where

$x$  represents

$\hat{y}$  represents

$b_0$  represents

$b_1$  represents

Fill in the following table with the appropriate notation for each summary measure.

Summary Measure	Parameter	Statistic
Correlation		
Slope		
$y$ -intercept		

Fill in the blanks below to define some of the properties of correlation:

The value of correlation must be between \_\_\_\_\_. (Includes the endpoints of the interval)

The sign of correlation gives the \_\_\_\_\_ of the linear relationship.

The magnitude of correlation gives the \_\_\_\_\_ of the linear relationship.

True or false: A scatterplot that shows random scatter would be considered non-linear.

True or false: If the correlation between two quantitative variables is equal to zero, then the two variables are not associated.

True or false: To calculate a predicted  $y$ -value from a given  $x$ -value, just look at the scatterplot and estimate the  $y$ -value.

True or false: A positive residual indicates the data point is above the regression line.

**Example: Brushtail possums**

1. What are the observational units?
2. Look at the scatterplot in Figure 6.5.
  - a) What is the explanatory variable? The response variable? What type is each?
  - b) What is the form of the scatterplot?
  - c) What is the direction of the scatterplot?
  - d) What is the strength of the scatterplot?
  - e) Are there any outliers on the scatterplot?
3. Write the equation of the regression line, in context (do not use  $x$  and  $y$ , use variable names instead).
4. Calculate the predicted head length for a possum with a 76.0 cm total length.
5. One of the possums in the data set has a total length of 76.0 cm and a head length of 85.1 mm. Calculate the residual for this possum. Does this possum lie above or below the regression line?

**Section 6.2 (Least squares regression)**

You may skip the special topic sections (6.2.7)

**Videos**

- 6.2

## Vocabulary

Least squares criterion:

Least squares line:

lm() R function: `name_of_model <- lm(response ~ explanatory, data = data_set_name)`

slope:

$y$ -intercept:

Extrapolation:

Coefficient of determination:

$s_y^2$  (or SST) represents

$s_{RES}^2$  (or SSE) represents

## Notes

Two methods for determining the best line:

1.

2.

Notation for the coefficient of determination:

Formulas for calculating the coefficient of determination:

True or false: A correlation between two quantitative variables implies a causal relationship exists between the variables.

True or false: The slope of the line tells us how much to expect the  $y$  variable to increase or decrease when the  $x$  variable increases by 1 unit.

True or false: The coefficient of determination is just the square of the correlation.

### Example: Elmhurst College

1. What are the observational units?
2. Look at the scatterplot in Figure 6.13.
  - a) What is the explanatory variable? The response variable?
  - b) What is the form of the scatterplot?
  - c) What is the direction of the scatterplot?
  - d) What is the strength of the scatterplot?
  - e) Are there any outliers on the scatterplot?
3. Write the equation of the regression line, in context (do not use  $x$  and  $y$ , use variable names instead).
4. Interpret the slope of the line, in the context of the problem. Remember that both family income and gift aid from the university are measured in \$1000s.
5. Interpret the  $y$ -intercept of the line, in the context of the problem. Remember that both family income and gift aid from the university are measured in \$1000s.
6. Is your interpretation in question 5 an example of extrapolation? Justify your answer.
7. Give and interpret, in context, the value of the coefficient of determination.

## Section 6.3 (Outliers in linear regression)

### Videos

- 6.3

## Vocabulary

Outlier:

Leverage:

Influential point:

## Notes

Investigate, but do not remove, outliers. Unless you find there was an actual error in the data collection, ignoring outliers can make models poor predictors!

True or false: All high leverage outliers are influential.

True or false: An outlier is considered high leverage if it is extreme in its  $x$ -value.

## Section 6.4 (Chapter 6 review)

Look at the table of vocabulary terms in the final section of each chapter. If there are any you do not know, be sure to review the appropriate section of your text.

