Lecture- 1

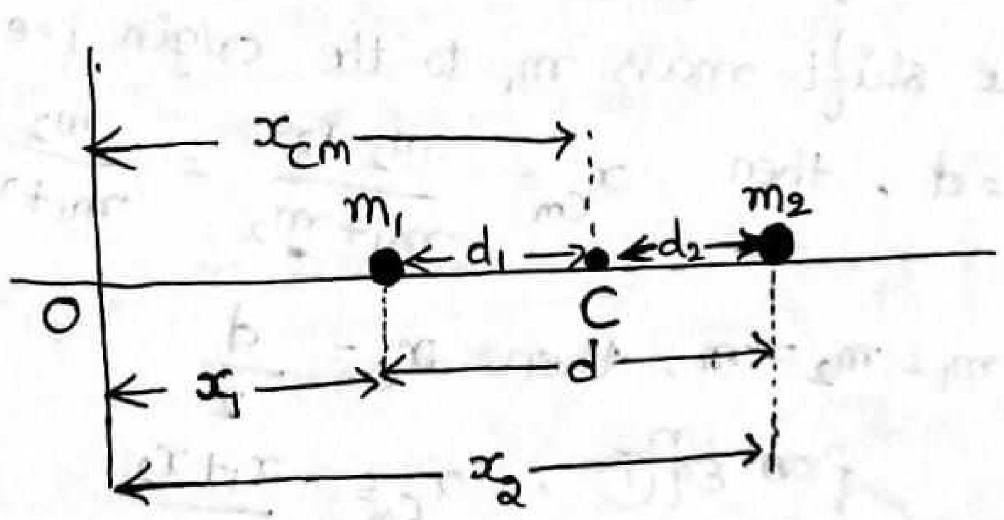
Centre of mass :-

· The centre of mass of a system (or a rigid body) is that point lying within the boundary of system where the entire mass appears to be concentrated".

centre of mass of a two particle system :-

Consider a system of two particles of mass m_1 and m_2 situated along the x-axis at positions x_1 and x_2 respectively w.r.t origin O.

The distance between the two particles is $d = x_2 - x_1$



Let the point 'C' be the centre of mass located at x_c in such a way that its distances d_1 and d_2 respectively from the first & 2nd particles are inversely proportional to their m

So, $\frac{d_1}{d_2} = \frac{m_2}{m_1}$

from the Fig $d_1 = x_{cm} - x_1$, $d_2 = x_2 - x_{cm}$

$$m_1 d_1 = m_2 d_2$$

$$m_1 [x_{cm} - x_1] = m_2 [x_2 - x_{cm}]$$

$$m_1 x_{cm} - m_1 x_1 = m_2 x_2 - m_2 x_{cm}$$

$$m_1 x_{cm} + m_2 x_{cm} = m_1 x_1 + m_2 x_2$$

$$x_{cm}[m_1 + m_2] = m_1 x_1 + m_2 x_2$$

$$\boxed{x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}} \quad \text{— (1)}$$

It is defined as the centre of mass of the system of two particles. It depends on masses of two particles and their positions.

⇒ If m_2 is zero i.e the system has only one particle with mass m_1 and its centre of mass is located at $x_c = x_1$

⇒ Similarly If m_1 is zero i.e $x_c = x_2$

⇒ If we shift mass m_1 to the origin i.e $x_1 = 0$

$x_2 = d$, then $x_{cm} = \frac{m_2 x_2}{m_1 + m_2} = \frac{m_2 d}{m_1 + m_2}$

⇒ If $m_1 = m_2 = m$, then $x_{cm} = \frac{d}{2}$

from Eqn (1), $x_{cm} = \frac{x_1 + x_2}{2}$

⇒ If $M = m_1 + m_2$, then $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{M}$

Eq^n (1) may be extended to a number of (3)
particles of masses m_1, m_2, m_3 ---- situated at
x_1, x_2, x_3 --- respectively, then the position of the
centre of mass is

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ----}{m_1 + m_2 + m_3 + ---} = \frac{\sum m_i x_i}{\sum m_i}$$

If $\sum m_i = M$, the entire mass may be assumed to be concentrated

$$\therefore x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M} = \frac{1}{M}\left[m_1x_1 + m_2x_2 + ----\right]$$

This is the centre of mass for a system of n-particles.

for a collection of n particles lying in x-y plane, the coordinates be (x_1, y_1) (x_2, y_2) --- (x_n, y_n) respectively

In three-dimensional space,

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + --- + m_nx_n}{m_1 + m_2 + --- + m_n} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + --- + m_ny_n}{m_1 + m_2 + --- + m_n} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

||ly $$z_{cm} = \frac{m_1z_1 + m_2z_2 + --- + m_nz_n}{m_1 + m_2 + --- + m_n} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

the position of the i^{th} particle (x_i, y_i, z_i)

the position vector of the centre of mass is

$$r_{cm} = \frac{m_1r_1 + m_2r_2 + -- + m_nr_n}{m_1 + m_2 + -- + m_n} = \frac{1}{M}\sum_{i=1}^{n} m_i r_i$$

where $r_c = x_c\bar{i} + y_c\bar{j} + z_c\bar{k}$

Lecture - 2

Motion of centre of mass :-

The centre of mass describes the motion of the entire system by considering that the total mass of the system is concentrated on the centre of mass and as all the external forces are acting at the centre of mass.

Let $r_1, r_2, ---$ be the position vectors of the particles of mass $m_1, m_2 ---$ respectively, forming a system. The position of the centre of mass of this system is

$$r_{cm} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2 + ---}{m_1 + m_2 + ---}$$

$$M r_{cm} = m_1\bar{r}_1 + m_2 r_2 + ---- , \quad \text{---} (1)$$

where M is the total mass of the system.

differentiating Eq^n (1) w.r.t time 't', we get

$$M \frac{dr_{cm}}{dt} = m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} + -----$$

i.e $M \bar{V}_{cm} = m_1 \bar{v}_1 + m_2 \bar{v}_2 + ---$

$$\bar{V}_{cm} = \frac{m_1 v_1 + m_2 v_2 + ---}{M} \quad \text{---} (2)$$

$$\boxed{V_{cm} = \frac{\sum_{i=1}^{n} m_i v_i}{M}}$$

where v_1, v_2, v_3 --- are the velocities of the particles at any time t. V_{cm} is the velocity of centre of mass at that time. ⑤

Again differentiating eqⁿ ②, we get

$$M \frac{d}{dt} V_{cm} = m_1 \frac{d}{dt} v_1 + m_2 \frac{d}{dt} v_2 + ----$$

As rate of change of velocity is acceleration,

$$a = \frac{dv}{dt}$$

$$M a_{cm} = m_1 a_1 + m_2 a_2 + m_3 a_3 + ----$$

Acceleration about the centre of mass, $a_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{M}$

But $m_i a_i$ is the resultant force on the i^{th} particle, so

$$M a_{cm} = \sum_{i=1}^{n} F_i$$

$$M a_{cm} = F_1 + F_2 + F_3 + ---- + F_n \quad — ③$$

F_i is the external force on i^{th} particle

where $F_{ext} = F_1 + F_2 + --- + F_n$ is net force of all external forces that act on the system.

The force on any particle may be due to internal or external agencies. By Newton's 3rd law, the action reaction pairs and they cancel out in pairs while the summation in eqⁿ ③ Hence the force on the system is only due to external forces were applied at that point.

from Eq^n (3) $F_{ext} = Ma_{cm} = \frac{dP_{cm}}{dt}$. [$\because$ Newton's 2nd law]

where 'p' is the linear momentum (6)

If external forces acting on the system sum upto zero i.e. $\sum F_{ext} = 0$, then $a_{cm} = 0$.

i.e. $F_{ext} = m \cdot \frac{dv_{cm}}{dt}$

$$0 = m\frac{dv_{cm}}{dt} \Rightarrow \frac{dv_{cm}}{dt} = 0$$

$$V_{cm} = \text{Constant}$$

velocity of the centre of mass is constant.

The individual particles may change their position, but the centre of mass remains at the same position.

Lecture - 3

centre of mass for different rigid bodies :- (7)

For a real body which is a continuous distribution of matter, point masses are then differential mass elements dm and centre of mass is defined as.

$$x_{cm} = \frac{1}{M}\int x\,dm, \quad y_{cm} = \frac{1}{M}\int y\,dm.$$

$$z_{cm} = \frac{1}{M}\int z\,dm$$

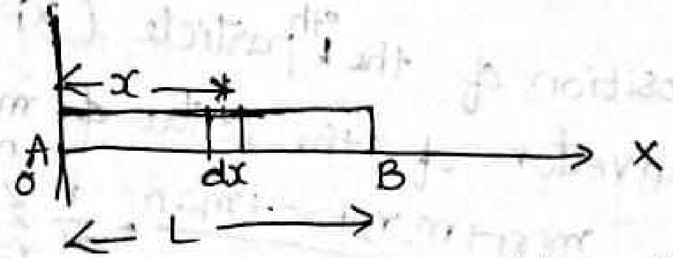
where M is total mass of that real body.

If we choose the origin of coordinates axes at centre of mass then,

$$\int x\,dm = \int y\,dm = \int z\,dm = 0.$$

centre of mass of a uniform thin Rod :-

Let us consider a uniform rod AB of mass M and length 'L'. The rod is held along x-axis with its end A at the origin where $x = y = 0$. Suppose the element is placed side by side of length 'dx' at distance 'x' from the origin



centre of mass of a uniform thin rod.

The centre of mass of uniform rod along x-axis is $x_{cm} = \frac{1}{M}\int x\, dm$ —(1) (8)

we know linear density $(\lambda) = \frac{M}{L}$ → (2)

for a mass "dm" and length "dx" is $\lambda = \frac{dm}{dx}$.

$\Rightarrow dm = \lambda dx$ → (3)

$$x_{cm} = \frac{1}{M}\int_0^L x\, dm = \frac{1}{M}\int_0^L x \cdot \lambda dx$$

$$= \frac{1}{M}\int_0^L x \boxed{\frac{M}{L}} dx \quad [\because \text{from } Eq^n (2)]$$

$$= \frac{1}{\cancel{M}} \cdot \frac{\cancel{M}}{L}\int_0^L x\, dx = \frac{1}{L}\int_0^L x\, dx$$

$$= \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{1}{\cancel{L}} \cdot \frac{L^{\cancel{2}}}{2} = \frac{L}{2}$$

$\boxed{x_{cm} = \frac{L}{2}}$ and $y_{cm} = \frac{1}{M}\int y\, dm = 0$

$y_{cm} = 0$ $[\because y = 0$ for thin rod$]$

Thus, It means that the centre of mass of thin rod AB lies at point $\left(\frac{L}{2}, 0\right)$ i.e centre between its ends A and B. Thus, the centre of mass coincides with geometric centre.

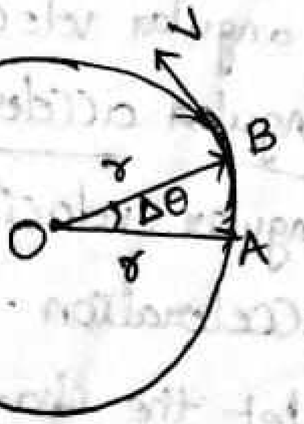
Rotatory Motion Lecture - 5.

In rotational motion, different particles of a rigid body move in circles and centers of all these circles lie on a line called axis of rotation.
The angular displacement and angular velocity are same for all the particles of the rigid body.

Rotatory Motion :- A body is said to be in rotatory motion if every particle moves in a circular path about a fixed point on a line which is called the axis of rotation.
Axis of rotation :- The locus of the centres of circular paths of the particles in a rotating body is called the axis of rotation.

Angular variables

Consider a particle moving along a circular path. Let the particle be displaced from A to B in a small interval of time "Δt". The radius vector "OA" joining the centre and the particle rotates through an angle "$\Delta\theta$" and occupies the position OB. This angle is called the angular displacement of the particle.



Angular displacement (θ):- "The angle through which the radius vector rotates in a given time is called angular displacement"

It is a vector and its direction is perpendicular to the plane of rotation as guided by the right hand thumb rule.

The unit of angular displacement is radian and represented by "rad".

Angular velocity:- The rate of angular displacement of a body is called angular velocity. (or) Angular displacement of a particle per unit time is called angular velocity

Angular velocity, $\omega = \frac{\Delta\theta}{\Delta t}$ as $\Delta\theta$ and Δt are very smalls and inlin

$$\omega = \underset{\Delta t \to 0}{Lt} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

This is called Instantaneous angular velocity at a particular instant of time.

Angular velocity is a vector. The unit of angular velocity is $rad\,s^{-1}$.

Angular acceleration :- The rate of change of angular velocity of a body is called angular acceleration.

Let the change in angular velocity during a small interval of time Δt be $\Delta \omega$.

Then angular acceleration $\alpha = \frac{\Delta \omega}{\Delta t}$.

Its Instantaneous angular acceleration is

$$\lim_{\Delta t \to 0} \left(\frac{\Delta \omega}{\Delta t}\right) = \frac{d\omega}{dt}.$$

its S.I unit is radian/sec^2

Angular acceleration is a vector whose direction is in the direction of change in angular velocity. When the angular velocity increases, the direction of angular acceleration is in the direction of angular velocity. When the angular velocity decreases, the direction of angular acceleration is in the opposite direction of angular velocity.

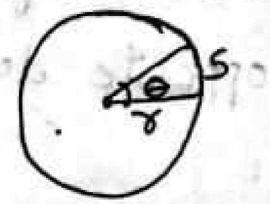
Lecture - 6

Relating linear & angular Quantities :-

Linear variables such as arc length covered by a particle, its linear speed and tangential acceleration can be obtained in terms of rotational variables and the distance of the particle from the axis of rotation.

consider a particle of a rigid body at a distance "r" from the axis of rotation. suppose it covers an arc length "S" in a time "t" and θ is the corresponding angular displacement. Then, from Geometry $S = r\theta$ — ①

Differentiating Eqⁿ ① w.r.t time "t"



$$\frac{ds}{dt} = r \cdot \frac{d\theta}{dt}$$

$$\boxed{V = r \cdot \omega} \quad \left[\because \frac{d\theta}{dt} = \omega\right]$$

In terms of vectors $\bar{V} = \bar{\omega} \times \bar{r}$

Since $v = r\omega$, differentiating it w.r.t time 't'

$$\frac{dv}{dt} = r \cdot \frac{d\omega}{dt}$$

$$\boxed{a = r \cdot \alpha}$$

$\frac{dv}{dt} = a$ is tangential acceleration and $\frac{d\omega}{dt} = \alpha$ the angular acceleration.

Equations of Rotational Motion :-

consider a rigid body rotating about an axis with a constant angular acceleration α.

Let ω_0 and ω represent its initial and final angular velocities and θ be the angular displacement after a time t. Then Eq^ns of motion are

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

These Eq^ns analogous to Equations of linear motion namely.

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2 \text{ and}$$

$$v^2 - u^2 = 2as, \text{ respectively.}$$

First Equation of motion :-

Let us consider a rigid body rotating about a fixed axis with constant angular acceleration α.

$$\frac{d\omega}{dt} = \alpha$$

$$d\omega = \alpha \cdot dt$$

If at $t=0$, $\omega = \omega_0$, then the angular velocity of the body at any time "t" is given by

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha \, dt$$

$$[\omega]_{\omega_0}^{\omega} = \alpha [t]_0^t$$

$$\omega - \omega_0 = \alpha t$$

$$\boxed{\omega = \omega_0 + \alpha t} \rightarrow (1)$$

Second Equation of motion:-

By definition, we have $\frac{d\theta}{dt} = \omega$

$$d\theta = \omega\, dt$$

$$\int_0^\theta d\theta = \int_0^t \omega\, dt$$

$$\int_0^\theta d\theta = \int_0^t (\omega_0 + \alpha t)\, dt \quad [\because \text{from eq } (1)]$$

$$[\theta]_0^\theta = \left[\omega_0 t + \alpha \frac{t^2}{2}\right]_0^t$$

$$\boxed{\theta = \omega_0 t + \frac{1}{2} \alpha t^2} \rightarrow (2)$$

Third Equation of motion:-

The angular acceleration can be expressed as

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \cdot \frac{d\theta}{dt}$$

$$\alpha = \omega \cdot \frac{d\omega}{d\theta}$$

$$\omega\, d\omega = \alpha\, d\theta$$

$$\int_{\omega_0}^{\omega} \omega\, d\omega = \int_0^{\theta} \alpha\, d\theta$$

$$\left[\frac{\omega^2}{2}\right]_{\omega_0}^{\omega} = \alpha\left[\theta\right]_0^{\theta}$$

$$\frac{1}{2}\left[\omega^2 - \omega_0^2\right] = \alpha\theta$$

$$\boxed{\omega^2 - \omega_0^2 = 2\alpha\theta}$$

For uniformly retarded motion, these Equations become

$$\omega = \omega_0 - \alpha t$$

$$\theta = \omega_0 t - \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 - 2\alpha\theta.$$

Lecture - 7

Energy in Rotational Motion :-

A rotating rigid body consists of mass in motion so it has kinetic energy. the mass of the i^{th} particle is m_i and its distance from the axis of rotation is r_i. r_i is the perpendicular distance from the axis to the i^{th} particle.

the speed of the i_{th} particle is given by

$$v_i = r_i \omega , \quad \omega \text{ is the body's angular speed.}$$

the kinetic energy of the i^{th} particle is

$$K.E = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$$

the total K.E of the body is the sum of the kinetic energies of all its particles.

$$K = \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \cdots = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$$K = \frac{1}{2} \left(\sum m_1 r_1^2 + m_2 r_2^2 + \cdots \right) \omega^2$$

$$= \frac{1}{2} \left[\sum m_i r_i^2 \right] \omega^2$$

multiplying the mass of each particle by the square of its distance from the axis of rotation and adding these products, is denoted by I and is called moment of Inertia of the body.

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots = \sum m_i r_i^2 \qquad (17)$$

$$\therefore K = \frac{1}{2} I \omega^2$$

The greater the moment of inertia, the greater the kinetic energy of a rigid body rotating with a given angular speed ω.

moment of Inertia :-

The property of a body by virtue of which it opposes the torque tending to change its state of rest or of uniform rotation about an axis is called rotational Inertia or moment of Inertia.

"The moment of Inertia of a rigid body about a given axis is the sum of the products of the mass of each particle and the square of its distance from the axis of rotation".

$$\boxed{I = mr^2} \quad kg\text{-}m^2$$

where r is the distance of particle from axis of rotation.

For a system of particles, the moment of Inertia is given by

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$$

$$\boxed{I = \sum_{i=1}^{n} m_i r_i^2}$$

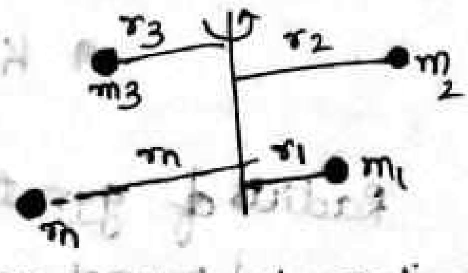
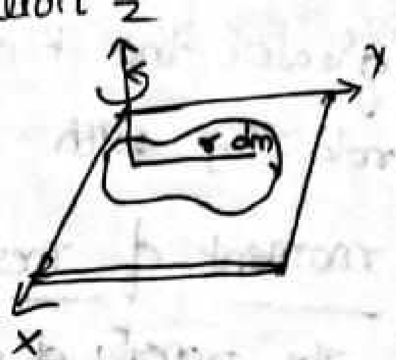


Fig: System of particles

moment of Inertia of a rigid body :-

consider a body rotating about an axis as shown in Fig. choose small element of mass "dm" at a distance 'r' from the axis, its moment of Inertia about the axis of rotation



$$dI = dm\, r^2$$

$$I = \int dm\, r^2$$

The moment of Inertia of a body depends on

i) Mass of the body

ii) size and shape of the body

iii) position and orientation of axis of rotation.

Radius of gyration :- we have $I = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$

Now suppose the whole mass of the system M is concentrated at a point and placed at a distance "K" from the axis, then

$$I = MK^2$$

$$MK^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2 = I$$

here K is called radius of gyration

$$MK^2 = I$$

$$\Rightarrow K = \sqrt{\frac{I}{M}}, \quad K = \sqrt{\frac{r_1^2 + r_2^2 + \cdots + r_n^2}{n}}$$

Radius of gyration of a body about a given axis is equal to root mean square distance of constituent particles from the given axis.

① **Moment of Inertia of a thin uniform rod :-**

consider a thin uniform rod of mass "m" and length "L", which is rotating about an axis passing through its centre of mass and perpendicular to its length.

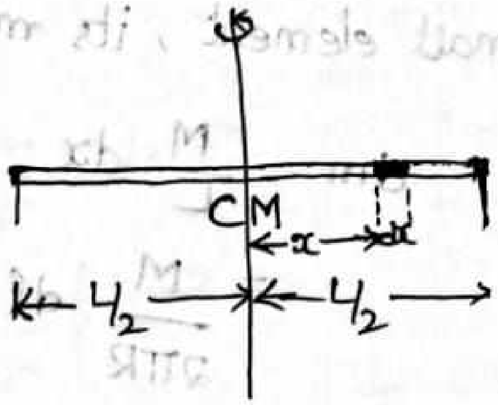
The moment of Inertia about an axis passing through its length will be zero.

consider a small element of length "dx" at a distance x from its C.M. The mass of the element $dm = \frac{M}{L} . dx$.

Moment of Inertia of the element $dI = dm . x^2$

Moment of Inertia of whole rod

$$I = \int_{-L/2}^{L/2} dm\, x^2$$



$$I = 2\int_{0}^{L/2} dm\, x^2$$

$$= 2\int_{0}^{L/2} \left[\frac{M}{L}\right] dx . x^2 = \frac{2M}{L}\int_{0}^{L/2} x^2\, dx$$

$$= \frac{2M}{L}\left[\frac{x^3}{3}\right]_{0}^{L/2} = \frac{2M}{L}\frac{1}{3}\left[\left[\frac{L}{2}\right]^3 - 0\right]$$

$$= \frac{2M}{3k}\left[\frac{L^3}{84}\right] = \frac{ML^2}{12}$$

$\therefore$ Moment of Inertia of a uniform rod, $\boxed{I = \frac{ML^2}{12}}$

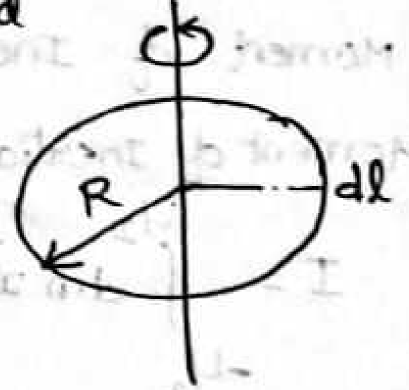
Radius of gyration of thin rod about C.M axis

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{ML^2}{12}}{M}} = \sqrt{\frac{L^2}{12}} = \frac{L}{\sqrt{12}}$$

$$\boxed{\therefore K = \frac{L}{\sqrt{12}}}$$

② Moment of Inertia of ring or hoop :-

consider a ring mass M and radius R, and take a small element, its mass



$$dm = \frac{M}{L}\,dx$$

$$= \frac{M}{2\pi R}\cdot dl \qquad [\because L = 2\pi R,\ dx = dl]$$

Moment of Inertia of this element about the axis, $dI = (dm)R^2$

Moment of Inertia of whole Ring

$$I = \int_0^L (dm)R^2$$

$$I = \int_0^{2\pi R} \frac{M}{2\pi R}\, dl \cdot R^2$$

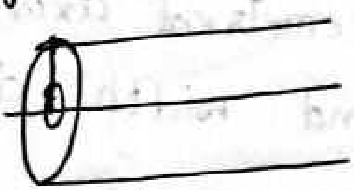
$$I = \frac{MR^2}{2\pi R} \int_0^{2\pi R} dl$$

$$= \frac{MR^2}{2\pi R} \left[l \right]_0^{2\pi R} = \frac{MR^2}{\cancel{2\pi R}} \left[\cancel{2\pi R} \right]$$

$$\therefore \boxed{I = MR^2}$$

(3) Moment of Inertia of a thick rod (or) cylinder :-

consider a solid circular cylinder of mass M and cross-sectional radius R. The length of the cylinder is L



$$dm = \frac{m}{L} dx$$

$$dm = \frac{M}{\pi R^2 L} \cdot 2\pi r L\, dr$$

$$\sigma = \frac{m}{A}$$

$$m = \sigma A$$

$$dm = \sigma\, dA$$

$$dm = \frac{M}{A} dA$$

$$dm = \frac{M}{\pi R^2 L} \cdot 2\pi r L\, dr$$

$$A = \pi r^2 L$$

$$dA = 2\pi r L\, dr$$

$$dm = \frac{2M}{R^2} r\, dr$$

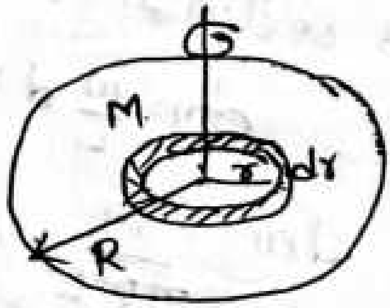
$$I = \int dm \cdot r^2 = \int_0^R \frac{2M}{R^2} r\, dr \cdot r^2$$

$$= \frac{2M}{R^2} \int_0^R r^3 dr = \frac{2M}{R^2} \left[\frac{r^4}{4} \right]_0^R$$

$$= \frac{2M}{R^2} \left[\frac{R^4}{4_2} \right] = \frac{1}{2} MR^2$$

$$\boxed{I = \frac{1}{2} MR^2}$$

(4) Moment of Inertia of disc :- Consider a disc of mass 'M' and Radius R rotating about its geometrical axis. choose an element of radius 'r' and width dr, its mass



$$dm = \frac{M}{\pi R^2}(2\pi r\, dr)$$

$$I = \int dm \cdot r^2 = \int \frac{M}{\pi R^2}(2\pi r dr) r^2$$

$dm = \frac{M}{A} \cdot dA$

$= \frac{M}{\pi R^2} \cdot 2\pi r dr$

$$= \frac{2M}{R^2}\int_0^R r^3 dr = \frac{2M}{R^2}\left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{2M}{R^2} \cdot \frac{R^4}{4} = \frac{1}{2}MR^2$$

Moment of Inertia of disc $= \frac{1}{2}MR^2$.

