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Onset of river ice breakup

S. Beltaos [1]

National Water Research Institute, Burlington, Ont. L7R 4A6, Canada

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Abstract

Predicting whether and where the ice cover of a river is going to break up as a result of increased runoff, is crucial in such concerns as environmental impact assessment of climatic and hydrologic changes, emergency flood warning and mitigation, and winter hydro-power production. The available predictive methods are generally empirical and site-specific. Their utility is limited by the need for local records, and the uncertainty introduced by changes in factors not explicitly accounted for in an empirical correlation. As a first step toward development of more general, transferable, breakup forecasting criteria, five data sets are used to test existing theoretical concepts of how breakup is initiated. One of these concepts results in acceptable consistency among the different sites, and shows promise for further testing. It involves such variables as water surface width, channel curvature, freeze-up stage, and ice competence. In addition to forecasting applications at a particular site, this method has the potential for application over entire reaches to address such problems as where ice jams form, when they release, and the magnitude of the ice clearing discharge.

Keywords: Hydrology; Ice; Ice breakup; Ice jams; Onset; Rivers

1. Introduction

The breakup of river ice is a brief but crucial event in the regime of northern rivers. Many biological processes are triggered by this event while being subject to potential disruptions caused by ice jamming and related phenomena (e.g. see Prowse and Gridley, 1993; Beltaos, 1995a). Changes in the timing and severity of the breakup event, caused by relatively abrupt channel regulation or by gradual climatic variation, are likely to have serious ecological impacts. However, current understanding of breakup processes is not sufficiently advanced to permit prediction of such impacts. In addition to its ecological dimension, river ice breakup interferes with many socio-economic activities. Ice-jam induced flooding of riverside communities is very frequent in northern parts of the globe. River structures are subject to ice damage while winter navigation and hydro-power production are inhibited by ice jams. A particularly vexing problem to the hydro industry is how to maximize winter energy production while making sure that concomitant flow releases will not initiate breakup and damaging jams downstream (NRCC, 1990).

Central to the study of breakup processes is the question of how the event is initiated. This is not only the key to forecasting the occurrence of breakup and issuing timely warnings, but it has a bearing on many other problems. Knowledge of the effects of

[1] Fax: + 1(905)336-4420. E-mail: spyros.beltaos@cciw.ca

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river morphology on the onset of breakup would help us assess its spatial variability, and thence predict where ice jams are likely to form, a question that remains intractable to date.

At the same time, identification of the hydraulic conditions leading to breakup would help determine the ice-clearing flow and forecast the severity of the event. Considering that the configuration and water levels caused by ice jams, given that they have formed somewhere, are now predictable (Pariset et al., 1966; Uzuner and Kennedy, 1976; Beltaos, 1983, 1993), improved knowledge of the onset of breakup would go a long way toward elucidating the entire event.

Most of the work done on this problem so far is highly empirical (e.g. see Shulyakovskii, 1966) and thus limited to site-specific applications. While there is much practical value in this, it provides little guidance when different sites within the same river or on other rivers are considered. Moreover, the empirical method implicitly relies on the temporal constancy of hydraulic and meteorologic parameters that do not appear in the empirical equations. If a change in these parameters occurs as a result of development or climatic alteration, the empirical relationship becomes uncertain.

Therefore, it is important to formulate more general criteria of breakup initiation, such that they explicitly consider all the relevant variables, or at least the ones that govern a large part of the answer. Herein, a first step is taken in this direction, by assessing where we stand at present. First, the literature is scanned for semi-empirical hypotheses and formulae that have the potential for "transferability", i.e. they could in principle apply to any river site. Next, field data from five gauged sites are used to evaluate each of these criteria. The implications of the results to breakup forecasting, as well as to prediction of ice jam occurrence and severity are discussed.

2. Definitions

Breakup in rivers ranges between the extremes of pre-mature and over-mature (Deslauriers, 1968). The latter involves low flow conditions with intense thermal inputs to the ice cover. The ice remains stationary and gradually disintegrates under the action of the sun and the increasing water temperature. This is a benign event, sometimes called *thermal breakup* though very little ice breaking actually takes place.

The opposite happens when high runoff is generated while the weather remains cool and the ice starts to break before thermal processes have a chance to significantly reduce its thickness and strength. Major and persistent ice jams attend a pre-mature breakup, occasionally called *mechanical breakup*, to signify the predominance of mechanical fracture processes. Herein, however, we will use this term to denote all events that are not thermal, ie, they involve fracture of the ice cover even though the latter may have already been subjected to partial thermal decay.

The onset or initiation of breakup (or OB for brevity) has been defined as the time when the ice cover begins a sustained movement at a particular site (e.g. see Beltaos, 1984a), which is quickly followed by breakdown into small blocks through ice-to-ice or ice-to-boundary impacts. This definition is akin to Shulyakovskii's (1966) "ice push", and not only appeals to intuition but is readily amenable to observation and quantification. It does not, however, apply to thermal events because then the ice does not move but slowly disappears by attrition. In the following discussion and analysis, thermal breakups are excluded, in order to concentrate on the more eventful mechanical ones.

Other instances where the preceding definition will not be meaningful include the in situ destruction of the ice cover by an advancing breaking front (Gerard et al., 1984; Prowse, 1986; Beltaos, 1990a); or the breakup of a cover that has frozen into the river bed and must first "pop up" via increasing temperature and uplift effects, as the water flows over it. Despite these restrictions, the "first sustained ice move" is by far the most common mode of breakup initiation.

3. Empirical criteria

Empirical predictions of the OB have relied on local records and invoked a multitude of variables such as air temperature, degree-days of thaw, indices of the accumulated heat input to the ice cover, ice thickness, ice strength, water level, rate of rise of water level, freeze-up stage, and others. A few examples may be found in Shulyakovskii (1966), Galbraith (1981), Murakami (1972), and Beltaos (1987).

