WWW, WIKIPEDIA AND OSNS

AP

NETWORKS OF HUMANS

Theme: no one controls the evolution of the network, which is self-organizing

What is represented (self, news, opinion, concept) and its **lifecycle** determines the structure and the research questions

look at how they connect and when

Direction of communication is important

```
1 import networkx as nx
2
3 eu_DG = nx.DiGraph()
```

GETTING DATA

WWW

- a Networkx digraph will represent connectivity
- a companion dictionary maps vertices to URLs of the relative pages
- source: a *scrape* of the 2005 ".eu" domain

TWITTER

- supported by the Twython module
- requires Twitter registration/API token
- alternative platforms exist, e.g. Tweety (+NLTK)
- interesting: the network of mentions as a voting system

WIKIPEDIA

- a network of concepts (lemmas/lemmata) maintained by humans (and some bot)
- time-stamped evolution of the network is available [here]
- contrary to *curated* taxonomies, e.g., [Linnaeus, 1735], this is not a tree

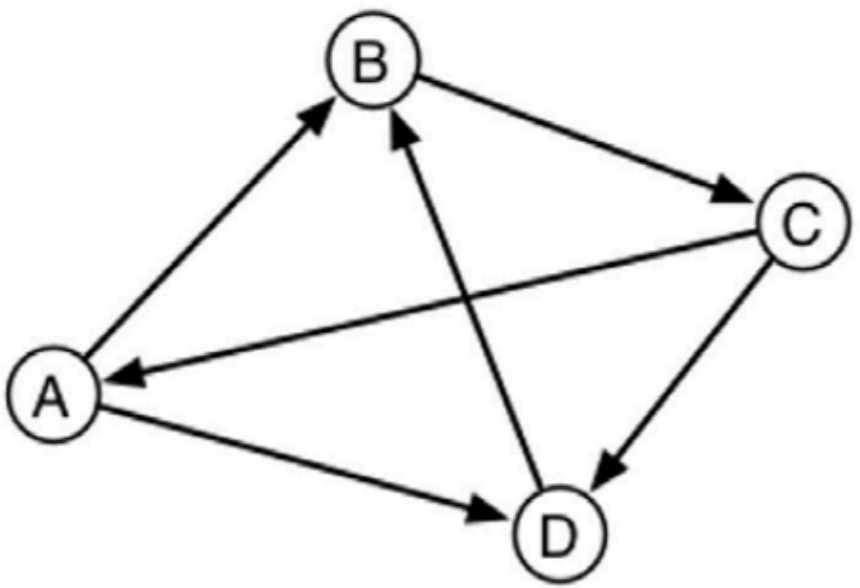
a **directed acyclic graph** is the reference model

RANKING ALGORITHMS: PAGERANK

PAGERANK IDEA

Rank vertices (page) on the basis of their *importance* for network *navigation*

Importance is captured by the relative value in the dominant Eigenvector of a new matrix P that represents *navigation*



	A	B	C	D
A	0	1	0	1
B	0	0	1	0
C	1	0	0	1
D	0	1	0	0

Fig. 4.1 A simple oriented graph with its adjacency matrix.

VARIABLES USED

A: the directed adjacency matrix (admits *dangling* ends)

$K0^{-1}$: 0 everywhere but $\frac{1}{k_i^{\text{out}}}$ on the main diagonal: $(K0^{-1})_{ii} = \frac{1}{k_i^{\text{out}}}$

$N = A \times K0^{-1}$

Matrix E: it models *escape* from the navigation and *forces* irreducibility in the sense of Perron-Frobenius

Set $E_{ii} = \frac{1}{n}$, where $n = |V|$; set 0 everywhere else

(current versions put different values on the main diagonal of E)

$$P = \alpha N + (1 - \alpha)E$$

Experimentally, Page and Brin set $\alpha = 0.85$

I.e., we assign a $1 - \alpha = 0.15$ probability to *jump out* of a navigation *path* and into an arbitrary *restart* node.

Compute the dominant eigenvector $\mathbf{r}_1$ of P

Perron-Frobenius applies so $\mathbf{r}_1$ will be a `proper' vector of real, positive values

Compute $\mathbf{r}_1$ by iterating the $\mathbf{r}_1^{(t+1)} = P\mathbf{r}_1^{(t)}$ eq. until it converges to a fixpoint value.

RANKING ALGORITHMS: HITS

HITS IDEA

Hyperlink-Induced Topic Search by [Kleinberg, 1999]

Sees importance of a node in a more nuanced way:

Pages that are important for consultation, e.g., train schedules, have *authority* and tend to be *terminal*

Well-connected *hub* pages that facilitate navigation, e.g., Time Out, are useful but not authoritative per se

1. authority score $\mathbf{au(i)}$
2. hub score $\mathbf{h(i)}$

HITS AS MUTUAL RECURSION

Hub-iness influences authority which in turns influences hub-iness:

$$au(i) \propto \sum_{j \to i} h(j)$$

page *i* is authoritative proportionally to the sum of the hub-iness of the pages that link to it.

$$h(i) \propto \sum_{i \to j} au(j)$$

page *i* is hub proportionally to the sum of the authoritativeness of pages that it links to.

COMPUTING HITS SCORES

We could start with assigning 1 everywhere and hoping that mutual recursion will converge to stable *au* and *h* values.

As with Von Mises' method, we normalise vectors to 1 at each iteration.

LINEAR ALGEBRA DERIVATIONS

$$\mathbf{h} \propto AA^T\mathbf{h} = \lambda_h AA^T\mathbf{h}$$

$$\mathbf{au} \propto A^TA\mathbf{au} = \lambda_{au} A^TA\mathbf{au}$$

I.e., we can find $\mathbf{h}$ and $\mathbf{au}$ separately by solving the eigenvalue problem for the matrices AA^T and A^TA

MAIN RESULT

For *primitive* matrices (i.e., connected networks, no dead-ends/sinks)

$$\mathbf{h} \propto AA^T\mathbf{h} = \lambda_h AA^T\mathbf{h}$$

$$\mathbf{au} \propto A^TA\mathbf{au} = \lambda_{au} A^TA\mathbf{au}$$

- convergence is assured;
- dominant λ is unique and
- values for $\mathbf{h}$ and $\mathbf{au}$ will be all positive, as desired.

(negative values have no interpretation here)

COMMUNITY DETECTION

FINDING SOCIAL STRUCTURES

this is an example of Provost-Fawcett's problems

- 4: Clustering
- 5: co-occurrence grouping

For homogeneous networks, eg., country-to-county of Ch. 2

Community: nodes that are closely connected with each other by *strenght* or *density*

Resolution limit: communities with less than $\sqrt{|V|}$ members cannot be properly identified.

GIVAN-NEWMAN

1. Rank edges by their *help to connectivity*
2. remove the top-ranking edge
3. repeat until loss of connection
4. now-isolated areas are called communities

Hyp: Betweenness centrality captures *help to connectivity*

MODULARITY

AS AN OPTIMIZATION PROB.

Istance: an adj. matrix A, a small integer g

Solution: a partition of V into g *groups*

Measure: maximise Q: the overall modularity measure

Interpretation: how likely is a random walker to leave the community?

THE Q FACTOR

Let $E_{g \times g}$ be the cross-group matrix and f_i the sum of col. i

Electrical conductance:

$$Q = \sum_{i=1}^{g} e_{ii} - f_i^2$$

Complexity: NP-complete

Even random networks might exibit densifications that might look as c.