## Notes

Statistics summarize:

Parameters summarize:

Determine whether each of the following statements about the correlation coefficient are true or false:

1. The correlation coefficient must be a positive number.
2. Stronger linear relationships are indicated by correlation coefficients far from 0.
3. The correlation coefficient is a robust statistic.
4. When two variables are highly correlated, that indicates a causal relationship exists between the variables.
5. The sign of the correlation coefficient will be the same as the sign of the regression line slope, though the values are typically different.

Fill in the blanks to correctly interpret:

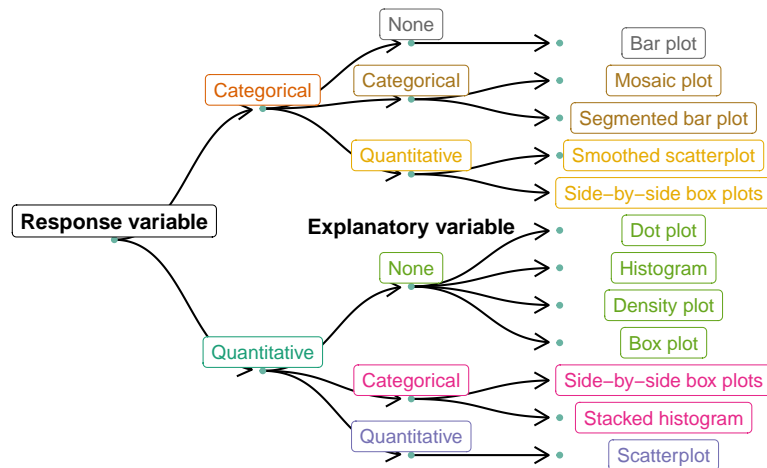
- Slope:

For every \_\_\_\_\_, we expect \_\_\_\_\_ to increase (if slope is \_\_\_\_\_) or decrease (if slope is \_\_\_\_\_) by the absolute value of the \_\_\_\_\_.

- $y$ -intercept:

If \_\_\_\_\_, we predict the \_\_\_\_\_ to equal \_\_\_\_\_.

Decision tree for determining an appropriate plot given a number of variables and their types from Chapter review:



## Section 7.1 (Gapminder world)

### Videos

- Chapter7

### Vocabulary

Interaction:

Aesthetic:

### Notes

Use color and a legend to add a third variable to a scatterplot. E.g., Color the dots to represent different levels of a categorical variable or use shading of the dots to represent different values of a quantitative variable.

If the response and one predictor are quantitative and the other predictor is categorical, we fit a regression line for each level of the categorical predictor.

- Parallel slopes would indicate that that the two predictors \_\_\_\_\_ in explaining the response.

- Non-parallel slopes would indicate that the two predictors \_\_\_\_\_ in explaining the response.

True or false: Scatterplots can only display two variables at a time.

## Section 7.2 (Simpson's Paradox, revisited)

### Videos

- Chapter7

### Reminder from Section 4.4

Simpson's Paradox: when the relationship between the explanatory and response variable is reversed when looking at the relationship within different levels of a confounding variable.

### Notes

True or false: Simpson's Paradox can only occur when the explanatory, response, and confounding variables are all categorical.

### Example: SAT scores

1. What are the observational units?
2. Look at the scatterplot in Figure 7.5.
  - a) What is the explanatory variable? The response variable?
  - b) What is the form of the scatterplot?
  - c) What is the direction of the scatterplot?
  - d) What is the strength of the scatterplot?
  - e) Are there any outliers on the scatterplot?
3. What would need to be done to the study design in order to eliminate the confounding variable: percent of eligible students taking the SAT?



4. What features of the scatterplots in Figure 7.6 demonstrate that the percent of eligible students taking the SAT is a confounding variable?
5. How does Figure 7.7 demonstrate Simpson's Paradox?

## 4.2 Activity 4A: Movie Profits — Linear Regression

### 4.2.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set with two quantitative variables.
- Use scatterplots to assess the relationship between two quantitative variables.
- Find the estimated line of regression using summary statistics and R linear model (`lm()`) output.
- Interpret the slope coefficient in context of the problem.

