STAT 216 Coursepack

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Montana State University

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This resource was developed by Melinda Yager, Jade Schmidt, and Stacey Hancock in 2021 to accompany the online textbook: Hancock, S., Carnegie, N., Meyer, E., Schmidt, J., and Yager, M. (2021). *Montana State Introductory Statistics with R*. Montana State University. https://mtstateintrostats.github.io/IntroStatTextbook/.

Contents

Preface

This coursepack accompanies the textbook for STAT 216: Montana State Introductory Statistics with R, which can be found at https://mtstateintrostats.github.io/IntroStatTextbook/. The syllabus for the course (including the course calendar), data sets, and links to D2L Brightspace, Gradescope, and the MSU RStudio server can be found on the course webpage: https://math.montana.edu/courses/s216/. Other notes and review materials are linked in D2L.

Each of the activities in this workbook is designed to target specific learning outcomes of the course, giving you practice with important statistical concepts in a group setting with instructor guidance. In addition to the in-class activities for the course, video notes are provided to aid in taking notes while you complete the required videos. Bring this workbook with you to class each class period, and take notes in the workbook as you would your own notes. A well-written completed workbook will provide an optimal study guide for exams!

All activities and labs in this coursepack will be completed during class time. Parts of each lab will be turned in on Gradescope. To aid in your understanding, read through the introduction for each activity before attending class each day.

STAT 216 is a 3-credit in-person course. In our experience, it takes six to nine hours per week outside of class to achieve a good grade in this class. By “good” we mean at least a C because a grade of D or below does not count toward fulfilling degree requirements. Many of you set your goals higher than just getting a C, and we fully support that. You need roughly nine hours per week to review past activities, read feedback on previous assignments, complete current assignments, and prepare for the next day’s class. A typical week in the life of a STAT 216 student looks like:

- *Prior to class meeting*:
  - Read assigned sections of the textbook, using the provided reading guides to take notes on the material.
  - Watch the provided videos, taking notes in the coursepack.
  - Read through the introduction to the day’s in-class activity.
  - Read through the week’s homework assignment and note any questions you may have on the content.
- *During class meeting*:
  - Work through the guided activity, in-class activity or weekly lab with your classmates and instructor, taking detailed notes on your answers to each question in the activity.
- *After class meeting*:
  - Complete any parts of the activity you did not complete in class.
  - Review the activity solutions in the Math and Stat Center, and take notes on key points.
  - Complete any remaining assigned readings for the week.
  - Complete the week’s homework assignment.

MODULE 1

Inference for a Single Quantitative Variable

1.1 Vocabulary Review and Key Topics

Review the Golden Ticket posted in the resources at the end of the coursepack for a summary of a single quantitative variable. Module 6 will cover hypothesis testing using both simulation and theory-based methods.

- The **summary measure** for one quantitative variable is the **mean**
- Additionally, we can find the five number summary (min, Q1, median, Q3, max) as well as the sample standard deviation
- R code to find the summary statistics for a quantitative variable

```
object %>% # Data set piped into...
    summarise(favstats(variable))
```

- Quartile 1, Q_1: value at the 25th percentile; approximately 25% of data values are at and below the value of Q_1
- Quartile 2, Q_3: value at the 75th percentile; approximately 75% of data values are at and below the value of Q_3
- Sample standard deviation, s: on average, each value in the data set is s units from the mean of the data set
- **Interquartile range**: $IQR = Q_3 - Q_1$

Types of plot for one quantitative variables

- **Histogram**: sorts a quantitative variable into bins of a certain width
- R code to create a histogram

```
object %>% # Data set piped into...
    ggplot(aes(x = variable)) +   # Name variable to plot
    geom_histogram(binwidth = 10) +  # Create histogram with specified binwidth
    labs(title = "Don't forget to title the plot!", # Title for plot
        x = "x-axis label", # Label for x axis
        y = "y-axis label") # Label for y axis
```

- **Boxplot**: plots the values of the five number summary and shows any outliers in the data set
- R code to create a boxplot

```
object %>% # Data set piped into...
    ggplot(aes(x = variable)) + # Name variable to plot
    geom_boxplot() + # Create boxplot
    labs(title = "Don't forget to title the plot!", # Title for plot
        x = "x-axis label", # Label for x axis
        y = "y-axis label") # Label for y axis
```

- **Dotplot**: plots each value as a dot along the x-axis

- Four characteristics of boxplots
  - Shape (symmetric or skewed)
  - Center
  - Spread
  - Outliers?

Simulation Hypothesis Testing

Hypotheses for a single quantitative variable:

$$H_0 : \mu = \mu_0$$

$$H_A : \mu \begin{Bmatrix} < \\ \neq \\ < \end{Bmatrix} \mu_0$$

- R code to use for **simulation methods** for one quantitative variable to find the p-value, one_mean_test, is shown below. Review the comments (instructions after the #) to see what each should be entered for each line of code.

```
one_mean_test(object$variable,#Enter the object name and variable
          null_value = xx, #Enter the null value for the study
          summary_measure = "mean",  #Can choose between mean or median
          shift = xx, #Difference between the null value and the sample mean
          as_extreme_as = xx, #Value of the summary statistic
          direction = "xx", #Specify direction of alternative hypothesis
          number_repetitions = 10000)
```

Theory-based Hypothesis Testing

- **Theory-based methods**: when specific conditions are met, a data can be fit with a theoretical distribution
- **Conditions for the sampling distribution of $\bar{x}$ to follow an approximate normal distribution**:
  - **Independence**: The sample's observations are independent, e.g., are from a simple random sample. (*Remember*: This also must be true to use simulation methods!)
  - **Large enough sample size: Normality Condition**: The sample observations come from a normally distributed population. To check use the the following rules of thumb:
    - $n < 30$: The distribution of the sample must be approximately normal with no outliers
    - $30 \geq n < 100$: We can relax the condition a little; the distribution of the sample must have no extreme outliers or skewness
    - $n > 100$: Can assume the sampling distribution of $\bar{x}$ is nearly normal, even if the underlying distribuion of individual observationals is not
- **t-distribution**: a theoretical distribution that is symmetric with a given degrees of freedom $(n-1)$
  - t_{n-1}
- **Standardized sample mean**: standardized statistic for a single quantitative variable calculated using:

$$T = \frac{\bar{x} - \mu_0}{SE(\bar{x})},$$

- **Standard error of the sample mean assuming the null is true**: measures the how far each possible sample mean is from the true mean, on average and is calculated using the formula below:

$$SE(\bar{x}) = \frac{s}{\sqrt{n}}$$

- The following R code is used to find the p-value using theory based methods for a single quantitative variables.
  - pt will give you a p-value using the t-distribution with n-1 df (enter for yy)
  - Enter the value of the standardized statistic for xx
  - If a greater than alternative, change lower.tail = TRUE to FALSE.
  - If a two-sided test, multiply by 2.

```
pt(xx, df = yy, lower.tail=TRUE)
```

Exploratory data analysis

At the end of this module, you should understand how to calculate a summary statistic and plot a single quantitative variable.

- Notation for a sample mean: $\bar{x}$
- Notation for a population mean: μ
- Types of plots for a single categorical variable:
  - Histogram
  - Boxplot
  - Dotplot

1.2 Video Notes: Exploratory Data Analysis of Quantitative Variables

Read Chapters 5 and 17 in the course textbook. Use the following videos to complete the video notes for Module 6.

1.2.1 Course Videos

- QuantitativeData
- 5.5to5.6
- 5.7
- 17.2
- 17.3TheoryTests

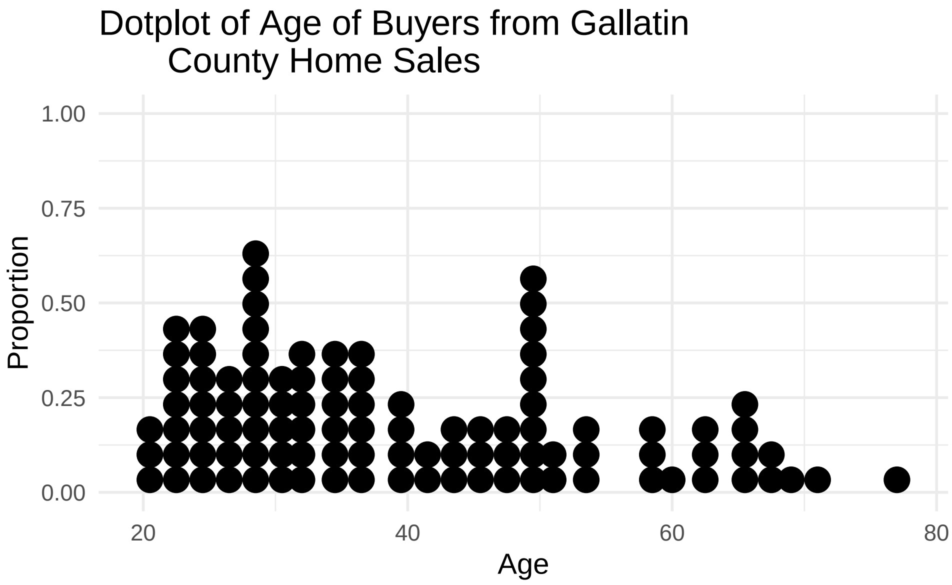
Summarizing quantitative data - Videos 5.2to5.4 and 5.5to5.6

Types of plots

We will revisit the moving to Montana data set and plot the age of the buyers.

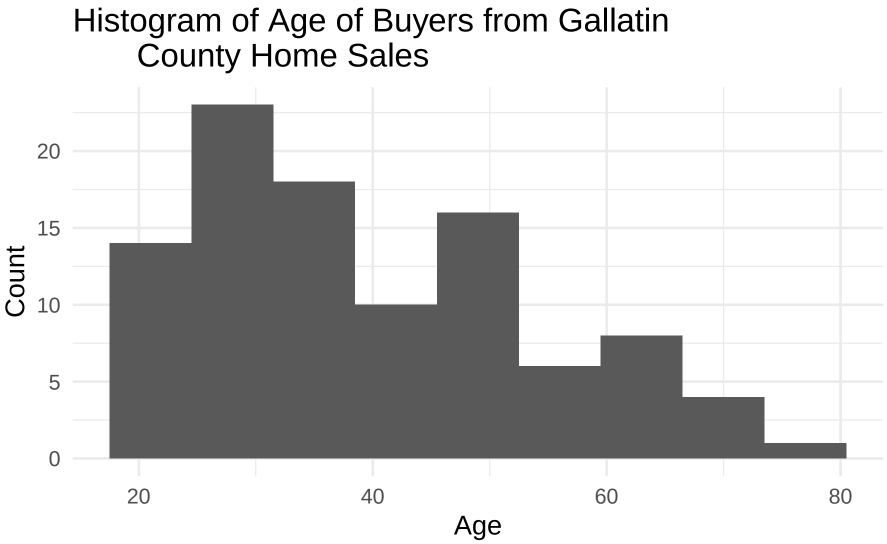
Dotplot:

```
moving %>%
  ggplot(aes(x = Age))+ #Enter variable to plot
  geom_dotplot() +
  labs(title = "Dotplot of Age of Buyers from Gallatin
       County Home Sales", #Title your plot
       x = "Age", #x-axis label
       y = "Proportion") #y-axis label
```



Histogram:

```r
moving %>%
  ggplot(aes(x = Age))+
  geom_histogram(binwidth = 7) +
  labs(title = "Histogram of Age of Buyers from Gallatin
       County Home Sales",
       #Title your plot
       x = "Age",
       y = "Count")
```



Quantitative data can be numerically summarized by finding:

Two measures of center:

- Mean: ____________________ of all the ______________________ in the data set.
  - Sum the values in the data set and divide the sum by the sample size
- Notation used for the population mean:
  - Single quantitative variable:

    - One categorical and one quantitative variable:

      - Subscripts represent the __________________ variable groups
- Notation used for the sample mean:

    - Single quantitative variable:

    - One categorical and one quantitative variable:

- Median: Value at the ____________________ percentile
    - _______________ % of values are at and _________________ and at and _________________ the value of the ______________________.
    - Middle value in a list of ordered values

Two measures of spread:

- Standard deviation: Average ______________________________ each data point is from the ______________________ of the data set.

