# **Sliding Window**

# 1. Longest Substring Without Repeating Characters

Pattern: Sliding Window

### **Problem Statement**

Given a string s, find the length of the **longest substring** without repeating characters.

# Sample Input & Output

Input: "abcabcbb"
Output: 3
Explanation: The answer is "abc", with length 3.

Input: "bbbbb"

Explanation: All characters are the same; longest valid substring is "b".

Input: "pwwkew"
Output: 3

Output: 1

Explanation: "wke" is the longest substring without repeats

(not "pwke", which is a subsequence).

```
from typing import List
class Solution:
   def lengthOfLongestSubstring(self, s: str) -> int:
       # STEP 1: Initialize structures
       # - Use a set to track characters in current window
       # - Use two pointers (left, right) to define window
       char set = set()
       left = 0
       \max len = 0
       # STEP 2: Main loop / recursion
           - Expand window by moving right pointer
           - If duplicate found, shrink from left until unique
       for right in range(len(s)):
           # STEP 3: Update state / bookkeeping
               - Why here? Ensures window always has unique chars
           # - What breaks if not? Duplicates would stay in set
           while s[right] in char_set:
               char_set.remove(s[left])
               left += 1
           char_set.add(s[right])
           max_len = max(max_len, right - left + 1)
       # STEP 4: Return result
       # - Handles empty string (max_len stays 0)
       return max_len
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
   # Test 1: Normal case
   assert sol.lengthOfLongestSubstring("abcabcbb") == 3
   # Test 2: Edge case
   assert sol.lengthOfLongestSubstring("bbbbb") == 1
   # Test 3: Tricky/negative
   assert sol.lengthOfLongestSubstring("pwwkew") == 3
```

```
# Extra: Empty string
assert sol.lengthOfLongestSubstring("") == 0
print(" All tests passed!")
```

# **Example Walkthrough**

```
We'll trace s = "pwwkew" step by step.
Initial state: char_set = {}, left = 0, max_len = 0.
```

```
Step 1: right = 0 → char 'p'
- 'p' not in char_set → skip while loop
- Add 'p' → char_set = {'p'}
- max_len = max(0, 0-0+1) = 1
State: left=0, right=0, set={'p'}, max_len=1
```

```
Step 2: right = 1 → char 'w'
- 'w' not in set → skip while
- Add 'w' → char_set = {'p','w'}
- max_len = max(1, 1-0+1) = 2
State: left=0, right=1, set={'p','w'}, max_len=2
```

```
Step 3: right = 2 \rightarrow \text{char 'w'}
- 'w' is in set \rightarrow enter while loop
- Remove s[left] = 'p' \rightarrow set = \{'w'\}, left = 1
- Check again: 'w' still in set \rightarrow remove s[1] = 'w'
\rightarrow set = {}, left = 2
- Now 'w' not in set \rightarrow exit while
- Add 'w' \rightarrow set = {'w'}
-\max_{n} = \max(2, 2-2+1) = 2
State: left=2, right=2, set={'w'}, max_len=2
Step 4: right = 3 \rightarrow \text{char 'k'}
- 'k' not in set \rightarrow skip while
- Add 'k' \rightarrow set = {'w', 'k'}
- max_len = max(2, 3-2+1) = 2 \rightarrow still 2
State: left=2, right=3, set={'w','k'}, max_len=2
Step 5: right = 4 \rightarrow char 'e'
- 'e' not in set \rightarrow skip while
- Add 'e' \rightarrow set = {'w','k','e'}
-\max_{n} e^{-n} = \max_{n} (2, 4-2+1) = 3
State: left=2, right=4, set={'w','k','e'}, max_len=3
Step 6: right = 5 \rightarrow \text{char 'w'}
- 'w' is in set \rightarrow enter while
- Remove s[2] = 'w' \rightarrow set = \{'k', 'e'\}, left = 3
- Now 'w' not in set \rightarrow exit while
- Add 'w' \rightarrow set = {'k', 'e', 'w'}
```

-  $\max_{\text{len}} = \max(3, 5-3+1) = 3 \text{ (unchanged)}$ 

Final State:  $max_len = 3 \rightarrow returned$ .