(5) Moment of Inertia of Solid sphere :-

Consider a sphere of mass M and radius R rotating about any of its diameter. choose an element in the form of a disc of radius r and thickness dx. The mass of the element

$$dm = \frac{M}{\frac{4}{3}\pi R^3} \cdot \pi r^2 dx = \frac{3M}{4R^3} r^2 dx$$

here $\boxed{r^2 = R^2 - x^2}$

$\rho = \frac{m}{V}$

$m = \rho V$

$dm = \frac{M}{V} dV$

$dV = \pi r^2 \cdot dx$

$dm = \frac{M}{V} \cdot \pi r^2 dx$

$dm = \frac{M}{\frac{4}{3}\pi R^3} \pi r^2 dx$

M.I of the disc $\boxed{I = \frac{MR^2}{2}}$

M.I of exceeding small thin disks M.I of sphere will give the M.I of sphere

$$dI = \frac{r^2}{2} dm$$

$$I = \int_{-R}^{R} dm \frac{r^2}{2}$$

$$= \frac{1}{2} \int_{-R}^{R} \frac{3M}{4R^3} r^2 dx \cdot r^2$$

$$= \frac{1}{\cancel{2}} \frac{3M}{4R^3} \cdot \cancel{2} \int_{0}^{R} r^4 dx$$

$$= \frac{3M}{4R^3} \int_{0}^{R} (R^2 - x^2)^2 dx$$

$$I = \frac{3M}{4R^3}\int_0^R \left(R^4 + x^4 - 2R^2x^2\right) dx$$

$$= \frac{3M}{4R^3}\left[R^4x + \frac{x^5}{5} - 2R^2\frac{x^3}{3}\right]_0^R$$

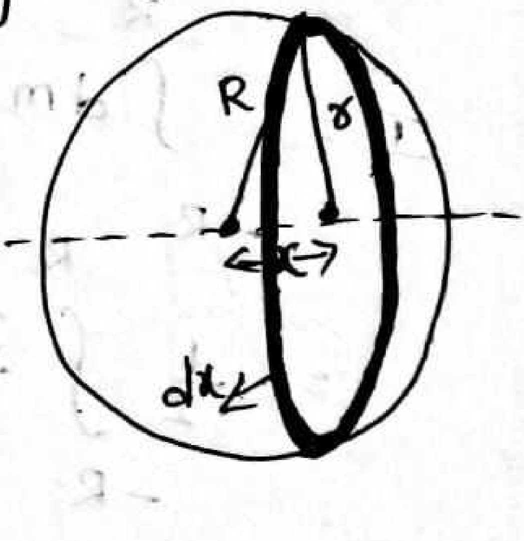
$$= \frac{3M}{4R^3}\left[R^4.R + \frac{R^5}{5} - 2R^2.\frac{R^3}{3}\right]$$

$$= \frac{3M}{4R^3}\left[R^5 + \frac{R^5}{5} - \frac{2}{3}R^5\right]$$

$$= \frac{3M}{4R^3}\left[\frac{15R^5 + 3R^5 - 10R^5}{15}\right]$$

$$= \frac{\cancel{3}M}{\cancel{4}R^{\cancel{3}}}\left[\frac{\overset{2}{\cancel{8}}R^5}{\underset{5}{\cancel{15}}}\right]$$

$$\boxed{I = \frac{2}{5}MR^2}$$

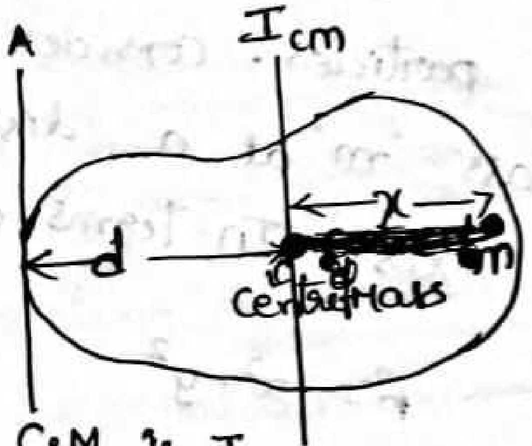


Lecture-9

parallel axes theorem :-

• The moment of Inertia of a rigid body about an axis is equal to the sum of moment of Inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of distance between the two parallel axes.

$$I_A = I_{cm} + Md^2$$



consider a body, whose moment of inertia about an axis passing through C.M is I_{cm}.

Let 'm' is the mass of the particle at a distance 'x' from the axis of rotation, then the moment of Inertia of whole body about C.M axis

$$I_{cm} = \Sigma mx^2 \quad \text{— (1)}$$

choose an axis 'A' parallel to the C.M, the moment of Inertia of body about axis A.

$$I_A = \Sigma M(d+x)^2$$

$$= \Sigma M(d^2 + x^2 + 2xd)$$

$$= \Sigma md^2 + \Sigma mx^2 + \Sigma m2xd$$

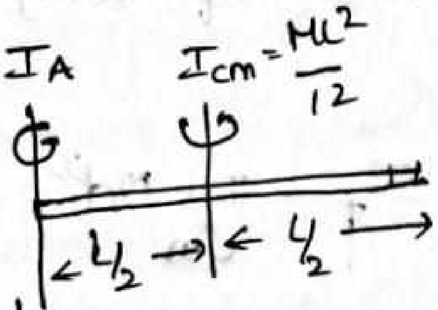
here $\Sigma md^2 = Md^2$, $\Sigma mx^2 = I_{cm}$, $\Sigma mx = 0$

$$\therefore \boxed{I_A = I_{cm} + Md^2} \quad (25)$$

Hence, the theorem is proved.

Applications of parallel axis Theorem :-

i) Moment of Inertia of a thin uniform rod $I_{cm} = \frac{ML^2}{12}$



Moment of Inertia of about any other axis parallel to I_{cm} can be obtained by using parallel axis theorem.

$$I_A = I_{CM} + Md^2$$

$$d = L/2, \quad I_{cm} = \frac{ML^2}{12}$$

$$\therefore I_A = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{4}$$

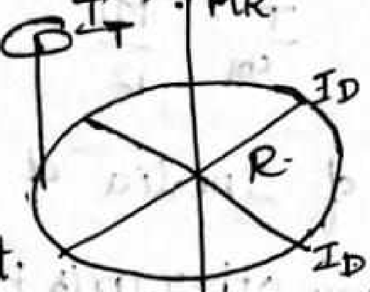
$$= \frac{ML^2 + 3ML^2}{12} = \frac{\cancel{4}ML^2}{\cancel{12}\,3}$$

$$\boxed{I_{end} = \frac{ML^2}{3}}$$

ii) M.I of Ring :-

M.I of Ring $I_{cm} = MR^2$.

$$I_T = I_{cm} + Md^2.$$



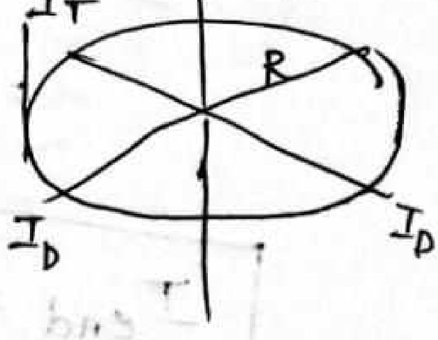
M.I about the tangent parallel to the geometrical axis is I_T

$$I_T = MR^2 + MR^2$$
$$= 2MR^2.$$

iii) M.I of disc :-

we know $I_{cm} = \frac{MR^2}{2}$ for a disc

M.I about the tangent parallel to geometrical axis : By parallel axis theorem ., $d = R$



$$I_T = I_{cm} + Md^2$$

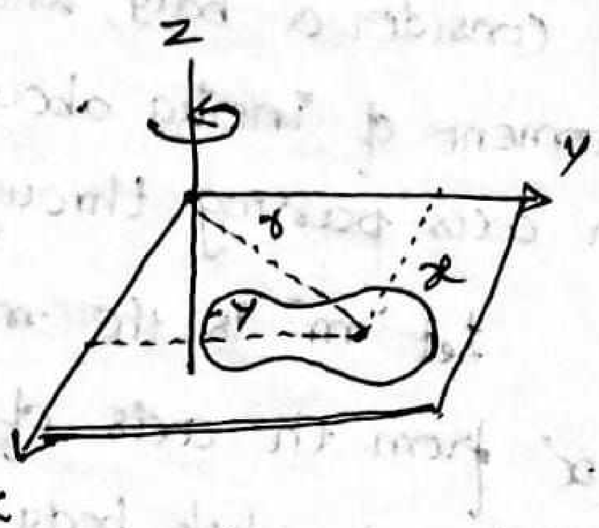
$$= \frac{MR^2}{2} + MR^2$$

$$\boxed{I_T = \frac{3}{2}MR^2}$$

Q. Perpendicular axis theorem :-

Consider a body lying in the xy-plane. It can be assumed to made of large number of particles. consider one such particle of mass 'm' at a distance r from the origin of axis. In terms of cartesian coordinate.

$$r^2 = x^2 + y^2$$



The moment of Inertia of the particle about x-axis

$$I_x = my^2$$

M.I of the body about x-axis is

$$I_x = \Sigma my^2$$

M.I of the body about Y-axis is

$$I_y = \Sigma mx^2$$

M.I of the body about Z-axis

$$I_z = \Sigma mr^2$$

$$I_z = \sum m(x^2+y^2)$$

$$= \sum mx^2 + \sum my^2$$

$$\boxed{I_z = I_x + I_y}$$

Thus the moment of Inertia of a body lying in a plane about an axis perpendicular to its plane is equal to the sum of the moment of Inertia of the body about any two mutually perpendicular axes in its plane and intersecting each other at the point where the perpendicular axis passes through the body.

Applications of perpendicular axis theorem

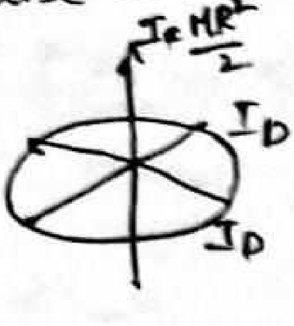
1) Moment of Inertia of disc :-

we have to find the M.I about the diameter of the disc, let us consider a system of three axes, such that z-axis is along the geometrical axis of the disc and x-axis & y-axis lie in the plane of the disc such that the centre of mass G lies at the origin of the system of axes.

$$I_z = \frac{MR^2}{2}$$

By ⊥ar axes theorem, $I_z = I_x + I_y$

Due to symmetry M.I of the disc about diameter is



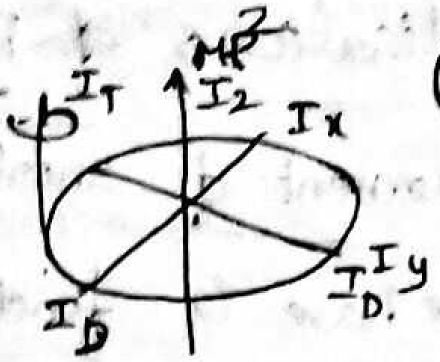
$$I_d = I_x = I_y$$

$$\frac{MR^2}{2} = I_d + I_d = 2I_d$$

$$\boxed{I_d = \frac{MR^2}{4}}$$

This is an expression for M.I of a thin uniform disc about its diameter

2: Moment of Inertia of Ring :-



we have to find out the M.I of about diameter of ring. let us consider a system of three axes

along z-axis $\quad I_z = MR^2$

From perpendicular axis theorem

$$I_x + I_y = I_z$$

$$I_x = I_y = I_D$$

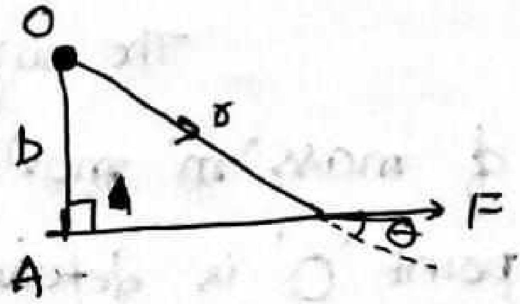
$$I_D + I_D = I_z$$

$$2I_D = MR^2$$

$$\boxed{I_D = \frac{MR^2}{2}}$$

Lecture -11

Torque:- Torque is also known as moment of force or couple. we can define the torque for a particle about a point as the vector product of position vector of the point where the force acts and with the force itself :



The Torque acting on the particle relative to the fixed point "O" is a vector Quantity.

$$\therefore \tau = r \times F$$

$$\tau = rF \sin\theta$$

$$= Fr \sin\theta$$

$$\tau = Fr_{\perp} = Fb$$

Here $r_{\perp}$ is the perpendicular distance of the line of action of F from the point O.

$$\tau = rF \sin\theta$$

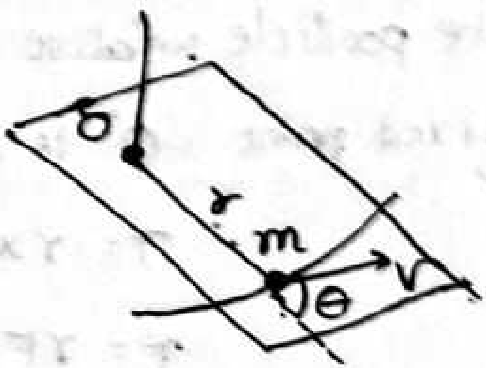
$$= rF_{\perp}$$

here $F_{\perp}$ is the component of F in the direction perpendicular to r.

Angular momentum of a particle :-

Angular momentum can be defined as moment of linear momentum about a point.

The angular momentum of a particle of mass 'm' moving with velocity 'v' about a point 'O' is defined as



$$L = r \times p$$

$$\therefore L = m(r \times v)$$

$$L = mvr\sin\theta$$

θ = angle between r and v

L can be represented as $L = rp\sin\theta = rp_{\perp}$

and $L = pr\sin\theta = pr_{\perp}$

$r_{\perp}$ is the perpendicular distance of the linear momentum vector (p) from origin.

Relation between Torque and Angular Momentum

$L = r \times p$ or $L = m(r \times v)$

Differentiating w.r.t time

$$\frac{dL}{dt} = \frac{d}{dt} m(r \times v)$$

$$\frac{dL}{dt} = m\left[\frac{dr}{dt} \times v + r \times \frac{dv}{dt}\right]$$

But $\frac{dr}{dt} = v$ and $v \times v = 0$

$$\frac{dL}{dt} = m\left[r \times \frac{dv}{dt}\right]$$

$$= m[r \times a] \quad \left[\because \frac{dv}{dt} = a\right]$$

$$= [r \times ma]$$

$$\frac{dL}{dt} = r \times F$$

$$\boxed{\frac{dL}{dt} = \tau}$$

Rate of change of angular momentum, $\frac{dL}{dt} = \tau$

the rate of change of angular momentum is equal to the torque applied.

Conservation of angular Momentum :-

As. $\frac{dL}{dt} = \tau_{net}$ for the rigid body which can be treated as a system of n- particles.

If $\tau_{net} = 0$, then

$$\frac{dL}{dt} = 0 \Rightarrow \boxed{L = \text{Constant}}$$

Hence, without any external torque, angular momentum of a system of particles remain constant. This is known as the conservation of angular momentum.

$$\text{If } \tau_{ext} = 0 \Rightarrow \boxed{L_{initial} = L_{final}}$$

If L is constant, then its all three components will also be constant

L_x = constant

L_y = constant

L_z = Constant.

Ex:- Skater, ballet dancer and a circus acrobat performs on bringing the arms and legs closer to the body, then M.I ↓ decreases, hence angular velocity increases.

Equilibrium of a rigid body :-

A rigid body is said to be in equilibrium, if both of its linear momentum and angular momentum are not changing with time.

Thus, equilibrium body does not possess linear acceleration or angular acceleration.

i, The total force, i.e the vector sum of all forces acting on the rigid body is zero

$$F_1 + F_2 + \cdots + F_n = \sum_{i=1}^{n} F_i = 0$$

If the total face is zero, then the linear momentum of body remains constant, so body must be in translatory motion i.e $F = 0 \Rightarrow \frac{dp}{dt} = 0$

$\boxed{P = \text{constant}}$

$$\sum_{i=0}^{n} F_{ix} = 0, \quad \sum_{i=0}^{n} F_{iy} = 0 \text{ and } \sum_{i=0}^{n} F_{iz} = 0$$

ii, The total torque i.e the vector sum of all torques acting on the body must be zero

$$\tau_1 + \tau_2 + \tau_3 + \cdots + \tau_n = \sum_{i=1}^{n} \tau_i = 0$$

$$\sum_{i=0}^{n} \tau_{ix} = 0, \quad \sum_{i=0}^{n} \tau_{iy} = 0 \; \& \; \sum_{i=0}^{n} \tau_{iz} = 0$$

then the body is in Rotation Equilibrium. A body may remain only in translational or Rotational Equilibrium

Weight :-

The weight of a body is the total gravitational force exerted on the body by all other bodies in the universe

Consider a body of mass "m" on the surface of a planet (or) very near to it so that the distance "r" of the body from the centre of the planet becomes the Radius "R" of the planet.

The weight of a body is $W = mg$

$$W = F \Rightarrow F = mg \quad \text{— ①}$$

F - Gravitational force

We know $F = \frac{G m_1 m_2}{r^2}$

here $m_1 = M$ mass of the earth

$m_2 = m$ mass of the body

$r = R$

$$\therefore F = \frac{GMm}{R^2} \quad \text{— ②}$$

from Eqns ① & ②

$$mg = \frac{GMm}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}}$$

g is independent of the mass of the body.

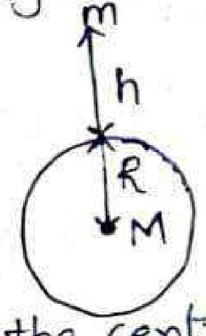
Variation of Acceleration due to gravity :-

i) Variation of g with Altitude

Consider a body of mass 'm' lying on the surface of the earth of mass 'M' and radius R. Then Acceleration due to gravity at the surface of earth is

$$g = \frac{GM}{R^2} \rightarrow (1)$$

Suppose the body is taken to a height 'h' above the surface of the earth, then



$$g_h = \frac{GM}{(R+h)^2} \rightarrow (2)$$

where (R+h) is the distance between the centres of body and the earth.

$$\frac{(2)}{(1)} \Rightarrow \frac{g_h}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}} = \frac{R^2}{(R+h)^2}$$

$$= \frac{R^2}{\left[R\left(1+\frac{h}{R}\right)\right]^2} = \frac{\cancel{R^2}}{\cancel{R^2}\left(1+\frac{h}{R}\right)^2} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \left[1+\frac{h}{R}\right]^{-2}$$

$$\boxed{g_h = g\left[1 - \frac{2h}{R}\right]}$$ [$\because h << R$, $\frac{h}{R}$ is very small using Binomial theorem]

The Acceleration due to gravity decreases with altitude.

$$W = F = \frac{GMm}{R^2}$$

$$\therefore \boxed{W = \frac{GMm}{R^2}}$$

The weight of a body decreases inversely with the square of its distance from the earth's center.

Acceleration Due to gravity :-

Acceleration of freely falling body is known as acceleration due to gravity (g).
The force acting on the falling body is 'mg' and the acceleration is net force divided by mass of the body

$$F = Mg$$

$$\boxed{g = \frac{F}{M}}$$

Relation b/n "g" and "G" :-

we know $F = mg$. —①, m = mass of the body

$F = \frac{GMm}{R^2}$ —②, M = mass of the earth

R = distance between earth & body

① = ② $mg = \frac{GM\cancel{m}}{R^2}$

$$\boxed{g = \frac{GM}{R^2}}$$

This is the relation b/n acceleration due to gravity (g) and gravitational constant (G)

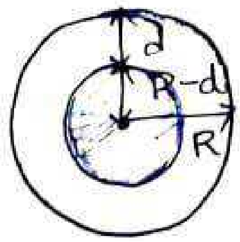
ii) Variation of g with depth:-

Assume the earth to be a homogeneous sphere. Let "ρ" be the mean density of the earth and a body be lying on the surface of the earth.

$$g = \frac{GM}{R^2} \quad (\text{or}) \quad g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$g = \frac{4}{3}\pi G R \rho \rightarrow (1) \quad \left[\because \text{mass of the earth, } M = \frac{4}{3}\pi R^3 \rho\right]$$

Now, the body be taken to a depth "d" below the free surface of the earth, then acceleration due to gravity is g_d, which is acting on the inner solid sphere of Radius $(R-d)$



$$\therefore \quad g_d = \frac{GM'}{(R-d)^2}$$

$$= \frac{G}{(R-d)^2} \times \frac{4}{3}\pi (R-d)^3 \rho$$

$$g_d = \frac{4}{3}\pi G (R-d)\rho \longrightarrow (2)$$

$$\frac{(2)}{(1)} \Rightarrow \frac{g_d}{g} = \frac{\frac{4}{3}\pi G (R-d)\rho}{\frac{4}{3}\pi G R \rho} = \frac{R-d}{R}$$

$$\boxed{g_d = g\left(1 - \frac{d}{R}\right)}$$

From the above Equation it becomes clear that the value of "g" decreases with the depth.

Lecture -18

Kepler's laws of planetary motion:-

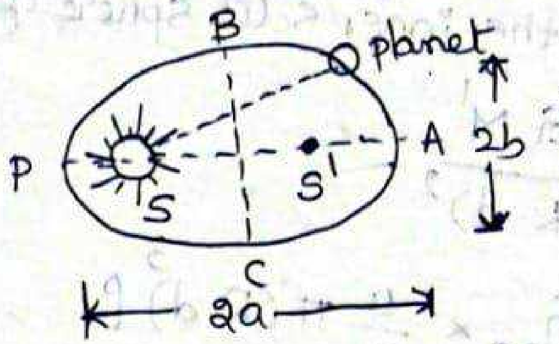
on the basis of observations Johannes Kepler (1571-1630) presented three famous laws about planetary motion These are

1. Law of orbits (Kepler's first law)

Each planet revolves around Sun in an elliptical orbit with the sun at one of the foci.

Elliptical orbit of a planet, PA = 2a = major axis
BC = 2b = minor axis

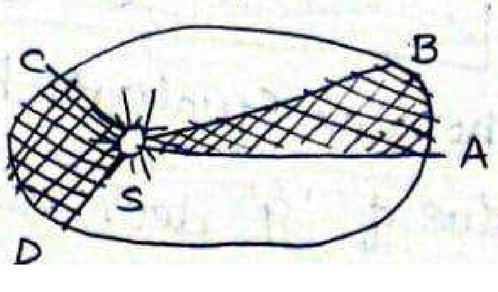
The distance of planet closest to Sun is called perihelion and farthest to sun is called aphelion.



2. Law of Areas (Kepler's Second law)

The radius vector drawn from the Sun to planet Sweeps out equal areas in equal intervals of time i.e the areal velocity of a planet around the sun is constant.

The linear speed of a planet is greater when it is closer to the sun than its linear speed when away from the sun.



3. Law of periods (Kepler's third law)

The square of the time period of revolution of a planet around the sun is proportional to the cube of the semi major axis of its elliptical orbit.

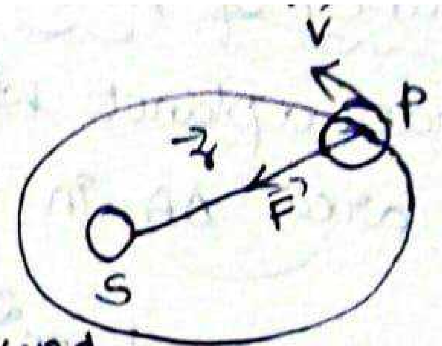
If 'T' is the time period of revolution of a planet and 'a' is the length of the semi major axis of it, then

$$T^2 \propto a^3$$

1. Two bodies one of which is twice as massive as the other are 0.2m apart. They attract each other with a force of 0.1mg wt. Find their masses.

2. Two bodies each of mass 25kg are separated by a distance 1m. Find the force between them. What will the force be, if the distance between the masses is doubled?

Derivation of Kepler's laws



Kepler's second law :-

Let the planet "p" revolves round sun. It experiencea force

$$\bar{F} = \frac{G M_S m_p}{r^2} \hat{r}$$

$$= \frac{G M_S m_p}{r^2} \cdot \frac{\vec{r}}{r} \qquad \left[\because \hat{r} = \frac{\bar{r}}{r}\right]$$

$$= \frac{G M_S m_p}{r^3} \cdot \vec{r}$$

The Torque exerted on the planet "p" about the sun 's'

$$\bar{\tau} = \bar{r} \times \bar{F} = \bar{r} \times \frac{G M_S m_p \bar{r}}{r^3}$$

$$= \frac{G M_S m_p}{r^3} (\bar{r} \times \bar{r})$$

$$\boxed{\bar{\tau} = \bar{0}} \qquad [\because \bar{r} \times \bar{r} = \bar{0}]$$

problems

(3) The gravitational force between two identical objects at a separation of 1m is 0.0667 mgwt. Find the masses of the objects.

($G = 6.67 \times 10^{-11}\ Nm^2/kg^2$ and $g = 10\ m/s^2$)

(4) If the Acceleration due to gravity on earth is $9.81\ m/s^2$ and the radius of the earth is 6370 km. Find the mass of the earth?

($G = 6.67 \times 10^{-11}\ Nm^2/kg^2$)

(5) If g on the Surface of the earth is $9.8\ m/s^2$, Find its value at a height of 6400km

(Radius of the earth = 6400km)

(6) Find the value of g at a height of 100 km from the Surface of the earth

(Radius of the earth = 6400km, $g = 9.8\ m/s^2$)

(7) If g on the surface of the earth is $9.8\ m/s^2$, Find its value at a depth of 3200km

(Radius of the earth = 6400km)

Problems

(3) The gravitational force between two identical objects at a separation of 1m is 0.0667 mgwt. Find the masses of the objects. $\boxed{\text{Ans:- } m = 100kg}$

($G = 6.67 \times 10^{-11} Nm^2/kg^2$ and $g = 10 m/s^2$)

(4) If the Acceleration due to gravity on earth is $9.81 m/s^2$ and the radius of the earth is 6370 Km. Find the mass of the earth?

($G = 6.67 \times 10^{-11} Nm^2/kg^2$) $\boxed{\text{Ans: } M = 5.97 \times 10^{24} kg}$

(5) If g on the Surface of the earth is $9.8 m/s^2$, Find its value at a height of 6400km

(Radius of the earth = 6400km) $\boxed{\text{Ans: } g_h = 2.45 m/s^2}$

(6) Find the value of g at a height of 100 km from the surface of the earth.

(Radius of the earth = 6400km, $g = 9.8 m/s^2$)

$\boxed{\text{Ans: } g_h = 9.494 m/s^2}$

(7) If g on the surface of the earth is $9.8 m/s^2$, Find its value at a depth of 3200km

(Radius of the earth = 6400km)

$\boxed{\text{Ans: } g_d = 4.9 m/s^2}$

Areal velocity $= \frac{L}{2m}$

$$L = 2m \times \text{areal velocity}$$

$$= 2 \times 6.3 \times 10^{24} \times 2.24 \times 10^{15}$$

$$= 2.8 \times 10^{40}\ kg\,m^2 s^{-1}$$

(10) Time period of Jupiter is 11.6 years. how far is Jupiter from the sun? Distance of the earth from the sun is 1.5×10^{11} m.

