UNIT- V

Magnetic effects of current and Magnetism

Magnetic field: The space around a magnet in which its influence is felt is called magnetic field. (or) The space around a magnet in which the other objects can experience a magnetic force is called magnetic field. It is denoted by B. S.I unit of magnetic field is wb/m^2 (or) tesla.

Uniform magnetic field: If both magnitude & direction of magnetic field is same at all points, then the magnetic field is said to be uniform magnetic field.

Magnetic lines of force: It is an imaginary line representing the direction of magnetic field such that the tangent drawn at any point is the direction of field vector at that point.

According to "Oersted" a magnetic field will be produced around a current carrying conductor. The direction of magnetic field depends upon the direction of current and its strength also depends on magnitude of current.

Ampere's Swimming Rule: Imagine a person swimming along a current carrying wire in the direction of current facing a magnetic needle below the wire, then the magnetic north pole of needle deflects towards his left hand.

Ampere's right hand thumb rule:

When a straight conductor carrying current is held in the right hand such that the thumb is pointing along the direction of current, then the direction in which the other fingers curl round it gives the direction of magnetic lines of force.

Maxwell Cork Screw Rule:

Imagine a right handed cork screw advancing in the direction of current, then the direction of rotation of screw head gives the direction of magnetic lines of force.

Force on a charged Particle in a uniform magnetic field:- Consider a charged particle having a charge 'q' is moving with a speed 'v' in uniform magnetic field of induction 'B' making an angle 'θ' with the field, is given force acting on the particle $F = q(\vec{V} \times \vec{B}) = qVB\sin\theta$

→ If $v=0$, $\theta = 0$ (or) 180, then $F=0$

i.e if the particle is at rest, moves parallel (or) anti parallel to the field then the magnetic field can't apply a force on it.

→ If $\theta = 90°$, (i.e if the particle is moving ⊥lr to the magnetic field), then $\boxed{F_{max} = qvB}$

∴ Force will be maximum.

In this situation force is always ⊥lr to the direction of motion of the particle. Hence the particle takes the circular path. (Fleming left hand rule)

The neccesary Centripetal force is provided by $\vec{B}$ the magnetic field.

$$\therefore F_{centripetal} = F_{mag}$$

$$\frac{mv^2}{r} = Bqv$$

$$\frac{mv^2}{r} = Bqv = \frac{m(r\omega)^2}{r}$$

$$\therefore \boxed{r = \frac{mv}{Bq}}$$

But $mv = P$, hence $r = \frac{P}{Bq}$

also $P = \sqrt{2m(K.E)} \Rightarrow r = \frac{\sqrt{2m(K.E)}}{Bq}$

Hence, the radius of circular path is given by

$$r = \frac{mv}{Bq} = \frac{P}{Bq} = \frac{\sqrt{2m \cdot KE}}{Bq}$$

We also know that $V = r\omega$

$\therefore \quad V = \frac{mV}{Bq}\omega \qquad \left(\because r = \frac{mV}{Bq}\right)$

$\Rightarrow \boxed{\omega = \frac{Bq}{m}}$ This is the formula for angular frequency 'ω'.

Time period $T = \frac{2\pi}{\omega}$

$\therefore \quad T = \frac{2\pi}{\left(\frac{Bq}{m}\right)} \Rightarrow \boxed{T = \frac{2\pi m}{Bq}}$

This is the time period.

$f = \frac{1}{T} = \frac{Bq}{2\pi m} \Rightarrow \boxed{f = \frac{Bq}{2\pi m}}$

→ If $\theta \neq 90^\circ$ (i.e. other than $0^\circ, 180^\circ, 90^\circ$)

Then the particle takes the Helix path (or) Ellipse. Helix is a combination of circular and straight line paths.

Radius of helix $(r) = \frac{mV\sin\theta}{Bq}$

$\omega = \frac{Bq}{m}, \quad T = \frac{2\pi m}{Bq}, \quad f = \frac{Bq}{2\pi m}$

<u>pitch of helix (Ellipse)</u>:

The distance travelled by a charged particle along the field in one time period is called pitch.