    - Notation used for the population standard deviation

    - Notation used for the sample standard deviation

- Interquartile range: middle 50% of data values

  Formula:

    Quartile 3 (Q3) - value at the 75th percentile

    - __________________ % of values are at and ____________________ the value of Q3

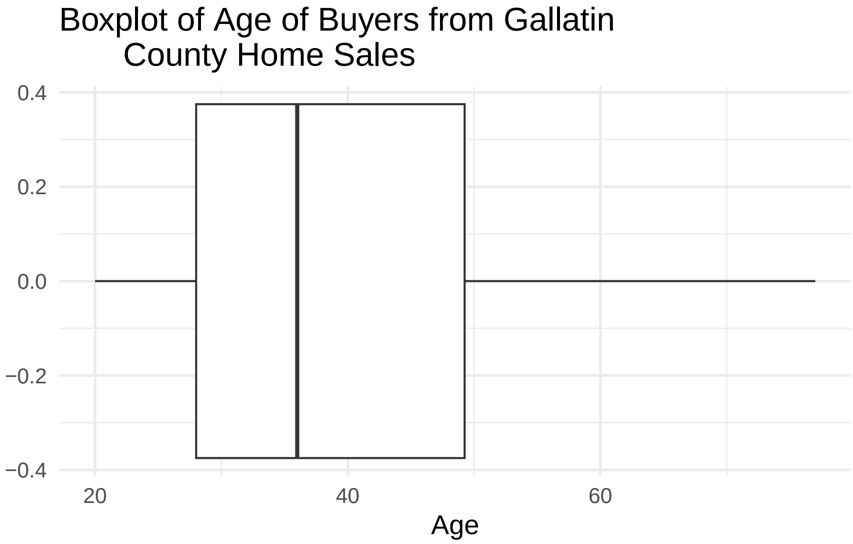
    Quartile 1 (Q1) - value at the 25th percentile

    - ____________________ % of values are at and ____________________ the value of Q1

Boxplot (3rd type of plot for quantitative variables)

```
- Five number summary: minimum, Q1, median, Q3, maximum
```

```
moving %>%
  ggplot(aes(x = Age))+ #Enter variable to plot
  geom_boxplot() +
  labs(title = "Boxplot of Age of Buyers from Gallatin
       County Home Sales", #Title your plot
       x = "Age", #x-axis label
       y = "") #y-axis label
```



```
favstats(moving$Age)
```

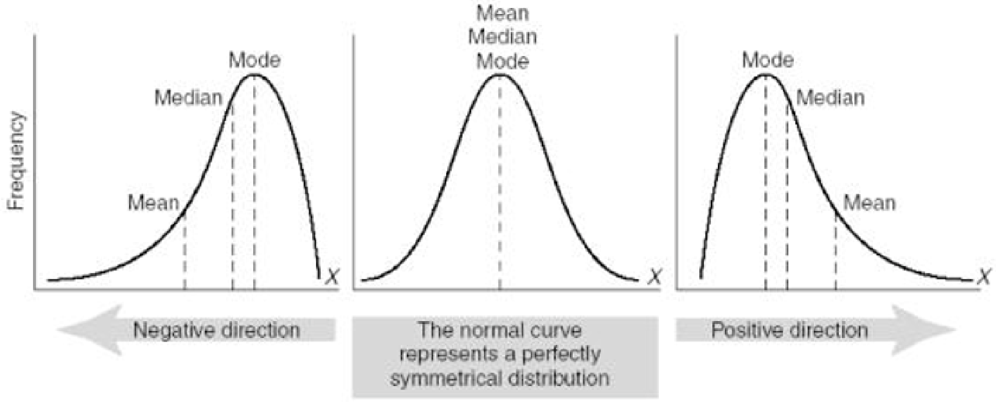
```
#>  min Q1 median    Q3 max  mean       sd   n missing
#>   20 28     36 49.25  77 39.77 14.35471 100       0
```

Interpret the value of Q_3 for the age of buyers.

Interpret the value of s for the age of buyers.

Four characteristics of plots for quantitative variables

- Shape: overall pattern of the data



- What is the shape of the distribution of age of buyers for Gallatin County home sales?

- Center:

  Mean or Median

  - Report the measure of center for the boxplot of age of buyers for Gallatin County home sales.

- Spread (or variability):

  Standard deviation or IQR

  - Report the IQR for the distribution of age of buyers from Gallatin County home sales.

- Outliers?

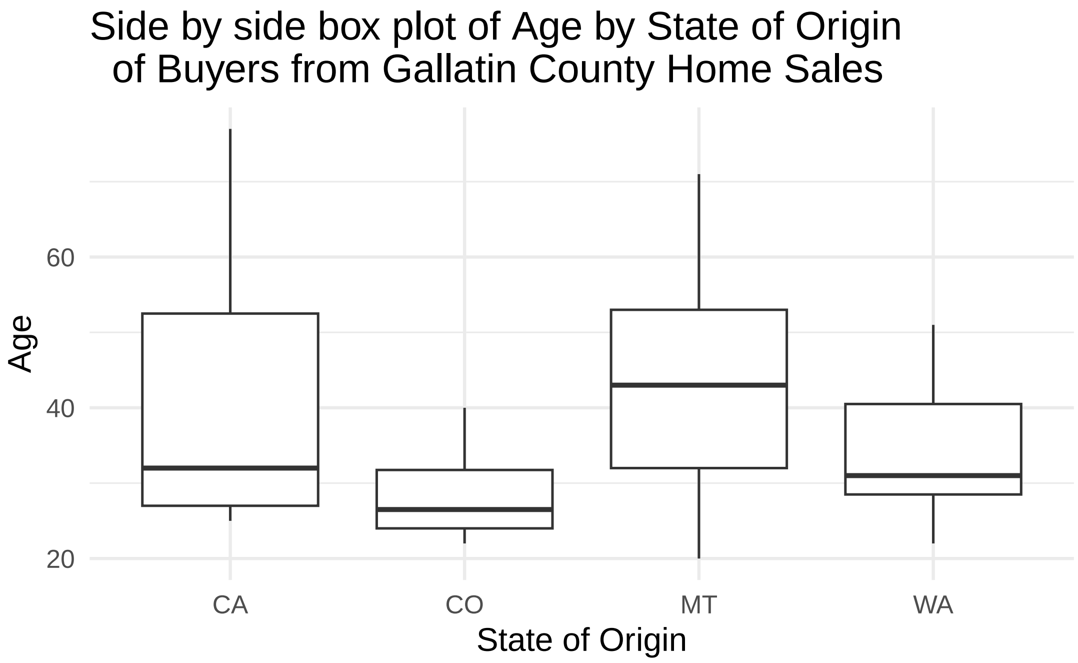
  values $< Q_1 - 1.5 \times IQR$

  values $> Q_3 + 1.5 \times IQR$

  - Use these formulas to show that there are no outliers in the distribution of age of buyers from Gallatin County home sales.

Let's look at side-by-side boxplot of the variable age by state of origin moved from.

```
moving %>%  # Data set piped into...
  ggplot(aes(y = Age, x = From))+  # Identify variables
  geom_boxplot()+  # Tell it to make a box plot
  labs(title = "Side by side box plot of Age by State of Origin
  of Buyers from Gallatin County Home Sales",  # Title
       x = "State of Origin",    # x-axis label
       y = "Age")  # y-axis label
```



- Which state of origin had the oldest median age of buyers from sampled home sales?

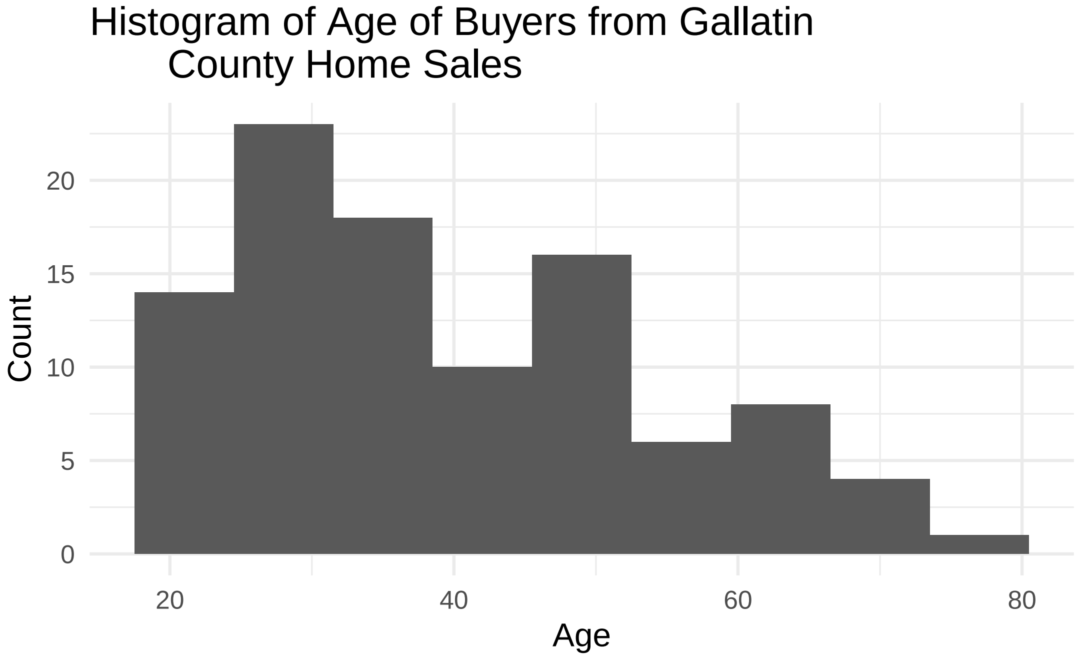
- Which state of origin had the most variability in age of buyers from sampled home sales?

- Which state of origin had the most symmetric distribution of ages of buyers from sampled home sales?

- Which state of origin had outliers for the age of buyers from sampled home sales?

