

Generation of Zoomable maps with Rivers and Fjords

Extended Abstract

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This paper presents a method for generating maps with rivers and fjords. The method is based on recursive subdivision of triangles and allows unlimited zoom on details without requiring generation of a full map at high resolution.

CCS Concepts: •**Computing methodologies** → **Computer graphics**;

Additional Key Words and Phrases: Procedural content generation, maps, rivers, zoom

ACM Reference format:

Anonymous Author(s). 2022. Generation of Zoomable maps with Rivers and Fjords. 1, 1, Article 1 (January 2022), 12 pages. DOI: 10.1145/nnnnnnnn.nnnnnnn

1 INTRODUCTION

Procedurally generated maps are often used in computer games, where they allow variability when playing a game multiple times. For strategy games, a full map of the entire playing area is typically generated, but for first-person-view games, storing a full map with all details can be unrealistic, so details are typically generated on the fly. This requires *zoomable* maps, where a high-resolution local map can be generated without having to generate the full map at this resolution. You may even want to generate different levels of detail for the near and far parts of the map. A similar requirement exists for pen-and-paper role-playing games, where you might want both a world map and local maps for individual islands and continents.

Many algorithms exist for generating topographical maps, basically generating altitude information for every point and then colouring the points according to some colour map. Examples include the diamond-square algorithm [4], the square-square algorithm [7], Fourier-transform-based methods [1], great circle displacement [2, 9, 12], simulated plate tectonics [3], and tetrahedral subdivision [8]. Most of these suffer from lack of zoomability: Generating a high resolution image of a small section of a map requires almost as much computation time as generating a high-resolution image of the full map. Exceptions are methods based on recursive subdivision such as diamond-square and tetrahedral subdivision.

None of these methods naturally support generation of rivers. For any fixed-size map, rivers can be placed after the map is generated, but this prevents zooming, as rivers can not be consistently placed on a local map without knowing their placement on the global map – we need, at least, to know where rivers flow into the selected map fragment. So the only zoom option is to pregenerate the full map at the resolution of the highest zoom and then show only the selected part. But that is very costly. Ideally, a zoomed map should be generated at a cost which is close to proportional to the size (in pixels) of the shown portion of the map and not of the full map.

Generation of realistic river networks using Strahler-Horton analysis has been proposed [11], but these do not follow any terrain. The challenge is to co-generate terrain and rivers while being zoomable.

This paper shows a method for doing just that. The method is based on triangle subdivision, so it suffers from the same kind of artefacts as the diamond-square algorithm, but it does generate zoomable maps with

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DOI: 10.1145/nnnnnnnn.nnnnnnn

