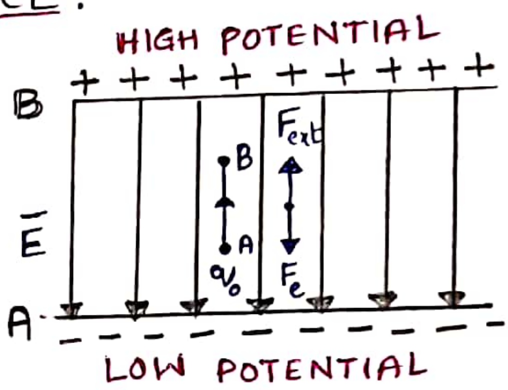
UNIT - III

ELECTROSTATICS - II

(Electric Potential & Capacitance)

* To define Electric potential we need to understand Electric potential difference.

ELECTRIC POTENTIAL DIFFERENCE:



" The work done by the external force in moving a unit charge 'q_0' from one point to another in an electric field is called Electric potential difference "

In the figure the charge 'q_0' is moved from A to B by an external force 'F_{ext}'

∴ Electric potential difference,

$$V_B - V_A = \frac{W_{A\to B}}{q_0}$$

But coulomb force is conservative, i.e., $W_{A\to B} = -\Delta U$

$$\Rightarrow \boxed{V_B - V_A = \frac{U_A - U_B}{q_0}}$$

Where U_A & U_B are Electrostatic P.E at A & B

SI Units: Volt (V), Joule/coulomb (J/C)

ELECTRIC POTENTIAL: The work done by the external force in bringing a unit positive charge from infinity to a point in the electric field is called Electric potential.

Here the point 'A' lies at Infinity means; $V_\infty = 0$

$$\therefore \; V_B - V_\infty = \frac{W_{\infty \to B}}{q_0}$$

$$\Rightarrow \; V_B = \frac{W_{\infty \to B}}{q_0}$$

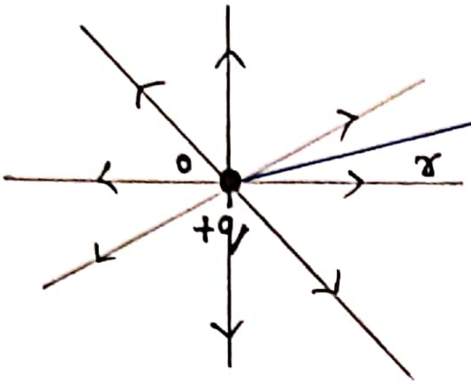
& simply, $\boxed{V = \frac{W}{q_0}}$

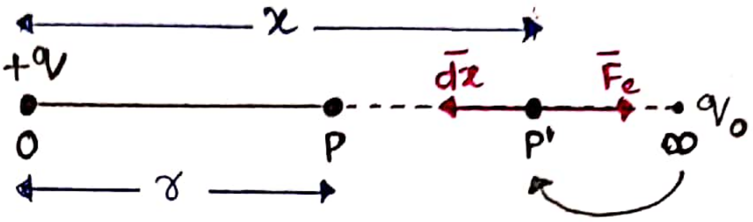
S.I units : Volt (V) ; $\frac{\text{Joule}}{\text{Coulomb}}$ (J/C)

VOLT: A potential difference of 1 volt exists between two points when 1 joule of work is done in bringing an unit positive charge from one point to another in an electric field.

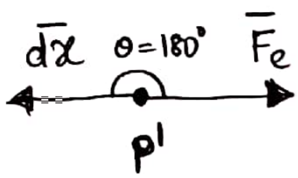
$$1\,V = \frac{1\,J}{1\,C}$$

ELECTRIC POTENTIAL DUE TO A POINT CHARGE





* Consider a point charge 'q' at origin 'O'. For definiteness take 'q' to be positive.
* Assume a point 'P' at a distance 'r' from 'q' where the electric potential has to be determined
* Let '$+q_0$' be the unit positive test charge lies at infinity
* First bring '$+q_0$' from '∞' to any intermediate point 'P'' (before P) at a distance 'x' from 'q'.
* From "P'", '$+q_0$' makes a small displacement $\overline{dx}$ against the Electrostatic force $\overline{F_e}$.



