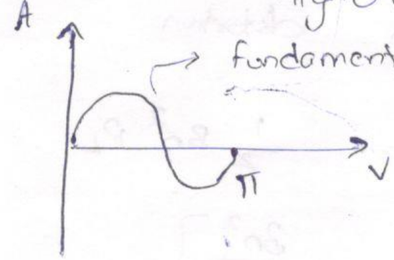
4

Harmonic :- It deals with waves with particular periodic waves. It is generally employed in music and acoustics, Electronic power transmission, Radio technology.

distortion :- This refers to distrobence in waves

As we know it is applied to repeating waves or signals, Such as sin, Cosine waves. A harmonic of such a wave is a wave with the freq that is the integral multiple of frequency of the original wave, known as fundamental frequency

* The original wave is the first harmonic, the following harmonics are higher harmonic.



How this harmonics Related to (Electrical power)

Harmonic voltages & current is an Elc power system are a result of non-linear Elc loads. Harmonic freq in the power grid are a freq cause of power quality problems. Harmonic in power system result in increased heating in the equipment and conductors, misfiring in variable speed drives, and torque pulsation in motors. Reduction of harmonic is considered desirable.

Power o/p due to distortion

$$P_{ac} = V \cdot I \qquad V = iR$$

$$= \frac{I_m V_m}{2} \qquad V_m = i_m R_L$$

$$P_{ac} = \frac{I_m^2 R_L}{2}$$

I_m = Peak value of the o/p current

$$= \frac{I_{pp}}{2} = \frac{I_{max} - I_{min}}{2}$$

$$B_1 = \frac{I_{max} - I_{min}}{2}$$

$$I_m = B_1$$

$$\boxed{P_{ac} = \tfrac{1}{2} B_1^2 R_L}$$

Hence o/p power with harmonic distortion

$$(P_{ac})_D = \frac{1}{2} B_1^2 R_L + \tfrac{1}{2} B_2^2 R_L + \cdots \frac{1}{2} B_n^2 R_L$$

$$= \frac{1}{2} B_1^2 R_L \left[1 + \frac{B_2^2}{B_1^2} + \frac{B_3^2}{B_1^2} + \cdots \frac{B_n^2}{B_1^2} \right]$$

$$(P_{ac})_D = P_{ac} \left[1 + D_2^2 + D_3^2 + \cdots D_n^2 \right]$$

$$D^2 = D_2^2 + D_3^2 + \cdots + D_n^2$$

$$(P_{ac})_D = P_{ac} \left[1 + D^2 \right]$$

If the total harmonic distortion $D = 0.15$

$$(P_{ac})_D = P_{ac} \left[1 + (0.15)^2 \right]$$

$$= 1.0225 \, P_{ac}$$

So that is 2.25 % increase in the power given to the load.

B_1, B_2, B_3 and B_4 are the Amp of the fundamental, second 3rd & 4th harmonic components.

At 5 instants

$\cos 0 = 1$		0
$\cos 30^\circ = \sqrt{3}/2$		$\pi/6$
$\cos 45 = 1/\sqrt{2}$		$\pi/4$
$\cos 60^\circ = 1/2$		$\pi/3$
$\cos 90 = 0$		$\pi/2$
$\cos 120 = -1/2$		$2\pi/3$
$\cos 135 = -1/\sqrt{2}$		
$\cos 150 = -\sqrt{3}/2$		
$\cos 180 = -1$		π
$\cos 3\pi/2 = 270 = 0$		

