# **Matrix**

### 1. Valid Sudoku

Pattern: Arrays & Hashing

### **Problem Statement**

Determine if a  $9 \times 9$  Sudoku board is valid. Only the filled cells need to be validated according to the following rules: 1. Each row must contain the digits 1-9 without repetition. 2. Each column must contain the digits 1-9 without repetition. 3. Each of the nine  $3 \times 3$  sub-boxes of the grid must contain the digits 1-9 without repetition.

### Note:

- A valid Sudoku board (partially filled) is not necessarily solvable.
- Only filled cells need validation.
- Empty cells are represented by '.'.

### Sample Input & Output

```
Input: board =
[["5","3",".",".","7",".",".","."]
,["6",".",".","1","9","5",".","."]
,[".","9","8",".",".",".","6","."]
,["8",".",".",".","6",".",".","3"]
,["4",".",".","8",".","3",".","1"]
```

```
,["7",".",".","2",".",".","6"]
,[".","6",".",".","2","8","."]
,[".",".",".","4","1","9",".","5"]
,[".",".",".",".","8",".","7","9"]]
Output: true
Explanation: All rows, columns, and 3x3 boxes satisfy Sudoku rules.
```

```
Input: board =
[["8","3",".",".","7",".",".","."]
,["6",".",".","1","9","5",".",".","."]
,[".","9","8",".",".",".","6","."]
,["8",".",".",".","6",".",".","3"]
,["4",".",".","8",".","3",".","1"]
,["7",".",".",".","2",".",".","6"]
,[".","6",".",".",".","2","8","."]
,[".",".",".","4","1","9",".","5"]
,[".",".",".",".","8",".","7","9"]]
Output: false
Explanation: There are two 8s in the top-left 3x3 sub-box.
```

```
Input: board =
[[".",".",".",".",".",".",".","."]
,[".","4",".",".",".",".",".","."]
,[".",".",".",".",".",".","."]
,["8",".",".",".",".",".","."]
,[".",".","2",".",".",".","."]
,[".",".",".",".",".",".","."]
,[".",".",".",".",".",".","."]
,[".",".",".",".",".",".","."]
)Output: false
Explanation: Column 5 has two 5s (at [0][4] and [8][4] is actually 8 -but here, two 2s appear in column 1).
```

```
from typing import List
class Solution:
    def isValidSudoku(self, board: List[List[str]]) -> bool:
       # STEP 1: Initialize structures
       # - Use sets to track seen digits in rows, cols, and boxes
       rows = [set() for _ in range(9)]
       cols = [set() for _ in range(9)]
       boxes = [set() for _ in range(9)]
       # STEP 2: Main loop / recursion
       # - Iterate over every cell (i, j)
       for i in range(9):
           for j in range(9):
               val = board[i][j]
               if val == '.':
                   continue # Skip empty cells
               # STEP 3: Update state / bookkeeping
               # - Compute box index: (i//3)*3 + j//3 maps 3x3 blocks
               box_idx = (i // 3) * 3 + (j // 3)
               # Check for duplicates in row, col, or box
               if (val in rows[i] or
                   val in cols[j] or
                   val in boxes[box_idx]):
                   return False
               # Add current value to trackers
               rows[i].add(val)
               cols[j].add(val)
               boxes[box_idx].add(val)
       # STEP 4: Return result
       # - If no duplicates found, board is valid
       return True
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
  # Test 1: Normal valid Sudoku
```

```
board1 = [
    ["5","3",".",".","7",".",".",".","."],
    ["6",".",".","1","9","5",".",".","."],
    [".","9","8",".",".",".",".","6","."],
    ["8",".",".","6",".",".",".","3"],
    ["4",".",".","8",".","3",".",".","1"],
    ["7",".",".",".","2",".",".",".","6"],
    [".","6",".",".",".","2","8","."],
    [".",".",".","4","1","9",".",".","5"],
    [".",".",".","8",".",".","7","9"]
]
print("Test 1:", sol.isValidSudoku(board1)) # Expected: True
   Test 2: Invalid due to duplicate in 3x3 box
board2 = [
    ["8","3",".",".","7",".",".",".","."],
    ["6",".",".","1","9","5",".",".","."],
    [".","9","8",".",".",".",".","6","."],
    ["8",".",".","6",".",".",".","3"],
    ["4",".",".","8",".","3",".",".","1"],
    ["7",".",".",".","2",".",".",".","6"],
    [".","6",".",".",".","2","8","."],
    [".",".",".","4","1","9",".",".","5"],
    [".",".",".","8",".",".","7","9"]
print("Test 2:", sol.isValidSudoku(board2)) # Expected: False
   Test 3: Invalid due to duplicate in column
board3 = [
    [".",".",".",".","5",".",".","1","."],
    [".","4",".","3",".",".",".",".","."],
    [".",".",".",".","3",".",".","1"],
    [".",".","2",".","7",".",".",".","."],
    [".","1","5",".",".",".",".",".","."],
    [".",".",".",".","2",".","."],
    [".","2",".","9",".",".",".",".","."],
    [".",".","4",".",".",".",".","."]
]
print("Test 3:", sol.isValidSudoku(board3)) # Expected: False
```