Sol:- $T_J = 11.6\,yr$, $r_e = 1.5 \times 10^{11}$ m

$$\frac{T_j^2}{T_e^2} = \frac{r_j^3}{r_e^3} \Rightarrow r_j = r_e \left(\frac{T_j}{T_e}\right)^{2/3}$$

$$r_j = 1.5 \times 10^{11} \left(\frac{11.6}{1}\right)^{2/3} = 7.68 \times 10^{11}\,m$$

$$\boxed{r_j = 7.68 \times 10^{11}\,m}$$

we know $\bar{\tau} = \frac{d\bar{L}}{dt}$

If $\bar{\tau} = 0 \Rightarrow \frac{d\bar{L}}{dt} = 0$

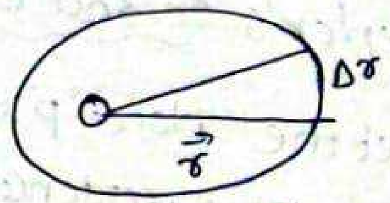
$\therefore \boxed{L = \text{Constant}}$

$\therefore$ so momentum is constant

Constancy of Areal velocity :-

consider a planet whose radius vector $\bar{r}$ sweeps out area ΔA in time Δt. we know that

$$\Delta A = \frac{1}{2} r^2 \Delta \theta$$



$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \frac{\Delta \theta}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \frac{1}{2} r^2 \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$\frac{d\bar{A}}{dt} = \frac{1}{2} r^2 \omega \quad \left[\because \omega = \frac{d\theta}{dt}\right]$$

$$= \frac{1}{2} \frac{r^2 \omega m}{m}$$

$$\frac{d\bar{A}}{dt} = \frac{1}{2} \frac{\bar{L}}{m} \quad [\because \bar{L} = m r^2 \omega]$$

$$\boxed{\frac{d\bar{A}}{dt} = \frac{\bar{L}}{2m}}$$

since $\bar{L}$ is constant so that arealvelocity $\left(\frac{d\bar{A}}{dt}\right)$ is constant.

1. Two bodies one of which is twice as massive as the other are 0.2 m apart. other with a force of 0.1 mg wt. Find their masses

Ans:- $m_1 = 17.14\,kg$, $m_2 = 34.28\,kg$

2. Two bodies each of mass 25 kg are separated by a distance 1m. Find the force between them. What will the force be, If the distance between the masses is doubled?

Ans:- $F_1 = 4.17 \times 10^{-8}\,N$

$F_2 = 1.04 \times 10^{-8}\,N$

Kepler's third law of period

The Gravitational force between sun and planet is

$$F = \frac{GMm}{r^2}$$ M- mass of the sun, m- mass of the planet.

$F = ma$ from Newton's 2nd law of motion

$$\frac{GMm}{r^2} = ma \Rightarrow \frac{GM}{r^2} = a$$

$$\frac{GM}{r^2} = r\omega^2 \quad [\because a = r\omega^2]$$

$$\frac{GM}{r^3} = \omega^2 \Rightarrow \omega = \sqrt{\frac{GM}{r^3}}$$

$\therefore T = \frac{2\pi}{\omega}$, here T = time period.

$$T = \frac{2\pi}{\sqrt{\frac{GM}{r^3}}}$$

$$T^2 = \frac{4\pi^2}{\left(\frac{GM}{r^3}\right)} \rightarrow \frac{4\pi^2}{GM} r^3$$

$\boxed{T^2 \propto r^3}$ here $\frac{4\pi^2}{GM}$ is constant

4) If the Acceleration due to gravity on earth (13) is $g = 9.8\ m/s^2$ and the radius of the earth is 6370 km. Find the mass of the earth ($G = 6.67 \times 10^{-11}\ Nm^2/kg^2$)

Sol:- $g = 9.8\ m/s^2$, $R = 6370\ km = 6370 \times 10^3\ m = 6.370 \times 10^6\ m$

$$g = \frac{GM}{R^2} \Rightarrow M = \frac{gR^2}{G} = \frac{9.8 \times (6.370 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M = 5.97 \times 10^{24}\ kg$$

(5) If g on the surface of the earth is $9.8\ m/s^2$. Find its value at a height of 6400 km. (Radius of the earth = 6400 km)

Sol:- Let 'g' at a height 'h' is g_h.

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$h = 6400\ km$, $R = 6400\ km$, $g = 9.8\ m/s^2$

$$g_h = \frac{9.8}{\left(1+\frac{6400}{6400}\right)^2} = \frac{9.8}{(1+1)^2} = \frac{9.8}{4}$$

$$\boxed{g_h = 2.45\ m/s^2}$$

For second part $\quad r_2 = 2r_1$

$$F_2 = \frac{Gm_1m_2}{r_2^2} = \frac{Gm_1m_2}{(2r_1)^2} = \frac{Gm_1m_2}{4r_1^2}$$

$$= \frac{1}{4}\left(\frac{Gm_1m_2}{r_1^2}\right) = \frac{1}{4} F_1$$

$$= \frac{1}{4} \times 4.17 \times 10^{-8}$$

$$\boxed{F_2 = 1.04 \times 10^{-8} N}$$

③ The Gravitational force b/n two identical objects at a separation of 1m is 0.0667 mg wt. Find the masses of the objects
[$G = 6.67 \times 10^{-11} Nm^2/kg^2$ and $g = 10 m/s^2$]

Sol:- $m_1 = m_2 = m, \quad r = 1 m$

$$F = 0.0667 \times 10^{-6} \times 10 N$$

$$F = \frac{Gm_1m_2}{r^2}$$

$$0.0667 \times 10^{-5} = \frac{6.67 \times 10^{-11} \times m^2}{1^2}$$

$$\frac{0.0667 \times 10^{-5}}{6.67 \times 10^{-11}} = m^2$$

$$\frac{\cancel{6.67} \times 10^{-7}}{\cancel{6.67} \times 10^{-11}} = m^2 \Rightarrow 10^4 = m^2$$

$$m = 10^2$$

$$\Rightarrow \boxed{m = 100 kg}$$

1, Two bodies one of which is twice as massive as the other are 0.2m apart. They attract each other with a force of 0.1 mg wt. Find their masses.

Sol:- Given $M_1 = 2m$, $M_2 = m$, $d = 0.2m$

$F = 0.1\ mg\,wt = 0.1 \times 10^{-6}\ kg\,wt = 0.1 \times 10^{-6} \times 9.8\,N$

$$F = \frac{GM_1M_2}{d^2}$$

$$0.1 \times 10^{-6} \times 9.8 = \frac{6.67 \times 10^{-11} \times 2m \times m}{(0.2)^2}$$

$$m^2 = 2.9385 \times 10^2$$

$\boxed{m = 17.14\,kg}$ and $\boxed{2m = 2 \times 17.14 = 34.28\,kg}$

2, Two bodies each of mass 25 kg are separated by a distance 1m. Find the force b/n them. what will the force be, If the distance b/n the masses is doubled? $G = 6.673 \times 10^{-11}\ Nm^2kg^{-2}$

Sol:- mass of the first body $m_1 = 25\,kg$

" " 2nd body $m_2 = 25\,kg$

separation b/n them $r_1 = 1\,m$

Gravitational Constant $G = 6.673 \times 10^{-11}$

$$\therefore F_1 = \frac{Gm_1m_2}{r_1^2} = \frac{6.673 \times 10^{-11} \times 25 \times 25}{1^2}$$

$$\boxed{F_1 = 4.17 \times 10^{-8}\,N}$$

(8) A sphere of mass 40 kg is attracted by a second sphere of mass 60 kg with a force equal to 4 mgf. If G is $6\times10^{-11}\,Nm^2/kg^2$, then calculate the distance b/n them. Consider g=10m/s² (15)

Sol:- Given $M = 40kg,\ m = 60kg$

$$F = 4\,mgf = 4\times10^{-6}\times10 = 4\times10^{-5}\,N$$

$$G = 6\times10^{-11}\,Nm^2/kg^2,\quad g = 10\,m/s^2$$

$$F = \frac{GMm}{r^2} \Rightarrow r^2 = \frac{GMm}{F}$$

$$r = \sqrt{\frac{GMm}{F}} = \sqrt{\frac{6\times10^{-11}\times40\times60}{4\times10^{-5}}}$$

$$r = 0.06m = 6cm$$

$$\therefore \boxed{r = 6cm}$$

(9) Mean distance ~~of~~ the earth of mass 6.3×10^{24} kg from the sun is 150 million km. Find the i, mean areal velocity (ii) angular momentum of the earth about the sun.

Sol:- Areal velocity $= \frac{\pi R^2}{T} = \frac{3.14\times(1.5\times10^{11})^2}{365\times24\times3600}$

$$= 2.24\times10^{15}\,m^2 s^{-1}$$

(6) Find the value of g at a height of 100km from the surface of the earth (Radius of the earth = 6400 km)

Sol:-

For smaller heights $g_h = g\left(1 - \frac{2h}{R}\right)$

$h = 100km$, $R = 6400km$, $g = 9.8\ m/s^2$

$$g_h = 9.8\left(1 - \frac{2 \times 100}{6400}\right) = 9.8\left(1 - \frac{1}{32}\right)$$

$$= 9.8\left(\frac{31}{32}\right) = 9.494\ m/s^2$$

$$\boxed{g_h = 9.494\ m/s^2}$$

(7) If g on the surface of the earth is $9.8\ m/s^2$. Find its value at a depth of 3200km (Radius of the earth = 6400 km)

Sol:- $g = 9.8\ m/s^2$, $d = 3200$ km, $R = 6400km$

g at a depth, $g_d = g\left(1 - \frac{d}{R}\right)$

$$= 9.8\left(1 - \frac{3200}{6400}\right)$$

$$= \frac{9.8}{2} = 4.9$$

$$\therefore \boxed{g_d = 4.9\ m/s^2}$$

Gravitational potential

Gravitational potential at a point in the gravitational field is defined as the amount of workdone in bringing a body of unit mass from infinity to that point without acceleration.

$$V = \frac{W}{m}$$

where 'W' is the amount of work done in bringing a body of mass 'm' from infinity to that point in the gravitational field.

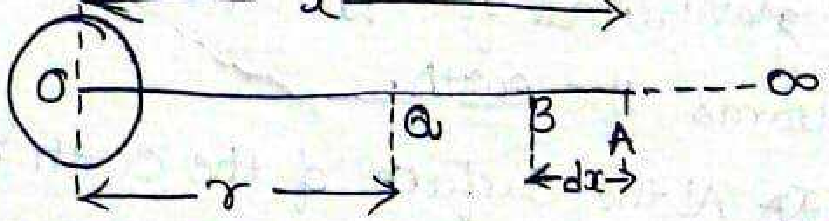
It is a scalar quantity and S.I unit is Joule/kg

If $m=1$, $\boxed{V = +W}$ for unit mass, it is equal to the negative of the workdone in the field

Expression for Gravitational potential :-

Suppose the earth be perfect sphere of radius R and mass 'M', can be supposed to be concentrated at its centre 'O'. Then, the gravitational potential at point 'Q' can be calculated, where $OQ = r$

[$\because r > R$]



Take two points A and B, so $OA = x$ and $AB = dx$.

At point A, gravitational force of attraction on a body of unit mass will be $F = \frac{GM \times 1}{x^2} = \frac{GM}{x^2}$

If we displace the unit mass through a small distance 'dx' from A to B, then small amount of work has to be done which is given by

$$dW = F\,dx = \frac{GM}{x^2}\,dx$$

total workdone in bringing the object of unit mass from ∞ to the point 'Q' is obtained by

$$\int dW = \int F\,dx$$

$$W = \int_{\infty}^{r} \frac{GM}{x^2}\,dx$$

$$= GM\left[-\frac{1}{x}\right]_{\infty}^{r} = -GM\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = -\frac{GM}{r}$$

This workdone remains in the form of gravitational potential at Q.

$$V = W = \frac{-GM}{r}$$

The negative sign shows that the work is done by gravitational force when the object moves towards the earth.

At the surface of the earth $r = R$

$$\boxed{V = \frac{-GM}{R}}$$

$$W = \int_{\infty}^{r} \frac{GMm}{x^2} dx$$

$$= GMm \left[\frac{-1}{x} \right]_{\infty}^{r} = -GMm \left[\frac{1}{r} - \frac{1}{\infty} \right]$$

$$\boxed{W = -\frac{GMm}{r}}$$

This is the workdone by the gravitational force when the body of mass 'm' moves from ∞ to p

$\therefore$ gravitational potential energy $U = W$

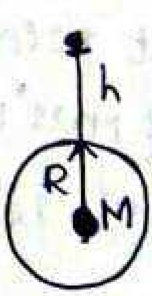
$$\boxed{U_A = -\frac{GMm}{r}}$$

$\therefore$ Gravitational potential energy = Gravitational potential x mass of the body.

Change in Potential energy :-

With respect to infinity as reference level the potential energy of particle at earth surface $r=R$

$$U = -\frac{GMm}{R}$$



The potential energy of particle at height (h) $\Big\}\ U_h = \dfrac{-GMm}{R+h}$

change in potential energy $\Delta U = U_h - U$

$$\Delta U = \frac{-GMm}{R+h} + \frac{GMm}{R} \Rightarrow GMm \left[\frac{1}{R} - \frac{1}{R+h} \right] = GMm \left[\frac{R+h-R}{R(R+h)} \right]$$

$$\Delta U = \frac{GMmh}{R(R+h)} = \frac{GMmh}{R^2+Rh} = \frac{GMmh}{R^2(1+\frac{h}{R})} = \frac{mgh}{1+\frac{h}{R}} \quad \left[\because g = \frac{GM}{R^2} \right]$$

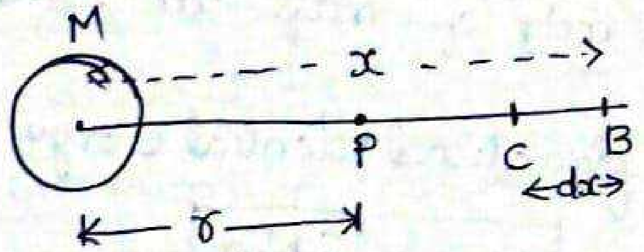
for $h << R$, $\frac{h}{R} \to 0$

$$\boxed{\Delta U = mgh}$$

Expression for Gravitational potential energy:-

The gravitational potential energy of a body at a point is defined as the amount of workdone in bringing the given body from infinity to that point against the gravitational force.

Consider a body of mass 'm' is placed at 'P' with a distance 'r' in the gravitational field of a body of mass 'M'.



We need to find the P.E of the body at point 'P'

By definition, the gravitational potential energy U_A of the body is $U_A = W$, workdone in bringing the body from infinity to point 'P'

Suppose at any instant the body is at point 'B' at a distance 'x' from the centre of mass 'M'. The gravitational force on the body at 'B' is

$$F = \frac{GMm}{x^2}$$

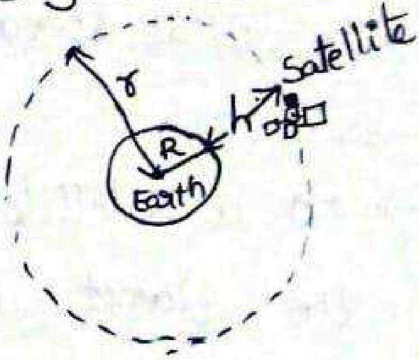
Small amount of workdone when body moves from B to C is

$$dW = F\,dx = \frac{GMm}{x^2}\,dx$$

Total workdone $\int dW = \int F\,dx$

Orbital velocity :- (or) orbital speed of a satellite

Satellites are natural (or) artificial bodies describing orbit around a planet under its gravitational attraction.



Condition for establishment of artificial satellite is that the centre of orbit of satellite must coincide with centre of earth or satellite must move around great circle of earth.

Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth

For revolution of satellite around the earth, the gravitational pull provides the required centripetal force.

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$v_0^2 = \frac{GM}{r} \Rightarrow v_0 = \sqrt{\frac{GM}{R}} \quad [\because \text{If } r = R]$$

If $GM = gR^2$ and $r = R+h$

$$\therefore v_0^2 = \frac{gR^2}{R+h}$$

$$v_0 = R\sqrt{\frac{g}{R+h}} \quad (\text{or}) \quad v_0 = \sqrt{\frac{GM}{R+h}}$$

Escape velocity:- The object escapes from earth's gravitational field for a particular velocity is called escape velocity.

"The minimum velocity with which an object must be projected from the earth's surface so that it escapes from earth's gravitational attraction is called escape velocity"

The gravitational potential energy of a body is equal to the workdone to displace a body from the surface of earth $(r=R)$ to infinity $(r=\infty)$ is

$$W = \int_R^\infty \frac{GMm}{x^2}\,dx = -GMm\left[\frac{1}{\infty}-\frac{1}{R}\right]$$

$$W = \frac{GMm}{R}$$

This work required to project the body so as to escape the gravitational pull is performed on the body by providing an equal amount of kinetic energy to it at the surface of the earth i.e

$K \cdot E = \frac{1}{2}mv_e^2$, v_e is the required escape velocity

$$\frac{1}{2}\cancel{m}v_e^2 = \frac{GM\cancel{m}}{R}$$

$$v_e^2 = \frac{2GM}{R} \Rightarrow V_e = \sqrt{\frac{2GM}{R}} \quad -(1)$$

We know, $g = \frac{GM}{R^2} \Rightarrow GM = gR^2 \quad -(2)$

Energy of an orbiting satellite

When a satellite revolves around a planet in its orbit, it possesses both potential energy and K.E

If 'm' is the mass of the satellite and v is its orbital velocity then K.E is

$$K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\frac{GM}{r} \qquad \left[\because v_0^2 = \frac{GM}{r}\right]$$

The satellite is placed at a certain height 'h' then $r = R+h$

$$\boxed{\therefore K = \frac{GMm}{2(R+h)}}$$

P.E of the satellite $U = mV$

$$\boxed{U = -\frac{GMm}{R+h}}$$

satellite is placed at certain height from the planet then gravitational potential energy is $-\frac{GMm}{R+h}$

Total Mechanical energy of satellite, $E = K+U$

$$E = \frac{GMm}{2(R+h)} - \frac{GMm}{R+h}$$

$$\boxed{E = -\frac{GMm}{2(R+h)}}$$

satellites are always at finite distance from the earth and hence, their energy cannot be positive or zero.

→ h is the height of the satellite above the surface of the earth of Radius R.

→ The orbital speed is independent of mass of the satellite.

→ For a satellite very close to the surface of the planet [As h=0]

$$v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

As the height of the satellite increases, its orbital speed decreases

Time period of satellite :-

It is the time taken by satellite to go once around the earth

$$T = \frac{\text{circumference of the orbit}}{\text{orbital velocity}}$$

$$= \frac{2\pi r}{v_0} = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$

It is clear that time period is independent of the mass of orbiting body and depends on the mass of central body and radius of the orbit.

Geostationary satellite :-

(27)

A satellite which appears stationary to an observer on earth's surface is called geostationary satellite (or) geosynchronous (or) communication satellite.

These satellites always stay over the same place above the earth, such a satellite is never at rest and appears stationary due to its zero relative velocity w.r.t that place on the earth.

The orbit of a geostationary satellite is known as the parking orbit.

It should revolve in an orbit concentric and coplanar with the equatorial plane. Its sense of rotation should be same as that of the earth about its own axis.

i) $\therefore T = 24\,hr = 86400\,sec$

this is the time period of Geostationary satellite

ii) By substituting the values of T, g, R, we will get height of geostationary satellite from the surface of earth $h = 36000\,Km$

iii) orbital velocity of geo stationary satellite can be obtained by $V = \sqrt{\frac{GM}{r}} \Rightarrow V = 3.08\,Km/sec$

Height of the satellite :-

We know $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$= 2\pi\sqrt{\frac{(R+h)^3}{gR^2}} \qquad \left[\because g = \frac{GM}{R^2} \quad r = R+h\right]$$

By squaring on both sides

$$T^2 = 4\pi^2\left(\frac{(R+h)^3}{gR^2}\right)$$

$$gR^2T^2 = 4\pi^2 (R+h)^3$$

$$(R+h)^3 = \frac{gR^2T^2}{4\pi^2}$$

$$R+h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3}$$

$$h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3} - R$$

By knowing the value of time period we can calculate the height of satellite from the surface of the earth.

substitute Eq^n ② in Eq^n ①

$$\Rightarrow V_e = \sqrt{\frac{2gR^2}{R}}$$

$$\boxed{V_e = \sqrt{2gR}}$$

This is the expression for escape velocity

it depends on

i, mass of the planet and (ii) Radius of the planet

Escape velocity is independent on

i) mass of the projected body and (ii) direction of its projection.

The escape velocity from earth's surface is

11.2 km/s

it is the initial kinetic energy given to the object that determines whether the object escapes from the earth's attraction.

Lecture - 25

Stress : when a deforming force is applied on a body, a restoring force is developed in the body due to the action of interatomic forces.

Since the body is in equilibrium, the restoring force developed is equal in magnitude but opposite in the direction to the applied deforming force.

" The restoring force per unit area is called stress ".

If 'F' is the force applied and 'A' is the area of crossection of the body, then

$$\boxed{\text{stress} = \frac{F}{A}}$$

The S.I unit of stress is N/m^2 (or) pascal

Its dimensional formula is $[M L^{-1} T^{-2}]$

on the basis of applied forces on the body, the stress can be classified as

1. longitudinal stress
2. Tangential (or) shearing stress
3. Volume (or) Bulk stress

③

2. Shearing stress:-

The stress which tends to change the shape of a body is called shear stress

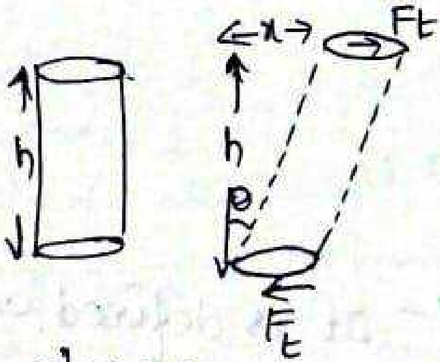
Here, a pair of forces, each of magnitude are applied parallel (or) tangential to the surface of the object.

It is also called tangential stress.

$$\text{shear stress} = \frac{\text{shearing force}}{\text{Area of cross-section}}$$

(or)

$$\text{tangential stress} = \frac{\text{tangential force } (F_t)}{\text{Area } (A)}$$

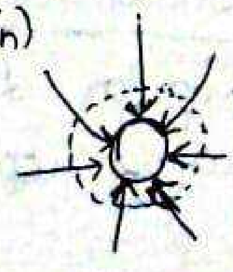


3. Bulk stress:-

The stress which tends to change the volume of a body is called bulk (or) volumetric stress.

Here normal inward forces are applied uniformly over the entire surface of the solid

$$\text{Bulk stress} = \frac{\text{Normal force } (F_n)}{\text{Area } (A)}$$



This type of stress is also called hydraulic stress.

1. Longitudinal stress :-

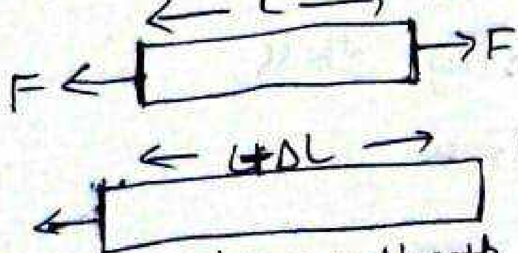
The stress which tends to change the length of a body is called longitudinal stress.

In this type, two forces each of Magnitude F_n are applied to opposite faces of a solid, Each force is normal to the face and is uniformly distributed over the surface.

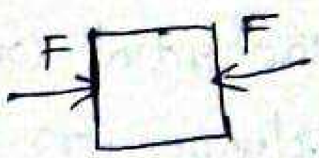
$$\text{Longitudinal stress} = \frac{F_n}{A} = \frac{\text{Normal force}}{\text{Area}}$$

The Longitudinal stress is said to be tensile stress (normal stress) ~~if the applied forces~~ If two equal and opposite forces are applied to a rod to increase the length of a rod, then restoring force is equal to the applied force, it is called tensile stress.

When two equal and opposite forces are applied at the end of a rod to decrease its length (or) compress it, then restoring force equal to the applied force, this force per unit area is known as compressive stress



increased length



Compressive stress

shear strain = $\tan\theta = \frac{x}{h} = \frac{\text{displacement}}{\text{height}}$

If θ is very small, $\tan\theta \approx \theta$

$$\therefore \theta = \frac{x}{h}$$

$$\boxed{\therefore \text{ shearing strain} = \theta}$$

it can be expressed in radian.

3. Bulk strain :- It is defined as the ratio of change in volume (ΔV) to the original volume (V) of a body.

$$\text{Bulk strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

It is also called volume strain. it is demensionless and has no unit.

HOOKE'S LAW :-

"Within the elastic limits, the stress is directly propotional to the corresponding strain"

i.e. stress $\propto$ strain

$$\therefore \text{stress} = E \times \text{strain}$$

$$\boxed{E = \frac{\text{stress}}{\text{strain}}}$$

here E is a constant for a given material and is called elastic constant or modulus of elasticity.

It's S.I unit is N/m^2 and dimensional formula is $[ML^{-1}T^{-2}]$

Strain :- ~~when a~~
Normal stress on a body causes change in length or volume and tengential stress produces change in shape of the body.

The ratio of the change in dimensions of a body to the original dimensions is called strain.

$$\text{strain} = \frac{\text{change in dimensions}}{\text{original dimensions}}$$

it is a ratio of two like quantities, so it has no unit and dimension.

strain is threes types

1. longitudinal strain
2. shear strain
3. Volumetric strain

1. longitudinal strain :- It is defined as the ratio of change in length (ΔL) to the original length (L) of a thin rod (or) a wire

$$\text{Longitudinal strain} = \frac{\text{Change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

2. shear strain :- It is defined as the small angular displacement of a reference line on a surface on which a shear stress is acting.

Young's Modulus of Elasticity :- (Y)

"It is defined as the ratio of longitudinal stress to longitudinal strain. within proportionality limit"

Let 'L' be the original length of a wire or a rod of area of cross section A. Its length changed by 'ΔL' when a force F is applied on it.

then Longitudinal stress $= \frac{F}{A}$

Longitudinal strain $= \frac{\Delta L}{L}$

Young's modulus $Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

$$= \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$\boxed{\therefore Y = \frac{FL}{A\Delta L}}$$

2. shear modulus (or) Rigidity modulus (η)

"It is defined as the ratio of shearing stress to the shearing strain of the body, within proportionality limit" is called lateral strain.

$$\therefore \sigma = \frac{-\Delta D/D}{\Delta L/L}$$

poisson's ration, $\sigma = -\frac{L}{D} \cdot \frac{\Delta D}{\Delta L}$

The negative sign indicates that longitudinal and lateral strain are in opposite sense, it has no unit and dimensions.

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\theta}$$

$$\boxed{\eta = \frac{F}{A\theta}}$$

where, F = tangential force

A = Area of the surface

θ = angular displacement

3. Bulk modulus (K)

"It is defined as the ratio of volume stress to volume strain within propotionality limit"

$$K = \frac{\text{volume stress}}{\text{Volume strain}} = \frac{F/A}{\Delta V/V} = \frac{P}{\Delta V/V}$$

$$\boxed{\therefore K = \frac{P \cdot V}{\Delta V}}$$

where P = Bulk stress = change in pressure

V = origenal volume

ΔV = change in volume.

