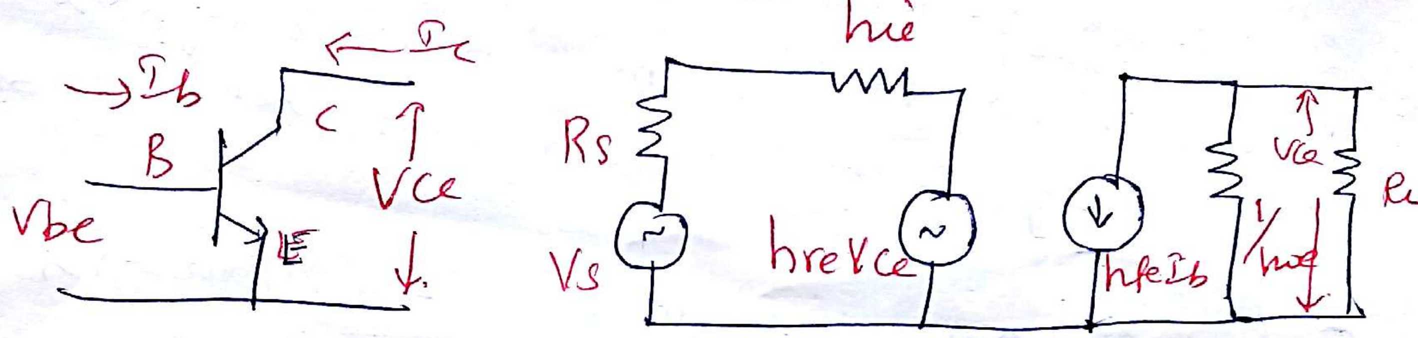
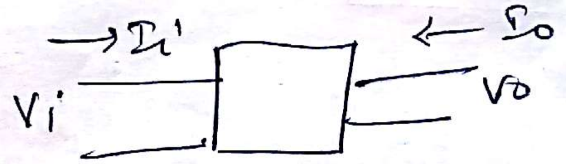
Small signal equivalent of CE amplifier (or) Hybrid model



Hybrid parameter (or) h-parameter

* The dimensions of the hybrid parameters are not alike, they are hybrid in nature so, they are called hybrid parameters.

* Transistor is a two port network



$$V_i = h_{11} I_i + h_{12} V_o$$

$$I_o = h_{21} I_i + h_{22} V_o$$

$\Rightarrow h_{11} = \left[\frac{V_i}{I_i}\right] / _{V_o = 0} = h_{ie} \rightarrow$ Input impedance

$h_{12} = \left[\frac{V_i}{V_o}\right] / _{I_i = 0} = h_{re}$ - Reverse voltage Gain

$h_{21} = \left[\frac{I_o}{I_i}\right] / _{V_o = 0} = h_{fe}$: Forward current Gain

$h_{22} = \left[\frac{I_o}{V_o}\right] / _{I_i = 0} = h_{oe}$: output admittance

Salient features of h-parameters are

1. h-parameters are real numbers
2. Easy to measure
3. Convient to use in circuit analysis & design
4. easily convertible from Configuration to other