instant feedback.

### **Example Walkthrough**

We'll walk through **Test 1** step by step.

Initial state: - rows = [set(), set(), ..., set()] (9 empty sets) - Same for cols and boxes.

Step 1: i=0, j=0, val = "5" - Not '.', so proceed. - box\_idx = (0//3)\*3 + (0//3) = 0 - Check: "5" not in rows[0], cols[0], or boxes[0]  $\rightarrow$  OK. - Add "5" to all three  $\rightarrow$  rows[0] = {"5"}, etc.

Step 2: i=0, j=1, val = "3" - box\_idx = 0 - "3" not seen in row 0, col 1, or box  $0 \rightarrow add$  it.

Step 3: i=0, j=2, val = "."  $\rightarrow$  skip.

•••

Step 10: i=1, j=0, val = "6" - box\_idx = (1//3)\*3 + 0 = 0\*3 + 0 = 0 - Check box 0: currently has {"5","3","8","9"} from previous rows? - Wait! Actually, we haven't processed row 3 yet. At this point (i=1), box 0 only has "5","3" from row 0 and now "6" from row 1  $\rightarrow$  no conflict.

Continue until all 81 cells are checked.

**Final state**: No duplicates found  $\rightarrow$  return True.

Key insight: Each digit is checked **exactly once** against its row, column, and box using hash sets for O(1) lookups.

# **Complexity Analysis**

• Time Complexity: 0(1)

The board is always  $9 \times 9 \to \text{fixed } 81 \text{ cells.}$  Each cell is processed once with O(1) set operations. So technically constant time. If generalized to  $n \times n$ , it would be  $O(n^2)$ .

• Space Complexity: 0(1)

We use 27 sets (9 rows + 9 cols + 9 boxes), each holding at most 9 digits. Total space is bounded by a constant  $(27 \times 9)$ . Thus, O(1).

### 2. Set Matrix Zeroes

Pattern: Arrays & Hashing (In-Place Modification)

#### **Problem Statement**

Given an  $m \times n$  integer matrix matrix, if an element is 0, set its entire row and column to 0's.

You must do it in place.

### Sample Input & Output

```
Input: matrix = [[1,1,1],[1,0,1],[1,1,1]]
Output: [[1,0,1],[0,0,0],[1,0,1]]
Explanation: The zero at (1,1) zeroes out row 1 and column 1.
```

```
Input: matrix = [[1]]
Output: [[1]]
Explanation: No zeros → no change (edge: 1x1 matrix).
```

```
from typing import List
class Solution:
    def setZeroes(self, matrix: List[List[int]]) -> None:
        # STEP 1: Initialize structures
           - Use first row and first col as markers.
        # - Track separately if first row/col originally had zeros.
        m, n = len(matrix), len(matrix[0])
        first_row_has_zero = any(matrix[0][j] == 0 for j in range(n))
        first_col_has_zero = any(matrix[i][0] == 0 for i in range(m))
        # STEP 2: Main loop / recursion
           - Scan inner matrix (from [1][1] onward).
        # - If cell is 0, mark its row head and col head as 0.
        for i in range(1, m):
           for j in range(1, n):
                if matrix[i][j] == 0:
                   matrix[i][0] = 0
                   matrix[0][j] = 0
        # STEP 3: Update state / bookkeeping
        # - Use markers in first row/col to zero out inner cells.
        for i in range(1, m):
           for j in range(1, n):
               if matrix[i][0] == 0 or matrix[0][j] == 0:
                   matrix[i][j] = 0
        # STEP 4: Return result
          - Handle edge cases: zero out first row/col if needed.
        if first_row_has_zero:
           for j in range(n):
               matrix[0][j] = 0
        if first_col_has_zero:
           for i in range(m):
               matrix[i][0] = 0
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
    # Test 1: Normal case
   mat1 = [[1,1,1],[1,0,1],[1,1,1]]
```

```
sol.setZeroes(mat1)
print("Test 1:", mat1)
# Expected: [[1,0,1],[0,0,0],[1,0,1]]

# Test 2: Edge case
mat2 = [[1]]
sol.setZeroes(mat2)
print("Test 2:", mat2)
# Expected: [[1]]

# Test 3: Tricky/negative
mat3 = [[0,1,2,0],[3,4,5,2],[1,3,1,5]]
sol.setZeroes(mat3)
print("Test 3:", mat3)
# Expected: [[0,0,0,0],[0,4,5,0],[0,3,1,0]]
```

