CALCULATE ENTROPY

AP

THE SETUP

WHEATHER NEWS

$X_{LDN} \in \{$ snow, showers, light rain, wet, misty, cloudy, breezy, bright, sunny$\}$

$Pr[X_{LDN} = x_i] = [\frac{1}{16}, \frac{1}{8}, \dots \frac{1}{8}, \frac{1}{16}]$

$X_{WH} \in \{$ cloudy w. light rain, sunny$\}$

$Pr[X_{WH} = y_i] = [\frac{1}{8}, \frac{7}{8}]$

LET'S CALCULATE

ENTROPY IN WADI HALFA

$$H[X_{WH}] = \sum Pr[X_{WH} = y_i] \cdot I[y_i]$$

$$H[X_{WH}] = -\sum Pr[X_{WH} = y_i] \cdot \log_2 Pr[X_{WH} = y_i]$$

$$H[X_{WH}] = -\frac{1}{8} \cdot \log \frac{1}{8} - \frac{7}{8} \cdot \log \frac{7}{8}$$

$$= -\tfrac{1}{8} \cdot \log \tfrac{1}{8} - \tfrac{7}{8}[\log 7 - \log 8]$$
$$= \tfrac{1}{8} \cdot \log 8 + \tfrac{7}{8}[\log 8 - \log 7]$$
$$= \tfrac{1}{8} \cdot 3 + \tfrac{7}{8}[3 - 2.8073]$$
$$= \tfrac{3}{8} + \tfrac{7}{8} \cdot 0.2 \text{ (I'm rounding!)}$$
$$= \tfrac{3}{8} + \tfrac{7}{8} \cdot \tfrac{1}{5}$$
$$= [\tfrac{3}{8} \cdot 5 + \tfrac{7}{8}]\tfrac{1}{5}$$
$$= \tfrac{22}{8} \cdot \tfrac{1}{5}$$
$$= 2.75 \cdot 0.2 = 0.55 \approx 1 \text{ bit needed.}$$

ENTROPY IN LONDON

$$H[X_{LDN}] = \sum Pr[X_{LDN} = x_i] \cdot I[x_i]$$
$$H[X_{LDN}] = -\sum Pr[X_{LDN} = x_i] \cdot \log_2 Pr[X_{LDN} = x_i]$$
$$H[X_{LDN}] = -\frac{1}{16} \cdot \log \frac{1}{16} - \frac{1}{8} \cdot \log \frac{1}{8} \dots$$

$$= -2\tfrac{1}{16} \cdot \log \tfrac{1}{16} - 7\tfrac{1}{8} \log \tfrac{1}{8}$$
$$= 2\tfrac{1}{16} \cdot \log 16 + 7\tfrac{1}{8} \log 8$$
$$= \tfrac{1}{8} \cdot \log 16 + \tfrac{7}{8} \log 8$$
$$= \tfrac{1}{8} \cdot 4 + \tfrac{7}{8} \cdot 3$$
$$= \tfrac{25}{8} = 3.125 \approx 4 \text{ bits needed.}$$