Small signal analysis of CE configuration

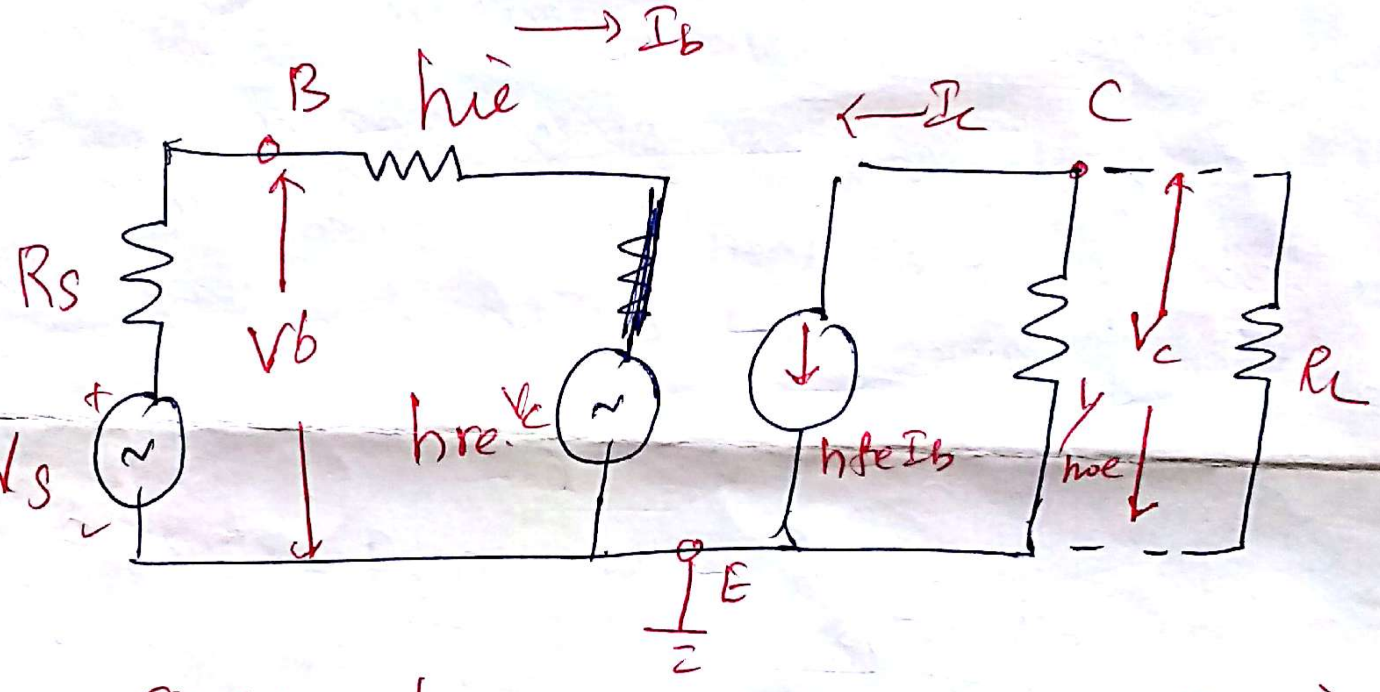
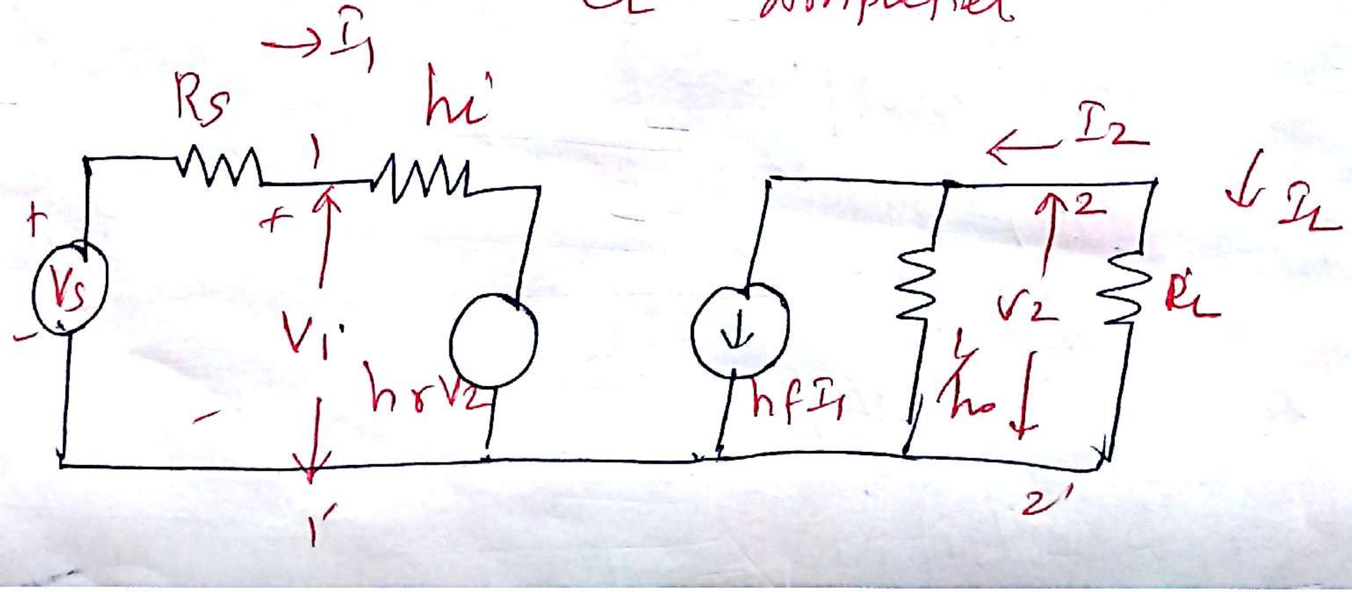


Fig (1) - h-parameters equivalent circuit for CE Amplifier



Current Gain (Ai)

$$A_i = \frac{I_L}{I_1} = \frac{-I_2}{I_1} \quad [\because I_L = -I_2]$$

$$I_2 = h_f I_1 + h_o V_2$$

Sub $V_2 = -I_2 R_L$.

$$I_2 = h_f I_1 - h_o I_2 R_L$$

$$I_2 + h_o I_2 R_L = h_f I_1$$

$$I_2 (1 + h_o R_L) = h_f I_1$$

$$I_2 / I_1 = \frac{h_f}{1 + h_o R_L}$$

$$A_i = \frac{-I_2}{I_1} = \frac{-h_f}{1 + h_o R_L}$$

$$\boxed{A_i = \frac{-h_f}{1 + h_o R_L}}$$

For CE Amp

$$\boxed{A_i = \frac{-h_{fe}}{1 + h_{oe} R_L}}$$

Voltag

Input Impedance (Z_i')