### 4.2.2 Terminology review

In today's activity, we will review summary measures and plots for two quantitative variables. Some terms covered in this activity are:

- Scatterplot
- Least-squares line of regression
- Slope and  $y$ -intercept
- Residuals
- Multivariable plots

To review these concepts, see Chapter 6 & 7 in the textbook.

### 4.2.3 Movies released in 2016

A data set was collected on movies released in 2016 (“IMDb Movies Extensive Dataset” 2016). Here is a list of some of the variables collected on the observational units, movies released in 2016.

Variable	Description
<code>budget_mil</code>	Amount of money (in US \$ millions) budgeted for the production of the movie
<code>revenue_mil</code>	Amount of money (in US \$ millions) the movie made after release
<code>duration</code>	Length of the movie (in minutes)
<code>content_rating</code>	Rating of the movie (G, PG, PG-13, R, Not Rated)
<code>imdb_score</code>	IMDb user rating score from 1 to 10
<code>genres</code>	Categories the movie falls into (e.g., Action, Drama, etc.)
<code>facebook_likes</code>	Number of likes a movie receives on Facebook

```
movies <- read.csv("https://math.montana.edu/courses/s216/data/Movies2016.csv") # Reads in data set
```

## Vocabulary review

To look at the relationship between two quantitative variables we will create a scatterplot with the explanatory variable on the x-axis and the response variable on the y-axis. We can also find three summary measures for the linear relationship between the two variables: regression slope, correlation and the coefficient of determination.

We will look at the relationship between budget and revenue for movies released in 2016. Upload and open the Movie Profits Activity 4A F22 Code R script file. Enter the explanatory variable name, `budget_mil`, for `explanatory` and the response variable name, `revenue_mil`, for `response` at line 7 in the R script file to create the scatterplot. (Note: both variables are measured in “millions of dollars” (\$MM).) Highlight and run lines 1–12.

```
movies %>% # Data set pipes into...
ggplot(aes(x = explanatory, y = response))+ # Specify variables
  geom_point() + # Add scatterplot of points
  labs(x = "Budget in Millions ($)", # Label x-axis
       y = "Revenue in Millions ($)", # Label y-axis
       title = "Revenue vs. Budget") + # Be sure to title your plots
  geom_smooth(method = "lm", se = FALSE) # Add regression line
```

1. Sketch the scatterplot created from the code.
2. Assess the four features of the scatterplot that describe this relationship. Describe each feature using a complete sentence!
  - Form (linear, non-linear)
  - Direction (positive, negative)
  - Strength
  - Unusual observations or outliers

3. Based on the plot, does there appear to be an association between budget and revenue? Explain.

## Slope

The linear model function in R (`lm()`) gives us the summary for the least squares regression line. The estimate for `(Intercept)` is the  $y$ -intercept for the line of least squares, and the estimate for `budget_mil` (the  $x$ -variable name) is the value of  $b_1$ , the slope. Run lines 16 – 17 in the R script file to reproduce the linear model output found in the coursepack.

```
# Fit linear model: y ~ x
revenueLM <- lm(revenue_mil ~ budget_mil, data=movies)
summary(revenueLM)$coefficients # Display coefficient summary
```

```
#>               Estimate Std. Error t value    Pr(>|t|)
#> (Intercept)  9.1693054   9.0175499  1.016829 3.119606e-01
#> budget_mil   0.9460001   0.1056786  8.951670 4.339561e-14
```

4. Write out the least squares regression line using the summary statistics provided above in context of the problem.

You may remember from middle and high school that slope =  $\frac{\text{rise}}{\text{run}}$ .

Using  $b_1$  to represent slope, we can write that as the fraction  $\frac{b_1}{1}$ .

Therefore, the slope predicts how much the line will *rise* for each *run* of +1. In other words, as the  $x$  variable increases by 1 unit, the  $y$  variable is predicted to change (increase/decrease) by the value of slope.

