UNITS AND DIMENSIONS - 1

*. **PHYSICAL QUANTITY** : All those quantities which can be measured di... or indirectly in terms of which the laws of physics can be expr... are called physical quantities

-> Physical quantities are two types :

*. **FUNDAMENTAL QUANTITIES** :

The physical quantity which can be treated as independent on other quantities called Fundamental Quantities.

-> There are 7 types.
1. Length
2. Mass
3. Time
4. Temperature
5. Electric current
6. Amount of Substance
7. Luminious intensity

*. **DERIVED PHYSICAL QUANTITY**:

The physical quantity whose defined operations are based on other physical quantities are called Derived Physical quantity

Ex: Speed, velocity, force, acceleration, momentum.

*. **MEASUREMENT** : It is the process of comparing quantity with a standard amount of a same kind.

-> Measure of a physical quantity = $n\,u$

n → Numerical Value; u → Size of unit

The numerical is inversely proportional to the size of unit.

ie.. $n \propto \frac{1}{u} \Rightarrow n = \frac{\text{Constant}}{u}$

$\therefore n_1 u_1 = n_2 u_2$

→ If n_1 and n_2 are numerical values for a physical quantity corresponding to units u_1 and u_2, then $n_1 u_1 = n_2 u_2$

* **FUNDAMENTAL UNITS** : The physical units which neither be derived from one another nor they can be further resolved into more simpler units.

* **DERIVED UNITS** : All the other physical units which can be expressed in terms of the fundamental units are called derived units.

* **SYSTEM OF UNITS** :

A complete set of units which is used to measure all kinds of fundmental and derived quantities is called system of units.

SYSTEM	C.G.S.	M.K.S.	F.P.S.	S.I.
LENGTH	Cm	m	Foot	m
MASS	g	kg	Pound	kg
TIME	s	s	s	s
TEMPERATURE	-	-	-	K
AMOUNT OF SUBSTANCE	-	-	-	mol
ELECTRIC CURRENT	-	-	-	A
LUMINOUS INTENSITY	-	-	-	Cd

SUPPLEMENTARY S.I. UNITS:

PHYSICAL QUANTITY	BASIC UNIT	SYMBOL
PLANE ANGLE	Radium	rad
SOLID ANGLE	Steradian	Sr

→ Rules for writing S.I. units in symbolic form

1. Small letters are used for symbols of units.
2. The initial letter of a symbol is 'capital' only when the unit named after a scientist.
3. The full name of a unit always begins with a small le
4. Symbols do not have plural form.

*. Prefixes for powers of 10:

MULTIPLE	PREFIX	SYMBOL
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	K
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	Femto	f
10^{-18}	atto	a

Capital Letters

Ex:-

1 mega ohm	1 MΩ	10^{6} Ω
1 kilo metre	1 Km	10^{3} m
1 deca gram	1 dag	10 g
1 centi metre	1 cm	10^{-2} m
1 mA	10^{-3} A	1 milli ampere
1 pF	10^{-12} F	1 pico farad
1 micro volt	10^{-6} V	1 μV

1 nano second

1 ns → 10^{-9} s

* DIMENSIONS OF THE PHYSICAL QUANTITY:

- -> Dimension of mass - [M]
- -> Dimension of length - [L]
- -> Dimension of time - [T]
- -> Dimension of temperature - [K]
- -> Dimension of Electric current - [mol]
- -> Dimension of amount of substance - [c.d]
- -> Dimension of luminous intensity - [A]

Dimensions are the powers to which the fundamental quantities must be raised to represent physical quantity completely.

Ex:- Area $= (\text{length})^2$

1. $[\text{area}] = [L^2]$ -> 2 in length
2. $[\text{Volume}] = [\text{length}^3] = [L^3]$ -> 3 in length
3. $[\text{Speed}] = \frac{[d]}{[t]} = \frac{L}{T} = [LT^{-1}]$

   Dimensional formula of speed $= [M^0 L T^{-1}]$

   Dimensional equation $\Rightarrow [\text{Speed}] = [M^0 L T^{-1}]$

4. $[\text{Acceleration}] = \frac{[V]}{[T]} = \frac{LT^{-1}}{T} \Rightarrow [M^0 L T^{-2}]$ -> m/s^2
5. $[\text{Force}] = [m][a] \Rightarrow [MLT^{-2}]$ -> N (or) $\frac{Kg\,m}{s^2}$
6. $[\text{Work}] = [F][S] \Rightarrow [ML^2T^{-2}]$ -> J (or) $\frac{Kg\text{-}m^2}{s^2}$
7. $[\text{Energy}] = [\text{Amount of work}] = [ML^2T^{-2}]$ -> J
8. $[\text{Power}] = \frac{[\text{Work}]}{[\text{Time}]} \Rightarrow \frac{[ML^2T^{-2}]}{[T]} \Rightarrow [ML^2T^{-3}]$ -> W
9. $[\text{Density}] = \frac{[\text{mass}]}{[\text{Volume}]} = [ML^{-3}]$ -> Kg/m^3
10. $[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{[MLT^{-2}]}{[L^2]} \Rightarrow [ML^{-1}T^{-2}]$ -> $\frac{N}{m^2}$ (or) Pa
11. $[\text{Angle}] = \frac{(\text{Arc length})}{(\text{radius})} \Rightarrow \frac{L}{L} \Rightarrow [M^0 L^0 T^0]$ -> dimensionless P.Q. -> "rad".

12. [Frequency] = $\frac{1}{[\text{Time Period}]} = T^{-1}$

13. Planck's constant $\Rightarrow [h] = \frac{[E]}{[\nu]} \Rightarrow \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}] \rightarrow J\,S$

14. Universal Gravitational constant $G = \frac{Fr^2}{m_1 m_2} \Rightarrow F = G\frac{m_1 m_2}{r^2}$

$$[G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{[MLT^{-2}][L^2]}{[M][M]} \Rightarrow [M^{-1}L^3T^{-2}] \rightarrow \frac{Nm^2}{Kg^2}$$

*. Applications of Dimensional analysis:

1. To write units of a physical quantity.
2. To convert physical quantity from one system to another.
3. To check the correctness of a given physical relation.
4. To derive relationship between different physical quantities.

*. Convertion of units from one system to another:

Numerical value $\propto \frac{1}{\text{Size of the units}}$

, i.e., $n \propto \frac{1}{u}$

$\Rightarrow nu = \text{Constant}$

*. If n_1 & n_2 are numerical values of a physical quantity corresponding to units u_1 & u_2,

then

$Q = n_1 u_1 = n_2 u_2$

$n_2 = n_1 \frac{u_1}{u_2}$

$u_1 = M_1^a L_1^b T_1^c$, $u_2 = M_2^a L_2^b T_2^c$

$$n_2 = n_1 \frac{M_1^a}{M_2^a} \frac{L_1^b}{L_2^b} \frac{T_1^c}{T_2^c} \Rightarrow n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Q. How many dynes are equal to 9N?

Sol: $9N = ?$ dynes

$n_1 u_1 = n_2 u_2$

$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$u_1 = M_1^a L_1^b T_1^c \Rightarrow M_1 L_1 T_1^{-2}$ → S.I. ⇒ $M \rightarrow 1kg$, $L \rightarrow 1m$, $T \rightarrow 1s$

$a = 1, b = 1, c = -2$

$u_2 = M_2^a L_2^b T_2^c \Rightarrow M_2 L_2 T_2^{-2}$ → C.G.S. ⇒ $M \rightarrow 1g$, $T \rightarrow 1s$, $L \rightarrow 1cm$

$\therefore n_2 = 9 \left[\frac{1kg}{1g}\right]^1 \left[\frac{1m}{1cm}\right]^1 \left[\frac{1s}{1s}\right]^{-2} \Rightarrow 9\left(\frac{1000g}{1g}\right)\left(\frac{100cm}{1cm}\right)$

$\Rightarrow 9 \times 10^3 \times 10^2$

$9N \Rightarrow 9 \times 10^5$ dynes

Note: $1N = 10^5$ dynes.

Q. How many joules are equal to 4 ergs?

Sol: $4\,ergs = ?$ Joules.

$n_1 u_1 = n_2 u_2$

$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$u_2 = M_1^a L_1^b T_1^c \Rightarrow M L_1^2 T_1^{-2}$ → S.I. → $M \rightarrow 1kg$, $L \rightarrow 1m$, $T \rightarrow 1s$

$a = 1, b = 2, c = -2$

$u_2 = M_2^a L_2^b T_2^c \Rightarrow M_2 L_2^2 T_2^{-2}$ → C.G.S. → $M \rightarrow 1g$, $T \rightarrow 1s$, $L \rightarrow 1cm$

$\therefore n_2 = 4 \left[\frac{1g}{1kg}\right]^1 \left[\frac{1cm}{1m}\right]^2 \left[\frac{1s}{1s}\right]^{-2} \Rightarrow 4\left(\frac{1g}{1000g}\right)\left(\frac{1cm}{100cm}\right)^2$

$\Rightarrow 4 \times 10^{-3} \times 10^{-4}$

$\Rightarrow 4 \times 10^{-7} J$

* Checking the dimensional corrections of equations.

-> Principle of Homogeneity:

A physical equation will be dimensionally correct, if the dimensions of all terms occuring on the both sides of the eq. is be same.

Q. Check the dimensional corrections of following equations.

1. $S = ut + \frac{1}{2}at^2$

L.H.S. $\Rightarrow [S] = [L]$

R.H.S. $\Rightarrow [ut] = [u][t]$

$= LT^{-1}T \Rightarrow [L]$

$[\frac{1}{2}at^2] = [\frac{1}{2}][a][t]^2$

$= LT^{-2}T^2 \Rightarrow [L]$

$\therefore [S] = [ut] = [\frac{1}{2}at^2]$

$\therefore$ given equations is dimensionally corrects.

2. $\frac{1}{2}mv^2 = mgh$

L.H.S $\Rightarrow [\frac{1}{2}mv^2] = [\frac{1}{2}][m][v^2]$

$= M[LT^{-1}]^2 = [ML^2T^{-2}]$

RH.S $\Rightarrow [mgh] = [m][g][h]$

$= MLT^{-2}L$

$= [ML^2T^{-2}]$

$\therefore [\frac{1}{2}mv^2] = [mgh]$

3. $v^2 - u^2 = \frac{2a}{S}$

L.H.S. $\Rightarrow [v^2] = [LT^{-1}]^2 = [L^2T^{-2}]$

$[u^2] = [LT^{-1}]^2 = [L^2T^{-2}]$

R.H.S. $\Rightarrow [\frac{2a}{S}] = \frac{[2][a]}{[S]} = \frac{LT^{-2}}{L} \Rightarrow [T^{-2}]$

$\therefore [v^2] - [u^2] \neq [\frac{2a}{S}]$

Given equation is dimensionally incorrect.

* Deriving relations between different physical quantities:

Q. Derive an expression for centripetal force F which depends on mass of body, velocity of body and radius of circle by using method of dimensions.

Centripetal force "F" depends on 1. mass of body "m"
2. velocity of body "v"
3. radius of circle "r".

let $F \propto m^a v^b r^c$

$F = K m^a v^b r^c \rightarrow (1)$

where K is dimension-less constant

$[F] = [K m^a v^b r^c]$

$MLT^{-2} = [K][m]^a [v]^b [r]^c$

$MLT^{-2} = [K] M^a (LT^{-1})^b L^c$

$MLT^{-2} = M^a L^{b+c} T^{-b} \rightarrow (2)$

from eq-(2), $\boxed{a=1}$, $\boxed{b+c=1}$, $-b=-2 \Rightarrow \boxed{b=2}$

$\boxed{c=-1}$

from eq-(1)

$F = K m^1 v^2 r^{-1}$

$F = K \frac{mv^2}{r}$

Q. Derive an expression for time period of oscillation of simple pendulum which depends on mass of bob, length of pendulum & acceleration due to gravity by using method of dimensions.

Time period of oscillation "T" depends on 1. mass of bob "m"
2. length of pendulum "l"
3. Acceleration due to gravity "g"

let $T \propto m^a l^b g^c$

$T = K m^a l^b g^c \rightarrow (1)$

where K is dimension-less constant

$$[T] = [K\, m^a l^b g^c]$$

$$[M^0L^0T^1] = [K][m]^a (l)^b [g]^c$$

$$M^0L^0T = M^a L^b [LT^{-2}]^c$$

$$M^0L^0T = M^a L^{b+c} T^{-2c} \rightarrow ②$$

from eq-② , $\boxed{a=0}$, $b+c=0$, $-2c=1$

$\boxed{b=\frac{1}{2}}$ $\boxed{c=-\frac{1}{2}}$

from eq-①.

$$T = K\, m^0 l^{\frac{1}{2}} g^{-\frac{1}{2}}$$

$$T = K\frac{\sqrt{l}}{\sqrt{g}} \Rightarrow \boxed{T = K\sqrt{\frac{l}{g}}}$$

*. Limitations of dimensional analysis:

-> The method doesnot give any information about constant K.

-> Its fails when a physical quantity depends on more than 3. P.Q.

-> It fails when a physical quantity is the sum or difference of 2 or more quantities.

-> It fails to derive relations which involve trignometric, logarithmic exponention functions.

*. ACCURACY: It refers to the close-ness of a measurement to the value.

PRECISSION: It refers to the resolution or the limit to which the is measured.

-> Precission is determined by least count of instrument.

-> The smaller the least count, greater is the precission.

* **TRUE VALUE**: Let $a_1, a_2, a_3, \ldots\ldots, a_n$ be the "n" measured values of "n" physical quantity.

Average of measured values is known as True Value.

$$\text{True value} = \frac{a_1 + a_2 + a_3 + \ldots\ldots + a_n}{n} \Rightarrow \bar{a} \text{ (or) } a_{mean}$$

* **ABSOLUTE ERROR**: The difference b/w true value & individual measured value of a physical quantity is known as absolute error.

-> It can be positive or negative.

$$\Delta a_1 = \bar{a} - a_1$$
$$\Delta a_2 = \bar{a} - a_2$$
$$\Delta a_n = \bar{a} - a_n$$

+ve (or) -ve.

* **MEAN ABSOLUTE VALUE**: The average of the positive magnitudes of absolute errors is called mean absolute error.

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots\ldots + |\Delta a_n|}{n}$$

* **RELATIVE ERROR**: The ratio of the mean absolute to true value of physical quantity is called relative error.

$$\delta_n = \frac{\Delta \bar{a}}{\bar{a}} = \frac{\text{MEAN ABSOLUTE}}{\text{TRUE VALUE}}$$

* **PERCENTAGE ERROR**: The relative error expressed in percentage is called percentage error.

$$\%\ \text{ERROR} = \delta a \times 100\% \Rightarrow \frac{\Delta \bar{a}}{\bar{a}} \times 100\%$$

Note: Final result $= \bar{a} \pm \Delta \bar{a}$

** Measured value "a" will be "$\bar{a} - \Delta \bar{a} < \bar{a} < \bar{a} + \Delta \bar{a}$".

* **ERROR**: The Error is equal to difference b/w true value and measured value.

Ex:

Q. In an experiment the readings of ... simple pendulum are 2.63 sec, 2.56 sec, 2.42 sec, 2.71 sec and 2.80 sec.. calculate mean value of

i. Period of oscillation.

ii. Absolute error.

iii. Mean absolute error

iv. Relative error

v. Percentage error.

vi. Express the result in proper form

Sol: i. True value:

$$\bar{T} = \frac{T_1 + T_2 + T_3 + T_4 + T_5}{5}$$

$$= \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= 2.624 \text{ Sec.}$$

$$= 2.62 \text{ Sec}$$

ii. Absolute error:

$$\Delta T_1 = T - T_1 \Rightarrow 2.62 - 2.63 = -0.01 \text{ Sec}$$

$$\Delta T_2 = T - T_2 \Rightarrow 2.62 - 2.56 = 0.06 \text{ Sec}$$

$$\Delta T_3 = T - T_3 \Rightarrow 2.62 - 2.42 = 0.20 \text{ Sec}$$

$$\Delta T_4 = T - T_4 \Rightarrow 2.62 - 2.71 = -0.09 \text{ Sec}$$

$$\Delta T_5 = T - T_5 \Rightarrow 2.62 - 2.80 = -0.18 \text{ Sec}$$

iii. Mean absolute error:

$$\Delta\bar{T} = \frac{|\Delta T_1| + |\Delta T_2| + |\Delta T_3| + |\Delta T_4| + |\Delta T_5|}{5}$$

$$= \frac{0.01 + 0.06 + 0.20 + 0.09 + 0.18}{5} \Rightarrow \frac{0.54}{5} \Rightarrow 0.108 \Rightarrow 0.11$$

iv. Relative error:

$$\delta T = \frac{\Delta\bar{T}}{T} \Rightarrow \frac{0.11}{2.62} \Rightarrow 0.0419$$

$$\Rightarrow 0.04 \text{ Sec}$$

v. PERCENTAGE ERROR: $\% \text{ error} = \frac{\Delta \bar{T}}{\bar{T}} \times 100$

$$= 0.04 \times 100$$

$$= 4\%$$

vi. FINAL RESULT : $\bar{T} \pm \Delta \bar{T}$

$$\Rightarrow (2.62 \pm 0.11)$$

{or}

$$\Rightarrow (2.62 \pm 4\%)$$

*.COMBINATION OF ERRORS:

i. Let $Z = A + B$

The maximum possible error in sum of quantities is $\Delta \bar{Z} = \Delta \bar{A} + \Delta \bar{B}$

$$Z = A + B$$

$$= (\bar{A} + \bar{B}) \pm (\Delta \bar{A} + \Delta \bar{B})$$

$$= \bar{Z} \pm \Delta \bar{Z}$$

ii. Let $Z = A - B$

The maximum possible error in difference of 2 quantities is

$$\Delta \bar{Z} = \Delta \bar{A} + \Delta \bar{B}$$

iii. Let $Z = A.B$

The maxium fractional error in the product of 2 quantities is

$$\text{Relative error} \Rightarrow \frac{\Delta \bar{Z}}{\bar{Z}} = \frac{\Delta \bar{A}}{\bar{A}} + \frac{\Delta \bar{B}}{\bar{B}}$$

iv. Let $Z = \frac{A}{B}$

The maximum fractional error in the division of 2 quantities is

$$\text{Relative error} \Rightarrow \frac{\Delta \bar{Z}}{\bar{Z}} = \frac{\Delta \bar{A}}{\bar{A}} + \frac{\Delta \bar{B}}{\bar{B}}$$

v. Let $z = A^n$

The fractional error in "n^{th}" power of a quantity is

$$\text{Relative error} \Rightarrow \frac{\Delta \bar{z}}{z} = n \frac{\Delta \bar{A}}{A}$$

Q. 2 resistance $R_1 = 100 \pm 3\Omega$, $R_2 = 200 \pm 4\Omega$ are connected in series. What is their equivalent resistant?