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$V_1 = h_i I_1 + h_r V_2$$

$$Z_i = \frac{V_1}{I_1} = h_i + \frac{h_r V_2}{I_1}$$

Sub $V_2 = -I_2 R_L = A_i I_1 R_L$ $\left[A_i = \frac{I_2}{I_1}\right]$

$$Z_i = h_i + \frac{h_r A_i I_1 R_L}{I_1}$$

$$Z_i = h_i + h_r A_i R_L$$

Sub $A_i = \frac{-h_f}{1 + h_o R_L}$

$$Z_i = h_i - \frac{h_r \cdot h_f \cdot R_L}{1 + h_o R_L}$$

$\div$ Nr & Dr by R_L

$$Z_i = h_i - \frac{h_r h_f}{h_o + 1/R_L}$$

$$\boxed{Z_i = h_i - \frac{h_r h_f}{h_o + Y_L}}$$ where $Y_L = 1/R_L$

Voltage Gain (Av)

$A_{ve} = V_2 / V_1$

WKT: $V_2 = -I_2 R_L = A_i I_1 R_L$

$$A_{ve} = \frac{A_i I_1 R_L}{V_1} = \frac{A_i R_L}{Z_1}$$

Voltage Gain with source:

$$A_{vs} = \frac{V_2}{V_s} = \frac{V_2}{V_1} \cdot V_1 / V_s$$

$\therefore A_{vs} = A_{ve} \cdot V_1 / V_s$

power Gain: (Ap)

$$A_p = P_2 / P_1 = \frac{-V_2 I_2}{V_1 I_1} = A_{ve} \cdot A_i = \frac{A_i^2 R_L}{Z_i}$$

$\left[\because A_{ve} = \frac{A_i R_L}{Z_i}\right]$

$$\boxed{A_p = \frac{A_i^2 R_L}{Z_i}}$$

Output Admittance (Y_o).

$$Y_o = \frac{1}{h_o} = I_2 / V_2 \text{ with } V_s = 0$$

$$I_2 = h_f I_i + h_o V_2.$$

$\div$ above eqn by V_2.

$$I_2 / V_2 = h_f I_1 / V_2 + h_o$$

$V_s = 0$, we can write.

$$R_s I_1 + h_i I_1 + h_r V_2 = 0.$$

$$\therefore (R_s + h_i) I_1 = -h_r V_2$$

$$\frac{I_1}{V_2} = \frac{-h_r}{R_s + h_i}.$$

$$Y_o = \frac{-h_f h_r}{R_s + h_i} + h_o$$

$$\boxed{Y_o = h_o - \frac{h_f h_r}{R_s + h_{ie}}}$$

Power Gain (A_p)

$$A_p = \frac{P_2}{P_1} = \frac{-V_2 I_2}{V_1 I_1} = A_v \cdot A_i$$

$$[P_2 = V_2 I_L = -V_2 I_2]$$

$$A_p = A_i^2 \frac{R_L}{R_i} \quad \left[\because A_v = \frac{A_i R_L}{Z_i}\right]$$

Summary.

$$A_i = \frac{-h_f}{1+h_o R_L}$$

$$A_{is} = \frac{A_i R_s}{Z_i + R_s}$$

$$Z_i = h_i + h_r A_i R_L = h_i - \frac{h_f h_r}{h_o + Y_L}$$

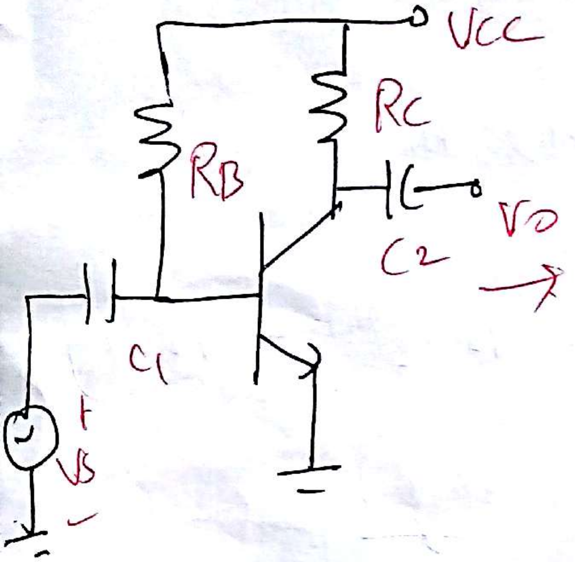
$$A_v = \frac{A_i R_L}{Z_i}$$

$$A_{vs} = \frac{A_{is} R_L}{R_s}$$

$$Y_o = h_o - \frac{h_f h_r}{h_i + R_s}$$

$$A_p = A_i^2 \cdot R_L / Z_i$$

Transistor Amplifier Analysis using re-model



re-model is also sensitive to the dc operating point of the amplifier circuit.
re model is used for low frequency analysis