Poisson's ration (σ) :-

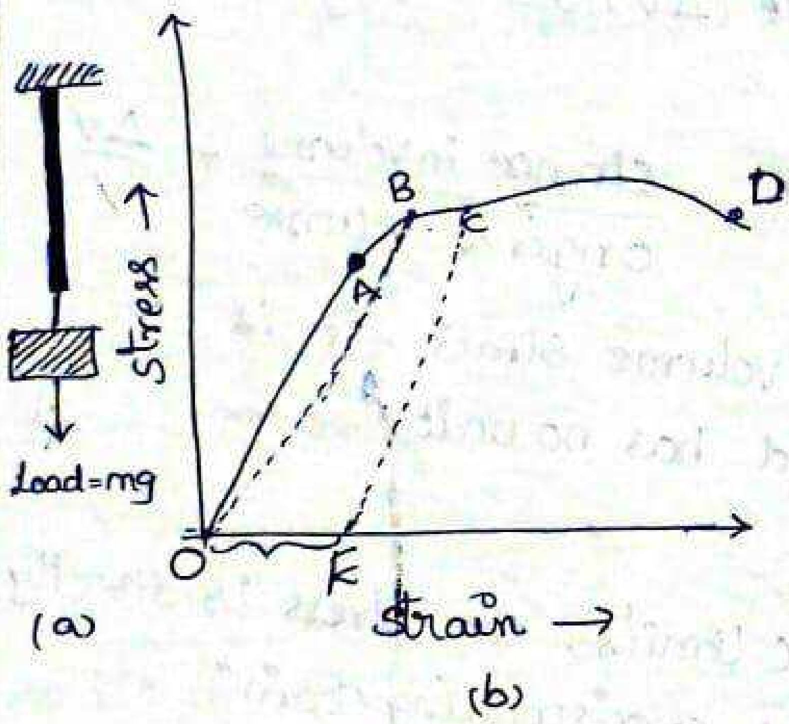
"It is defined as the ratio of the lateral strain to the longitudinal strain"

When a wire is stretched by tensile stress, there will be a contraction or decrease in the thickness of the wire. The ratio of the change in the thickness or diameter (ΔD) to the original thickness or diameter (D)

Stress - Strain curve :- To study the behaviour of a metal wire under increasing load, a metal wire is suspended from a rigid support and loaded at the other end.

The load is increased gradually until it breaks.

A graph is plotted between the stress on the Y-axis and strain on the X-axis.



A - proportional limit
B - elastic limit
CD - plastic region
D - Breaking point
OE - permanent set

i) Between O and A, the curve is linear and hence stress is proportional to strain, which obey's Hooke's law

• "The value of stress upto which stress and strain are proportional to each other is called proportional limit". Here A is known as proportional limit.

ii) When stress is increased beyond A, then for small stress, there is a large strain in the wire upto point B.

"The minimum value of stress at which permanent deformation occurs is called the elastic limit"

here "B" is known as point of elastic limit or yield point.

iii) If the stress or load increases beyond point B, the strain further increases. Now, If the load is removed, the wire does not regain its original length.

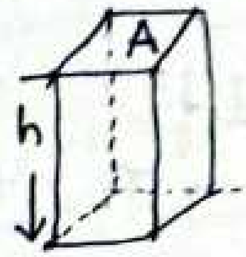
- "The permanent strain produced in the wire when the stress is removed is called permanent set"
- "OE" represents permanent set.

iv) when the stress is increased beyond the yield point the strain increases more rapidly and breaks at point D. "D" is called the fracture point. the corresponding stress is called "breaking strength"

The material is said to exhibit plastic behaviour in the region b/n elastic limit and the breaking point.

The pressure from the weight of a column of liquid of area 'A' and height 'h' is

$$\text{pressure} = \frac{\text{weight}}{\text{area}}$$



$$W = mg = \rho V g \quad \left[\because \rho = \frac{m}{V}\right]$$

here V = volume = hA

volume = hA

weight = mg

$$\therefore W = \rho h A g$$

$$\therefore \text{pressure } (P) = \frac{W}{A} = \frac{\rho h \cancel{A} g}{\cancel{A}} = \rho h g$$

$$\therefore \boxed{P = \rho g h}$$

Static fluid pressure does not depend on the shape, total mass or surface Area of the liquid. It depends on the depth 'h' within the fluid.

Lecture - 28

Fluid statics

pressure :- The pressure may be defined as the normal force exerted on a unit area around that point.

If the force F acts normally over a flat area A, then the pressure is

$$\boxed{P = \frac{F}{A}} \quad N/m^2 \text{ (or) pascal}$$

$$1 \text{ pascal} = 10 \text{ dyne}/cm^2$$

another common unit of pressure is atmosphere (atm).

$$1 \text{ atm} = 1.013 \times 10^5 \text{ pa}$$

It is a scalar Quantity.

pressure due to a fluid column :-

The pressure exerted by a static fluid depends only upon the depth of the fluid, the density of the fluid, and the acceleration of gravity

$$P_{\text{static fluid}} = \rho g h$$

where $\rho = \frac{m}{V}$ = fluid density

g - acceleration of gravity

h - depth of fluid

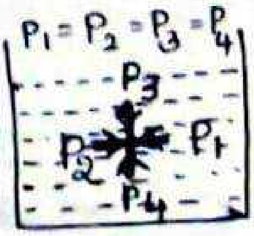
Lecture - 29

pascal's law:- This law tells about the transmission of pressure in a liquid. It can be stated in the following equivalent ways

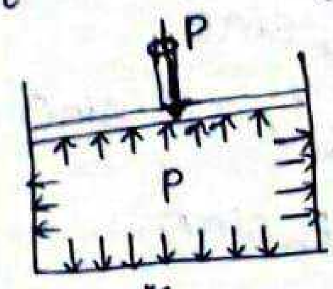
i) The pressure exerted at any point on an enclosed liquid is transmitted equally in all directions.

ii) A change in pressure applied to an enclosed incompressible liquid is transmitted undiminished to every point of the liquid and the walls of the container

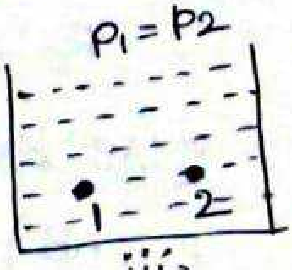
iii) The pressure in a liquid at rest is same at all points if we ignore gravity.



(i)



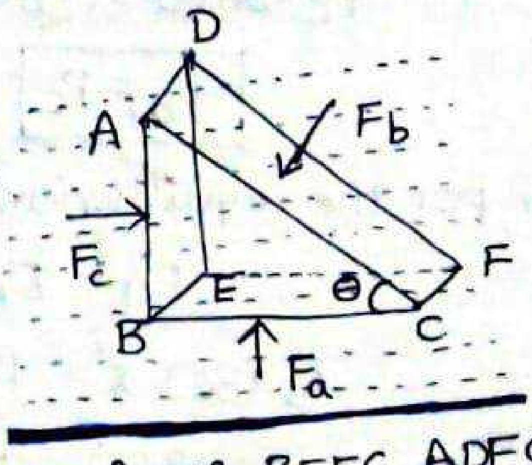
(ii)



(iii)

proof of pascal's law:-

consider a small element ABC-DEF in the form of a right angled prism inside a liquid at rest.



Suppose the exerts pressure P_a, P_b and P_c on the faces BEFC, ADFC ADEB respectively of the element.

If F_a, F_b, F_c are the corresponding forces on these faces, then

$P = \frac{F}{A}$

$F = PA$

$= P\, l \times b$

$F = P\, b \times l$

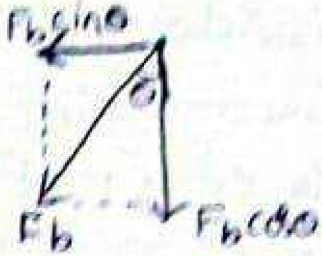
$F_a = P_a (BC)\, l$

$F_b = P_b (AC)\, l$

$F_c = P_c (AB)\, l$

As the element is at rest, so net force on it must be zero. we can write

~~for the equilibrium in horizontal direction~~



Along horizontal direction

$$F_c = F_b \sin\theta$$

Along vertical direction

$$F_a = F_b \cos\theta$$

i) For the equilibrium horizontal direction

$$F_c = F_b \sin\theta$$

$$P_c (AB) \cancel{l} = P_b (AC) \cancel{l} \sin\theta$$

$$P_c \left(\frac{AB}{AC}\right) = P_b \sin\theta$$

$$P_c \cancel{\sin\theta} = P_b \cancel{\sin\theta}$$

$$\boxed{P_b = P_c}$$

ii) For the equilibrium vertical direction

$$F_a = F_b \cos\theta$$

$$P_a (BC) \cancel{l} = P_b (AC) \cancel{l} \cos\theta$$

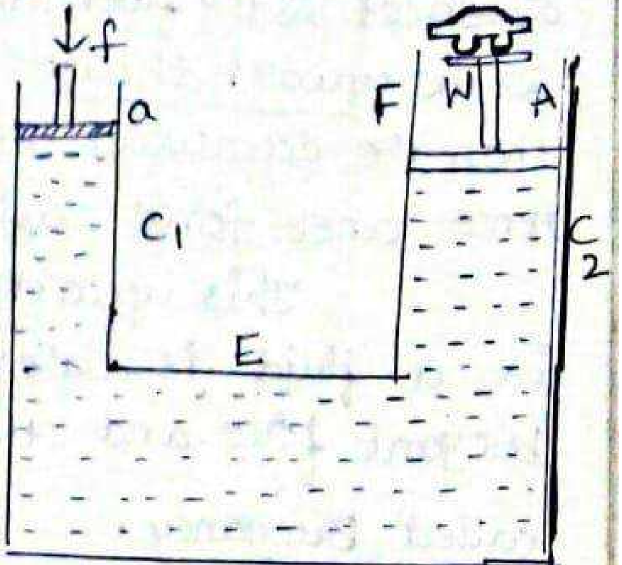
$$P_a \left(\frac{BC}{AC}\right) = P_b \cos\theta$$

$$P_a \cancel{\cos\theta} = P_b \cancel{\cos\theta}$$

$$\boxed{P_a = P_b}$$

Hence, the pressure exerted by the fluid at rest on a body in the fluid is same in all directions. This proves pascal's law.

Application of pascal's law [Hydraulic Lift]



It is used to lift heavy loads (cars, trucks) for small height.

A piston of small cross sectional area 'a' is used to exert a small effort "f" on a liquid such as oil.

The pressure $p = \frac{f}{a}$.

This pressure is transmitted to a larger cylinder (C_2) equipped with a larger piston of area 'A' through a pipe (E).

According to pascal's law

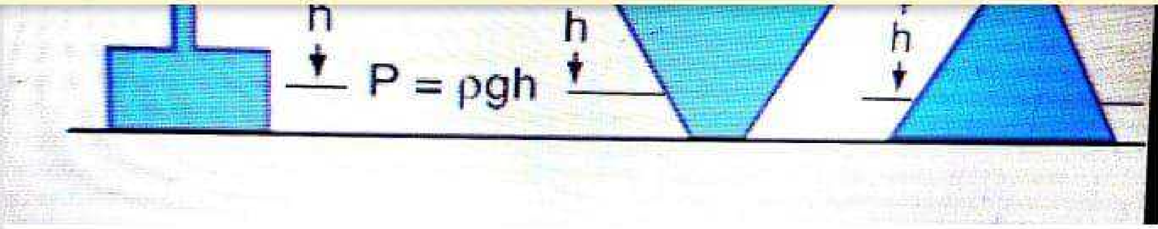
pressure at smaller piston = pressure at larger piston

$$\frac{f}{a} = \frac{W}{A}$$

$$W = f\left(\frac{A}{a}\right)$$

as $A > a$, $\therefore$ $W > f$

Hence by making $\left(\frac{A}{a}\right)$ larger, heavy loads can be lifted by applying small effort.



EXAMPLE |1| Pressure Exerted by Human Body
The two thigh bones (femurs), each of cross-sectional area 10 cm^2 support the upper part of a human body of mass 40 kg. Estimate the average pressure sustained by the femurs. [NCERT]

Sol. Given, $A = 20 \times 10^{-4} m^2$

Weight of body acting vertically downwards

Force on bones, $F = 40$ kg - wt $= 400$ N $\quad [\because g = 10 \, m/s^2]$

$$p_{av} = \frac{F}{A} = \frac{400}{20 \times 10^{-4}}$$

$$= 2 \times 10^5 \, N/m^2$$

Pascal's law

Pascal's law states that "A change in pressure exerted at any part of

$$\Rightarrow \qquad \Delta L = \frac{FL}{AY} = \frac{3\times 10 \times 4}{2\times 110 \times 10^{9}} = 0.0545\times 10^{-6} \text{ m}$$

$$\Delta L = 54.5\times 10^{-3} \text{ mm}$$

EXAMPLE |3| Finding Young's Modulus

The ball of 200 g is attached to the end of a string of an elastic material (say rubber) and having length and cross-sectional area of 51 cm and 22 mm², respectively. Find the Young's modulus of this material if string is whirled round, horizontally at a uniform speed of 50 rpm in a circle of diameter 104 cm.

Sol. Mass of the ball, $M = 200\text{ g} = 0.2\text{ kg}$

Area of cross-section, $A = 22\text{mm}^2 = 22\times 10^{-6}\text{ m}^2$

Radius of the circle, $r = \dfrac{D}{2} = \dfrac{104}{2} = 52\text{ cm} = 0.52\text{ m}$

Length of the string, $l = 51\text{ cm} = 0.51\text{ m}$

Revolution per second, $= 50\times 60\text{ rps} = 3000\text{ rps}$

Certain petal force, $F = mr\omega^2 = 0.2\times 0.52\times(2\pi\times N)^2$

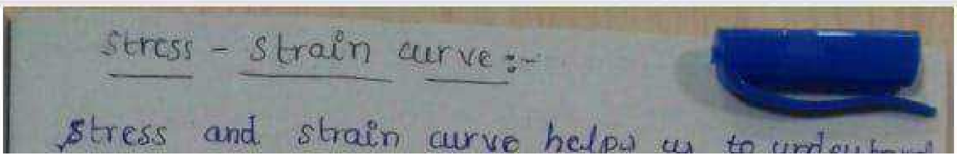
$$F = 36.95\times 10^6\text{ N}$$

The change in length Δl

$$\Delta l = \text{radius of the circle} - \text{length of the string}$$
$$= 0.52 - 0.51$$
$$\Delta l = 0.01\text{ m}$$

Young's modulus of the material

$$Y = \frac{F}{A}\frac{l}{\Delta l} = \frac{36.95\times 10^6}{22\times 10^{-6}}\times\frac{0.51}{0.01} = 85.67\times 10^{12}\text{ Nm}^{-2}$$



Vo)) LTE 1 4:06 p.m.

8. **Find the increase in pressure required to decrease the volume of a water sample by 0.05%. Bulk modulus of water = 2.1 × 10⁹ Pa**

Solution

$$\frac{dV}{V} = -0.05\% = \frac{-0.05}{100}$$

$$B = -\frac{dp}{\left(\frac{dV}{V}\right)} \Rightarrow dp = 2.1\times10^{9}\times\frac{0.05}{100} = 1.05\times10^{6}\text{ Pa.}$$

Hence, to decrease the volume of water by 0.05% pressure should be increased by 1.05×10^6 Pa.

9. **Compute the bulk modulus of water from the following data:**

Initial volume = 100.5 litre

Pressure increase = 100.0 atm

Final volume = 100.0 litre

Solution

$\Delta V = -0.5$ litre $= -0.5 \times 10^{-3}$ m³

$\Delta P = 100.0$ at $m = 100 \times 1.013 \times 10^5$ Pa

$V = 100.5$ litre $= 100.5 \times 10^{-3}$ m³

$$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{1.013\times10^{7}\times100.5\times10^{-3}}{0.5\times10^{-3}} = 2.04\times10^{9}\text{ N m}^{-2}$$

13. **Calculate the work done in stretching a 2 m long wire uniformly by 0.5 cm. Given : Young's modulus of the material of the wire is 8 × 10¹⁰ N m⁻². Radius of the wire is 0.89 mm.**

Solution

$$Work\ done = \frac{1}{2}\times Stress\times Strain\times Volume$$

$$= \frac{1}{2}\times Y\times Strain\times Strain\times \pi r^2 l$$

$$= \frac{1}{2}\times Y\times Strain^2\times \pi r^2 l$$

$$= \frac{1}{2}\times\left(\frac{e}{l}\right)^2\times \pi r^2 l\times Y$$

$$= \frac{1}{2}\times\frac{e^2\times \pi r^2}{l}\times Y$$

$$= \frac{1}{2}\times\frac{(0.5\times10^{-2})^2\times\pi\times(0.89\times10^{-3})^2}{2}\times8\times10^{10}$$

$= 1.24$ J

EXAMPLE |1| Stress in a Wire

EXAMPLE |1| Stress in a Wire

Calculate the value of stress in a wire of steel having radius of 2 mm of 10 kN of force is applied on it.

Sol. Force, $F = 10\text{ kN} = 1\times10^{4}\text{ N}$

Radius, $r = 2\text{ mm} = 2\times10^{-3}\text{ m}$

Area, $A = \pi r^2 = \pi\times(2\times10^{-3})^2$

$$= 12.56\times10^{-6}\text{ m}^2$$

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{1\times10^{4}\text{ N}}{12.56\times10^{-6}\text{ m}^2}$$

$$= 0.0796\times10^{10}$$

$$= 7.96\times10^{8}\text{ N/m}^2$$

EXAMPLE |1| An Elongated Wire

If a wire of length 4 m and cross-sectional area of 2m^2 is stretched by a force of 3 kN, then determine the change in length due to this force. Given Young's modulus of material of wire is $110\times10^{9}\text{ N/m}^2$.

Sol. Given, area of cross-section, $A = 2\text{ m}^2$

Force, $F = 3\text{ kN} = 3\times10^{3}\text{ N}$

Length, $L = 4\text{ m}$

Young's modulus, $Y = 110\times10^{9}\text{ N/m}^2$

Change in length, $\Delta L = ?$

Apply, $Y = \dfrac{FL}{A\Delta L}$

$$\Rightarrow \quad \Delta L = \frac{FL}{AY} = \frac{3\times10^{3}\times4}{2\times110\times10^{9}} = 0.0545\times10^{-6}\text{ m}$$

$$\Delta L = 54.5\times10^{-3}\text{ mm}$$

EXAMPLE |3| Finding Young's Modulus

The ball of 200 g is attached to the end of a string of an elastic material (say rubber) and having length and cross-sectional area of 51 cm and 22 mm^2 respectively

$T = mg = 10 \times 9.8 = 98$ N (as both masses matter)

$$\text{Young's modulus } (Y) = \frac{Stress}{Strain} = \frac{\left(\frac{F}{\pi r^2}\right)}{\left(\frac{e}{l}\right)} = \frac{Fl}{\pi r^2 e}$$

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{98 \times 1.5}{\pi (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.5 \times 10^{-4} \text{ m} = 0.15 \text{ mm}$$

(b) Elongation in brass

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{6 \times 98 \times 1.0}{\pi (0.125 \times 10^{-2})^2 \times 1.3 \times 10^{11}} = 9.21 \times 10^{-5} \text{ m} = 92.1 \text{ µm.}$$

4. A copper wire of cross-sectional area 0.001 cm² is under a tension of 15 N. Find the decrease in the cross-sectional area. Young's modulus of copper = 1.2×10^{11} N m^{-2} and Poisson's ratio = 0.31.

Solution

$T = 15$ N, $A = 1 \times 10^{-7}$ m², $Y = 1.2 \times 10^{11}$ N m^{-2},

$\sigma = 0.31$.

$$Y = \frac{Stress}{Longitudinal\ strain\ (\alpha)} = \frac{\frac{T}{A}}{\alpha}$$

$$\alpha = \frac{T}{AY}$$

$$\sigma = \frac{Lateral\ strain\ (\beta)}{Longitudinal\ strain (\alpha)}$$

$$\beta = \sigma\alpha = \frac{\sigma T}{AY}$$

$$\frac{dr}{r} = \frac{\sigma T}{AY}$$ where dr = decrease in radius and r = original radius

$$\frac{dr}{r} = \frac{0.31 \times 15}{1 \times 10^{-7} \times 1.2 \times 10^{11}}$$

$= 3.875 \times 10^{-4}$

$A = \pi r^2$

$\therefore dA = 2\pi r dr$

$$\frac{dA}{A} = \frac{2\pi r dr}{\pi r^2} = 2\frac{dr}{r} = 2 \times 3.875 \times 10^{-4}$$

Decrease in area $= 7.75 \times 10^{-4} \times 10^{-7} = 7.75 \times 10^{-11}$ m²

$\therefore\ e = 12 \times 10^{-?} \times 3 = 36 \times 10^{-?}$ m.

2. **Calculate the longest length of steel wire that can hang vertically without breaking. Breaking stress for steel = 7.982 × 10⁸ N m⁻² and density of steel = 8.1 × 10³ kg m⁻³.**

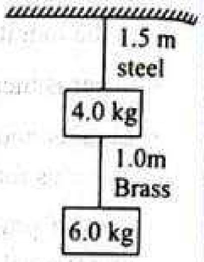
Solution

$$Stress = \frac{Force}{Area} = \frac{mg}{A} = \frac{\rho(Al)g}{A}$$

$\therefore$ Breaking stress $= \rho(l_{max})g$.

$$\Rightarrow l_{max} = \frac{7.982\times10^8}{8.1\times10^3\times9.8} = 1.01\times10^4\,\text{m} = 10.1\text{ km}$$

3. **Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and brass wires.**



Young's modulus of steel = 2.0×10^{11} N m^{-2}

Young's modulus of brass = 1.3×10^{11} N m^{-2}

Solution

(a) Elongation in steel

$T = mg = 10 \times 9.8 = 98$ N (as both masses matter)

$$Young's\ modulus\,(Y) = \frac{Stress}{Strain} = \frac{\left(\frac{F}{\pi r^2}\right)}{\left(\frac{e}{l}\right)} = \frac{Fl}{\pi r^2 e}$$

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{98\times1.5}{\pi(0.125\times10^{-2})^2\times2\times10^{11}} = 1.5\times10^{-4}\text{ m} = 0.15\text{ mm}$$

(b) Elongation in brass

$$\therefore e = \frac{Fl}{\pi r^2 Y} = \frac{6\times98\times1.0}{\pi(0.125\times10^{-2})^2\times1.3\times10^{11}} = 9.21\times10^{-5}\text{ m} = 92.1\ \mu\text{m}.$$

4. **A copper wire of cross-sectional area 0.001 cm² is under a tension of 15 N. Find the decrease in the cross-sectional area. Young's modulus of copper = 1.2×10^{11} N m^{-2} and Poisson's ratio = 0.31.**

Solution

$T = 15$ N, $A = 1 \times 10^{-7}$ m², $Y = 1.2 \times 10^{11}$ N m^{-2},

$\sigma = 0.31$.

$$Y = \frac{Stress}{Longitudinal\ strain\ (\alpha)} = \frac{\frac{T}{A}}{\alpha}$$

$$\alpha = \frac{T}{AY}$$

$$\sigma = \frac{Lateral\ strain\ (\beta)}{Longitudinal\ strain(\alpha)}$$

1. A load of 12 kg is suspended by a metal wire 3 m long and having a cross-sectional area 5 mm². Find (a) the stress (b) the strain and (c) the elongation. Given : Young's modulus of the metal is 2.0×10^{11} N m^{-2}; g = 10 m s^{-2}.

Solution

$F = mg = 12 \times 10 = 120$ N; $l = 3$ m, $A = 5 \times 10^{-6}$ m².

(a) $\text{Stress} = \frac{F}{A} = \frac{120}{5\times10^{-6}} = 24\times10^{6}\,\text{N m}^{-2}$

(b) $Y = \frac{Stress}{Strain} \Rightarrow Strain = \frac{Stress}{Y}$

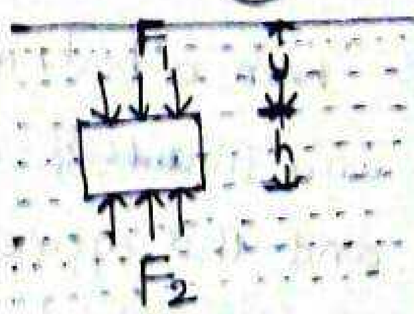
$\therefore Strain = \frac{24\times10^{6}}{2\times10^{11}} = 12\times10^{-5}$

(c) $Strain = \frac{Elongation}{Original\ length} \Rightarrow e = Strain \times l$

$\therefore\ e = 12 \times 10^{-5} \times 3 = 36 \times 10^{-5}$ m.

2. Calculate the longest length of steel wire that can hang vertically without breaking. Breaking

The top face of the body is at a distance 'y' from the free surface of the liquid



pressure at the top face of the body

$$P_1 = \rho g y$$

pressure at the bottom face of the body

$$P_2 = \rho g (y+h)$$

Thrust on the top face of the body

$$F_1 = P_1 A = \rho g y A$$ acting vertically downwards

Thrust acting on the bottom face of the body

$$F_2 = P_2 A = \rho g (y+h) A$$ acting vertically upwards

The resultant force $(F_2 - F_1)$ acts on the body in the upward direction and is called buoyant force (F_b). Thus

$$F_b = F_2 - F_1 = \rho g (y+h) A - \rho g y A$$

$$= \rho g h A$$

$$F_b = \rho g V \quad [\because V = Ah]$$

$V = Ah$, the volume of the body, which is equal to the volume of the liquid displaced.

$$\therefore F_b = (\rho V) g = mg \qquad [\because \rho = \frac{m}{V}]$$

$$\boxed{F_b = mg}$$

Thus buoyant force is equal to the weight of the fluid displa-

Lecture -30

Buoyancy :-

Body inside a fluid experiences pressure on its all faces. As the fluid pressure increases with depth, so the upward thrust at the bottom is more than the downward thrust on the top.
Hence a net force acts in upward direction.
This upward force acting on a body in a fluid is called upthrust or buoyant force and the phenomenon is called Buoyancy.

Archimede's principle :-

The Archimede's principle gives the magnitude of buoyant force on a body. It states that
"When a body is immersed in a fluid, partially or wholly, it experiences an upward force equal to the weight of the volume of the fluid displaced by the body"

Proof :- consider a body of height "h" and area "A", lying inside a liquid of density "ρ"
The upward force (F_2) on the bottom of the body is more than the downward force (F_1) on its top. If P_1 & P_2 are the pressures at upper face and lower face of the body respectively

When a body is immersed completely or partially in a liquid it appears to lose a part of its weight and this apparent loss of weight is equal to the weight of the liquid displaced by the body.

i.e The apparent loss of weight of the body in the liquid = weight of the liquid displaced by the body.

Lecture -31

Ideal Fluid :-

"An ideal fluid is one which is incompressible and non-viscous".