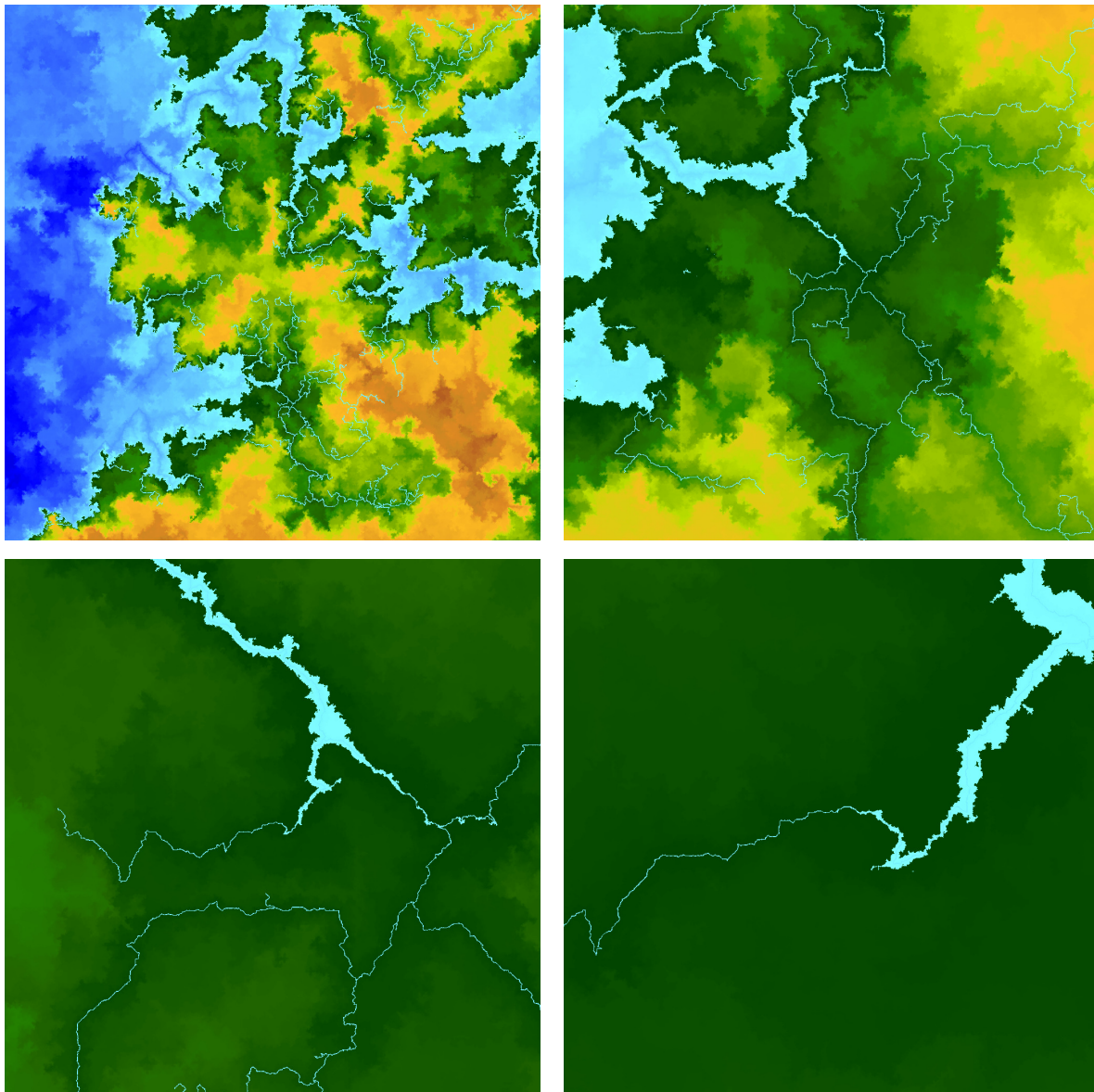
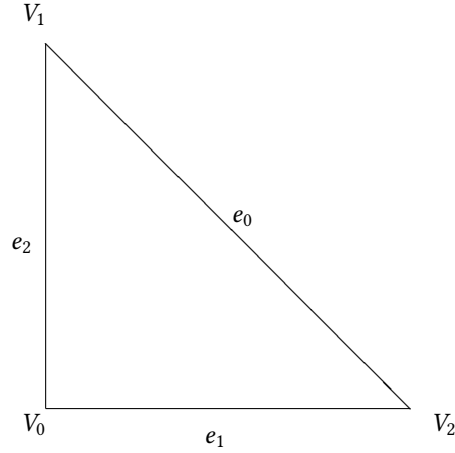
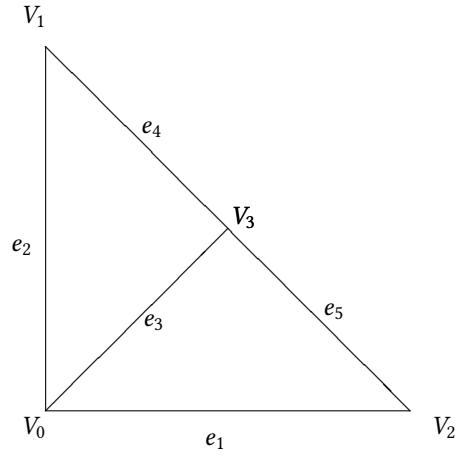


Fig. 1. Map showing rivers and fjords at zoom 1, 5, 25 and 125

decent-looking river networks and fjords, which to the authors' knowledge has not been done previously. Figure 1 shows an example of such a map at four different degrees of zoom. On the top-left picture it is clear that the river network actually continues as trenches in the fjord.



(a) Isosceles, right-angled triangle.



(b) Subdivision of triangle.

Fig. 2. Triangle Subdivision

2 MAP GENERATION BY TRIANGLE SUBDIVISION

The first step is to present a simple map generation method using triangular subdivision. We use isosceles, right-angled triangles as shown in Figure 2(a). We divide this into two half-size triangle by drawing a new edge e_3 from the vertex V_0 to the middle of the opposite edge e_0 , splitting this into two new edges e_4 and e_5 , as shown in Figure 2(b). The new vertex at the middle of e_0 is called V_3 .