Robust statistics - Video 5.7

Let’s review the summary statistics and histogram of age of buyers from sampled home sales.



```
#>  min Q1 median    Q3 max  mean       sd   n missing
#>   20 28     36 49.25  77 39.77 14.35471 100       0
```

Notice that the ____________________ has been pulled in the direction of the _______________________.

- The _________________ is a robust measure of center.
- The _________________ is a robust measure of spread.
- Robust means not __________________________ by outliers.

When the distribution is symmetric use the ___________________ as the measure of center and the _________________ as the measure of spread.

When the distribution is skewed with outliers use the ____________________ as the measure of center and the ___________________ as the measure of spread.

1.2.2 Video notes single quantitative variable inference

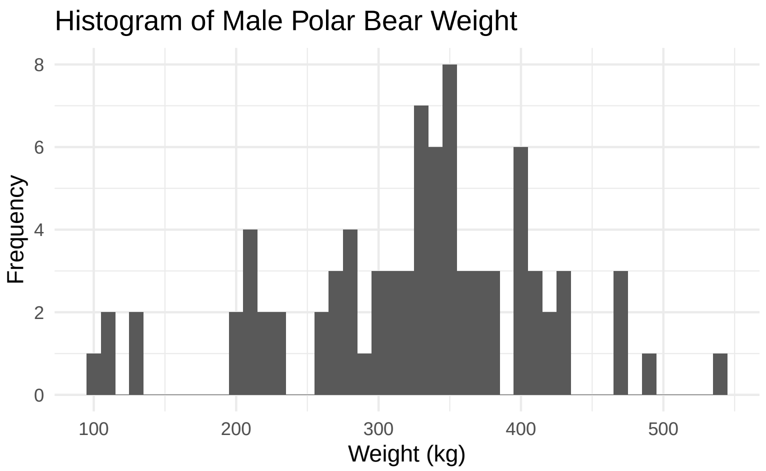
Example: What is the average weight of adult male polar bears? The weight was measured on a representative sample of 83 male polar bears from the Southern Beaufort Sea.

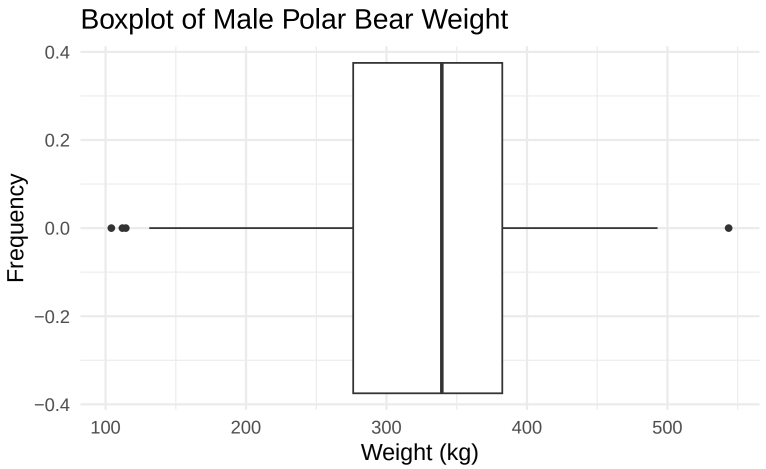
```
pb <- read.csv("https://math.montana.edu/courses/s216/data/polarbear.csv")
```

Plots of the data:

```
pb %>%
    ggplot(aes(x = Weight)) +    # Name variable to plot
    geom_histogram(binwidth = 10) +  # Create histogram with specified binwidth
    labs(title = "Histogram of Male Polar Bear Weight", # Title for plot
       x = "Weight (kg)", # Label for x axis
       y = "Frequency") # Label for y axis

pb %>% # Data set piped into...
ggplot(aes(x = Weight)) +    # Name variable to plot
  geom_boxplot() +  # Create boxplot
  labs(title = "Boxplot of Male Polar Bear Weight", # Title for plot
       x = "Weight (kg)", # Label for x axis
       y = "Frequency") # Label for y axis
```





Summary Statistics:

```
pb %>%
  summarise(favstats(Weight)) #Gives the summary statistics
#>     min    Q1 median     Q3   max     mean       sd  n missing
#> 1 104.1 276.3  339.4 382.45 543.6 324.5988 88.32615 83       0
```

Hypothesis testing

- Hypotheses are always written about the ____. For a single mean we will use the notation __________________.

Null Hypothesis:

H_0 :

Alternative Hypothesis:

H_A :

- Direction of the alternative depends on the _____________________________ ______________________________.

Simulation-based method

- Simulate many samples assuming $H_0 : \mu = \mu_0$
  - Shift the data by the difference between μ_0 and $\bar{x}$
  - Sample with replacement n times from the shifted data
  - Plot the simulated shifted sample mean from each simulation
  - Repeat 1000 times (simulations) to create the null distribution
  - Find the proportion of simulations at least as extreme as $\bar{x}$

Example: Is there evidence that male polar bears weigh less than 370kg (previously recorded measure), on average? The weight was measured on a representative sample of 83 male polar bears from the Southern Beaufort Sea.

Hypotheses:

In notation:

H_0 :

H_A :

In words:

H_0 :

H_A :

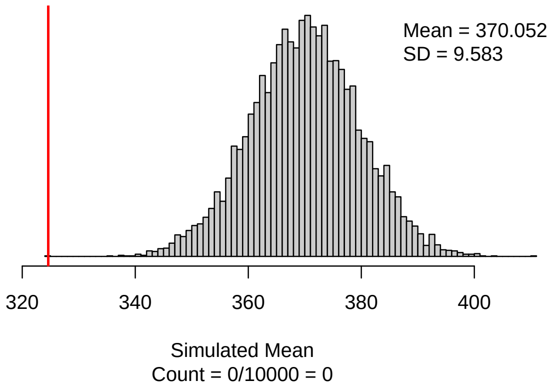
Reminder of summary statistics:

```
pb %>%
  summarise(favstats(Weight)) #Gives the summary statistics
#>     min     Q1 median     Q3   max     mean       sd  n missing
#> 1 104.1 276.3  339.4 382.45 543.6 324.5988 88.32615 83       0
```

Find the difference:

$\mu_0 - \bar{x} =$

```
set.seed(216)
one_mean_test(pb$Weight,    #Enter the object name and variable
              null_value = 370, #Enter null value for the study
              summary_measure = "mean",  #Can choose between mean or median
              shift = 45.4,   # Shift needed for bootstrap hypothesis test
              as_extreme_as = 324.6,  # Observed statistic
              direction = "less",  # Direction of alternative
              number_repetitions = 10000)  # Number of simulated samples for null distribution
```



Interpretation of the p-value:

- Statement about probability or proportion of samples
- Statistic (summary measure and value)
- Direction of the alternative
- Null hypothesis (in context)

Conclusion:

- Amount of evidence
- Parameter of interest
- Direction of the alternative hypothesis

Theory-based method

Conditions for inference using theory-based methods:

- Independence:

- Large enough sample size:

T - distribution

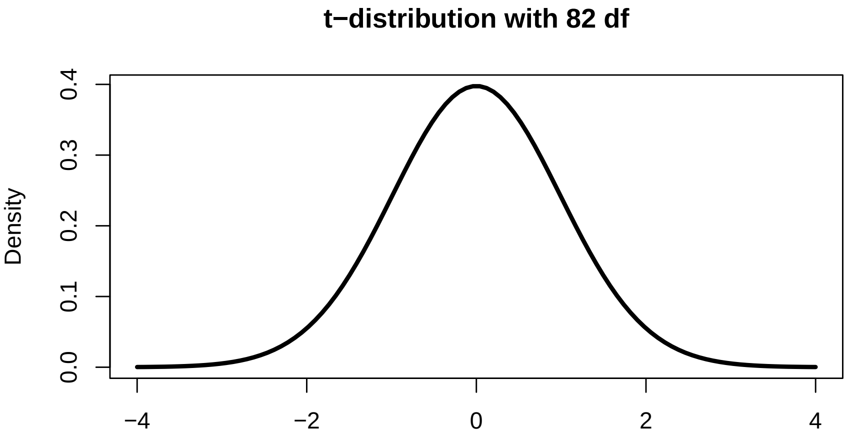
In the theoretical approach, we use the CLT to tell us that the distribution of sample means will be approximately normal, centered at the assumed true mean under H_0 and with standard deviation $\frac{\sigma}{\sqrt{n}}$.

$$\bar{x} \sim N(\mu_0, \frac{\sigma}{\sqrt{n}})$$

- Estimate the population standard deviation, σ, with the ____ standard deviation, ____________.

- For a single quantitative variable we use the ______ - distribution with ______________________ degrees of freedom to approximate the sampling distribution.

The t^* multiplier is the value at the given percentile of the t-distribution with $n - 1$ degrees of freedom.

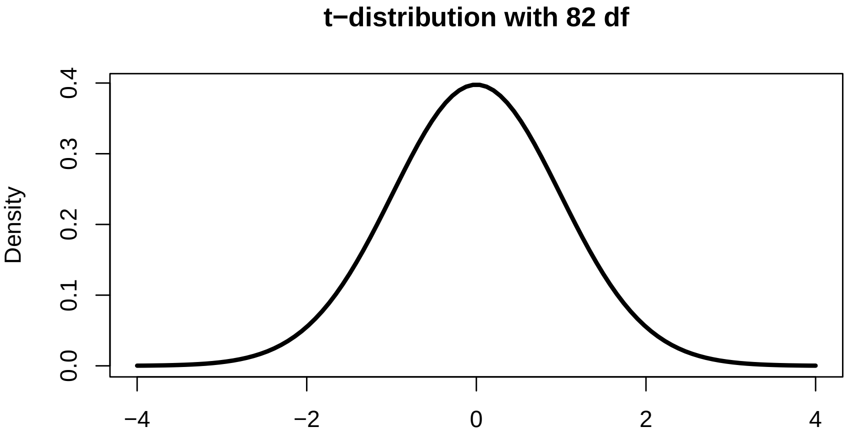


- Calculate the standardized statistic
- Find the area under the t-distribution with $n-1$ df at least as extreme as the standardized statistic

Equation for the standard error of the sample mean:

Equation for the standardized sample mean:

Calculate the standardized sample mean weight of adult male polar bears:



Interpret the standardized sample mean weight:

To find the theory-based p-value:

```
pt(-4.683, df=82, lower.tail=TRUE)
#> [1] 5.531605e-06
```

1.2.3 Concept Check

Be prepared for group discussion in the next class. One member from the table should write the answers to the following on the whiteboard.

1. What plots can be used to summarize quantitative data?

2. Which measure of center is robust to outliers?

1.3 Activity 11: Summarizing Quantitative Variables

1.3.1 Learning outcomes

- Identify and create appropriate summary statistics and plots given a data set or research question for quantitative data.
- Interpret the following summary statistics in context: median, lower quartile, upper quartile, standard deviation, interquartile range.