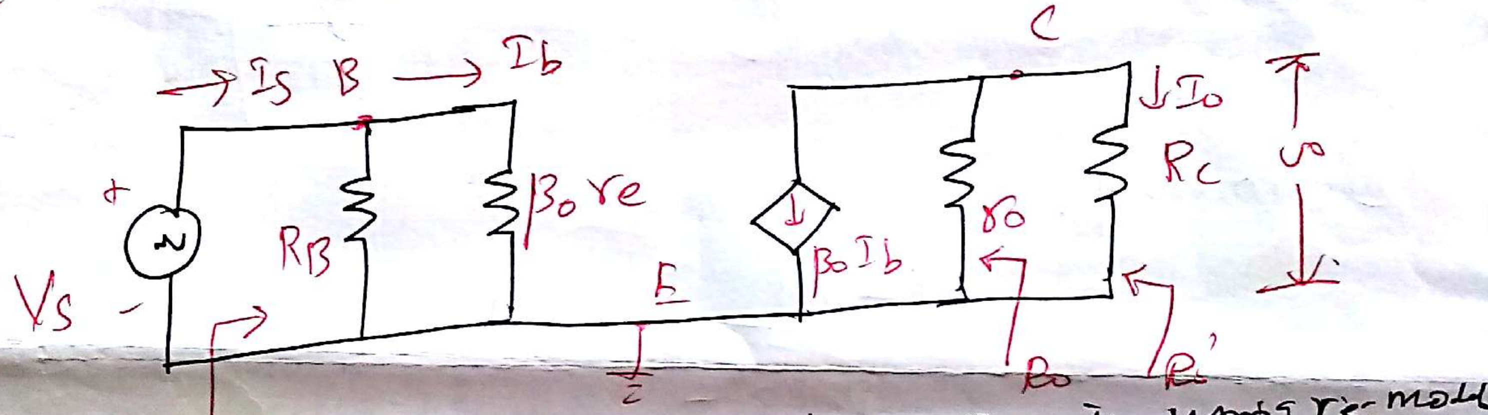


Fig: AC equivalent circuit using re-model

DC Analysis

$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

$$I_E = (1+\beta) I_B$$

$$r_e = \frac{26\,mV}{I_E}$$

I/p resistance

$$R_i = \beta r_e$$

$$R_i' = R_i \parallel R_B$$

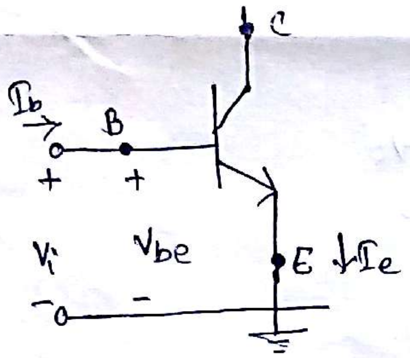
O/p resistance

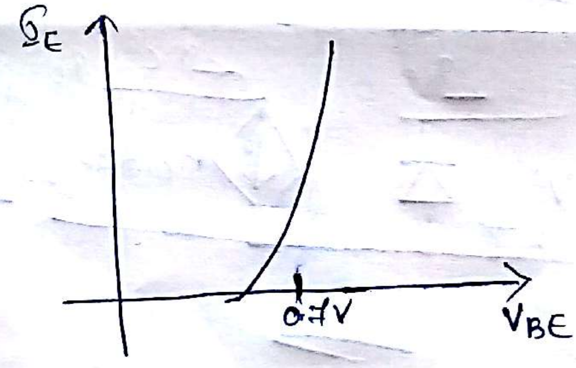
$$R_o = r_o$$

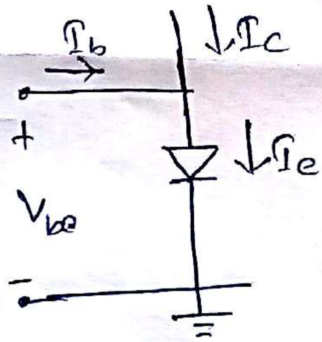
$$R_o' = R_c \parallel r_o$$

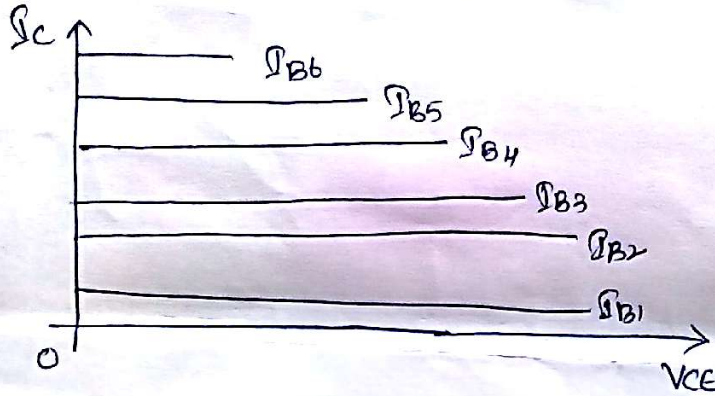
The r_e Transistor Model:

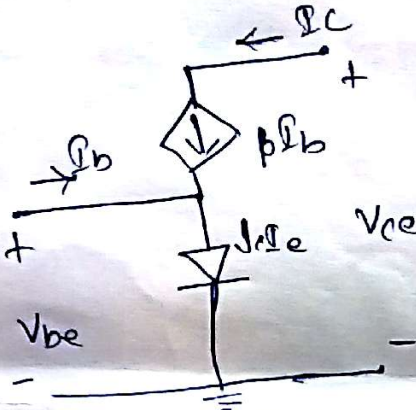
C.E Configuration









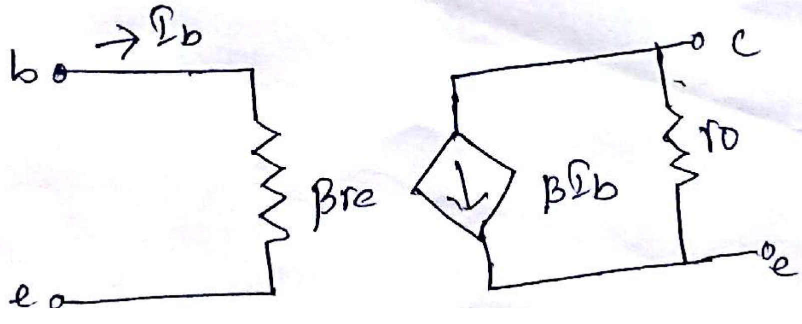


$$Z_i = \frac{V_i}{I_b} = \frac{V_{be}}{I_b}$$

$$V_{be} = I_e r_e = (I_c + I_b) r_e = (\beta I_b + I_b) r_e$$
$$= (\beta + 1) I_b r_e$$

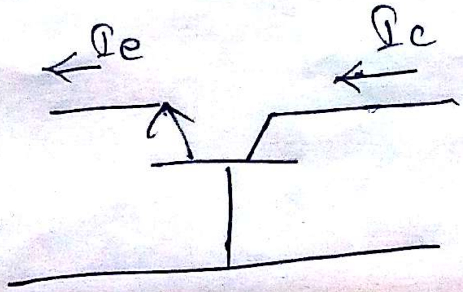
$$Z_i = \frac{V_{be}}{I_b} = \frac{(\beta+1) I_b r_e}{I_b} = (\beta+1) r_e \cong \beta r_e$$

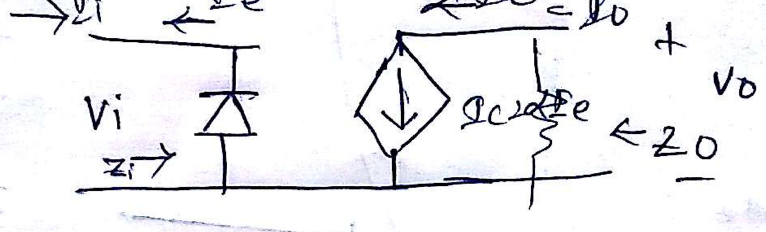
$$r_e = \frac{26\,mV}{I_E}$$



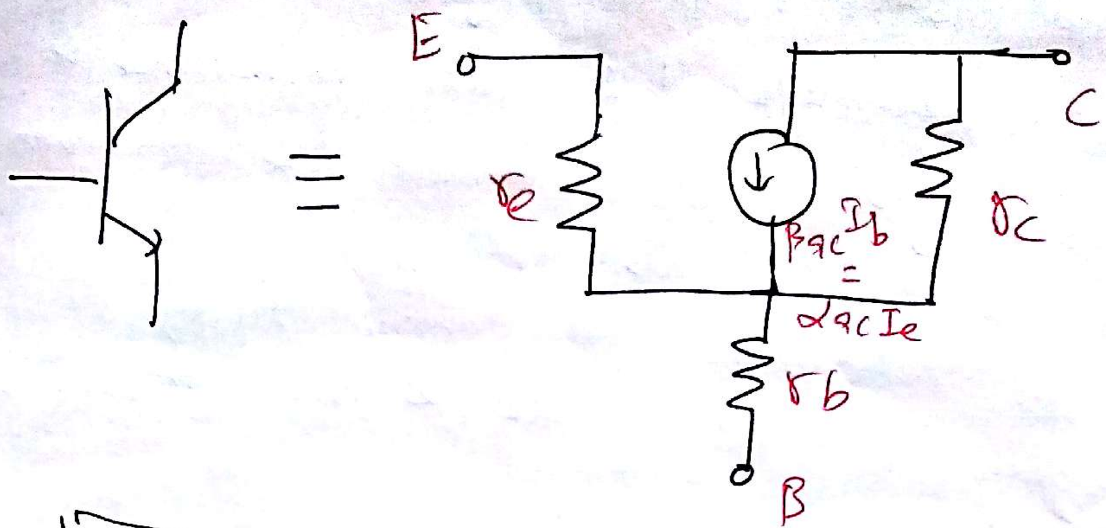
Common Base Configuration:

$I_c = \alpha I_e$





(6)



r-parameter	Description
α_{ac}	ac alpha (I_c/I_e)
β_{ac}	ac Beta (I_c/I_b)
re	ac emitter resistance.
rc	ac collector "
rb	ac base "