5. Interpret the value of slope in context of the problem.

6. Using the least squares line from question 4, predict the revenue for a movie with a budget of 165 \$MM.

7. Predict the revenue for a movie with a budget of 500 \$MM.

8. The prediction in question 7 is an example of what?

## Residuals

The model we are using assumes the relationship between the two variables follows a straight line. The residuals are the errors, or the variability in the response that hasn't been modeled by the line (model).

$$\begin{aligned}\text{Data} &= \text{Model} + \text{Residual} \\ \implies \text{Residual} &= \text{Data} - \text{Model} \\ e_i &= y_i - \hat{y}_i\end{aligned}$$

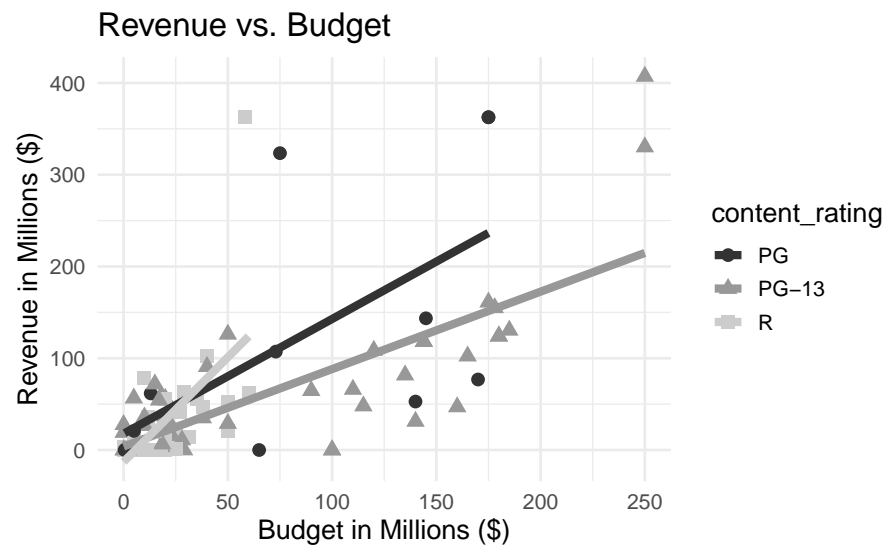
9. The movie *Independence Day: Resurgence* had a budget of 165 \$MM and revenue of 102.315 \$MM. Find the residual for this movie.

10. Did the line of regression overestimate or underestimate the revenue for this movie?

## Multivariable plots

What if we wanted to see if the relationship between movie budget and revenue differs if we add another variable into the picture? The following plot visualizes three variables, creating a **multivariable** plot.

```
movies %>% # Data set pipes into...
  filter(content_rating != "Not Rated") %>% # Remove Not Rated movies
  ggplot(aes(x = budget_mil, y = revenue_mil, color = content_rating)) + # Specify variables
  geom_point(aes(shape = content_rating), size = 3) + # Add scatterplot of points
  labs(x = "Budget in Millions ($)", # Label x-axis
       y = "Revenue in Millions ($)", # Label y-axis
       color = "content_rating", # Label legend
       title = "Revenue vs. Budget") + # Be sure to title your plots
  geom_smooth(method = "lm", se = FALSE, lwd = 2) + # Add regression lines
  scale_color_grey() # Make black and white
```



11. Identify the three variables plotted in this graph.
  
12. Does the *relationship* between movie budget and revenue differ among the different content ratings? Explain.

#### 4.2.4 Take-home messages

1. Two quantitative variables are graphically displayed in a scatterplot. The explanatory variable is on the  $x$ -axis and the response variable is on the  $y$ -axis. When describing the relationship between two quantitative variables we look at the form (linear or non-linear), direction (positive or negative), strength, and for the presence of outliers.
2. There are three summary statistics used to summarize the relationship between two quantitative variables: correlation ( $R$ ), slope of the regression line ( $b_1$ ), and the coefficient of determination ( $R^2$ ).
3. We can use the line of regression to predict values of the response variable for values of the explanatory variable. Do not use values of the explanatory variable that are outside of the range of values in the data set to predict values of the response variable (reflect on why this is true.). This is called **extrapolation**.