1.3.2 Terminology review

In today's activity, we will review summary measures and plots for quantitative variables. Some terms covered in this activity are:

- Two measures of center: mean, median
- Two measures of spread (variability): standard deviation, interquartile range (IQR)
- Plots of quantitative variables: dotplots, boxplots, histograms
- Given a plot or set of plots, describe and compare the distribution(s) of quantitative variables (center, variability, shape, outliers).

To review these concepts, see Chapter 5 in the textbook.

1.3.3 The Integrated Postsecondary Education Data System (IPEDS)

These data were collected on a subset of institutions that met the following selection criteria (Education Statistics 2018):

- Degree granting
- United States only
- Title IV participating
- Not for profit
- 2-year or 4-year or above
- Has full-time first-time undergraduates

Some of the variables collected and their descriptions are below. Note that several variables have missing values for some institutions (denoted by "NA").

Variable	Description
`UnitID`	Unique institution identifier
`Name`	Institution name
`State`	State abbreviation
`Sector`	whether public or private
`LandGrant`	Is this a land-grant institution (Yes/No)
`Size`	Institution size category based on total student enrolled for credit, Fall 2018: Under 1,000, 1,000$-4, 999, 5, 000-9, 999, 10, 000-$19,999, 20,000 and above
`Cost_OutofState`	Cost of attendance for full-time out-of-state undergraduate students
`Cost_InState`	Cost of attendance for full-time in-state undergraduate students
`Retention`	Retention rate is the percent of the undergraduate students that re-enroll in the next year
`Graduation_Rate`	6-year graduation rate for undergraduate students

Variable	Description
`SATMath_75`	75th percentile Math SAT score
`ACT_75`	75th percentile ACT score

Identifying Variables in a data set

Look through the provided chart showing the description of variables measured. The UnitID and Name are identifiers for each observational unit, *US degree granting institutions in 2018.*

1. Identify in the chart which variables collected on the US institutions are categorical (C) and which variables are quantitative (Q).

Summarizing quantitative variables

The `favstats()` function from the `mosaic` package gives the summary statistics for a quantitative variable. The `R` output below provides the summary statistics for the variable `Graduation_Rate`. The summary statistics provided are the two measures of center (mean and median) and two measures of spread (standard deviation and the quartile values to calculate the IQR) for IMDb score.

- Highlight and run lines 1 – 12 in the provided `R` script file to load the data set. Check that the summary statistics match the output given in the coursepack.
- Notice that the 2-year institutions were removed so the observational units for this study are **4-year higher education institutions.**

```
IPEDS <- read.csv("https://www.math.montana.edu/courses/s216/data/IPEDS_2018.csv")
IPEDS <- IPEDS %>%
  filter(Sector != "Public 2-year") # Filters the data set to remove Public 2-year
IPEDS <- IPEDS %>%
  filter(Sector != "Private 2-year") # Filters the data set to remove Private 2-year
IPEDS %>%
    summarize(favstats(Graduation_Rate))
```

```
#>   min Q1 median Q3 max     mean       sd    n missing
#> 1   0 38     53 67 100 52.48749 20.63192 1918      49
```

2. Report the values for the two measures of center (mean and median).

3. Calculate the interquartile range (IQR $=$ Q3 $-$ Q1) of Graduation Rates.

4. Report the value of the standard deviation and interpret this value in context of the problem.

5. Interpret the value of Q_3 in context of the study.

Displaying a single quantitative variable

There are three type of plots used to plot a single quantitative variable: a dotplot, a histogram or a boxplot. A dotplot of graduation rate would plot a dot for the graduation rate for each 4-year US higher education institution.

First, let's create a histogram of the variable `Graduation_Rate`.

- Enter the name of the variable in line 19 for `variable` in the R script file.
- Replace the word title for the plot in line 21 between the quotations with a descriptive title. **A title should include: type of plot, variable or variables plotted, and observational units.**
- Highlight and run lines 18 – ?? to create the histogram.

```r
IPEDS %>% # Data set piped into...
ggplot(aes(x = xx)) +   # Name variable to plot
  geom_histogram(binwidth = 10) +  # Create histogram with specified binwidth
  labs(title = "Don't forget to title the plot!", # Title for plot
       x = "Graduation Rate", # Label for x axis
       y = "Frequency") # Label for y axis
```

Notice that the **bin width** for the histogram is 10. For example the first bin consists of the number of institutions in the data set with a graduation rate of 0 to 10%. It is important to note that a graduation rate on the boundary of a bin will fall into the bin above it; for example, 20 would be counted in the bin 20–30.

6. Which range of Graduation Rates have the highest frequency?

Next we will create a boxplot of the variable `Graduation_Rate`.

- Enter the name of the variable in line 19 for `variable` in the R script file.
- Highlight and run lines....

```r
IPEDS %>% # Data set piped into...
ggplot(aes(x = variable)) +   # Name variable to plot
  geom_boxplot() +  # Create boxplot with specified binwidth
  labs(title = "Boxplot of Graduation Rates for 4-year Higher Education Institutions", # Title for plot
       x = "Graduation_Rate", # Label for x axis
       y = "") + # Remove y axis label
    theme(axis.text.y = element_blank(),
          axis.ticks.y = element_blank()) # Removes y-axis ticks
```

7. Sketch the boxplot created and identify the values of the 5-number summary (minimum value, Q1, median, Q3, maximum value) on the plot. Use the following formulas to find the invisible fence on both ends of the distribution. Draw a dotted line at the invisible fence to show how the outliers were found.

$$\text{Lower Fence: values} \leq \text{Q1} - 1.5 \times \text{IQR}$$

$$\text{Upper Fence: values} \geq \text{Q3} + 1.5 \times \text{IQR}$$

When describing plots of quantitative variables we discuss the shape (symmetric or skewed), the center (mean or median), spread (standard deviation or IQR), and if there are outliers present.

8. What is the shape of the distribution of graduation rates?

9. From which plot (histogram or boxplot) is it easier to determine the shape of the distribution?

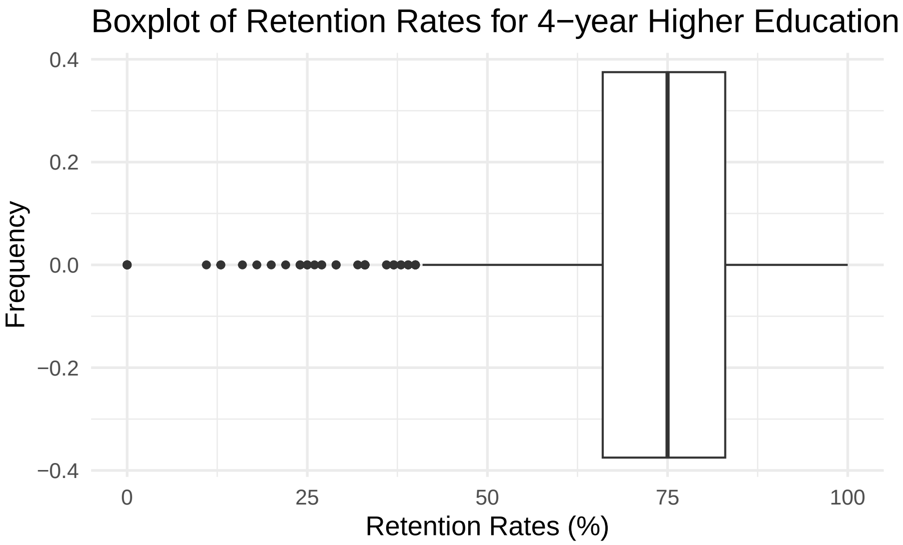
10. From which plot is it easier to determine if there are outliers?

Robust Statistics

Let's examine how the presence of outliers affect the values of center and spread. For this part of the activity we will look at the variable retention rate in the IPEDS data set.

```
IPEDS %>% # Data set piped into...
    summarise(favstats(Retention))
#>   min Q1 median Q3 max    mean       sd    n missing
#> 1   0 66     75 83 100 73.8525 15.14323 1817     150

IPEDS %>% # Data set piped into...
    ggplot(aes(x = Retention)) + # Name variable to plot
    geom_boxplot() + # Create boxplot
    labs(title = "Boxplot of Retention Rates for 4-year Higher Education Institutions", # Title for plot
         x = "Retention Rates (%)", # Label for x axis
         y = "Frequency") # Label for y axis
#> Warning: Removed 150 rows containing non-finite outside the scale range
#> (`stat_boxplot()`).
```



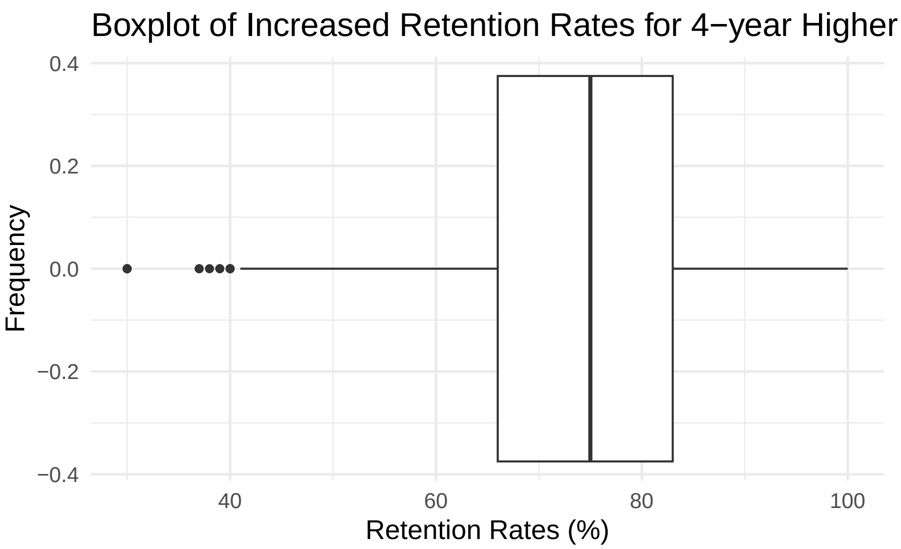
11. Report the two measures of center for these data.

12. Report the two measures of spread for these data.

To show the effect of outliers on the measures of center and spread, the smallest values of retention rate in the data set were increased by 30%. This variable is called `Retention_Inc`.

```
IPEDS %>% # Data set piped into...
    summarise(favstats(Retention_Inc))
#>   min Q1 median Q3 max     mean       sd    n missing
#> 1  30 66     75 83 100 74.49642 13.41255 1817     150

IPEDS %>% # Data set piped into...
    ggplot(aes(x = Retention_Inc)) + # Name variable to plot
    geom_boxplot() + # Create histogram
labs(title = "Boxplot of Increased Retention Rates for 4-year Higher Education Institutions", # Title for
x = "Retention Rates (%)", # Label for x axis
y = "Frequency") # Label for y axis
#> Warning: Removed 150 rows containing non-finite outside the scale range
#> (`stat_boxplot()`).
```



13. Report the two measures of center for this new data set.

14. Report the two measures of spread for this new data set.

15. Which measure of center is robust to outliers? Explain your answer.

16. Which measure of spread is robust to outliers? Explain your answer.

1.3.4 Take-home messages

1. Histograms, box plots, and dot plots can all be used to graphically display a single quantitative variable.
2. The box plot is created using the five number summary: minimum value, quartile 1, median, quartile 3, and maximum value. Values in the data set that are less than $Q_1 - 1.5 \times \text{IQR}$ and greater than $Q_3 + 1.5 \times \text{IQR}$ are considered outliers and are graphically represented by a dot outside of the whiskers on the box plot.
3. Data should be summarized numerically and displayed graphically to give us information about the study.
4. When comparing distributions of quantitative variables we look at the shape, center, spread, and for outliers. There are two measures of center: mean and the median and two measures of spread: standard deviation and the interquartile range, $\text{IQR} = \text{Q3} - \text{Q1}$.

