FINANCIAL NETWORKS

AP

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INTRODUCTION

Theme: discover a relationship among traded shares (equity)

look at historical market data to see whether price variations relate to each other.

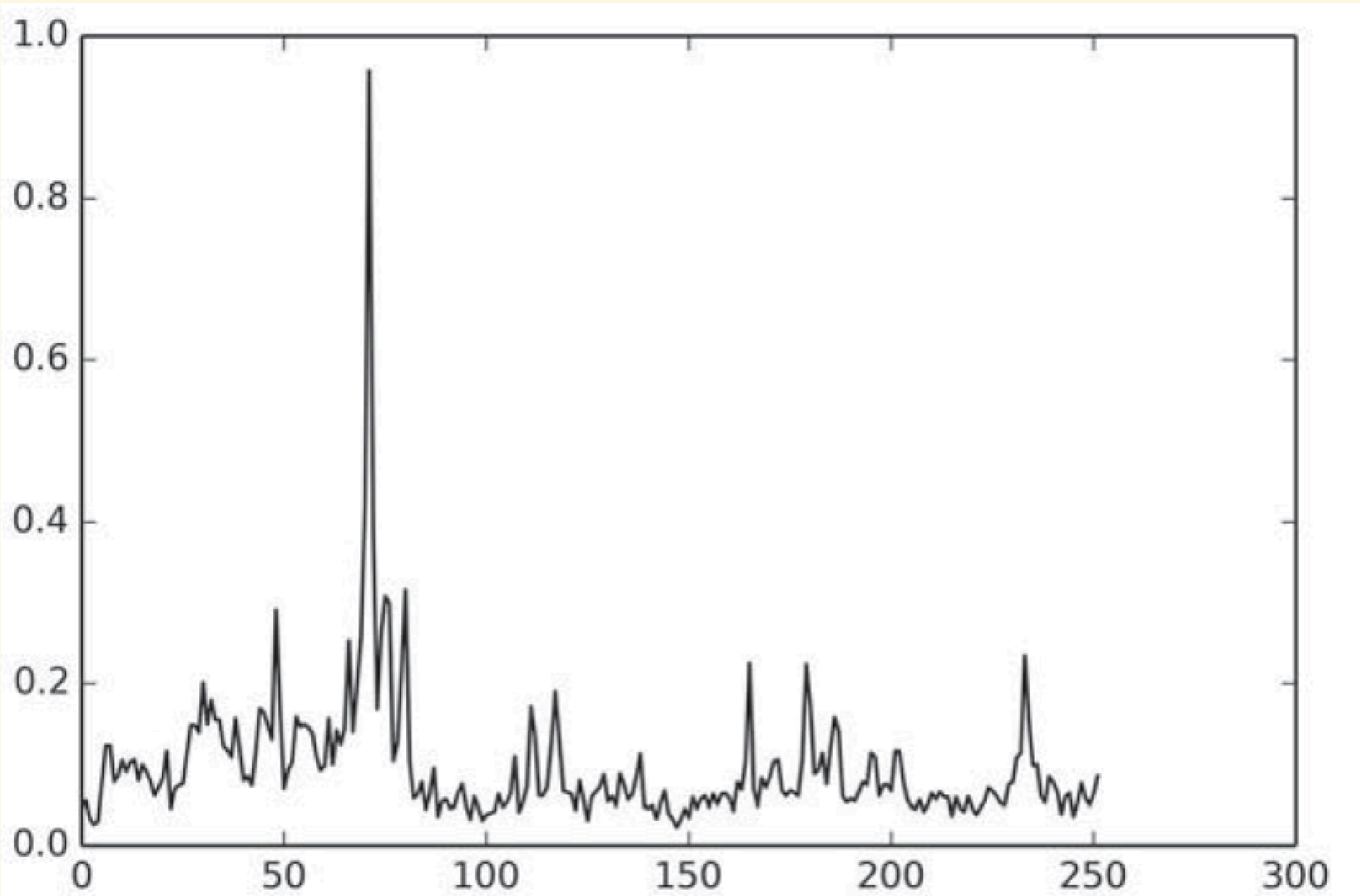
Are there *regularities* that could anticipate the future behaviour of price?

In Food Networks (Ch. 1) we discovered a regularity:

$$\frac{\#pred}{\#prey} \approx 1$$

IMPORTANT ASSUMPTION

When markets are *calm*, investment becomes somewhat `mathematical’



PRICE TIME SERIES

PROPORTIONAL RETURN ON INVESTMENT

- depends on time
- essentially, the discrete counterpart of the time derivative of price:

$$r(\Delta t) = \frac{p(t_0 + \Delta t) - p(t_0)}{p(t_0)}$$

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as $\Delta t \to 0$ returns tend to 0

We can apply the well-known Taylor expansion:

$$\ln(1 + x) = x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \dots$$

take only the first element of the expansion, x, and substitute:

$$\ln(1 + r(\Delta t)) \approx r(\Delta t)$$

$$r(\Delta t) \approx \ln\left(1 + \frac{p(t_0 + \Delta t) - p(t_0)}{p(t_0)}\right)$$

$$r(\Delta t) \approx \ln \frac{p(t_0) + p(t_0 + \Delta t) - p(t_0)}{p(t_0)}$$

$$= \ln \frac{p(t_0 + \Delta t)}{p(t_0)} = \ln p(t_0 + \Delta t) - \ln p(t_0)$$

in conclusion,

$$r(\Delta t) \approx \ln p(t_0 + \Delta t) - \ln p(t_0)$$

Also, Quant Finance considers the *instantaneous* returns function as the derivative of the logged prices:

$$r(t) \simeq \frac{d \ln p(t)}{dt}$$

CORRELATION OF RETURNS

- correlations in time series (or simply *comovements*) are valuable indicators
- Two shares are correlated if historically they *moved in a similar way.*
- To qualify such a relation compute the correlation between their price returns over Δt.