At point 1 $\omega t = 0$ $i_c = I_{max}$

$$I_{max} = I_{CQ} + B_0 + B_1 + B_2 + B_3 + B_4 \rightarrow (6)$$

At point 2 $\omega t = \pi/3$ $i_c = I_{1/2}$

$$i_c = I_{CQ} + B_0 + B_1 \cos \pi/3 + B_2 \cos \frac{2\pi}{3} + B_3 \cos \frac{3\pi}{3} + B_4 \cos \frac{4\pi}{3}$$

$$I_{1/2} = I_{CQ} + B_0 + 0.5 B_1 - 0.5 B_2 - B_3 - 0.5 B_4 \rightarrow (7)$$

At point 3 $\omega t = \pi/2$, $i_c = i_{CQ}$

$$i_c = I_{CQ} + B_0 + B_1 \cos \frac{\pi}{2} + B_2 \cos 2\pi/2 + B_3 \cos \frac{3\pi}{2} + B_4 \cos 4\pi/2$$

$$I_{CQ} = I_{CQ} + B_0 - B_2 + B_4 \rightarrow (8)$$

At point 4 $\omega t = \frac{2\pi}{3}$ $i_c = I_{-1/2}$

$$I_{-1/2} = I_{CQ} + B_0 - 0.5 B_1 - 0.5 B_2 + B_3 - 0.5 B_4 \rightarrow 9$$

At point 4 $\omega t = \pi$, $i_c = I_{min}$

$$i_c = I_{CQ} + B_0 + B_1 \cos \pi + B_2 \cos 2\pi + B_3 \cos 3\pi + B_4 \cos 4\pi$$

$$I_{min} = I_{CQ} + B_0 - B_1 + B_2 - B_3 + B_4 \rightarrow (10)$$

Solving (5) eq we get B_0, B_1, B_2, B_3, B_4 we get.

$$I_{max} + I_{min} = I_{CQ} + B_0 + \cancel{B_1} + B_2 + \cancel{B_3} + B_4$$
$$= I_{CQ} + B_0 - \cancel{B_1} + B_2 - \cancel{B_3} + B_4$$

$$I_{max} + I_{min} = 2I_{CQ} + 2B_0 + 2B_2 + 2B_4 \rightarrow 11$$

$$2I_{1/2} = 2I_{CQ} + 2B_0 + \cancel{B_1} - B_2 - \cancel{2B_3} - B_4$$
$$2I_{-1/2} = 2I_{CQ} + 2B_0 - \cancel{B_1} - B_2 + \cancel{2B_3} - B_4$$

Adding Eqs (11) & (12) to find B_0

$$I_{max} + I_{min} + 2I_{1/2} + 2I_{-1/2} = 6I_{CQ} + 6B_0$$

$$= 6[I_{CQ} + B_0]$$

$$\approx 6B_0$$

$$\therefore B_0 = \frac{1}{6}[I_{max} + I_{min} + 2I_{1/2} + 2I_{-1/2}]$$

$$I_{max} - I_{min} = 2B_1 + 2B_3$$

$$2I_{-1/2} - 2I_{1/2} = -2B_1 + 4B_3$$

$$I_{max} - I_{min} + 2I_{-1/2} - 2I_{1/2} = \cancel{2B_1} + 2B_3 - \cancel{2B_1} + 4B_3$$

$$6B_3$$

$$\therefore B_3 = \frac{1}{6}[I_{max} - I_{min} + 2I_{-1/2} - 2I_{1/2}]$$

finally

$$B_0 = \frac{1}{6}[I_{max} + 2I_{1/2} + 2I_{-1/2} + I_{min}]$$

$$B_1 = \frac{1}{3}[I_{max} + I_{1/2} - I_{-1/2} - I_{min}]$$

$$B_2 = \frac{1}{4}[I_{max} - 2I_{CQ} + I_{min}]$$

$$B_3 = \frac{1}{6}[I_{max} - 2I_{1/2} + 2I_{-1/2} - I_{max}]$$

$$B_4 = \frac{1}{12}[I_{max} - 4I_{1/2} + 6I_{CQ} - 4I_{-1/2} + I_{min}]$$

Here the harmonic distortion co-efficient can be obtained by

$$D_n = \frac{|B_n|}{|B_1|}$$

$$B_2 = \frac{I_{max} + I_{min} - 2I_{CQ}}{4}$$

As the Amp of the fundamental & 2nd harmonic are

...on, the 2nd harmonic distortion can be cal as

$$\% D_2 = \frac{|B_2|}{|B_1|} \times 100$$

As the method uses three point on the collector

...rrent wave form to obtain the amplitude of the harmonics

This is called 3-point method of determining 2nd harmonic distor

...er order harmonic distortion (five point method

As the non-linearity present in dynamic characte...