1.3.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

1.4 Activity 12: Hypothesis Testing of a Single Quantitative Variable

1.4.1 Learning outcomes

- Given a research question involving one quantitative variable, construct the null and alternative hypotheses in words and using appropriate statistical symbols.
- Investigate the process of creating a null distribution for one quantitative variable
- Find, evaluate, and interpret a p-value from the null distribution

1.4.2 Terminology review

In today's activity, we will simulation and theory-based methods to analyze a single quantitative variable. Some terms covered in this activity are:

- Null hypothesis
- Alternative hypothesis

To review these concepts, see Chapter 17 in the textbook.

1.4.3 College student sleep habits

According to the an article in *Sleep* (Watson 2015), experts recommend adults (>18) get at least 7 hours of sleep per night. A survey was sent to students in four sections of Stat 216 asking about their sleep habits. Is there evidence that sleep college students get less than the recommended 7 hours of sleep per night, on average?

Summarizing quantitative variables

- Download the R script file and data file for this activity
- Upload both files to the RStudio server and open the R script file
- Enter the name of the dataset for datasetname.csv
- Highlight and run lines 1–8 to load the data

```
sleep <- read.csv("datasetname.csv")
```

Ask a research question

1. Write the parameter of interest in context of the study.

2. Write the null hypothesis in words in context of the study.

3. Write the alternative hypothesis in notation.

Summarize and visualize the data

The `favstats()` function from the `mosaic` package gives the summary statistics for a quantitative variable.

- Enter the variable name, `SleepHours` for variable in line 13
- Highlight and run lines 12–13

```
sleep %>%
    summarize(favstats(variable))
```

4. How far is each number of hours of sleep for a Stat 216 student from the mean number of hours of sleep, on average?

Create a boxplot of the variable `SleepHours`.

- Enter the name of the variable in line 19 for `variable` in the R script file.
- Enter a title in line 21 for the plot between the quotations
- Highlight and run lines 18 - 25

```
sleep %>% # Data set piped into...
    ggplot(aes(x = variable)) +   # Name variable to plot
    geom_boxplot() +  # Create boxplot with specified binwidth
    labs(title = "Don't forget to title your plot!", # Title for plot
       x = "Amount of sleep (hrs)", # Label for x axis
       y = "") + # Remove y axis label
    theme(axis.text.y = element_blank(),
          axis.ticks.y = element_blank()) # Removes y-axis ticks
```

5. Describe the boxplot using the four characteristics of boxplots.

Simulation methods

To simulate the null distribution of sample means we will use a bootstrapping method. Recall that the null distribution must be created under the assumption that the null hypothesis is true. Therefore, before bootstrapping, we will need to *shift* each data point by the difference $\mu_0 - \bar{x}$. This will ensure that the mean of the shifted data is μ_0 (rather than the mean of the original data, $\bar{x}$), and that the simulated null distribution will be centered at the null value.

6. Calculate the difference $\mu_0 - \bar{x}$. Will we need to shift the data up or down?

- Open the data set (sleep_college) in Excel
- Create a new column labeled Shift
- In the column, Shift, add the shifted value to each value in the column, SleepHours
- Save the file and upload again to the RStudio server
- Find the favstats of the variable, Shift

- Highlight and run lines 30–32

```
sleep <- read.csv("sleep_college.csv")
sleep %>%
    summarize(favstats(Shift))
```

7. Report the mean of the Shift variable. Why does it make sense that this value is the same as the null value?

8. Report the standard deviation of the Shift variable. How does this compare to the standard deviation for the variable SleepHours? Explain why these values are the same?

9. What inputs should be entered for each of the following to create the simulation?

- Null Value (What is the null value for the study?):

- Summary measure ("mean" or "median"):

- Shift (Difference between $\mu_0 - \bar{x}$):

- As extreme as (enter the value for the sample difference in proportions):

- Direction (`"greater"`, `"less"`, or `"two-sided"`):

- Number of repetitions:

Using the R script file for this activity…

- Enter your answers for question 9 in place of the `xx`'s to produce the null distribution with 10000 simulations
- Highlight and run lines 361–42.

```
one_mean_test(sleep$SleepHours,#Enter the object name and variable
              null_value = xx,
              summary_measure = "xx",  #Can choose between mean or median
              shift = xx, #Difference between the null value and the sample mean
              as_extreme_as = xx, #Value of the summary statistic
              direction = "xx", #Specify direction of alternative hypothesis
              number_repetitions = 10000)
```

10. Interpret the p-value of the test in context of the problem.

11. Write a conclusion to the test in context of the problem.

1.4.4 Take-home messages

1. Histograms, box plots, and dot plots can all be used to graphically display a single quantitative variable.
2. The box plot is created using the five number summary: minimum value, quartile 1, median, quartile 3, and maximum value. Values in the data set that are less than $Q_1 - 1.5 \times \text{IQR}$ and greater than $Q_3 + 1.5 \times \text{IQR}$ are considered outliers and are graphically represented by a dot outside of the whiskers on the box plot.
3. Data should be summarized numerically and displayed graphically to give us information about the study.
4. When comparing distributions of quantitative variables we look at the shape, center, spread, and for outliers. There are two measures of center: mean and the median and two measures of spread: standard deviation and the interquartile range, $\text{IQR} = \text{Q3} - \text{Q1}$.

1.4.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

1.5 Activity 13: Body Temperature

1.5.1 Learning outcomes

- Given a research question involving a quantitative variable, construct the null and alternative hypotheses in words and using appropriate statistical symbols.
- Describe and perform a theory-based hypothesis test for a single mean.
- Interpret and evaluate a p-value for a theory-based hypothesis test for a single mean.

1.5.2 Terminology review

In today's activity, we will analyze quantitative data using theory-based methods. Some terms covered in this activity are:

- Normality
- t-distribution
- Degrees of freedom
- T-score

To review these concepts, see Chapter 5and? in the textbook.

1.5.3 Body Temperature

It has long been reported that the mean body temperature of adults is 98.6°F. There have been a few articles that challenge this assertion. In 2018, a sample of 52 Stat 216 undergraduates, were asked to report their body temperature. Is there evidence that body temperatures of adults differ from the known temperature of 98.6°F?

Ask a research question

1. Write out the null hypothesis in proper notation for this study.

2. Write out the null hypothesis in words for this study.

In general, the sampling distribution for a sample mean, $\bar{x}$, based on a sample of size n from a population with a true mean μ and true standard deviation σ can be modeled using a Normal distribution when certain conditions are met.

Conditions for the sampling distribution of $\bar{x}$ to follow an approximate Normal distribution:

- **Independence**: The sample's observations are independent. For paired data, that means each pairwise difference should be independent.
- **Normality**: The data should be approximately normal or the sample size should be large.
  - $n < 30$: If the sample size n is less than 30 and the distribution of the data is approximately normal with no clear outliers in the data, then we typically assume the data come from a nearly normal distribution to satisfy the condition.

- $30 \leq n < 100$: If the sample size n is betwe 30 and 100 and there are no particularly extreme outliers in the data, then we typically assume the sampling distribution of $\bar{x}$ is nearly normal, even if the underlying distribution of individual observations is not.
- $n \geq 100$: If the sample size n is at least 100 (regardless of the presence of skew or outliers), we typically assume the sampling distribution of $\bar{x}$ is nearly normal, even if the underlying distribution of individual observations is not.

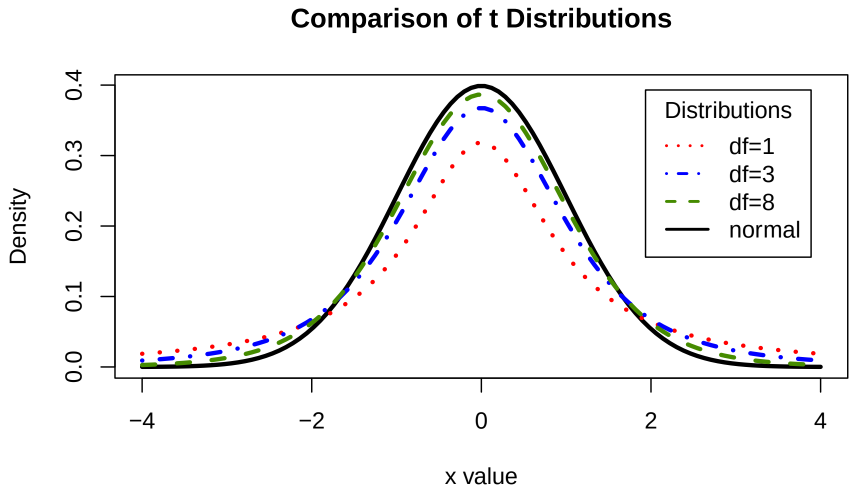


Figure 1.1: Comparison of the standard Normal vs t-distribution with various degrees of freedom

Like we saw in Chapter **5**, we will not know the values of the parameters and must use the sample data to estimate them. Unlike with proportions, in which we only needed to estimate the population proportion, π, quantitative sample data must be used to estimate both a population mean μ and a population standard deviation σ. This additional uncertainty will require us to use a theoretical distribution that is just a bit wider than the Normal distribution. Enter the ***t*-distribution**!

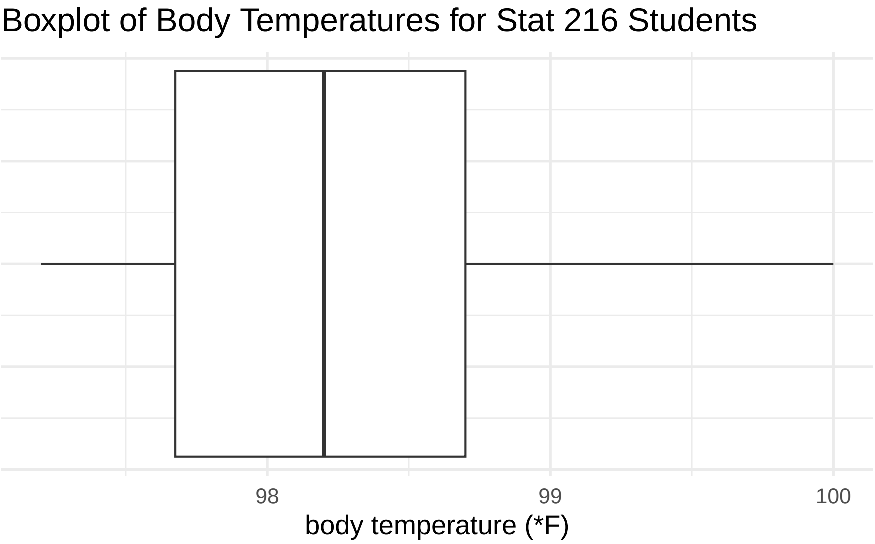
As you can seen from Figure 1.1, the t-distributions (dashed and dotted lines) are centered at 0 just like a standard Normal distribution (solid line), but are slightly wider. The variability of a t-distribution depends on its degrees of freedom, which is calculated from the sample size of a study. (For a single sample of n observations or paired differences, the degrees of freedom is equal to $n - 1$.) Recall from previous classes that larger sample sizes tend to result in narrower sampling distributions. We see that here as well. The larger the sample size, the larger the degrees of freedom, the narrower the t-distribution. (In fact, a t-distribution with infinite degrees of freedom actually IS the standard Normal distribution!)