Sol:
$$R = R_1 + R_2$$
$$= (100 \pm 3) + (200 \pm 4)$$
$$= (300 \pm 7)\Omega$$

Q. The reading of initial and final temperatures of water are $(18 \pm 0.3)^\circ C$ & $(40 \pm 0.5)^\circ C$ respectively. Find the raise in temperature.

Sol: Raise in temperatue = final temperature - initial temperature

$$= (40 \pm 0.5) - (18 \pm 0.3)$$
$$= (40 - 18) \pm (0.5 + 0.3)$$
$$= (22 \pm 0.8)^\circ C$$

Q. The Resistance $R = \frac{V}{I}$ when $V = (100 \pm 5)\,V$, $I = (10 \pm 0.2)\,A$. Find percentage error in R.

Sol:
$$R = \frac{V}{I}.$$

$$\frac{\Delta \bar{R}}{\bar{R}} \times 100 = \left(\frac{\Delta \bar{V}}{\bar{V}} + \frac{\Delta \bar{I}}{\bar{I}}\right) \times 100$$

$$= \left(\frac{5}{100} + \frac{0.2}{10}\right) \times 100$$

$$= \left(\frac{5+2}{100}\right) \times 100 \Rightarrow \frac{7}{100} \times 100 \Rightarrow 7\%$$

Q. The percentage errors in the measurement of mass & speed are 2% & 3% respectively. Find % error in Kinetic Energy.

Sol. $K.E. = \frac{1}{2}mv^2$

$$\frac{\Delta \bar{m}}{\bar{m}} \times 100 = 2\% , \quad \frac{\Delta \bar{v}}{\bar{v}} \times 100 = 3\%$$

$$\frac{\Delta \bar{K}}{\bar{K}} \times 100 = \left(\frac{\Delta \bar{m}}{\bar{m}} \times 100\right) + \left(2 \times \frac{\Delta \bar{v}}{\bar{v}} \times 100\right)$$

$$= 2\% + (2 \times 3\%)$$

$$= 8\%$$

Q. Find Relative error in "Z" if $Z = \frac{A^4 B^{1/3}}{C\, D^{3/2}}$.

Sol. $\frac{\Delta \bar{Z}}{\bar{Z}}$ = Relative error in A^4 + Relative error in $B^{1/3}$ + Relative error in C + Relative error in $D^{3/2}$.

$$= 4\frac{\Delta \bar{A}}{\bar{A}} + \frac{1}{3}\frac{\Delta \bar{B}}{\bar{B}} + \frac{\Delta \bar{C}}{\bar{C}} + \frac{3}{2}\frac{\Delta \bar{D}}{\bar{D}}$$

*. Different types of errors:

1. CONSTANT ERROR: The error which affect the each observation by same amount.

Ex: Wearing of Scale.

2. SYSTEMATIC ERRORS: The error which tend to occur in one direction either positive or negative.

-> There are 4 types of systematic errors. They are:

i. INSTRUMENTAL ERRORS: This is due to inbuilt defect of instrument.

Ex: Wearing of scale.

ii. IMPERFECTIONS IN EXPERIMENTAL: This is due to limitation in experimental arrangement.

Ex: Calorimetric experiment

iii. PERSONAL ERROR : This is due to care-less-ness in taking observation and improper arrangement.

iv. EXTERNAL ERROR : This is due to change in external conditions like pressure, temperature, etc..

Ex: Expansion of a metal scale due to rise in temperature.

3. RANDOM ERRORS : The error which occurs irregularly and at random in magnitude and direction are called random errors.

4. LEAST COUNT ERRORS : This error is due to least count of measuring instrument.

Q. The period of oscillation of a pendulum is $T = 2\pi\sqrt{l/g}$ measured value of "l" is 20.0 cm known to 1mm accuracy and time for hundred oscillations of the Pendulum is 90 sec. using a watch of 1 sec. resolution. What is the accuracy in the determination / measurement of value g.

Sol: Given, $T = 2\pi\sqrt{l/g}$

Squaring on both sides

$$T^2 = 4\pi^2 \cdot l/g$$

$$g = 4\pi^2 \frac{l}{T^2}$$

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

$$= \left(\frac{0.1}{20} + 2 \cdot \frac{1}{90}\right)$$

$$= \left(\frac{1}{200} + \frac{2}{900}\right)$$

$$= \left(\frac{9+40}{1800}\right)$$

$$\Rightarrow \frac{49}{1800}$$

$$\Rightarrow 0.027$$

Percentage error = Relative error $\times 100$

$$= 0.027 \times 100$$

$$= 2.7$$

$$\Rightarrow 3\%$$

VECTORS

*. SCALAR QUANTITY : A physical quantity which have magnitude but not have direction.

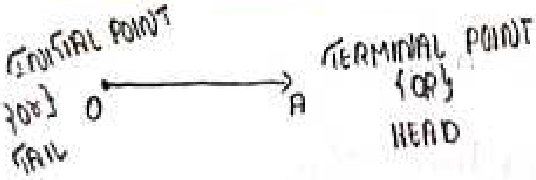
Ex: Length, time, distance, electric current, etc..

*. VECTOR QUANTITY : A physical quantity which have both magnitude, direction & obey the law of vector addition are called Vector Quantity.

Ex: Force, acceleration, displacement, velocity, etc..

-> REPRESENTATION OF VECTOR : A vector quantity is represented by a straight line with an arrow head over it.

Ex: 40 Km/h towards East.



length => magnitude = 40 Km/h

Vector notation = $\overline{OA}$

*. A vector is represented by single letter either in bold or it an arrow over it.

Ex: Velocity vector represents as $\overline{V}$ {or} $\vec{V}$ {or} **V**.

TYPES OF VECTORS:

-> EQUAL VECTORS: Vectors having same direction & same magnitude are called equal vectors

Ex: $\xrightarrow{\bar{A}}$ 5N $\quad \bar{A} = \bar{B}$

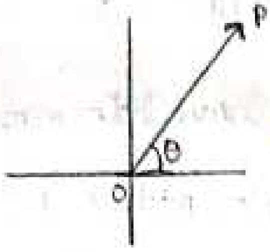
$\xrightarrow{\bar{B}}$ 5N

-> **MODULAS OF A VECTOR**: It is magnitude of a vector.

Ex: Modulus of vector $\bar{F}$ => $|\bar{F}|$ {or} F

-> **POSITION VECTOR**: A vector which gives position of an object w.r.t respect to origin.

Ex:



Position vector = $\overline{OP}$

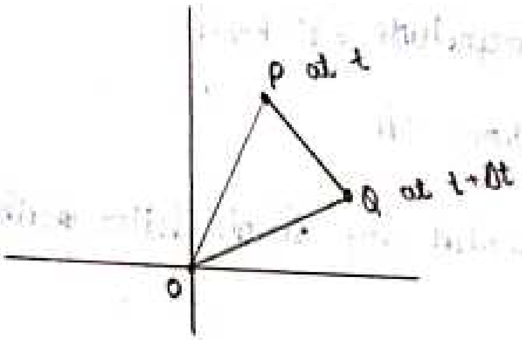
*. It tells about straight line distance b/w origin & object.

*. It tells about direction of object with respect to origin.

-> **DISPLACEMENT VECTOR**: A vector which tells how much & in which direction object changes it position in given time interval is called Displacement Vector.

Ex: Let object at point P when time t

and it moves to point Q when time $t+\Delta t$.

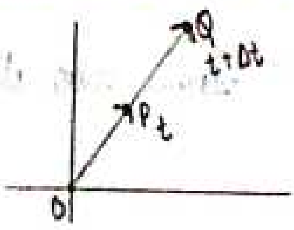


$\left.\begin{array}{l}\overline{OP} \\ \overline{OQ}\end{array}\right\}$ POSITION VECTOR

$\overline{PQ}$ - DISPLACEMENT VECTOR

-> **FIXED VECTOR {OR} LOCALISED VECTOR**: If initial point of a vector is fixed then it is called fixed vector.

Ex:



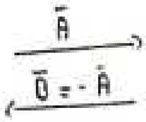
POSITION VECTOR - $\overline{OP}, \overline{OQ}$

→ **FREE VECTOR {or} NON-LOCALISED VECTOR:**

If initial point of a vector is not fixed then it is called Free vector.

Ex: Velocity vector.

→ **NEGATIVE OF A VECTOR:** It is another vector which have same magnitude but opposite in direction.

Ex: 

} → Negative of a vector.

→ **COLLINEAR VECTOR:** Vectors which either act along same straight line or along parallel lines are called collinear vectors.

* Two collinear vectors have same direction $\{\theta = 0°\}$ are called like vectors {or} parallel vectors:

Ex: 

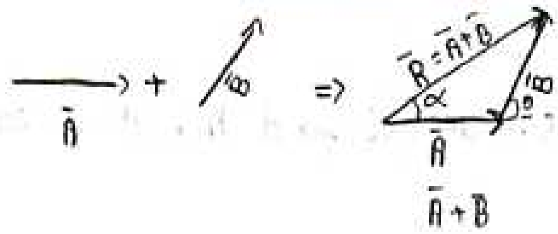
* Two collinear vectors having opposite $(\theta = 180°)$ are called unlike {or} anti-parallel vectors.

Ex: 

→ **COPLANAR VECTORS:** The vectors which act in the same plane are called co-planar vectors.

* **ADDITION OF VECTORS:**

→ **TRIANGLE LAW:** If two vectors acting at a point are represented in magnitude & direction by two sides of a triangle in order, then their resultant vector represented by third side of triangle but opposite in order.



- Magnitude of resultant vector

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$, θ is angle b/w $\bar{A}$ & $\bar{B}$, i.e., which always acute.

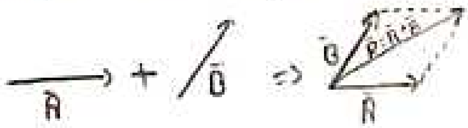
- Direction of resultant vector

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$; α is the angle b/w $\bar{R}$ & $\bar{A}$.

(7)

→ PARALLELOGRAM VECTOR LAW :

If two vectors acting at a point are represented in magnitude & direction by two adjacent sides of parallelogram drawn from that point, then their resultant vector represented by diagonal of parallelogram drawn from common tail of vectors.



- MAGNITUDE OF RESULTANT VECTOR : {$\bar{R}$}

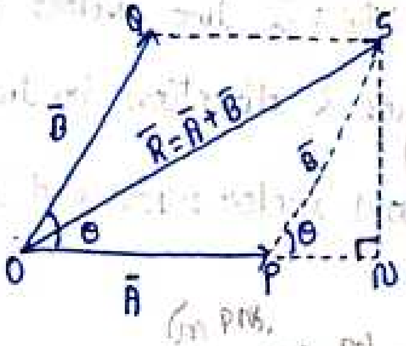
In llgm OPQS,
OP//QS & OP=QS ⇒ $\bar{A}$
OQ//PS & OQ=PS ⇒ $\bar{B}$
OS = R In ΔONS

In llgm OPQS,

OP // QS & OP = QS ⇒ $\bar{A}$

OQ // PS & OQ = PS ⇒ B

OS = R



In ΔONS, $(OS)^2 = (ON)^2 + (SN)^2$

$$R^2 = (OP + PN)^2 + (SN)^2 \rightarrow ①$$

In PNS,
$\cos\theta = \frac{PN}{PS}$ = PS cos θ
PN = B cos θ

In ΔPNS,

$\cos\theta = \frac{PN}{PS} \Rightarrow PN = PS\cos\theta$ $\sin\theta = \frac{SN}{PS} = B\sin\theta$

$\boxed{PN = B\cos\theta} \rightarrow (2)$

$\sin\theta = \frac{SN}{PS} \Rightarrow SN = PS\sin\theta$

$\boxed{SN = B\sin\theta} \rightarrow (3)$

from eq-(1),(2),(3)

$$R^2 = (OP + PN)^2 + (SN)^2$$

$$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$$

$$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$

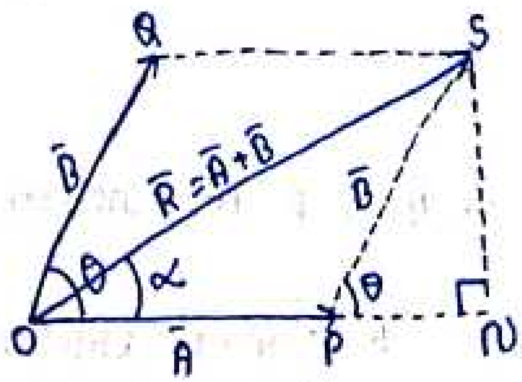
$$R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta \quad \{\because \cos^2\theta + \sin^2\theta = 1\}$$

$$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

• DIRECTION OF RESULTANT VECTOR $\{\bar{R}\}$:

Let α be the angle b/w $\bar{R}$ & $\bar{A}$.



In ΔONS,

$$\tan\alpha = \frac{SN}{ON}$$

$$= \frac{SN}{OP + PN}$$

$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$ or $\boxed{\alpha = \tan^{-1}\frac{B\sin\theta}{A + B\cos\theta}}$

*SPECIAL CASES :

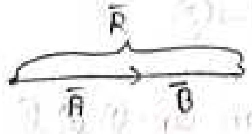
CASE - I : If $\bar{A}$ & $\bar{B}$ are acting along same direction, i.e., $\theta = 0°$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB}$$

$\boxed{R = A + B}$ → Sum of the two vectors magnitude.

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow 0$$



$$\alpha = 0°$$

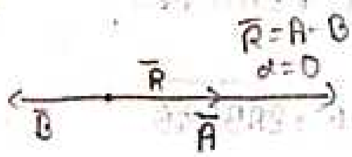
CASE - II : If $\bar{A}$ & $\bar{B}$ are acting along mutually opposite direction, i.e., $\theta = 180°$

$$R = \sqrt{A^2 + B^2 + 2AB\cos 180°}$$

$$R = \sqrt{A^2 + B^2 - 2AB}$$

$\boxed{R = A \sim B \text{ (or) } |A - B|}$ → Difference of magnitude.

$$\tan\alpha = \frac{B\sin 180}{A + B\cos 180}$$



$\tan\alpha = 0$

$\alpha = 0°$ (or) $180°$ } along longer vectors.

CASE - III : If $\bar{A}$ & B are acting perpendicular to each other, i.e., $\theta = 90°$

$$R = \sqrt{A^2 + B^2 + 2AB\cos 90°}$$

$$R = \sqrt{A^2 + B^2 + 2AB(0)}$$

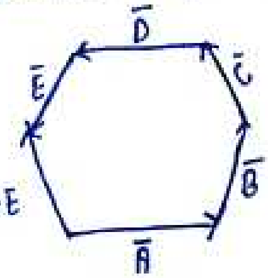
$$\boxed{R = \sqrt{A^2 + B^2}}$$

$$\tan\alpha = \frac{B\sin 90}{A + B\cos 90}$$

$$\Rightarrow \frac{B(1)}{A + B(0)} \Rightarrow \frac{B}{A} \qquad \boxed{\therefore \tan\alpha = \frac{B}{A}}$$

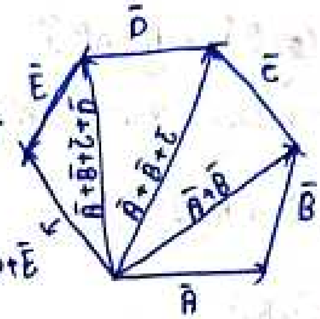
→ POLYGON LAW: If no. of vectors acting on a particle at a time are represented in magnitude & direction by the various sides of an open polygon taken in same order, then an resultant vector represented magnitude & direction by the closing side of polygon taken in opposite sides

"n" vectors are present, then n+1 is resultant side.



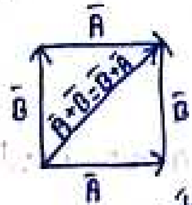
$\bar{R} = \bar{A}+\bar{B}+\bar{C}+\bar{D}+E$

Simpler way:

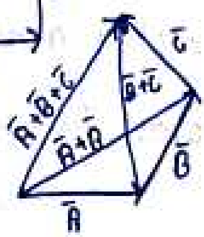


$\bar{R} = \bar{A}+\bar{B}+\bar{C}+\bar{D}+\bar{E}$

→ PROPERTIES OF VECTOR ADITION:



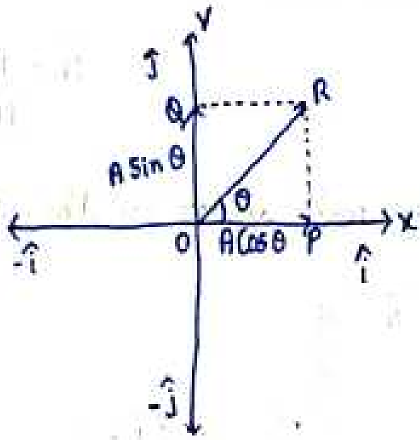
1. It is commutative.
   $\bar{A}+\bar{B} = \bar{B}+\bar{A}$



2. It is associative.
   $\bar{A}+(\bar{B}+\bar{C}) = (\bar{A}+\bar{B})+\bar{C}$
3. It is distributive
   $\lambda(\bar{A}+\bar{B}) = \lambda\bar{A}+\lambda\bar{B}$
4. $\bar{A}+\bar{0} = \bar{A}$

*. RESOLUTION OF A VECTOR: It is a process of splitting a vector into sum of two or more vectors.

$\bar{A} = \bar{OP} + \bar{OQ}$



In ΔOPR,

$\cos\theta = \frac{OP}{OR}$

$OP = OR\cos\theta$

$\boxed{OP = A\cos\theta}$

$$\sin\theta = \frac{PR}{OR}$$

$$PR = OR\sin\theta$$

$$PR = A\sin\theta$$

$$\boxed{OQ = PR = A\sin\theta}$$

$$\overline{OP} = A\cos\theta\,\hat{i}$$

$$\overline{OQ} = A\sin\theta\,\hat{j}$$

$$\bar{A} = \underbrace{A\cos\theta\,\hat{i}}_{\text{x-component}} + \underbrace{A\sin\theta\,\hat{j}}_{\text{y-component}}$$

$$\bar{A} = A_x\hat{i} + A_y\hat{j}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A\sin\theta}{A\cos\theta} \Rightarrow \frac{A_y}{A_x}$$

Note: 1. $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along positive x, y, z -axis respectively

2. Generally vectors represented as $\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\Rightarrow A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

UNIT VECTOR: It is a vector of unit magnitude drawn in the direction of a given vector.