Then the small work done;

$$dW = \overline{F_e} \cdot \overline{dx}$$

$$\Rightarrow dW = F_e\,dx\cos\theta$$

$$\Rightarrow dW = F_e\,dx\cos 180^\circ \Rightarrow \boxed{dW = -F_e\,dx} \longrightarrow (1)$$

Applying Coulomb's law for points 'O' & 'P''

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{(+q_0)(+q)}{x^2} \Rightarrow \boxed{F_e = \frac{q q_0}{4\pi\varepsilon_0}\left[\frac{1}{x^2}\right]} \longrightarrow (2)$$

Substitute eq ② in ①

$$\Rightarrow dW = -\frac{qq_0}{4\pi\varepsilon_0}\frac{dx}{x^2}$$

To determine Electric potential at 'P', Integrate the above equation on both sides with limits ∞ to r on r.H.S.

$$\Rightarrow \int dW = -\int_{\infty}^{r}\frac{qq_0}{4\pi\varepsilon_0}\frac{dx}{x^2}$$

$$\Rightarrow W = -\frac{qq_0}{4\pi\varepsilon_0}\int_{\infty}^{r}\frac{1}{x^2}\,dx$$

$$\Rightarrow W = -\frac{qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$\Rightarrow W = -\frac{qq_0}{4\pi\varepsilon_0}\left[\left(-\frac{1}{r}\right)-\left(-\frac{1}{\infty}\right)\right]$$

$$\Rightarrow W = -\frac{qq_0}{4\pi\varepsilon_0}\left[\left(\frac{-1}{r}\right)-0\right]$$

$$\Rightarrow W = \frac{qq_0}{4\pi\varepsilon_0 r}$$

But from the definition of Electric Potential.

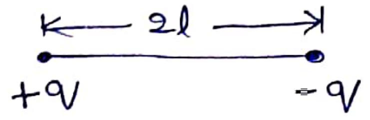
$$V_p = \frac{W}{q_0} \Rightarrow V_p = \frac{q\cancel{q_0}}{4\pi\varepsilon_0 r\times \cancel{q_0}}$$

$$\therefore \boxed{V_p = \frac{q}{4\pi\varepsilon_0 r}}$$

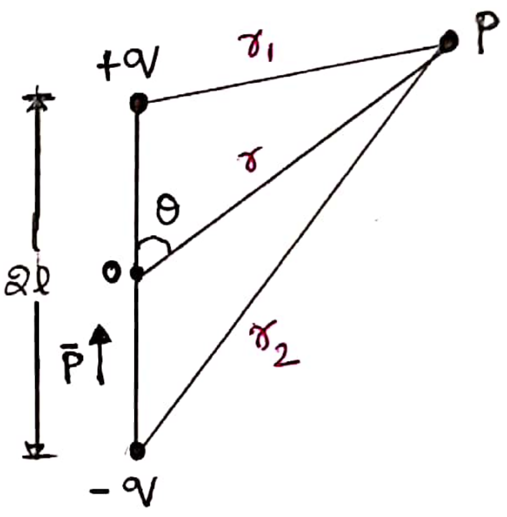
Where 'V_p' is the electric potential at point 'P'

ELECTRIC POTENTIAL DUE TO AN DIPOLE

* An electric dipole consists of two equal and opposite charges separated by a (small) distance '$2l$' with dipole moment; $\boxed{P = q2l}$



* Let 'O' be the origin at the center of dipole.
* Like electric field electrostatic potential also obeys the principle of superposition.
* The electric potential due to charge $+q$ at point 'P' is



$$V_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{r_1}\right)$$

The electric potential due to charge '$-q$' at 'P' is,

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r_2}\right)$$

Net Electric potential; $V = V_1 + V_2$

$$\Rightarrow \boxed{V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]} \longrightarrow (1)$$

From figure (apply parallelogram law of vectors)

$$r_1^2 = r^2 + l^2 - 2lr\cos\theta$$

$$r_2^2 = r^2 + l^2 + 2lr\cos\theta$$

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But $r >> l$, so apply Binomial theorem and retain only the first order terms of '$\frac{l}{r}$'

$$\therefore \quad r_1^2 = r^2 - 2lr\cos\theta$$

$$\Rightarrow r_1^2 = r^2\left[1 - \frac{2l\cos\theta}{r}\right]$$

$$\Rightarrow r_1 = r\left[1 - \frac{2l\cos\theta}{r}\right]^{1/2}$$

$$\Rightarrow \frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2l\cos\theta}{r}\right]^{-1/2}$$

Applying Binomial theorem

$$\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{\cancel{2}l\cos\theta}{\cancel{2}r}\right] \Rightarrow \boxed{\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{l}{r}\cos\theta\right]} \rightarrow 2$$

Similarly; $\boxed{\frac{1}{r_2} = \frac{1}{r}\left[1 - \frac{l}{r}\cos\theta\right]} \longrightarrow 3$