$\therefore$ Pitch $= V\cos\theta \times \frac{2\pi m}{Bq}$

Lorentz force: when a charged particle is moving in a region containing both electric field and magnetic field, in this case

electric force acting on it, $F_E = \vec{E} q$

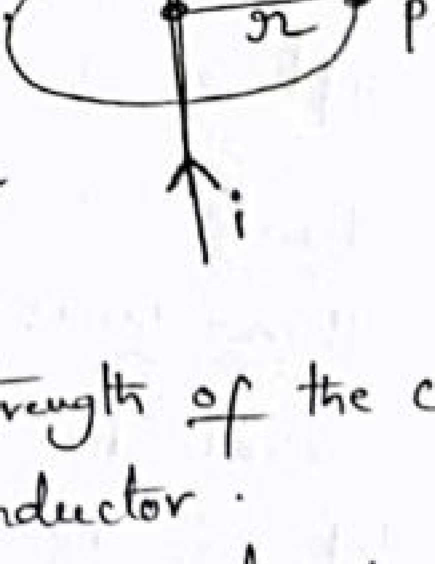
Magnetic force acting on it, $F_B = q(\vec{v} \times \vec{B})$

$\therefore$ Total force $= \vec{F}_E + \vec{F}_m = \vec{E} q + q(\vec{v} \times \vec{B})$

This force is called Lorentz force.

Ampere's law:

When a current is passed through a straight conductor, then a magnetic field will be developed in a plane perpendicular to the direction of current. The induced magnetic field will be in the form of concentric circles with the conductor as the centre.

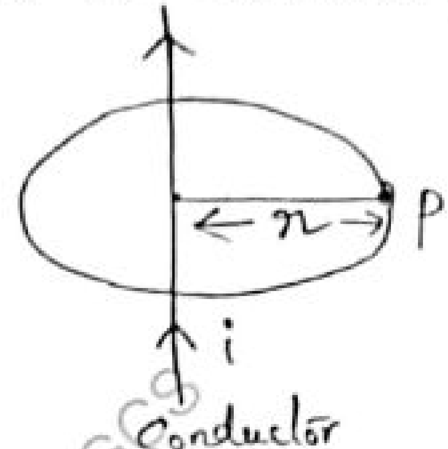


According to Ampere's law, the magnetic induction at a point due to straight conductor carrying current (B) is

(i) directly proportional to the strength of the current passing through the conductor.

(ii) inversely proportional to the perpendicular distance of the point under consideration (P).

Explanation:- As shown in the figure, Consider a conductor carrying a current of 'i' amperes. let 'p' be a point at a distance of 'r' from the conductor.



let 'B' be the magnitude of magnetic field induced at 'p' due to conductor.

$\therefore$ According to Ampere's law, $B \propto i$

$\propto \frac{1}{r}$

$$\therefore B \propto \frac{i}{r} \Rightarrow B = \frac{\mu_0}{2\pi}\frac{i}{r}, \quad \frac{\mu_0}{2\pi} \text{ is a const.}$$

$$\therefore \boxed{B = \frac{\mu_0 i}{2\pi r}}$$

It can also be written as $B \times 2\pi r = \mu_0 i$

$$\therefore \boxed{\int B \cdot dl = \mu_0 i} \qquad \left(\because \int dl = 2\pi r\right)$$

Applications of Ampere's law:

Determination of 'B' due to infinitely long straight thin wire:-

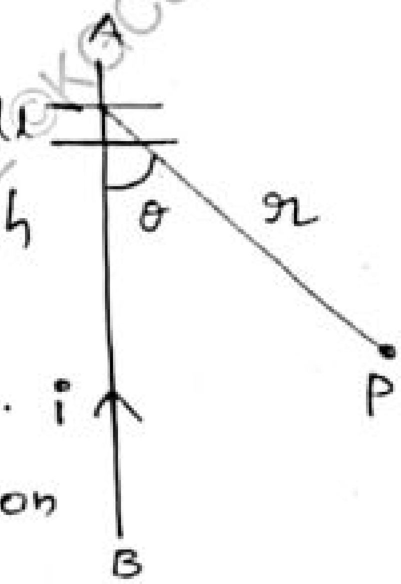
Consider an infinitely long straight thin wire. let 'i' be the current passing through the wire. let 'B' be the magnetic field induced around the wire which is in the form of concentric circles. Through out the path of the circle, 'B' will have the same magnitude.

$$\therefore \int B \cdot dl = B \int dl = B \times 2\pi r$$

From Ampere's law, it is equal to $\mu_0 i$.

$$\therefore B \times 2\pi r = \mu_0 i \Rightarrow \boxed{B = \frac{\mu_0}{2\pi} \frac{i}{r}}$$

Force on a current Carrying Conductor in a Uniform magnetic field:-



Consider a straight conductor AB. Let 'i' be the current passing through it. Let 'dl' be the infinite small element of conductor of length 'l'.