### **Example Walkthrough**

```
We'll walk through Test 3:
matrix = [[0,1,2,0],[3,4,5,2],[1,3,1,5]]

Initial state:
- m = 3, n = 4
- first_row_has_zero = True (because matrix[0][0] == 0 and matrix[0][3] == 0)
- first_col_has_zero = True (because matrix[0][0] == 0)

Step 1: Mark inner zeros

Loop over i=1..2, j=1..3:
- At (1,1): 4 → no mark
- At (1,2): 5 → no mark
- At (1,3): 2 → no mark
- At (2,1): 3 → no mark
- At (2,2): 1 → no mark
- At (2,3): 5 → no mark
- At (2,3): 5 → no mark
- No new markers added (only original zeros in row 0).
```

### Step 2: Zero out inner cells using markers

Check each inner cell:

```
- For i=1, j=1: matrix[1][0]=3 0, matrix[0][1]=1 0 \rightarrow keep 4
- j=2: same \rightarrow keep 5
- j=3: matrix[0][3]=0 \to set matrix[1][3] = 0
- For i=2, j=1: matrix[0][1]=1, matrix[2][0]=1 \rightarrow keep 3
- j=2: keep 1
- j=3: matrix[0][3]=0 \rightarrow set matrix[2][3] = 0
Now matrix looks like:
[[0,1,2,0], [3,4,5,0], [1,3,1,0]]
```

### Step 3: Zero out first row and first column

- first\_row\_has\_zero = True  $\rightarrow$  set entire row 0 to 0
- first\_col\_has\_zero = True o set matrix[0][0], matrix[1][0], matrix[2][0] to 0

Final matrix:

[[0,0,0,0], [0,4,5,0], [0,3,1,0]]

### **Complexity Analysis**

• Time Complexity: O(m \* n)

We scan the matrix a constant number of times (3 full passes):

- once to check first row/col,
- once to mark,
- once to apply zeros,
- and two partial passes for first row/col cleanup.

All are linear in total elements.

• Space Complexity: 0(1)

We use only a few boolean flags (first\_row\_has\_zero, first\_col\_has\_zero). No extra arrays or hash maps — all marking is done **in-place** using the matrix's own first row and column.

### 3. Spiral Matrix

Pattern: Matrix Traversal (Simulation)

#### **Problem Statement**

Given an  $m \times n$  matrix, return all elements of the matrix in spiral order.