Each vertex is represented by a record that contains the coordinates (x, y) of the vertex (each ranging from 0.0 to 1.0), the altitude h at the vertex (ranging from -1.0 to 1.0), and a pseudorandom value s in the range -1.0 to 1.0. We generate the attributes of V_3 by the formulae:

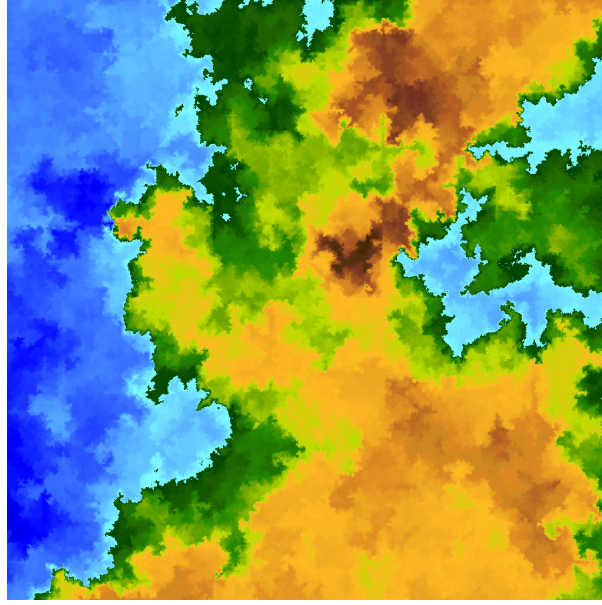


Fig. 3. Map generated by triangle subdivision

$$\begin{aligned}
 V_3.x &= (V_1.x + V_2.x)/2 \\
 V_3.y &= (V_1.y + V_2.y)/2 \\
 V_3.s &= \mu(V_1.s, V_2.s) \\
 \delta &= k_1|V_1 - V_2| + k_2|V_1.h - V_2.h| \\
 V_3.h &= (V_1.h + V_2.h)/2 + \delta V_3.s
 \end{aligned}$$

where μ is a mixing function that takes two pseudorandom values and produces a new pseudorandom value. It must be the case that $\mu(s_1, s_2) = \mu(s_2, s_1)$ for all s_1 and s_2 . We will also use $v(s) = \mu(s, s)$.

δ is a measure of how much the altitude can differ from the average of $V_1.h$ and $V_2.h$. k_1 and k_2 are constants that determine how much the altitude varies as a function of the distance between the vertices ($|V_1 - V_2|$) and the altitude difference between the vertices ($|V_1.h - V_2.h|$), respectively. For the examples used in this paper, we use $k_1 = 0.32$ and $k_2 = 0.55$. It is possible (though unlikely) that this gives a $V_3.h$ outside the range -1.0 to 1.0. If this happens, we cap it to be inside the range.

Note that $V_0.h$ is not used when finding $V_3.h$. This is because the edge e_0 can be shared with another triangle that does not have access to V_0 . This is the main difference between this method and the diamond-square method: Only two altitudes are averaged when finding the altitude of a new point, where the diamond-square method averages four altitudes when finding the altitude of a new point.

We continue recursively subdividing the triangle into ever-smaller triangles, until a triangle is the size of one pixel. At this point, we use $V_3.h$ to colour the corresponding point on the map. The maps shown in this paper uses blue shades for negative altitudes and green, brown, and white shades for positive altitudes. A map produced by this simple algorithm is shown in Figure 3.

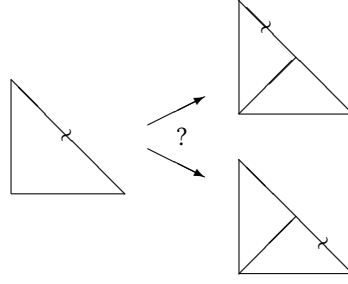


Fig. 4. When there is a river on the long edge

3 ADDING RIVERS

We will now extend the simple map generator to include rivers. We will do so using the edges of the triangle: An edge e_i has an *optional* altitude attribute $e_i.h$. If this attribute is present, it indicates that *somewhere* on edge e_i a river crosses at altitude $e_i.h$. If the attribute is absent, no river flows through the edge. When subdividing a triangle, we use the attributes on e_0 , e_1 and e_2 together with the attributes on V_0 , V_1 and V_2 to generate the attributes for e_3 , e_4 , and e_5 . Moreover, we allow the altitude attribute $e_0.h$ to influence $V_3.h$. Basically, we want that, when the subdivision finishes, $V_3.h$ denotes the altitude of the corresponding point of the river (if any).

