

# FINANCIAL NETWORKS

AP

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# INTRODUCTION

Theme: discover a relationship among traded shares (equity)

look at historical market data to see whether price variations relate to each other.

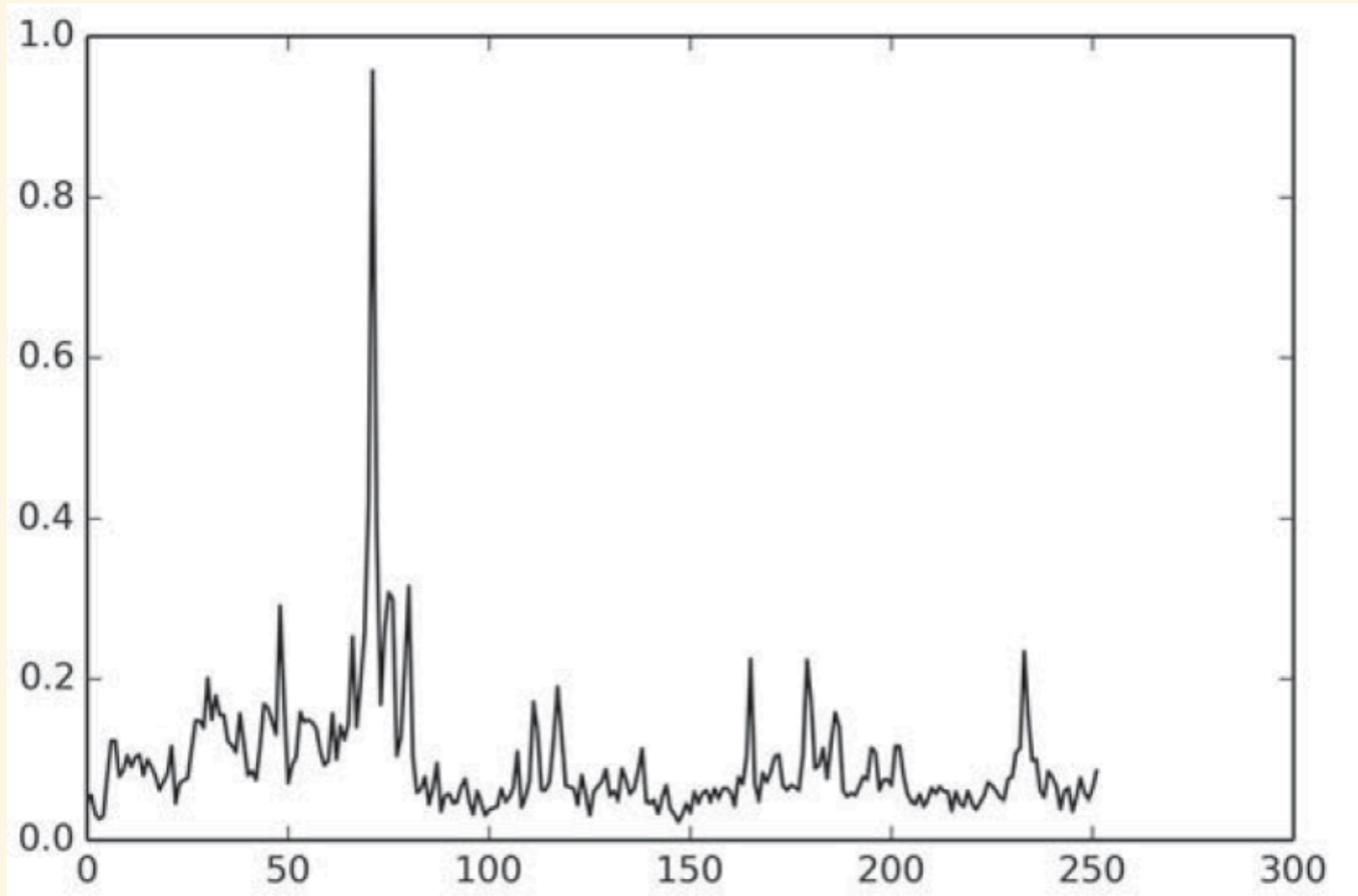
Are there *regularities* that could anticipate the future behaviour of price?

In Food Networks (Ch. 1) we discovered a regularity:

$$\frac{\#pred}{\#prey} \approx 1$$

# IMPORTANT ASSUMPTION

When markets are *calm*, investment becomes somewhat 'mathematical'



# PRICE TIME SERIES

# PROPORTIONAL RETURN ON INVESTMENT

- depends on time
- essentially, the discrete counterpart of the time derivative of price:

$$r(\Delta t) = \frac{p(t_0 + \Delta t) - p(t_0)}{p(t_0)}$$

$$r(\Delta t) = \frac{p(t_0 - \Delta t) - p(t_0)}{p(t_0)}$$

in the limit  $\Delta t \rightarrow 0$  it can be rewritten:

$$r(t) \simeq \frac{d \ln p(t)}{dt}$$

For discrete time:

$$r = \ln p(t_0 + \Delta t) - \ln p(t_0)$$

# CORRELATION OF PRICES

- correlations in time series (or simply *comovements*) are valuable indicators
- Two shares are correlated if historically their price varied *in a similar way*.
- To qualify such a relation compute the correlation between their price returns over  $\Delta t$ .