Relationships of r-parameter & h-parameters

$$\alpha_{ac} = h_{fb}$$

$$\beta_{ac} = h_{fe}$$

$$r_e = \frac{h_{re}}{h_{oe}}$$

$$r_c = \frac{h_{re}+1}{h_{oe}}$$

$$r_b = h_{ie} - \frac{h_{re}(1+h_{fe})}{h_{oe}}$$

$$r_e = \frac{KT}{q I_E}$$

$K = 1.38 \times 10^{-23}$ J/K

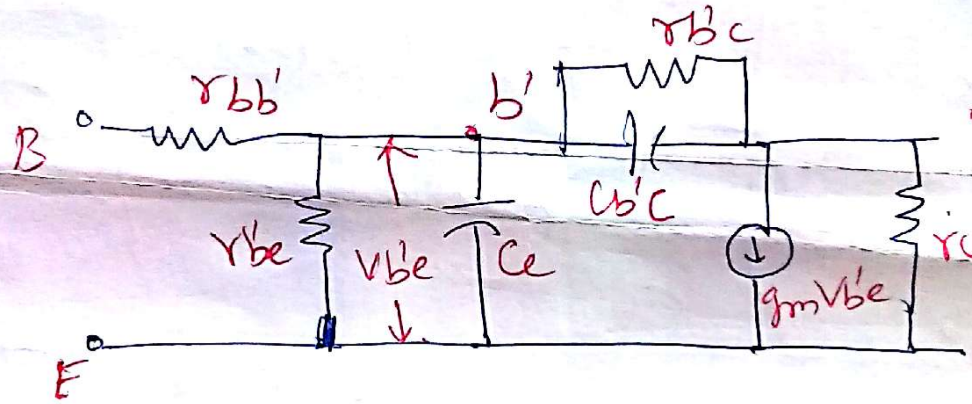
$T = 273°C$

$q = 1.602 \times 10^{-19}$ C.

$r_e = 26\,mV$ at $25°C$

The re model is sufficiently accurate and requires one parameter hfe. I/p impedance is derived from just one parameter hfe. However the re model does not have parameters for output admittance & reverse voltage ratio. It is only suitable at dc & mid frequencies.

Common Emitter Short Circuit Current Gain of Transistor at high frequency using hybrid-π model



B' - Internal Node in Base
Rbb' - Base spreading resistance
rb'e - Internal base node to emitter resistance

The hybrid-π model with resistive load

Hybrid-π model is used to analyze the BJT in high frequency range.

rce = Collector to emitter resistance
Ce = diffusion capacitance of emitter base Jn
rb'c = Feedback resistance from internal base node to collector node
gm = trans conductance

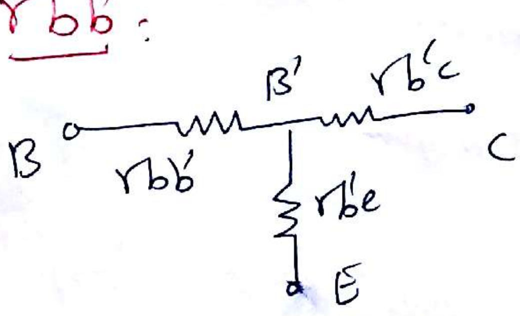
The main advantage of hybrid π model is that it can be simplified to obtain low frequency model of BJT. This is done by eliminating capacitance. So that BJT responds without any significant delay to the input signal.

Elements of hybrid - π - model.

$C_{b'e}$ & $C_{b'c}$: Forward biased PN Junction exhibits a capacitive effect called the diffusion capacitance. This is represented as $C_{b'e}$ (or) C_e.

$|||^{ly}$ the reverse bias PN junction exhibits a capacitive effect called the transition capacitance & it is represented by $C_{b'c}$ (or) C_c.

$r_{bb'}$:



The internal node b' is physically not accessible, bulk node b represents External base terminal.

$r_{bb'}$ is called as base spreading resistance

$r_{b'c}$: Due to early effect, the varying voltage across the collector to emitter junction results in base-width modulation. A change in the effective base width causes the emitter current to change. This feedback effect between o/p to I/p is represented by $r_{b'c}$

$r_{b'e}$: This resistance is in series with the collector junction.

g_m: Due to small changes in voltage $V_{b'e}$ across the emitter junction, there is excess minority carrier concentration injected into the base which is proportional to $V_{b'e}$