3.1 The long edge

We recall that the attributes of V_3 , e_4 and e_5 can only depend on the attributes of V_1 , V_2 and e_0 , since the edge e_0 can be shared with another triangle that does not have access to V_0 , e_1 and e_2 . So let us start with V_3 , e_4 and e_5 and then handle e_3 later.

We noted that e_0 has an attribute $e_0.h$ if and only if a river crosses e_0 . If this is the case, we have to choose whether this river crosses the half-edge e_4 or the half-edge e_5 , as shown in Figure 4. We do not allow a river to cross both e_4 and e_5 , as that could potentially lead to circular river segments that do not connect to sea. We use a simple method for this: If $e_0.h$ is closer to $V_1.h$ than to $V_2.h$, the river flows through e_4 , otherwise it flows through e_5 . The attribute of e_0 is copied to the chosen half-edge, and the other half-edge has no attribute. Ties can be broken by looking at the other attributes of V_1 and V_2 , but ties are extremely rare when floating-point numbers are used. If a river flows through e_4 , $e_4.h$ is used instead of $V_1.h$ when determining $V_3.h$, and if a river flows through e_5 , $e_5.h$ is used instead of $V_2.h$. If no river flows through e_0 , we determine $V_3.h$ as before. We can describe the determination of e_4 , e_5 , and $V_3.h$ with the following pseudocode:

```

if  $e_0$  has an attribute  $e_0.h$ , then
  if  $|e_0.h - V_1.h| < |e_0.h - V_2.h|$  then
     $e_4.h = e_0.h$ ;
     $e_5.h = \text{Absent}$ ;
     $V_3.h = (e_4.h + V_2.h)/2 + \delta V_{3.s}$ 
  else
     $e_5.h = e_0.h$ ;
     $e_4.h = \text{Absent}$ ;
     $V_3.h = (e_5.h + V_1.h)/2 + \delta V_{3.s}$ 
else
   $e_5.h = \text{Absent}$ ;
   $e_4.h = \text{Absent}$ ;
   $V_3.h = (V_1.h + V_2.h)/2 + \delta V_{3.s}$ 

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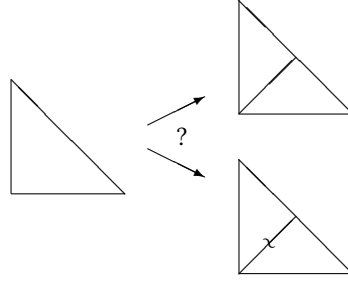


Fig. 5. When there is no river on any outer edge

The other attributes of V_3 are calculated as before.

3.2 The short edge e_3

The complicated part is determining the attribute (if any) of e_3 . Note that this can (and does) depend on all vertices and all edges.

3.2.1 No surrounding edge has a river. We start with the case where there is no river on e_1, e_2, e_4 or e_5 . We then need to determine if a river should flow through e_3 or not, as illustrated in Figure 5. If V_1 is above water and V_2 is below water, there is a possibility of a river flowing from somewhere in the triangle V_1, V_0, V_3 to somewhere in the triangle V_2, V_0, V_3 , but only if there is no closer exit. So we rule that both V_0 and V_3 should be higher than V_2 . We also prefer rivers with some steepness over very flat rivers, so we want V_1 to be higher above water than V_2 if they are far apart than if they are close. Additionally, we want the height of the river at e_3 to be between $V_2.h$ and the minimum of $V_0.h$ and $V_3.h$. In pseudocode, this is

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if  $V_1.h > k_3 \wedge V_2.h < k_4 \wedge V_2.h < V_0.h \wedge V_2.h < V_3.h$ 
then
   $e_3.h = \beta(V_2.h, \min(V_0.h, V_3.h), \mu(V_0.s, V_3.s));$ 
else
   $e_3.h = \text{Absent};$ 

```

Where k_3 and k_4 are the minimal above-land and below-sea altitudes for $V_1.h$ and $V_2.h$. In the examples in this paper, we have use $k_3 = 0.1$ and $k_4 = -0.1$. $\beta(h_1, h_2, s)$ finds an altitude between h_1 and h_2 using the pseudorandom value s . This should tend towards the middle of the interval, so (since $-1 \leq s \leq 1$), we define $\beta(x, y, s) = (x + y + s^3(x - y))/2$. Other definitions can be used as long as the result tends towards the average of x and y .