An ideal fluid has the following characteristics

1. incompressible :- volume can't be changed by application of pressure. So it is incompressible (density is constant)

2. non-viscous :- As the fluid moves, there is no friction between its layers. So no loss of energy. It is non-viscous.

3. Flow of Ideal fluid is irrotational.

4. velocity of flow and pressure do not change with time. It exhibits a steady flow.

5. They have no surface tension

6. water is an example for Ideal fluid

To understand fluid dynamics, we assume that the fluid is Ideal.

properties of streamline :-

1. In stream line flow, no two stream lines can cross each other.

2. the tangent at any point on the line of flow gives the direction of flow.

3. The path that the fluid follows in stream line flow starting from anypoint, is called a stream line.

4. The stream lines may curve and bend, but they cannot cross each other.

5. No two streamlines can cross each other.

6. velocity is independent of time.

7. the liquid having stream line flow in form of parallel layers one above the other.

The flow of fluids is divided into two types, namely

i) streamline flow and (ii) Turbulent flow

1. Streamline flow :- If a fluid flows such that the velocity of its particles at a given point is always the same in magnitude and direction, the fluid is said to have a streamline flow. it is also called steady flow.

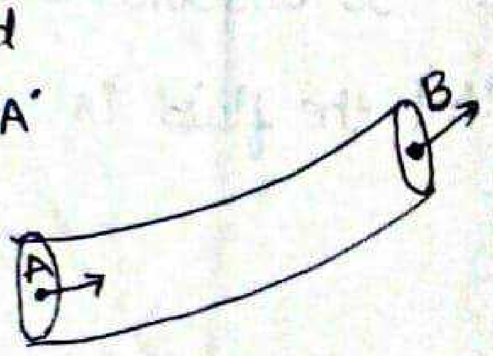
In this each particle of the liquid passing through a point travels along the same path and with same velocity as the preceeding particle passing through the same point.

It is also defined as

a curve whose tangent at any point is in the direction of the fluid velocity at that point.

Consider a liquid passing through a tube. If the velocity of flow is small, all the particles which come to 'A' will have the same speed & directi... As a particle goes from A to another point B, its speed and direction may change, but all the particles at 'B' will have the same speed.

If one particle passing through 'A' has gone through 'B' then all the particles passing through A go through B. Such, a flow is called steady flow.



Turbulent flow:-

when the speed of the flow of fluid is within certain limits, the flow will be steady.

• when the speed of flow exceeds a limiting value called critical velocity. The orderly motion of the fluid is lost and it acquires an unsteady motion called turbulent flow.

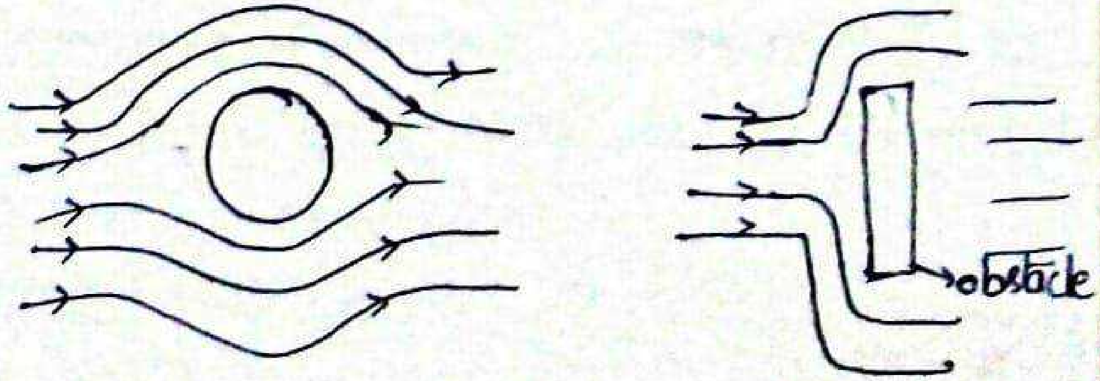
when you have obstacles through the path of flow of liquid, velocity is very high

→ There will be a sudden change in direction

→ motion of fluid becomes irregular

→ The fluid exerts very high thrust on the obstacle

→ the velocity of all particles crossing a given point is not same and motion of the fluid is disorder. Ex:- Floods, whirlpools etc.



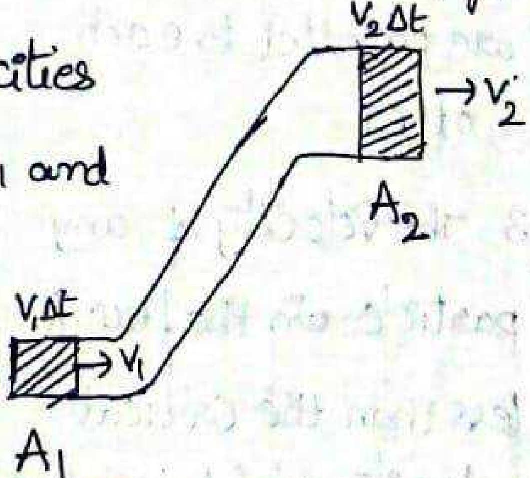
Turbulent flow of liquid

Streamline flow	Turbulent flow
1. The flow is regular and orderly	The flow is irregular and disorderly
2. The lines of flow are parallel to each other	The lines of flow are not parallel to each other.
3. The velocity of any particle in the flow is less than the critical velocity which is defined for the flow.	The velocity of particles exceeds a certain value called critical velocity.
4. Different particles cross a given point with the same velocity.	Different particles cross a given point with different velocities.

Equation of continuity :-

The equation of continuity is the mathematical statement of the principle of conservation of mass.

consider a fluid is flowing in a pipe of varying area of Cross-section as shown in fig.



Let V_1 & V_2 are the velocities of flow at cross-sections A_1 and A_2 respectively.

The mass of the fluid enters into section 1 in time Δt

$$m_1 = \rho_1 V_1$$
$$= \rho_1 (\text{Area of cross-section} \times \text{length})$$
$$= \rho_1 (A_1 V_1 \Delta t)$$

velocity $= \frac{\text{dist}}{\text{time}}$

$V = \frac{l}{t}$

$V_1 = \frac{l}{\Delta t}$

$l = V_1 \Delta t$

The mass of the fluid leaving the section '2' in the same interval of time

$$m_2 = \rho_2 V_2$$
$$= \rho_2 (A_2 V_2 \Delta t)$$

By Conservation of mass $\quad m_1 = m_2$

$$\rho_1 (A_1 V_1 \Delta t) = \rho_2 (A_2 V_2 \Delta t)$$
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For incompressible fluid density is constant

$$\rho_1 = \rho_2$$

$$\therefore \quad A_1 V_1 = A_2 V_2$$

or $$\boxed{AV = \text{constant}}$$

This is known as equation of continuity.

It states that for an incompressible fluid product of flow velocity (V) and corresponding area of cross-section (A) of a pipe remains constant throughout the flow.

Bernoulli's Theorem :-

The swiss scientist Daniel Bernoulli in 1738 first derived the principle which is based on the law of conservation of energy and applies to ideal fluid.

According to this theorem the total energy (pressure energy, potential energy & kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow.

Mathematically, it can be expressed as in terms of an equation,

$$\boxed{P + \rho g h + \frac{1}{2}\rho v^2 = \text{Constant}}$$

where, P presents pressure energy per unit volume, $\rho g h$ for potential energy per unit volume and $\frac{1}{2}\rho v^2$ for kinetic energy per unit volume and ρ is density of flowing fluid (Ideal)

This Equation is called Bernoulli's Equation.

The network done on the element during this displacement

$$W = F_1 \Delta S_1 - F_2 \Delta S_2$$

$$= P_1 A_1 \Delta S_1 - P_2 A_2 \Delta S_2$$

$$= P_1 \Delta V - P_2 \Delta V = (P_1 - P_2) \Delta V \quad \text{--- (1)}$$

change in K.E from a to b:

The mass of the fluid between a and a'

$$\Delta m = \text{density} \times \text{volume}$$

$$= \rho \Delta V \quad \text{--- (2)}$$

The K.E of the fluid between a and a'

$$K_1 = \frac{1}{2} \Delta m v_1^2$$

$$= \frac{1}{2} \rho \Delta V v_1^2$$

similarly, at the end of Δt, the K.E of the fluid between b and b'.

$$K_2 = \frac{1}{2} \rho \Delta V v_2^2$$

Thus the change in kinetic energy of the fluid between a and b.

$$\Delta K = K_2 - K_1 = \frac{1}{2} \rho \Delta V v_2^2 - \frac{1}{2} \rho \Delta V v_1^2$$

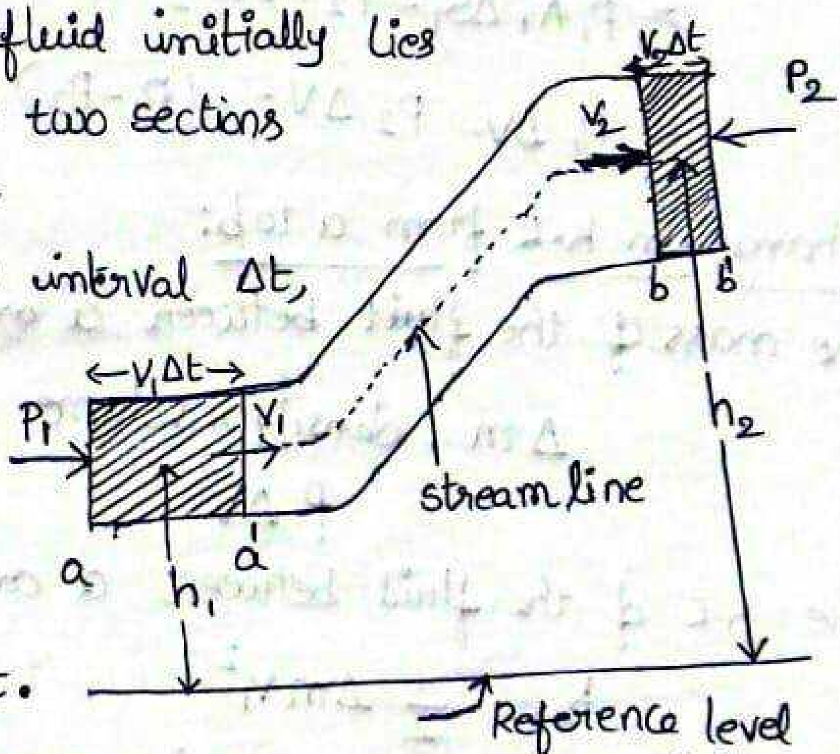
$$= \frac{1}{2} \rho \Delta V [v_2^2 - v_1^2] \quad \text{--- (3)}$$

Derivation of Bernoulli's Equation :-

To derive Bernoulli's equation, we can apply the work-energy theorem to the fluid in a section of a flow tube.

Consider the fluid initially lies between the two sections respectively.

In a time interval Δt, the fluid was initially at 'a' moves to a', a distance $v_1 \Delta t$.



If the same time the fluid initially at 'b' moves to b', a distance is $V_2 \Delta t$.

If A_1 & A_2 are the cross-sectional areas at the two ends, then by Equation of Continuity, the volume of fluid ΔV passing any cross-section in time Δt is

$$\Delta V = A_1(V_1 \Delta t) = A_2(V_2 \Delta t)$$
$$= A_1 \Delta S_1 = A_2 \Delta S_2$$

If P_1 and P_2 are the pressures at two ends, then force at the cross-section 'a' is $P_1 A_1$, and that 'b' is $P_2 A_2$.

change in potential energy:-

The potential energy of mass entering at 'a' in time Δt is,

$$U_1 = \Delta m g h_1 = \rho \Delta V g h_1$$

The potential energy of the mass leaving at 'b' is

$$U_2 = \Delta m g h_2 = \rho \Delta V g h_2$$

The change in potential energy between a & b is

$$\Delta U = U_2 - U_1 = \rho \Delta V g (h_2 - h_1)$$

Now using work-energy theorem

$$W = \Delta K + \Delta U$$

$$(P_1 - P_2) \cancel{\Delta V} = \frac{1}{2} \rho \cancel{\Delta V} (v_2^2 - v_1^2) + \rho \cancel{\Delta V} (h_2 - h_1) g$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$$

$$\boxed{P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}}$$

Bernoulli's equation can also be written as

$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

~~Limitations~~ of Bernoulli's theorem (30)

1. Bernoulli's Equation ideally applies to non-~~fluids~~ viscous fluids.

2. The fluids must be incompressible.

3. Bernoulli's equation is applicable only to streamline flow of a fluid.

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------------------------> **Problems** <------------------------

Problem 1:

A 500 g solid cube having an edge of length 20 cm floats in water. How much volume of the cube is outside the water? Density of water is 1000 kg/m^3.

Solution:

Step 1: Given data:

$m = 500\text{ g} = 0.5\text{ kg}$

$\rho = 1000\text{ kg/m}^3$

$l = 20\text{ cm}$

Step 2: To find:

Volume of the cube

$V\rho g = mg$

$$V = \frac{m}{\rho}$$

$$= \frac{0.5}{1000} = 5\text{x}10^{-4}\text{ m}^3 = 500\text{ cm}^3$$

The total volume of the cube = $(20\text{ cm})^3 = 8000\text{ cm}^3$

$\therefore$ The volume outside the water is $8000\text{ cm}^3 - 500\text{ cm}^3 = 7{,}500\text{ cm}^3$.

p 3: Answer:

10:41 a.m.

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Ex. 3 In a car lift compressed air exerts a force F_1 on a small piston having a radius of 5 cm. This pressure is transmitted to a second piston of radius 15 cm. If the mass of the car to be lifted is 1350 kg, what is F_1? What is the pressure necessary to accomplish this task? Take $g = 9.81 m/s^2$.

Sol.

As the pressure through air is transmitted equally on both the pistons, so

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Here $F_2 = mg = 1350 \times 9.81$ N

$$\therefore \quad F_1 = \frac{A_1}{A_2} F_2$$

$$= \frac{\pi r_1^2}{\pi r_2^2} F_2 = \frac{r_1^2}{r_2^2} F_2$$

$$= \frac{5^2}{15^2} \times 1350 \times 9.81$$

$$= 1.47 \times 10^3 \text{ N} \qquad \textit{Ans.}$$

Required air pressure,

$$P = \frac{F_1}{A_1} = \frac{1.47 \times 10^3}{\pi (0.05)^2}$$

$$= 1.87 \times 10^5 \text{ N/m}^2 \qquad \textit{Ans.}$$

(2) a slab of ice floats on a fresh water lake. what minimum volume must the slab have for a 60 kg woman to be able to stand on it without getting her feet wet? [$\rho_{water} = 1000\ kg/m^3$, $\rho_{ice} = 920\ kg/m^3$]

Sol:-

$m = 60\ kg$

$\rho_{water} = 1000\ kg/m^3$

$\rho_{ice} = 920\ kg/m^3$

$\rho V g = m g$

$\rho_{water} V_{ice}\ g = m_{total}\ g$

$\rho_{water} V_{ice}\ g = [60 + m_{ice}]\ g$

$\rho_{water} V_{ice} = 60 + \rho_{ice} V_{ice}$ $\left[\begin{array}{c}\rho = \frac{m}{V} \\ m = \rho V\end{array}\right]$

$\rho_{water} V_{ice} - \rho_{ice} V_{ice} = 60$

$V_{ice}\left[\rho_{water} - \rho_{ice}\right] = 60$

$$V_{ice} = \frac{60}{\rho_{water} - \rho_{ice}} = \frac{60}{1000 - 920}$$

$V_{ice} = 0.75\ m^3$

Area of cross section at N ...

To find the velocity at N; using the formula $a_1 v_1 = a_2 v_2$, we get

$1.2 \times 10^{-4} \times 9.8 \times 10^{-2} = 10 \times 10^{-6} \times v_2$

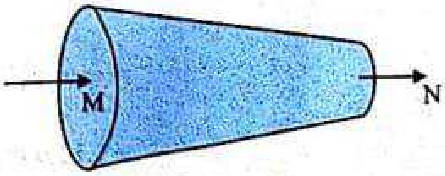
$$v_2 = \frac{1.2 \times 9.8}{10} = 1.176 \text{ m s}^{-1}$$

By Bernoulli's theorem

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

For a horizontal flow $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$= \frac{1}{2}\rho\,[(1.176)^2 - (0.098)^2]$$

$$= \frac{1}{2} \times 1048\,[1.383 - 0.0096]$$

$$= \frac{1}{2} \times 1048 \times 1.373 = 719.45 \text{ Pa}$$

$$\therefore P_1 - P_2 = 719.45 \text{ N m}^{-2}.$$

5. The level of water in a tank is 5 m high. A hole of area 1 cm² is made in the bottom of the tank. Calculate the rate of leakage of water from the hole ($g = 10$ m s^{-2}).

EXAMPLE |1| **Water Flowing Through Two Pipes**
Consider the two horizontal pipes of different diameters which are connected together and the water is flowing through these two pipes. In the first pipe, the pressure is $3.0 \times 10^4 \text{ N/m}^2$ and the speed of the water flowing is 5 m/s. If the diameters of the pipes are 4 cm and 6 cm, respectively, then what will be the speed and the pressure of the water in the second pipe? Density of the water is 10^3 kg/m^3.

Sol. According to the equation of continuity, we get

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\therefore \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Given,

$$r_1 = \frac{4}{2} = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$r_2 = \frac{6}{2} = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

$$v_1 = 5 \text{ m/s}$$

$$v_2 = \left(\frac{2}{3}\right)^2 \times 5 = 2.22 \text{ m/s}$$

If 'ρ' is the density of the flowing fluid and P_1 and P_2 are the pressures of fluid at inlet and throat, then by Bernoulli's equation :

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$V_2^2 - V_1^2 = \frac{2(P_1 - P_2)}{\rho} \longrightarrow (2)$$

substituting Eq^n (1) in Eq^n (2)

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = \frac{2(P_1 - P_2)}{\rho}$$

$$Q^2\left[\frac{A_1^2 - A_2^2}{A_1^2 A_2^2}\right] = \frac{2(P_1 - P_2)}{\rho}$$

$$Q^2 = A_1^2 A_2^2 \frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}$$

$$Q = A_1 A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}} \longrightarrow (3)$$

From the manometer $P_1 - P_2 = \rho_m g h$

$$\therefore \quad Q = A_1 A_2 \sqrt{\frac{2\rho_m g h}{\rho(A_1^2 - A_2^2)}}$$

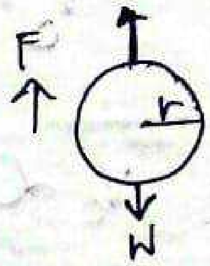
If simple manometers are inserted in inlet & throat, then $\rho_m = \rho$

$$\Rightarrow \quad Q = A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

Terminal velocity and stoke's law :-

when a body moves through a fluid, the fluid in contact with body is also dragged with it and the relative motion between fluid layer oppose the motion of body.

According to stoke's law, the viscous force on a sphere of radius 'r' moving with speed 'v' through a fluid of co-efficient of viscosity (η) is given by :-



$$\Rightarrow \boxed{F_V = 6\pi\eta r v}$$

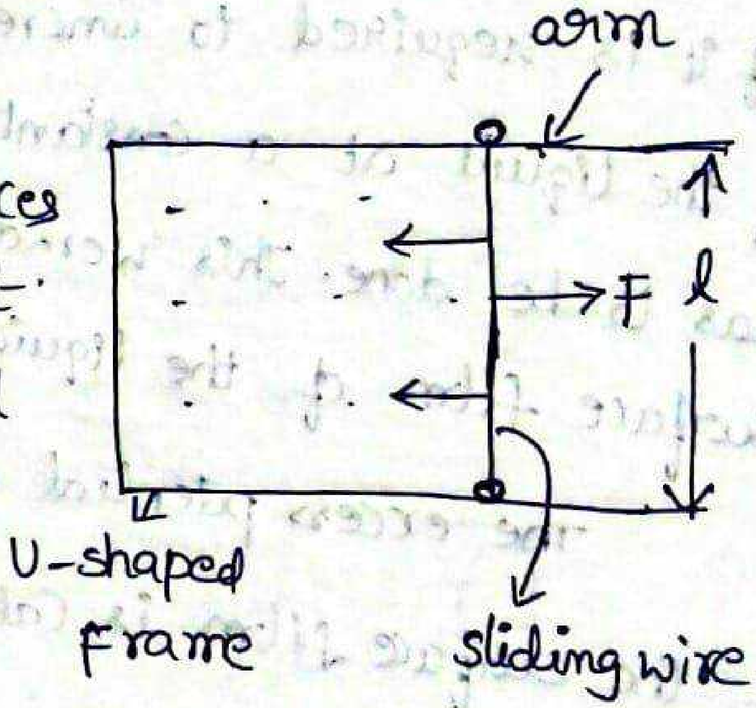
$6\pi\eta$ is constant of proportionality

v = terminal velocity.

The velocity attend after the viscous force balances the difference of weight and Buoyant force and the velocity becomes constant after this state achieved.

Relation between surface tension and Surface energy

Consider a rectangular wire frame. It has two surfaces. Both surface in contact with the sliding wire.



→ both surface apply forces of surface tension on it.

→ Net force (or) pull by surface of soap on wire

$$F = 2(Tl)$$

we apply an equal and opposite face to Keep the sliding wire in equilibrium.

suppose the sliding wire is slowly pulled by the force "F" through a distance 'x' so that the area of the soapfilm increases by lx. The new created surface area of the soap film is $2(lx)$.

Surface Energy :-

The free surface of a liquid always has a tendency to contract and possess minimum surface area.

If it is required to increase the surface area of the liquid at a constant temperature, work has to be done. This workdone is stored in the surface film of the liquid as its potential energy.

The excess potential energy per unit area of the surface film is called surface energy.

Hence, the surface energy may be defined as the amount of workdone in increasing the area of the liquid surface by unity. Thus,

$$\text{surface energy} = \frac{\text{work done in increasing the surface area}}{\text{Increase in surface area}}$$

Its S.I unit is J/m^2. Its CGS unit is erg/cm^2.

$$1\ J/m^2 = \frac{10^7 erg}{10^4 cm^2} = 10^3 erg/cm^2$$

Its dimensional formula $= [M^1 L^0 T^{-2}]$

If 'F' is the force acting perpendicular to the imaginary line of length l, then (41)

$$F \propto l$$

$$F = Tl$$

$$\boxed{T = \frac{F}{l}}$$

here T is called surface tension.

Its SI unit is N/m

Its CGS unit is dyne/cm

Its dimensional formula = $[M L^0 T^{-2}]$

* It is a Scalar
* It depends on nature of fluid & intermolecular forces
* surface Tension decreases with rise in temperature and becomes zero at critical temperature.
* The surface tension of liquids changes appreciably with addition of impurities.

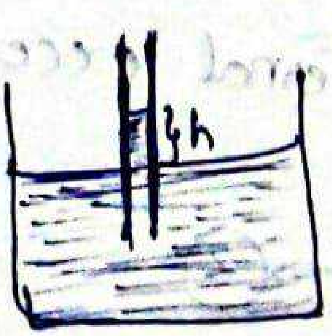
Capillarity

→ A tube of very narrow bore is called a capillary tube.

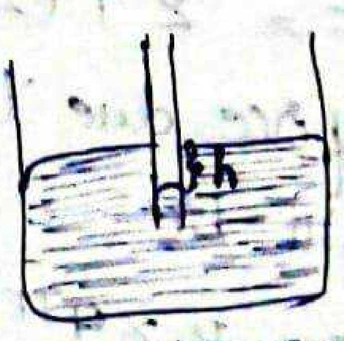
→ capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

→ The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary fall.

→ It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.



Water



Mercury

Example 7: What should be the radius of a capillary tube if the water has to rise to a height of 6 cm in it. Surface tension of water = $7.2 \times 10^{-2}Nm^{-1}$.

Solution

Height (h) = 6 cm = 6×10^{-2} m.

Surface tension of water (T) = 7.2×10^{-2} Nm^{-1}.

for water $\theta = 0 \therefore \cos\theta = 1$

Density of water (d) = 1000 kg m^{-3}.

$$\text{Surface tension (T)} = \frac{hrdg}{2\cos\theta}$$

Substituting the values

$$7.2 \times 10^{-2} = \frac{(6 \times 10^{-2})(r)(1000)(9.8)}{2 \times 1}$$

③ A circular disc of radius $7/\pi$ cm is placed horizontally on the surface of water. Find the vertical force required to separate it from the water surface ($T = 0.070$ N/m Surface tension of water)

Sol:- The downward force on the circumference of the circular disc $= T \times 2\pi R$

The vertical force required to separate it from the water surface = Downward force

$$F = T \times 2\pi R$$

$$F = 0.070 \times 2 \times \pi \times \frac{7}{\pi}$$

$$F = 9.8 \times 10^{-2} N$$

EXAMPLE |2| Water Rise in Capillary

A capillary of radius 0.05 cm is immersed in water. Find the value of rise of water in capillary if value for the surface tension is 0.073 N/m and angle of contact is 0°.

Sol. Given, $S = 0.073$ N/m,

$$R = 0.05\,\text{cm} = 5 \times 10^{-4}\,\text{m}$$

$$\theta = 0°, \; h = ?$$

From the formula, $h = \dfrac{2S\cos\theta}{\rho g R}$

$$= \frac{2 \times 0.073 \times \cos 0°}{10^3 \times 9.8 \times 5 \times 10^{-4}} = 0.02979\,\text{m}$$

2) A horizontal circular loop of wire of diameter 0.08m is lowered into a oil. The force due to surface tension required to pull the loop out of the liquid is 0.0226 N. Calculate the surface tension of oil.

Sol:- $d = 0.08\ m$

$r = 0.04\ m$

$F = 0.0226N, \quad T = ?$

$$F = TL$$

$$\therefore T = \frac{F}{L} = \frac{F}{2 \cdot 2\pi r}$$

$$= \frac{0.0226}{2 \times 2 \times 3.14 \times 0.04}$$

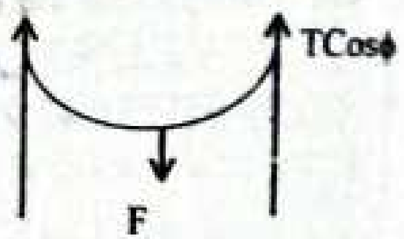
$$T = 0.0449\ N/m$$

$$\therefore r = \frac{2\times 7.2\times 10^{-2}}{6\times 10^{-2}\times 1000\times 9.8} \qquad r = 0.24\times 10^{-3}\,m$$

Example 8: In an experiment with capillary tube a liquid of density 0.75 x 10^3 kgm^{-3} rises to a height of 19.8 x 10^{-3} m in a capillary tube of diameter 2 mm. Find the surface tension of the liquid (angle of contact is 8°).