#### 4.2.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## 4.3 Activity 4B: Movie Profits — Correlation and Coefficient of Determination

### 4.3.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set with two quantitative variables.
- Calculate and interpret  $r^2$ , the coefficient of determination, in context of the problem.
- Find the correlation coefficient from R output or from  $r^2$  and the sign of the slope.

### 4.3.2 Terminology review

In today's activity, we will review summary measures and plots for two quantitative variables. Some terms covered in this activity are:

- Correlation ( $r$ )
- Coefficient of determination ( $r$ -squared or  $r^2$ )

To review these concepts, see Chapter 6 in the textbook.

### 4.3.3 Movies released in 2016

We will revisit the movie data set collected on Movies released in 2016 (“IMDb Movies Extensive Dataset” 2016) to further explore the relationship between budget and revenue. Here is a reminder of the variables collected on these movies.

Variable	Description
budget_mil	Amount of money (in US \$ millions) budgeted for the production of the movie
revenue_mil	Amount of money (in US \$ millions) the movie made after release
duration	Length of the movie (in minutes)
content_rating	Rating of the movie (G, PG, PG-13, R, Not Rated)
imdb_score	IMDb user rating score from 1 to 10
genres	Categories the movie falls into (e.g., Action, Drama, etc.)
facebook_likes	Number of likes a movie receives on Facebook



## Correlation

Correlation measures the strength and the direction of the linear relationship between two quantitative variables. The closer the value of correlation to +1 or −1, the stronger the linear relationship. Values close to zero indicate a very weak linear relationship between the two variables. The following output shows a correlation matrix between several pairs of quantitative variables. Upload and open the Movie Profits Activity 4B F22 Code R script file. Highlight and run lines 1–12 to produce the same table as below.

```
movies <- read.csv("https://math.montana.edu/courses/s216/data/Movies2016.csv") # Reads in data set
movies %>% # Data set pipes into
  select(c("budget_mil", "revenue_mil",
           "duration", "imdb_score",
           "facebook_likes")) %>%
  cor(use="pairwise.complete.obs") %>%
  round(3)
```

```
#>
#>      budget_mil revenue_mil duration imdb_score facebook_likes
#> budget_mil      1.000      0.686    0.463      0.292          0.678
#> revenue_mil      0.686      1.000    0.227      0.398          0.723
#> duration         0.463      0.227    1.000      0.261          0.438
#> imdb_score        0.292      0.398    0.261      1.000          0.309
#> facebook_likes    0.678      0.723    0.438      0.309          1.000
```

1. Using the output above, which two variables have the *strongest* correlation? What is the value of this correlation?
2. What is the value of correlation between budget and revenue?
3. Based on the value of correlation found in question 2, what would the sign of the slope be? Positive or negative? Explain.
4. Does your answer to question 3 match the direction you choose in question 2 in Activity 4A?
5. Explain why the correlation values on the diagonal are equal to 1.

### Coefficient of determination (squared correlation)

Another summary measure used to explain the linear relationship between two quantitative variables is the coefficient of determination ( $r^2$ ). The coefficient of determination,  $r^2$ , can also be used to describe the strength of the linear relationship between two quantitative variables. The value of  $R^2$  (a value between 0 and 1) represents the **proportion of variation in the response that is explained by the least squares line with the explanatory variable**. There are two ways to calculate the coefficient of determination:

Square the correlation coefficient:  $r^2 = (r)^2$

Use the variances of the response and the residuals:  $r^2 = \frac{s_y^2 - s_{RES}^2}{s_y^2} = \frac{SST - SSE}{SST}$

6. Use the correlation,  $r$ , found in question 2 of the activity, to calculate the coefficient of determination between budget and revenue,  $r^2$ .
7. The variance of the response variable, revenue in \$MM, is about  $s_{revenue}^2 = 8024.261$  \$MM<sup>2</sup> and the variability in the residuals is about  $s_{RES}^2 = 4244.832$  \$MM<sup>2</sup>. Use these values to calculate the coefficient of determination. Verify that your answers to 6 and 7 are the same.