...rease, the order of the harmonic distortion also increases

Let the mathematical expression for the collector

...urrent due to higher order harmonic be

$$i_c = K_1 i_b + K_2 i_b^2 + K_3 i_b^3 + K_4 i_b^4 \rightarrow (1)$$

sub i/p signal $i_b = I_{bm} \cos\omega t$ we get.

$$i_c = K_1 I_{bm} \cos\omega t + K_2 I_{bm} \cos^2\omega t + K_3 I_{bm} \cos^3\omega t + K_4 I_{bm} \cos^4\omega t \rightarrow (2)$$

Sub $\cos^2\omega t$, $\cos^3\omega t$, & $\cos^4\omega t$ & doing trigometric oper, we get.

$$i_c = B_0 + B_1 \cos\omega t + K_2 I_{bm} \frac{1+\cos 2\omega t}{2} \longrightarrow (3)$$

$$i_c = B_0 + B_1 \cos\omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + B_4 \cos 4\omega t \longrightarrow 4$$

The total current at collector including dc bias can be written as

$$i_c = i_{CQ} + B_0 + B_1 \cos\omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + B_4 \cos 4\omega t \rightarrow (5)$$

where $I_{CQ} + B_0$ is the dc component

②

It can be seen that due to presence of harmonics, the dc current increases practically the presence of harmonic can be detected by connecting millimeter in the collector circuit. The readings can be observed without an a.c input signal and with ac input signal. If the two readings are almost same there are no harmonic present. But if milliammeter shown an increase in current, when an input is applied, then the harmonics can be concluded to present in the o/p signal.

Now to find the value of total collector current at the various instant 1, 2 & 3

At point 1, $\omega t = 0$, sub in 5

$$\therefore\ i_C = I_{CQ} + B_0 + B_1 + B_2 = I_{max} \rightarrow 6$$

Hence the Eq

At point 2 $\omega t = \pi/2$

$$\therefore\ i_C = I_{CQ} + B_0 - B_2 = I_{CQ} \rightarrow 7$$

At point 3 $\omega t = \pi$

$$\therefore\ i_C = I_{CQ} + B_0 - B_1 + B_2 = I_{min} \rightarrow 8$$

But at

$$\omega t = 0,\ i_C = I_{max}$$
$$\omega t = \pi/2,\ i_C = I_{CQ}$$
$$\omega t = \pi,\ i_C = I_{min}$$

Hence the Eq

from Eq ⑦

$$B_0 = B_2$$

Now

$$I_{max} - I_{min} = \cancel{I_{CQ}} + \cancel{B_0} + B_1 + \cancel{B_2} - \cancel{I_{CQ}} - \cancel{B_0} + B_1 - \cancel{B_2}$$

$$I_{max} - I_{min} = 2B_1$$

$$B_1 = \frac{I_{max} - I_{min}}{2}$$

$$I_{max} + I_{min} = I_{CQ} + B_0 + \cancel{B_1} + B_2 + I_{CQ} + B_0 - \cancel{B_1} + B_2$$
$$= 2I_{CQ} + 2B_0 + 2B_2$$

$$I_{max} + I_{min} = 2I_{CQ} + 4B_2$$

$$B_2 = \frac{I_{max} + I_{min} - 2I_{CQ}}{4}$$

As the Amp of the fundamental & 2nd harmonic are known, the 2nd harmonic distortion can be cal as

$$\% D_2 = \frac{|B_2|}{|B_1|} \times 100$$

As the method uses three point on the collector current wave form to obtain the amplitude of the harmonics. This is called 3-point method of determining 2nd harmonic distortion.