Substitute 3 & 2 in eq 1

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{l}{r}\cos\theta\right) - \frac{1}{r}\left(1 - \frac{l}{r}\cos\theta\right)\right]$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0 r}\left[\cancel{1} + \frac{l}{r}\cos\theta - \cancel{1} + \frac{l}{r}\cos\theta\right]$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0 r}\left(\frac{2l\cos\theta}{r}\right) \Rightarrow V = \frac{(q\,2l)\cos\theta}{4\pi\varepsilon_0 r^2}$$

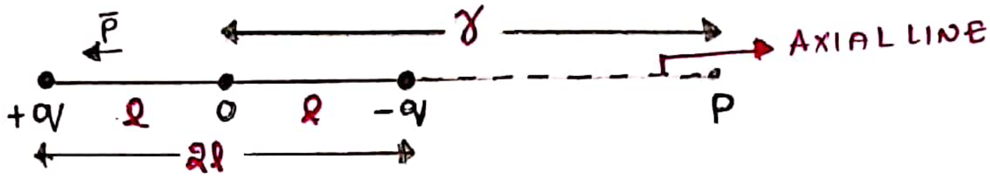
Electrical potential due to dipole at point 'P' is

$$\therefore \quad \boxed{V_P = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}}$$

'$P = q2l$' is the dipole moment
'θ' is the angle b/n dipole moment & Position Vector 'r'

NOTE :

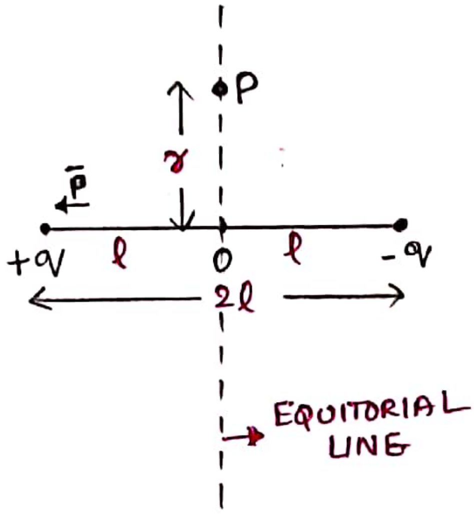
1. Electric potential due to the dipole when point 'P' lies on the axial line,



Here, $\theta = 0°$

$$\therefore \quad V_P = \frac{P\cos 0°}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{V_P = \frac{P}{4\pi\varepsilon_0 r^2}}$$

2. Electric potential due to the dipole when point 'P' lies on the equitorial line,

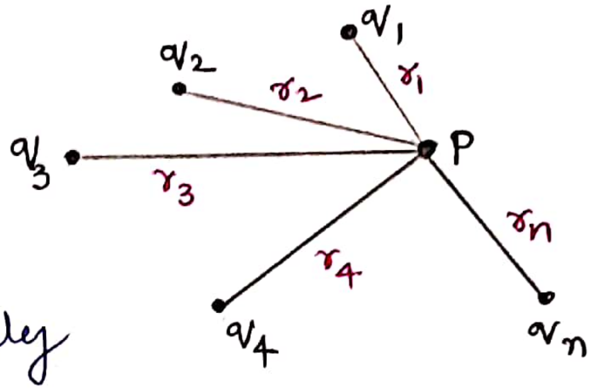


Here, $\theta = 90°$

$$\therefore \quad V_P = \frac{P\cos 90°}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{V_P = 0}$$

ELECTRIC POTENTIAL DUE TO A SYSTEM OF CHARGES

* Consider a system of 'n' charges $q_1, q_2, q_3, \ldots, q_n$ with position vectors $r_1, r_2, r_3 \ldots\ldots r_n$ respectively



* The Electric potential due to charge q_1 at P is,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_1}$$

* The Electric potential due to charge q_2 at P is,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_2}$$

* Similarly Electric potential due to charge q_n at P is

$$V_n = \frac{1}{4\pi\varepsilon_0} \frac{q_n}{r_n}$$

* According to principle of superposition net electric potential at 'P' due to 'n' charges is,

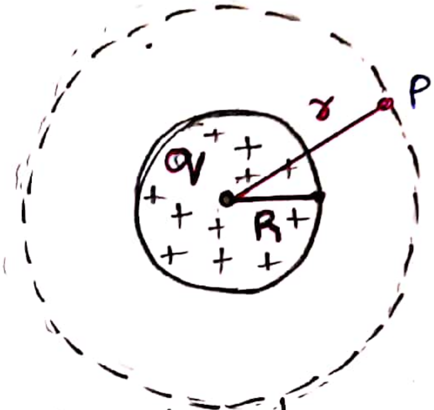
$$V = V_1 + V_2 + V_3 + \ldots\ldots + V_n$$

$$\Rightarrow \boxed{V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots + \frac{q_n}{r_n}\right]}$$