The most consistent results appear to be obtained with the following type of equation (Beltaos, 1990a, 1995b)

$$H_B - H_F \approx K h_{i0} - F(S_5) \quad (1)$$

in which $H_B =$ water level at which the ice cover starts to move; $H_F =$ "freeze-up" water level, intended to provide an indication for the level that must be exceeded in the spring before the ice is detached from the banks and other river boundary supports — herein it is taken as the average level during the week following the formation of a complete ice cover; $h_{i0} =$ ice cover thickness prior to the start of melt; K is a dimensionless site-specific coefficient, often close to 3, though steep-bank rivers may have much higher values (Beltaos, 1990a); S_5 is an index of the accumulated heat input to the ice cover but often defined simply as the accumulated degree-days of thaw referred to a base air temperature of -5°C as suggested by Bilello (1980); the function F has the dimension of length and is site-specific, with $F(0) = 0$. For premature events, breakup is thus initiated when the water level rises above the freeze-up stage by an amount proportional to the ice thickness.

Other empirical criteria, such as illustrated by Shulyakovskii (1966), can be shown to be special versions of Eq. (1). For instance, cases where H_B is simply a function of H_F imply relatively stable winter and spring conditions so that neither ice thickness nor thawing degree-days change appreciably from year to year. Similarly, where $H_B - H_F$ is found to be a function of accumulated heat inputs to the ice cover, it is likely that the end-of-winter ice thickness is relatively constant.

4. Semi-empirical criteria

The main drawback of empirical breakup criteria derives from the assumption that all relevant factors not explicitly accounted for, remain the same. For instance, the effects of channel morphology do not appear in Eq. (1), hence, it has to be specifically calibrated at any given river site. Practical application, however, is reliable so long as the local morphology does not change.

In recent years, several hypotheses have been advanced as to the actual mechanism of the initiation of breakup. While all of them are "naive", because they do not entirely describe the many complex phenomena at work, they are potentially *transferable* among different sites. Herein, such criteria are called semi-empirical because they have a basis on physical reasoning while still requiring empirical evaluation of one or more parameters. Five semi-empirical criteria were found in the literature, as described below.

4.1. Model ice experiments

Michel and Abdelnour (1975) carried out laboratory experiments in a rectangular flume using a wax-based model ice material in order to determine the velocity at which an ice cover will be completely destroyed by hydrodynamic forces. In all cases, the edge of the cover became submerged prior to failure. With increasing discharge, the cover started to oscillate until finally cracks would develop and pieces break off and be carried downstream. The test results were described by the following equation:

$$V_F = \sqrt{2g(1-s_i)h_i} + 0.055\left(100\frac{h_i}{W_i}\right)^{3.82}\sqrt{\frac{\sigma_f}{\rho}} \quad (2)$$

in which $V_F =$ average flow velocity at failure; $s_i =$ specific gravity of ice; $g =$ acceleration due to gravity; $W_i =$ width of the cover; $\rho =$ density of water; $h_i =$ thickness of the cover; and $\sigma_f =$ flexural ice strength. Eq. (2) appears to be designed so as to account for water spilling over the cover's edge (first term on the right-hand-side of the equation) and subsequent flexural failure (second term on the RHS). It is not clear how the influence of the ice cover width, W_i, was delineated, since W_i was fixed at 1.8 m for all tests. For normal values of σ_f (≤ 600 kPa) and h_i (≥ 0.3 m), the flexure term in Eq. (2) is negligible when $W_i > 200\ h_i$.

4.2. Side resistance

Ferrick and Mulherin (1989) have utilized the *side resistance* as a criterion for the dislodgment of the ice cover. Using a dynamic, one-dimensional model of the flow in reaches with ice covers, they calculated the tangential stress, τ, applied on the ice cover as a function of location and time. The shear stress which develops at the sides of the ice cover as a result of the action of τ, is given by:

$$\tau_s = \frac{\tau W_i}{2h_i} \quad (3)$$

in which $\tau_s =$ side shear stress or side resistance; $h_i =$ ice cover thickness; $W_i =$ width of ice sheet (by the time the ice is about to move, longitudinal fractures near the river banks, known as hinge cracks, have already developed so that W_i represents the width of the middle strip of the cover); and $\tau =$ total tangential stress = shear stress applied by the flow on the underside of the ice cover augmented by the downslope component of the weight of the cover per unit surface area.

By comparing calculated values with observations pertaining to the start of ice motion in a few river reaches, it was found that τ_s was about 1.6 kPa. This is far too low to represent the shear strength of the ice but could reflect a residual frictional resistance that may be available at the hinge cracks. According to this criterion, reaches with relatively high velocity and slope (high τ) will open up first while those with tranquil flow would be prone to ice jamming. Similarly, thinner ice will tend to break up first. Moreover, we would expect τ_s to decrease with increasing thermal inputs and disappear once the side strips of ice (which remain attached to the river banks after hinge cracking occurs) loose their integrity.

4.3. Flexure and buckling

Kozitskiy (1989) presents a review of Russian literature on breakup, including the fracture and dislodgment caused by horizontal forces. A formula due to D.F. Panfilov gives the average flow velocity, V_B, above a hydro-development, at which the ice begins to run at the headrace

$$V_B = 1.8\sqrt{\frac{\sigma_f h_i}{\rho W_i\left(1 + 10\dfrac{h_i}{y}\right)}} \quad (4)$$

in which $y =$ average flow depth. This equation was derived from observations and measurements during the passage of ice through regulated Siberian rivers, using two dimensionless parameters, $\rho V_B^2 W_i/\sigma_f h_i$ and h_i/y. (Very similar formulae have also been proposed by V.A. Korenkov and V.A. Buzin, as quoted by Kozitskiy, 1989). It is not known what physical mechanism is described by Eq. (4), beyond the fact that it must somehow involve flexure. If we use an average value of 1.65 for the quantity $1 + (10h_i/y)$, as applies to the data analyzed by Panfilov, and ignore the weight of the ice cover in the term τ of Eq. (3), we can show that the criteria expressed by Eqs. (3) and (4) are numerically equivalent, provided the side resistance, τ_s, is set equal to $f_i \sigma_f/8$ (with $f_i =$ friction factor of the underside of the ice cover at the time of breakup initiation). For a typical pre-breakup value of 0.07 for f_i, the two formulae will coincide when $\sigma_f = 180$ kPa, even though they are based on different concepts of how breakup is initiated, i.e. overcoming of hinge crack resistance (Eq. (3)) versus flexural failure (Eq. (4)). Another formula, due to Kozitskiy (1989), derives from a semi-empirical formulation of the failure of the ice cover in buckling:

$$V_B = \sqrt{40\frac{E}{\rho}\left(\frac{h_i}{W_i}\right)^3} \quad (5)$$

in which $E =$ modulus of elasticity of the ice.