Higher order harmonic distortion (five point method)

As the non-linearity present in dynamic character increase, the order of the harmonic distortion also increases.

Let the mathematical expression for the collector current due to higher order harmonic be

$$i_c = K_1 i_b + K_2 i_b^2 + K_3 i_b^3 + K_4 i_b^4 \rightarrow (1)$$

sub i/p signal $i_b = I_{bm} \cos\omega t$ we get.

$$i_c = K_1 I_{bm} \cos\omega t + K_2 I_{bm} \cos^2\omega t + K_3 I_{bm} \cos^3\omega t + K_4 I_{bm} \cos^4\omega t \rightarrow (2)$$

Sub $\cos^2\omega t$, $\cos^3\omega t$, & $\cos^4\omega t$ & doing trigonometric opr, we get.

$$i_c = B_0 + B_1 \cos\omega t + K_2 I_{bm} \frac{1 + \cos 2\omega t}{2} \longrightarrow (3)$$

$$i_c = B_0 + B_1 \cos\omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + B_4 \cos 4\omega t \longrightarrow 4$$

The total current at collector including dc bias can be written

$$i_c = i_{CQ} + B_0 + B_1 \cos\omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + B_4 \cos 4\omega t \rightarrow (5)$$

where $I_{CQ} + B_0$ is the dc component

B_1, B_2, B_3 and B_4 are the Amp of the fundamental, second 3^{rd} & 4^{th} harmonic Components.

At 5 instants

$\cos 0 = 1$		0
$\cos 30° = \sqrt{3}/2$		$\pi/6$
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$\cos 135 = -1/\sqrt{2}$		
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$\cos 180 = -1$		π
$\cos 3\pi/2 = 270 = 0$		

At point 1 $\omega t = 0$ $i_c = I_{max}$

$$I_{max} = I_{CQ} + B_0 + B_1 + B_2 + B_3 + B_4 \rightarrow (6)$$

At point 2 $\omega t = \pi/3$ $i_c = I_{1/2}$

$$i_c = I_{CQ} + B_0 + B_1 \cos \pi/3 + B_2 \cos 2\pi/3 + B_3 \cos \frac{3\pi}{3} + B_4 \cos \frac{4\pi}{3}$$

$$I_{1/2} = I_{CQ} + B_0 + 0.5 B_1 - 0.5 B_2 - B_3 - 0.5 B_4 \rightarrow (7)$$

At point 3 $\omega t = \pi/2$, $i_c = i_{CQ}$

$$i_c = I_{CQ} + B_0 + B_1 \cos \frac{\pi}{2} + B_2 \cos 2\pi/2 + B_3 \cos \frac{3\pi}{2} + B_4 \cos 4\pi/2$$

$$I_{CQ} = I_{CQ} + B_0 - B_2 + B_4 \rightarrow (8)$$

At point 4 $\omega t = \frac{2\pi}{3}$ $i_c = I_{-1/2}$

$$I_{-1/2} = I_{CQ} + B_0 - 0.5 B_1 - 0.5 B_2 + B_3 - 0.5 B_4 \rightarrow 9$$

At point 4 $\omega t = \pi$, $i_c = I_{min}$

$$i_c = I_{CQ} + B_0 + B_1 \cos \pi + B_2 \cos 2\pi + B_3 \cos 3\pi + B_4 \cos 4\pi$$

$$I_{min} = I_{CQ} + B_0 - B_1 + B_2 - B_3 + B_4 \rightarrow (10)$$

Solving (5) eq we get B_0, B_1, B_2, B_3, B_4 we get.