We have in the examples shown here chosen that there is always a river if the preconditions are met, but you can add a pseudorandom element if you want fewer rivers.

The case where V_1 is below water and V_2 is above water is handled symmetrically.

3.2.2 One surrounding edge has a river. If only one of the edges e_1, e_2, e_4 and e_5 is crossed by a river, we want to see if we should extend this through e_3 . We show the case if a river passing through e_1 (s shown in Figure 6, as the other cases are symmetric.

If there is a river through e_1 , $V_1.h < 0$, and $V_1.h < e_1.h$, the river might flow down towards V_1 , but only if there is not a shorter path to the sea near V_3 . So if $V_1.h < 0$, $V_1.h < e_1.h$ and $V_3.h > 0$, we add a river to e_3 with altitude between that of e_1 and V_1 , so, $e_3.h = \beta(e_1.h, V_1.h, \mu(V_3.s, V_0.s))$.

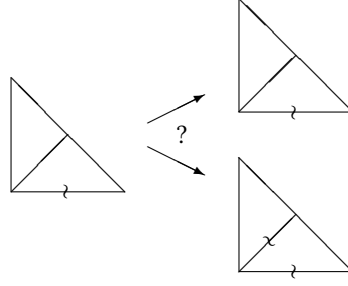


Fig. 6. When there is a river on one outer edge

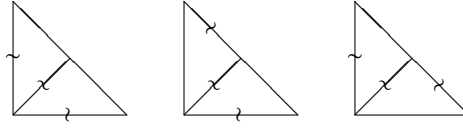


Fig. 7. When there are rivers on two opposing outer edges

If, on the other hand, $V_1.h > e_1.h$, there is a possibility that the river flows from somewhere in the triangle V_1 , V_0 , V_3 towards e_1 , but only if all three vertices are higher up than e_3 . Also, we might not want to always extend the river all the way to the top of a mountain, so we add a chance that it will not extend across e_3 .

In pseudocode, we handle the case of a river only through e_1 by

```

if  $V_1.h < 0 \wedge V_1.h < e_1.h \wedge V_3.h > 0$  then
   $e_3.h = \beta(V_1.h, e_1.h, \mu(V_0.s, V_3.s))$ ;
else if  $V_1.h > e_1.h \wedge V_0.h > e_1.h \wedge V_3.h > e_1.h$ 
  then if  $|\mu(V_0.s, V_3.s)| < k_5$  then
     $e_3.h = \beta(e_1.h, \min(V_1.h, V_0.h, V_3.h, v(V_1.s)))$ ;
  else
     $e_3.h = \text{Absent}$ ;
else
   $e_3.h = \text{Absent}$ ;

```

Where k_5 is the probability that we extend a river upwards if the preconditions are met. In the examples used in this paper, $k_5 = 0.7$.

There are symmetric cases for rivers going through e_2 , e_4 , or e_5 . Where we use V_3 , above, we use V_0 when extending rivers through e_4 or e_5 . I.e., we do not use a vertex on the same edge as the river.

3.2.3 Two surrounding edges have rivers. If the two edges that have rivers are on opposite sides of e_3 , the case is simple: There *will* be a river on e_3 and its altitude is between that of the two other river edges. There are three such cases: rivers on e_1 and e_2 , rivers on e_1 and e_4 , and rivers on e_2 and e_5 (since there can not be rivers on both e_4 and e_5). We illustrate these cases in Figure 7.

The case where the two river-carrying edges are on the same side of e_3 is more interesting. It is clear that a river flows from one edge to the other without crossing e_3 , but there is a possibility that the river branches with a branch entering through e_3 , as illustrated in figure 8. This branch must be higher up than the lowest of the river-carrying outside edges, as we do not allow a river to split downwards.¹ We want all three vertices of the

¹This can happen in river deltas, but we do not handle this case here.

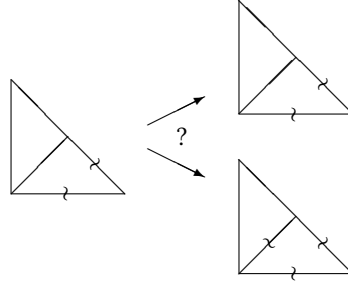


Fig. 8. Potentially branching river

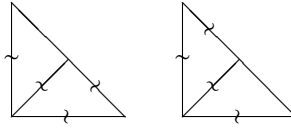


Fig. 9. When there are rivers on three outer edges, they must connect

triangle on the opposite edge of e_3 to be higher than the lowest of the river-carrying outside edges, and the river on e_4 must have an altitude between the lowest of the three vertices and the lowest of the river-carrying outside edges. Additionally, we might not always want a branch, so we add a probability.