### Sample Input & Output

```
Input: [[1,2,3],[4,5,6],[7,8,9]]
Output: [1,2,3,6,9,8,7,4,5]
Explanation: Traverse top row → right column → bottom row (rev)
→ left column (rev), then repeat inward.

Input: [[1,2,3,4],[5,6,7,8],[9,10,11,12]]
Output: [1,2,3,4,8,12,11,10,9,5,6,7]
Explanation: Spiral continues layer by layer until all cells visited.

Input: [[1]]
Output: [1]
Explanation: Single-element edge case.
```

```
from typing import List

class Solution:
    def spiralOrder(self, matrix: List[List[int]]) -> List[int]:
        # STEP 1: Initialize boundaries and result list
        # - top, bottom, left, right define current layer
        # - result collects elements in spiral order
        if not matrix or not matrix[0]:
            return []

        top, bottom = 0, len(matrix) - 1
        left, right = 0, len(matrix[0]) - 1
```

```
result = []
       # STEP 2: Main loop - traverse while boundaries valid
       # - Invariant: [top, bottom] and [left, right] form a valid submatrix
       while top <= bottom and left <= right:</pre>
           # Traverse top row (left → right)
           for col in range(left, right + 1):
               result.append(matrix[top][col])
           top += 1 # Move top boundary down
           # Traverse right column (top → bottom)
           for row in range(top, bottom + 1):
               result.append(matrix[row][right])
           right -= 1 # Move right boundary left
           # Traverse bottom row (right → left), if row exists
           if top <= bottom:</pre>
               for col in range(right, left - 1, -1):
                   result.append(matrix[bottom][col])
               bottom -= 1 # Move bottom boundary up
           # Traverse left column (bottom → top), if column exists
           if left <= right:</pre>
               for row in range(bottom, top -1, -1):
                   result.append(matrix[row][left])
               left += 1 # Move left boundary right
       # STEP 4: Return result
       # - All elements collected in spiral order
       return result
# ----- INLINE TESTS ------
if __name__ == "__main__":
   sol = Solution()
   # Test 1: Normal case
   assert sol.spiralOrder([[1,2,3],[4,5,6],[7,8,9]]) == [1,2,3,6,9,8,7,4,5]
   # Test 2: Edge case - single element
   assert sol.spiralOrder([[1]]) == [1]
   # Test 3: Tricky/negative - wide rectangle
```

```
assert (sol.spiralOrder([[1,2,3,4],[5,6,7,8],[9,10,11,12]]) ==
        [1,2,3,4,8,12,11,10,9,5,6,7])
print(" All tests passed!")
```

# **Example Walkthrough**

We'll trace spiralOrder([[1,2,3],[4,5,6],[7,8,9]]) step by step.

#### Initial state:

```
- matrix = [[1,2,3],[4,5,6],[7,8,9]]

- top = 0, bottom = 2

- left = 0, right = 2

- result = []
```

# Step 1: Top row (left $\rightarrow$ right)

```
Loop: col from 0 to 2
```

- Append matrix[0][0] = 1  $\rightarrow$  result = [1]

- Append matrix[0][1] =  $2 \rightarrow \text{result} = [1,2]$ 

- Append matrix[0][2] = 3  $\rightarrow$  result = [1,2,3]

Then: top += 1  $\rightarrow$  top = 1

State: top=1, bottom=2, left=0, right=2, result=[1,2,3]

# Step 2: Right column (top $\rightarrow$ bottom)

```
Loop: row from 1 to 2
```

- Append matrix[1][2] =  $6 \rightarrow \text{result} = [1,2,3,6]$
- Append matrix[2][2] =  $9 \rightarrow \text{result} = [1,2,3,6,9]$

Then: right  $-= 1 \rightarrow \text{right} = 1$ 

State: top=1, bottom=2, left=0, right=1, result=[1,2,3,6,9]

# Step 3: Bottom row (right $\rightarrow$ left)

Check: top (1) <= bottom (2)  $\rightarrow$ 

Loop: col from 1 down to 0

- Append matrix[2][1] = 8  $\rightarrow$  result = [1,2,3,6,9,8]
- Append matrix[2][0] =  $7 \rightarrow \text{result} = [1,2,3,6,9,8,7]$

Then: bottom  $-= 1 \rightarrow bottom = 1$ 

State: top=1, bottom=1, left=0, right=1, result=[1,2,3,6,9,8,7]

### Step 4: Left column (bottom $\rightarrow$ top)

Check: left (0) <= right (1)  $\rightarrow$ 

Loop: row from 1 down to 1 (only one iteration)

- Append matrix[1][0] =  $4 \rightarrow \text{result} = [1,2,3,6,9,8,7,4]$ 

Then: left += 1  $\rightarrow$  left = 1

State: top=1, bottom=1, left=1, right=1, result=[1,2,3,6,9,8,7,4]

### Next loop iteration:

top (1) <= bottom (1) and left (1) <= right (1)  $\rightarrow$  continue

### Step 5: Top row again

Loop: col from 1 to 1

- Append matrix[1][1] =  $5 \rightarrow \text{result} = [1,2,3,6,9,8,7,4,5]$ 

Then: top += 1  $\rightarrow$  top = 2

Now: top (2) > bottom (1)  $\rightarrow$  loop ends.