$$I_{max} + I_{min} = I_{CQ} + B_0 + \cancel{B_1} + B_2 + \cancel{B_3} + B_4$$
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$$I_{max} + I_{min} = 2I_{CQ} + 2B_0 + 2B_2 + 2B_4 \rightarrow 11$$

$$2I_{1/2} = 2I_{CQ} + 2B_0 + \cancel{B_1} - B_2 - \cancel{2B_3} - B_4$$
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$$2I_{1/2} + 2I_{-1/2} = 4I_{CQ} + 4B_0 - 2B_2 - 2B_4 \rightarrow 12$$

Adding Eqs (11) & (12) to find B_0

$$I_{max} + I_{min} + 2I_{1/2} + 2I_{-1/2} = 6I_{CQ} + 6B_0$$

$$= 6[I_{CQ} + B_0]$$

$$= 6B_0$$

$$\therefore B_0 = \frac{1}{6}\left[I_{max} + I_{min} + 2I_{1/2} + 2I_{-1/2}\right]$$

$$I_{max} - I_{min} = 2B_1 + 2B_3$$

$$2I_{-1/2} - 2I_{1/2} = -2B_1 + 4B_3$$

$$I_{max} - I_{min} + 2I_{-1/2} - 2I_{1/2} = \cancel{2B_1} + 2B_3 - \cancel{2B_1} + 4B_3$$

$$6B_3$$

$$\therefore B_3 = \frac{1}{6}\left[I_{max} - I_{min} + 2I_{-1/2} - 2I_{1/2}\right]$$

finally

$$B_0 = \frac{1}{6}\left[I_{max} + 2I_{1/2} + 2I_{-1/2} + I_{min}\right]$$

$$B_1 = \frac{1}{3}\left[I_{max} + I_{1/2} - I_{-1/2} - I_{min}\right]$$

$$B_2 = \frac{1}{4}\left[I_{max} - 2I_{CQ} + I_{min}\right]$$

$$B_3 = \frac{1}{6}\left[I_{max} - 2I_{1/2} + 2I_{-1/2} - I_{max}\right]$$

$$B_4 = \frac{1}{12}\left[I_{max} - 4I_{1/2} + 6I_{CQ} - 4I_{-1/2} + I_{min}\right]$$

Here the harmonic distortion co-efficient can be obtained by

$$D_n = \frac{|B_n|}{|B_1|}$$

Power o/p due to distortion

$$P_{ac} = V \cdot I \qquad V = iR$$

$$= \frac{I_m V_m}{2} \qquad V_m = i_m R_L$$

$$P_{ac} = \frac{I_m^2 R_L}{2}$$

I_m = Peak value of the o/p current

$$= \frac{I_{pp}}{2} = \frac{I_{max} - I_{min}}{2}$$

$$B_1 = \frac{I_{max} - I_{min}}{2}$$

$$I_m = B_1$$

$$\boxed{P_{ac} = \tfrac{1}{2} B_1^2 R_L}$$

Hence o/p power with harmonic distortion

$$(P_{ac})_D = \tfrac{1}{2} B_1^2 R_L + \tfrac{1}{2} B_2^2 R_L + \cdots \tfrac{1}{2} B_n^2 R_L$$

$$= \tfrac{1}{2} B_1^2 R_L \left[1 + \frac{B_2^2}{B_1^2} + \frac{B_3^2}{B_1^2} + \cdots \frac{B_n^2}{B_1^2} \right]$$