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### **Complexity Analysis**

• Time Complexity: O(n)

Each character is visited at most twice — once by right pointer, once by left pointer. The inner while loop may seem nested, but it's amortized constant per character.

• Space Complexity: O(min(m, n))

m = size of charset (e.g., ASCII = 128). The set stores at most all unique characters in the string, which is bounded by alphabet size or string length n, whichever is smaller.

# 2. Longest Repeating Character Replacement

Pattern: Sliding Window

#### **Problem Statement**

You are given a string **s** and an integer **k**. You can choose **any character** in the string and change it to **any other uppercase English character** at most **k** times.

Return the length of the **longest substring** containing the same letter after performing the above operations.

# Sample Input & Output

```
Input: s = "ABAB", k = 2
Output: 4
Explanation: Replace the two 'A's with 'B's or vice versa → "BBBB".
```

```
Input: s = "AABABBA", k = 1
Output: 4
Explanation: Replace one 'B' in "AABABB" → "AAAABB" →
longest valid is "AABA" → "AAAA" (length 4).

Input: s = "AAAA", k = 0
Output: 4
Explanation: No replacements needed; entire string is already uniform.
```

```
from typing import List
class Solution:
    def characterReplacement(self, s: str, k: int) -> int:
        # STEP 1: Initialize structures
           - freq: tracks count of each char in current window
        # - max_freq: highest freq of any char in window
           - left: start of sliding window
        # - max_len: result to return
        freq = [0] * 26
        max_freq = 0
        left = 0
        \max len = 0
        # STEP 2: Main loop / recursion
        # - Expand window by moving right pointer
        # - Invariant: window is valid if (window_size - max_freq) <= k</pre>
        for right in range(len(s)):
           # Update frequency of current character
           idx = ord(s[right]) - ord('A')
           freq[idx] += 1
           max_freq = max(max_freq, freq[idx])
           # STEP 3: Update state / bookkeeping
           # - If window becomes invalid, shrink from left
            # - Why? We want largest valid window ending at 'right'
           window_size = right - left + 1
```

```
if window_size - max_freq > k:
               left_idx = ord(s[left]) - ord('A')
               freq[left_idx] -= 1
               left += 1
           # Update max length after possible adjustment
           max_len = max(max_len, right - left + 1)
       # STEP 4: Return result
       # - max_len holds the answer; handles empty string via init
       return max_len
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
   # Test 1: Normal case
   assert sol.characterReplacement("ABAB", 2) == 4, \
       f"Expected 4, got {sol.characterReplacement('ABAB', 2)}"
   # Test 2: Edge case - no replacements needed
   assert sol.characterReplacement("AAAA", 0) == 4, \
       f"Expected 4, got {sol.characterReplacement('AAAA', 0)}"
   # Test 3: Tricky/negative - limited replacements
   assert sol.characterReplacement("AABABBA", 1) == 4, \
       f"Expected 4, got {sol.characterReplacement('AABABBA', 1)}"
   print(" All tests passed!")
```

#### **Example Walkthrough**

```
We'll trace s = "AABABBA", k = 1 step by step.

Initial state:

- freq = [0]*26 \rightarrow all zeros
- max_freq = 0
```

```
-left = 0
-\max_{n} = 0
```

Step 1: right =  $0 \rightarrow char 'A'$ -idx = 0-freq[0] = 1 $-\max_{\text{freq}} = \max(0, 1) = 1$ - window\_size = 1, 1 - 1 = 0 <= 1  $\rightarrow$  valid  $- \max_{n=1}^{\infty} e^{-n} = e^{-n} = e^{-n}$ 

State: left=0, max\_len=1, freq[A]=1

```
Step 2: right = 1 \rightarrow \text{char 'A'}
-freq[0] = 2
-\max_{\text{freq}} = 2
- window_size = 2, 2 - 2 = 0 <= 1 \rightarrow valid
-\max_{n} = 2
```