→ A unit vector in the direction of ~~vector~~ given vector is coined by

i.e., unit vector in the direction of vector $\bar{A}$ is given by

$$\hat{A} = \frac{\bar{A}}{|\bar{A}|}$$

Q. Find unit vector in the direction of $\bar{A} = 3\hat{i} + 4\hat{j}$.

Sol:

$$\hat{A} = \frac{\bar{A}}{|\bar{A}|}$$

$$= \frac{3\hat{i} + 4\hat{j}}{\sqrt{3^2 + 4^2}} \Rightarrow \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

verification:

$$|\hat{A}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9+16}{25}} \Rightarrow 1$$

Q. Find unit vector parallel to resultant of vectors $\bar{A} = \hat{i} + 4\hat{j} - 2\hat{k}$ & $\bar{B} = 3\hat{i} - 5\hat{j} + 4\hat{k}$.

Sol:

$$\bar{A} = \hat{i} + 4\hat{j} - 2\hat{k}$$
$$\bar{B} = 3\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\bar{R} = \bar{A} + \bar{B} = 4\hat{i} - \hat{j} + 2\hat{k}$$

$$\hat{R} = \frac{\bar{R}}{|\bar{R}|} \Rightarrow \frac{4\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{4^2 + (-1)^2 + 2^2}}$$

$$\Rightarrow \frac{4\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{21}}$$

Q. Two forces of 5N & 7N act on a particle an angle of 60° b/w them, find the resultant force.

Sol:

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{5^2 + 7^2 + 2(5)(7)\cos 60}$$

$$= \sqrt{25 + 49 + 2(5)(7)\left(\tfrac{1}{2}\right)}$$

$$= \sqrt{25 + 49 + 35}$$

$$= \sqrt{109}\ N$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$= \frac{7\sin 60}{5 + 7\cos 60}$$

$$= \frac{7(\sqrt{3}/2)}{5 + 7(1/2)}$$

$$=$$

$$\tan\alpha = 0.7132$$

$$\alpha = 35.4^\circ$$

Q. 2 vectors both equal in magnitude have their resultant equal to magnitude of the either, find the angle b/w the vectors?

Sol: Given,

$$A = B$$

$$R = A = B$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$A = \sqrt{A^2 + A^2 + 2AA\cos\theta}$$

$$A = \sqrt{2A^2 + 2A^2\cos\theta}$$

Squaring on both sides,

$$A^2 = 2A^2 + 2A^2\cos\theta$$

$$A^2 - 2A^2 = 2A^2\cos\theta$$

$$-A^2 = 2A^2\cos\theta$$

$$\cos\theta = \frac{-A^2}{2A^2}$$

$$\cos\theta = \frac{-1}{2}$$

$$\cos\theta = \cos 120^\circ$$

$$\theta = 120^\circ$$

Q. Two forces $F_1 = 3\hat{i} + 4\hat{j}$ & $F_2 = 3\hat{j} + 4\hat{k}$ acting simultaneously at a point what is the magnitude of resultant force.

Sol:

$$\bar{F}_1 = 3\hat{i} + 4\hat{j} + 0\hat{k}$$

$$\bar{F}_2 = 3\hat{j} + 0\hat{j} + 4\hat{k}$$

$$\bar{F}_R = \bar{F}_1 + \bar{F}_2 = 3\hat{j} + 7\hat{j} + 4\hat{k}$$

$$\bar{F}_R = \sqrt{3^2 + 7^2 + 4^2}$$

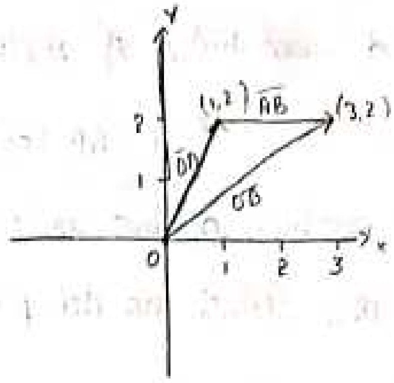
$$= \sqrt{9 + 49 + 16}$$

$$= \sqrt{74} \text{ units.}$$

Q. Find vector $\overline{AB}$ and its magnitude if it has initial point A(1,2) & B(3,2) final point.

Sol: Given,

A (1,2), B(3,2)



$$\overline{OA} + \overline{AB} = \overline{OB}$$

$$\overline{AB} = \overline{OB} - \overline{OA}$$

$$\overline{AB} = (3\hat{i} + 2\hat{j}) - (\hat{i} + 2\hat{j})$$

$$\overline{AB} = 2\hat{i}$$

$$R_{AB} = \sqrt{2^2} => 2$$

Note: 1. (x_1, y_1, z_1) is a point,

Position vector $= x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

2. (x_1, y_1, z_1), (x_2, y_2, z_2)

Displacement vector $= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Subtraction of vectors:

Subtraction of vectors is equal to addition of first vector to the negative of second vector.

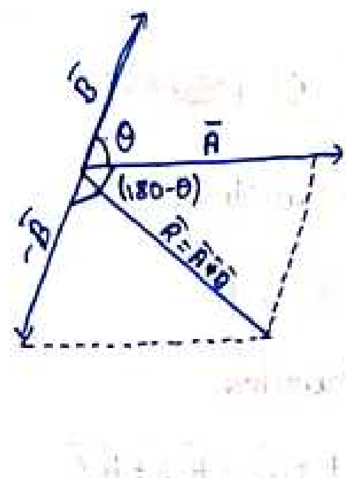
$$\bar{R} = \bar{A} - \bar{B}$$

$$= \bar{A} + (-\bar{B})$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos(180 - \theta)}$$

(or)

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$



$$\tan\alpha = \frac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)}$$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

PRODUCT OF VECTORS:

1. SCALAR PRODUCT: Scalar product of two vectors is defined as the product of magnitudes of vectors & cosine of angle b/w them.

$$\bar{A}.\bar{B} = AB\cos\theta$$

*. Scalar product of two vector is always scalar.

*. It is also called as dot product.

$$\bar{A}.\bar{B} = A(B\cos\theta)$$

=> magnitude of $\bar{A}$. magnitude of $\bar{B}$ along $\bar{A}$

{or}

=> magnitude of $\bar{A}$. projection of $\bar{B}$ along/onto $\bar{B}$

$$\bar{A}.\bar{B} = AB\cos\theta \Rightarrow B(A\cos\theta)$$

=> magnitude of $\bar{B}$. magnitude of $\bar{A}$ onto $\bar{B}$

{or}

=> magnitude of $\bar{B}$. projection of $\bar{A}$ onto $\bar{B}$

"Thus, scalar product defined as, product of magnitude of 1st vector & magnitude of 2nd along 1st vector."

Properties of dot product:

1. It is commutative.

$$\bar{A}.\bar{B} = \bar{B}.\bar{A}$$

2. It is distributive.

$$\bar{A}.(\bar{B}+\bar{C}) = \bar{A}.\bar{B}+\bar{A}.\bar{C}$$

3. Dot product of 2 parallel vectors is maximum {+ve value}.

$$\bar{A}.\bar{B} = AB\cos 0 \Rightarrow AB$$

4. Dot product of 2 anti-parallel vector is maximum {-ve value}.

$$\bar{A}.\bar{B} = AB \cos 180 \Rightarrow -AB$$

5. Dot product of 2 ⊥ vectors is zero.

$$\bar{A}.\bar{B} = AB \cos 90 \Rightarrow 0$$

6. Dot product of a vector itself is equal to square of its magnitude.

$$\bar{A}.\bar{A} = AA \cos 0 \Rightarrow A^2$$

7. $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

8. If $\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then

$$\bar{A}.\bar{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_xB_x + A_yB_y + A_zB_z$$

Examples:

1. Work done ⇒ dot product of force vector ($\bar{F}$) and displacement vectors

$$W = \bar{F}.\bar{S}$$

2. Power ⇒ dot product of froce vector ($\bar{F}$) & velocity vector ($\bar{v}$)

$$P = \bar{F}.\bar{v}.$$

Problems:

Q. A force of $7\hat{i} + 6\hat{k}$ N makes a dolly move on a rough plane with a velocity of $3\hat{j} + 4\hat{k}$ m/s. Calculate the power.

Sol:

$\bar{F} = 7\hat{i} + 6\hat{k}$ N

$\bar{v} = 3\hat{j} + 4\hat{k}$ m/s

$$\bar{P} = \bar{F}.\bar{v}$$

$$= (7\hat{i} + 0\hat{j} + 6\hat{k}).(0\hat{i} + 3\hat{j} + 4\hat{k})$$

$$= 7(0) + 0(3) + 6(4)$$

$$= 24 \text{ W (or) } N\text{-}\frac{m}{s} \text{ (or) } J/s$$

Q. A body constained to move along z-axis is subjected to a force $\bar{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N. Calculate the work done by the force in displacing the dody to a distance of 4m along the z axis

$$\bar{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\bar{S} = 4\hat{k}$$

$$W = \bar{F}.\bar{S}$$

$$= (-\hat{i} + 2\hat{j} + 3\hat{k}).(0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= 12 \text{ J}$$

Q. Prove that the vectors $\bar{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\bar{B} = 2\hat{i} - \hat{j}$ are $\perp^{er}$ to each other.

Sol: $\bar{A}.\bar{B} = (\hat{i} + 2\hat{j} + 3\hat{k}).(2\hat{i} - \hat{j})$

$$= 2 - 2 + 0$$

$$= 2$$

$\bar{A}.\bar{B} = 0 \Rightarrow \bar{A}$ is $\perp^{er}$ to $\bar{B}$

Q. Find the value of λ. so that vectors $\bar{A} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\bar{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$. are $\perp^{er}$ to each other.

$$\bar{A}.\bar{B} = (2\hat{i} + \lambda\hat{j} + \hat{k}).(4\hat{i} - 2\hat{j} - 2\hat{k}) \Rightarrow 0$$

$$\Rightarrow 2(4) - 2(\lambda) + 1(-2) \Rightarrow 0$$

$$\Rightarrow 8 - 2\lambda - 2 \Rightarrow 0$$

$$\Rightarrow -2\lambda \Rightarrow -6$$

$$2\lambda = 6$$

$$\lambda = \frac{6}{2}$$

$$\boxed{\lambda = 3}$$

Q. Find the angle b/w vectors $\bar{A} = \hat{i} + 2\hat{j} - \hat{k}$ & $\bar{B} = -\hat{i} + \hat{j} - 2\hat{k}$.

$$\bar{A} = \hat{i} + 2\hat{j} - \hat{k}$$

$$\bar{B} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\bar{A}.\bar{B} = (\hat{i} + 2\hat{j} - \hat{k}).(-\hat{i} + \hat{j} - 2\hat{k})$$

$$= -1 + 2 + 2$$

$$= 3$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \Rightarrow \sqrt{(1)^2 + (2)^2 + (-1)^2}$$

$$\Rightarrow \sqrt{1+4+1} = \sqrt{6}$$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} \Rightarrow \sqrt{(-1)^2 + (1)^2 + (-2)^2} \Rightarrow \sqrt{6}$$

$$\bar{A}.\bar{B} = AB \cos\theta$$

$$\cos\theta = \frac{\bar{A}.\bar{B}}{AB}$$

$$\Rightarrow \frac{3}{\sqrt{6}.\sqrt{6}}$$

$$\Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

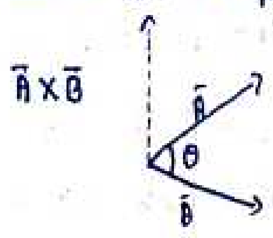
$$\theta = 60^\circ$$

2. CROSS PRODUCT (or) VECTOR PRODUCT:

Vector product of two vectors is a vector whose magnitude is product of magnitudes of vectors & sine of angle of b/w them and direction in the plane $\perp^{er}$ to two vectors.

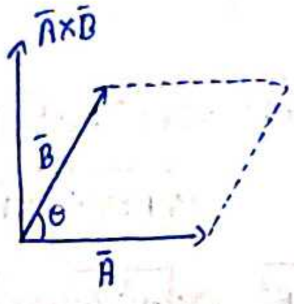
$$\bar{A} \times \bar{B} = AB \sin\theta \ \hat{n}$$

where $\hat{n}$ is unit vector in the plane $\perp^{er}$ to $\bar{A}$ & $\bar{B}$



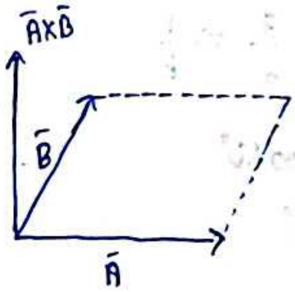
RIGHT-HANDED SCREW RULE:

If a right handed screw is placed with its axis $\perp^{r}$ to the plane of vectors $\bar{A}$ & $\bar{B}$ and rotated from A to B through the smaller angle, then the direction in which the screw advances gives the direction of $\bar{A}\times\bar{B}$.



RIGHT-HAND THUMB RULE:

Curl the fingers of right hand in such a way that the point in the direction of rotation from $\bar{A}$ to $\bar{B}$ through the smaller angle then the streched come from the points in the direction from $\bar{A}$ to $\bar{B}$.



PROPERTIES OF VECTOR PRODUCT:

1. It is anti-commutative

$$\bar{A}\times\bar{B} = -(\bar{B}\times\bar{A})$$

2. It is distributive over the addition

$$\bar{A}\times(\bar{B}+\bar{C}) = (\bar{A}\times\bar{B})+(\bar{A}\times\bar{C})$$

3. Vector product of two parallel or anti parallel vectors is 'null vector'

Parallel $\rightarrow$ If $\theta = 0 \Rightarrow \bar{A}\times\bar{B} = AB\sin 0\ \hat{n} \Rightarrow 0\hat{n} \Rightarrow \bar{0}$

Anti-Parallel $\rightarrow$ If $\theta = 180 \Rightarrow \bar{A}\times\bar{B} = AB\sin 180\ \hat{n} \Rightarrow 0\hat{n} \Rightarrow \bar{0}$

4. The magnitude of cross product of two mutually $\perp^{ler}$ vectors is equal to "product of magnitudes of vectors".

$$|\bar{A}\times\bar{B}| = AB\sin 90 = AB$$

5. Vector product of a vector itself is null vector

$$\bar{A}\times\bar{A} = AA\sin 0\ \hat{n} = \bar{0}$$

6. i. $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \bar{0}$

ii. +ve {clock wise}

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{k}\times\hat{i} = \hat{j}$

$\hat{j}\times\hat{k} = \hat{i}$

-ve {anti-clock wise}

$\hat{j}\times\hat{i} = -\hat{k}$

$\hat{i}\times\hat{k} = -\hat{j}$

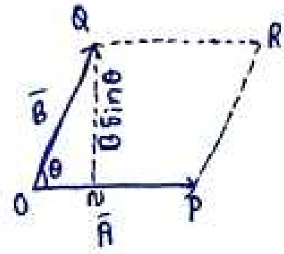
$\hat{k}\times\hat{j} = -\hat{i}$

7. Unit vector $\perp^{ler}$ to the plane of two vectors $\bar{A}$ & $\bar{B}$ is given by

$$\bar{A}\times\bar{B} = AB\sin\theta\ \hat{n}$$

$$\boxed{\hat{n} = \frac{\bar{A}\times\bar{B}}{AB\sin\theta}} \quad \{or\} \quad \boxed{\hat{n} = \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|}}$$

8. Magnitude of cross product of two vectors = "Area of parallelogram formed by two vectors as its adjacent sides."



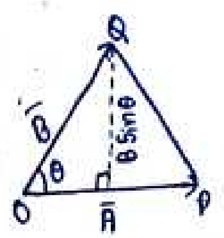
Proof: $|\bar{A}\times\bar{B}| = AB\sin\theta$

$= A(B\sin\theta)$

$=$ base $\times\ \perp^{ler}$ distance b/w parallel sides

$\Rightarrow$ area of parallelogram OPRQ.

9. Magnitude of cross product = "2 × Area of Δ^{le} formed by 2 vectors as its adjacent sides."



Proof: $|\bar{A}\times\bar{B}| = AB\sin\theta$

$= A(B\sin\theta)$

$= 2[\frac{1}{2}A(B\sin\theta)]$

$= 2[\frac{1}{2}(\text{base} \times \text{height})]$

$= 2 \times$ area of Δ^{le} OPQ

Problems:

Examples:

1. Torque acting on particle = "Cross product of position vector ($\bar{r}$) & force vector ($\bar{F}$)".

$$\boxed{\bar{\tau} = \bar{r} \times \bar{F}}$$

2. Angular momentum of particle = "Cross product of position vector ($\bar{r}$) & linear momentum ($\bar{p}$)".

$$\boxed{\bar{L} = \bar{r} \times \bar{p}}$$

3. Velocity of particle = "Cross product of angular velocity ($\bar{\omega}$) & position vector ($\bar{r}$)".

$$\boxed{\bar{v} = \bar{\omega} \times \bar{r}}$$

Q. Find the vector product of 2 vectors $\bar{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$
$\bar{B} = -2\hat{i} + \hat{j} - 3\hat{k}$.

Sol:

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -4 & 5 \\ -2 & 1 & -3 \end{vmatrix}$$

$= \cancel{\hat{i}(-4 \times -3)} - \hat{j}(3 \times -3)$

$$\Rightarrow \hat{i} \begin{vmatrix} -4 & 5 \\ 1 & -3 \end{vmatrix} - \hat{j} \begin{vmatrix} 3 & 5 \\ -2 & -3 \end{vmatrix} + \hat{k} \begin{vmatrix} 3 & -4 \\ -2 & 1 \end{vmatrix} \quad \left\{ \because \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \right\}$$

$$\Rightarrow \hat{i}\,|12 - 5| - \hat{j}\,|-9 + 10| + \hat{k}\,|3 - 8|$$

$$\Rightarrow 7\hat{i} - \hat{j} - 5\hat{k}$$

Q. Find the torque of a force $\bar{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$ about the origin. The force acts on a particle whose position vector is $\hat{i} - \hat{j} + \hat{k}$.