Summarize and visualize the data

The following code is used to create a boxplot of the data.

- Download the R script file upload to the R studio server.
- Open the R script file and highlight and run lines 1–14

```r
bodytemp <- read.csv("https://math.montana.edu/courses/s216/data/normal_temperature.csv")
bodytemp %>%
  ggplot(aes(x = Temp))+
  geom_boxplot()+
  labs(title="Boxplot of Body Temperatures for Stat 216 Students",
       x = "body temperature (*F)") +
        theme(axis.text.y = element_blank(),
          axis.ticks.y = element_blank()) # Removes y-axis ticks
```



- Highlight and run lines 17 - 18 to get the summary statistics for the variable Temp.

```
bodytemp %>%
  summarise(favstats(Temp))
```

```
#>    min     Q1 median   Q3 max     mean        sd  n missing
#> 1 97.2 97.675   98.2 98.7 100 98.28462 0.6823789 52       0
```

Check theoretical conditions

3. Report the sample size of the study. Give appropriate notation.

4. Report the sample mean of the study. Give appropriate notation.

5. How do you know the independence condition is met for these data?

6. Is the normality condition met to use the theory-based methods for analysis? Explain your answer.

Use statistical inferential methods to draw inferences from the data

To find the standardized statistic for the mean we will use the following formula:

$$T = \frac{\bar{x} - \mu_0}{SE(\bar{x})},$$

where the standard error of the sample mean difference is:

$$SE(\bar{x}) = \frac{s}{\sqrt{n}}.$$

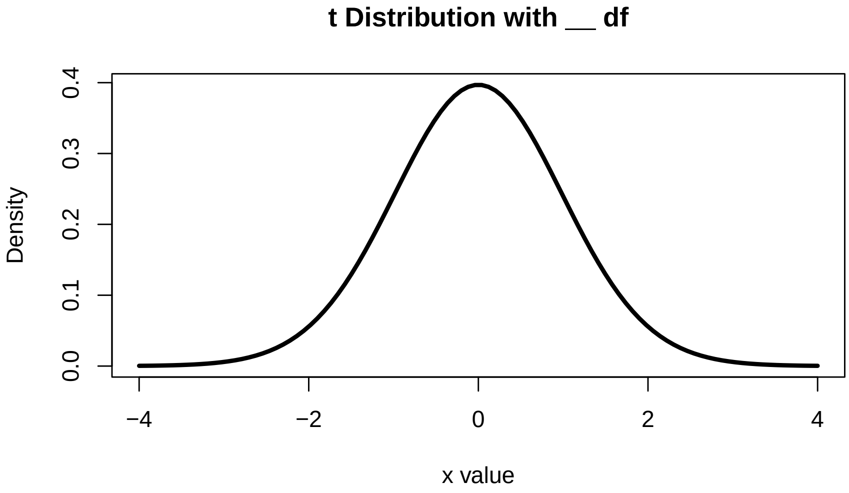
7. Calculate the standard error of the sample mean.

8. Interpret the standard error in context of the study.

9. Calculate the standardized mean.

10. We model a single mean with a t-distribution with $n - 1$ degrees of freedom. Calculate the degrees of freedom for this study.

11. Mark the value of the standardized statistic on the t-distribution and illustrate how the p-value is found.



To find the p-value for the theory-based test:

- Enter the value for the standardized statistic for xx in the pt function.
- Enter the df for yy in the pt function.
- Highlight and run line 24

```
pt(xx, df=yy, lower.tail=FALSE)
```

12. What does this p-value mean, in the context of the study? Hint: it is the probability of what…assuming what?

Next we will calculate a theory-based confidence interval. To calculate a theory-based confidence interval for the paired mean difference, use the following formula:

$$\bar{x} \pm t^{*} \times SE(\bar{x}).$$

We will need to find the t^* multiplier using the function `qt()`.

- Enter the appropriate percentile in the R code to find the multiplier for a 90% confidence interval.
- Enter the df for yy.
- Highlight and run line 30

```
qt(percentile, df = yy, lower.tail=TRUE)
```

13. Report the t^* multiplier for the 90% confidence interval.

14. Calculate the margin of error for the true mean using theory-based methods.

15. Calculate the confidence interval.

16. Interpret the confidence interval in context of the study.

17. Write a conclusion to the test in context of the study.

1.5.4 Take-home messages

1. In order to use theory-based methods for dependent groups (paired data), the independent observational units and normality conditions must be met.
2. A T-score is compared to a t-distribution with $n - 1$ df in order to calculate a one-sided p-value. To find a two-sided p-value using theory-based methods we need to multiply the one-sided p-value by 2.
3. A t^* multiplier is found by obtaining the bounds of the middle X% (X being the desired confidence level) of a t-distribution with $n - 1$ df.

1.5.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered

MODULE 2

Inference for a Single Quantitative Variable

2.1 Vocabulary Review and Key Topics

Review the Golden Ticket posted in the resources at the end of the coursepack for a summary of a single quantitative variable. Module 7 will cover creating confidence interval using both simulation and theory-based methods. Additionally, we learn about errors and power.

Simulation Confidence Interval

- R code to find the simulation confidence interval using the `onemean_CI` function from the `catstats` package.

```
one_mean_CI(object$variable, #Enter the name of the variable
            summary_measure = "mean", #choose the mean or median
            number_repetitions = 10000,  # Number of simulations
            confidence_level = xx)
```

- Interpretation of the confidence interval is very similar as for a single proportion only the context and summary measure has changed
  - To write in context include:
    - How confident you are (e.g., 90%, 95%, 98%, 99%)
    - Parameter of interest
    - Calculated interval

Theory-based Confidence Interval

- Calculation of the confidence interval for a sample mean

$$\bar{x} \pm t^* \times SE(\bar{x})$$

- R code to find the multiplier for the confidence interval using theory-based methods.
  - qt will give you the multiplier using the t-distribution with $n-1$ df (enter for yy)
  - Enter the percentile for the given confidence level

```
qt(percentile, df=yy, lower.tail=FALSE)
```

Errors and Power

- **Significance level (α): a given cut-off value that we compare the p-value to determine a decision of a test.
- **Decisions**:
  - If the p-value is less than the significance level, we make the decision to reject the null hypothesis
  - If the p-value is greater than the significance level, we make the decision to fail to reject the null hypothesis

- **Type I Error**: concluding there is evidence to reject the null hypothesis, when the null is actually true.
- **Type II Error**: concluding there is no evidence to reject the null hypothesis, when the null is actually false.
- **Power**: probability of concluding there is evidence to reject the null hypothesis, when the null is actually false

2.2 Video Notes: Theory-based Inference for a single quantitative variable

Read Chapters 5 and 17 in the course textbook. Use the following videos to complete the video notes for Module 7.

2.2.1 Course Videos

- 17.1
- 17.3TheoryIntervals

2.2.2 Single quantitative variable

- Reminder: review summary measures and plots discussed in the Module 6 material and Chapter 5 of the textbook.
- The summary measure for a single quantitative variable is the ____________________.

Notation:

- Population mean:

- Population standard deviation:

- Sample mean:

- Sample standard deviation:

- Sample size:

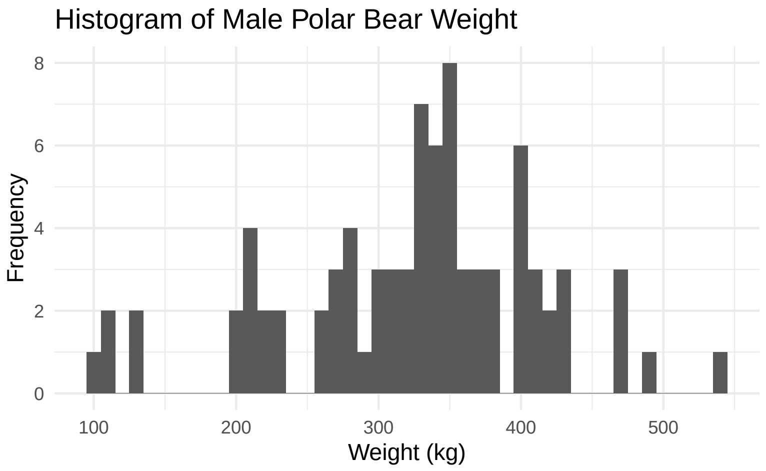
Example: What is the average weight of adult male polar bears? The weight was measured on a representative sample of 83 male polar bears from the Southern Beaufort Sea.

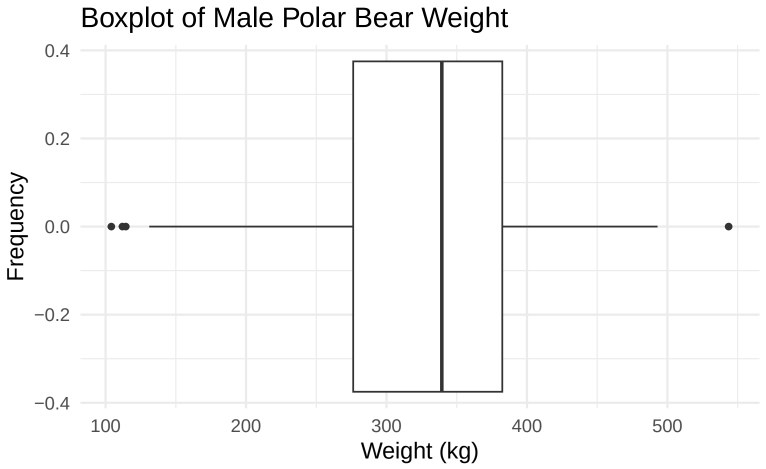
```r
pb <- read.csv("https://math.montana.edu/courses/s216/data/polarbear.csv")
```

Plots of the data:

```r
pb %>%
    ggplot(aes(x = Weight)) +   # Name variable to plot
    geom_histogram(binwidth = 10) +  # Create histogram with specified binwidth
    labs(title = "Histogram of Male Polar Bear Weight", # Title for plot
       x = "Weight (kg)", # Label for x axis
       y = "Frequency") # Label for y axis

pb %>% # Data set piped into...
ggplot(aes(x = Weight)) +   # Name variable to plot
  geom_boxplot() +  # Create boxplot
  labs(title = "Boxplot of Male Polar Bear Weight", # Title for plot
       x = "Weight (kg)", # Label for x axis
       y = "Frequency") # Label for y axis
```