4.4. Transverse cracking and movement of ice sheets

Shulyakovskii (1972) noted that the flow shear stress applied on the ice cover must cause horizontal bending moments, owing to the meandering river planform. Assuming that bending stresses in excess of the flexural strength of the ice can develop to cause transverse cracking; and that transverse crack formation signifies the onset of breakup, Shulyakovskii deduced that site-specific relationships, similar to Eq. (1), should apply.

Beltaos (1984a, 1990a) confirmed the first hypothesis by means of field observations on transverse crack spacing and large-scale horizontal bending tests, but found that transverse crack formation is not sufficient to initiate ice movement. The river geometry may be such as to prevent the downstream passage of the separate ice sheets that have formed by successive transverse cracks (see Fig. 1). Considering a meandering channel, Beltaos (1990a) calculated the water surface width needed to allow a curved ice sheet to move in the straight river segment downstream of a bend (a very similar relationship is obtained when considering a relatively straight ice sheet going past a bend):

$$\frac{W_B - W_i}{h_{i0}} = \frac{\beta(m - 0.50)\sigma_f h_i}{8m^2\tau h_{i0}} \quad (6)$$

in which $m = R/W_i$ with $R =$ radius of curvature of a river bend; and $\beta =$ dimensionless constant expected to be between 0.3 and 1.5, the upper end of this range being the more likely. This parameter was introduced to quantify the assumption $a_b \propto l_s^2$ ($l_s =$ length of a curved ice sheet; $a_b =$ area of sector contributing to the horizontal bending stress) and was evaluated from transverse fracture data on the Thames and MacKenzie Rivers (Beltaos, 1990a). Its applicability to other rivers is adopted as a working hypothesis, subject to further confirmation.

Eq. (6), herein referred to as the boundary constraint criterion, appears to explain several trends known by experience. For instance, one may note that the width "clearance", $W_B - W_i$, is roughly proportional to the water level rise, $H_B - H_F$, since W_i is usually close to W_F, the river width at freeze-up, and the cross sectional shapes of natural streams are roughly trapezoidal. Substituting these results in Eq. (4) and re-arranging it, a form similar to the empirically established Eq. (1) is obtained (Beltaos, 1990a). The effect of river planform shape is expressed by the dimensionless radius of curvature, m. For relatively straight reaches, m is in the ballpark of 10, and the quantity $(m - 0.50)/m^2$ is about 0.1; for a sharp bend, with $m = 3$, it increases three-fold to about 0.3. Straight reaches are thus expected to break up first and jamming is very likely to occur at sharp bends, as is also known by experience. Though not

Table 1
Characteristics of five sites used in breakup data analysis

Site and years of record	Channel description	Latitude (N) R [a] (m) Q_m [b] (m^3/s)	Slope (m/km)	m [c]
Thames River at Thamesville, 1980–1986	Straight single channel, deep, steep banks	42°32′42″ 1160 51	0.23	31.4
Grand River near Marsville, 1981–1984	Moderate bends, single channel shallow	43°51′43″ 240 7.7	2.30	6.4
Moose River at Moose River, 1961–1980	Straight channel, islands, wide	50°48′50″ 10800 780	0.38	14.4
Nashwaak River at Durham Bridge, 1965–1983	Straight channel, large island above gauge	46°07′33″ 1250 35	0.73	21.6
Restigouche River above Rafting Ground Brook, 1970–1992	Wide meander, single channel	47°54′29″ 1100 165	0.82	7.9

[a] Radius of curvature.
[b] Long-term mean discharge.
[c] Dimensionless radius of curvature, defined as ratio of average radius in the reach to the long-term mean river width under open water conditions.

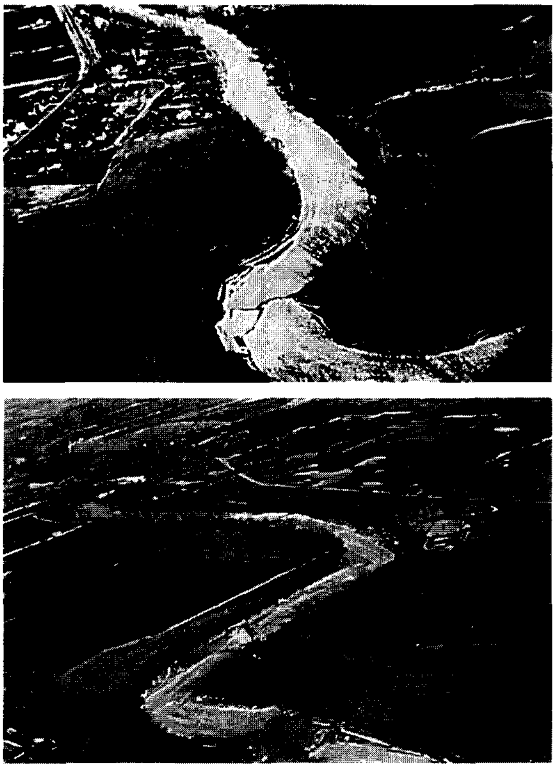

Fig. 1. Ice sheets formed by transverse fractures in Thames R, Ontario. Top: pre-breakup condition, sheets restrained by the channel boundaries; bottom: sheets moving, breakup initiated.

explicitly shown in Eq. (6), $W_B - W_i$ depends on the distance between consecutive cracks which, under natural conditions, depends on flow shear as well as ice thickness, strength, and width. Cutting trenches in the ice cover before hand, would reduce the crack spacing and hasten breakup initiation and ice clearance. Jolicoeur et al. (1984) experimented with different trench patterns, comprising transverse, longitudinal, and diagonal trenches. All patterns had the desired effect, that is, to cause fracture into relatively small, easily-dislodged ice floes. The most effective pattern involved diagonal trenches, oriented roughly perpendicular to each other, and extending from near the bank to the mid-stream.