$\bar{F} = 7\hat{i} + 3\hat{j} - 5\hat{k}$

$\bar{r} = \hat{i} - \hat{j} + \hat{k}$

$\bar{\tau} = \bar{r} \times \bar{F}$

$$\bar{r} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -1 & 1 \\ 3 & -5 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 1 \\ 7 & -5 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & -1 \\ 7 & 3 \end{vmatrix}$$

$$= \hat{i}[5-3) - \hat{j}(-5-7) + \hat{k}(3+7)$$

$$= 2\hat{i} + 12\hat{j} + 10\hat{k}$$

Q. Calculate the area of $\square^{le}$ and a triangle whose 2 adjacent sides are formed by vectors $\bar{A} = 3\hat{i} + 4\hat{j}$, $\bar{B} = -3\hat{i} + 7\hat{j}$.

Sol:

$$\bar{A} = 3\hat{i} + 4\hat{j} + 0\hat{k}$$

$$\bar{B} = -3\hat{i} + 7\hat{j} + 0\hat{k}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -3 & 7 & 0 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 4 & 0 \\ 7 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 0 \\ -3 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 4 \\ -3 & 7 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(0) + \hat{k}(21 + 12)$$

$$\bar{A} \times \bar{B} = 33\hat{k}$$

$$|\bar{A} \times \bar{B}| = \sqrt{33^2} \Rightarrow 33 \text{ units}$$

i. area of parallelogram $= |\bar{A} \times \bar{B}|$

$$= 33 \text{ units}^2$$

ii. area of Δ^{le} $= \frac{1}{2}|\bar{A} \times \bar{B}|$

$$= \frac{1}{2}(33) \Rightarrow 16.5 \text{ units}^2$$

Q. Determine the unit vector $\perp^{ler}$ to $\bar{A}$ & $\bar{B}$. $\bar{A} = 2\hat{i} + \hat{j} + \hat{k}$, $\bar{B} = \hat{i} - \hat{j} + 2\hat{k}$

let $\bar{A} \times \bar{B} = AB \sin\theta \ \hat{n}$

$$\hat{n} = \frac{\bar{A} \times \bar{B}}{AB\sin\theta}$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$$

$$\Rightarrow \hat{i}(2+1) - \hat{j}(4-1) + \hat{k}(-2-1)$$

$$\bar{A}\times\bar{B} \Rightarrow 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$|\bar{A}\times\bar{B}| = \sqrt{(3)^2+(-3)^2+(-3)^2}$$

$$= \sqrt{27}$$

$$= 3\sqrt{3}$$

$$\therefore \hat{n} = \frac{3\hat{i} - 3\hat{j} - 3\hat{k}}{3\sqrt{3}}$$

$$= \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

Q. Prove that the vectors $\bar{A} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\bar{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$ are parallel.

Sol:

$$\bar{A} = 2\hat{i} - 3\hat{j} - \hat{k}$$

$$\bar{B} = -6\hat{i} + 9\hat{j} + 3\hat{k}$$

$$\bar{A}\times\bar{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$$

$$= \hat{i}\begin{vmatrix} -3 & -1 \\ 9 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -1 \\ -6 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -3 \\ -6 & 9 \end{vmatrix}$$

$$\Rightarrow \hat{i}(-9+9) - \hat{j}(6-6) + \hat{k}(18-18)$$

$$\Rightarrow 0\hat{i} + 0\hat{j} + 0\hat{k}$$

$=$

CH-3

KINEMATICS

MECHANICS: Mechanics is the branch of physics that deals with the conditions of rest or motion of the material objects around us.

SUB-BRANCHES OF MECHANICS:

i. STATICS: It is the branch of mechanics that deals with the study of objects at rest or in equilibrium.

ii. KINEMATICS: It is the branch of mechanics that deals with the study of motoin of objects without considering the cause of motion.

iii. DYNAMICS: It is the branch of mechanics that deals with the study of motion of objects taking into consideration the cause of their motion.

*REST: An object is said to be at rest if it does not change its position i.e., with respect to its surroundings. {with the passage of time.}

Ex: A book lying on a table.

*MOTION: An object is said to be at motion if it changes its position with respect to its surroundings. {with the passage of time.}

Ex: A train moving on rails.

-> Rest and motion are relative terms.

-> Absolute rest and motion are unknown.

*POINT OBJECT: If the position of an object changes by distances much greater than its own size

Ex: i. Earth can be regarded as a point object for studying its motion around the sun.

ii. A train under a journey of several hundred Kilometers can be regarded as a point object.

TYPES OF DIMENSIONAL FORCES:

-> One Dimensional motion: The motion of an object is said to be one dimensional if only one of the three co-ordinates specifying the position of the object changes with time.

Ex: 1. Motion of a train along a straight track.

2. Motion of a freely falling body.

-> TWO DIMENSIONAL MOTION:

If only two of the three co-ordinates specifying the position of the object changes with time.

Ex:- 1. MOTION OF Planets around the sun.

2. A car moving along a zig-zag path on a level road.

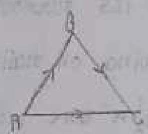
=> THREE DIMENSIONAL MOTION: If only all the 3 co-ordinates specifying its position change with time

Ex: 1. A kite flying on a windy day.

2. Motion of an aeroplane in space.

*. DISTANCE OR PATH LENGTH: It is the length of the actual path travelled by a body b/w its initial & final positions.

Distance covered = AB + BC



-> It is a scalar quanity, because it has only magnitude and no direction.

-> Distance covered is always positive (or) zero.

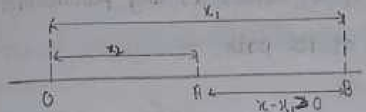
-> S.I. units => m

-> C.G.S. units => cm

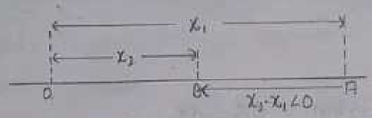
*. DISPLACEMENT: The shortest pathe measured in the direction from initial point to final point.

-> It is vector quantity.

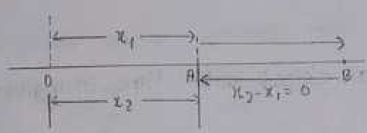
->It may be positive, negative or zero.
->S.I. units => m
->C.G.S. units => cm
->CHARACTERISTICS OF DISPLACEMENT :
=>POSITIVE DISPLACEMENT :



=> NEGATIVE DISPLACEMENT :



=> ZERO DISPLACEMENT :



=> Displacement is not dependent on the choice of the origin O of the position co-ordinates.

*SPEED :
->"Rate of change of position of an object with time in any direction."
->It is equal to the distance travelled by the object per unit time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

->It is scalar quantity.
->It is neither positive or zero, but it never be negative.
->S.I. units => m/s
->C.G.S. units => cm/s
->Dimensional formula => $[M^0L^1T^{-1}]$
->Different types of speed :
1. UNIFORM SPEED : If it covers equal distance in equal intervals of time.

ii. VARIABLE SPEED : An object is said to be moving with variable speed if it covers unequal distances in equal interval of time.

iii. AVERAGE SPEED : The average speed is the total distance travelled by the total time taken to cover that distance.

iv. INSTANTANEOUS SPEED : The speed of an object at any particular instant of time or at a particular point of its path.

Distance covered $= \Delta x$

In a small time interval $\Rightarrow \Delta t$

$$\text{Average speed} = \frac{\Delta x}{\Delta t}$$

$$\text{INSTANTANEOUS SPEED} = v \Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

*. VELOCITY :

-> Rate of change of position of an object with time in a given direction.

-> It can be defined as the speed of an object in a given direction.

-> It is equal to the displacement covered per unit time

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

-> It has both magnitude & direction, i.e., vector quantity.

-> It is positive, zero or negative depending on the displacement is +ve, -ve, zero.

-> S.I. units $\Rightarrow$ m/s

-> C.G.S. units $\Rightarrow$ cm/s

-> Dimensional formula $\Rightarrow [M^0 T^{-1} L^1]$

-> UNIFORM VELOCITY : If it covers equal displacement in equal interval of time.

-> VARIABLE VELOCITY : A body is said to be moving with variable velocity if either its speed changes or direction of motion changes or both change with time.

→ AVERAGE VELOCITY: the ratio of its total displacement to the total time interval in which that displacement occurs. If x_1 & x_2 are positions of an object at times t_1 & t_2. $v_{av} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$.

→ INSTANTANEOUS VELOCITY: The velocity of an object at a particular instant of time or at a particular point of its path. $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} \Rightarrow \frac{d\vec{x}}{dt}$

* ACCELERATION:

→ The rate of change of velocity of an object with time is called its acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

→ It is a vector quantity.

→ It has the same direction as that of the change in velocity.

→ S.I. units ⇒ m/s^2

→ C.G.S. units ⇒ cm/s^2

→ Dimensional formula ⇒ $[M^0L^1T^{-2}]$

→ UNIFORM ACCELERATION: If its velocity changes by equal amounts in equal intervals of time, however small these time interval may be.

→ VARIABLE ACCELERATION: If its velocity changes by unequal amounts in equal intervals of time.

→ AVERAGE ACCELERATION: The ratio of the total change in velocity of the object to the total time interval taken.

$$a_{av} = \frac{v_2 - v_1}{t_2 - t_1} \Rightarrow \frac{\Delta v}{\Delta t}$$

→ INSTANTANEOUS ACCELERATION: The acceleration of an object at a given instant of time or at a given point of its motion.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$v = \frac{dx}{dt}$$

$$\therefore a = \frac{d}{dt}\left(\frac{dx}{dt}\right) \Rightarrow \frac{d^2x}{dt^2}$$

POSITIVE ACCELERATION: If the velocity of an object increases with time, its acceleration is positive. When a bus leaves a bus-stop, its acceleration is positive.

NEGATIVE ACCELERATION: If the velocity of an object decreases with time, its acceleration is negative. Negative acceleration is also called "retardation or deceleration".

Problems:

Q. A car is moving along x-axis as shown in the figure, it moves from O to P in 18 sec & returns from P to Q in 6 sec. What are the average velocity & average speed of car. in going from

a. from O to P

b. from O to P & back to Q.

Sol: i. from O to P

average speed = $\frac{\text{distance}}{\text{time}} \Rightarrow \frac{360}{18} \Rightarrow 20\ m/s$

average velocity = $\frac{\text{displacement}}{\text{time}} \Rightarrow \frac{360}{18} \Rightarrow 20 m/s$

ii. from O to P & back to Q.

average speed = $\frac{\text{Total distance}}{\text{time}} \Rightarrow \frac{360+120}{18+6} \Rightarrow \frac{480}{24} \Rightarrow 20 m/s$

average velocity = $\frac{\text{displacement}}{\text{time}} \Rightarrow \frac{240}{18+6} \Rightarrow \frac{240}{24} \Rightarrow 10 m/s$

Q. A man walks on a straight road from his home to a market 2.5 Kms away, with a speed of 5 Km/h. finding the market closed he instantly turns & walks back home with a speed 7.5 Km/h. What is the

a. magnitude of average velocity.

b. average speed of the man over the interval time

i. 0 to 30 min

ii. 0 to 50 min

iii. 0 to 40 min

Sol:

i. 0 to 30 min

Distance covered in 30 min $= \frac{1}{2}$ h

$$\text{Distance} = \text{speed} \times \text{time}$$
$$= 5 \frac{\text{km}}{\text{h}} \times \frac{1}{2} \text{h}$$
$$\Rightarrow 2.5 \text{ km}$$

a. average velocity $= \frac{\text{displacement}}{\text{time}} \Rightarrow \frac{2.5 \text{ km}}{(\frac{1}{2})\text{h}} \Rightarrow 5 \text{ km/h}$

average speed $= \frac{\text{distance}}{\text{time}} \Rightarrow \frac{2.5 \text{ km}}{(\frac{1}{2})\text{h}} \Rightarrow 5$ km/h.

ii. 0 to 50 min

man covers 2.5 km distance in 1st 30min then man covers distance in last 20 min $= \frac{20}{60} \Rightarrow \frac{1}{3}$ h

$$\text{Distance} = \text{speed} \times \text{time}$$
$$= 7.5 \times \frac{1}{3} \Rightarrow 2.5 \text{ km}$$

a. average velocity $= \frac{\text{displacement}}{\text{time}} \Rightarrow 0$ km/h

average speed $= \frac{\text{distance}}{\text{time}} \Rightarrow \frac{5 \text{ km}}{(\frac{50}{60})\text{h}} \Rightarrow \frac{5 \times 60}{50} \Rightarrow 6$ km/h

iii. 0 to 40 min.

man covers 2.5 km distance in 1st 30min, then he covers distance in 10 min

$$= 7.5 \times \frac{10}{60} = 1.25 \text{ km}$$

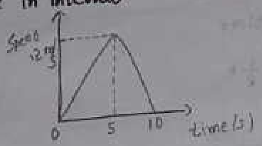
a. average velocity $= \frac{\text{displacement}}{\text{time}} \Rightarrow \frac{1.25}{(\frac{40}{60})} \Rightarrow 1.25 \times \frac{60}{40} \Rightarrow 1.875$ km/h.

average speed $= \frac{\text{distance}}{\text{time}} \Rightarrow \frac{2.5 + 1.25}{(\frac{40}{60})} \Rightarrow 3.75 \times \frac{60}{40} \Rightarrow 5.625$ km/h.

Q. Speed-time graph of a partical moving along a fixed direction is showed in figure. Calculate distance travelled by the particle & average speed of the particle in intervals

i. t=0 to 10s

ii. t=2 to 6s



Sol: i. t=0 to 10s

distance = area under v-t graph

$= \frac{1}{2} bh$

$= \frac{1}{2}(10)(12)$

$= 60m$

average speed = $\frac{\text{total distance}}{\text{time}}$

$\Rightarrow \frac{60}{10} = 6m/s$

ii. t = 2 to 6s

b/w 0 to 5s

initial speed $u = 0$

final speed $v = 12m/s$

$v = u + at$

$12 = 0 + a(5)$

$a = \frac{12}{5} \Rightarrow 2.4 m/s^2$

from t=0 to 2s

$v = u + at$

$v = 0 + (2.4)(2)$

$v = 4.8 m/s$

from 2 to 5s

$S_1 = ut + \frac{1}{2}at^2 \Rightarrow (4.8)(3) + \frac{1}{2}(2.4)(3^2) \Rightarrow 25.2m$

direction is
le & averege

5 to 10s

$v = u + at$

$0 = 12 + a(5)$

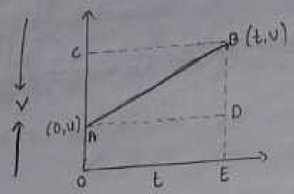
$a = \frac{-12}{5}$

from 5 to 6s

$s = ut + \frac{1}{2}at^2$

=

i. $v=u+at$ {ii}. $s=ut+\frac{1}{2}at^2$ {iii}. $v^2-u^2=2as$



acceleration = slope of v-t graph AB

$$a = \frac{DB}{AD} \Rightarrow \frac{BD}{OE} = \frac{EB-ED}{OE}$$

$$a = \frac{v-u}{t}$$

$$at = v-u$$

$$\boxed{v = u+at}$$

from part-1, where

$$a=\frac{BD}{AD} = \frac{BD}{t} \Rightarrow BD=at$$

Distance travelled by object in time "t" is

s = Area of trapezium OABE

= Area of rectangle OADE + Area of Δ^{le} ABD

$= OA\times OE + \frac{1}{2}\times BD\times AD$

$= ut + \frac{1}{2}\times at\times t$

$= ut+\frac{1}{2}at^2$

$\therefore \boxed{s=ut+\frac{1}{2}at^2}$

Distance travelled by object in time "t" is

s = Area of trapezium OABE

$= \frac{1}{2}\times(EB+OA)\times OE$

$= \frac{1}{2}(EB+ED)\times OE \rightarrow (1)$

a = Slope of velocity - time graph AB

$$a = \frac{BD}{AD} = \frac{EB-ED}{OE} \Rightarrow OE = \frac{EB-ED}{a}$$

$$S = \frac{1}{2}(EB+ED)\left(\frac{EB-ED}{a}\right) \quad \{\text{from eq - ①}\}$$

$$= \frac{1}{2}\frac{(EB^2-ED^2)}{a}$$

$$S = \frac{v^2-u^2}{2a}$$

$$\boxed{v^2-u^2 = 2as}$$

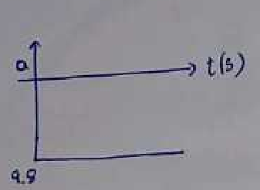
Free fall: An object released near the surface of the earth is accelerated downward under the influence of the force of gravity.
{if air resistance is neglected}.

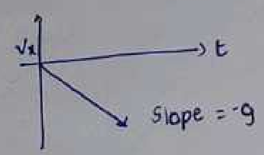
$s = ut + \frac{1}{2}at^2 \Rightarrow s = -\frac{1}{2}gt^2$ $\quad u = 0,\ a = -g \Rightarrow -9.8$

$y = mx^2$

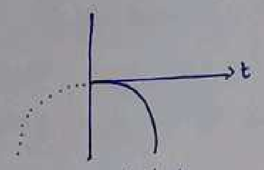
$v = u + at \Rightarrow v = -gt \Rightarrow y = -mx$

$v^2 - u^2 = 2as \Rightarrow v^2 = -2gs$





If body is dropped from height 'H' u=0



neglected

$S = ut + \frac{1}{2}at^2$

$S = \frac{1}{2}(g)t^2$

$S = -\frac{1}{2}gt^2$

Note: If body is dropped from height "H" $u = 0$

* Vertically downward direction taken as "-ve" then $S = \frac{1}{2}gt^2$

$u = 0,\ a = -g,\ s = -H$ $\quad H = -S$, $S = -H$

Time of flight: The sum of the time of acent and 0 time of decent

* Total time for which the body remains in air.

$$s = ut + \frac{1}{2}at^2$$

$$-H = 0 - \frac{1}{2}gt^2$$

$$\Rightarrow \frac{2H}{g} = T^2$$

$$\boxed{T = \sqrt{2H/g}}$$

Final velocity: $u = 0,\ a = -g,\ T = \sqrt{2H/g},\ v = ?$

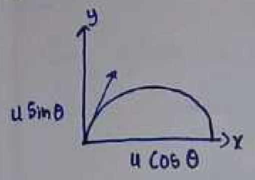
$v^2 - u^2 = 2as \Rightarrow \boxed{V = \sqrt{2gH}}$

$v^2 - 0 = +2gH$

PROJECTILE MOTION:

Any body thrown into space with some initial velocity moves there after under the influence of gravitational force only.

Ex: Cricket ball, Javeling throw by a athelete.