Summary Statistics:

```
pb %>%
  summarise(favstats(Weight)) #Gives the summary statistics
#>     min    Q1 median     Q3   max     mean       sd  n missing
#> 1 104.1 276.3  339.4 382.45 543.6 324.5988 88.32615 83       0
```

Confidence interval

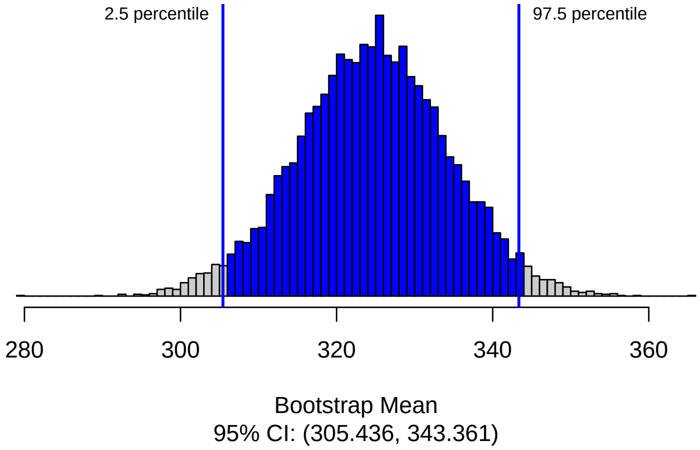
Simulation-based method

- Label cards with the values from the data set
- Sample with replacement (bootstrap) from the original sample n times
- Plot the simulated sample mean on the bootstrap distribution
- Repeat at least 1000 times (simulations)
- Find the cut-offs for the middle X% (confidence level) in a bootstrap distribution.
- ie. 95% CI = (2.5th percentile, 97.5th percentile)

Conditions for inference for a single mean:

- Independence:

```r
set.seed(216)
one_mean_CI(pb$Weight,
  summary_measure = "mean",
  number_repetitions = 10000,
  confidence_level = 0.95)
```



The confidence interval estimates the ______________________ of ____________________________.

Confidence interval interpretation:

- How confident you are (e.g., 90%, 95%, 98%, 99%)
- Parameter of interest
- Calculated interval
- Order of subtraction when comparing two groups

Theory-based method

- Calculate the interval centered at the sample statistic

  statistic ± margin of error

Conditions for inference using theory-based methods:

- Independence:

- Large enough sample size:

T - distribution

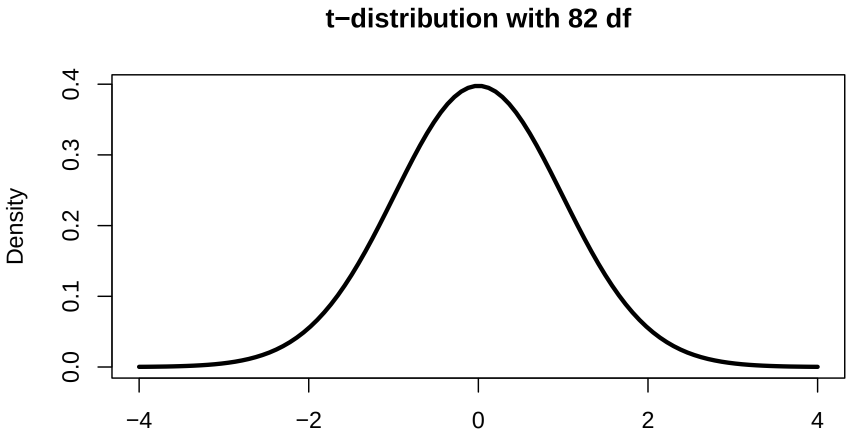
In the theoretical approach, we use the CLT to tell us that the distribution of sample means will be approximately normal, centered at the assumed true mean under H_0 and with standard deviation $\frac{\sigma}{\sqrt{n}}$.

$$\bar{x} \sim N(\mu_0, \frac{\sigma}{\sqrt{n}})$$

- Estimate the population standard deviation, σ, with the ____ standard deviation, ____________.

- For a single quantitative variable we use the ______ - distribution with ______________________ degrees of freedom to approximate the sampling distribution.

The t^* multiplier is the value at the given percentile of the t-distribution with $n - 1$ degrees of freedom.



To find the t^* multiplier for a 95% confidence interval:

```
qt(0.975, df = 82)
#> [1] 1.989319
```

Calculation of the confidence interval for the true mean weight of polar bears from the Southern Beaufort Sea:

2.3 Activity 14: Danceability of Songs

2.3.1 Learning outcomes

- Use simulation methods to find a confidence interval for a single mean
- Use theory-based methods to find a confidence interval for a single mean.
- Interpret a confidence interval for a single mean.
- Use a confidence interval to determine the conclusion of a hypothesis test.

2.3.2 Terminology review

In today's activity, we will estimate the parameter of interest using simulation and theory-based methods. Some terms covered in this activity are:

- Bootstrap distribution
- t-distribution
- Degrees of freedom
- T-score

To review these concepts, see Chapter 15 in the textbook.

2.3.3 Danceability

Spotify created a list of the top songs around the world for the past 10 years and several different audio features of those songs. One of the variables measured on these songs is Danceability. Danceability measures how easy it is to dance to a song; the higher the point value the easier it is to dance to the song. Estimate the average danceability of top songs from Spotify.

- Download the R script file for this activity from D2L and upload to the RStudio server
- Open the R script file, highlight and run

```
#>   min Q1 median Q3 max     mean       sd   n missing
#> 1   0 57     66 73  97 64.37977 13.37872 603       0
```

```
songs %>% # Data set piped into...
    ggplot(aes(x = Danceability)) +   # Name variable to plot
    geom_boxplot() +  # Create boxplot with specified binwidth
    labs(title = "Boxplot of Danceability Score for Top Spotify Songs", # Title for plot
         x = "danceability score (points)", # Label for x axis
         y = "") + # Remove y axis label
    theme(axis.text.y = element_blank(),
          axis.ticks.y = element_blank()) # Removes y-axis ticks
```

Summarizing quantitative variables

1. Describe the boxplot of danceability of top songs over the past 10 years on Spotify.

2. Write the parameter of interest in context of the study.

Simulation methods to create a confidence interval

Unlike creation of the null, the bootstrap distribution is found by sampling with replacement from the original sample.

- Write the original values for the variable on the cards
- Sample with replacement n times
- Plot the mean from each resampled sample on the distribution

Use the provided R script file to find a 95% confidence interval

- Enter the name of the variable for variable
- Enter the appropriate confidence interval
- Highlight and run lines 22–25

```
one_mean_CI(songs$variable, #Enter the name of the variable
            summary_measure = "mean", #choose the mean or median
            number_repetitions = 10000,  # Number of simulations
            confidence_level = xx)
```

3. Report the 95% confidence interval for the parameter of interest.

Theory-based methods to create a confidence interval

- **Conditions for the sampling distribution of $\bar{x}$ to follow an approximate normal distribution**:
  - **Independence**: The sample's observations are independent, e.g., are from a simple random sample. (*Remember*: This also must be true to use simulation methods!)
  - **Large enough sample size: Normality Condition**: The sample observations come from a normally distributed population. To check use the the following rules of thumb:
    - $n < 30$: The distribution of the sample must be approximately normal with no outliers
    - $30 \geq n < 100$: We can relax the condition a little; the distribution of the sample must have no extreme outliers or skewness
    - $n > 100$: Can assume the sampling distribution of $\bar{x}$ is nearly normal, even if the underlying distribuion of individual observational is not

Next we will calculate a theory-based confidence interval. To calculate a theory-based confidence interval for the paired mean difference, use the following formula:

$$\bar{x} \pm t^* \times SE(\bar{x}).$$

We will need to find the t^* multiplier using the function `qt()`.

- Enter the appropriate percentile in the R code to find the multiplier for a 95% confidence interval.

- Enter the df for yy. *The degrees of freedom for a single mean is* $n - 1$
- Highlight and run line 31

```
qt(percentile, df = yy, lower.tail=TRUE)
```

4. Mark on the t-distribution found below the values of $\pm t^*$. Draw a line at each multiplier and write the percentiles used to find each.

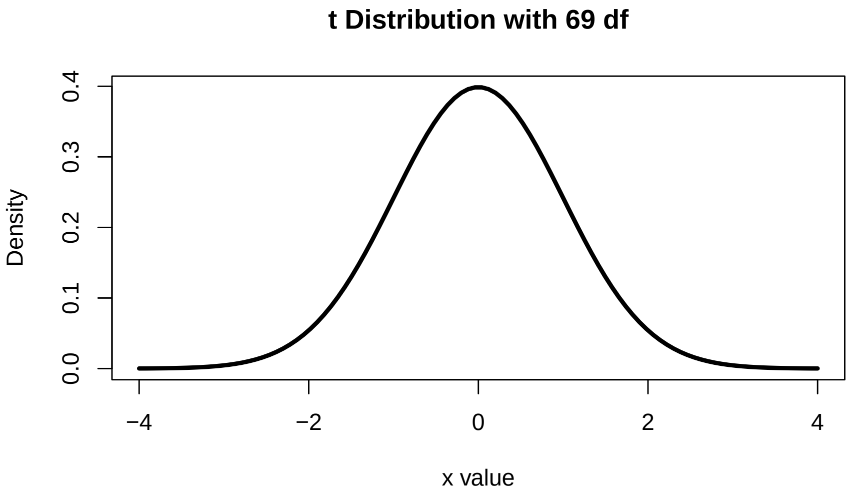


Figure 2.1: t-distribution with 602 degrees of freedom

5. Calculate the margin of error for the true mean using theory-based methods.

6. Calculate the confidence interval for the true mean using theory-based methods.

7. Interpret the confidence interval in context of the study.

8. Explain why the CI with theory-based methods is similar to the simulation CI.

2.3.4 Take-home messages

1. In order to use theory-based methods for a single mean, the independent observational units and normality conditions must be met.
2. The simulation based confidence interval and theory-based confidence interval should be similar if the normality condtion is met.
3. A t^* multiplier is found by obtaining the bounds of the middle X% (X being the desired confidence level) of a t-distribution with $n - 1$ df.

2.3.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered

2.4 Activity 15: Errors and Power

2.4.1 Learning outcomes

- Explain Type I and Type 2 Errors in the context of a study.
- Explain the power of a test in the context of a study.
- Understand how changes in sample size, significance level, and the difference between the null value and the parameter value impact the power of a test.
- Understand how significance level impacts the probability of a Type 1 Error.
- Understand the relationship between the probability of a Type 2 Error and power.
- Be able to distinguish between practical importance and statistical significance.

2.4.2 Terminology review

In this activity, we will examine the possible errors that can be made based on the decision in a hypothesis test as well as factors influencing the power of the test. Some terms covered in this activity are:

- Significance level
- Type 1 Error
- Type 2 Error
- Power

To review these concepts, see Chapter 12 in the textbook.