5. Description of data

Five data sets are used to test the above criteria, obtained at gauged river sites, and representing wide ranges of river size and slope, as indicated in Table 1. For the Moose and Nashwaak Rivers, these data are entirely based on analysis of gauging station records (Water Survey of Canada) as described by Beltaos (1990b). The data for the Thames and Grand Rivers are based solely on direct field observations while those for the Restigouche (Beltaos and Burrell, 1992) include both gauge records (1972–1987) and in situ measurements (1988–1992). In all cases, the reach-average river bathymetry and slope have been derived from several cross-sections surveyed in the vicinity of the gauge.

The tangential stress, τ, which appears in Eqs. (3) and (6) can be calculated as:

$$\tau = \tau_i + \rho g S_w \{ s_i h_i + [s_i (1 - p) + p] h_p \} \qquad (7)$$

with $S_w =$ slope of the water surface; p, $h_p =$ porosity and thickness of any ice accumulation that may be present under the solid ice cover (h_p was assumed to be zero in the present data sets because such accumulations appeared to melt away before the breakup, based on the limited ice thickness data available from discharge measurement notes during the winter); and $\tau_i =$ flow shear stress applied on the bottom surface of the cover. Herein, it was calculated using the river slope and the applicable flow depth, assuming equality between the bed- and ice-controlled hydraulic radii. This was considered a reasonable approximation, since the Manning roughness coefficient for the ice cover just before breakup is close to 0.03 (Carey, 1966, 1967; Shen and Ackermann, 1981), a value also considered typical for river beds.

To determine the breakup velocity, V_B, which appears in the Panfilov and Kozitskiy criteria (Eqs. (4) and (5)), one would normally use the discharge and flow area as obtained from cross-sectional and water stage data. However, the published discharge data (Water Survey of Canada) just before and during breakup are mere estimates and can be uncertain, owing to changing ice cover thickness and roughness. While analyzing the present data sets, it was found on many instances that published flows resulted in implausibly low or high velocities. Consequently, the velocity was estimated by the slope–area method, using an assumed value of 0.03 for the composite Manning coefficient, or previously cali-

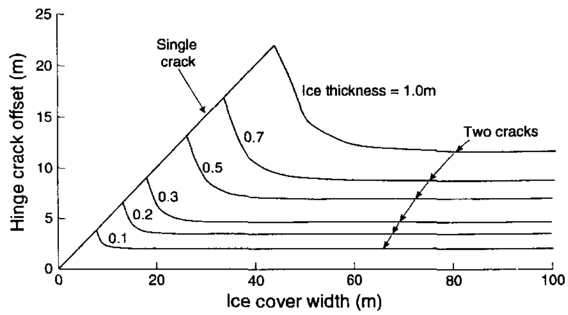

Fig. 2. Location of a hinge crack relative to the river bank (after Beltaos, 1995b).

brated values (Restigouche River; Beltaos and Burrell, 1990). As already mentioned, typical river beds have a Manning value of about 0.03 while the same is true for pre-breakup sheet ice cover that has been subjected to some degree of thermal erosion. Of course, this approach involves some uncertainty about the calculated velocities but, as shown later, this does not alter the conclusions regarding the transferability of the velocity-based criteria (Eqs. (4) and (5)).

The ice cover width, W_i, was obtained by calculating the hinge crack distance from the river banks and subtracting twice this amount from the width corresponding to the freeze-up level. Fig. 2 can be used for convenient determination of the hinge-crack offset distance; it is based on the theory of elastic beams on elastic foundations with a field-calibrated value of 1.4 GPa for E (Beltaos, 1990a). The low value of E was attributed to viscous creep effects. In fair-sized rivers, the offset is independent of channel width, and amounts to 10–20 ice thicknesses. A single central crack forms in smaller streams with thick ice cover (linear portion of graph in Fig. 2) though none of the test sites considered herein are in this category.

Using water level information and channel bathymetry, it is possible to calculate a strength-related parameter for each breakup event, depending on which of Eqs. (2)–(6) is being used. Because the ice thickness at the time of breakup is not generally known, it is best to use the known values, h_{i0}, which occur at the end of winter before melt begins. Consequently, the quantities computed from Eqs. (2)–(6) are ''strength'' values (e.g. σ_f or τ_s) multiplied by a power of the ratio h_i/h_{i0}.

These modified-strength parameters reflect the combined effects of thickness loss and strength decline due to thermal deterioration which can be represented by a thermal index, such as degree-days of thaw. For the present data, S_5 has been calculated from air temperatures recorded at nearby weather stations. It is emphasized that S_5 is a mere surrogate for the interaction of several complex hydro-climatic processes, and its use is a practical necessity at present. Ice strength decreases due to preferential melting at the crystal boundaries which is in turn controlled by short-wave radiation absorption and dissipation within the ice sheet. Computation of this effect is uncertain without detailed, and usually unavailable, data (e.g. see Bulatov, 1972; Ashton, 1985; Prowse et al., 1990). Even the relatively simple question of top and bottom ice melting would require numerical modelling and field calibration.

6. Testing of criteria

In each case the calculated strength parameter can be plotted versus the thermal index S_5 and the resulting relationship can be compared to those obtained at other sites.