Horizontal component of initial velocity $u_x = u\cos\theta$

vertical component of initial velocity $u_y = u\sin\theta$

along x-axis — along y-axis

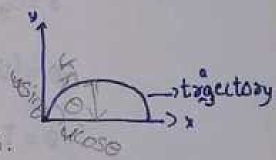
moving with uniform velocity — moving with uniform acceleration

$u_x = u\cos\theta$ — $u_y = u\sin\theta$

$a_x = 0$ — $a_y = -g$

Equation of trajectory of a projectile:-

path followed by projectile is called trajectory.



x- direction

$s = ut + \frac{1}{2}at^2$

$x = u_x t + \frac{1}{2}(a_x)t^2$

$x = (u\cos\theta)t \longrightarrow ①$

y- direction.

$s = ut + \frac{1}{2}at^2$

$y = u_y t + \frac{1}{2}a_y t^2$

$y = (u\sin\theta)t - \frac{gt^2}{2} \longrightarrow ②$

from ① $t = \frac{x}{u\cos\theta}$ substitute in ②

$$y = (u\sin\theta)\frac{x}{u\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2 \quad ③$$

Similar to $y = ax - bx^2$, where a,b are constants.

$\Rightarrow$ y is a quadratic equation.

$\Rightarrow$ Trajectory of a projectile is a parabola.

Maximum height: It is the maximum vertical distance attained by the projectile,

at maximum height $v_y = 0,\ u_y = u\sin\theta,\ a_y = -g$

$s \Rightarrow y = H_{max}$

$v_y^2 - u_y^2 = 2a_y s$

$0 - (u\sin\theta)^2 = -2g H_{max}$

$$\boxed{H_{max} = \frac{u^2 \sin^2\theta}{2g}}$$

Time of flight: It is the total time for which the projectile remains in its air.

$y = s = 0,\quad u_y = u\sin\theta,\quad a_y = -g,\ t = T$

$s = u_y t + \frac{1}{2} a_y t^2$

$0 = (u\sin\theta)T - \frac{1}{2} g T^2$

$0 = T\left[u\sin\theta - \frac{1}{2} g t\right]$

$0 = u\sin\theta - \frac{1}{2} g T$

$\frac{1}{2} g T = u\sin\theta$

$$\boxed{T = \frac{2u\sin\theta}{g}}$$

Horizontal Range: It is the horizontal distance covered by the projectile during its time of flight.

$s = x = R,\ u_x = u\cos\theta,\ t = T = \frac{2u\sin\theta}{g},\ a_x = 0$

$x = u_x t + \frac{1}{2} a_x t^2$

$R = (u\cos\theta)\, t + 0$

$= (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right)$

$$= \frac{u^2\, 2\sin\theta\cos\theta}{g}$$ {∵ Where $2\sin\theta\cos\theta = \sin 2\theta$}

$$\therefore \boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Time of asecent : {Ta}

Time taken to reach maximum height from ground

$$t = T_a,\ V_y = 0,\ u_y = U\sin\theta,\ a_y = -g$$

$$V_y = U_y + a_y t$$

$$\Rightarrow 0 = U\sin\theta - gt$$

$$gt = U\sin\theta$$

$$\boxed{T_a = \frac{U\sin\theta}{g}}$$

Time of decent: {Td}

Time taken to reach ground from maximum height

$$t = T_d,\ U_y = 0,\ a_y = -g,\ S = -H_{max}$$

$$S = U_y t + \frac{1}{2} a_y t^2$$

$$-H_{max} = -\frac{1}{2} g t^2$$

$$T^2 = \frac{2H}{g}$$

$$T^2 = \frac{2}{g} \times \frac{u^2 \sin^2\theta}{2g}$$

$$T = \sqrt{\frac{u^2 \sin^2\theta}{g^2}}$$

$$T_d = \frac{U\sin\theta}{g}$$

Time of flight $T = T_a + T_d = \frac{U\sin\theta}{g} + \frac{U\sin\theta}{g}$

$$\Rightarrow \frac{2U\sin\theta}{g}$$

Velocity of projectile at any instant:

Horizontal component $V_x = U\cos\theta$

vertical component $V_y = U_y + a_y t$

$= U\sin\theta - gt$

$\therefore$ Velocity of avg instant $= V = \sqrt{V_y^2 + V_x^2}$

$= \sqrt{(u\cos\theta)^2 + (U\sin\theta - gt)^2}$

In vector form $u\cos\theta\,\hat{i} + (U\sin\theta - gt)\hat{j}$

let velocity makes "α" angle with x-axis then

$$\boxed{\tan\alpha = \frac{U\sin\theta - gt}{u\cos\theta}}$$

Velocity of projectile at the end point:

{or} final velocity {or} impact velocity.

In this case $t = T \Rightarrow$ Time of Height $= \frac{2u\sin\theta}{g}$

$V_x = U\cos\theta,\ V_y = U\sin\theta - gt$

$= U\sin\theta - g\left(\frac{2u\sin\theta}{g}\right)$

$= U\sin\theta - 2u\sin\theta$

$= -U\sin\theta$

$V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta}$

$= \sqrt{u^2(\cos^2\theta + \sin^2\theta}$

$= \sqrt{u^2(1)} \Rightarrow u$

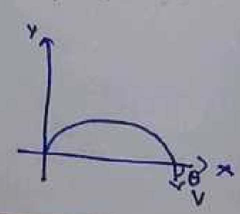
$\boxed{V = u}$ final speed = initial speed

$\tan\alpha = \frac{V_y}{V_x} = \frac{-U\sin\theta}{U\cos\theta}$

$\tan\alpha = -\tan\theta$

$\tan\alpha = \tan(-\theta)$

$\boxed{\alpha = -\theta}$



Condition for maximum Range:

$$R = \frac{u^2 \sin 2\theta}{g}$$

Clearly R will be maximum

$\sin 2\theta = 1 \Rightarrow \sin 2\theta = \sin 90^\circ$

$2\theta = 90^\circ$

$\theta = 45^\circ$

$$\boxed{R_{max} = \frac{u^2}{g}}$$

Two angles of projectile for the same horizontal range:

$$R = \frac{u^2 \sin^2 \theta}{g}$$

Replacing θ to $(90-\theta)$

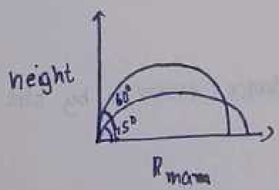
$$R' = \frac{u^2 \sin 2(90-\theta)}{g}$$

$$R' = \frac{u^2 \sin (180-2\theta)}{g}$$

$$R' = \frac{u^2 \sin 2\theta}{g}$$

$$R' = R$$

for a given velocity projectile has same horizontal range for angle projectile θ & $90-\theta$

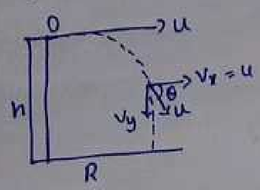


Horizontal projectile:

Suppose a body is projected horizontally with a velocity 'u' from point O at a certain height above the ground level. It can be treated as combined effect of 2 independent motions.

$\Rightarrow$ Uniform horizontal velocity 'u'.

=> Vertically downward accelerated (uniform) motion.



x-direction

$u_x = u$

$a_x = 0$

y-direction

$u_y = 0$

$a_y = -g$

Trajectory of the projectile:

x-direction

$x = u_x t + \frac{1}{2} a_x t^2$

$\boxed{x = ut}$

$t = \frac{x}{u}$

y-direction

$y = u_y t + \frac{1}{2} a_y t^2$

$\boxed{y = \left(\frac{-g}{2u^2}\right) x^2}$

$y = -Kx^2$

Parabola.

Time of flight: It is the total time for which the projectile remaining in air.

$y = -h,\ a_y = -g,\ u_y = 0,\ t = T$

$y = u_y t + \frac{1}{2} a_y t^2$

$-h = 0 - \frac{1}{2} g t^2$

$T = \sqrt{\frac{2h}{g}}$

Horizontal Range: It is the horizontal distance covered by the projectile during its time of flight.

$x = u_x t + \frac{1}{2} a_x t^2$

$x = R = ut$

$R = u\sqrt{\frac{2h}{g}}$

Velocity of particle at any instant: along x-direction $v_x = u$

y-direction $v_y = u_y + a_y t$

$v_y = -gt$

In vector notation,

$$\bar{v} = u\hat{i} - gt\hat{j}$$

Magnitude of $\bar{v} = \sqrt{u^2+g^2t^2}$

angle "α" is the angle between velocity horizontal

$$\tan\alpha = \frac{v_y}{v_x} \Rightarrow \frac{-gt}{u}$$

Velocity of projectile at end point:

x-direction $\quad v_x = u$

y-direction $\quad v_y = u_y + a_y t$

$$v_y = 0 - gt$$

$$v_y = -g\sqrt{\frac{2h}{g}}$$

$$v_y = -\sqrt{2gh}$$

$$\boxed{v = \sqrt{u^2+2hg}}$$

LAWS OF MOTION

FORCE : Force may be defined as an agency or a push or pull which changes or tends to change the state of rest or of uniform motion or the direction of motion of a body.

*. Force can change speed of an body.

*. force can change direction of motion of an object.

*. force can change the shape of an object.

INERTIA AND ITS DIFFERENT TYPES:

*. INERTIA: The inherent property of a material body by virtue of which it can't change by itself, its state of rest or of uniform motion in a straight line is called inertia. {Inertia means resistance to change}.

DIFFERENT TYPES OF INERTIA:

i. INERTIA OF REST : The tendency of a body to remain in its position of rest is called Inertia of rest.

Ex: A person standing in a bus falls backword when the bus suddenly starts moving forward.

ii. INERTIA OF MOTION: The tendency of a body to remain in its state of uniform motion in a straight line is called inertia of motion.

Ex: When a moving bus suddenly stops, a person sitting in its falls forward.

iii. INERTIA OF DIRECTION: The inability of a body to change by itself, its direction of motion is called inertia of direction.

Ex: When a bus takes a sharp turn, a person sitting in the bus experiences a force acting away from the centre of the curved path due to his tendency to move in the original direction.

MASS AS THE MEASURE OF INERTIA : Mass of a body is the measure of its inertia.

*. MOMENTUM : It is equal to product of mass and velocity of the body.

Momentum = Mass x Velocity

$$\boxed{\vec{P} = m\vec{v}}$$

S.I. units of momentum = Kg m/s

C.G.S. units of momentum = g.cm/s

The dimensional formula of momentum is $[MLT^{-1}]$

*. NEWTON'S LAWS OF MOTION:

→ FIRST LAW : Every body continues in its state of rest or of uniform motion in a straight line unless it is compiled by some external force to change its state.

→ SECOND LAW : The rate of change of linear momentum of body is directly proportional to the applied force and change take place in the direction of the applied force.

Third LAW :- To every action, there is always an equal and opposite reaction.

*. ILLUSTRATIONS OF NEWTON'S FIRST LAW OF MOTION:

A. Based on inertia of rest.

i. When a horse suddenly starts running, the raider falls backward.

ii. Dust is removed from a hanging carpet by beating it with a stick.

iii. When we shake the branch of tree, its fruits and dry leaves fall down.

iv. Coin falls into the tumbler when the card is given a sudden jerk.

B. Based on inertia of motion.

i. when a horse running fast and suddenly stops, the raider is thrown forward.

ii. A person getting out of a moving bus or train falls into the forward direction.

iii. An athelete runs for a certain distance before taking a long jump.

iv. A fireman in a railway engine quickly moves his coal shovel near the furnace and then suddenly stops it. The shovel comes to rest but the coal continues moving due to inertia and falls into fusmance.

v. A ball thrown upward in a moving train comes back into the thrower's hands.

C. Based on inertia of direction.

i. During the sharpening of a knife, the sparkes coming from the grind stone fly off tangentially to the rim of the rotating stone.

ii. When a vehicle moves, the mud sticking to its wheels flies off tangentially.

iii. When a dog chases a hare, the hare runs along a zig-zag path. It becomes difficult for the dog to catch the hare. This is because the dog has more mass and hence has more inertia of direction that of hare.

*. MEASUREMENT OF FORCE FROM NEWTON'S SECOND LAW OF MOTION:

If a body of mass "m" is moving with velocity $\vec{v}$, then its linear momentum is

$$\vec{P} = m\vec{v}$$

Differentiating both sides with respect to time t, we get

$$\frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) \Rightarrow m\frac{d\vec{v}}{dt} \Rightarrow m\vec{a}$$

where $\vec{a}$ is the acceleration produced in the body.

According to Newton's Second Law,

Applied force $\propto$ Rate of change of momentum.

$$\vec{F} \propto \frac{d\vec{P}}{dt}$$

$$\vec{F} \propto m\vec{a}$$

$$\boxed{\vec{F} = K m\vec{a}}$$

S.I. units = Newton

C.G.S. units = Dyne.

Dimensional formula of force is $[MLT^{-2}]$.

GRAVITATIONAL UNITS OF FORCE: A gravitational unit of force is that force which produces an acceleration equal to g (acceleration due to gravity) in a body of unit mass. It may also be defined as the weight of body of unit mass.

In S.I. the gravitational unit of force is kilogram weight (kg wt) or kilogram force (kgf). It is defined as that force which produces an acceleration of 9.8 m/s² in a body of mass 1kg.

In C.G.S. the gravitational unit of force is gram weight (g wt) or gram force (gf). It is defined as that force which produces an acceleration of 980 cm/s² in a body of mass 1 g.

$$\boxed{\begin{array}{l} 1\ kg\ wt = 1\ kgf = 9.8\ N \\ 1\ g\ wt = 1\ gf = 980\ dyne \end{array}}$$

Q. Find the no. of dyne in 1 Newton.

$1N = 1\,kg \times 1\,m/s^2$

$= 1000\,g \times 100\,cm/s^2$

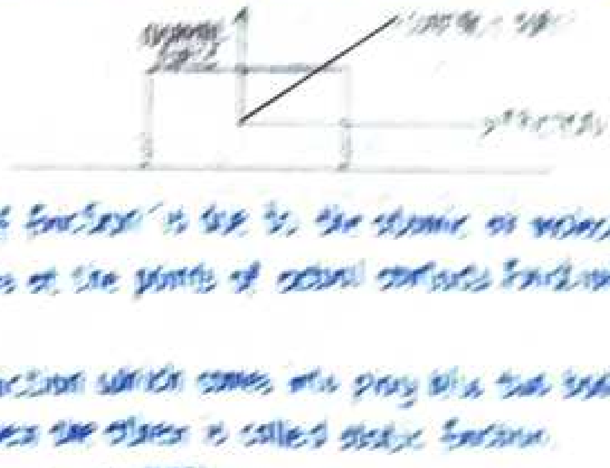
$= 10^5\,g\,cm/s^2$

FRICTION

FRICTION: Whenever a body moves or tends to move over the surface of another body, a force comes into play which acts parallel to the surface of contact and opposes the relative motion. This opposing force is called friction.

FRICTION AS THE COMPONENT OF CONTACT FORCE:

The component of a contact force F normal to the contact surface is called normal force or normal reaction N. The component parallel to the contact surface is called friction.



ORIGIN OF FRICTION: The force of friction is due to the atomic or molecular force of attraction b/w the two surfaces at the points of actual contact. Friction is due to surface irregularities.

STATIC FRICTION: The force of friction which comes into play b/w two bodies before one body actually starts moving over the other is called static friction.

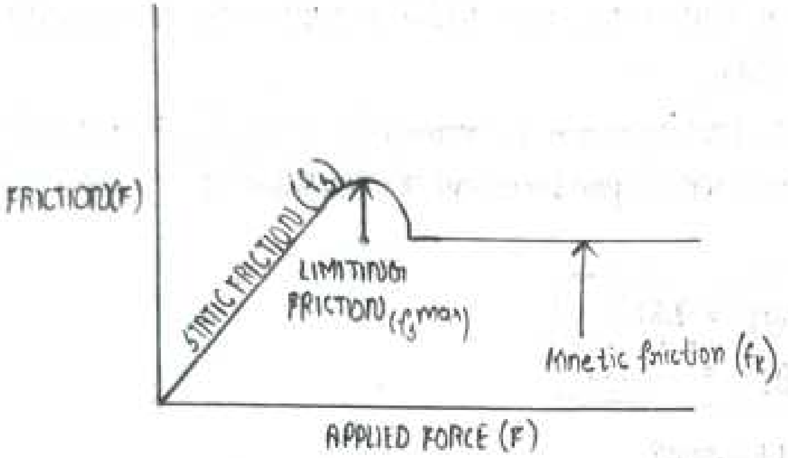
The maximum force of static friction (f_s^{max}) which comes into play when a body just starts moving over the surface of another body is called limiting friction. Clearly $f_s \le f_s^{max}$

Once the motion has begun, the force of friction decreases.

KINETIC FRICTION: The force of friction which comes into play when a body is in a state of steady motion over the surface of another body is called kinetic friction.

friction (f_K). When $F = f_K$, the body moves with constant velocity V. The kinetic friction opposes the actual relative motion, when the applied force is faster than the kinetic friction, the block accelerates with acceleration equal to $(F - f_K)/m$.

→ Static friction is a self adjusting friction.



*. LAWS OF LIMITING FRICTION:

i. The limiting friction depends on the nature of the surface in contact and their state of polish.

ii. The limiting friction acts tangential to the two surfaces in contact and in a direction opposite to the direction of motion of the body.

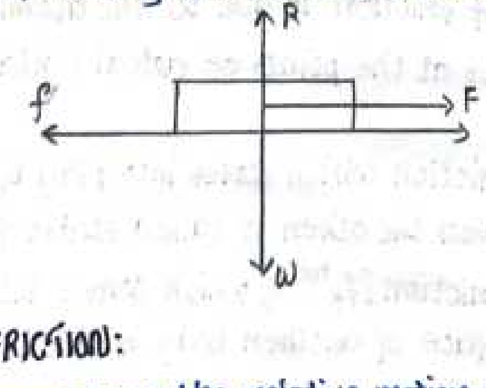
iii. The value of limiting friction is independent of the area of the surface in contact so long as the normal reaction remains the same.

iv. The limiting friction (f_s^{max}) is directly proportional to the normal reaction R b/w the two surfaces.

i.e., $f_s^{max} \propto R$ {or} $f_s^{max} = \mu_K R$

$$\mu_K = \frac{f_s^{max}}{R} = \frac{\text{Limiting friction}}{\text{Normal reaction}}.$$

The proportionality constant μ_K is called co-efficient of static friction. It is defined as the ratio of limiting friction to the normal reaction.