Solution

Radius of Capillary tube (r) = $\frac{2\text{ mm}}{2} = 1\text{ mm} = 1\times 10^{-3}\,m$



Height (h) = 19.8 x 10^{-3} m

Density of the liquid (d) = 0.75 x 10^{3} kg m^{-3}

acceleration due to gravity (g) = 9.8 ms^{-2}

Angle of contact (θ) = 8°

$\therefore \cos\theta = \cos 8^\circ = 0.9003$

$$T = \frac{hrdg}{2\cos\theta} = \frac{(19.8\times 10^{-3})\times(1\times 10^{-3})\times(0.75\times 10^{3})\times 9.8}{2\times 0.9903}$$

Surface tension (T) = $73.5\times 10^{-3}\,Nm^{-1}$.

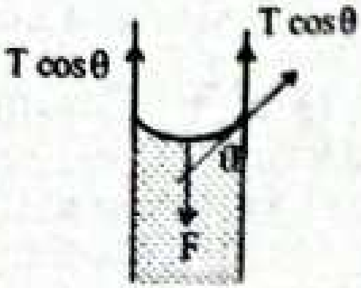
Example 9: Find the weight of water supported by surface tension in a capillary tube with a radius of 0.2 mm surface tension of water is 0.072 Nm^{-1}; and angle of contact of water is 0°.

Solution

Assume the weight of water to be 'F'.

weight of water in a capillary tube = upward force due to surface tension (see figure).

i.e. $F = 2\pi r\,(T\cos\theta)$



Surface tension of water T = 0.072Nm^{-1}

Angle of contact $\theta = 0^\circ$

Radius of Capillary tube (r) = $\frac{0.2}{1000}\,m = 0.2\times 10^{-3}\,m$.

$F = 2\pi r(T\cos\theta) \;= 2\times\frac{22}{7}\times 0.2\times 10^{-3}\times 0.072\times 1 \;= 90.51\times 10^{-6}\,N$. $F = 90.51\times 10^{-6}\,N$.

Example 12: The diameter of a soap bubble is 8 mm and the surface tension of soap solution is 4×10^{-2} Nm^{-1}. Find the excess pressure inside the bubble.

Solution

Diameter of the bubble	=	8 mm = 8×10^{-3} m
Radius of the bubble (r)	=	4×10^{-3} m
Surface tension (T)	=	4×10^{-2} Nm^{-1}

$$\text{Excess pressure inside the bubble} = \frac{4T}{r}$$

$$= \frac{4 \times 4 \times 10^{-2}}{4 \times 10^{-3}} = 40 Nm^{-2}$$

$\therefore$ Excess pressure inside the bubble = 40 Nm^{-2}.

- units of α, β, γ - $^{\circ}C$ or K

In general with change in volume, the density will also change.

Relation among the coefficients of expansion

$$\alpha = \frac{\Delta l}{l} \cdot \frac{1}{\Delta T}$$

$$\beta = \frac{\Delta A}{A} \cdot \frac{1}{\Delta T}$$

$$\gamma = \frac{\Delta V}{V} \cdot \frac{1}{\Delta T}$$

Its ratio is given by

$$\alpha : \beta : \gamma = \alpha : 2\alpha : 3\alpha = 1:2:3$$

$$\boxed{\alpha : \beta : \gamma = 1:2:3}$$ [$\because \beta = 2\alpha, \gamma = 3\alpha$]

2 Expansion of Liquids :-

In liquids only expansion in volume takes place on heating because liquids have no shape of their own.

If v is the volume of the liquid is heated and its temperature is raised by $\Delta\theta$ then

$$V_L = V(1 + \gamma_L \Delta\theta)$$

γ_L = coefficient of real expansion or coefficient of volume expansion of liquid.

As liquid is always taken in a vessel for heating so if a liquid is heated, the vessel also gets heated and it also expands.

$$V_S = V(1 + \gamma_S \Delta)$$

γ_S - Coefficient of volume expansion for solid vessel.

So the change in volume of liquid relative to vessel.

$$V_L - V_S = V[\gamma_L - \gamma_S] \Delta T$$

$$\Delta V_{app} = V \gamma_{app} \Delta T$$

$\gamma_{app} = \gamma_L - \gamma_S$ = Apparent Coefficient of volume expansion for liquid.

If $\gamma_L > \gamma_S$ - Level of liquid in vessel will rise on heating $(\gamma = +ve]$

$\gamma_L < \gamma_S$ - Level of liquid in vessel will fall on heating $[\gamma = -ve]$

$\gamma_L = \gamma_S$ - level of liquid in vessel will remain same. $[\gamma = 0]$

Anomalous Expansion of water :—

Generally matter expands on heating and contracts on cooling.

Incase of water, it expands on heating if its temperature is greater than $4^{\circ}C$.

In the range $0^{\circ}C$ to $4^{\circ}C$, water contracts on heating and expands on cooling i.e γ is negative.

This behaviour of water in the range from $0^{\circ}C$ to $4^{\circ}C$ is called anomalous expansion.

Expansion of Gases :—

Gases have no definite shape, therefore gases have only volume expansion. Since the expansion of container is negligible in comparison to the gases, therefore gases have only real expansion.

Coefficient of volume expansion :-

At constant pressure, the unit volume of a given mass of a gas, increases with $1^{\circ}C$ rise of temperature, is called coefficient of volume expansion

$$\alpha = \frac{\Delta V}{V} \times \frac{1}{\Delta T}$$

$\therefore$ Final volume $V' = V(1+\alpha\Delta T)$

Coefficient of pressure expansion:

$$\beta = \frac{\Delta P}{P} \times \frac{1}{\Delta T}$$

Final pressure $P' = P(1+\beta\Delta T)$

For an ideal gas, coefficient of volume expansion is equal to the coefficient of pressure expansion.

Lecture - 38

- **Temperature** :- Temperature is the measure of degree of hotness or coldness of a body.

⇒ It is a relative measurement of temperature. It is a scalar physical quantity.

Consider two bodies with temperatures T_1 and T_2 where $T_1 > T_2$, then the body with T_1 is called hotter one with respect to another one which is known as colder body.

⇒ A device which measures the thermal condition of a body is called thermometer.

⇒ SI unit :- Kelvin (K)

⇒ commonly used unit : $°C$ (or) $°F$

⇒ Conversion : $t_K = t_{°C} + 273.15$

→ Branch of physics dealing with production and measurement of temperatures close to 0K is known as Cryogenics while that dealing with the measurement of very high temperature is called as Pyrometry.

→ Normal temperature of human body is 310.15 K ($37°C = 98.6°F$)

when mechanical energy (work) is converted into heat, the ratio of workdone (W) to heat produced (Q) always remains the same and constant, represented by J.

$$\frac{W}{Q} = J \quad (\text{or}) \quad W = JQ$$

J is called Mechanical equivalent of heat and has value 4.2 J/cal

Different Scales of Temperature :-

The measurement of temperature is done by some specified scales are commonly called thermometers. It measures the temperature of the body in the unit as Kelvin (K), degree centigrade (°C), degree fahrenheit (°F), etc..., among which Kelvin (K) Scale is Absolute temperature Scale.

observation of Thermometric property with the change in temperature and comparing it with certain reference situations.

Reference situation is generally ice point or steam point.

Name of the scale	Symbol for each degree	Ice point (or) Lower fixed point (L.F.P)	Steam point (or) Upper fixed point (U.F.P)	Number of divisions on the scale.
Celsius scale	°C	0°C	100°C	100
Fahrenheit scale	°F	32°F	212°F	180
Kelvin scale	K	273.15 K	373.15 K	100

temperature on one scale can be converted into other scale by using the following identity.

$$\frac{\text{Reading on any scale} - LFP}{UFP - LFP} = \text{Constant for all scales}$$

$$\frac{C-0}{100-0} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15}$$

$$\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5}$$

i) $\frac{C}{5} = \frac{F-32}{9}$

$$\frac{9}{5}C = F-32 \Rightarrow \boxed{F = \frac{9}{5}C + 32}$$

(or) $C = \frac{5}{9}(F-32)$

Heat :- Heat is a form of energy flow
i) between two bodies or
ii) between a body and its surroundings by virtue of temperature difference between them.

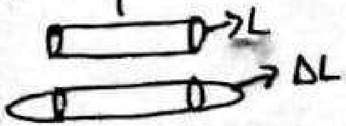
S.I unit :- Joule (J)

Commonly used unit : Calorie (cal)

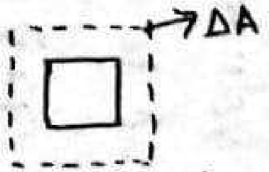
Conversion :- 1 cal = 4.186 J

Heat always flows from a higher temperature system to a lower temperature system.

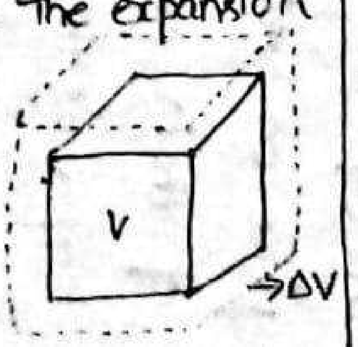
Linear Expansion:- The expansion in length of a body due to increase in its temperature is called the linear expansion.



Area Expansion:- The expansion in the area of a surface due to increase in its temperature is called area expansion.



Volume Expansion:- The expansion in the volume of an object due to increase in its temperature is known as volume expansion.



$$\boxed{\frac{\Delta L}{L} = \alpha \Delta T}$$

$$\alpha = \frac{\Delta L}{L} \cdot \left(\frac{1}{\Delta T}\right)$$

$$\frac{\Delta A}{A} = \beta \Delta T$$

$$\beta = \frac{\Delta A}{A} \cdot \left(\frac{1}{\Delta T}\right)$$

$$\frac{\Delta V}{V} = \gamma \Delta T$$

$$\gamma = \frac{\Delta V}{V} \left(\frac{1}{\Delta T}\right)$$

Coefficient of Linear expansion (α): Increase in length per unit length per degree rise in temperature.

Coefficient of Area expansion (β): Increase in area per unit area per degree rise in temperature.

Coefficient of Volume expansion (γ):- Increase in area per unit volume per degree rise in temperature.

Thermal Expansion :-

The phenomenon of change in dimensions of an object due to heat supplied is known as thermal expansion.

There are three types of thermal expansion.

1. Expansion of solids
2. Expansion of liquids
3. Expansion of gases.

1. **Expansion of solids** :- There are three types of thermal expansion takes place in solid.

(ii) $\frac{C}{5} = \frac{K-273}{5}$

$$K - 273 = \frac{5C}{5} \Rightarrow K = C + 273$$

$$(\text{or}) \quad T_K = T_C + 273$$

problems :-

1. convert $-10^\circ C$ into Fahrenheit scale.

Sol:- $C = -10^\circ C$

$$F = \frac{9}{5}C + 32$$

$$= \frac{9}{5}(-10) + 32 = -18 + 32 = 14^\circ F.$$

2. convert the normal temperature of a human body $98.4^\circ F$ into Celsius scale.

Sol:- $F = 98.4^\circ F$

$$C = \frac{5}{9}(F-32) = \frac{5}{9}[98.4 - 32]$$

$$= 36.9^\circ C$$

Example 13: The radius of mercury drop at 20°C is 3 mm. If the surface tension of mercury at this temperature is 4.65 x10^{-1} Nm^{-1} fFind the excess pressure inside the liquid drop.

Solution

Excess pressure (P) = $\frac{2T}{r}$

Surface tension (T) = 4.65×10^{-1} Nm^{-1}

radius of the mercury drop = r = 3 mm = 3×10^{-3} m

$$\therefore P = \frac{2T}{r} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 Nm^{-2}.$$

$10^{3} \times 9.8 \times 5 \times 10^{-4}$

EXAMPLE |3| Different Liquid in Capillary Tube

In a glass capillary tube, water rises upto a height of 10.0 cm while mercury fall down by 5.0 cm in the same capillary. If the angles of contact for mercury glass is 60° and water glass is 0°, then find the ratio of surface tension of mercury and water.

Sol. For water, $h_1 = 10.0\,\text{cm} = 0.1\text{ m}$

$$\rho_1 = 10^3\ \text{kg/m}^3, \theta = 0°$$

For mercury, $h_2 = 5.0\,\text{cm} = 0.05\text{ m}$

$$\rho_2 = 13.6 \times 10^3\ \text{kg/m}^3, \theta = 60°$$

Suppose S_1 and S_2 are the surface tensions for water and mercury, respectively, then

$$S_1 = \frac{h_1 R \rho_1 g}{2\cos\theta_1} \quad \text{and} \quad S_2 = \frac{h_2 R \rho_2 g}{2\cos\theta_2}$$

The ratio of surface tension of mercury and water,

$$\frac{S_2}{S_1} = \frac{h_2 R \rho_2 g}{2\cos\theta_2} \times \frac{2\cos\theta_1}{h_1 R \rho_1 g}$$

$$\frac{S_2}{S_1} = \frac{h_2 \rho_2 \cos\theta_1}{h_1 \rho_1 \cos\theta_2}$$

$$= \frac{0.05 \times 13.6 \times 10^3 \times \cos 0°}{0.1 \times 1000 \times \cos 60°} = 13.6 : 1$$

$$\gamma = \frac{\Delta V}{V} \cdot \frac{1}{\Delta T}$$

$$\gamma = \frac{V_L - V}{V} \frac{1}{\Delta T}$$

$$\gamma V \Delta T = V_L - V$$

$$V_L = V + V \gamma \Delta T$$

$$\boxed{V_L = V [1 + \gamma \Delta T]}$$

Example 4: An aluminium rod of length 50 cm is heated so that its temperature increases from 20°C to 80°C. If the linear coefficient of expansion of aluminium is 24 x 10^{-6} /°C find the increase in the length of the aluminium rod.

Solution

$\ell_1 = 50$ cm; $t_1 = 80°C$; $t_2 = 20°C$; $\alpha = 24 \times 10^{-6}$ /° C

coefficent of linear expansion, $\alpha = \dfrac{\ell_2 - \ell_1}{\ell_1(t_2 - t_1)}$.

The increase in length of the rod, $(\ell_2 - \ell_1) = \alpha \ell_1 (t_2 - t_1)$

$= 24 \times 10^{-6} \times 50 \times (80 - 20)$

$= 7.2 \times 10^{-2}$ cm.

EXAMPLE |1| A Hardworking Blacksmith

A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameter of the rim and the iron ring are 5.243 m and 5.231 m, respectively at 27°C. To what temperature should the ring be heated so as to fit the rim of the wheel? $(\alpha = 1.20 \times 10^{-5} \text{K}^{-1})$ [NCERT]

Sol. Given, initial temperature $T_1 = 27°\text{C}$

Initial length, $l_1 = 5.231$ m

Final length, $l_2 = 5.243$ m

Now, $$\alpha_l = \frac{\Delta l}{l_1 \Delta T} = \frac{l_2 - l_1}{l_1 \Delta T}$$

$\Rightarrow$ $$l_2 = l_1 [1 + \alpha_l (T_2 - T_1)]$$

i.e. $5.243 \text{ m} = 5.231\text{m} [1 + 1.20 \times 10^{-5} \text{ K}^{-1} (T_2 - 27°\text{C})]$

This gives $T_2 = 218°\text{C}$

EXAMPLE |5| Expansion in a Glass Block

The volume of a glass block initially was 15000 cm^3 but when the temperature of that glass block increases from 20°C to 45°C, then its volume increases by 5cm^3. Determine the coefficient of linear expansion.

Sol. Given, $V_1 = 15000 \text{ cm}^3$,

$$\Delta T = T_2 - T_1 = 45°C - 20°C = 25°C$$

Change in volume, $\Delta V = 5 \text{ cm}^3$

Coefficient of volume expansion,

$$\alpha_V = \frac{\Delta V}{V_1 \, \Delta T} = \frac{5}{15000 \times 25}$$

$$\alpha_V = 0.0133 \times 10^{-3} \text{ °C}^{-1}$$

Coefficient of linear expansion

$$\alpha_l = \frac{\alpha_V}{3} = \frac{0.0133 \times 10^{-3}}{3}$$

$$\alpha_l = 4.4 \times 10^{-6} \text{ °C}^{-1}$$

(1) A needle of length 5cm com just rest on the surface of water of surface tension 0·073 N/m. Find the vertical force required to detach this floating needle from the surface of water.

Sol:- $L = 5\ cm = 5 \times 10^{-2}\ m$

$T = 0{\cdot}073\ N/m$

$F = ?$

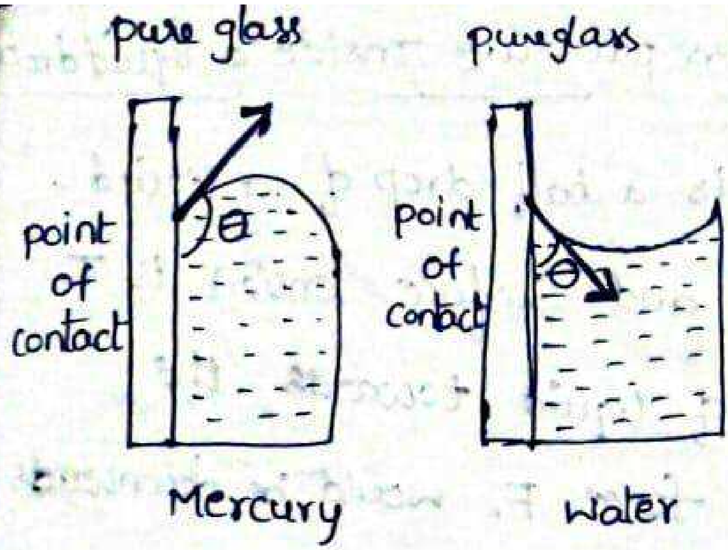
$T = \frac{F}{L} \Rightarrow F = TL$

The total length of the needle in contact with water $= 2L$

$\therefore\ F = T \times 2L$

$= 0{\cdot}073 \times 2 \times 5 \times 10^{-2}$

$F = 7{\cdot}3 \times 10^{-3}\ N$



* For pure water and perfectly clean glass, the angle of contact is 0°. For ordinary water and glass, it lies between 8° and 18°.

* For the liquids which wet the surface of solid (Such as water and glass). the angle of contact is acute [adh > coh]
  i.e $\theta < 90^\circ$. The molecules of the liquid are strongly attracted to those of solid. and weakly attracted to themselves.
  Ex:- Soap water

* For the liquids which do not wet the surface of Solid [Such as mercury and glass], the angle of contact is obtuse. [coh > adh]
  i.e $\theta > 90^\circ$. The molecules of the liquid are attracted strongly to themselves and weakly to those of solid.
  Ex:- water-leaf or glass-mercury Interface.

Example 7: The coefficient of linear expansion of nickel is 13 x 10^{-6}/°C. Find its coefficients of areal and cubical expansion.

Solution

$\alpha = 13 \times 10^{-6}$ /° C.

Its coefficient of areal expansion, $\beta = 2\alpha = 2 \times 13 \times 10^{-6} = 26 \times 10^{-6}$ /° C.

Coefficient of cubical expansion, $\gamma = 3\alpha = 3 \times 13 \times 10^{-6} = 39 \times 10^{-6}$ /° C.

EXAMPLE |3| Expansion in Metal Ball

A metal ball having a diameter of 0.4 m is heated from 273 to 360 K. If the coefficient of area expansion of the material of the ball is 0.000034 K^{-1}, then determine the increase in surface area of the ball.

Sol. Given, Diameter = 0.4 m

Radius, $r = \frac{0.4}{2} = 0.2$ m

$$\Delta T = T_2 - T_1 = 360 \text{ K} - 273 \text{ K} = 87 \text{ K}$$

$$\alpha_A = 0.000034 \text{ K}^{-1}$$

$$\Delta A = ?$$

Apply, $\Delta A = \alpha_A A_1 \Delta T$

where, $A_1 = 4\pi r^2 = 4 \times \pi \times (0.2)^2$

$$= 0.5024 \text{ m}^2$$

$$\Delta A = 0.000034 \times 0.5024 \times 87$$

$$\Delta A = 0.001486$$

$$= 1.486 \times 10^{-3} \text{ m}^2$$

1. At constant pressure 100 cc of hydrogen at $0^\circ C$ expands to 136.6 cc at $100^\circ C$. Find the volume coefficient of hydrogen.

Sol:- $V_0 = 100\,cc$, $V_t = 136.6\,cc$; $\Delta V = V_t - V_0$

$\Delta t = 100^\circ C$

$$\gamma = \frac{\Delta V}{V}\left(\frac{1}{\Delta T}\right)$$

$$= \frac{V_t - V_0}{V_0}\left(\frac{1}{\Delta T}\right)$$

$$= \frac{136.6 - 100}{100}\left(\frac{1}{100}\right)$$

$$\boxed{\gamma = 0.00366\ /^\circ C}$$

Surface Energy :-

The free surface of a liquid always has a tendency to contract and possess minimum surface area.

If it is required to increase the surface area of the liquid at a constant temperature, work has to be done. This workdone is stored in the surface film of the liquid as its potential energy.

The excess potential energy per unit area of the surface film is called surface energy.

Hence, the surface energy may be defined as the amount of workdone in increasing the area of the liquid surface by unity. Thus,

$$\text{surface energy} = \frac{\text{work done in increasing the surface area}}{\text{Increase in surface area}}$$

Its S.I unit is J/m^2. Its CGS unit is erg/cm^2.

$$1\ J/m^2 = \frac{10^7 erg}{10^4 cm^2} = 10^3\ erg/cm^2$$

Its dimensional formula $= [M^1 L^0 T^{-2}]$

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Angle of contact :-

The angle between tangent to the liquid surface at the point of contact and the solid surface inside the liquid is called as angle of contact.

It is denoted by θ.

The value of angle of contact depends on the following factors.

i) Nature of the solid and liquid in contact.

ii) cleanliness of the surface in contact.

iii) Medium above the free surface of the liquid.

iv) Temperature of the liquid.

Heat capacity :- If heat is supplied to a substance, its internal energy may increase, then the temperature of the substance increases. The quantity of heat required to raise the temperature is depends on mass of the substance.

To change the temperature of substance, a given quantity of heat is absorbed or rejected by it, is known as heat capacity.

The heat capacity is defined as amount of heat needed to change the temperature by unity i.e 1C.

It is denoted by 'S'

Heat capacity $\boxed{S = \frac{\Delta Q}{\Delta T}}$

Its S.I unit is Joule/Kelvin (J/K)

where ΔQ = heat absorbed or rejected by body

ΔT = change in temperature

The mass of water having an equivalent heat capacity of the substance expressed in CGS units is known as the water equivalent of the substance.

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specific heat capacity :-

. The specific heat capacity of a substance is the amount of energy required to raise the temperature of 1 kg of the substance by $1^\circ C$.

$$\boxed{C = \frac{\Delta Q}{m \Delta T}}$$

where m = mass of given substance (kg)

C = specific heat capacity (J/kg°C)

ΔQ = heat energy transfered (J) (or) change in thermal energy

ΔT = change in temperature (°C)

From the above equation, that the quantity of heat ΔQ required by a body of mass 'm' to produce a temperature rise "Δt" is given by, $\boxed{\Delta Q = mc\,\Delta T}$

∴ The heat capacity of the substance } $\boxed{\frac{\Delta Q}{\Delta T} = mc}$

Hence heat capacity of a body is equal to the product of mass and specific heat of the body.

Types of Latent Heat :-

There are two types of latent heat of materials :

i) Latent heat of Fusion or Melting :-

It is latent heat for solid - liquid state change. It is denoted by L_f and is given by

$$\boxed{\text{Latent heat of fusion, } L_f = \frac{Q}{m}}$$

$$\text{Solid} \underset{\substack{\text{Heat taken} \\ -\Delta Q}}{\overset{\substack{\text{Heat given} \\ +\Delta Q}}{\rightleftharpoons}} \text{Liquid} \quad L_f$$

Its SI unit is J/kg.

ii) Latent heat of Vaporisation :-

It is latent heat for liquid - gas state change.

It is denoted by L_v.

$$\boxed{\text{Latent heat of vaporisation } L_v = \frac{Q}{m}}$$

Its S.I. unit is J/kg

$$\text{Liquid} \underset{\substack{-\Delta Q \\ \text{Heat taken}}}{\overset{\substack{\text{Heat given} \\ +\Delta Q}}{\rightleftharpoons}} \text{Gas} \quad \cdots (-L_v)$$

Molar specific Heat capacity :-

Molar specific heat of a substance is defined as the amount of heat required to rise the temperature of 1 gram mole of the substance through a unit degree.

The gram mole of any substance quantity is numerically equal to the molecular mass (M)

Thus $C_m = M\left(\frac{\Delta Q}{m\Delta T}\right)$ —— ①

$$C_M = MC$$

i.e $n = \frac{m}{M}$ —— ②

where m = mass of the substance

M = molecular mass

n = number of moles

C_m = molar specific heat capacity

from eq^n ①, $C_m = M\left(\frac{\Delta Q}{m\Delta T}\right)$

$$C_m = \frac{M}{m}\left(\frac{\Delta Q}{\Delta T}\right)$$

$$\boxed{C_m = \frac{1}{n}\left(\frac{\Delta Q}{\Delta T}\right)} \quad \left[\because n = \frac{m}{M} \Rightarrow \frac{1}{n} = \frac{M}{m}\right]$$

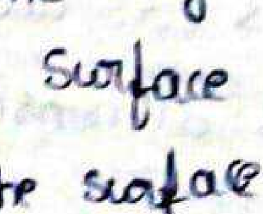
Molar specific heat capacity = Molecular mass x specific heat capacity

Surface Tension :-

According to molecular theory, it is observed that the surface of a liquid behaves like a stretched elastic membrane or a stretched rubber sheet.

If we imagine a line on the surface of a liquid, it will divide the surface into two parts. The part of the surface on the left side pulls the part of the surface on the right side with a force propotional to the length of the line. Similarly the part of the surface on the right side pulls the part of the surface on the left side with an equal and opposite force. This force, F is perpendicular to the line and tangential to the Surface as shown in Fig.

" Surface tension is defined as the tengential force per unit length, of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line".



If 'F' is the force acting perpendicular to the imaginary line of length l, then (41)

$$F \propto l$$

$$F = T l$$

$$\boxed{T = \frac{F}{l}}$$

here T is called surface tension.

Its SI unit is N/m

Its CGS unit is dyne/cm

Its dimensional formula = $[M L^0 T^{-2}]$

* It is a Scalar

* It depends on nature of fluid & intermolecular forces

* surface Tension decreases with rise in temperature and becomes zero at critical temperature.

* The surface tension of liquids changes appreciably with addition of impurities.

poiseuille's Expression :-

poiseuille suggested a method for the determination of coefficient of low viscous liquids like water. The discharge rate of a viscous fluid through a tube can be given as

$$\boxed{\eta = \frac{\pi p a^4}{8VL}}$$

p - pressure difference b/n the ends of the Capillary tube.

L - length of the capillary tube.

V - Volume of liquid flowing per second

a - radius of the capillary tube.

Latent heat :- The amount of heat required to change the state of the mass m of the substance is called latent heat.

The change in state occurs at a constant temperature.