$$(P_{ac})_D = P_{ac} \left[1 + D_2^2 + D_3^2 + \cdots D_n^2 \right]$$

$$D^2 = D_2^2 + D_3^2 + \cdots + D_n^2$$

$$(P_{ac})_D = P_{ac} \left[1 + D^2 \right]$$

If the total harmonic distortion $D = 0.15$

$$(P_{ac})_D = P_{ac} \left[1 + (0.15)^2 \right]$$

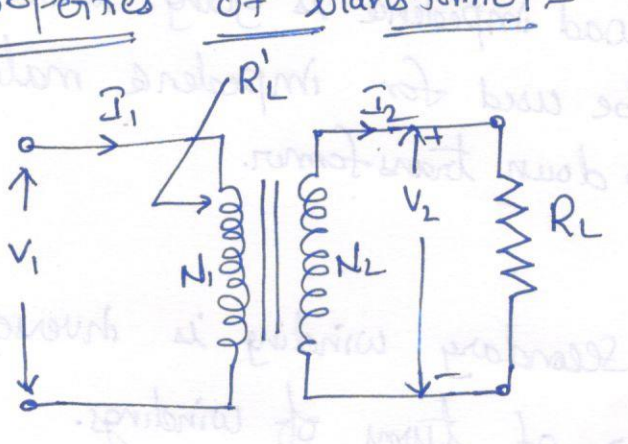
$$= 1.0225 \, P_{ac}$$

So that is 2.25 % increase in the power given to the load.

Transformer Coupled class A Amplifier:-

* For maximum power transfer to the load, the impedence matching is necessary.
* For loads like loudspeaker, having low impedence values, impedence matching is difficult using directly coupled amplifies circuit.
* This is because loudspeaker resistance is in the range of 3 to 4 ohms to 16 ohms while the output impedence of series fed directly coupled class A amplifier is very much high.
* This problem can be eliminated by using transformer to deliver power to the load. The transformer is called an output transformer and amplifier is called transformed coupled class A amplifier.

Properties of transformer:-



* Consider a transformer as shown in the fig. which is connected to a load resistance R_L.
* while analysing transformer, is is assumed that the transformer is ideal and there are no loses in transformer. Similarly the winding resistances are assumed to be zero.

N_1 = Number of turns on primary
N_2 = Number of turns on secondary
V_1 = Voltage applied to primary
V_2 = Voltage on secondary

I_2 = primary current.

1) Turns ratio:-

The ratio of no. of turns on secondary to the no. of turns on primary is called turns ratio and is denoted by n.

$$\boxed{n = \frac{N_2}{N_1}}$$

* Sometimes it is specified as $\frac{N_2}{N_1} : 1$ (or $\frac{N_1}{N_2} : 1$

2) Voltage transformation:-

The transformer transforms the voltage applied on one to other side proportional to turns ratio. The transformer be step up (or step down transformer.

$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = n}$$

In amplifier analysis, load impedance is going to be small. And the transformer is to be used for impedance matching. Hence it has to be a step down transformer.

3) Current transformation:-

The current in the secondary winding is inversely proportional to the number of turns of windings.

$$\boxed{\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}}$$

4) <u>Impedence transformation:-</u>

As current and voltage get transformed from primary to secondary, an impedence seen from either side also changes. The impedence of the load on secondary is R_L.

$$R_L = \frac{V_2}{I_2} \quad \text{and} \quad R_L = \frac{V_1}{I_1}$$

$$V_1 = \frac{N_1}{N_2} V_2 \quad \text{and} \quad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$I_1 = \frac{N_2}{N_1} I_2$$

$$R_L' = \frac{\frac{N_1}{N_2} V_2}{\frac{N_2}{N_1} I_2} = \left[\frac{N_1}{N_2}\right]^2 \times \frac{V_2}{I_2}$$

$$\boxed{R_L' = \frac{R_L}{n^2}}$$

The R_L' is reflected impedence.

<u>Problem</u>

The load of 4Ω is connected to the secondary of a transformer having primary turns of 200 and secondary turns of 20. Calculate the <u>reflected load impedence</u> on primary.

<u>solution</u> Given that

$$R_L = 4\Omega \quad N_1 = 200 \quad N_2 = 20$$

$$n = \frac{N_2}{N_1} = \frac{20}{200} = 0.1$$

$$R_L' = \frac{R_L}{n^2} = \frac{4}{(0.1)^2} = 400\Omega$$