Consider a small element (dl) on the conductor. Let a point 'P' be at a distance of 'r' from the small element.

Let a magnetic pole of pole strength 'm' be placed at a point 'P'. Let 'θ' be the angle made by the line joining the point 'P' and the element (dl).

According to Biot - Savart's law,

The magnetic induction (dB) at 'P' due to small element (dl) is $dB = \frac{\mu_0}{4\pi} \frac{i\, dl \sin\theta}{r^2}$ — ①

The force on the magnetic pole due to the small element (dl) is $dF = m\, dB$ — ② $(\because F = mB)$

From ① & ②, $dF = m \cdot \frac{\mu_0}{4\pi} \frac{i\,dl \sin\theta}{r^2}$

i.e $dF = \frac{\mu_0}{4\pi} \frac{m\,i\,dl \sin\theta}{r^2}$ — ③

but $B = \frac{\mu_0}{4\pi} \frac{m}{r^2}$, (magnetic induction at 'P' due to magnetic pole)

Therefore equation ③ becomes,

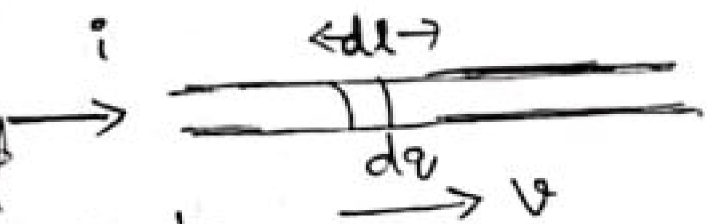
$dF = B\,i\,dl \sin\theta$

The force on a current carrying conductor is

$$\boxed{F = B\,i\,l \sin\theta}.$$

(or)

Consider a conductor carrying a current of 'i' amperes. Let 'dl' be the length of a small element of a conductor..



Let 'dq' be the charge passed through a distance of 'dl' in time 'dt'. ∴ The current flowing through this small element $i = \frac{dq}{dt} \Rightarrow dq = i\,dt$ — ①

Let 'v' be the velocity of charge passing through 'dl'. ∴ Time taken for the charge to move through it is $dt = \frac{dl}{v}$ ② ($\because$ time = $\frac{\text{distance}}{\text{velocity}}$)

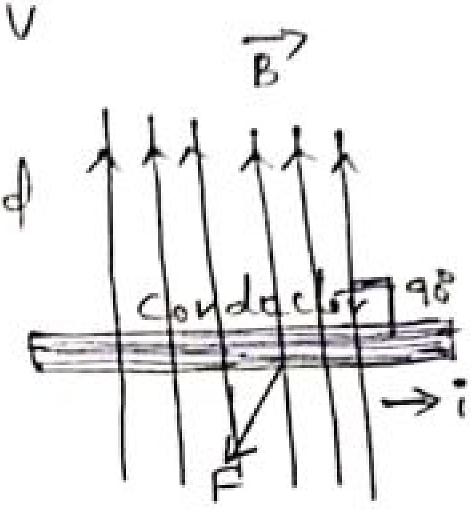
put ② in ①, we get $dq = i\frac{dl}{v}$.

If the current carrying conductor is placed at right angle to magnetic field (B).

Then the magnetic force on the conductor

$dF = B \cdot dq \cdot V \Rightarrow dF = B\, i \frac{dl}{V} \cdot V$

$\therefore \; dF = B i\, dl$



If θ is the angle between B and current carrying conductor, then

$dF = B i\, dl \sin\theta$

The total force acting on a entire conductor is

$F = \int dF = \int B i\, dl \sin\theta$

$\therefore \; \boxed{F = B i l \sin\theta}$

Biot-Savart's law:

Statement: According to Biot-Savart's law the magn-etic induction (B) of the induced magnetic field at a point due to infinite small element of a condu-ctor carrying current (i)