State: left=0, max\_len=2, freq[A]=2

```
Step 3: right = 2 \rightarrow \text{char 'B'}
-idx = 1, freq[1] = 1
-\max_{\text{freq}} = \max(2, 1) = 2
- window_size = 3, 3 - 2 = 1 <= 1 \rightarrow valid
-\max_{n} = 3
```

State: left=0, max\_len=3, freq[A]=2, B=1

```
Step 4: right = 3 \rightarrow char 'A'
-freq[0] = 3
-\max_{\text{freq}} = 3
- window_size = 4, 4 - 3 = 1 <= 1 \rightarrow valid
-\max_{n} = 4
```

State: left=0, max\_len=4, freq[A]=3, B=1

```
Step 5: right = 4 \rightarrow \text{char 'B'}
-freq[1] = 2
-\max_{\text{freq}} = \max(3, 2) = 3
- window_size = 5, 5 - 3 = 2 > 1 \rightarrow invalid!
- Shrink window:
-s[left] = 'A' \rightarrow freq[0] = 2
-left = 1
- New window: indices 1-4 \rightarrow \text{size} = 4
-\max_{n} = \max(4, 4) = 4
State: left=1, max_len=4, freq[A]=2, B=2
Step 6: right = 5 \rightarrow \text{char 'B'}
-freq[1] = 3
-\max_{\text{freq}} = \max(2, 3) = 3
- window_size = 5 (1-5), 5 - 3 = 2 > 1 \rightarrow invalid
- Shrink:
-s[1] = 'A' \rightarrow freq[0] = 1
-left = 2
- New window: 2-5 \rightarrow \text{size} = 4
-\max_{n} = 4
State: left=2, freq[A]=1, B=3
Step 7: right = 6 \rightarrow \text{char 'A'}
-freq[0] = 2
-\max_{\text{freq}} = \max(3, 2) = 3
- window_size = 5 (2-6), 5 - 3 = 2 > 1 \rightarrow invalid
- Shrink:
-s[2] = 'B' \rightarrow freq[1] = 2
-left = 3
- New window: 3-6 \rightarrow \text{size} = 4
```

Final Output: 4

 $-\max_{n} = 4$ 

# Key Insight:

We never reduce max\_freq when shrinking — it may become stale, but that's okay! Because we only care about longer windows, and a stale max\_freq only makes the condition stricter (safe).

# **Complexity Analysis**

• Time Complexity: O(n)

We traverse the string once with right pointer. Each character is visited at most twice (once by right, once by left). All operations inside loop are O(1).

• Space Complexity: 0(1)

The frequency array has fixed size 26 (uppercase English letters). No other space scales with input.

# 3. Minimum Window Substring

Pattern: Sliding Window + Hash Map (Two Pointers with Character Frequency Tracking)

#### **Problem Statement**

Given two strings **s** and **t** of lengths **m** and **n** respectively, return the **minimum substring** of **s** such that every character in **t** (including duplicates) is included in the window.

If there is no such substring, return the empty string "".

The testcases will be generated such that the answer is **unique**.