Final result: [1,2,3,6,9,8,7,4,5]

### **Complexity Analysis**

• Time Complexity: O(m \* n)

Every element is visited exactly once. Total elements = m \* n.

• Space Complexity: O(1) (excluding output)

Only a few boundary variables (top, bottom, left, right) are used. The output list is not counted toward auxiliary space.

# 4. Rotate Image

 ${\bf Pattern} \hbox{:} \ {\bf Matrix} \ {\bf Manipulation} \ (\hbox{In-Place Transformation})$ 

### **Problem Statement**

You are given an n x n 2D matrix representing an image. Rotate the image by 90 degrees (clockwise).

You have to rotate the image **in-place**, which means you have to modify the input 2D matrix directly. **DO NOT** allocate another 2D matrix and do the rotation.

### Sample Input & Output

```
Input: [[1]]
Output: [[1]]
Explanation: Single-element matrix remains unchanged.
```

```
from typing import List
class Solution:
    def rotate(self, matrix: List[List[int]]) -> None:
        # STEP 1: Initialize structures
           - n is the size of the square matrix
            - We'll perform in-place rotation using layer-by-layer
              swaps (like peeling an onion)
        n = len(matrix)
        # STEP 2: Main loop / recursion
        # - Loop over layers from outer to inner
        # - For an n x n matrix, there are n // 2 layers
        for layer in range(n // 2):
           # Define first and last index of current layer
           first = layer
           last = n - 1 - layer
           # STEP 3: Update state / bookkeeping
              - For each element in the current layer's top row
                  (excluding the last, which is handled by rotation),
```

```
perform a 4-way swap
           for i in range(first, last):
               offset = i - first
               # Save top element
               top = matrix[first][i]
               # Move left \rightarrow top
               matrix[first][i] = matrix[last - offset][first]
               # Move bottom → left
               matrix[last - offset][first] = \
                   matrix[last] [last - offset]
               # Move right → bottom
               matrix[last][last - offset] = \
                   matrix[i][last]
               # Move saved top → right
               matrix[i][last] = top
       # STEP 4: Return result
       # - Nothing to return; matrix is modified in-place
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
   # Test 1: Normal case (3x3)
   mat1 = [[1,2,3],[4,5,6],[7,8,9]]
   sol.rotate(mat1)
   expected1 = [[7,4,1],[8,5,2],[9,6,3]]
   assert mat1 == expected1, f"Test 1 failed: got {mat1}"
   print(" Test 1 passed")
   # Test 2: Edge case (1x1)
   mat2 = [[1]]
   sol.rotate(mat2)
   expected2 = [[1]]
   assert mat2 == expected2, f"Test 2 failed: got {mat2}"
   print(" Test 2 passed")
```

### **Example Walkthrough**

We'll walk through **Test 1**: [[1,2,3],[4,5,6],[7,8,9]].

Initial matrix:

```
[1, 2, 3]
[4, 5, 6]
[7, 8, 9]
```

### Step-by-step execution:

```
1. n = 3 \rightarrow n // 2 = 1, so 1 layer (layer = 0).
2. first = 0, last = 2.
3. Loop i from 0 to 1 (since range(0, 2)).
```

First iteration (i = 0): - offset = 0 - 0 = 0 - top = matrix[0][0] = 1Now perform 4-way swap:

```
• Left \rightarrow Top:
```

matrix[0][0] = matrix[2 - 0][0] = matrix[2][0] = 7 
$$\rightarrow$$
 Row 0 becomes [7, 2, 3]

• Bottom  $\rightarrow$  Left:

matrix[2][0] = matrix[2][2 - 0] = matrix[2][2] = 9 
$$\rightarrow$$
 Row 2 becomes [9, 8, 9]

• Right  $\rightarrow$  Bottom:

$$matrix[2][2] = matrix[0][2] = 3$$
  
 $\rightarrow Row 2 becomes [9, 8, 3]$ 

• Top (saved)  $\rightarrow$  Right:

matrix[0][2] = top = 1 
$$\rightarrow$$
 Row 0 becomes [7, 2, 1]

Matrix now:

Second iteration (i = 1): - offset = 1 - 0 = 1 - top = matrix[0][1] = 2Swaps:

• Left  $\rightarrow$  Top:

matrix[0][1] = matrix[2 - 1][0] = matrix[1][0] = 4 
$$\rightarrow$$
 Row 0: [7, 4, 1]

• Bottom  $\rightarrow$  Left:

matrix[1][0] = matrix[2][2 - 1] = matrix[2][1] = 8 
$$\rightarrow$$
 Row 1: [8, 5, 6]

• Right  $\rightarrow$  Bottom:

matrix[2][1] = matrix[1][2] = 6 
$$\rightarrow$$
 Row 2: [9, 6, 3]

• Top  $\rightarrow$  Right:

matrix[1][2] = top = 2 
$$\rightarrow$$
 Row 1: [8, 5, 2]

Final matrix:

[7, 4, 1]

[8, 5, 2]

[9, 6, 3]

Matches expected output!

**Key insight**: Each layer is rotated by moving elements in groups of 4 — top  $\leftarrow$  left  $\leftarrow$  bottom  $\leftarrow$  right  $\leftarrow$  top.

### **Complexity Analysis**

• Time Complexity: O(n<sup>2</sup>)

We visit each element exactly once. The outer loop runs n // 2 times, and the inner loop runs up to n - 1 times per layer. Total operations  $n^2$  / 4 \* 4 =  $n^2$ .

• Space Complexity: 0(1)

Only a constant amount of extra space is used (top, offset, loop indices). The rotation is done **in-place**.

### 5. Sudoku Solver

Pattern: Backtracking

### **Problem Statement**

Write a program to solve a Sudoku puzzle by filling the empty cells.

A sudoku solution must satisfy all of the following rules:

- Each of the digits 1-9 must occur exactly once in each row.
- Each of the digits 1-9 must occur exactly once in each column.
- Each of the digits 1-9 must occur exactly once in each of the 9 3x3 sub-boxes of the grid.

The '.' character indicates empty cells.

The input board is guaranteed to be solvable. Modify the board in-place.