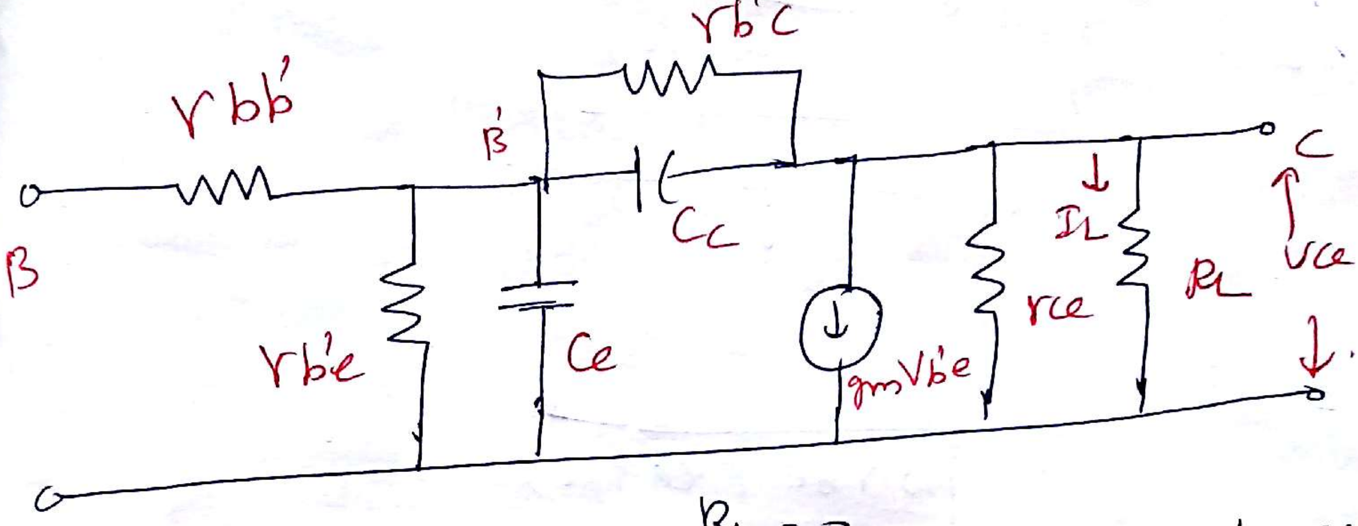
Hybrid π parameters value.

Parameter	Meaning	Value.
g_m	Mutual conductance	50 mA/V
$r_{bb'}$	Base spreading resistance	100 Ω
r_{ce}	o/p resistance	80 kΩ
$r_{b'e}$	Resistance between b' & e	1 kΩ
$r_{b'c}$	Reverse biased pn junction between base & collector	4 MΩ
C_e	junction capacitance	100 pF
C_c	junction capacitance	3 pF.

common gain

Common Emitter Short circuit current gain using hybrid-π-model.

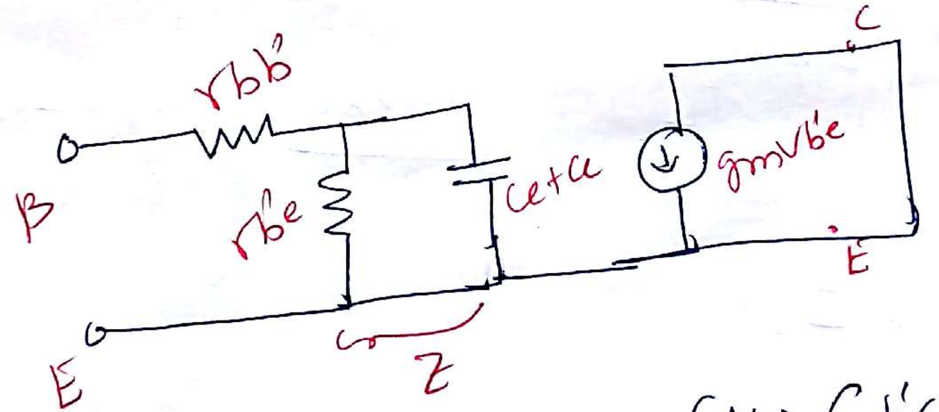
(8)



Let us assume $R_L = 0$

If $R_L = 0$, o/p short circuited r_{ce} becomes zero, C_e & $C_{b'c}$ becomes parallel. $C_{b'c}$ appears between base & emitter, it is known as Miller capacitance. (C_M)

$$C_M = C_{b'c}(1 + g_m R_L)$$



fig(2) simplified hybrid-π model for short circuit

Here $R_L = 0$; $C_M = C_{b'c}$.

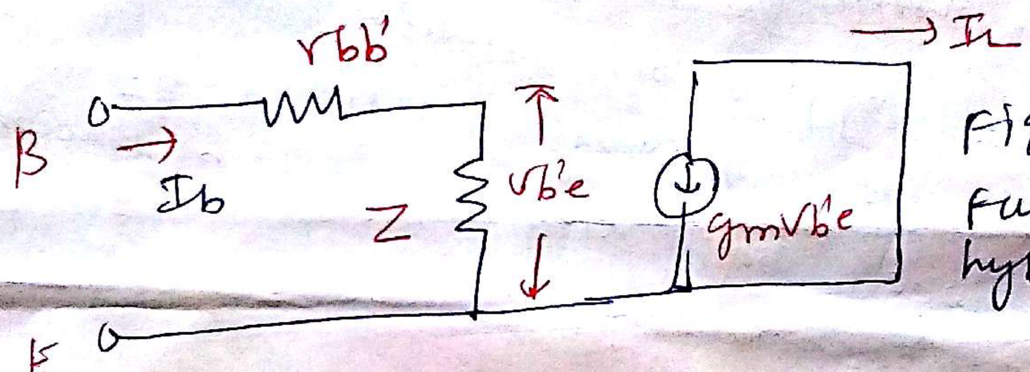
As $r_{b'c} >> r_{b'e}$, $r_{b'e}$ is neglected.