Fig. 3 shows a test of the Panfilov criterion. Of the five data sets, only the Thames and Nashwaak Rivers exhibit the expected trend. The three other sets show no perceptible decreasing trend of the calculated strength with increasing degree-days of thaw. Moreover, there seems to be no agreement among the values of $\sigma_f h_i/h_{i0}$ obtained by Eq. (4) for the different sites. A thirty-fold spread is apparent and implies that additional factors need to be

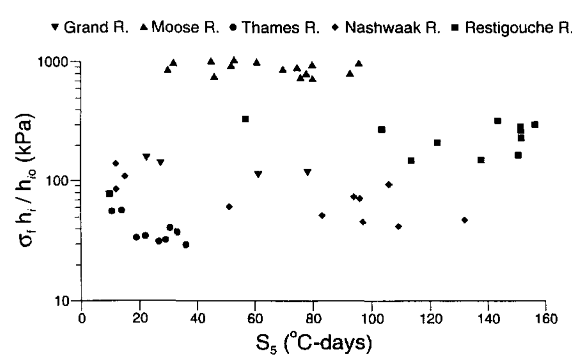

Fig. 3. Test of Panfilov's criterion (Eq. (4)).

accounted for. Note also that this magnitude of spread cannot be explained by possible errors made in estimating the breakup velocity V_B, using an assumed Manning coefficient. Similar spread was obtained by the same data but with the velocities taken from published discharges.

The criterion proposed by Ferrick and Mulherin (1989) is tested in Fig. 4. The spread of the data points is similar to that of Fig. 3. For three sites, the data points fall between 0.2 and 2 kPa while for the remaining two sites they fall mostly between 2 and 7 kPa. Moreover, there is no clear indication that the side shear resistance actually decreases with increasing degree-days of thaw.

The boundary constraint criterion (Eq. (6)) is examined in Fig. 5 which exhibits two encouraging features. First, the data points from all five data sets collapse in a relatively small (two-fold) range of scatter. And second, the modified strength, $\sigma_f h_i/h_{i0}$, shows a clear trend to decrease with increasing S_5. A part of the scatter in Fig. 5 could be caused by the use of degree-days of thaw to represent thermal deterioration effects, as discussed earlier.

In the special case of pre-mature breakup, $S_5 = 0$ and $h_i/h_{i0} = 1$. Therefore the intercept of the data-defined relationship with the vertical axis in Fig. 5 should represent the undeteriorated flexural ice strength, σ_{f0}, multiplied by the constant β. Backward extrapolation suggests that $\beta\sigma_{f0}$ is between 70 and 100 kPa and this range could be partly due to β being reach-dependent, as discussed in conjunction with Eq. (6). Noting that β can be as much as 1.5, the flexural strength could be as little as 50–70 kPa. Such values are much less than what is expected from bending tests on small specimens ($\approx$ 600 kPa

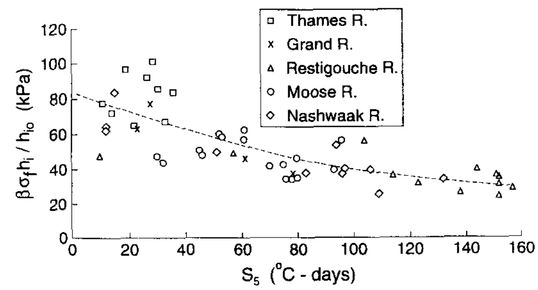

Fig. 5. Test of the boundary constraint criterion (Eq. (6)).

or more) but this is likely due to the "scale" effect on ice strength, as shown by Beltaos (1990a). Conventional ice strength is known to decrease with increasing specimen size, a property that is generally attributed to the presence of flaws in ice (Sanderson, 1988). In the present case, the important dimension is the width of the ice cover which is much greater than the size of the commonly tested cantilever ice beams. It may be noted that the ordinates of Figs. 3 and 5 appear to be the same except for the coefficient β (Fig. 5). However, they represent different quantities, respectively calculated from the hydraulic data to fit the Panfilov and boundary constraint criteria (Eqs. (4) and (6)).

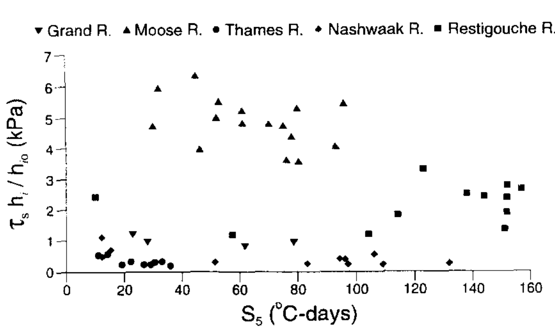

Fig. 4. Test of the side resistance criterion (Eq. (3)).

The criteria by Michel and Abdelnour (1975) and Kozitskiy (1989), respectively expressed by Eqs. (2) and (5), resulted in highly inconsistent findings, and are thus deemed to be far less applicable than the above three. For instance, the modulus of elasticity obtained from Eq. (5) ranges from 0.001 to 100 GPa while the flexural strength obtained from Eq. (2) ranges from 0 to over 20,000 kPa.

7. Implications to jam formation and ice clearance

The results described in the previous sections suggest that the initiation of breakup is governed by quantifiable physical processes. The development of a generally applicable and transferable criterion would greatly improve current forecasting procedures by reducing or eliminating the amount of site-specific data that are normally required for empirical forecasting applications. Moreover, transferable OB criteria have a variety of other applications that go far beyond the initiation of breakup.

To date, it has not been possible to predict where breakup jams form. We know by experience that certain morphological features are more conducive to ice jamming than others but ice jams are known to form anywhere, so long as there is competent ice cover to impede the downstream passage of ice blocks. This problem can be, at least partially (see additional discussion at the end of this section) addressed by means of a transferable OB criterion. Because of the natural variations of shear stress, width, and curvature along a river, Eq. (6), for instance, indicates that breakup will be initiated first at certain sites of a given reach. The ice cover will remain in place elsewhere. Once the large ice slabs that are formed by transverse cracks are set in motion, they quickly break down into small ice blocks which are arrested by intact ice cover segments to instigate ice jams. Essentially, then, we could predict the sites of ice jamming by applying the OB criteria to an entire reach.

It should be stressed that this concept applies to a natural stream with a meandering planform and sloping banks. The breakup process would be very different, though perhaps more predictable, in a straight channel of rectangular cross section. In that case, we would still expect to see hinge cracks and eventual loss of lateral restraint through thermal deterioration at the hinge. As the channel is straight, horizontal bending will not occur. Increasing flow velocities, however, may produce vertical bending and fracture at the upstream end of the cover. Fragments would likely be carried under the ice and eventually form a jam, which may then advance in a series of releases and arrests as the flow rises and thermal deterioration of the ice progresses.