NOTE :

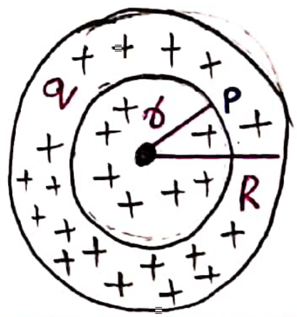
1. Electric potential for a uniformly charged ~~sp~~ spherical shell outside the shell ~~is~~ of radius 'R' is

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}} \quad (r > R)$$



2. Electric potential for a uniformly charged spherical shell inside the shell equals its value at the surface. Because electric field inside the shell is zero and hence the potential remains constant

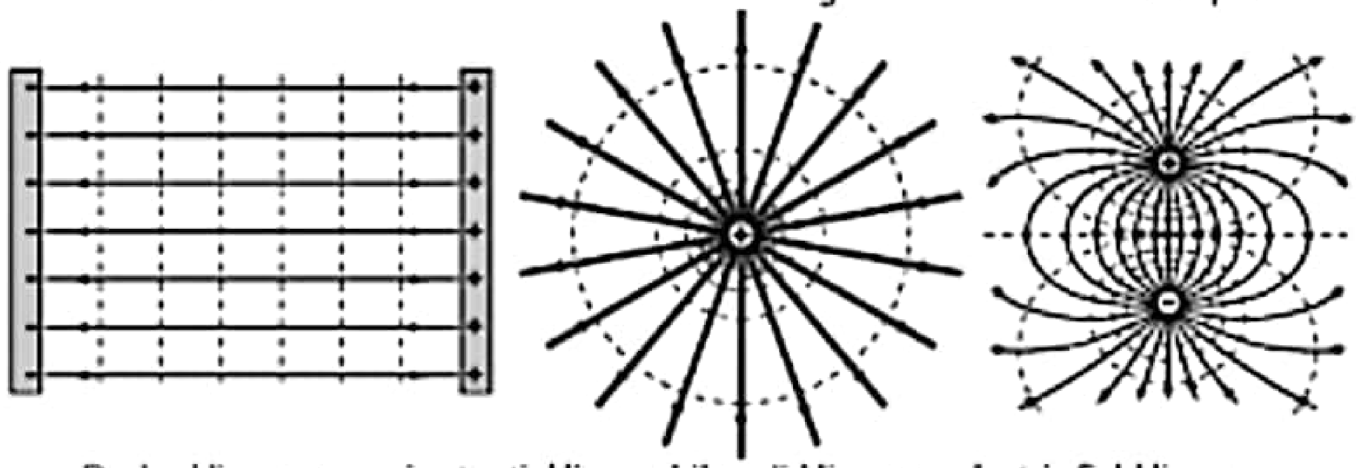
$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}} \quad (r \leq R)$$



EQUIPOTENTIAL SURFACES :

"An equipotential surface is a surface with a constant value of potential at all points on its surface."

- ~~An~~ Equipotential surfaces of a single point charge are concentric spherical surfaces centered at the charge
- Whether the charge configuration is positive or negative, the electric field at every point is normal to the equipotential surface.



Dashed lines are equipotential lines while solid lines are electric field lines.
Click on one of the diagrams for further detail.

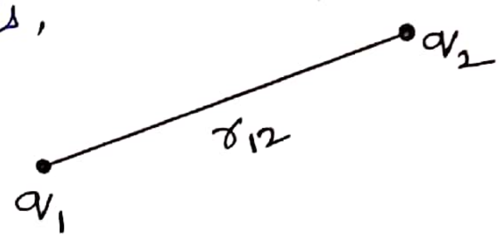
ELECTRIC POTENTIAL ENERGY

ELECTRIC POTENTIAL ENERGY WITH NO EXTERNAL FIELD

→ The electric potential energy of a single charge 'q' without any external electric field is '<u>Zero</u>'

→ The Electric potential Energy of two charges q_1 & q_2 without any external field is,

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}}$$



ELECTRIC POTENTIAL ENERGY WITH EXTERNAL FIELD ($\vec{E}$)

→ The Electric potential Energy of a single charge 'q' in an external field $\vec{E}$ at a point 'P' in the field is,

$$\boxed{U = qV}$$

Where, 'V' is the electric potential at Point 'P' due to '$\vec{E}$'