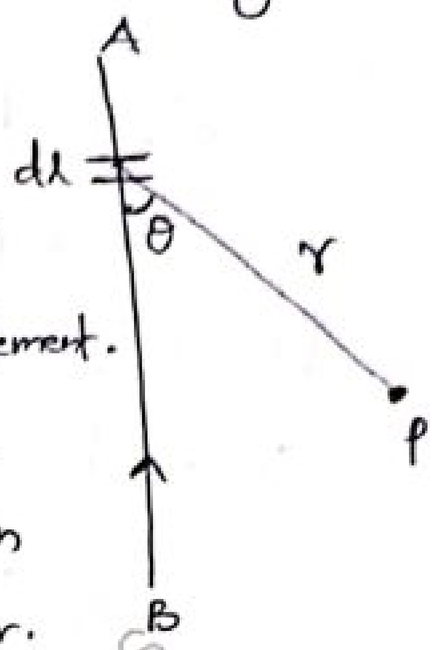
(i) is directly proportional to the strength of current

(ii) is directly proportional to length of small element under consideration

(iii) is directly proportional to 'Sine' of the angle made by the line joining the point under consid-eration (P) and small element (dl) with the direc-tion of current.

(iv) inversely proportional to square of distance of point from the small element.

<u>Explanation</u>: Consider a Conductor Carrying a current of 'i' amperes as shown. Consider infinite small element (dl) of the conductor. Consider a point 'p' at a distance 'r' from the small element. Let 'θ' be the angle made by the line Joining the point under Cousideration and Small element with the Conductor.



Let 'dB' be the magnitude of induced magnetic field due to small element at a point 'P',

According to Biot-savart's law

$$dB \propto i$$
$$\propto dl$$
$$\propto \sin\theta$$
$$\propto \frac{1}{r^2}$$

$$\therefore \quad dB \propto \frac{i\,dl\sin\theta}{r^2} \Rightarrow \boxed{dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}}$$

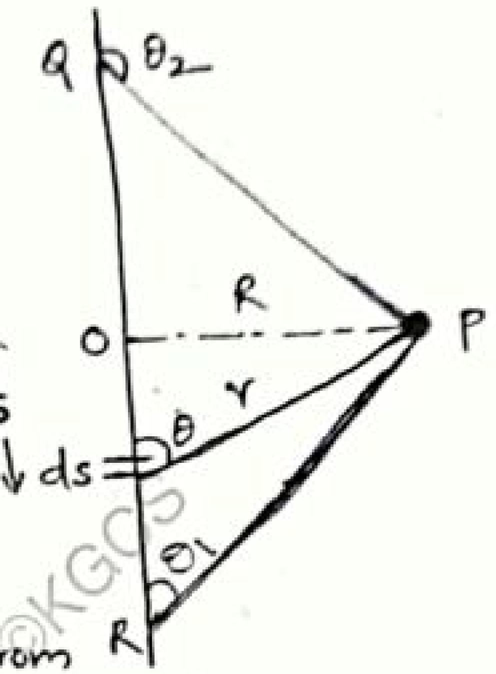
where $\frac{\mu_0}{4\pi} = 10^{-7}$ Henry/meter

$\therefore$ The magnetic field Induction (dB) at 'p' due to small element of Conductor is $dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$

Hence, the magnetic field <u>Induction</u> (B) at 'p' due to entire Conductor is $\boxed{B = \frac{\mu_0}{4\pi}\int\frac{i\,dl\sin\theta}{r^2}}$

Applications :

(i) Determination of 'B' at a point due to finite length of the wire :-



Consider a straight wire of finite length as shown. Let θ_1 and θ_2 be the angles made by the point 'P' with the two ends of the wire respectively. Consider a small element 'ds' on the wire at a distance of 'r' from the point 'P'.

From the figure, $\tan\theta = \frac{R}{-s}$ $\left(\because \text{As } \theta < \pi/2, \text{ then } s = -s\right)$

$$\therefore \quad s = -R\frac{1}{\tan\theta} = -R\cot\theta$$

$$\therefore \quad s = -R\cot\theta \Rightarrow ds = -R\left(-\operatorname{cosec}^2\theta\right)d\theta$$

$$\therefore ds = R\operatorname{cosec}^2\theta\, d\theta = \frac{R}{\sin^2\theta}d\theta$$

$$\therefore \quad \boxed{ds = \frac{R}{\sin^2\theta}d\theta}$$

and from the figure, $\sin\theta = \frac{R}{r} \Rightarrow \boxed{r = \frac{R}{\sin\theta}}$