*. LAWS OF KINETIC FRICTION:

i. The kinetic friction opposes the relative motion and has a constant value depending on the nature of the two surfaces in contact.

ii. The value of kinetic friction f_K is independent of the area of contact so long as the normal reactions remains the same.

iii. The kinetic friction does not depend on velocity, provided the velocity is neither too large nor too small.

iv. The value of kinetic friction f_K is directly proportional to the normal reaction R b/w the two surfaces, i.e., $\rightarrow f_K \propto R$ (or) $f_K = \mu_K R$

$$\mu_K = \frac{f_K}{R} = \frac{\text{Kinetic friction}}{\text{Normal Reaction}}.$$

The proportional constant μ_K is called co-efficient of kinetic friction. It is defined as the ratio of kinetic friction and normal reaction.

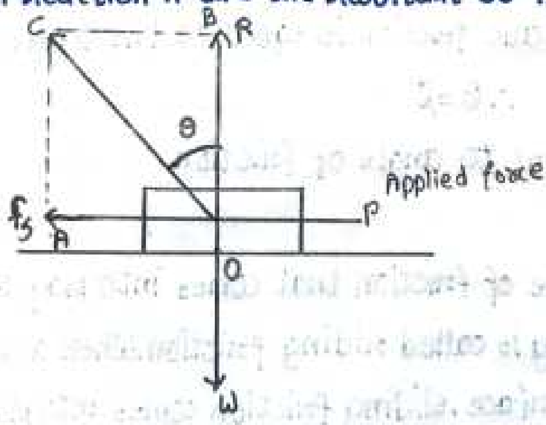
$$\text{As } f_K < f_s^{max} \text{ (or) } \mu_K R < \mu_s R \quad \therefore \boxed{\mu_K < \mu_s}$$

Thus co-efficient of kinetic friction is less than the co-efficient of static friction.

*ANGLE OF FRICTION: The angle of friction may be defined as the angle which the resultant of limiting friction and the normal reaction make with the normal reaction.

*RELATION B/w ANGLE OF FRICTION AND CO-EFFICIENT OF FRICTION:

In the figure, W is the weight of body, R is the normal reaction, f_s^{max} is the limiting friction, P is the applied force and $\vec{OC}$ is the resultant of f_s^{max} and R. The angle θ b/w the normal reaction R and the resultant $\vec{OC}$ is, by definition, the angle of friction.



Angle of friction,

$$\therefore \tan\theta = \frac{BC}{OB} = \frac{OA}{OB} = \frac{f_s^{max}}{R}$$

But $\frac{f_s^{max}}{R} = \mu_s$ = Co-efficient of static friction.

$$\therefore \tan\theta = \mu_s$$

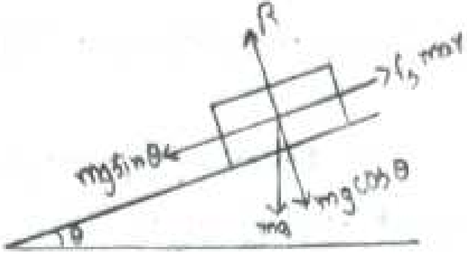
Thus the co-efficient of static friction is equal to the tangent of angle of friction.

*. ANGLE OF REPOSE: It is the minimum angle that an inclined plane makes with the horizontal when a body placed on it just begins to slide down

*. RELATION B/w ANGLE OF REPOSE AND CO-EFFICIENT OF FRICTION:

In the figure, consider a body of mass 'm' placed on an inclined plane. The angle of inclination of the inclined plane is so adjusted that a body placed on it just begins to slide down.

Thus, ø is angle of ~~respon~~ repose



Various forces acting on the body are:-

i. Weight mg of the body acting vertically downward.

ii. The limiting friction f_s^{max} in upward direction along the inclined plane. It balances the component $mg\sin\phi$ of the weight mg along the inclined plane. Thus,

$$\boxed{f_s^{max} = mg\sin\phi.}$$

iii. The normal reaction R perpendicular to the inclined plane. It balances the component $mg\cos\theta$ of weight mg perpendicular to the inclined plane.

Thus, $\boxed{R = mg\cos\phi}$

Dividing equation ① & ②, we get

$$\frac{f_s^{max}}{R} = \frac{mg\sin\phi}{mg\cos\phi} \Rightarrow \mu_s = \tan\phi.$$

Thus, the co-efficient of static friction is equal to the tangent of angle of repose.

As, $\mu_s = \tan\theta = \tan\phi \quad \therefore \theta = \phi$

Thus, angle of repose is equal to angle of friction.

*. TYPES OF KINETIC FRICTION:

i. SLIDING FRICTION: The force of friction that comes into play when a body slides over the surface of another body is called sliding friction. When a wooden block is pulled or pushed over a horizontal surface, sliding friction comes into play.

ii. ROLLING FRICTION: The force of friction that comes into play when a body rolls over the surface of another body is called rolling friction. When a wheel rolls over a road, rolling friction comes into play.

∴ Rolling friction is always much smaller than the sliding friction.

CAUSE OF ROLLING FRICTION: Consider a wheel rolling along a road, as the wheel rolls, it exerts a large pressure (weight/area) due to its small area. This cause a slight depression of the road below and a small elevation or mount in front of it. In addition to this the rolling wheel has to continuously detach itself from the surface which it rolls.

*. LAWS OF ROLLING FRICTION:

i. Rolling friction is directly proportional to the normal reaction, i.e., $f_r \propto R$.

ii. Rolling friction is inversely proportional to the radius of the rolling cylinder or wheel, i.e., $f_r \propto \frac{1}{r}$

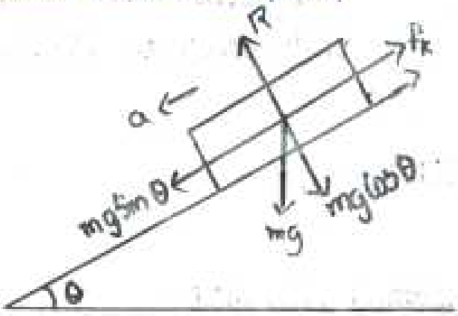
Combining the two laws, we get

$$f_r \propto \frac{R}{r} \quad \text{or} \quad \boxed{f_r = \mu_r \frac{R}{r}}$$

Here, μ_r is the co-efficient of rolling friction, unlike μ_s and μ_k {which is pure ratio and has no dimensions}, μ_r has the dimensions of length and its S.I. unit is meter. The above equation is applicable only when there is rolling without slipping.

* ~~RELATION~~ ACCELERATION OF A BODY SLIDING DOWN A ROUGH INCLINED PLANE:

From the figure, consider a body of weight mg placed on an inclined plane. Suppose the angle of inclination θ be greater than the angle of repose. Let "a" be the acceleration with which the body slides down the inclined plane



The weight mg has two rectangular components.

i. mg cos θ perpendicular to the inclined plane. It balances the normal reaction R.

Thus, $\boxed{R = mg\cos\theta}$

ii. mg sin θ down the inclined plane.

If f_k is the kinetic friction, then the net force acting down the plane is

$$\boxed{F = mg\sin\theta - f_k}$$

But, $f_k = \mu_k R = \mu_k mg\cos\theta$

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$

$$\boxed{a = g(\sin\theta - \mu_k\cos\theta)}$$

FRICTION IS A NECESSARY EVIL:

FRICTION IS NECESSARY:-
{ADVANTAGES}

1. It is due to friction b/w the ground and the feet that we are able to walk.
ii. The brakes of a vehicle cannot work without friction.
iii. Various parts of a machine are able to rotate because of friction b/w belt and pulley.
iv. The types of vehicle are made rough to increase friction.
v. Nails and screws joins various ~~wod~~ wooden parts due to friction.
vi. In the absence of friction, it will not be possible to write on a paper with a pen or pencil.

FRICTION IS AN EVIL:-{DISADVANTAGES}

i. Wear and tear of machinery is due to friction.

ii. A large amount of power is wasted in over coming friction and the efficiency of the machine decreases considerably.

iii. Excessive friction b/w rotating parts of a machine produces enough heat and causes damage to the machinery.

METHODS OF REDUCING FRICTION:

i. By polishing
ii. Lubrication
iii. Streamlining
iv. By using ball-bearing
v. By using anti-friction alloys.
vi. By using air cushion.

METHODS OF INCREASING FRICTION:

i. Treading of tyres.
ii. Sand is thrown on tracks covered with snow.
iii. On a rainy day, we throw some sand on the slippery ground.

PULLING A LAWN ROLLER IS EASIER THAN TO PUSH IT:

It is easier to pull a body than to push it from the figure. Suppose a force F is applied to pull a lawn roller of weight ω. The force F has two rectangular components

i. Horizontal component, $F\cos\theta$ helps to move the roller forward.

ii. Vertical component $F\sin\theta$ acts in the upward direction.

If R is the normal reaction, then,

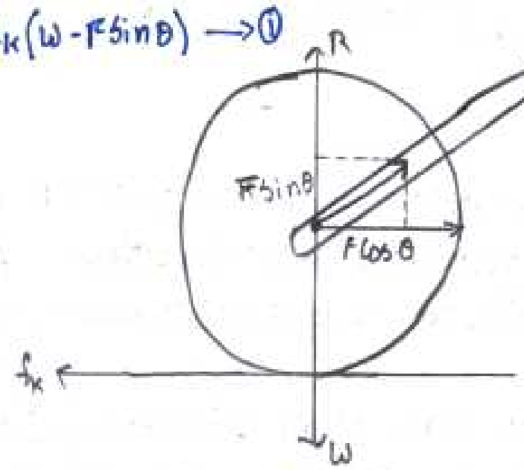
$R + F\sin\theta = \omega$ {Equating the vertical components}

$R = \omega - F\sin\theta$

{or}

force of kinetic friction,

$f_k = \mu_k R = \mu_k(\omega - F\sin\theta) \longrightarrow ①$



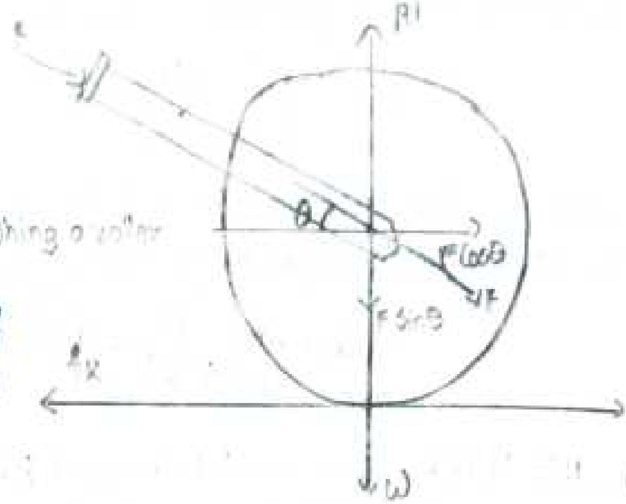
Pulling a roller

from the figure, if a force F is applied to push a roller of weight ω, then the normal reaction is

$R' = \omega + F\sin\theta$

force of Kinetic friction.

$$f_K = \mu_K R' = \mu_K(W + F\sin\theta) \rightarrow ②$$



Comparing ① and ②, we find that $f_K > f_K$
i.e., the force of friction is more in case of push than in case of pull. So, it is easier to pull a body than to push it.

UNIFORM CIRCULAR MOTION:-

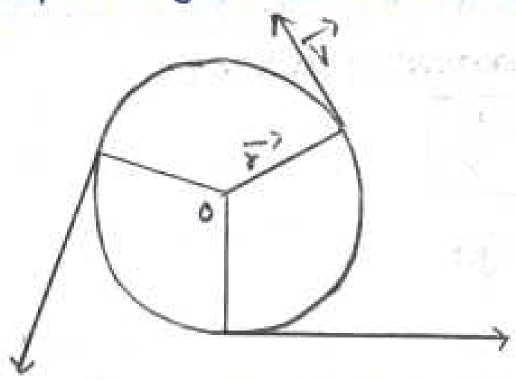
If a particle moves along a circular path with a constant speed {i.e. at covers equal distances along the circumference of the circle in equal intervals of time} then its motion is said to be in a uniform circular motion.

Ex: 1. Motion of the tip of the second hand of a clock.

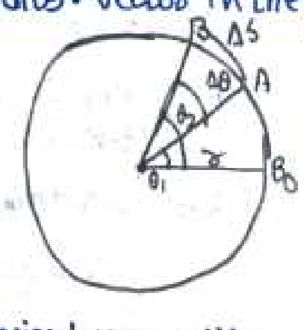
2. Motion of the point on the rim of a wheel rotating uniformly.

UNIFORM CIRCULAR MOTION IS AN ACCELERATED MOTION:

In uniform circular motion, the speed of the body remains the same but the direction of motion changes at every point. Thus the velocity of the body changes continuously due to the continuous change in the direction of motion of the body. As the rate of change of velocity is acceleration, so a uniform circular motion is an accelerated motion.



i. ANGULAR DISPLACEMENT: The angular displacement of a particle ~~moving~~ moving along a circular path is defined as the angle swept out by its radius vector in the given time interval.



$$\Delta\theta = \frac{\Delta s}{r}$$

$$\left[\because \text{Angle} = \frac{\text{Arc}}{\text{Radius}}\right]$$

The unit of angular displacement is radian. It is dimensionless quantity.

ii. ANGULAR VELOCITY: The time rate of change of angular displacement of a possible particle is called its angular velocity. It is denoted by ω. It is measured in radian per second (rad/s) and its dimensional formula is $[M^0 L^0 T^{-1}]$. If Δθ is the angular displacement of a particle is time Δt, then its average angular velocity is

$$\boxed{\vec{\omega} = \frac{\Delta\theta}{\Delta t}}$$

When the time interval $\Delta t \to 0$, the limiting value of the average velocity is called the instantaneous angular velocity, which is given by.

$$\boxed{\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}}$$

iii. **TIME PERIOD**: The time taken by a particle to complete one revolution along its circular path is called its period of revolution. It is denoted by T and is measured in seconds.

iv. **FREQUENCY**: The frequency of an object in circular motion is defined as the no. of revolutions completed per unit time. It is denoted by ν (nu).

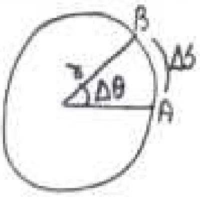
If ν is the frequency of revolution of a particle, then time taken to complete ν revolutions = 1 second, then time taken to complete 1 revolution $= \frac{1}{\nu}$ second. But time taken to complete 1 revolution is the time period, T so

$$\boxed{T = \frac{1}{\nu} \text{ or } \nu = \frac{1}{T}}$$

RELATION B/w LINEAR VELOCITY AND ANGULAR VELOCITY:

Consider a particle moving along a circular path of radius r from the fig. suppose the particle moves from A to B in time Δt covering distance Δs along the area B. Hence the angular displacement of particle is

$$\boxed{\Delta\theta = \frac{\Delta s}{r}}$$



Dividing both sides by Δt, we get

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$$

Taking the limit $\Delta t \to 0$ on both sides,

$$\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{1}{r} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

But, $\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{ds}{dt} = V$

is the instantaneous linear velocity,

$$\omega = \frac{1}{r} \cdot V$$

$$\boxed{V = \omega r}$$

linear velocity = Angular velocity × radius.

In vector notation, we have the relation

$$\vec{v} = \vec{\omega} \times \vec{r}$$

For a given angular velocity (ω), the linear velocity v of a particle is directly proportional to its distance from the centre.

RELATION B/W ω, V and T :-

By the definition of time period, a particle completes one revolution in time T, i.e., it traverse an angle of 2π radian in Time T.

When time $t = T$

angular displacement $\theta = 2\pi$ radians.

$$\text{Angular velocity} = \frac{\text{Angular displacement}}{\text{Time}}$$

(or) $$\boxed{\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu} \quad \left\{\because \frac{1}{T} = \nu\right\}$$

ANGULAR ACCELERATION: The time rate of change of angular velocity of a particle is called its angular acceleration. If $\Delta\omega$ is change in angular velocity in time Δt, then the average angular acceleration is

$$\vec{\alpha} = \frac{\Delta\omega}{\Delta t}$$

The instantaneous acceleration is equal to the limiting value of the average acceleration $\Delta\omega/\Delta t$ when Δt approaches zero. It is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The angular acceleration is measured in radian per second^2 (rad/s^2) and has the dimensions $[M^0L^0T^{-2}]$.

The relation b/w linear velocity V and the angular velocity is $\boxed{V = r\omega}$

Differentiating both sides w.r.t time t, we get

$$\frac{dV}{dt} = \frac{d}{dt}(r\omega)$$

$$\frac{dV}{dt} = r\left(\frac{d\omega}{dt}\right)$$

$$\boxed{a = \alpha r}$$

Linear acceleration = Angular acceleration x radius.

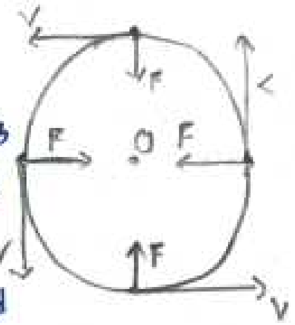
In vector notation, we have the relation

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

CENTRIPETAL ACCELERATION: When a body is in uniform circular motion, its speed remains constant but its velocity changes continuously due to the change in its direction. Hence the motion is accelerated. A body undergoing uniform circular motion is acted upon by an acceleration which is directed along radius towards the centre of the circular path. This acceleration is called centripetal acceleration {centre seeking}.

CENTRIPETAL FORCE: A force required to make a body move along a circular path with uniform speed is called centripetal force. It always acts along the radius and towards the centre of the circular path.

From the diagram, such a force continuously deflets a particle from its straight line path to make it move along a circle.



EXPRESSION FOR CENTRIPETAL FORCE:

when a body is in uniform circular motion, its velocity changes continuously due to change in the direction of motion. Hence it undergoes an acceleration which acts radially inwards. It is called Centripetal acceleration and is given by

$$\boxed{a = \frac{v^2}{r} = r\omega^2}$$

where v and ω are the linear and angular speeds. of the body and r is the radius of the circular path. According to Newton's 2nd law, the centripetal force required to move a body of mass m along a circular path of radius r is given by,

F = mass × Centripetal acceleration

$$\boxed{F = \frac{mv^2}{r} = mr\omega^2}$$

Ex:- 1. For a stone rotated in circle, the tension in the string provides the centripetal force.

ii. The centripetal force for the motion of the planet around the sun is provided by gravitational force exerted by the sun on the planet.

FORMULAE:

1. For a body moving along a horizontal circular path, centripetal force is

$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi\nu)^2 = mr\left(\frac{2\pi}{T}\right)^2$$

2. Centripetal force is equal to centripetal force in magnitude but acts away from the centre.

Units: Force F in newton, velocity v in m/s, angular frequency ω in rad/s.