→ **ELECTRON VOLT (eV):** When an electron with charge $q = e = 1.6\times10^{-19}C$ is accelerated by a potential difference of $V = 1\,Volt$, it would gain an energy of one electron Volt.

i.e., $1\,eV = (1.6\times10^{-19}C)\times(1V)$

$$\boxed{1\,eV = 1.6\times10^{-19}J}$$

ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF TWO CHARGES IN AN ELECTRIC FIELD ($\vec{E}$)

* Let q_1 & q_2 are the two charges located at $\bar{r}_1$ & $\bar{r}_2$ in an external field $\vec{E}$
* Work done in bringing the charge q_1 from infinity to $\bar{r}_1$ is $q_1 V_1 \rightarrow ①$
* Work done in bringing the charge q_2 from infinity to $\bar{r}_2$ is due to both $\vec{E}$ and charge q_1 which is already in the field.

$\therefore$ Work done on q_2 due to $\bar{E}$ is, $q_2 V_2 \rightarrow ②$

Work done on q_2 due to q_1 is $= q_2 V_{12}$

$$= q_2 \left[\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{12}}\right] \rightarrow ③$$

Where; V_{12} is the electric potential difference b/n q_1 & q_2

According to principle of Superposition,

Work done on q_2 is, ② + ③ $= q_2 V_2 + \dfrac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} \rightarrow ④$

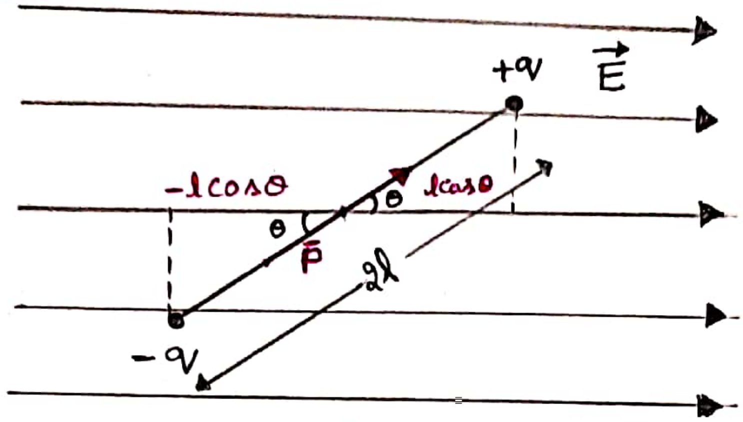
Now, Electric potential Energy of the system of two charges in $\vec{E}$ is,

$$\boxed{U = q_1 V_1 + q_2 V_2 + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}}$$

'r_{12}' is distance between q_1 & q_2

When there is no external field the first two terms in the above equation becomes zero & reduces to the equation discussed in no external field case.

ELECTRIC POTENTIAL ENERGY OF A DIPOLE IN AN ELECTRIC FIELD ($\vec{E}$) :



* Consider a dipole placed in an external field $\vec{E}$
* In an external field the dipole always aligns with $\vec{E}$.
* The torque on a dipole due to uniform electric field is, $\bar{\tau} = \bar{P} \times \bar{E} \Rightarrow \boxed{\tau = PE\sin\theta}$
* An external torque (τ_{ext}) ~~equal in magnitude to 'PEsinθ'~~ is applied in such a manner that it neutralises the torque due to $\vec{E}$ and rotates it in a plane of paper from θ_1 to θ_2 without angular acceleration.
* The amount of work done by 'τ_{ext}' is

$$W = \int_{\theta_1}^{\theta_2} \tau_{ext}\, d\theta = \int_{\theta_1}^{\theta_2} PE\sin\theta\, d\theta$$

This workdone stores in the form of change in Potential Energy.

i.e., $$U_2 - U_1 = \int_{\theta_1}^{\theta_2} PE\sin\theta \, d\theta$$

$$= PE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\boxed{U_2 - U_1 = PE\left[\cos\theta_1 - \cos\theta_2\right]}$$

To get the Electric potential Energy at any one point consider taking either θ_1 or θ_2 as $\frac{\pi}{2}$.

At $\theta_1 = \frac{\pi}{2} \Rightarrow U_1 = -PE\cos\frac{\pi}{2} = 0$

$$\therefore \quad U_2 - 0 = PE\left[0 - \cos\theta_2\right]$$

$$\Rightarrow \quad U_2 = -PE\cos\theta_2$$

In general at any value of 'θ' the electric Potential Energy in external field $\vec{E}$ is,

$$\boxed{U = -PE\cos\theta}$$

(or)

$$\boxed{U = -\bar{P}\cdot\bar{E}}$$

CONDUCTORS: In some materials electrons are weakly bound to atoms & molecules, these electrons are free to move throughout the body known as free electrons. When such material is placed in an external electric field the free electrons moves opposite to the applied field. These materials are known as conductors

Eg: All metals, Graphite etc.