The case where the edges e_1 and e_5 carry rivers is shown in pseudocode below. The case where the edges e_1 and e_5 carry rivers is symmetric.

```

if  $\min(V_1.h, V_0.h, V_3.h) > \min(e_1.h, e_5.h)$ 
then if  $|\mu(V_0.s, V_3.s)| < k_6|V_1 - V_2|$  then
   $e_3.h = \beta(\min(V_1.h, V_0.h, V_3.h), \min(e_1.h, e_5.h), v(V_1.s))$ 
else
   $e_3.h = \text{Absent};$ 
else
   $e_3.h = \text{Absent};$ 

```

where $k_6|V_1 - V_2|$ is the probability that the river branches if the preconditions are met. Note that this means that, on a larger scale, rivers are more likely to branch than on a smaller scale. We use $k_6 = 2$.

3.2.4 Three surrounding edges have rivers. This case is also relatively simple: There is always a river crossing e_3 , as illustrated in Figure 9. The only nontrivial part is finding the altitude of this river. The two higher river edges will flow towards the lowest river edge, so the river on e_3 must be higher than the lowest river edge but lower than the river(s) on the opposite side of the lowest river.

The case where there are rivers on e_1 , e_2 , and e_4 is handled by the pseudocode

$$e_3.h = \beta(e_1.h, \min(e_2.h, e_4.h))$$

The case where there are rivers on e_1 , e_2 , and e_5 is handled symmetrically.

There are no cases with rivers on both e_4 and e_5 , so we have handled all possible cases.

Figure 1 shows a map generated by these rules. Compare the top-left image to Figure 3, which is the same map but without rivers. Note how the rivers “carve” fjords and river valleys into the land. Fjords are generated when the river has an altitude that is below sea level and “pulls” nearby land below sea as well.

4 VARIANT: ISLANDS IN FJORDS

If we allow rivers on both halves (e_4 and e_5) of a split long edge (e_0) of the isosceles triangle, it can potentially lead to disconnected circular rivers. But if we do so only if $e_0.h < k_7$, for some negative k_7 (the examples use $k_7 = -0.1$), such circularity can only happen in fjords. So instead of circular rivers, we get islands in fjords and occasional narrow straits crossing land. Using this variant, we get the maps shown in Figure 10. The islands are in the fjord near the top of the map.

We have to modify splitting of e_0 to

```

if  $e_0$  has an attribute  $e_0.h$ , then
  if  $e_0.h < k_7$  and  $|v(e_0.s)| < k_8$  then
     $e_4.h = e_0.h$ ;
     $e_5.h = e_0.h$ ;
     $V_3.h = (e_4.h + e_5.h + V_2.h)/3 + \delta V_3.s$ 
  else if  $|e_0.h - V_1.h| < |e_0.h - V_2.h|$  then
     $e_4.h = e_0.h$ ;
     $e_5.h = \text{Absent}$ ;
     $V_3.h = (e_4.h + V_2.h)/2 + \delta V_3.s$ 
  else
     $e_5.h = e_0.h$ ;
     $e_4.h = \text{Absent}$ ;
     $V_3.h = (e_5.h + V_1.h)/2 + \delta V_3.s$ 
else
   $e_5.h = \text{Absent}$ ;
   $e_4.h = \text{Absent}$ ;
   $V_3.h = (V_1.h + V_2.h)/2 + \delta V_3.s$ 

```

where k_8 is a probability of copying the river. We have used $k_8 = 0.15$. It is not entirely realistic that $e_4.h = e_5.h$, but if we change one or the other, we can get rivers flowing upward.

We also need to add cases where there are rivers on both e_4 and e_5 . If no other edges or all other edges have rivers, we do not add a river on e_3 . If one other edge (e_1 or e_2) has a river, we add a river on e_3 with an altitude between $e_1.h$ and $e_4.h$ or between $e_2.h$ and $e_5.h$, depending on whether e_1 or e_2 has a river.