From Biot-Savart's law, we have

$$dB = \frac{\mu_0}{4\pi}\frac{i\, ds \sin\theta}{r^2} \Rightarrow B = \int_{\theta_1}^{\theta_2} dB = \int_{\theta_1}^{\theta_2}\frac{\mu_0}{4\pi}\frac{i\, ds\sin\theta}{r^2}$$

$$\therefore B = \frac{\mu_0}{4\pi}i\int_{\theta_1}^{\theta_2}\frac{\cancel{R}}{\cancel{\sin^2\theta}}d\theta \times \frac{\sin\theta}{R\cancel{^2}} \times \cancel{\sin^2\theta}$$

$$\therefore \quad B = \frac{\mu_0}{4\pi}\frac{i}{R}\int \sin\theta\, d\theta = \frac{\mu_0}{4\pi}\frac{i}{R}\left[-\cos\theta\right]_0^{\pi}$$

$$\therefore B = \frac{\mu_0}{4\pi}\frac{i}{R}\left[\cos 0 - \cos\pi\right] = \frac{\mu_0}{4\pi}\frac{i}{R}\times 2$$

$$\therefore \boxed{B = \frac{\mu_0}{2\pi}\frac{i}{R}}$$

(ii) Determination of 'B' at the centre of circular loop of radius 'R'.

Consider a circular loop of radius 'R' as shown. Consider a small element 'dl' of the loop. Consider a point 'P' at a distance of 'R' from small element 'dl'.

Now magnetic field induction at the centre of loop due to small element 'dl' of the loop is given by

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{R^2} \qquad (\because r = R)$$

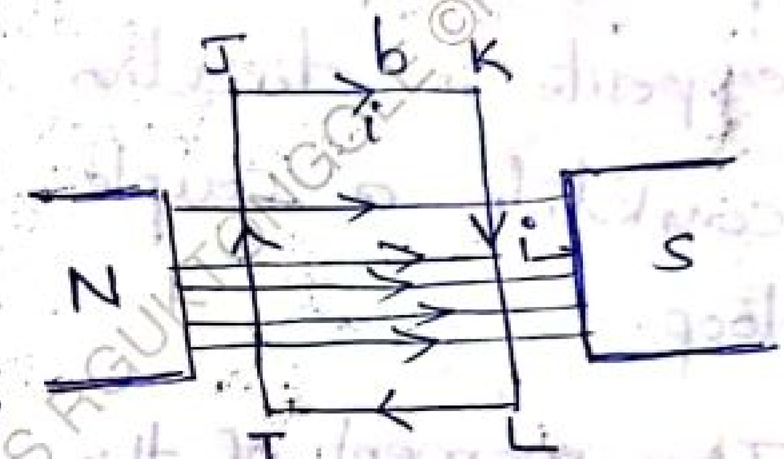
The magnetic field induction due to entire loop is given by $B = \int dB = \int \frac{\mu_0}{4\pi}\frac{i\,dl}{R^2}$ $\quad (\because \theta = 90^\circ)$

$$\therefore B = \frac{\mu_0}{4\pi}\frac{i}{R^2}\int dl = \frac{\mu_0}{4\pi}\frac{i}{R^2}\times 2\pi R$$

$$\therefore \boxed{B = \frac{\mu_0}{2}\frac{i}{R}}$$

Torque on a loop placed in a magnetic field

Consider a rectangular coil (loop) of length 'l', breadth 'b' carrying a current of 'i' amps be placed in a magnetic field of induction B. N & S are the poles of a power-ful horse shoe magnet.



Let 'n' be the no. of turns of the coil.

According to Biot-Savart's law, the force acting on a conductor placed in a magnetic field is $F = Bil \sin\theta$

The force on the conductor along IJ (length wise) is $F_1 = Bil \sin 90° = Bil$

The force on the loop along JK (breadth wise) is $F_2 = Bib \sin 0 = 0$

The force on the loop along KL (length wise) is $F_3 = -Bil \sin 90° = -Bil$

The force on the loop along LI (breadth wise) $F_4 = B \times i \times b \times \sin 180° = 0$.

It can be concluded that there is no net force along the breadth of the loop.

Along the length, two '||' forces (parallel forces) are acting at the two ends which are equal in magnitude and opposite in direction.

These two parallel forces equal in magnitude, opposite in direction acting at different points constitute a couple and rotates the coil (or) loop.

The moment of the couple (or) Torque τ = one of the forces $\times$ $\perp$ lr distance between the forces.

$\therefore$ Torque on one turn of the coil $\tau = Bil \times b$

$\therefore \tau = BiA \quad (\because l \times b = A)$

The torque due to 'n' turns is given by

$$\boxed{\tau = BiAn}.$$