INSULATORS: Another class of materials in which electrons are tightly bound to their atoms & molecules and have no free electrons. When such materials are subjected to external electric field, the electrons cannot move freely. Such materials are known as Insulators.

Eg: Glass, wood, rubber etc.

FREE CHARGES & BOUND CHARGES INSIDE A CONDUCTOR

→ The loosely bound electrons with in atoms & molecules of conductor on applying external electric field starts moving freely are called free charges.

→ When the free charges are ejected out of the atoms & molecules they leave back positive ions which cannot move in the external electric field and remains static. They are called bound charges.

DIELECTRICS & POLARISATION

DIELECTRIC : It is an insulating material or a very poor conductor of electricity. When such materials placed in an electric field almost no current flows because they donot have free electrons like metals. But Polarisation can be observed in dielectric materials due to electric field.

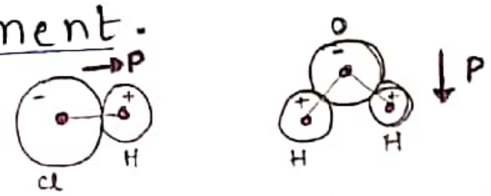
Eg : Porcelian, Ceramic, mica, paraffin wax etc.

→ Dielectric materials are classified into Polar & Non-polar materials.

~~ELECTRIC POLARISATION~~ :

POLAR MATERIALS : In polyatomic materials, the center of mass of negative charge distribution does not coincide with center of mass of positive charge distribution and each molecule of polar materials has permanent dipole moment.

Eg : H_2O, HCl etc.



ASYMMETRIC MOLECULES

NON-POLAR MATERIALS : In non-polar materials the center of mass of positive charge distribution coincides with negative charge distribution. So, non-polar materials have no permanent dipole moment.

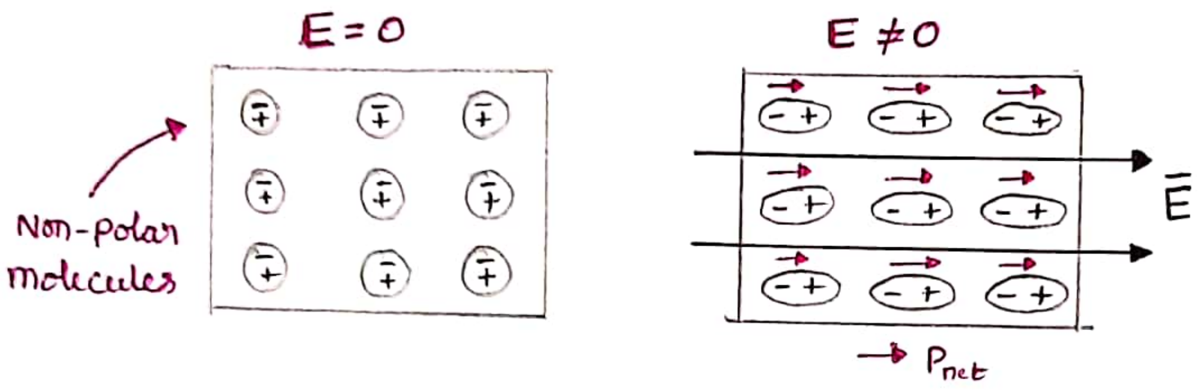
→ But, dipole moment can be induced in non-polar molecules by applying external electric field.

Eg: CO_2, H_2 etc.