5 VARIANT: PENROSE TILES

To reduce the axis-aligned artefacts, we can use Penrose tiles [5, 6, 10], which form aperiodic tilings. We use the P2-tiling that uses “kites and darts”, both formed by joining two Robinson triangles: isosceles triangles where the ratio of the long and short sides is the golden ratio $\phi = \frac{\sqrt{5}+1}{2}$. These can be subdivided into smaller Robinson triangles as shown in Figure 11. The divided edge is split at a ratio of ϕ to 1. Since the split is uneven, mirroring matters, so we have indicated the orientation of triangles with arrows. Two of the top-type (acute) triangles form a kite shape, and a map in this shape created by subdividing these is shown in Figure 12. We have used the same rules for river placement as for the right-angles triangles, but the altitude of a new vertex on a split edge without rivers use a weighted average of the endpoints (with random displacement), so the altitude of the new point is (statistically) closer to the nearest of its neighbours. We use

$$V_3.h = (\phi - 1)V_1.h + (2 - \phi)V_2.h + \delta V_3.s$$

where V_1 is the vertex closest to the cut point.

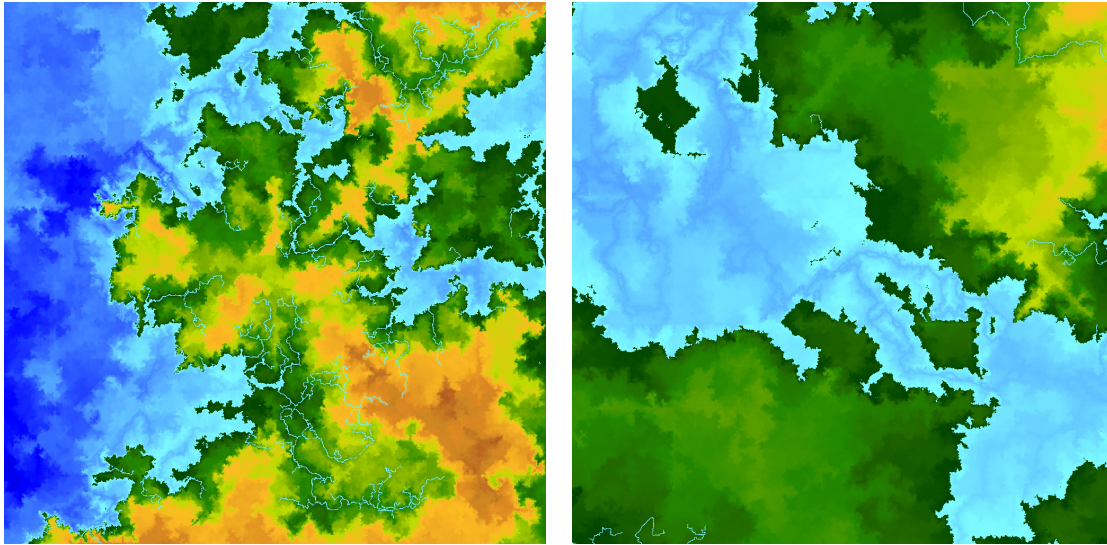


Fig. 10. Islands in fjords

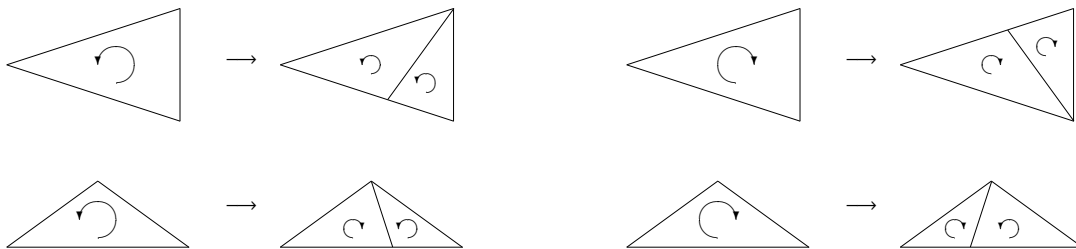


Fig. 11. Subdividing Robinson triangles

6 ASSESSMENT

The method generates zoomable maps with rivers that look somewhat realistic, but they suffer from the same problems that maps generated by recursive subdivision usually do: There are some artefacts due to the chosen grid, and the rivers do not always flow naturally: They do not always take the steepest route down, and sometimes there are no rivers where you would expect rivers to be. Only altitudes and rivers are generated, other details such as vegetation and climate are not handled.

Nevertheless, we believe the maps generated by the method can be used for gaming – both computer and table-top games – possibly with extensions for creating vegetation and climate details.