# Sample Input & Output

```
Input: s = "ADOBECODEBANC", t = "ABC"
Output: "BANC"
Explanation: "BANC" is the smallest substring containing all chars of "ABC".

Input: s = "a", t = "a"
Output: "a"
Explanation: Single character match - edge case with minimal input.

Input: s = "a", t = "aa"
Output: ""
Explanation: t has two 'a's but s only has one → impossible.
```

```
from typing import List
from collections import Counter
class Solution:
    def minWindow(self, s: str, t: str) -> str:
        # STEP 1: Initialize structures
        # - t_count tracks required char frequencies from t
        # - window_count tracks current window char frequencies
        # - have = chars in window meeting required count
        # - need = total unique chars we must satisfy
        if not t or not s:
           return ""
        t count = Counter(t)
        window_count = {}
       have = 0
        need = len(t_count)
       res = ""
        res_len = float('inf')
        left = 0
```

```
# STEP 2: Main loop / recursion
   - Expand right pointer to include new char
   - Update window_count and check if char requirement met
for right in range(len(s)):
    char = s[right]
   window_count[char] = (
        window_count.get(char, 0) + 1
   # STEP 3: Update state / bookkeeping
   # - Only increment 'have' when count exactly matches need
   if (
        char in t_count
        and window_count[char] == t_count[char]
   ):
       have += 1
    # Contract window from left while valid
   while have == need:
        # Update result if current window smaller
        current_length = right - left + 1
        if current_length < res_len:</pre>
           res = s[left:right + 1]
            res_len = current_length
        # Remove leftmost char and update counts
        left_char = s[left]
        window_count[left_char] -= 1
        if (
            left_char in t_count
            and window_count[left_char] < t_count[left_char]</pre>
        ):
           have -= 1
        left += 1
# STEP 4: Return result
# - Return empty string if no valid window found
if res_len == float('inf'):
   return ""
return res
    ----- INLINE TESTS -----
```

```
if __name__ == "__main__":
    sol = Solution()

# Test 1: Normal case
    result1 = sol.minWindow("ADOBECODEBANC", "ABC")
    assert result1 == "BANC", f"Expected 'BANC', got '{result1}'"

# Test 2: Edge case
    result2 = sol.minWindow("a", "a")
    assert result2 == "a", f"Expected 'a', got '{result2}'"

# Test 3: Tricky/negative
    result3 = sol.minWindow("a", "aa")
    assert result3 == "", f"Expected '', got '{result3}'"

    print(" All tests passed!")
```

### **Example Walkthrough**

```
We'll trace s = "ADOBECODEBANC", t = "ABC" step by step.

Initial State:
- t_count = {'A':1, 'B':1, 'C':1}
- window_count = {}
- have = 0, need = 3
- res = "", res_len = \omega
- left = 0
```

```
Step 1: right = 0 → char 'A'
- window_count = {'A':1}
- 'A' in t_count and count matches → have = 1
- have (1) < need (3) → skip while loop
- State: left=0, have=1</pre>
```

```
Step 2: right = 1 \rightarrow 'D'
- window_count = {'A':1, 'D':1}

    'D' not in t_count → have unchanged

- State: left=0, have=1
Step 3: right = 2 \rightarrow 0
- Add 'O' \rightarrow window_count['O']=1
- Not in t \rightarrow have=1
Step 4: right = 3 \rightarrow 'B'
- window_count['B']=1
- 'B' in t and count matches \rightarrow have = 2
- Still < 3 \rightarrow \text{continue}
Step 5: right = 4 \rightarrow 'E'
- Add 'E' \rightarrow no effect on have
Step 6: right = 5 \rightarrow 'C'
- window_count['C']=1
- 'C' in t and count matches \rightarrow have = 3
- Now have == need \rightarrow enter while loop
      Inside while loop (valid window: "ADOBEC")
      - Length = 6 < \infty \rightarrow \mathrm{update} res = "ADOBEC", res_len = 6
      - Remove s[0] = 'A':
      - window count['A'] = 0
      - 'A' in t and now 0 < 1 \rightarrow have = 2
      - Exit while loop
      - State: left=1, have=2, res="ADOBEC"
```

Continue expanding right...

Eventually, at right = 12 ('C'), window "ODEBANC" becomes valid again.

Then we contract:

- Remove 'O'  $\rightarrow$  still valid
- Remove 'D'  $\rightarrow$  still valid
- Remove 'E'  $\rightarrow$  still valid
- Remove 'B' → window\_count['B']=1 (still ok)
- Remove 'A'  $\rightarrow$  window\_count['A']=0  $\rightarrow$  invalid!
- Before removal: window = "BANC"  $\rightarrow$  length  $4 < 6 \rightarrow$  update res = "BANC"

Final res = "BANC".

### **Complexity Analysis**

• Time Complexity: O(m + n)

We traverse s with right pointer once (O(m)), and left moves forward at most m times total (each char visited at most twice). Building  $t\_count$  is O(n). Total linear.