Returning to the natural stream conditions, we may also note that the release of ice jams is another unsolved problem that can be studied by applying an OB criterion. A jam will release when the intact ice cover segment that holds it in place is set in motion, ie when breakup is initiated at the site downstream of the jam. Of course, this is only a part of the answer because the downstream ice cover is also subject to thermal action and development of open leads which may hasten the release. Nevertheless, application of the OB criterion would give useful results where the release occurs before such advanced stages of ice cover decay appear.

Closely related to ice jam release is the concept of the *ice clearing discharge*. There is anecdotal evidence suggesting that all the ice is flushed out of a reach at sufficiently high flows, which implies that ice jams cannot form beyond a certain discharge. Recalling Eq. (6), we see that, in any one river reach, there will be a site that is least amenable to breakup initiation, depending on the spatial variation of freeze-up level, bathymetry, river curvature, flow shear stress, ice thickness and ice strength. This site will be the last to break up in the reach under consideration, and when it does the reach will be cleared of ice. Since the water surface width depends on stage and thence on discharge, it is reasonable to postulate that there is a limiting value, the ice clearing flow Q_{cl}, such that breakup is completed when the flow in the river exceeds Q_{cl} and the last remaining ice segment moves out.

A natural extension of the ice clearing flow concept is in predicting the severity of breakup. If Q_{in} is the flow that initiates the first ice dislodgment in a reach, then ice jamming within the reach is possible for flows between Q_{in} and Q_{cl}. These limiting values are subject to hydro-climatic influences as suggested by Eq. (6), and will change from reach to reach and from year to year. Fig. 6 illustrates how the influence of the freeze-up level on breakup severity can be predicted by means of Eq. (6). For instance, a low

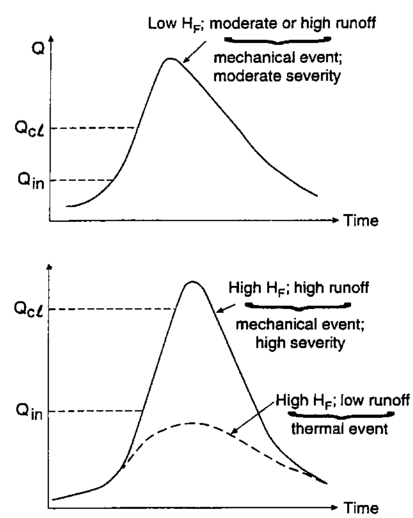

Fig. 6. Different types of breakup events produced by different combinations of freeze-up level and runoff hydrograph.

freeze-up level, H_F, implies relatively low values for both Q_{in} and Q_{cl}. Occurrence of a mechanical event is now likely, due to the low Q_{in}, but the worst that can happen is an ice jam persisting until the (moderate) flow Q_{cl} is attained. On the other hand, a high H_F implies relatively high values of both Q_{in} and Q_{cl}. In this case, a thermal event is probable, if the actual runoff is moderate. However, if the runoff is intense, producing river flows equal to or in excess of Q_{cl}, major ice jams should be expected. Similar analysis applies to other relevant factors such as ice thickness or degree-days of thaw.

It follows that, if site-specific historical data on the maximum breakup stage, H_m, are plotted versus the applicable freeze-up stages, the result would be a diagram with much scatter but with a well-defined upper envelope, imposed by the ice clearing process. This would explain why such trends are found in actual case studies (e.g. see BC Hydro, 1975; Beltaos, 1984b; Burrell et al., 1986).

Another major application of transferable OB criteria is in assessing the impacts of hydro-climatic changes on the ice regime, and thence on the aquatic ecosystem or on the socio-economic fabric of nearby communities (see also next section). Such changes may be due to natural climatic variability or caused by human activities such as development and regulation of northern rivers, basin resources exploitation, and emission of greenhouse gases.

More site-specific data are needed to further test the breakup criteria and extend the range of validation. At the same time, emphasis should be placed on the spatial application of the criteria with a view towards prediction of ice jam formation and release. This is not a simple matter because it requires knowledge of freeze-up levels, ice thicknesses, bathymetry, and ice conditions throughout the length of a study reach. A combination of in situ observations, numerical modelling, and satellite imagery, or other remote sensing technique, may prove fruitful.

Before closing this section it is worth stressing that not all jams form by rubble accumulation behind undislodged ice sheets. Ice runs, ensuing when jams let go, may encounter intact ice cover and yet keep going through the formation of a *breaking front*. The front ploughs into the cover, incorporating the newly created rubble, and often moves at surprising speeds (Gerard et al., 1984; Prowse, 1986; Beltaos, 1990a). Breaking fronts usually come to a halt, thus initiating a new jam, but not before they have cleared the ice for many kilometres. Little is known about this phenomenon (Beltaos, 1995b), except that it seems to occur after the breakup process has started and progressed to the point of creating enough rubble to form a major ice jam.

8. Examples of practical application

In view of the limited data sets available at present, the best approach to breakup forecasting is to rely on past observations and measurements over a period of a few years, so as to calibrate one or more of the criteria discussed previously. However, this is rarely feasible. Where no previous breakup data exist but site reconnaissance suggests that the breakup is governed by mechanical fracture and ice movement processes, the average relationship from Fig. 5 could be used as a first approximation. To determine the flow shear stresses and the widths W_B and W_i as functions of water level, it is necessary to survey the river bathymetry and calculate reach-averaged areas, widths and depths at different elevations.

As an example, consider a site where the slope is 0.0005, the dimensionless radius of curvature, m, is 10.0, while the reach-averaged cross-section is trapezoidal so that the width and average depth are given by

$$W = 200 + 15(E-22)$$

$$y = \frac{(E-22) + 0.0375(E-22)^2}{1 + 0.075(E-22)} \tag{8}$$

in which E = elevation; y = average flow depth under the ice cover; 22 is the elevation of the river bottom; and all quantities are expressed in metres. The river froze in December, and the average water surface elevation during the five days following the formation of a full ice cover was 24.0 m. This is taken as the freeze-up level H_F. By the middle of March, the ice has attained a thickness of 0.50 m while air temperatures have remained below −5°C. A warm trend is forecast for March 16 to 20, including some rainfall. The anticipated flows and air temperatures are shown in Table 2.