SYMMETRIC MOLECULES

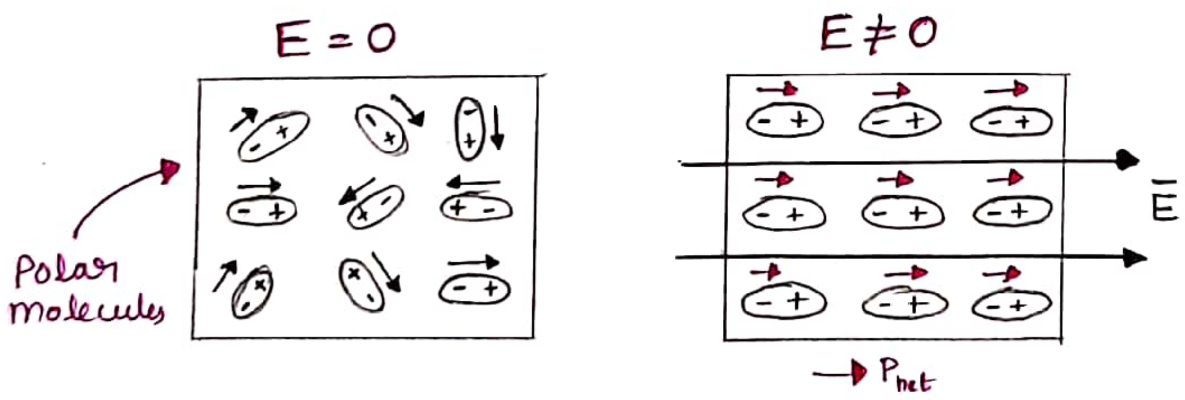
DIELECTRIC POLARISATION

→ When a non-polar dielectric is held in an external electric field $\vec{E}$, the centre of positive charges in each molecule is pulled in the direction of $\vec{E}$ and the centre of negative charge is pulled opposite to $\vec{E}$. Hence, inducing polarisation.



→ Under no external electric field the polar dielectric molecules having permanent dipole moment are oriented randomly with in the material making the total dipole moment of the material zero.

→ When external electric field is applied, the individual molecule with dipole moments tends to align with the field when all the dipole moments of individual molecules are added the total dipole moment of the material becomes non-zero. Hence, inducing polarisation.



ELECTRIC SUSCEPTIBILITY (χ)

The dielectric polarisation in dielectric materials is directly proportional to the external electric field (E)

i.e., $P \propto E_0$

or $$\boxed{P = \chi E_0}$$

The constant 'χ' is called Electric Susceptibility of dielectric medium.

CAPACITOR : A capacitor is a system of two conductors (Plates) separated by an insulating medium. One is positively charged (+Q) and the other is negatively charged (-Q), by using a battery.

- The charge on positive plate is called the charge of the capacitor.
- The potential difference between the plates is called potential of capacitor ($V = V_1 - V_2$)

CAPACITANCE : It is the ability of a conductor to store the electric charge.

The electric charge is proportional to applied potential difference 'V'

i.e., $Q \propto V \Rightarrow \boxed{Q = CV}$

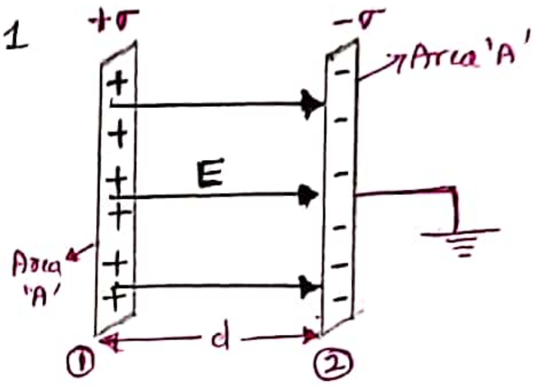
'C' is called capacitance.

$$\boxed{C = \frac{Q}{V}}$$

- S.I unit of capacitance is, farad or coulomb/volt.
- Symbol of fixed capacitance is —||—
- Symbol of variable capacitor is

PARALLEL PLATE CAPACITOR (without Dielectric)

- A parallel plate capacitor consists of two thin conducting plates 1 and 2 each of area 'A' held parallel to each other separated by distance 'd'
- The medium between the plates is taken as Vacuum.
- A charge $+Q$ is given to the plate 1 & charge $-Q$ to plate 2.



- Surface charge density, $\boxed{\sigma = \frac{Q}{A}}$
- Surface charge density for plate 1 & 2 is σ & $-\sigma$ respectively
- In the region between the plates electric field is,

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow E = \frac{Q}{A\varepsilon_0}$$

- Potential difference 'V' between the plates is

$$V = Ed \Rightarrow V = \frac{Q}{A\varepsilon_0} d$$

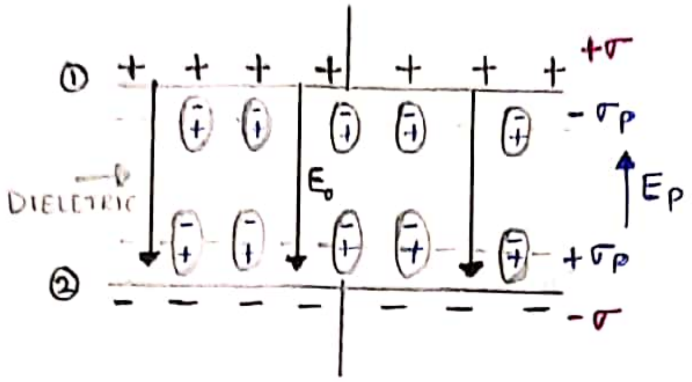
$$\Rightarrow \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

W.K.T, Capacitance, $C = \frac{Q}{V}$.