• Space Complexity: O(k)

Where k is the number of unique characters in t (at most 52 for upper+lower letters). window\_count and t\_count store only chars from t, so space is bounded by alphabet size, **not** input length. Technically O(1) for fixed alphabet, but O(k) generally.

# 4. Find All Anagrams in a String

Pattern: Sliding Window

### **Problem Statement**

Given two strings s and p, return an array of all the start indices of p's anagrams in s. You may return the answer in any order.

An **Anagram** is a word or phrase formed by rearranging the letters of a different word or phrase, typically using all the original letters exactly once.

# Sample Input & Output

```
Input: s = "cbaebabacd", p = "abc"
Output: [0,6]
Explanation: The substring starting at index 0 ("cba")
and index 6 ("bac") are anagrams of "abc".
```

```
Input: s = "abab", p = "ab"
Output: [0,1,2]
Explanation: "ab", "ba", and "ab" (starting at 0,1,2)
are all anagrams of "ab".

Input: s = "a", p = "aa"
Output: []
Explanation: p is longer than s → no anagram possible.
```

```
from typing import List
class Solution:
    def findAnagrams(self, s: str, p: str) -> List[int]:
        # STEP 1: Initialize structures
        # - Use fixed-size frequency maps for 26 lowercase letters
        # - Track how many characters have matching counts (matches)
        if len(p) > len(s):
            return []
        p_{\text{count}} = [0] * 26
        s_{count} = [0] * 26
        # Build initial frequency map for p and first window in s
        for char in p:
            p_count[ord(char) - ord('a')] += 1
        for i in range(len(p)):
            s_{ount}[ord(s[i]) - ord('a')] += 1
        # Count how many letters have equal frequency
        matches = 0
        for i in range(26):
            if p_count[i] == s_count[i]:
                matches += 1
        result = []
        if matches == 26:
```

```
result.append(0)
       # STEP 2: Main loop / sliding window
       # - Slide window one char at a time: remove left, add right
       # - Maintain matches count to detect anagram in O(1)
       left = 0
       for right in range(len(p), len(s)):
           # Add new char on the right
           idx_r = ord(s[right]) - ord('a')
           s_{count}[idx_r] += 1
           # Update matches for new char
           if s_count[idx_r] == p_count[idx_r]:
               matches += 1
           elif s_count[idx_r] == p_count[idx_r] + 1:
               matches -= 1
           # Remove old char on the left
           idx_l = ord(s[left]) - ord('a')
           s_count[idx_1] -= 1
           # Update matches for removed char
           if s_count[idx_1] == p_count[idx_1]:
               matches += 1
           elif s_count[idx_1] == p_count[idx_1] - 1:
               matches -= 1
           left += 1
           # STEP 3: Check if current window is an anagram
           # - All 26 letter counts match → valid anagram
           if matches == 26:
               result.append(left)
       # STEP 4: Return result
       # - Handles empty result for edge cases (e.g., p longer than s)
       return result
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
```

```
# Test 1: Normal case
assert sol.findAnagrams("cbaebabacd", "abc") == [0, 6], \
    f"Test 1 Failed: {sol.findAnagrams('cbaebabacd', 'abc')}"

# Test 2: Edge case - overlapping anagrams
assert sol.findAnagrams("abab", "ab") == [0, 1, 2], \
    f"Test 2 Failed: {sol.findAnagrams('abab', 'ab')}"

# Test 3: Tricky/negative - p longer than s
assert sol.findAnagrams("a", "aa") == [], \
    f"Test 3 Failed: {sol.findAnagrams('a', 'aa')}"

print(" All tests passed!")
```

### **Example Walkthrough**

We'll trace s = "abab", p = "ab" step by step.

### Step 0: Initial Checks

- len(p) = 2, len(s) =  $4 \rightarrow \text{proceed}$ .
- Initialize p\_count and s\_count as 26-zero lists.