CIRCULAR MOTION OF A CAR ON A LEVEL ROAD:

When a car negotiates a curved level road, the force of friction b/w the road and the tyres provides the centripetal force required to keep the car in motion around the curve.

Consider a car weight mg going around a circular level road of radius 'r' with constant speed v.

forces acting on car are,

weight of car $= \omega = mg$

normal reaction $= R$

frictional force $= f_s$

along y-direction :- $F_{net} = 0$

$$R - W = 0 \Rightarrow R = \omega$$

$$\boxed{R = mg}$$

along x-direction :- frictional force acts as centripetal force

We know that $f_s = \mu_s R$

$$\boxed{f_s = \mu_s mg}$$

for the car to stay on the road,

$$f_s \geq \frac{mv^2}{r}$$

$$\mu_s mg \geq \frac{mv^2}{r}$$

$$\frac{\mu_s g r m}{m} \geq v^2$$

$$v^2 \leq \mu r g$$

$$v \leq \sqrt{\mu_s r g}$$

$\therefore$ The maximum speed with when the car turn safely is $V_{max} = \sqrt{\mu_s r g}$

$\therefore V_{max}$ depends on r & μ_s.

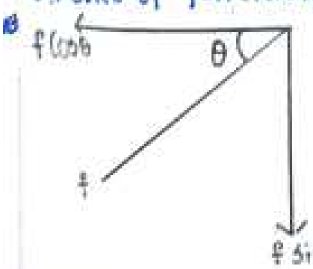
CIRCULAR MOTION OF A CAR ON A BANKED ROAD:

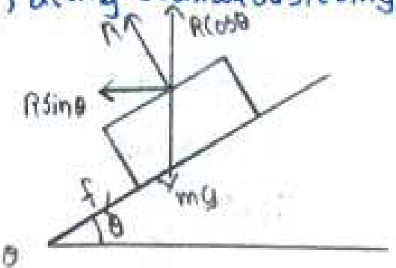
The system of raising the outer edge of a curved road above its inner edge is called banking of the curved road. The angle through which the outer edge of the curved road is raised above the inner edge is called angle of banking. From the figure, consider a car of weight mg going along a curved path of radius r with speed v on a road banked at an angle θ. The forces acting on the vehicle are

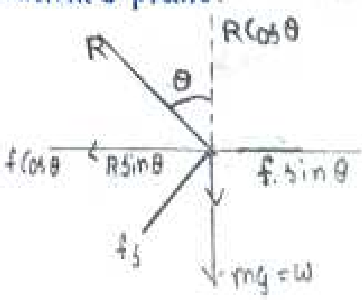
1. Weight mg acting vertically downwards.

2. Normal reaction R of the road acting at an angle θ with the vertical.

3. Force of friction f acting downwards, doing the inclined plane.







along y-direction:

$$F_{net} = 0$$

$$R\cos\theta - mg - f\sin\theta = 0$$

$$\boxed{R\cos\theta - f\sin\theta = mg} \rightarrow (1)$$

along x-direction

$$F_{net} = \frac{mv^2}{r}$$

$$\boxed{R\sin\theta + f_s\cos\theta = \frac{mv^2}{r}} \rightarrow (2)$$

$$\frac{(2)}{(1)} = \frac{R\sin\theta + f\cos\theta}{R\cos\theta - f\sin\theta} = \frac{\left(\frac{mv^2}{r}\right)}{mg}$$

divide with $R\cos\theta$

$$\frac{\frac{R\sin\theta}{R\cos\theta} + \frac{f\cos\theta}{R\cos\theta}}{\frac{R\cos\theta}{R\cos\theta} - \frac{f\sin\theta}{R\cos\theta}} = \frac{v^2}{rg}$$

$$\frac{\tan\theta + \frac{f_s}{R}}{1 - \frac{f_s}{R}\tan\theta} = \frac{v^2}{rg} \qquad \{\because f = \mu_g\}$$

$$\frac{\tan\theta + \frac{\mu R}{R}}{1 - \frac{\mu R}{R}\tan\theta} = \frac{v^2}{rg} \Rightarrow \frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta} = \frac{v^2}{rg}$$

$$v^2 = rg\left[\frac{\tan\theta + \mu_s}{1 - \mu_s\tan\theta}\right]$$

$$v = \sqrt{rg\left(\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right)}$$

SPECIAL CASE: When there is no friction b/w the road and the tyres $\mu = 0$

$$\Rightarrow v = \sqrt{rg\tan\theta}$$

→ The angle of banking θ for minimum, wear and tear of tyre is given by,

$$\tan\theta = \frac{v^2}{rg} \qquad\qquad v = \sqrt{rg\tan\theta}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \qquad\qquad v^2 = rg\tan\theta$$

$$\tan\theta = \frac{v^2}{rg}$$

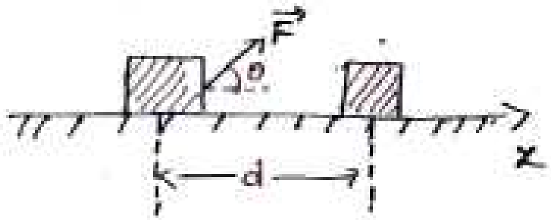
Chapter 6

WORK ENERGY AND POWER

1. WORK :-

An object can be displaced or set in to motion by the application of force on it. If a constant force of magnitude 'F' acts on a body and produces a displacement 'd' in the positive x-direction as shown in fig

The work done by the force is defined to be the product of component of the force in the direction of the displacement and magnitude of this displacement.



An object undergoes a displacement d under the influence of force

$$W = F \times d$$

Force and displacement are vectors. The displacement need not be always along the direction of the force. If they are in different directions, then teir scalar product is defined as work. If 'θ' is the angle between them.

$$W = \vec{F} \cdot \vec{d} = FS\cos\theta$$

No work is done if

* the displacement is zero, there is no work done even if the force is large

* The force is zero. A block moving on a smooth horizontal table is not acted up on by a horizontal force (since there is not friction), but may undergo a large displacement.

* If $\theta = 90^\circ$, then $Fd\cos\theta = 0$. It means that no work has been done.

For Ex: consider a person who is walking horizontally with a box on his head. Now the work done by the gravitational force on the box is zero However, the box possesses displacement in the horizontal direction. This work is done by the force of his muscles.

Units of work:

Joule is the unit of work in S.I. system one joule 'J' of work is said to be done by a force of 1 N If the force moves an object through a distance of 1m along its direction.

$$1J = 1N \times 1m = 1Nm$$

Dimensional formula for work is $[ML^2T^{-2}]$

2. Energy:

Energy is defined as the 'capacity' to do a work. Energy is measured basing on the amount of work done by a force acting on a body. As the work done on the body is equal to its energy, accordingly the unit of energy is "Joule".

Kinetic Energy:-

Kinetic energy is defined as the energy possessed by a body by virtue of its motion. If an object of mass 'm' has velocity 'V', its kinetic energy K is

$$k = \frac{1}{2} mv \cdot v = \frac{1}{2} mv^2$$

Kinetic energy is a scalar quantity.

For ex: a moving car possesses kinetic energy, An object falling from a height has maximum kinetic energy before it touches the ground. Soon after it strikes the ground that energy is transformed in to heat energy and a small portion of the energy may be transformed into sound energy.

3. NOTIONS OF WORK AND KINETIC ENERGY: THE WORK - ENERGY THEOREM :-

The following relation for rectilinear motion under constant acceleration.

Let us consider a body of mass 'm', which is moving with an initial velocity 'u'. If a constant force 'F' acts on it in the direction of motion, the body acquires an acceleration 'a' and velocity of the body increases to 'v' in 't' seconds. In this process let the displacement of the body be 'd'

Acceleration $a = \frac{v-u}{t}$ $(v = u + at)$

displacement d = average velocity × time

$$= \left(\frac{v+u}{2}\right) t$$

Work done 'W' by the force 'F' on the body

$$= Fd = mad \quad \text{(From Newton's second law)}$$

$$= m\left(\frac{v-u}{2}\right)t\left(\frac{v+u}{t}\right)$$

$$= \frac{m}{2}\left(v^2 - u^2\right)$$

$$\boxed{W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2}$$

Work done by the net force on the body = final kinetic energy - initial kinetic energy

From the above equation it is evident that the work done by an unbalanced or resultant external force on a body is equal to the change in its kinetic energy.

This is called "work energy theorem"

Problem:- It is well known that a raindrop falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop of mass 1.00g falling from a height 1.00 km. It hits the ground with a speed of $50.0\,ms^{-1}$

a) What is the work done by the gravitational force?

b) What is the work done by the unknown resistive force?

Sol :-

a] The change in kinetic energy of the drop is

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Here drop of mass $m = 1.00g$

The speed of water drop when it hits the ground $V = 50.0 ms^{-1}$

Water drop falling from a height $h = 1.00 km$

(its a freely falling body so $u=0$).

$$\therefore \Delta K = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 10^{-3} \times 50 \times 50$$

$$= 1.25 J$$

The work done by the gravitational force

$$W = mgh$$

$$= 10^{-3} \times 10 \times 10^3 = 10.0 J$$

b] From work-energy theorem

$$\Delta K = W_g + W_r$$

where W_r is the work done by the resistive force on the raindrop.

Thus $W_r = \Delta K - W_g$

$$= 1.25 - 10$$

$$W_r = -8.75 J$$

Problem: A cyclist comes to skidding stop in 10m. During this process, the force on the cycle due to the road is 200N and is directly opposed to the motion. a] How much work does the road do on the cycle b] How much work does the cycle do on the road?

Sol:- work done on the cycle by the road is the work done by the stopping force (friction) on the cycle due to the road

a] The stopping force and the displacement make an angle of 180° with each other

Thus, work done by the road.

$$W_r = Fd\cos\theta$$

Here $F = 200N$

$d = 10m$

$$W_r = 200 \times 10 \times \cos 180^\circ$$

$$= -2000J$$

It is the negative work that brings the cycle to a halt in accordance with WE theorem

b] From Newton's Third law an equal and opposite force acts on the road due to the cycle. Its magnitude is 200N. However, the road undergoes no displacement. Thus, workdone by cycle on the road is zero.

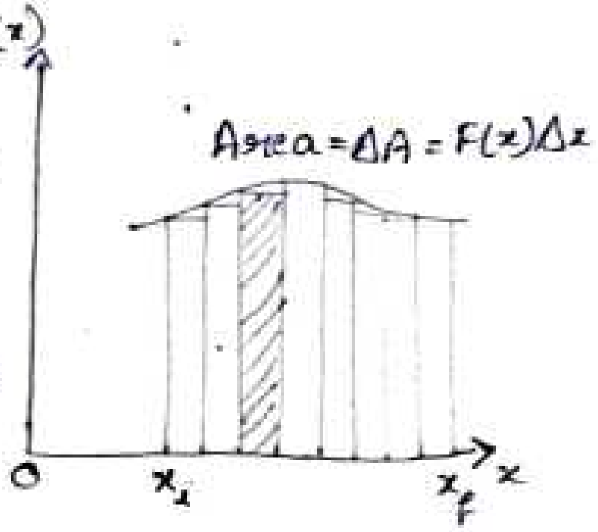
4. Work done By a Variable Force:-

Workdone by a constant force depends only on the initial and final positions

Workdone by a constant force $F(x)$ for a small displacement Δx is $\Delta W = F(x)\Delta x$.

Adding successive rectangular areas in fig below, we get the total work done as

$$W = \sum_{x_i}^{x_f} F(x)\,\Delta x$$



If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit, but the sum approaches a definite value equal to the area under the curve in fig. Then workdone is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\,\Delta x$$

$$\boxed{W = \int_{x_i}^{x_f} F(x)\,dx}$$

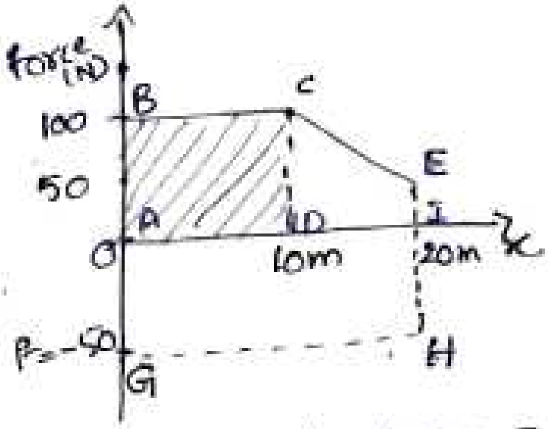
Thus for a variable force the work-done can be expressed as a definite integral of force over displacement.

problem: A woman pushes a trunk on a railway platform which has a rough surface she applies a force of 100N over a distance of 10m. There after, she gets progressively tired and her applied force reduces linearly with distance to 50N. The total distance through which the trunk has been moved is 20m. plot a force applied by the woman and the frictional force, which is 50N versus displacement. Calculate the workdone by the two forces over 20m.

sol:

The work done by the woman is

$W_F \rightarrow$ area of the rectangle ABCD + area of the trapezium CEID



plot of the force F applied by the woman and the opposing frictional force f versus displacement

$$W_F = 100 \times 10 + \frac{1}{2}(100+50) \times 10$$

$$= 1000 + 750$$

$$= 1750 J$$

The work done by the ~~fo~~ frictional force is

$W_f \rightarrow$ area of the rectangle AGHI

$$W_f = (-50) \times 20m$$

$$= -1000 J$$

5. The WORK - ENERGY THEOREM FOR A VARIABLE FORCE:

The time rate of change of kinetic energy is $\frac{dk}{dt} = \frac{d}{dt}(\frac{1}{2}mv^2)$

$$= \frac{1}{2} 2m v \frac{dv}{dt}$$

$$= m\frac{dv}{dt} v$$

From Newton's second law i.e $F = ma$

$$\frac{dk}{dt} = FV$$

Thus $\frac{dk}{dt} = F\frac{dx}{dt}$

$$dk = F\,dx$$

Integrating from the initial position (x_i) to final position (x_f), we have.

$$\int_{k_i}^{k_f} dk = \int_{x_i}^{x_f} F\,dx$$

$$[k]_{k_i}^{k_f} = \int_{x_i}^{x_f} F\,dx$$

$$\therefore W = k_f - k_i$$

Thus, the work Energy theorem is proved for a variable force.

6 POTENTIAL ENERGY :-

The word potential suggests capacity for action.

potential energy is defined as the energy possessed by a body by virtue of the position or state.

The gravitational force on a ball of mass m is mg. Let us raise the ball up to a height h. The work done by the external agency against the gravitational force is mgh. This work gets stored as potential energy.

Gravitational potential energy of an object, as a function of the height h, is denoted by v(h). Negative work done by the gravitational force in raising the object to that height.

$$\boxed{V(h) = mgh}$$

If h is taken as variable

$$F = -\frac{d}{dh} v(h) = -mg$$

The negative sign, indicates that the gravitational force is downward.

The velocity or speed of the ball is given by the kinematic relation.

$$v^2 - u^2 = 2gh$$

$$v^2 = 2gh$$

$$\frac{1}{2}mv^2 = mgh$$

when the object is released itself as kinetic energy of the object on reaching the ground.

The potential energy $V(x)$ is defined if the force $F(x)$ can be written as

$$F(x) = -\frac{dv}{dx}$$

$$\Rightarrow \int_{x_i}^{x_f} F(x)\, dx = -\int_{v_i}^{v_f} dv = v_i - v_f$$

The work done by a conservative force such as gravity depends on the initial and final positions only.

The dimensions of potential energy are $[ML^2T^{-2}]$ and the unit is Joule (J).

7 **The Conservation of Mechanical Energy :-**

Suppose that a body undergoes displacement Δx under the action of a conservative force F. Then from work energy theorem we have,

$$\Delta k = F(x)\,\Delta x \rightarrow ①$$

If the force is conservative, the potential energy function $V(x)$ can be defined such that

$$-\Delta V = F(x)\,\Delta x \rightarrow ②$$

From above ① & ② eq's we get

$$\Delta k + \Delta V = F(x)\,\Delta x - F(x)\,\Delta x$$

$$\Delta k + \Delta v = 0$$

The sum of the kinetic and potential energies of the body is constant.

* A Force $F(x)$ is conservative if it can be derived from a scalar quantity $V(x)$ by the relation given by $\Delta V = -F(x)\,\Delta x$.
* The work done by the conservative force depends only on the end points. This can be seen from the relation

$$W = k_f - k_i = V(x_i) - V(x_f).$$

* A third definition states that the work done by this force in a closed path is zero.

Thus the principle of conservation of total mechanical energy can be stated as

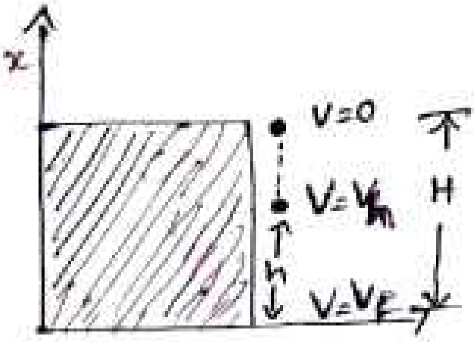
"The total mechanical energy of a system is conserved if the forces, doing work on it, are

Conservative

* According to the law of conservation of energy, the total energy of an isolated system does not change. Energy may be transformed from one form to another but the total energy of an isolated system remains constant.

Energy can neither be created, nor destroyed.

Derivation:-



The conservation of potential energy to kinetic energy for a ball of mass 'm' dropped from a height H.

The total mechanical energies E_0, E_h and E_H of the ball at the indicated heights zero, h and H are

$$E_H = mgH$$
$$E_h = mgh + \frac{1}{2}mV_h^2$$
$$E_0 = (\frac{1}{2})mV_f^2$$

The mechanical energy is conserved, Thus

$$E_H = E_0$$
$$mgH = \frac{1}{2}mV_f^2 \Rightarrow V_f = \sqrt{2gH}$$

For a freely falling body

$$E_H = E_h \Rightarrow mgH = mgh + \frac{1}{2}mV_h^2$$
$$\Rightarrow V_h^2 = 2g(H-h)$$

At a height H, the energy is purely potential. It is partially converted to kinetic at height h and is fully kinetic at ground level. This illustrates the conservation of Mechanical energy.

problem: A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity V_0 at the lowest point A. Such that it completes a semi circular trajectory in the vertical plane with string becoming slack only on reaching the top most point, C. This is shown in fig. obtain expression for 1] V_0 2] the speeds at points B and C 3] The ratio of the kinetic energies (K_B/K_C) at B and C comment on the nature of trajectory of the bob after it reaches the point C.