In the next part of the activity we will explore what the coefficient of determination measures.

- Go to the website [www.rossmanchance.com/ISlapplets.html](http://www.rossmanchance.com/ISlapplets.html) and click on Corr/Regression under Quantitative Response.
  - Click **Clear** below the box containing the sample data.
  - Download and open the csv file “Movie2016” from D2L. Copy the two columns containing **budget\_mil** and **revenue\_mil** including the headers and paste into the sample data box.
  - Click ‘Use Data’.
8. Click on **Show Moveable Line**. Write down the equation of the line given. Why is the slope zero for this line?
  9. Click on **Show Squared Residuals**. Write down the value for SSE. Since this is the sum of squared errors (SSE) for the horizontal line we call this the total sum of squares (SST).

10. Click on **Show Regression Line**. Write down the equation of the line given. Does this match the least squares line found in Activity 4A question 4?

11. Click on **Show Squared Residuals**. Write down the value for SSE.

12. Calculate the value for  $r^2$  using the values found for SST and SSE.

13. Write a sentence interpreting the coefficient of determination in context of the problem.

#### 4.3.4 Take-home messages

1. The sign of correlation and the sign of the slope will always be the same. The closer the value of correlation is to  $-1$  or  $+1$ , the stronger the relationship between the explanatory and the response variable.
2. The coefficient of determination multiplied by 100 ( $r^2 \times 100$ ) measures the percent of variation in the response variable that is explained by the relationship with the explanatory variable. The closer the value of the coefficient of determination is to 100%, the stronger the relationship.

#### 4.3.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

## 4.4 Week 4 Lab: Penguins

### 4.4.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set with two quantitative variables.
- Use scatterplots to assess the relationship between two quantitative variables.
- Find the estimated line of regression using summary statistics and R linear model (`lm()`) output.
- Interpret the slope coefficient in context of the problem.
- Calculate and interpret  $R^2$ , the coefficient of determination, in context of the problem.
- Find the correlation coefficient from R output or from  $R^2$  and the sign of the slope.

### 4.4.2 Penguins

The Palmer Station Long Term Ecological Research Program sampled three penguin species on islands in the Palmer Archipelago in Antarctica. Researchers took various body measurements on the penguins, including flipper length and body mass. The researchers were interested in the relationship between flipper length and body mass and wondered if flipper length could be used to accurately predict the body mass of these three penguin species.

Upload and import the `Antarctica_Penguins` csv file and the provided R script file for week 4 lab. Enter the name of the data set (see the environment tab) for `datasetname` in the R script file in line 4.

First we will create a scatterplot of the flipper length and body mass. Notice that we are using flipper length to predict body mass. This makes flipper length the explanatory variable. **Make sure to give your plot a descriptive title.** Highlight and run lines 1–13 in the R script file. **Upload a copy of your scatterplot to Gradescope.**

```
penguins <- datasetname #Creates the object penguins
penguins %>%
  ggplot(aes(x = flipper_length_mm, y = body_mass_g))+ # Specify variables
  geom_point() + # Add scatterplot of points
  labs(x = "flipper length (mm)", # Label x-axis
       y = "body mass (g)", # Label y-axis
       title = "Title") + # Be sure to title your plots
  geom_smooth(method = "lm", se = FALSE) # Add regression line
```

1. Assess the four features of the scatterplot that describe this relationship.

- Form (linear, non-linear)
- Direction (positive, negative)
- Strength

- Unusual observations or outliers

Highlight and run lines 16–20 to get the correlation matrix in the R script file.

```
penguins %>% # Data set pipes into
  select(c("bill_length_mm", "bill_depth_mm",
           "flipper_length_mm", "body_mass_g")) %>%
  cor(use="pairwise.complete.obs") %>%
  round(3)
```

2. Using the R output, which two variables have the *strongest* correlation? What is the value of this correlation?
3. Using the value of correlation found in question 2, calculate the value of the coefficient of determination.
4. Interpret the coefficient of determination in context of the problem.