Latent heat is denoted by 'L' and having S.I unit J/kg. Its dimension is $[L^2 T^{-2}]$

If a mass "m" of a substance undergoes a change from one state to the other, then the quantity of heat required is given by

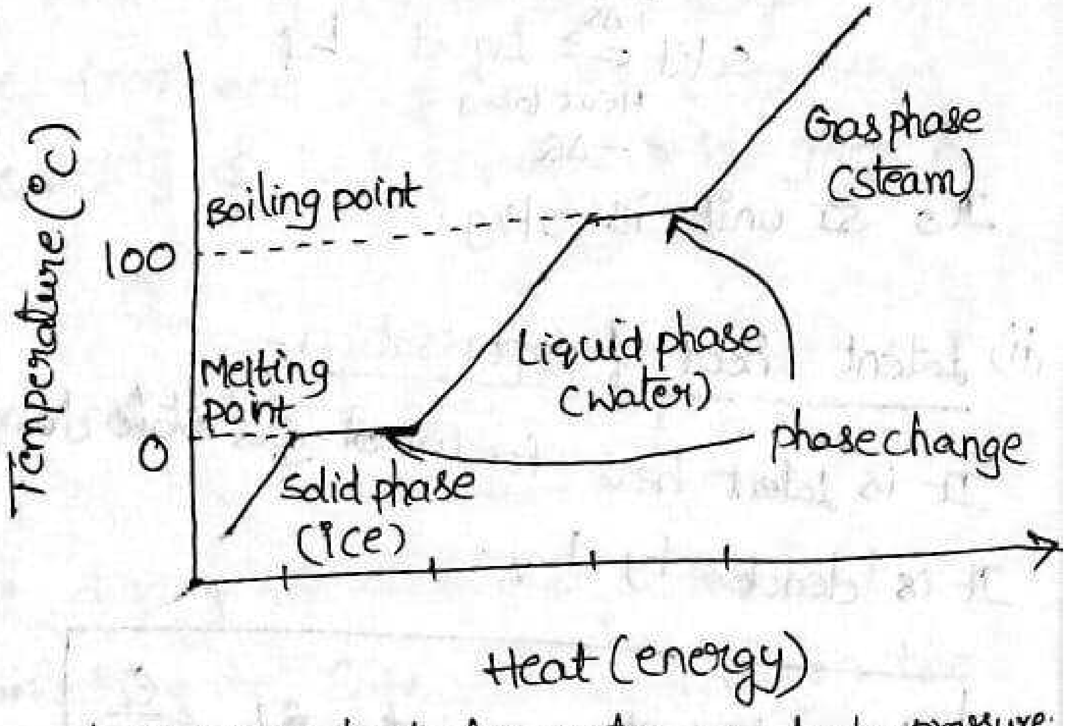
$$Q = mL$$

Latent heat, $\boxed{L = \frac{Q}{m}}$

Hence, during the phase change, the heat required by the substance depends on the mass (m) of the substance and heat of Transformation (Q).

Variation of temperature during change of state

It is observed from the above, as heat is added or removed during a change of state, the temperature remains constant. hence the slopes of the phase lines are not all the same, which indicates the specific heats of the various states are not equal.



Temperature versus heat for water at 1 atm pressure

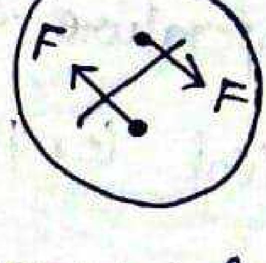
Ex:- water having $L_f = 3.33 \times 10^5$ J/kg and
$L_v = 22.6 \times 10^5$ J/kg,

it means 3.33×10^5 J of heat is needed to melt 1 kg of ice at 0°C and 22.6×10^5 J of heat is needed to convert 1 kg of water to steam at 100°C.

Surface Tension :-

According to molecular theory, it is observed that the surface of a liquid behaves like a stretched elastic membrane or a stretched rubber sheet.

If we imagine a line on the surface of a liquid, it will divide the surface into two parts. The part of the surface on the left side pulls the part of the surface on the right side with a force proportional to the length of the line. Similarly the part of the surface on the right side pulls the part of the surface on the left side with an equal and opposite force.. This force, F is perpendicular to the line and tangential to the Surface as shown in Fig.



" Surface tension is defined as the tangential force per unit length, of an imaginary line drawn on the liquid surface, the direction of force being perpendicular to this line".

$\frac{dV}{dy}$ is called velocity gradient

Coefficient of viscosity :- For unit velocity gradient the amount of viscous force that acts per unit area of a fluid is called coefficient of viscosity of that fluid.

This force acts along tangent to the layers of the fluid.

$$F = \eta A\left(\frac{dv}{dy}\right)$$

(or) $$\eta = \frac{F}{A\left(\frac{dv}{dy}\right)}$$

its dimensional formula is $[ML^{-1}T^{-1}]$.

unit of η in M.K.S and S.I method is NSm^{-2}. In many cases poise is used as the unit of η.

$$10' \text{ poise} = 1\ Ns/m^2$$

in C.G.S, 1 poise = 1 dyne-s/cm^2

sometimes coefficient of viscosity is called dynamic viscosity.

Critical velocity and Reynold's Number

As we know upto a specific flow velocity, fluid flow remains streamline and beyond a critical velocity it becomes turbulent.

It depends on some factors,

$V_c \propto \eta$ {coefficient of viscosity of fluid

$V_c \propto \frac{1}{\rho}$ [density of the fluid]

$V_c \propto \frac{1}{d}$ [diameter of tube]

$$V_c \propto \frac{\eta}{\rho d}$$

(or) $$V_c = \frac{R_n \eta}{\rho d}$$

$$\rightarrow \boxed{R_n = \frac{V_c \rho d}{\eta}}$$

R_n – Reynolds number

→ R_n less than 1000, the flow of fluid is streamline.

Viscosity :-

When a layer of fluid slides over another layer of the same fluid, a force of friction comes into play, which is called viscous force.

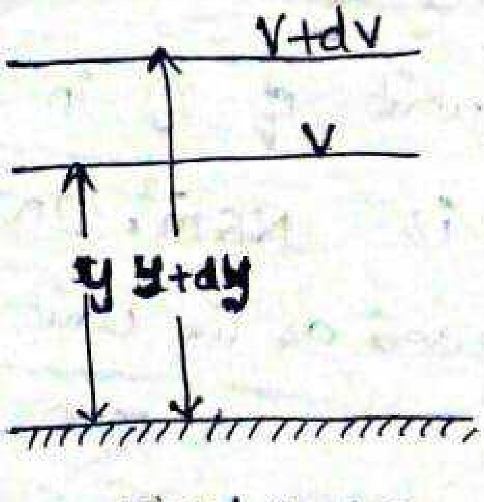
This force opposes the relative motion of the different layers of a fluid.

"The property of a liquid by virtue of which it opposes relative motion between its different layers is called viscosity".

viscosity depends upon i) surface area of the liquid layer (ii) velocity gradient of layer

$$F_v \propto A$$

$$F_v \propto \frac{dv}{dy}$$



Fixed surface

viscous force on fluid layer is

$$F_v \propto A \frac{dv}{dy}$$

$$\boxed{F_v = \eta A \frac{dv}{dy}}$$

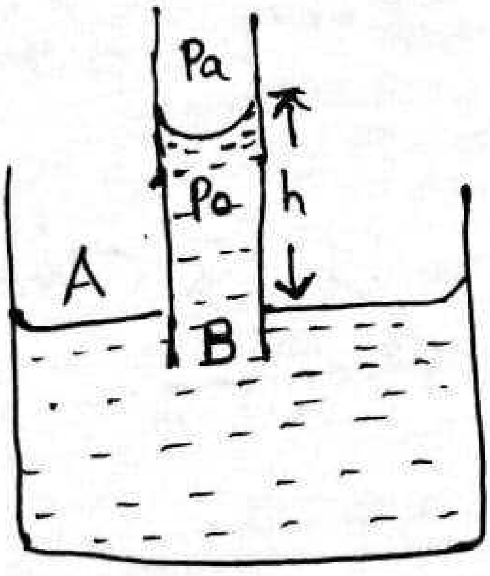
where 'η' is coefficient of viscosity.

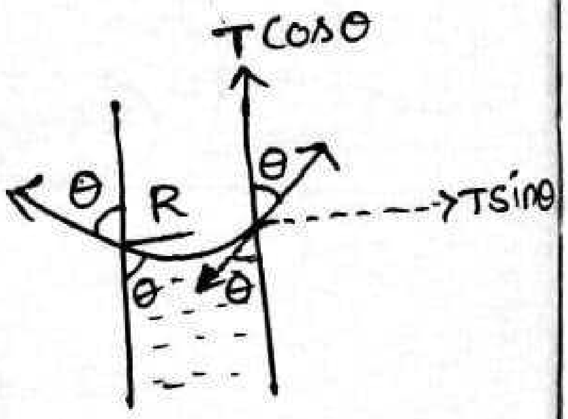
when a glass capillary tube is dipped vertically in a liquid of angle of contact less than 90°, the liquid rises in the tube until the force due to surface tension is balanced by the weight of the liquid lifted up.

when the angle of contact is greater than 90°, the liquid gets depressed due to surface tension forces acting downwards.

Ex:- 1. we use towels for drying our skin

Determination of Surface Tension by Capillary Rise Method :-





The phenomenon of rise or fall of liquid in a capillary tube is called capillarity. (47)

Let one end of the capillary tube of radius 'r' immersed into a liquid of density 'ρ'. 'h' is the height of the liquid column. The surface of water in the capillary is concave.

Let R = radius of curvature of liquid meniscus.

P_a = atmospheric pressure.

There must be a pressure difference between the two sides of the meniscus.

$$P_a - P_o = \frac{2T}{r} = \frac{2T\cos\theta}{R} \quad \text{—①}$$

Now consider two points A and B. According to pascal's law, they must be at the same pressure

$$P_A = P_B$$

$$P_a - P_o = \rho g h \quad \text{—②}$$

from eqns ① & ②

$$\rho g h = \frac{2T\cos\theta}{R}$$

$$\boxed{h = \frac{2T\cos\theta}{\rho R g}}$$

This is the formula for the rise of liquid in a capillary.

According to the principle of Calorimetry (20)

Heat gained by cold bodies = Heat lost by hot bodies.

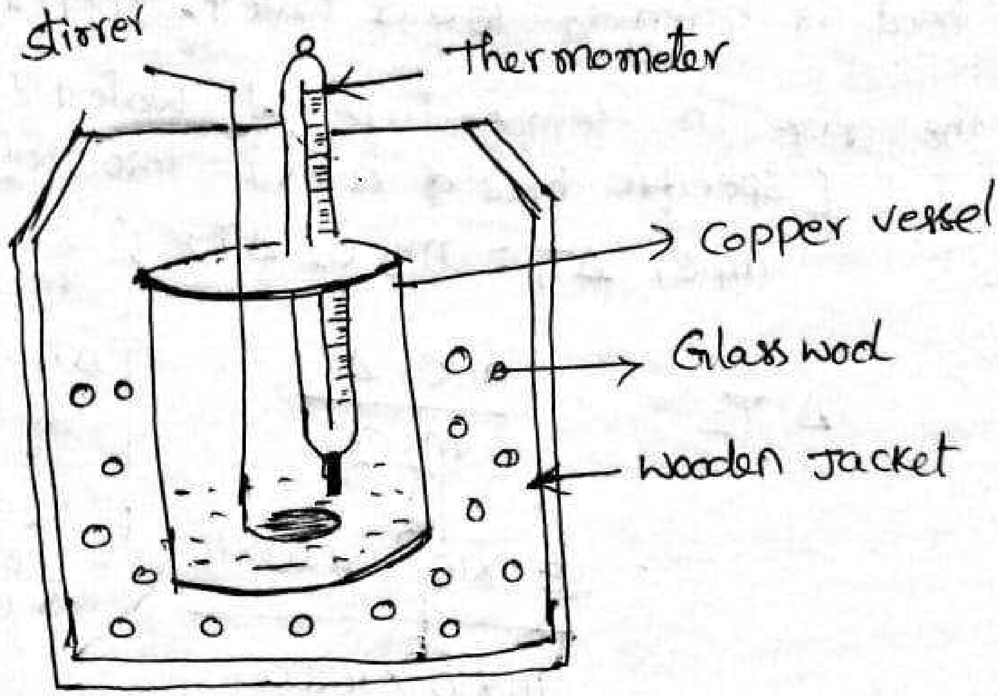


Fig: Calorimeter

$$\therefore T = \frac{m_1 c_1 T_1 + m_2 c_2 T_2}{m_1 c_1 + m_2 c_2}$$

i) If bodies are of same material $c_1 = c_2$, then

$$T = \frac{m_1 T_1 + m_2 T_2}{m_1 + m_2}$$

ii) If bodies are of same mass $(m_1 = m_2)$, then

$$T = \frac{T_1 c_1 + T_2 c_2}{c_1 + c_2}$$

iii) If bodies are of same material and of equal masses $(m_1 = m_2, c_1 = c_2)$ then $T = \frac{T_1 + T_2}{2}$

Lecture - 43

Heat transfer :- There are three modes of heat transfer.

1. conduction 2. convection 3. Radiation.

Conduction :-

The transfer of heat taking place due to molecular vibrations is known as heat conduction.

In this process, there is no change in the position of the molecule and hence there is no mass movement of matter in solids.

Ex:- a metallic rod being heated at the end and the other end of the rod gets heated automatically.

(3) Radiation :-

1, It is a process of transmission of heat in which heat travels directly from one place to another without the agency of any intervening medium.
Ex:- Sun.

2, This radiation of heat energy occurs in the form of E.M waves.

3, These radiators are emitted by virtue of its temperature, like the radiation by a red hot iron or light from a filament lamp.

4) Every body radiates energy as well as absorbs energy from surroundings.

5, The propotion of energy absorbed depends upon the colour of the body.

Calorimeter:-

1. It is a device used for measuring the Quantities of heat.
2. It consists of a Cylindrical vessel of copper provided with a stirrer.
3. The vessel is kept inside a wooden Jacket. The space between the calorimeter and the Jacket is packed with a heat insulating material like glass wool, etc.
4. Thus, the Calorimeter gets thermally isolated from the surroundings.
5. The loss of heat due to radiation is reduced by polishing the outer surface of calorimeter and inner surface of the Jacket.
6. The lid is provided with holes for inserting a thermometer and a stirrer into the Calorimeter.
7. when bodies at different temperatures are mixed together in the calorimeter, heat is exchanged between the bodies as well as with the Calorimeter

Calorimetry principle :-

calorimetry is the branch of Science that deals with the measurement of heat.

when a body at higher temperature is brought in contact with another body at lower temperature, then the heat flows from the high temperature body to low temperature body till the both bodies acquire the same temperature.

"The principle of calorimetry states that total heat given by a hotter body equals to the total heat received by the colder body."

i.e. Heat lost by hotter body = Heat gained by colder body.

If there are two bodies of masses m_1 and m_2 and having values of specific heats C_1 and C_2 respectively, then for temperature difference ΔT.

$$\boxed{m_1 C_1 \Delta T = m_2 C_2 \Delta T}$$

[$\because \Delta Q = mc\Delta T$]

$$m_1 c_1 (T_1 - T) = m_2 C_2 (T - T_2)$$

$$\boxed{T = \frac{m_1 C_1 T_1 + m_2 C_2 T_2}{m_1 C_1 + m_2 C_2}},$$ T = Temperature equilibrium

(2) Convection :-

1. The process in which heat is transferred from one point to another by the actual movement of the heated material particles from a place at higher temperature to another place of lower temperature is called as thermal convection.

2. If the medium is forced to move with the help of a fan or a pump, it is called as forced convection. Ex:- circulatory system, cooling system of an automobile heat convector

3. If the material moves because of the differences in density of the medium, the process is called natural or free convection.

Ex:- Trade winds, sea Breeze/ Land Breeze.

Wien's Displacement Law :-

Wien's displacement law states that the wavelength (λ_m) corresponding to which the energy emitted by a black body is maximum and is inversely proportional to its absolute temperature (T).

Thus $\lambda_m \propto \frac{1}{T}$ (or) $\boxed{\lambda_m T = b}$

where b = Wien's constant $= 2.9 \times 10^{-3}$ mK.

Stefan- Boltzmann Law :-

Stefan- Boltzmann Law states that the total energy emitted per second by a unit area of a perfect black body is proportional to the fourth power of its absolute temperature.

i.e $E \propto T^4$

Total energy, $\boxed{E = \sigma T^4}$

where 'σ' is a universal constant called Stefan- Boltzmann constant.

$\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$

If the body is not a perfect black body

$\boxed{E = \epsilon \sigma T^4}$, ϵ = emissivity of that body

Rate of flow $\frac{\Delta Q}{\Delta t} = H$

$$\therefore \boxed{H = KA\frac{\Delta T}{L}}$$

Heat transfer $\boxed{Q = KA\frac{\Delta T}{L}t}$

The greater value of K implies that material will conduct the heat more rapidly.

The S.I unit of K is $Js^{-1}m^{-1}K^{-1}$ or $Wm^{-1}K^{-1}$

The term $\left(\frac{\Delta T}{L}\right)$ is known as temperature gradient.

Coefficient of thermal conductivity of a material may be defined as the quantity of heat that flows per unit time through a unit area of the material under unit temperature gradient, the heat flow being normal to the area.

<u>Thermal current</u> <u>and</u> <u>thermal Resistance</u> :-

The rate of flow of heat is known as heat current. It is denoted by H. Its S.I unit is J/s or watt.

$$H = \frac{\Delta Q}{\Delta t}$$

Newton's Law of cooling :-

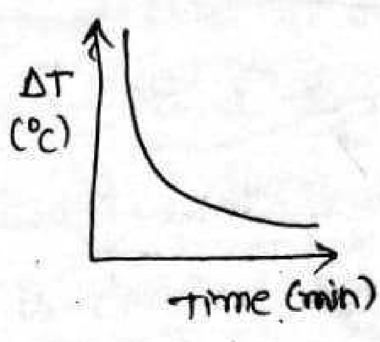
Newton's law of cooling states that the rate of cooling of a body is directly proportional to the temperature difference between the body and its surroundings, provided the temperature difference is small.

i.e Rate of loss of heat $\propto$ Temperature difference between the body and its surroundings.

$$-\frac{dQ}{dt} \propto (T - T_0)$$

Rate of loss of heat, $-\frac{dQ}{dt} = K(T - T_0)$

where K is a positive constant depending upon area and nature of the surface of the body.



For small temperature difference, the rate of cooling due to conduction, convection & radiation combined is proportional to difference in temperature.

Black body radiation laws:-

Black body radiation:-

A body that absorbs all the radiations falling on it is known as a black body. It emits the radiations at the fastest rate.

The radiations emitted by a black body is known as black body radiation.

The black body is also called the ideal radiator. A perfect body absorbs 100% of radiations falling on it, is only an ideal concept because within the universe there is no existence of black body.

i) **Emissive power**:- The amount of heat energy radiated per unit area of the surface of a body, per unit time and per unit wavelength range is constant which is called as the "emissive power" (e_λ) of the given surface, given temperature and wavelength. Its S.I unit is $J s^{-1} m^{-2}$.

planck showed that the spectral radiance of a body for frequency 'ν' at absolute temperature is given by

$$\boxed{E(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\left(\frac{hc}{\lambda KT}\right)} - 1}}$$

h – plank's constant $= 6.625 \times 10^{-34}$ J.S

C – speed of light $= 3 \times 10^8$ m/s

λ – wavelength (m)

K – Boltzmann constant $= 1.38 \times 10^{-23}$ J/K

T – temperature (K)

Rayleigh- Jeans law :- For radiations of long wavelength $\lambda \gg \frac{hc}{KT}$, Rayleigh Jeans Energy distribution law can be used

$$E(\lambda, T) = \frac{8\pi KT}{\lambda^4} d\lambda \text{ (or)}$$

$\nu = \frac{c}{\lambda}$

$\Rightarrow \lambda = \frac{c}{\nu}$

ii) Absorptive power :- when any radiation is incident over a surface of a body, a part of it gets reflected, a part of it gets refracted and the rest of it is absorbed by that surface. Therefore, the "absorptive power" of a surface at a given temperature and for given wavelength is the ratio of the heat energy absorbed by a surface to the total energy incident on it.

It has no unit as it is a ratio

$$a_\lambda = \frac{\text{Amount of heat energy absorbed}}{\text{Total heat energy incident}}$$

iii) perfect Black body :- A body is said to be a perfect black body, if its absorptivity is 1. It neither reflects nor transmits but absorbs all the thermal radiations incident on it irrespective of their wavelengths.

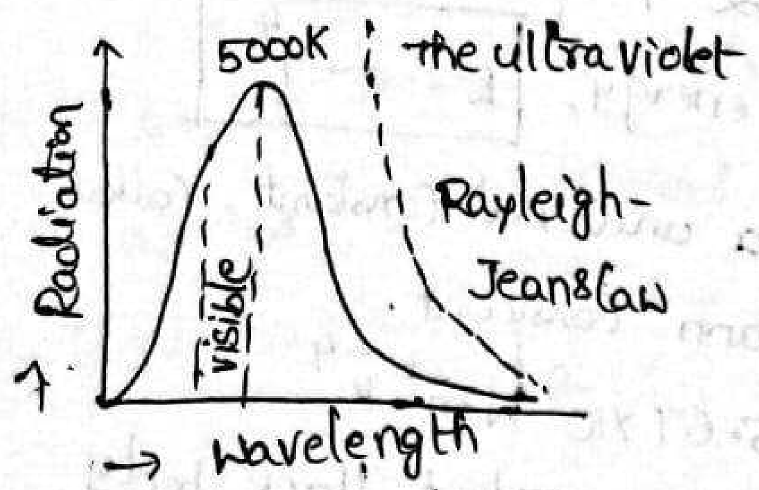
—

Planck's law :-

Max. Planck explain the spectral - energy distribution of radiation emitted by a black body.

Planck assumed that the sources of radiations are atoms in a state of oscillation and that the vibrational energy of each oscillator. He assumed that when an atom changes from a state of energy E_1 to a state of lower energy E_2, the discrete amount of energy $E_1 - E_2$, or quantum radiation is equal to the product of frequency radiation. It is released in the form of energy packets called quanta.

$$E_1 - E_2 = h\nu$$



Example 3: Find the temperature of an oven if it radiates 8.28 cal per second through an opening, whose are is 6.1 cm^2. Assume that the radiation is close to that of a black body.

Solution

The emittance of the oven (the energy radiates in one second by unit surface area) is

$$E = \frac{8.28 \text{ cal/sec}}{6.1 \text{cm}^2} = \frac{8.28 \times 4.2 \text{ J/s}}{6.1 \times 10^{-4} \text{m}^2} = 5.7 \times 10^4 \text{ Watt/m}^2.$$

From Stefan-Boltzmann formula

$$E = \sigma T^4, \text{ where } = \sigma = 5.67 \times 10^{-8} \text{ W/} \times \text{ m}^2.\text{ K}^4$$

$$T^4 = \frac{E}{\sigma}$$

$$T^4 = \frac{5.7 \times 10^4}{5.67 \times 10^{-8}} = 1 \times 10^{12} \text{ K}^4$$

$$\therefore T = 10^3 \text{ K} = 1000 \text{ K}.$$

Example 4: If a black body is radiating at T = 1650 K, at what wavelength is the maximum intensity?

Solution

According to Wien's law

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ mK}$$

Substituting the value of the temperature T = 1650 K

$$\lambda_{max} = \frac{2.9 \times 10^{-3} \text{ m.K}}{1650 \text{ K}} = 1.8 \,\mu\text{m}.$$

EXAMPLE |11| Heat Lost by the Sheet

A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600°C, and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.0250? Neglect heat loss due to convection. (Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$).

[NCERT]

Sol. As the energy is radiated from both surfaces of the sheet, so

$$A = 2 \times 15.0 \times 12.0 \times 10^{-4} \text{ m}^2$$

$$= 3.60 \times 10^{-2} \text{ m}^2$$

$$T = 600 + 273 = 873 \text{ K}$$

$$\varepsilon = 0.250,$$

$$\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}.$$

The rate of heat loss by the sheet,

$$H = \frac{Q}{t} = \varepsilon \sigma A T^4$$

$$= 0.250 \times 5.67 \times 10^{-8} \times 3.60 \times 10^{-2} \times (873)^4$$

$$= 296 \text{ Js}^{-1} = 296 \text{ W}.$$

EXAMPLE |7| Heat Flow through a Glass

Calculate the rate of loss of heat through a glass window of are 1000 cm^2 and thickness 0.4 cm. When temperature inside i 37°C and outside is −5°C. Coefficient of thermal conductivity of glass is 2.2×10^{-3} cal s^{-1} cm^{-1} K^{-1}.

Sol. Given, $A = 1000\ \text{cm}^2$, $L = 0.4$ cm

$$\Delta T = T_1 - T_2 = 37 - (-5) = 42°\text{C}$$

$$K = 2.2 \times 10^{-3}\ \text{cal s}^{-1}\text{cm}^{-1}\text{K}^{-1}$$

Rate of loss of heat,

$$H = \frac{Q}{T} = \frac{KA(T_1 - T_2)}{L} = \frac{2.2 \times 10^{-3} \times 1000 \times 42}{0.4}$$

$$H = 231\ \text{cal s}^{-1}$$

EXAMPLE |12| A Hot Body Radiating Energy

A hot body having the surface temperature 1327°C. Determine the wavelength at which it radiates maximum energy. Given Wien's constant $= 2.9 \times 10^{-3}$ mK.

Sol. Given, $T = 1327 + 273 = 1600$ K

Wien's constant, $b = 2.9 \times 10^{-3}$ m K

$$\lambda_m = \frac{b}{T} = \frac{2.9 \times 10^{-3}}{1600} = 1.81 \times 10^{-6} \text{ m}$$

EXAMPLE |11| Heat Lost by the Sheet

A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600°C, and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.0250? Neglect heat loss due to convection. (Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$).

[NCERT]

Sol. As the energy is radiated from both surfaces of the sheet, so

$$A = 2 \times 15.0 \times 12.0 \times 10^{-4}\ \text{m}^2$$

$$= 3.60 \times 10^{-2}\ \text{m}^2$$

$$T = 600 + 273 = 873\ \text{K}$$

$$\varepsilon = 0.250,$$

$$\sigma = 5.67 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}.$$

The rate of heat loss by the sheet,

$$H = \frac{Q}{t} = \varepsilon \sigma A T^4$$

$$= 0.250 \times 5.67 \times 10^{-8} \times 3.60 \times 10^{-2} \times (873)^4$$

$$= 296\ \text{Js}^{-1} = 296\ \text{W}.$$

Connection of Rods with Different Thermal Conductivities

a) **Series Combination** :- we know the series combination of Resistances gives the equivalent Resistance as.

$$R_{eq} = R_1 + R_2$$

$$\frac{L_1 + L_2}{K_{eq} A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} \quad \left[\because R = \frac{L}{KA}\right]$$

If $L_1 = L_2 = L$, then $\frac{2L}{K_{eq} A} = \frac{L}{K_1 A} + \frac{L}{K_2 A}$

Equivalent thermal conductivity $\boxed{K_{eq} = \frac{2 K_1 K_2}{K_1 + K_2}}$

b) **parallel combination** :- we know

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{\left(\frac{L}{K_{eq} 2A}\right)} = \frac{1}{\left(\frac{L}{K_1 A}\right)} + \frac{1}{\left(\frac{L}{K_2 A}\right)}$$

$$\frac{2A K_{eq}}{L} = \frac{K_1 A}{L} + \frac{K_2 A}{L}$$

($\because$ for equal area of cross-sections and length)

$$\boxed{K_{eq} = \frac{K_1 + K_2}{2}}$$

EXAMPLE |13| Rate of Energy Lost

Consider a filament which is indirectly heated. The radiating maximum energy having wavelength 3.4×10^{-5} cm. Determine the amount of heat energy lost per second per unit area if the temperature of surrounding air is 17°C.