The values for the constants k_1, \dots, k_6 are chosen somewhat arbitrarily by looking at generated maps and see what “looks good”. There are possibilities for varying these and even adding more “magic” constants to modify the statistics of the produced maps.

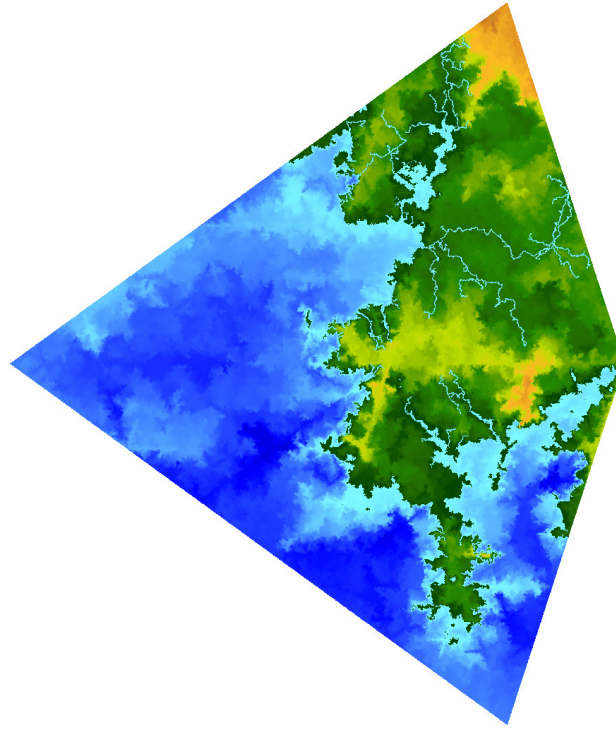


Fig. 12. Penrose-tiled map

The maps shown in the paper are all made by a straight-forward implementation of the algorithm in F# running on Mono, and all use 1023×1023 pixels.

Relatively few edges have rivers, so even though handling rivers is fairly complex, this is done relatively rarely, so rivers add almost nothing to the running time. The time to generate unzoomed maps with or without rivers is about 1.4 seconds. The $125\times$ zoomed map takes about 1.5 seconds, so the time is also almost independent on the zoom factor. All executions are done using single-threaded F# compiled to .NET and executed using Mono on an Intel Core i7-10510U processor running at 1.80GHz.

Using integers instead of floating points for coordinates, seeds, and altitudes can speed calculation up, but the reference implementation uses floats to be close to the description in this paper. It is also fairly easy to exploit multiple cores: The first few subdivisions can make one of the recursive calls run on a different core.

The Penrose-tile variant takes a bit longer to generate, and it can not (easily) use integer coordinates. But it generates fewer grid-aligned artefacts than the regular subdivision method.

7 FUTURE WORK

Future work may include extending the method to tetrahedral subdivision [8]. This generates a planet map by projecting each map point onto a sphere contained in an irregular tetrahedron, and then recursively subdividing this while discarding tetrahedra that do not contain the point. Figure 13 illustrates tetrahedral subdivision.

While tetrahedral subdivision resembles triangle subdivision, adding rivers is not trivial: With triangles, rivers can float from edges to edges, as all edges will be part of the final map. With tetrahedra, an edge may or may not intersect the surface of the sphere (and may do so twice). It may work to say that a river can only be generated

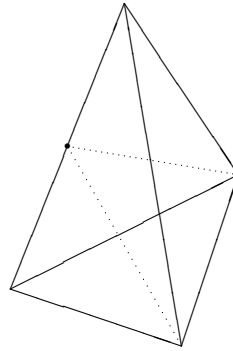


Fig. 13. Cutting a tetrahedron in two

on an edge that intersects the sphere exactly one, and when subdividing the edge, the river is put on the half edge that intersects the sphere, but that is unlikely to give natural-looking river flows. Additionally, an edge can be shared by more than two tetrahedra, so placing a river there is likely to give three-way or four-way river joins. It may be better to place rivers on faces instead of edges, as a face is shared only by two tetrahedra. Again, we can generate a river on a face only if it intersects the sphere, and when subdividing a face (which happens when a tetrahedron is cut in two), the river can be placed on a sub-face that intersects the sphere. There may be a choice, which can be resolved similar to how it is done with triangle subdivision. A problem is that rivers on faces can not affect altitudes on the map, as the altitude on the new point can only depend on attributes of the edge on which it is positioned and the end points of this edge. So it may be impossible to make rivers flow downwards only.

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