Enter the variable `body_mass_g` for response and the variable name `flipper_length_mm` for explanatory in line 23 in the R script file. Highlight and run lines 23–24 to get the linear model output.

```
# Fit linear model: y ~ x
penguinsLM <- lm(response~explanatory, data=penguins)
summary(penguinsLM)$coefficients # Display coefficient summary
```

5. Write out the least squares regression line using the summary statistics from the R output in context of the problem.
6. Interpret the value of slope in context of the problem.

7. Using the least squares regression line from question 5, predict the body mass for a penguin with a flipper length of 181 mm.
8. One penguin had a flipper length of 181 mm and a body mass of 3750 g. Find the residual for this penguin.
9. Did the line of regression overestimate or underestimate the body mass for this penguin?

Highlight and run lines 27–34 to get the multivariate plot.

```
penguins %>%  
  ggplot(aes(x = flipper_length_mm, y = body_mass_g, color=species))+ # Specify variables  
  geom_point(aes(shape = species), size = 3) + # Add scatterplot of points  
  labs(x = "flipper length (mm)", # Label x-axis  
       y = "body mass (g)", # Label y-axis  
       color = "species",  
       title = "TITLE") + # Be sure to tile your plots  
  geom_smooth(method = "lm", se = FALSE) # Add regression line
```

10. What three variables are plotted on this plot?
11. Does adding the variable species affect the relationship between body mass and flipper length? Explain your answer.
12. Explain the association between species and each of the other two variables.

---

## Group Exam 1 Review

---

Use the provided data set from the Islands (ExamReviewData.csv) and the appropriate Exam 1 Review R script file to answer the following questions. Each adult (>21) islander was selected at random from all adult islanders. Variables and their descriptions are listed below. Music type (classical or heavy metal) was randomly assigned to the Islanders. Time to complete the puzzle cube was measured after listening to music for each Islander. Heart rate and blood glucose levels were both measured before and then after drinking a caffeinated beverage.

Variable	Description
Island	Name of Island that the Islander resides on
City	Name of City in which the Islander resides
Population	Population of the City
Name	Name of Islander
Consent	Whether the Islander consented to be in the study
Gender	Gender of Islander (M = male, F = Female)
Age	Age of Islander
Married	Marital status of Islander
Smoking_Status	Whether the Islander is a current smoker
Children	Whether the Islander has children
weight_kg	Weight measured in kg
height_cm	Height measured in cm
respiratory_rate	Breaths per minute
Type_of_Music	Music type (Classical or Heavy Metal) Islander was randomly assigned to listen to
After_PuzzleCube	Time to complete puzzle cube (minutes) after listening to assigned music
Education_Level	Highest level of education completed
Balance_Test	Time balanced measured in seconds with eyes closed
Blood_Glucose_before	Level of blood glucose (mg/dL) before consuming assigned drink
Heart_Rate_before	Heart rate (bpm) before consuming assigned drink
Blood_Glucose_after	Level of blood glucose (mg/dL) after consuming assigned drink
Heart_Rate_after	Heart rate (bpm) after consuming assigned drink
Diff_Heart_Rate	Difference in heart rate (bpm) for Before - After consuming assigned drink
Diff_Blood_Glucose	Difference in blood glucose (mg/dL) for Before - After consuming assigned drink

1. What are the observational units?
2. In the table above, indicate which variables are categorical (C) and which variables are quantitative (Q).
3. What type of bias may be present in this study? Explain.



4. Use the appropriate Exam 1 Review R script file to find the appropriate summary statistic and graphical display of the data to assess the following research question, “Is there a difference in proportion of Islanders who have children for those who completed high school and those that completed university?” Use high school – university as the order of subtraction.