[Motion of a body in vertical plane:]

Sol: 1] There are two external forces on the bob: gravity and tension (T) in the string.

The potential energy of the bob is thus associated with the gravitational force only.

The potential energy of the system to be zero at the lowest point A. Thus at A K.E of the bob

$$E = \frac{1}{2}mv_0^2 \longrightarrow ①$$

$$T_A - mg = \frac{mv_0^2}{L} \longrightarrow ② \text{ (Newton's second Law) (From centrifugal force).}$$

At the highest point C the string slackens then

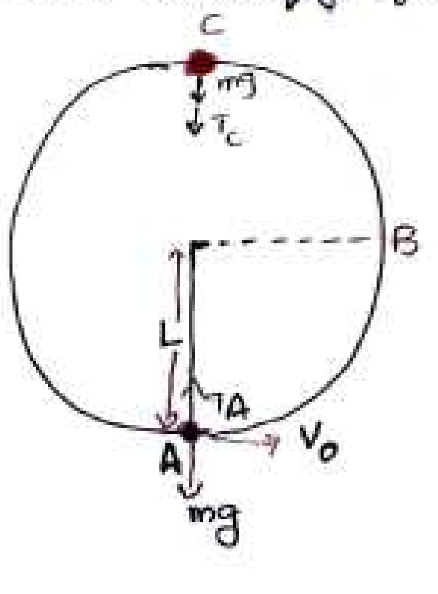
$T_C = 0$

Thus, at C

$$E = \frac{1}{2}mv_c^2 + 2mgL \longrightarrow ③$$

$$mg = \frac{mv_c^2}{L} \longrightarrow ④$$

where v_c is the speed at C



From eqn's ③ and ④

We get $E = \frac{1}{2} mgL + 2mgL$

$$E = \frac{5}{2} mgL$$

Equating this to the energy at A

$$\frac{5}{2} mgL = \frac{m}{2} V_0^2$$

or $v_0 = \sqrt{5gL}$

2] From eqn ④

$$mg = \frac{mv_c^2}{L}$$

$$v_c = \sqrt{gL}$$

At B, the energy is

$$E = \frac{1}{2} mv_B^2 + mgL$$

Equating this to the energy at A

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} mv_0^2$$

We know $v_0^2 = 5gL$

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} \times 5mgL$$

$$\frac{1}{2} mv_B^2 = \frac{5}{2} mgL - mgL$$

$$v_B^2 = 3gL$$

3] $\frac{K_B}{K_C} = \frac{\frac{1}{2} mv_B^2}{\frac{1}{2} mv_c^2} = \frac{3gL}{gL} = \underline{\underline{3}}$

8 The potential energy of a spring:

The spring force is an example of a variable force which is conservative. fig shows a block attached to a spring and resting on a smooth horizontal surface. The other end of the spring is attached to a rigid wall. The spring is light and may be treated as massless. The spring force F_s is proportional to x, where x is the displacement of the block from the equilibrium position. This force law for the spring is called Hooke's law

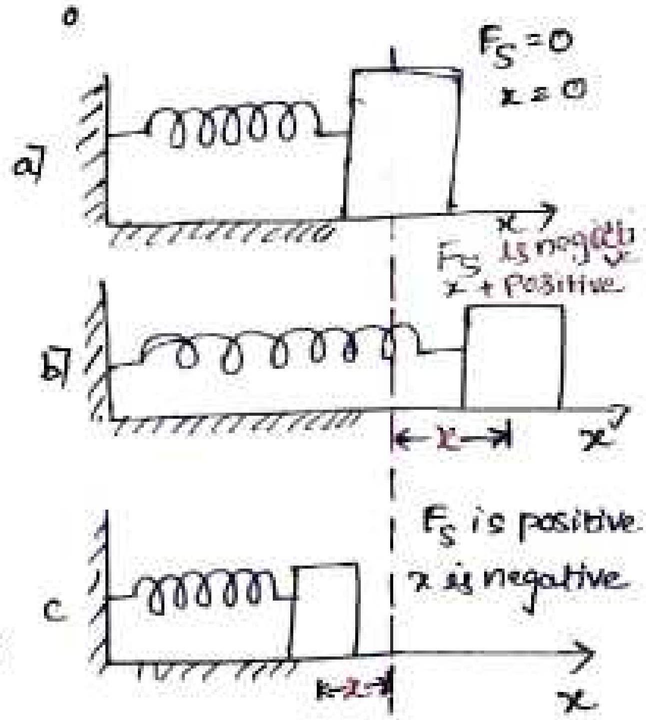
$$F_s = -kx$$

The constant k is called the spring constant. Its unit is Nm^{-1}

If the extension is x_m, the work done by the spring force is

$$W_s = \int_0^{x_m} F_s \, dx = -\int_0^{x_m} kx \, dx$$

$$W_s = -\frac{kx_m^2}{2}$$



The work done by the external pulling force F is positive since it overcomes the spring force

$$W = +\frac{kx_m^2}{2}$$

The same is true when the spring is compressed with a displacement x_c

The spring force does work

$$W_s = \frac{-kx_i^2}{2}$$

If the block is moved from an initial displacement x_i to final displacement x_f, the work done by the spring force W_s is

$$W_s = -\int_{x_i}^{x_f} kx\,dx = \frac{kx_i^2}{2} - \frac{kx_f^2}{2}.$$

If the block is pulled from x_i and allowed to return to x_i:

$$W_s = -\int_{x_i}^{x_i} kx\,dx = \frac{kx_i^2}{2} - \frac{kx_i^2}{2} = 0$$

$$W_s = 0$$

The work done by the spring force in a cyclic process is zero.

The work done by the spring force depends on position ($F_s = -kx$) and depends on the initial and final positions. Thus the spring force is a "conservative force"

We define the potential energy $V(x)$ of the spring to be zero when block and spring system is in the equilibrium position

For an extension (or compression) x the potential energy of the spring

$$V(x) = \frac{kx^2}{2}$$

The total mechanical energy at point x, where x lie's between $-x_m$ and $+x_m$ will be given by

$$\frac{1}{2}kx_m^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

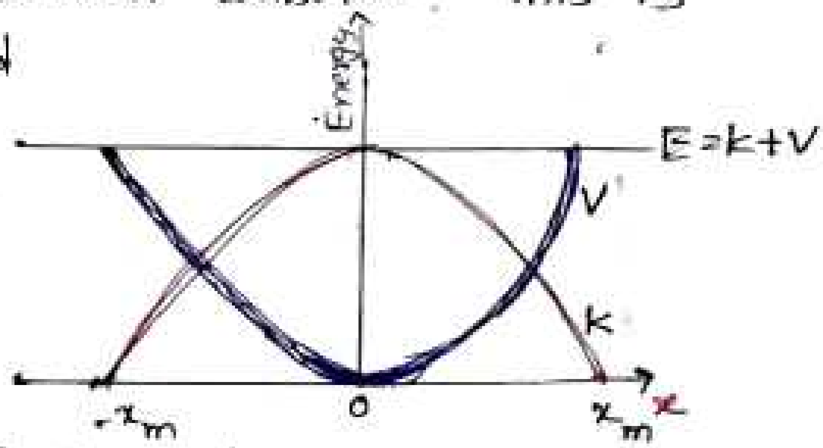
This suggests that the speed and the kinetic energy will be maximum at the equilibrium position, $x=0$, ie,

$$\frac{1}{2}mv_m^2 = \frac{1}{2}kx_m^2$$

Where v_m is the maximum speed

or $v_m = \sqrt{\frac{k}{m}}\, x_m$

The kinetic energy get converted in to potential energy and vice versa, the total mechanical energy remain constant. This is graphically depicted



parabolic plots of potential energy V and kinetic energy k of a block attached to a spring obeying Hooke's law. The two plots are complementary, one decreasing as the other increases. The total mechanical energy $E = k+V$ remains constant.

9 Various Forms OF ENERGY: The Law of Conservation OF ENERGY:-

Mechanical energy classified in to two distinct categories: one based on motion, namely kinetic energy; the other on configuration (position), namely potential energy. Energy comes in many form which transform in to one another.

1. **Heat :**

The frictional force is not a conservative force. However work is associated with the force of friction. The kinetic energy of the block lost due to frictional force. on examine the block and table we would detect a slight increase in temperatures. The work done by friction is not lost, but is transferred as heat energy. This raises the internal energy of the block and the table.

2. **Chemical energy :-**

Chemical energy arises from the fact that the molecules participating in the chemical reaction have different binding energies. chemical energy is associated with the forces that gives rise to stability of substances. These forces bind atoms in to molecules, molecules in to polymeric chain. The chemical energy arises from cooking gas, wood, combustion of coal etc...

3. **Electrical energy:**

The flow of electrical current causes bulbs to glow, fans to rotate and bells to ring. Energy is associated with an electric current called Electrical energy.

4. The Equivalence of Mass and Energy:-

Albert Einstein showed that mass and energy are equivalent are are related by the relation

$$\boxed{E = mc^2}$$

Where c is the speed of light in vaccum is approximately $3\times10^8 ms^{-1}$. The mass can be transformed in to energy and vice-versa.

5. Nuclear Energy:-

The most destructive weapons made by man, the fission and fusion bombs are manifestations of the equivalence of mass and energy. The energy ΔE released in a chemical reaction related to the mass defect $\Delta m = \Delta E/c^2$ for a chemical reaction, this mass defect is much smaller than for a nuclear reaction.

10 POWER:-

power is defined as the time rate at which work is done or energy is transferred.

The average power of a force is defined as the ratio of the work, W. to the total time t taken

$$\boxed{P_{av} = \frac{W}{t}}$$

The instantaneous power is defined as the limiting value of the average power as time interval approaches zero.

$$P = \frac{dw}{dt}$$

The work dw done by a force F for a displacement dr is $dw = F.dr$. The instantaneous power can also be expressed as

$$P = F.\frac{dr}{dt} = F.V$$

where v is the instantaneous velocity when the force is F.

power is a scalar. Watt (W) is the unit of power in SI system and it is 1 Watt, if 1 Joule of work is done in 1 second

$$\therefore 1W = 1Js^{-1}$$

Watt is a small unit and therefore to measure the larger quantities of power kilo-watt (KW) and Mega watt (MW) are used.

$$1\,KW = 10^3 W \; ; \quad 1\,MW = 10^6 W$$

The power of an electric motor some times a unit called "Horse-power" is used.

$$1\,hp = 746W$$

our electricity bills carry the energy consumption in units of KWh.

problem:- An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of $2ms^{-1}$. The friction force opposing the motion is 4000N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horse power

Sol: The downward force on the elevator is $F = mg + F_f$

Here $m = 1800\,kg$ $F_f = 4000\,N$

$$F = (1800 \times 10) + 4000 = 22000\,N$$

The motor must supply enough power to balance this force, Hence.

$$P = F\,v$$

Speed $v = 2\,ms^{-1}$

$$P = 22000 \times 2 = 44000\,W = 59\,hp. \quad (1\,watt = \frac{1}{746}\,hp)$$

11 COLLISIONS:

Two objects moving in the same directions, or in opposite directions, are said to collide if they hit each other. In the process of collision, the directions of motion of the colliding bodies changes abruptly due to mutual interaction between the bodies

Consider two masses m_1 and m_2. The particle m_1 is moving with speed v_i, the subscript 'i' initial. We can consider m_2 to be at rest. The mass m_1 collides with the stationary mass m_2 and this is depicted in below fig

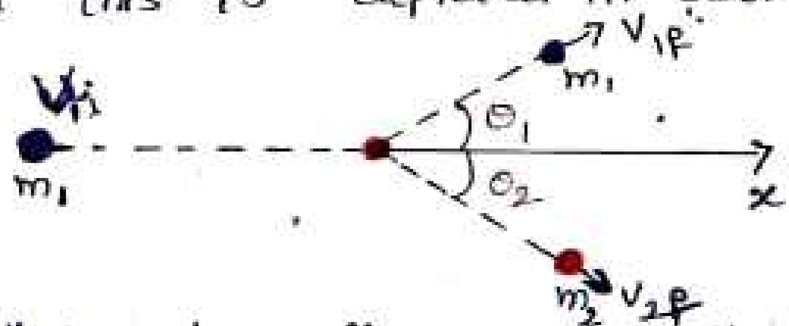


fig a Collision of mass m_1 with stationary mass m_2

The masses m_1 and m_2 fly-off in different directions.

i) Elastic and Inelastic collision :-

In all collisions the total linear momentum is conserved ; the initial momentum of the system is equal to the final momentum of the system.

When two objects collide, the mutual impulsive forces acting over the collision time Δt causes a change in their respective momenta.

$\Delta P_1 = F_{12}\Delta t$ (F_{12} is the force exerted on the first particle by the second particle)

$\Delta P_2 = F_{21}\Delta t$

Now from Newton's third law,

$$F_{12} = -F_{21}$$

$$\Rightarrow \Delta P_1 + \Delta P_2 = 0$$

The kinetic energy of the system is not necessarily conserved. The impact and deformation during collision may generate heat and sound. Part of the initial kinetic energy is transformed in to other forms of energy.

The collisions are classified in to two types

i) Elastic collision :

In this type of collision, the total kinetic energy of the system after collision is equal to the total kinetic energy of the system before collision.

$$= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2} \frac{m_1^2}{m_1+m_2} v_{1i}^2 \quad \text{(using equ ①)}$$

$$= \frac{1}{2} m_1 v_{1i}^2 \left[1 - \frac{m_1}{m_1+m_2}\right]$$

$$= \frac{1}{2} m_1 v_{1i}^2 \left[\frac{\cancel{m_1}+m_2-\cancel{m_1}}{m_1+m_2}\right]$$

$$\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1+m_2} v_{1i}^2$$

Consider next an elastic collision. using $\theta_1 = \theta_2 = 0$, the momentum and kinetic energy conservation equations are

Total momentum before collision = Total momentum after collision

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \rightarrow ②$$

Total initial kinetic energy = Total final kinetic energy

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2 \rightarrow ③$$

From equ ② and equ ③, we get

From equ ② $\quad m_2 v_{2f} = m_1 v_{1i} - m_1 v_{1f} \rightarrow ④$

subs equ ④ in equ ③, we get.

$$m_1 v_{1i}^2 = m_1 v_{1f}^2 + (m_1 v_{1i} - m_1 v_{1f}) v_{2f}$$

$$m_1 v_{1i}^2 - m_1 v_{1i} v_{2f} = m_1 v_{1f}^2 - m_1 v_{1f} v_{2f}$$

$$m_1 v_{1i} (v_{1i} - v_{2f}) = m_1 v_{1f} (v_{1f} - v_{2f})$$

$$m_2 v_{1i}(v_{2f} - v_{1i}) = m_1 v_{1f}(v_{2f} - v_{1f})$$

$$v_{1i} v_{2f} - v_{1i}^2 = v_{1f} v_{2f} - v_1^2$$

$$v_{1i} v_{2f} - v_{1f} v_{2f} = -v_{1f}^2 + v_{1i}^2$$

$$v_{2f}(v_{1i} - v_{1f}) = v_{1i}^2 - v_{1f}^2$$

$$v_{2f}\cancel{(v_{1i} - v_{1f})} = (v_{1i} + v_{1f})\cancel{(v_{1i} - v_{1f})}$$

$$\therefore\ v_{2f} = v_{1i} + v_{1f} \rightarrow (5)$$

Substitute equ(5) in equ(2) we obtain

$$\boxed{v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} v_{1i}}$$

Substitute v_{1f} in equ (5), we get

$$v_{2f} = v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$$

$$v_{2f} = v_{1i}\left[1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$= v_{1i}\left[\frac{m_1 + \cancel{m_2} + m_1 - \cancel{m_2}}{m_1 + m_2}\right]$$

$$\boxed{v_{2f} = \frac{2 m_1 v_{1i}}{m_1 + m_2}}$$

Thus, the unknown's (v_{1f}, v_{2f}) are obtained in terms of the knowns (m_1, m_2, v_{1i})

Case:1: If two masses are equal $m_1 = m_2$

$$V_{1f} = \frac{(m_1 - m_2)}{m_2 + m_2} v_{1i} = 0$$

$$V_{2f} = \frac{2m_2 V_{1i}}{m_2 + m_2} = \frac{2m_2 V_{1i}}{2m_2}$$

$$V_{2f} = V_{1i}$$

The first mass comes to rest and pushes out the second mass with its initial speed on collision.

Case 2: If one mass dominates eg $m_2 >> m_1$

$$V_{1f} \simeq -V_{1i} \qquad V_{2f} \simeq 0$$

The heavier mass is undisturbed while the lighter mass reverses its velocity.

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a one-dimensional collision or head on collision.

13 <u>Collisions in Two Dimension</u> :-

The above fig also depicts the collision of a moving mass m_1 with the stationary mass m_2. Linear momentum is conserved in such a collision. Since momentum is a vector this implies three equations for the three directions (x, y, z).

Consider a plane determined by the final velocity directions of m_1 and m_2 and choose it to be the x-y plane. The conservation of the z - component of the linear momentum implies that the entire collision is in x-y plane The x and y component equations are

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \rightarrow (1)$$

$$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2 \rightarrow (2)$$

If $\theta_1 = \theta_2 = 0$, the above equations become

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

If, further the collision is elastic

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

At least one of the four unknowns (v_{1f}, v_{2f}, θ_1 an θ_2) say θ_1 must be made known for the problem to be solvable.

Coefficient of Restitution or Coefficient of Resilience:

coefficient of restitution is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision It is represented by 'e'

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

Alternative units of work / Energy in J

erg - 10^{-7} J

electron volt eV - 1.6×10^{-19} J

Calorie cal - 4.186 J

kilowatt hour (kWh) - 3.6×10^{6} J

4 The scalar product:-

The scalar product or dot product of any two vector A and B denoted as A.B is defined as

$$A.B = AB\cos\theta$$

where θ is the angle between the two vectors shown in fig

$$A.B = A(B\cos\theta)$$
$$= B(A\cos\theta)$$

Geometrically $B\cos\theta$ is the projection of B on to A and $A\cos\theta$ is the projection of A onto B So A.B is the product of the magnitude of A and component of B along A

Scalar product obeys the commutative law

$$A.B = B.A$$

scalar product obeys distributive law.

$$A.(B+C) = A.B + A.C$$

The proof of the above Equations

For unit vectors $\hat{i},\hat{j},\hat{k}$, we have

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$
$$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$$