Let  $\langle r_i \rangle$  be the average return of  $i$  over  $\Delta t$



$$\rho_{ij}(\Delta t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}$$

- high  $\rho$ 's might uncover hidden links between stocks.
- however, monitoring  $n(n - 1)$  correlations quickly becomes unfeseable
- we focus on high  $\rho$  values.

# THE SPANNING TREE OF STOCKS

# SIMILAR-BEHAVIOUR SHARES

Correlation (or lack of it) induces a *distance* b/w shares:

$$d_{ij}(\Delta t) = \sqrt{2(1 - \rho_{ij}(\Delta t))}$$

Let  $D(\Delta t)$  be the complete matrix of pairwise distances:

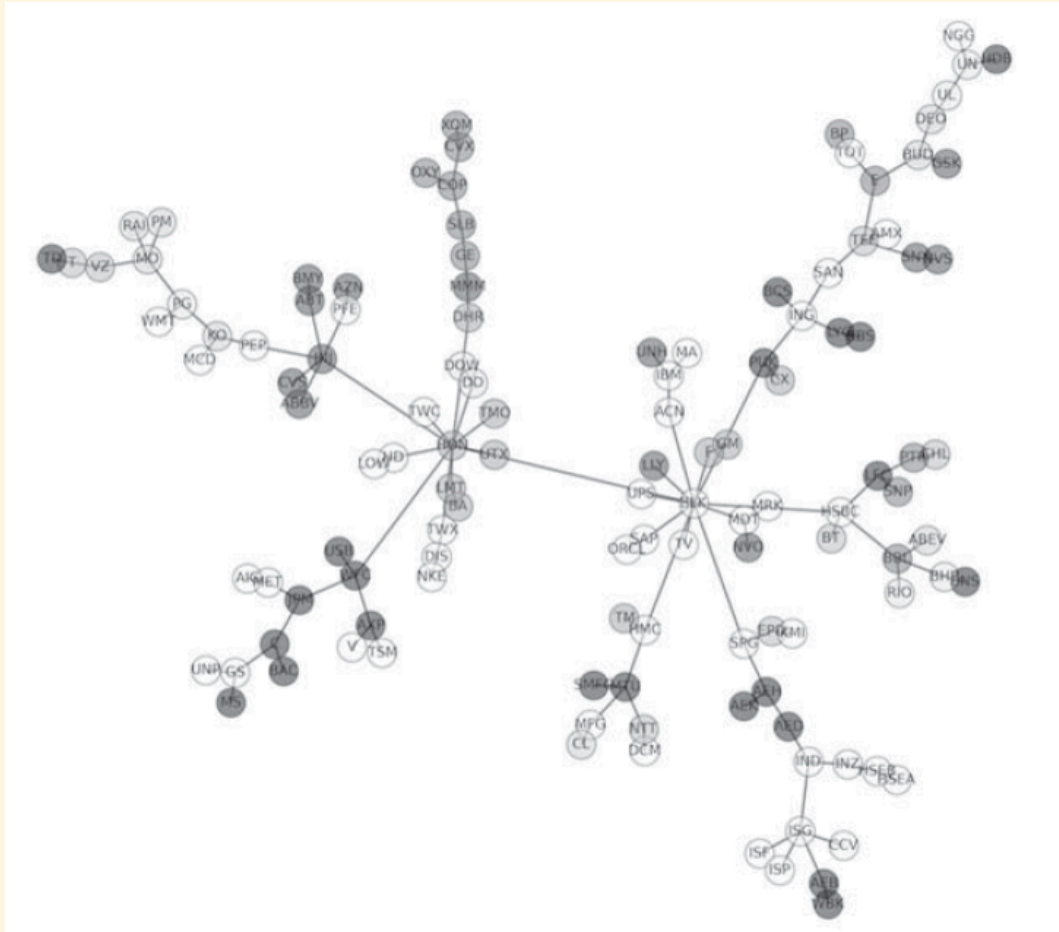
it describes a complete, weighted network!

*Prune* it to create its Minimum Spanning Tree (MST)

The MST has only  $n-1$  *heavy* connections while maintaining connectivity.

# RESULTING MODEL

The MST of 141 NYSE high-cap stocks,  $\Delta t = 6\text{h}:30\text{min}$



Some shares are *hubs* for local clusters of highly-correlated shares.

# CONSEQUENCES

- Network analysis helps indentifying local clusters
- Each clusters will have a *hub share* at its center
- Hub shares can *signal* the beaviour of the whole cluster:
- they provide leads in forecasting how sections of the market will move.