Existing data has indicated that the composite

Table 2
Example of breakup forecasting computations

Day in March	Air temp. (°C)	Discharge (cms)	Water surface elev. (m)	Degree-days, S_5 (base of -5°C)	Value of X from Fig. 7 (kPa)	Value of $\beta\sigma_f h_i/h_{i0}$ from Fig. 5 (kPa)
16	−3	200	24.00	2	10	84
17	0	250	24.25	7	15	79
18	2	500	25.15	14	42	75
19	5	1000	26.46	23	98(B) [a]	72
20	8	2000	28.46	36	210	66

[a] (B) indicates that breakup has been initiated.

Manning roughness coefficient at the time just prior to breakup is 0.03, equal to both the bed- and ice-associated values. This can be used to generate stage–discharge and stage–shear stress relationships, after accounting for the submerged portion of the ice cover thickness ($0.92 \times 0.5 = 0.46$ m). With this information, we can calculate the quantity $X \equiv 8m^2\tau(W_B - W_i)/(m - 0.50)h_{i0}$ as a function of the stage, E, as shown in Fig. 7. [Channel width at H_F is 230 m; subtract $2L_s$ (Fig. 2 gives $L_s = 7$ m) to obtain the width of the ice cover between the hinge cracks, $W_i = 216$ m.] Note that, according to Eq. (6), breakup is initiated when X exceeds a value equal to $\beta\sigma_f h_i/h_{i0}$, which is a function of the degree-days of thaw (Fig. 5). Fig. 7 is then used to fill the sixth column of Table 2. The last column is based on the "average" line drawn in Fig. 5, and is compared to the sixth column. Breakup is expected to occur on March 19, it being the first day when the value of X exceeds $\beta\sigma_f h_i/h_{i0}$. By interpolation, it can be estimated that the breakup stage and discharge are respectively equal to 25.9 m and 750 m^3/s.

To estimate the ice clearing flow at the site under consideration, one may examine breakup initiation parameters at downstream locations where ice jams are likely to occur and cause backwater effects. For simplicity let it be assumed that the river maintains its slope and cross-sectional geometry, but there is a sharp bend downstream, with $m = 2.5$. Ice clearance at the first site is then governed by breakup initiation at this bend. Using the new value of m, we can calculate new values of X and compare them with those in the last column of Table 2 (simply multiply previous values of X by the factor 0.3 which takes account of the effect of the changed curvature). On Mar. 20, the new X will be 62.3, just short of 66, indicated in the last column. Ice clearance will thus take place on the 21st, at a flow slightly greater than 2000 m^3/s and at a stage over 28.46 m.

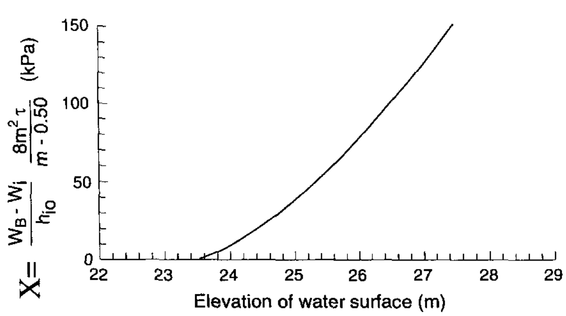

Fig. 7. Example of breakup forecasting graph: variation of quantity X with elevation of water surface, E.

Using the same approach, many other questions can be investigated, such as what would the effect on breakup be, if regulation were to increase freeze-up flows and levels while reducing the spring flows. Though the above described computations are not difficult to implement, it may be more illustrative to discuss the following examples in terms of the familiar hydraulic properties of stage and discharge, instead of channel width. The following analysis can be used as a first approximation within stage ranges where the width varies linearly with stage (ie, cross-sectional shape is roughly trapezoidal).

Consider a trapezoidal channel with average side slope, n, and longitudinal slope S. Let Y designate the water depth ($E_w - E_0$, $E_w =$ water surface elevation, $E_0 =$ channel bed elevation), and $h =$ average flow depth $= cY$, with $c \approx$ constant, commonly near 0.90 (this is a first approximation because c depends weakly on stage). By expressing the water surface width in terms of Y in Eq. (6), and neglecting certain small quantities, it is possible to obtain and solve a quadratic for the dimensionless breakup depth (Y_B/Y_F), i.e.:

$$\frac{Y_B}{Y_F} = \frac{1}{2}\left(B + \sqrt{B^2 + 4C}\right) \tag{9}$$

in which the quantities B and C are defined by

$$B = 1 - \frac{h_{i0}}{Y_F}\left(\frac{h_i}{h_{i0}} + n\frac{L_s}{h_{i0}}\right) - k\frac{h_p}{Y_F} \tag{10}$$

$$C = \frac{h_{i0}}{Y_F}\left\{n\frac{m-0.5}{m^2}f(S_5)\frac{\beta\sigma_{f0}}{8c\rho g Y_F S} + \left[1 - n\frac{L_s}{h_{i0}}\frac{h_{i0}}{Y_F}\left(1 + k\frac{h_p}{h_{i0}}\right)\right]\right\} \tag{11}$$

The coefficient k depends on c and p [$k = \{(2/c) - 1\}\{s_i + p(1 - s_i)\}$] but has a fairly constant value of about 1.20 when c is near 0.9 and p is in the range 0.4 to 0.8. The function $f(S_5)$ represents the relative loss of ice competence $= \sigma_f h_i/\sigma_{f0} h_{i0}$ and can estimated from the following equation, obtained by curve-fitting, after replotting the five data sets in the form $\sigma_f h_i/\sigma_{f0} h_{i0}$ versus S_5.

$$\frac{\sigma_f h_i}{\sigma_{f0} h_{i0}} \equiv f(S_5) \approx e^{-0.0071 S_5} \tag{12}$$