$\therefore$ The capacitance, of a parallel plate capacitor without dielectric is,

$$\boxed{C = \frac{A\varepsilon_0}{d}}$$

PARALLEL PLATE CAPACITOR (With Dielectric)



Net electric field

$E = E_0 - E_p$

→ The medium between the plates is filled with dielectric. which, on the application of electric field E_0 is Polarised.

→ After polarisation a surface charge density of $-\sigma_p$ develops in the dielectric at plate 1 & $+\sigma_p$ at plate 2.

→ Already surface charge density of $+\sigma$ & $-\sigma$ are present at plate 1 & plate 2.

→ ∴ Net surface charge density at plate 1 & 2 are $+(\sigma-\sigma_p)$ and $-(\sigma-\sigma_p)$ respectively.

→ The net electric field, $E = \dfrac{\sigma-\sigma_p}{\varepsilon_0}$

$E = E_0 - E_p$

$= \dfrac{+\sigma}{\varepsilon_0} - \dfrac{+\sigma_p}{\varepsilon_0}$

$E = \dfrac{\sigma-\sigma_p}{\varepsilon_0}$

→ The potential difference 'V' between the plates is; $V = Ed \Rightarrow \boxed{V = \dfrac{\sigma-\sigma_p}{\varepsilon_0} d}$

For linear dielectrics $(\sigma-\sigma_p)$ is proportional to σ

$\therefore \quad \sigma-\sigma_p \propto \sigma \Rightarrow$ ~~$\sigma-\sigma_p = K\sigma$~~

$\Rightarrow \sigma-\sigma_p = \dfrac{\sigma}{K}$

Where 'K' is Dielectric constant & $K > 1$

Now, $V = \frac{\sigma d}{K \varepsilon_0}$

$\Rightarrow V = \frac{Q d}{A K \varepsilon_0}$ $\left(\because \sigma = \frac{Q}{A} \right)$
Surface charge density

$\Rightarrow \frac{Q}{V} = \frac{K A \varepsilon_0}{d}$

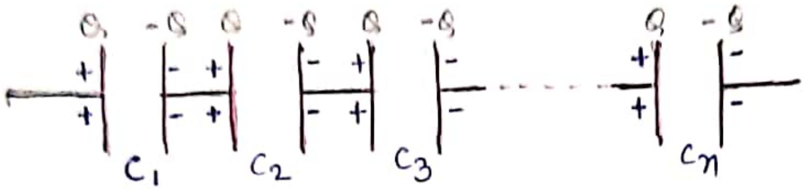
W.K.T, Capacitance, $C = \frac{Q}{V}$

$\therefore$ The capacitance, of a parallel plate capacitor with dielectric medium is,

$$\boxed{C = \frac{K A \varepsilon_0}{d}}$$

Where 'K' is the dielectric constant

COMBINATION OF CAPACITORS IN SERIES :



The total potential drop across the combination is the sum of the potential drops at each individual capacitor.

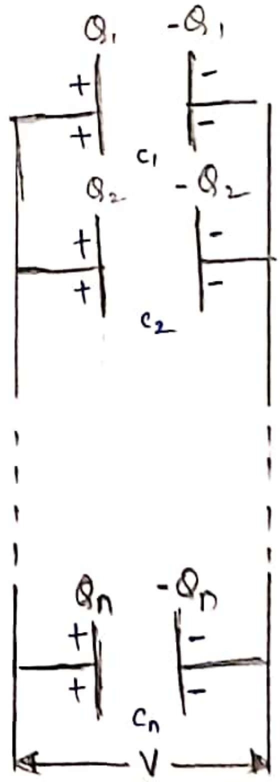
i.e., $V = V_1 + V_2 + V_3 + \cdots + V_n$

$$\Rightarrow \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots + \frac{Q}{C_n}$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

⊛ In series combination the charges on the plates ~~are~~ must be same on every capacitor.

COMBINATION OF CAPACITORS IN PARALLEL



→ In parallel combination same potential is applied across all the capacitors.

→ The charges on individual capacitors are not neccessarilly same.

→ The total charge of the combination of capacitors in parallel is,

$$Q = Q_1 + Q_2 + Q_3 + \cdots + Q_n$$

$$\Rightarrow CV = C_1V + C_2V + C_3V + \cdots + C_nV$$

$$\Rightarrow \boxed{C = C_1 + C_2 + C_3 + \cdots + C_n}$$