2.4.3 College Textbook Cost

A college student spends on average $280 on textbooks per year. Many universities have starting using opensource resources to help defray the cost of textbooks. One such university is hoping to show they have successfully reduced costs by $100, on average.

1. Write the parameter of interest (μ) in words, in the context of this problem.

2. Use proper notation to write the null and alternative hypothesis the university would need to test in order to check their claim.

After determining hypotheses and prior to collecting data, researchers should set a **significance level** for a hypothesis test. The significance level, represented by α and most commonly 0.01, 0.05, or 0.10, is a cut-off for determining whether a p-value is small or not. The *smaller* the p-value, the *stronger* the evidence against the null hypothesis, so a p-value that is smaller than or equal to the significance level is strong enough evidence to *reject the null hypothesis*. Similarly, the *larger* the p-value, the *weaker* the evidence against the null hypothesis, so a p-value that is larger than the significance level does not provide enough evidence against the null hypothesis and the researcher would *fail to reject the null hypothesis*. Rejecting the null hypothesis or failing to reject the null hypothesis are the two **decisions** that can be made based on the data collected.

As you have already learned in this course, sample size of a study is extremely important. Often times, researchers will conduct what is called a power analysis to determine the appropriate sample size based on the goals of their research, including a desired **power** of their test. Power is the probability of correctly rejecting the null hypothesis, or the probability of the data providing strong evidence against the null hypothesis *when the null hypothesis is false.*

The remainder of this activity will be spent investigating how different factors influence the power of a test, after which you will complete a power analysis for this physical therapy company.

- Navigate to https://istats.shinyapps.io/power/.
- Choose the tab `Population Mean`
- Use the scale under "Null Hypothesis value μ_0" to change the value to your null value from question 2. *Note we will convert this to a scale $100 dollars. In other words, use the null value of 2.8.
- Change the "Alternative Hypothesis" to the direction you wrote in question 2.
- Leave all boxes un-checked.
- Set the "True value of μ" to 2.8 as well
- Do not change the scales for "Sample size n" or "Type I Error α"

The red distribution you see is the scaled-Normal distribution representing the null distribution for this hypothesis test, if the sample size was 30 and the significance level was 0.05. This means the red distribution is showing the probability of each possible sample mean of college students who spent $280 on textbooks per year ($\bar{x}$) if we assume the null hypothesis is true.

3. Based off this distribution and your alternative hypothesis, give one possible sample mean which you think would lead to rejecting the null hypothesis. Explain how you decided on your value.

4. Check the box for "Show Critical Value(s) and Rejection Region(s)". You will now see a vertical line on the plot indicating the *minimum* sample mean which would lead to reject the null hypothesis. What is this value?

5. Notice that there are some sample means under the red line (when the null hypothesis is true) which would lead us to reject the null hypothesis. Give the range of sample means which would lead to rejecting the null hypothesis when the null hypothesis is true? What is the statistical name for this mistake?

Check the "Type I Error" box under **Display**. This should verify (or correct) your answer to question 5! The area shaded in red represents the probability of making a **Type 1 Error** in our hypothesis test. Recall that a Type 1 Error is when we reject the null hypothesis even though the null hypothesis is true. To reject the null hypothesis, the p-value, which was found assuming the null hypothesis is true, must be less than or equal to the significance level. Therefore the significance level is the maximum probability of rejecting the null hypothesis when the null hypothesis is true, so the significance level IS the probability of making a Type 1 Error in a hypothesis test!

6. Based on the current applet settings, What percent of the null distribution is shaded red (what is the probability of making a Type 1 Error)?

Let's say this university believes their program can reduce the cost of textbooks for college students by $100 per year. In the applet, set the scale under "True value of μ" to 1.8.

7. Where is the blue distribution centered?

The blue distribution that appears represents what the university believes, that \$180 (not \$280) is the true mean textbook cost for college students at this university. This blue distribution represents the idea that the **null hypothesis is false**.

8. Consider the definition of power provided earlier in this lab. Do you believe the power of the test will be an area within the blue distribution or red distribution? How do you know? What about the probability of making a Type 2 Error?

- Check the “Type II Error” and “Power” boxes under **Display**. This should verify (or correct) your answers to question 8! The area shaded in blue represents the probability of making a **Type 2 Error** in our hypothesis test (failing to reject the null hypothesis even though the null hypothesis is false). The area shaded in green represents the power of the test. Notice that the Type 1 and Type 2 Error rates and the power of the test are provided above the distribution.

9. Complete the following equation: Power + Type 2 Error Rate = . Explain why that equation makes sense. *Hint: Consider what power and Type 2 Error are conditional on.*

Now let’s investigate how changes in different factors influence the power of a test.

10. Using the same sample size and significance level, change the “True value of μ” to see the effect on Power.

True value of p	2.0	1.5	1.0	0.05
Power				

11. What is changing about the simulated distributions pictured as you change the “True value of μ”?

12. How does increasing the distance between the null and believed true mean affect the power of the test?

13. Using the same significance level, set the “True value of mu” to 1.8 and change the sample size to see the effect on Power.

Sample Size	20	40	50	60	80
Power					

14. What is changing about the simulated distributions pictured as you change the sample size?

15. How does increasing the sample size affect the power of the test?

16. Using the same "True value of μ", set the sample size to 30 and change the "Type I Error α" to see the effect on Power.

Type I Error α	0.01	0.03	0.05	0.10	0.15
Power					

17. What is changing about the simulated distributions pictured as you change the significance level?

18. How does increasing the significance level affect the power of the test?

19. Complete the power analysis for this university. The university believes they can reduce the cost of textbooks for their students by $100. They want to limit the probability of a type 1 error to 10% and the probability of a type 2 error to 15%. What is the minimum number of students the university will need to collect data from in order to meet these goals? Use the applet to answer this question, then download your image created and upload the file to Gradescope.

20. Based on the goals outlined in question 19, which mistake below is the university more concerned about? In other words, which error were the researchers trying to minimize. Explain your answer.

- Not being able to show their textbook cost is lower, on average, when their textbook cost really is lower.
- Advertising their textbook cost is lower, on average, even though it is not.

2.4.4 Take-home messages

1. There is a possibility of Type I Error when we make the decision to reject the null hypothesis. Type I Error - reject the null hypothesis when the null hypothesis is true.
2. There is a possibility of Type II Error when we make the decision to fail to reject the null hypothesis. Type II Error - fail to reject the null hypothesis when the null hypothesis is false.
3. Increasing the sample size will increase the power of the test.

2.4.5 Additional notes

Use this space to summarize your thoughts and take additional notes on today's activity and material covered.

2.5 Module 6 and 7 Lab: Arsenic

2.5.1 Learning outcomes

2.5.2 Arsenic

Scientists have devised a new way to measure a person's level of arsenic poisoning by examining toenail clippings. Scientists measured the arsenic levels (in parts per million or ppm) in toenail clippings from 19 randomly selected individuals with private wells in New Hampshire (data in the table below). An arsenic level greater than 0.150 ppm is considered hazardous. Is there evidence the ground water in New Hampshire has hazardous levels of arsenic concentration (as seen in the arsenic levels of New Hampshire residents)? How high is the arsenic concentration for New Hampshire residents with a private well?

1. What does μ represent in the context of this study?

2. Notice that there are two research questions for this study. Identify which research question is best answered by finding a confidence interval and which is best answered by completing a hypothesis test?

3. Write out the null hypothesis in proper notation for this study.

4. What sign ($<$, $>$, or $\neq$) would you use in the alternative hypothesis for this study? Explain your choice.

- Upload and open the R script file for Week 12 lab.
- Upload and import the csv file, `arsenic`.
- Enter the name of the data set (see the environment tab) for datasetname in the R script file in line 8.
- Highlight and run lines 1–9 to load the data and create a paired plot of the data.

```
swearing <- datasetname
```

5. Based on the plots, does there appear to be some evidence in favor of the alternative hypothesis? How do you know?

6. What is the value of $\bar{x}$? What is the sample size?

7. **How far, on average, is each arsenic level from the mean arsenic level? What is the appropriate notation for this value?**

Use statistical inferential methods to draw inferences from the data

8. Using the provided graphs and summary statistics, determine if both theory-based methods and simulation methods could be used to analyze the data. Explain your reasoning.

Hypothesis test

Remember that the null distribution is created based on the assumption the null hypothesis is true. In this study, the null hypothesis states that swearing does not affect pain tolerance, or that the length of time a subject kept their hand in the water would be the same whether the patient was swearing or not.

We will use the `one_mean_test()` function in R (in the `catstats` package) to simulate the null distribution of sample mean differences and compute a p-value.

10. Simulate a null distribution and compute the p-value. Using the R script file for this lab, enter your answers for question 9 in place of the xx's to produce the null distribution with 1000 simulations. Highlight and run lines 23–29.

```r
one_mean_test(object$variable,    # Vector of differences
                                  # or data set with column for each group
        shift = xx,    # Shift needed for bootstrap hypothesis test
        as_extreme_as = xx,  # Observed statistic
        direction = "xx",  # Direction of alternative
        number_repetitions = xx,  # Number of simulated samples for null distribution
        which_first = 1)  # Not needed when using calculated differences
```

Sketch the null distribution created using the `paired_test` code.

Communicate the results and answer the research question

11. **Report the p-value. Based off of this p-value and a 1% significance level, what decision would you make about the null hypothesis? What potential error might you be making based on that decision?**

12. Do you expect the 98% confidence interval to contain the null value of zero? Explain.

Confidence interval

We will use the `paired_bootstrap_CI()` function in R (in the `catstats` package) to simulate the bootstrap distribution of sample mean differences and calculate a confidence interval.

13. Using bootstrapping and the provided R script file, find a 98% confidence interval. Fill in the missing values/numbers in the `paired_bootstrap_CI()` function to create the 98% confidence interval. Highlight and run lines 34–37. **Upload a copy of the bootstrap distribution created to Gradescope for your group.**

```r
paired_bootstrap_CI(data = swearing_diff$differences, # Enter vector of differences
                    number_repetitions = 1000, # Number of bootstrap samples for CI
                    confidence_level = xx,  # Confidence level in decimal form
                    which_first = 1)  # Not needed when entering vector of differences
```

Report the 98% confidence interval in interval notation.

14. Interpret the *confidence level* of the interval found in question 12.

15. Write a paragraph summarizing the results of the study. **Upload a copy of your group's paragraph to Gradescope.** Be sure to describe:

- Summary statistic and interpretation
  - Summary measure (in context)
  - Value of the statistic
  - Order of subtraction when comparing two groups
- P-value and interpretation
  - Statement about probability or proportion of samples
  - Statistic (summary measure and value)
  - Direction of the alternative
  - Null hypothesis (in context)
- Confidence interval and interpretation
  - How confident you are (e.g., 90%, 95%, 98%, 99%)
  - Parameter of interest
  - Calculated interval
  - Order of subtraction when comparing two groups
- Conclusion (written to answer the research question)
  - Amount of evidence
  - Parameter of interest
  - Direction of the alternative hypothesis

- Scope of inference
  - To what group of observational units do the results apply (target population or observational units similar to the sample)?
  - What type of inference is appropriate (causal or non-causal)?

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