For the above example, $n = 2/15 = 0.13$ (see Eq. (8)), $h_p = 0$, and $\beta\sigma_{f0} = 84$ kPa (from Fig. 5, average line). With $L_s \approx 7.0$ m, the ratio L_s/h_{i0} is equal to 14. For the expected range of Y (2 to 5 m), the coefficient c changes from 0.94 to 0.86, thus permitting the assumption $c \approx 0.90$. Let $Y_F = 2.0$ m, a value that corresponds to a freeze-up elevation of 24 m. With $S_5 \approx 20$°C-days, as found for Mar. 20 in Table 2, Eq. (12) gives $f(S_5) = 0.87$. Using Eqs. (10) and (11), we find $B = 0.30$ and $C = 3.33$. Substituting in Eq. (9) gives $Y_B/Y_F = 2.0$, hence $Y_B = 4.0$ m; this corresponds to a breakup level of 26.0 m, similar to what was found by the more elaborate method (25.9 m).

Because the quantity B is usually considerably less than C, while C is mostly determined by the first term on the RHS of Eq. (11), it can be shown that the ratio Y_C/Y_B is given approximately by, provided channel bathymetry and slope, as well as ice thickness and strength are the same at the two sites:

$$\frac{Y_C}{Y_B} \approx \sqrt{\frac{f_C}{f_B}\frac{m_d - 0.5}{m - 0.5}\left(\frac{m}{m_d}\right)^2} \tag{13}$$

in which the suffixes B and C denote breakup initiation and ice clearing values while m_d is the value of m at the downstream bend. For the present example, Eq. (13) would give $Y_C \approx 1.7 Y_B \approx 6.9$ m, and thence $Q_C \approx 2300$ m^3/s, both in agreement with earlier results obtained with the more detailed method.

Now, suppose that freeze-up flows are increased due to regulation, and the typical value of Y_F becomes 3 m instead of 2 m. Using Eqs. (10) and (11), and retaining the value $f_B \approx 0.87$, we find $B = 0.53$ and $C = 1.54$; and from Eq. (9), $Y_B/Y_F = 1.53$ and thence, $Y_B = 4.6$ m (compare with 4 m found for the unregulated condition). This will result in more frequent occurrence of thermal breakup events, a trend compounded by a simultaneous reduction in spring flows, a usual consequence of regulation.

Another interesting question with regard to the regulated case is how soon and by how much can the flow be increased following freeze-up. Let us assume that a week after ice cover formation, the solid ice thickness is 0.15 m while there is a 1 m thick layer of dense slush under the solid ice. The Manning roughness coefficient of this layer is about 0.04 (Nezhikhovskiy, 1964), hence $n_0 \approx 0.035$ (recall that the bed Manning coefficient is 0.03). With $Y_F = 3.0$ m, the freeze-up discharge can be calculated as 240 m^3/s. To find the discharge that would initiate breakup of this ice cover, put $h_{i0} = 0.15$, $S_5 = 0$ (hence $f = 1.0$), $h_p = 1$ m in Eqs. (9)–(11), to find $B = 0.46$, $C = 0.90$, $Y_B = 3.6$ m, and $Q_B \approx 380$ m^3/s. The flow can thus be increased by about 60% and the stage raised by 0.6 m. As the winter progresses, both the thickness and roughness of the frazil layer will decrease while the solid ice layer will thicken. Suppose that, a few weeks later, h_p is only 0.5 m while n_0 drops to 0.03 and h_{i0} is now 0.30 m. With these values, we can calculate $B = 0.52$, $C = 1.23$, $Y_B = 4.2$ m, and $Q_B \approx 1100$ m^3/s. If the increased flow is maintained for a sufficient time, it is possible that a higher freeze-up level will be established, allowing further discharge increases. It should be stressed that the results described in this section are meaningful so long as the flow rise leading to breakup is due to runoff processes. The rate of rise of the river stage is then relatively slow and does not significantly augment the water surface slope over and above the river slope. This can be confirmed by means of the relationship:

$$\Delta S \approx \frac{1}{C_w} \frac{\partial E_w}{\partial t} \tag{14}$$

in which $\Delta S =$ added slope due to unsteadiness; $C_w =$ celerity of runoff wave, often near 2 m/s; and $\partial E_w / \partial t =$ rate of rise of the water level, commonly less than 0.2 m/h. Hence, ΔS should be less than about 3×10^{-5}, a negligible quantity relative to the slope of most rivers.

On the other hand, a surge produced by the release of an upstream ice jam, may be attended by much higher rates of rise, resulting in greatly augmented slopes and flow shear stresses (e.g. see Beltaos, 1990a). In turn, Eq. (6) suggests that the ice cover may be dislodged at lower water levels than indicated in the above examples if a surge were to go by. Surges may also result from peaking hydro-plant operations in the winter, and their effects on the ice cover will depend on the abruptness of a flow release and the length of the open-water reach downstream of the plant. Detailed quantification of surge characteristics can be accomplished using numerical models.

9. Summary and conclusions

As a first step toward the development of generalized, site-transferable methods to predict breakup occurrence and severity, semi-empirical criteria for the onset of river ice breakup, that are available in the literature, are reviewed and tested. This is accomplished using historical and observational data at five gauged river sites in Ontario and New Brunswick, located between the latitudes of 42°N and 51°N. The boundary constraint criterion, which takes into account freeze-up conditions, ice competence, and channel curvature, appears to give fairly consistent results. The implied flexural ice strength is roughly in the range of 50 kPa to 70 kPa prior to the start of thermal deterioration, and decreases consistently with increasing degree-days of thaw. The relatively low undeteriorated values are attributed to the well-known scale effect on ice strength.

It is shown further that simultaneous application of the boundary constraint criterion throughout an entire river reach would help resolve such problems as the prediction of ice jamming locations, release of ice jams, ice clearing, and winter hydro-plant operation. Impact assessment of natural and anthropogenic changes in one or more hydro-climatic parameters can also be addressed with a transferable breakup criterion.

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