# Step 1: Build Frequency Maps

- Process first window of s ("ab"):

```
- s_{count[0]} = 1, s_{count[1]} = 1
```

- Compare all 26 indices  $\rightarrow$  only indices 0 and 1 differ? No they match!
  - All others are  $0=0 \rightarrow \text{matches} = 26$
- So, append start index  $0 \rightarrow \text{result} = [0]$

# Step 2: Slide Window — Iteration 1 (right = 2)

- New char:  $s[2] = 'a' \rightarrow index 0$ 
  - s\_count[0] becomes 2
  - Before:  $s_{\text{count}}[0] == p_{\text{count}}[0] (1==1) \rightarrow \text{match}$
  - Now: 2  $1 \rightarrow \mathbf{breaks\ match} \rightarrow \mathbf{matches} = 25$
- Remove leftmost ( $s[0] = 'a' \rightarrow index 0$ )
  - s\_count[0] becomes 1
  - Now  $1 == 1 \rightarrow \mathbf{restores} \ \mathbf{match} \rightarrow \mathbf{matches} = 26$
- left becomes  $1 \rightarrow \text{window is s[1:3]} = \text{"ba"}$
- matches ==  $26 \rightarrow \text{append 1} \rightarrow \text{result} = [0,1]$

# Step 3: Slide Window — Iteration 2 (right = 3)

- New char:  $s[3] = 'b' \rightarrow index 1$ 
  - s\_count[1] becomes 2
  - Was  $1==1 \text{ (match)}, \text{ now } 21 \rightarrow \text{matches} = 25$
- Remove  $s[1] = 'b' \rightarrow index 1$ 
  - s\_count[1] becomes  $1 \rightarrow$  matches again  $\rightarrow$  matches = 26
- left =  $2 \rightarrow \text{window} = s[2:4]$  = "ab"
- Append  $2 \rightarrow \text{result} = [0,1,2]$

#### **Final Result**

• Return [0,1,2]

**Key Insight**: Instead of comparing full maps every time (O(26)), we track **how many letters match**. Each slide only changes **two letters**, so we update matches in O(1).

### **Complexity Analysis**

• Time Complexity: O(n)

We traverse s once with a sliding window. Each character is added and removed at most once. The initial setup is O(m) where m = len(p), but m-n, so overall O(n). The 26-letter comparison is constant.

• Space Complexity: 0(1)

We use two fixed-size arrays of length 26 (for lowercase letters). No scaling with input size beyond constant space.

# 5. Sliding Window Maximum

Pattern: Sliding Window

### **Problem Statement**

You are given an array of integers nums, there is a sliding window of size k which is moving from the very left of the array to the very right. You can only see the k numbers in the window. Each time the sliding window moves right by one position.

Return the max sliding window.

# Sample Input & Output

```
Input: nums = [1,3,-1,-3,5,3,6,7], k = 3
Output: [3,3,5,5,6,7]
Explanation:
Window position
                         Max
                         ----
[1 3 -1] -3 5 3 6 7
                       3
1 [3 -1 -3] 5 3 6 7
1 3 [-1 -3 5] 3 6 7
                       5
1 3 -1 [-3 5 3] 6 7
                        5
1 3 -1 -3 [5 3 6] 7
                         6
1 3 -1 -3 5 [3 6 7]
```

```
Input: nums = [1], k = 1
Output: [1]
Explanation: Only one window possible.
```

```
Input: nums = [1,-1], k = 1
Output: [1, -1]
Explanation: Window size is 1, so each element is its own max.
```

```
from typing import List
from collections import deque

class Solution:
    def maxSlidingWindow(
        self, nums: List[int], k: int
) -> List[int]:
        # STEP 1: Initialize structures
        # - Use deque to store indices of potential max values
        # - Maintain decreasing order: front = current max
        dq = deque()
        result = []
```

```
for i in range(len(nums)):
           # STEP 2: Remove indices outside current window
           \# - Window is [i - k + 1, i]
           # - If front index <= i - k, it's out of bounds
           if dq and dq[0] \le i - k:
               dq.popleft()
           # STEP 3: Maintain decreasing order in deque
           # - Remove from back while current num >= back val
               - Ensures front always holds max for current win
           while dq and nums[dq[-1]] <= nums[i]:
               dq.pop()
           # STEP 4: Add current index to deque
           dq.append(i)
           # STEP 5: Record max once first window is complete
           # - First valid window ends at index k - 1
           if i >= k - 1:
               result.append(nums[dq[0]])
       return result
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
   # Test 1: Normal case
   assert sol.maxSlidingWindow(
       [1,3,-1,-3,5,3,6,7], 3
   = [3,3,5,5,6,7], "Normal case failed"
   # Test 2: Edge case - single element
   assert sol.maxSlidingWindow([1], 1) == [1], \
       "Single element failed"
   # Test 3: Tricky case - window size 1
   assert sol.maxSlidingWindow([1,-1], 1) == [1, -1], \
       "Window size 1 failed"
   print(" All tests passed!")
```