With these approximation, we get simplified hybrid-π - model for short circuit CE transistor as shown in fig.

|| el combination of $r_{b'e}$ & (C_e+C_c) is given

$$Z = \frac{r_{b'e} \times \frac{1}{j\omega(C_e+C_c)}}{r_{b'e} + \frac{1}{j\omega(C_e+C_c)}}$$

$$Z = \frac{r_{b'e}}{1+j\omega r_{b'e}(C_e+C_c)}$$



Fig(3) further simplified hybrid-π-model

$$V_{b'e} = I_b \cdot Z$$

$$\Rightarrow Z = \frac{V_{b'e}}{I_b}$$

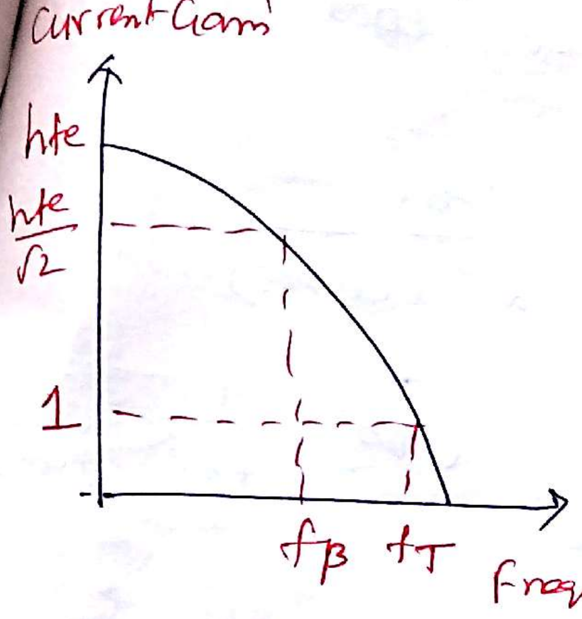
Current gain $A_i = \frac{I_L}{I_b} = \frac{-g_m V_{b'e}}{I_b}$ $[\because I_L = -g_m V_{b'e}]$

$$A_i = -g_m \cdot Z$$

$$A_i = \frac{-g_m \cdot r_{b'e}}{1+j\omega r_{b'e}(C_e+C_c)}$$

Already WKT $g_m V_{b'e} = \cancel{V_b}$ h_{fe}

$$A_i = \frac{-h_{fe}}{1 + j\omega r_{b'e}(C_e + C_c)}$$



Current is not constant in the above equation. It depends on frequency. When frequency is small, the term containing f is very small compared to 1 & hence at low frequency $A_i = -h_{fe}$. But as frequency increases A_i reduces as shown in fig.

Let us take

$$f_\beta = \frac{1}{2\pi r_{b'e}(C_e + C_c)}$$

$$A_i = \frac{-h_{fe}}{1 + jf/f_\beta}$$

$$|A_i| = \frac{h_{fe}}{\sqrt{1 + (f/f_\beta)^2}}$$

As $f = f_\beta$

$$A_i = \frac{h_{fe}}{\sqrt{2}}$$

f_β (cut off frequency)

It is the frequency atwhich the transistor's short circuit CE current gain drops by 3dB (or) $1/\sqrt{2}$ times from its value at low frequency.

$$f_\beta = \frac{1}{2\pi r_{b'e}(C_e + C_c)} = \frac{g_{b'e}}{2\pi(C_e + C_c)}$$

$$\boxed{f_\beta = \frac{1}{h_{fe}} \cdot \frac{g_m}{2\pi(C_e + C_c)}}$$ $$\left[g_{b'e} = \frac{1}{r_{b'e}} = \frac{g_m}{h_{fe}}\right]$$

f_α (Cut-off frequency)

It is the frequency atwhich the transistor's short circuit CB current gain drops by 3dB (or) $1/\sqrt{2}$ times from its value at low frequency.

$$A_i = \frac{-h_{fb}}{1 + j(f/f_\alpha)}$$

where $$f_\alpha = \frac{1}{2\pi r_{b'e}(1+h_{fe})\cdot C_e} = \frac{1+h_{fe}}{2\pi r_{b'e} C_e}$$

$$\simeq \frac{h_{fe}}{2\pi r_{b'e} C_e}$$

$$|A_i| = \frac{h_{fb}}{\sqrt{1 + (f/f_\alpha)^2}}$$ when $f = f_\alpha$

$$A_i = \frac{h_{fb}}{\sqrt{2}}$$

The parameter f_T:

It is the frequency at which short circuit CE current gain becomes unity

at $f = f_T$

$$1 = \frac{h_{fe}}{\sqrt{1+\left(f_T/f_\beta\right)^2}}$$

$f_T/f_\beta \gg 1$, then

$$1 = \frac{h_{fe}}{f_T/f_\beta}$$

$$\Rightarrow f_T = f_\beta \cdot h_{fe}$$

$$f_T = \frac{\cancel{h_{fe}} \times g_m}{\cancel{h_{fe}} \cdot 2\pi (C_e + C_c)}$$

$$f_T = \frac{g_m}{2\pi (C_e + C_c)} \qquad \left[\because f_\beta = \frac{1}{h_{fe}} \cdot \frac{g_m}{2\pi(C_e+C_c)}\right]$$

since $C_e \gg C_c$

$$\boxed{f_T = \frac{g_m}{2\pi C_e}}$$