# Sample Input & Output

```
Input: board = [
    ["5","3",".","","","","","","",""],
    ["6",".",".","1","9","5",".",""],
    ["8",".",".",".","6",".",".","3"],
    ["4",".",".",8",".","3",".","","1"],
    ["7",".",".",".","2",".",".","6"],
    [".","6",".",".","1","9",".","5"],
    [".",".",".","4","1","9",".","5"],
    [".",".",".","",","8",".","7","9"]
]
Output: board filled with valid digits (in-place)
Explanation: The puzzle has a unique valid solution that satisfies all Sudoku rules.
```

```
Input: board = [["."]*9 for _ in range(9)]
Output: A fully filled valid Sudoku grid
Explanation: Even an empty board is solvable; backtracking will
find one valid configuration.
```

```
Input: board = [
    ["1",".",".",".",".",".",".","."],
    [".","2",".",".",".",".",".","."],
    [".",".",".","4",".",".",".","."],
    [".",".",".",".",".",".","."],
    [".",".",".",".",".",".","."],
    [".",".",".",".",".",".","."],
    [".",".",".",".",".",".","."],
    [".",".",".",".",".",".",".","."],
    [".",".",".",".",".",".",".","."],
    [".",".",".",".",".",".",".","."],
    [".",".",".",".",".",".",".","."]]
]
Output: Completed valid Sudoku
Explanation: Diagonal initial values still allow a unique solution.
```

```
from typing import List
class Solution:
    def solveSudoku(self, board: List[List[str]]) -> None:
        Do not return anything, modify board in-place.
        11 11 11
        # STEP 1: Initialize structures
        # - rows[i]: set of digits in row i
           - cols[j]: set of digits in col j
        # - boxes[box_id]: set of digits in 3x3 box
        rows = [set() for _ in range(9)]
        cols = [set() for _ in range(9)]
        boxes = [set() for _ in range(9)]
        # Pre-fill known digits
        for i in range(9):
            for j in range(9):
                if board[i][j] != '.':
                    num = board[i][j]
                    rows[i].add(num)
                    cols[j].add(num)
                    box_id = (i // 3) * 3 + (j // 3)
                    boxes[box_id].add(num)
        # STEP 2: Main backtracking function
        # - Tries digits 1-9 in empty cells
          - Backtracks if conflict arises
        def backtrack(i, j):
            # Base: reached end of board
            if i == 9:
                return True
            # Move to next cell
            next_i, next_j = (i, j + 1) if j < 8 else (i + 1, 0)
            # Skip filled cells
            if board[i][j] != '.':
                return backtrack(next_i, next_j)
            # Try digits '1' to '9'
```

```
box_id = (i // 3) * 3 + (j // 3)
           for d in '123456789':
               if d in rows[i] or d in cols[j] or d in boxes[box_id]:
                   continue # conflict → skip
               # STEP 3: Update state / bookkeeping
               board[i][j] = d
               rows[i].add(d)
               cols[j].add(d)
               boxes[box_id].add(d)
               # Recurse to next cell
               if backtrack(next_i, next_j):
                   return True
               # Undo changes (backtrack)
               board[i][j] = '.'
               rows[i].remove(d)
               cols[j].remove(d)
               boxes[box_id].remove(d)
           # STEP 4: Return result
           # - No digit worked → signal failure to caller
           return False
       # Start backtracking from top-left
       backtrack(0, 0)
# ----- INLINE TESTS -----
if __name__ == "__main__":
   sol = Solution()
   # Test 1: Normal case
   board1 = [
       ["5","3",".",".","7",".",".",".","."],
       ["6",".",".","1","9","5",".",".","."],
       [".","9","8",".",".",".",".","6","."],
       ["8",".",".","6",".",".",".","3"],
       ["4",".",".","8",".","3",".",".","1"],
       ["7",".",".",".","2",".",".",".","6"],
       [".","6",".",".",".","2","8","."],
       [".",".",".","4","1","9",".",".","5"],
```

```
[".",".",".","8",".",".","7","9"]
1
sol.solveSudoku(board1)
assert all('.' not in row for row in board1), "Test 1 failed"
print(" Test 1 passed: Normal Sudoku solved")
# Test 2: Edge case - empty board
board2 = [["."]*9 for _ in range(9)]
sol.solveSudoku(board2)
assert all('.' not in row for row in board2), "Test 2 failed"
print(" Test 2 passed: Empty board solved")
# Test 3: Tricky/negative - diagonal start
board3 = [
    ["1",".",".",".",".",".",".","."]
    [".","2",".",".",".",".","."],
    [".",".","3",".",".",".",".","."],
    [".",".",".","4",".",".",".",".","."],
    [".",".",".",".","5",".",".",".","."],
    [".",".",".",".","6",".",".","."],
    [".",".",".",".",".","."],
    [".",".",".",".",".",".","8","."],
    [".",".",".",".",".",".",".","."]
sol.solveSudoku(board3)
assert all('.' not in row for row in board3), "Test 3 failed"
print(" Test 3 passed: Diagonal-start Sudoku solved")
```

### **Example Walkthrough**

We'll trace **Test 1** at a high level (full step-by-step would be thousands of steps due to backtracking):

#### 1. Initialization:

```
• rows[0] = \{'5', '3', '7'\}, cols[0] = \{'5', '6', '8', '4', '7'\}, etc.
```

- All pre-filled digits are recorded in rows, cols, boxes.
- 2. Start at (0,2) first empty cell.
  - Tries '1': not in row 0, col 2, or box  $0 \rightarrow$  place it.
  - Proceeds to next empty cell.
- 3. Later, a conflict arises (e.g., no digit fits at some cell):
  - Backtrack: undo last placement, try next digit.
  - This repeats until a valid path fills the board.
- 4. Eventually, a full assignment satisfies all constraints.
  - backtrack returns True up the call stack.
  - Original board is modified in-place with solution.

**Key Insight**: Backtracking explores possibilities **depth-first**, pruning invalid paths early using the sets (rows, cols, boxes) for O(1) conflict checks.

# **Complexity Analysis**

• Time Complexity:  $O(9^{n})$  where n = number of empty cells (worst case)

In worst case, each empty cell tries up to 9 digits. With  $\sim$ 50–60 empties, this is exponential — but pruning via constraint sets makes it feasible for standard Sudoku.

• Space Complexity: 0(1)

We use 3 fixed-size structures (rows, cols, boxes) of size 9 each. Recursion depth—81 (cells), so stack space is bounded by constant. Input board is modified in-place.