### **Example Walkthrough**

```
We'll trace nums = [1,3,-1,-3,5,3,6,7], k = 3.

Initial state:
dq = deque() (empty), result = []
```

```
i = 0 (num = 1)
- dq empty → skip popleft
- dq empty → skip while loop
- Append 0 → dq = [0]
- i=0 < 2 (k-1) → skip append to result</li>
→ State: dq=[0], result=[]
```

```
 \begin{split} \mathbf{i} &= \mathbf{1} \text{ (num = 3)} \\ -\text{ dq[0]=0} &> 1\text{-3}\text{=-2} \rightarrow \text{no popleft} \\ -\text{ While: nums[0]=1} &<= 3 \rightarrow \text{pop 0} \rightarrow \text{dq=[]} \\ -\text{ Append 1} \rightarrow \text{dq = [1]} \\ -\text{ i=1} &< 2 \rightarrow \text{skip result} \\ \rightarrow \text{ State: dq=[1], result=[]} \end{split}
```

```
i = 2 (num = -1)

- dq[0]=1 > 2-3=-1 \rightarrow no popleft

- While: nums[1]=3 > -1 \rightarrow don't pop

- Append 2 \rightarrow dq = [1,2]

- i=2 >= 2 \rightarrow add nums[1]=3 to result

\rightarrow State: dq=[1,2], result=[3]
```

```
    i = 3 (num = -3)
    - dq[0]=1 <= 3-3=0? No (1>0) → keep
    - While: nums[2]=-1 > -3 → don't pop
    - Append 3 → dq = [1,2,3]
    - Add nums[1]=3 → result=[3,3]
    → State: dq=[1,2,3], result=[3,3]
    i = 4 (num = 5)
```

```
1 = 4 (num = 5)

- dq[0]=1 <= 4-3=1 \rightarrow yes! \rightarrow popleft \rightarrow dq=[2,3]

- While:

- nums[3]=-3 <= 5 \rightarrow pop \rightarrow dq=[2]

- nums[2]=-1 <= 5 \rightarrow pop \rightarrow dq=[]

- Append 4 \rightarrow dq=[4]

- Add nums[4]=5 \rightarrow result=[3,3,5]

\rightarrow State: dq=[4], result=[3,3,5]
```

```
i = 7 (num = 7)
- dq[0]=6 > 7-3=4 → keep
- While: nums[6]=6 <= 7 → pop → dq=[]
- Append 7 → dq=[7]
- Add nums[7]=7 → result=[3,3,5,5,6,7]
→ Final result: [3,3,5,5,6,7]</pre>
```

**Key insight**: The deque always keeps indices of elements in **decreasing order**, so the front is always the max of the current window. Out-of-window indices are removed from the front; smaller-or-equal elements are removed from the back before adding the new one.

# **Complexity Analysis**

• Time Complexity: O(n)

Each element is pushed and popped from the deque at most once. The outer loop runs n times, and inner while loop operations are amortized constant time.

• Space Complexity: O(k)

The deque stores at most k indices (one per window position). The output list is O(n - k + 1), but auxiliary space is dominated by the deque, which is O(k).