- a. What is the name of the explanatory variable to be assessed in this research question?

What type of variable (categorical or quantitative) is the variable you identified?

- b. What is the name of the response variable to be assessed in this research question?

What type of variable (categorical or quantitative) is the variable you identified?

- c. Use the R script file to get the counts for each level and combination of variables. Fill in the following table with the variable names, levels of each variable, and counts using the values from the R output.

	<b>Explanatory Variable</b>		
<b>Response variable</b>	Group 1	Group 2	Total
Success			
Failure			
Total			

- d. Calculate the value of summary statistic to answer the research question. Give appropriate notation.

- e. Interpret the value of the summary statistic in context of the problem:
  
- f. What type of graph(s) would be appropriate for this research question?
  
- g. Using the provided R file create a graph of the data. Sketch the graph below:
  
  
  
  
  
  
  
  
  
  
- h. Based on the graph, does there appear to be an association between the two variables? Explain your answer.
  
  
  
  
  
  
  
  
  
  
- i. Is this an observational study or a randomized experiment? Explain your answer.
  
  
  
  
  
  
  
  
  
  
- j. What is the scope of inference for this study?

5. Use the appropriate Exam 1 Review R script file to find the appropriate summary statistic and graphical display of the data to assess the following research question: “Do Islanders who listen to classical music take less time to complete the puzzle cube after listening to the music than for Islanders that listen to heavy metal music?” Use classical – heavy metal as the order of subtraction.

- a. What is the name of the explanatory variable to be assessed in this research question?

What type of variable (categorical or quantitative) is the variable you identified?

- b. What is the name of the response variable to be assessed in this research question?

What type of variable (categorical or quantitative) is the variable you identified?

- c. Use the R script file to get the summary statistics for each level of the explanatory variable. Fill in the following table with the variable name, levels of the variable, and the summary statistics from the R output.

	<b>Explanatory Variable</b>	
<b>Summary value</b>	Group 1	Group 2
Mean		
Standard deviation		
Sample size		

- d. Calculate the value of the summary statistic to answer the research question. Give appropriate notation.

- e. Interpret the value of the summary statistic in context of the problem:
  
- f. What type of graph(s) would be appropriate for this research question?
  
- g. Using the provided R file create a graph of the data. Sketch the graph below:
  
  
  
  
  
  
  
  
  
  
- h. Based on the graph, does there appear to be an association between the two variables? Explain your answer.
  
  
  
  
  
  
  
  
  
  
- i. Compare the two plots using the four characteristics to describe plots of quantitative variables.  
Shape:  
  
Center:  
  
Spread:  
  
Outliers:
  
  
- j. Is this an observational study or a randomized experiment? Explain your answer.
  
  
  
  
  
  
  
  
  
  
- k. What is the scope of inference for this study?

6. Use the appropriate Exam 1 Review R script file to find the appropriate summary statistic and graphical display of the data to assess the following research question: “Do Islanders who are heavier tend to take more breaths per minute?”

- a. What is the name of the explanatory variable to be assessed in this research question?

What type of variable (categorical or quantitative) is the variable you identified?

- b. What is the name of the response variable to be assessed in this research question?

What type of variable (categorical or quantitative) is the variable you identified?

- c. Use the R script file to get the summary statistics for this data. Fill in the following table using the values from the R output:

	y-intercept	slope	correlation
Summary value			

- d. Interpret the value of slope in context of the problem.
- e. Interpret the value of correlation in context of the problem.
- f. Calculate the value of the coefficient of determination.
- g. Interpret the coefficient of determination in context of the problem.
- h. What type of graph(s) would be appropriate for this research question?

- i. Using the provided R file create a graph of the data. Sketch the graph below:
- j. Based on the graph, does there appear to be an association between the two variables? Explain your answer.
- k. Describe the plot using the four characteristics to describe scatterplots.
- Form:
- Direction:
- Strength:
- Outliers:
- l. Is this an observational study or a randomized experiment? Explain your answer.
- m. What is the scope of inference for this study?

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