Let $\langle r_i \rangle$ be the average return of *i* over Δt

$$\rho_{ij}(\Delta t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{(\langle r_i^2 \rangle - \langle r_i \rangle^2)(\langle r_j^2 \rangle - \langle r_j \rangle^2)}}$$

- high ρ's might uncover hidden links between stocks.
- however, monitoring $n(n-1)$ correlations quickly becomes unfeseable
- we focus on high ρ values.

THE SPANNING TREE OF STOCKS

SIMILAR-BEHAVIOUR SHARES

Correlation (or lack of it) induces a *distance* b/w shares:

$$d_{ij}(\Delta t) = \sqrt{2(1 - \rho_{ij}(\Delta t))}$$

Let $D(\Delta t)$ be the complete matrix of pairwise distances:

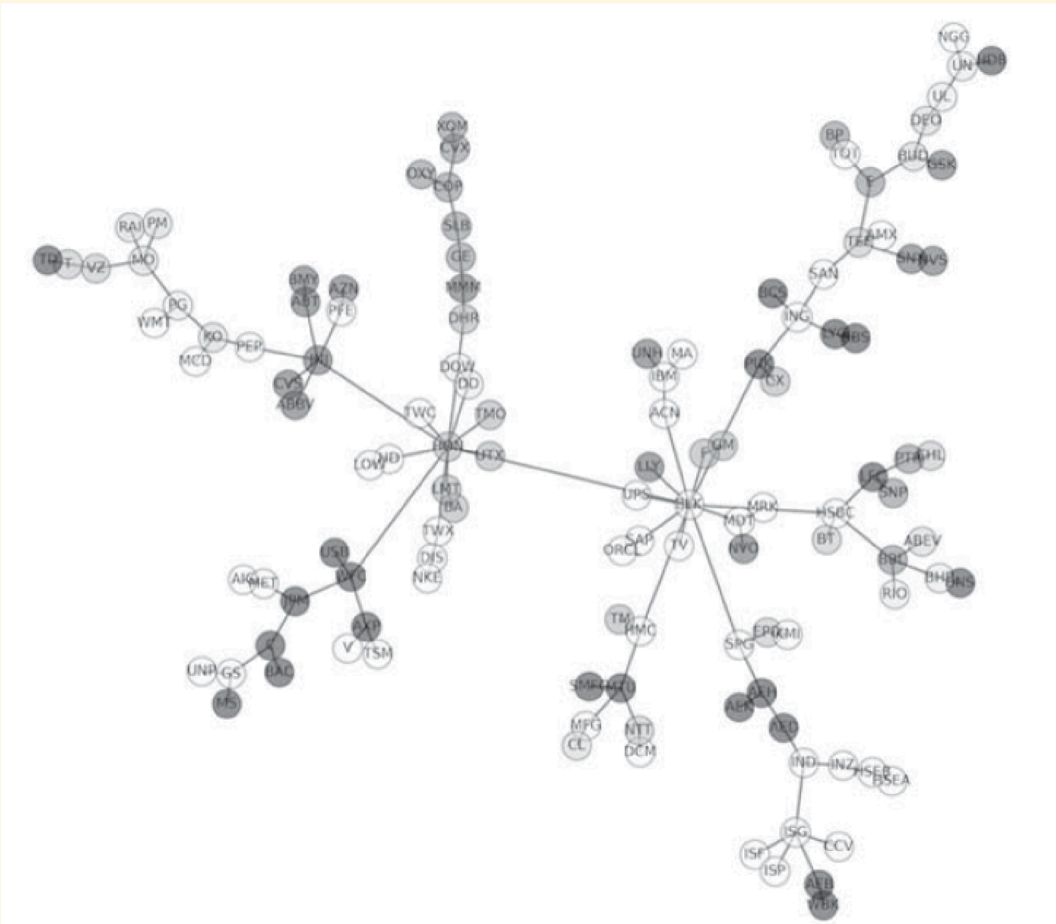
it describes a complete, weighted network!

Prune it to create its Mimimum Spanning Tree (MST)

The MST has only n-1 *heavy* connections while maintaining connectivity.

RESULTING MODEL

The MST of 141 NYSE high-cap stocks, $\Delta t = $ 6h:30min



Some shares are *hubs* for local clusters of highly-correlated shares.

CONSEQUENCES

- Network analysis helps indentifying local clusters
- Each clusters will have a *hub share* at its center
- Hub shares can *signal* the beaviour of the whole cluster:
- they provide leads in forecasting how sections of the market will move.