Given, $\sigma = 5.67 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$, $b = 2.9 \times 10^{-3}$ mK

Sol. Given, wavelength, $\lambda_m = 3.4 \times 10^{-5}\ \text{cm} = 3.4 \times 10^{-7}$ m

Temperature surrounding,

$$T_0 = 17°\text{C} + 273 = 290\ \text{K}$$
$$\sigma = 5.67 \times 10^{-8}\ \text{Wm}^{-2}\ \text{K}^{-4}$$
$$b = 2.9 \times 10^{-3}\ \text{mK}$$

According to Wien's law

$$T = \frac{b}{\lambda_m} = \frac{2.9 \times 10^{-3}\ \text{mK}}{3.4 \times 10^{-7}} = 8529.4\ \text{K}$$

According to Stefan-Boltzmann law,

$$E = \sigma(T^4 - T_0^4) = 5.67 \times 10^{-8}\ [(8529.4)^4 - (290)^4]$$

$$E = 30016.98 \times 10^4\ \text{W/m}^2$$

$$H = KA \frac{\Delta T}{L}$$

$$H = \frac{\Delta T}{\left(\frac{L}{KA}\right)}$$

obstruction offered to the flow of heat current by the medium is known as Thermal Resistance

Thermal Resistance $R = \frac{\Delta T}{H}$

$$R = \frac{\cancel{\Delta T}}{\frac{\cancel{\Delta T}}{\left(\frac{L}{KA}\right)}} = \frac{L}{KA}$$

$$\therefore \boxed{R = \frac{L}{KA}}$$

Its S.I unit is K/W

Thermal conductivity :-

→ The ability of material to conduct the heat through it is known as thermal conductivity.

⇒ Heat conduction is defined as the time rate of heat flow in a material for a given temperature difference.

⇒ consider a metal rod of length (l) and area of cross-section (A) Rate of heat transfer through any section (In steady state) of the rod is directly proportional to the temperature difference (ΔT) and area of cross-section (A) and is inversely proportional to the length (L) of the rod

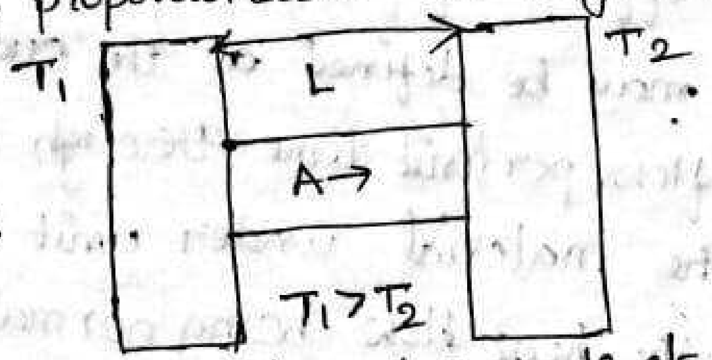


Fig : Thermal conductivity of metal rod

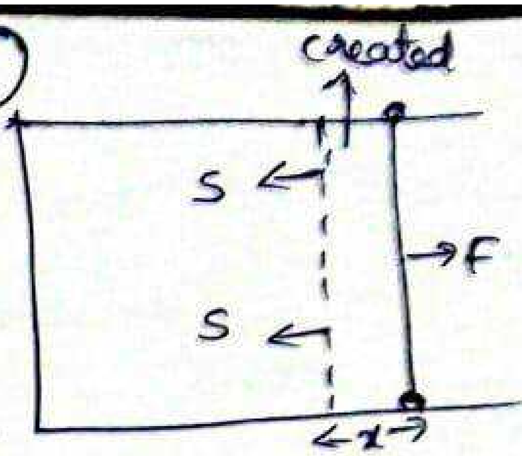
Rate of heat transfer $= \dfrac{\Delta Q}{\Delta t}$

$$\frac{\Delta Q}{\Delta t} \propto A \frac{\Delta T}{L}$$

$$\frac{\Delta Q}{\Delta t} = K A \frac{(T_1 - T_2)}{L}$$

Here 'K' is known as coefficient of thermal conductivity workdone $= F \cdot x$

$= 2Tlx$



This workdone is stored as the additional potential energy of the new surface,

$\therefore$ surface energy $= \dfrac{\text{Additional potential energy}}{\text{surface area}}$

$$E = \frac{\cancel{2Tlx}}{\cancel{2lx}}$$

$$\boxed{E = T}$$

Hence, surface energy is equal to the surface tension of the liquid.

$\therefore$ surface tension is the energy required to increase the surface area by unity.

Surface Energy = surface tension × surface Area created

The force acts on the ball which makes it follow a curved path, as shown in Fig(b)

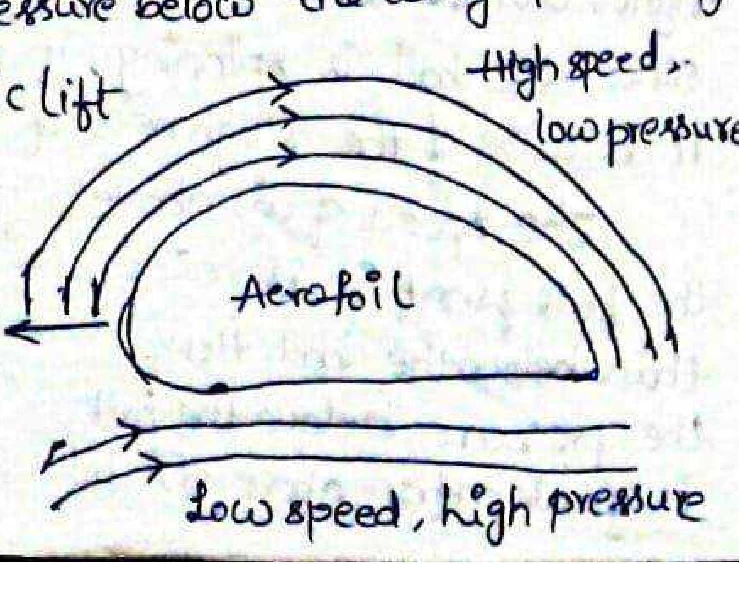
The difference in lateral pressure, which causes a spinning ball to take a curved path which is curved towards the greater pressure side, is called Magnus effect

(C) Aerofoil, Lift of an Aircraft wing :-

Aerofoil is a solid object shaped to provide an upward dynamic lift as it moves horizontally through air. This upward force makes aeroplane fly.

When the aeroplane moves through air, the air in the region above the wing moves faster than the air below as seen from the streamlines above the wing.

The difference in speed in the two regions makes the pressure in the region above lower than the pressure below the wing producing thereby a dynamic lift



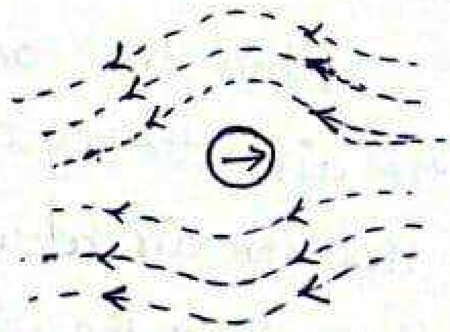
2) **Dynamic lift** :- Dynamic lift is the force that acts on a body by virtue of its motion through a fluid.

It acts on a body due to pressure difference caused by the motion of the body.

It is responsible for the curved path of a spinning ball and the lift of aircraft wing (Aerofoil).

a) **Ball moving without spin** :- when the velocity of the air above the ball is same as below the ball at the corresponding points resulting in zero pressure difference.

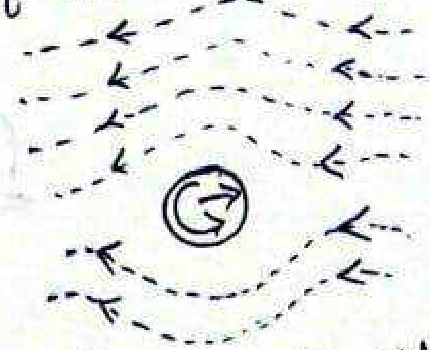
The air therefore exerts no upward or downward force on the ball.



(a) Ball moving without spin

b) **Ball Moving with spin** :- As the ball moves to the right, air rushes to the left with respect to the ball. since the ball is spinning, it drags some air with it because of the roughness of its surface.

The speed of air above the ball w.r.t it is greater than below the ball. Hence, the pressure below the ball is greater than above the ball.



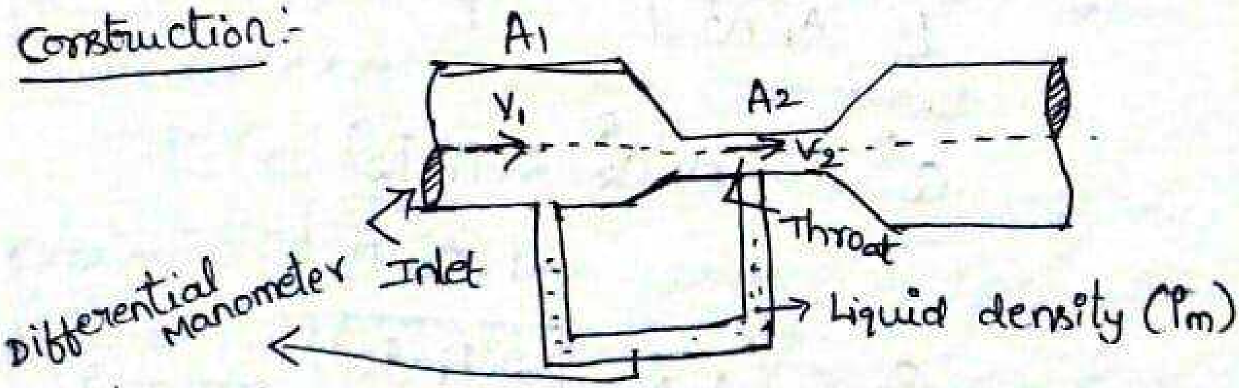
(b) Ball Moving with sp

Applications of Bernoulli's Equation

1. **venturimeter:-** It is a device used to measure the rate of flow of a incompressible fluid through a pipe. It is also known as flowmeter or venturi tube.

principle:- The basic principle of venturimeter is that by reducing the cross-sectional area of a flow passage. a pressure difference is created and this ~~pr~~ measurement of pressure difference enables the determination of rate of flow through the pipe

Construction:-



A venturimeter consists of ① an inlet section ② a cylindrical throat and ③ outlet section (divergent core) The diameter of ~~of~~ the inlet section pipe is of the same diameter as that of divergent cone.

From continuity Equation

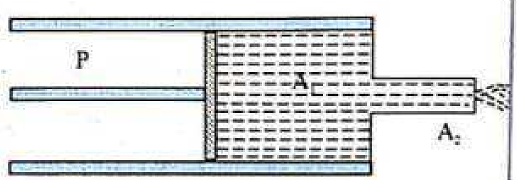
$$A_1V_1 = A_2V_2 = \frac{\Delta V}{\Delta t} \quad [\because \Delta V = A_1V_1\Delta t]$$

$$A_1V_1 = A_2V_2 = Q \quad \left[\text{Rate of flow } \frac{\Delta V}{\Delta t} = Q\right]$$
(or) flowspeed

$$V_1 = \frac{Q}{A_1} \text{ and } V_2 = \frac{Q}{A_2} \quad \text{—(1)}$$

$$= \sqrt{800} = 28.28 \text{ m/s}$$

1. The figure shows a liquid being pumped out using a piston of area of cross section 0.9×10^{-4} m². The area of cross section of the outlet is 18×10^{-6} m². If the speed of the motion of the piston is 1.8×10^{-2} m s⁻¹, calculate the speed of the outgoing liquid.



Solution

From the equation of continuity, we have

$$a_1 v_1 = a_2 v_2$$

$$0.9 \times 10^{-4} \times 1.8 \times 10^{-2} = 10 \times 10^{-6} \times v_2$$

$$v_2 = \frac{0.9 \times 1.8}{18} = 0.09 \text{ m s}^{-1}.$$

11:33 a.m.

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Sol. According to the equation of continuity, we get

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\therefore \quad v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1$$

Given,

$$r_1 = \frac{4}{2} = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$r_2 = \frac{6}{2} = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

$$v_1 = 5 \text{ m/s}$$

$$v_2 = \left(\frac{2}{3}\right)^2 \times 5 = 2.22 \text{ m/s}$$

Now, applying the Bernoulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

$$= 3.0 \times 10^4 + \frac{1}{2} \times 10^3 (5^2 - 2.22^2)$$

$$= 3 \times 10^4 + 500 \times 20.08$$

$$p_2 \approx 4 \times 10^4 \text{ N/m}^2$$

Ex. 46 **Water enters a house through a pipe 2.0 cm inside diameter, at an absolute pressure of 4×10^5 pa. The pipe leading to the second - floor bathroom 5 m above is 1.0 cm in diameter. When the flow velocity at the inlet pipe is 4 m/s, find the flow velocity and pressure in the bathroom.**

Sol.

By continuity equation the flow velocity

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi(0.01)^2}{\pi(0.005)^2}\times 4 = 16\,\text{m/s} \quad \textit{Ans}$$

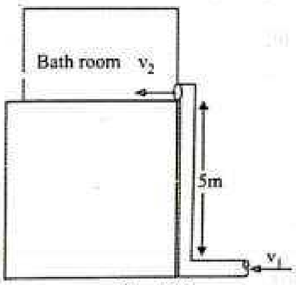


Fig. 3.89

Using Bernoulli's equation between 1 & 2, we have

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or

$$P_2 = P_1 - \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) - \rho g\left(h_2 - h_1\right)$$

$$= 4\times10^5 - \frac{1}{2}\times1000\left(16^2 - 4^2\right) - 1000\times9.8\times5$$

$$= 2.3\times10^5\ \text{Pa} \quad \textit{Ans.}$$

Ex. 47 **The reading of pressure-meter attached with a closed pipe is 3.5×10^5 N/m². On opening the valve of the pipe, the reading of the pressure-meter is reduced to 3.0×10^5 N/m². Calculate the speed of the water flowing in the pipe.**

Sol.

Before opening the valve

$$P_1 = 3.5\times10^5\,N/m^2,\quad v_1 = 0$$

After opening the valve

$$P_2 = 3.0\times10^5\,N/m^2$$

Let v_2 is the speed of the water after opening of the valve, then for the horizontal pipe

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore \quad v_2 = \left[v_1^2 + \frac{2(P_1 - P_2)}{\rho}\right]^{1/2}$$

As $\quad v_1 = 0$

$$\therefore \quad v_2 = \left[\frac{2(P_1 - P_2)}{\rho}\right]^{1/2}$$

$$= \left[\frac{2\left(3.5\times10^5 - 3.0\times10^5\right)}{1000}\right]^{1/2}$$

$$= 10\ \text{m/s} \quad \textit{Ans.}$$

2. **A liquid of density 1048 kg m^{-3} is flowing steadily in a tube of varying cross section. The cross section at the mouth is 1.2×10^{-4} m^2 and that at the end is 10×10^{-6} m^2. Calculate the difference in pressure for a horizontal flow, if the speed of flow at the mouth of the tube is 9.8×10^{-2} m s^{-1}.**

Solution

Plan

The cross-sectional area of each end and fluid velocity at left end is known. Therefore, using the equation of continuity fluid velocity at right end can be calculated. By using Bernoulli's theorem pressure difference can be calculated.

Let *MN* represent the tube in which the liquid flows.

Velocity at M, $v_1 = 9.8 \times 10^{-2}$ m s^{-1}

Area of cross section at $M = 1.2 \times 10^{-4}$ m^2

Area of cross section at $N = 10 \times 10^{-6}$ m^2

To find the velocity at N; using the formula $a_1 v_1 = a_2 v_2$, we get

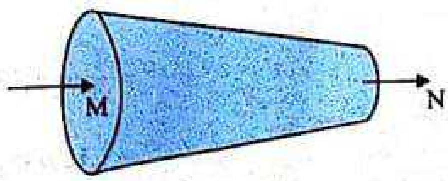
$1.2 \times 10^{-4} \times 9.8 \times 10^{-2} = 10 \times 10^{-6} \times v_2$

$$v_2 = \frac{1.2 \times 9.8}{10} = 1.176 \text{ m s}^{-1}$$

By Bernoulli's theorem

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

For a horizontal flow $h_1 = h_2$



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

(1) A 500g solid cube having an edge of length 20cm floats in water. How much volume of the cube is outside the water? Density of water is $1000 kg/m^3$.

Sol:- $m = 500g = 0.5 Kg$

$\rho = 1000 kg/m^3$

$l = 20 cm$

$v\rho g = mg$

$V = \frac{m}{\rho} = \frac{0.5}{1000} = 5 \times 10^{-4} m^3 = 500 cm^3$

total volume of the cube = $(20cm)^3 = 8000 cm^3$

$\therefore$ the volume outside the water is $8000 cm^3 - 500 cm^3$

$= \underline{7500 cm^3}$

$F = 3234.2$ N

2. A 40 kg girl wearing high heel footwear balances on a single heel. If the diameter of the circular heel is 1.2 cm, calculate the pressure exerted by the heel on a horizontal floor. (Given: $g = 10$ m s^{-2})

Solution

Plan

Use $P = \frac{F}{A}$, $F = mg$; $A = \frac{\pi d^2}{4}$

Given: $m = 40$ kg, diameter of the heel $= d = 1.2 \times 10^{-2}$ m, $P = ?$

$$F = mg$$
$$= 40 \times 10 = 400 \text{ N}$$

$$A = \frac{\pi d^2}{4} = \frac{3.14 \times (1.2 \times 10^{-2})^2}{4} = 1.1304 \times 10^{-4} \text{ m}^2$$

$$P = \frac{F}{A} = \frac{400}{1.1304 \times 10^{-4}} = 3.54 \times 10^6 \text{ Pa}$$

. A multistoreyed building has an overhead water tank connected to taps on each floor. The pressure of water at the ground floor is 196000 Pa and on the third floor is 98000 Pa. Find the height of the third floor (ρ of water $= 10^3$ kg m^{-3}, $g = 9.8$ m s^{-2}).

tion The pressure difference is due to the difference in the height. If the height of the third floor is h from the ground level, then pressure difference $= \rho g h$

Pressure difference = (196000 − 98000) Pa

$$\therefore \rho g h = 98000$$

$$\therefore h = \frac{98000}{\rho g} = \frac{98000}{10^3 \times 9.8}$$

$$h = 10 \text{ m}$$

3.12 Buoyant force and buoyancy

Body inside a fluid experiences pressure on its all faces. As the fluid pressure increases with depth, so the upward thrust at the bottom is more than the downward thrust on the top. Hence a net force acts in upward direction. This upward force acting on a body in a ... is called **upthrust or buoyant force** and the phenomenon is called buoyancy. The

Unit - IV
Heat

Heat & temperature:

Temperature:

Temperature may be defined as the degree of hotness or coldness of a body.

Heat:

Heat is a form of energy which flows from a body at higher temperature to another body at lower temperature, when two bodies are in contact with each other.

NOTE: Heat energy called as thermal energy.

Concept of heat:

Every body is made up of a large number of tiny particles, called molecules. Depending on the nature of substance (solid, liquid or gas) and temperature of the substance, its molecules may possess,

(a) Translatory motion : motion along a straight line.

(b) Vibratory motion : To and fro motion about mean position

(c) Rotatory motion. Rotations of molecules about their axis.

NOTE: Each type of motion provides some kinetic energy to its molecule.

Heat possessed by a body is its total thermal energy of the body, which is sum of kinetic energy of all the individual molecules of the body, on account of translational, vibrational & rotational motion of molecules.

Convention:

a) Heat energy supplied to a body is taken positive.
b) Heat energy lost by a body is taken as negative

S.I unit of Heat energy: Joule.
Practical unit: calorie
1 calorie = 4.186 Joule.

Measurement of temperature:

(Different Scales of Temperature).

Thermometer:

A device which is used for quantitive measurement of temperature is called thermometer.

Ex: a) Mercury thermometer.
b) Electrical resistance thermometers
c) Radiation thermometer.

Different Scales of temperature in use are

(a) Celsius Scale (Centigrade Scale)
(b) Fahrenheit Scale
(c) Reaumer Scale
(d) Kelvin Scale

ⓐ Celcius Scale:

In this Scale, the melting point of ice is regarded as $0°c$ and the boiling point of water as $100°c$. The space between these two fixed points is divided into 100 equal parts. Each part represents $1°c$.

ⓑ Fahrenheit scale:

In this scale, the lower fixed point on the scale as $32°F$, which is the melting point of ice at standard atmospheric pressure. The upper fixed point on the scale is $212°F$ which is boiling point of water at standard atmospheric pressure. The space between the two fixed points is divided into 180 equal parts. Each part is represented $1°$.

ⓒ Reaumer Scale:

Lower fixed point = $0°R$ (melting point of ice)
Upper fixed point = $80°R$ (Boiling point of water)
The space between two fixed points = 80 equal parts.
Each represent $1°R$.

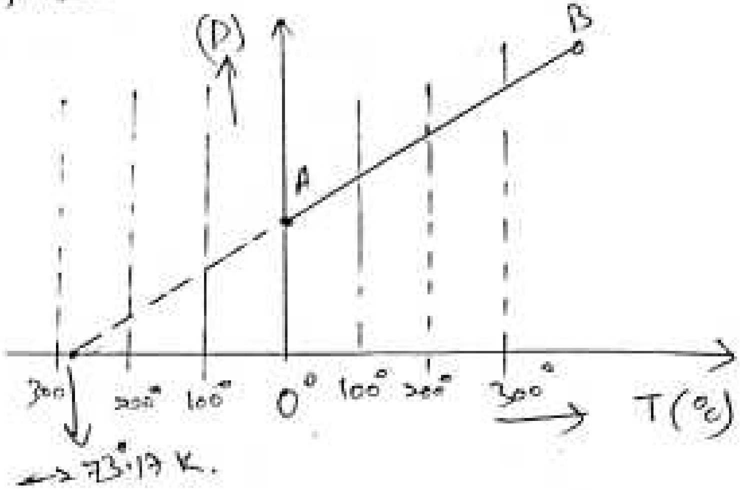
Relationship of T_C, T_F & T_R

T_C, T_F, T_R are temperature values of a body on Celsius Scale, Fahrenheit Scale & Reaumer Scale respectively.

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80}.$$

(d) Kelvin Scale

It is also called as Absolute Scale of temperature



Ideal gas Equation,

$PV = nRT$

For a given volume V of a gas its pressure P exerted by the gas

$P \propto T$.

Here n is number of moles of gas.

When the graph is extrapolated, it meets the temperature axis at $-273°C$.

Kelvin called this as "absolute zero" (0K).

as temperature and pressure of gas would reduce to zero. This indicate lowest attainable temperature is absolute zero. i.e: -273.

$$t\,K = t°C + 273.$$

NOTE: (Scale of division is same as that of Celsius)

$$t°C = t\,K - 273.$$

$\therefore$

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_R - 0}{80} = \frac{T_K - 273}{100}.$$

Thermal expansion:

When matter is heated without any change in its state, it usually expands. As with rise in temperature, the amplitude of vibration of the atoms & molecules increases. Therefore, effective interatomic separation increases. So the matter as a whole expands.

NOTE:

a) Thermal expansion is minimum in case of solids but maximum in case of gases because intermolecular forces are maximum in solids but minimum in gases.

b) Solids can expand in one dimension (linear) two dimension (Superficial expansion) & three dimension (volumetric expansion) while liquids and gases usually suffers change in volume only.

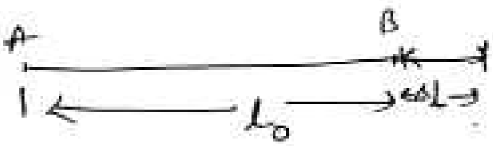
(a) Thermal Expansion of Solids is of three types:

(i) Linear expansion.
(ii) Superficial expansion.
(iii) Cubical expansion.

(i) Linear Expansion:

It is increase in length of a solid on heating while other dimensions show a negligible change.

Let l_0 is original length of the rod and it got elongated by a small length Δl upon increase in temperature by ΔT.



It is found that,

$\therefore \Delta l \propto \Delta T$.

& $\Delta l \propto l_0$.

$\therefore \Delta l \propto l_0 \Delta T$.

$\Delta l = \alpha l_0 \Delta T$ (α = coefficient of linear expansion).

$\therefore \alpha = \dfrac{\Delta l}{l_0 \Delta T}$; $l = l_0 + \Delta l$
$l = l_0 + \alpha l_0 \Delta T = l_0(1+\alpha\Delta T)$

α is defined as a small change in length per unit original length per °c change in temperature.

Final length, $\boxed{l = l_0(1+\alpha\Delta T)}$

Unit : $°c^{-1}$ or K^{-1}.

(ii) Superficial (areal) expansion:

We know, $l = l_0(1+\alpha\Delta T) = (l_0 + \Delta l)$

For areal expansion; Area $= l^2 = \cancel{[l_0(1+\alpha\Delta T)]^2}$.

New area; $l^2 = (l_0 + \Delta l)^2$
$l^2 = l_0^2 + 2.l_0.\Delta l + (\Delta l)^2$

change in area $= l^2 - l_0^2 = 2 l_0 \Delta l + (\Delta l)^2$

$l^2 - l_0^2 = 2 l_0 (\Delta l)$ [$\because (\Delta l)^2$ is too small, Hence neglected.]

$\therefore \beta = \dfrac{2 l_0 \Delta l}{l_0^2 \Delta T}$.

($\because \Delta A \propto A_0$.
$\Delta A \propto A_0 \Delta T$
$\Delta A = \beta A_0 \Delta T$
$\therefore \beta = \dfrac{\Delta A}{A_0 \Delta T}$)

$\beta = \dfrac{2\Delta l}{l_0 \Delta T} = 2\alpha$

$\therefore \beta = 2\alpha$.

[$A = A_0 + \Delta A$.
$= A_0 + \beta A_0 \Delta T$
$A = A_0(1+\beta\Delta T)$]

Note:
β = coefficient of superficial expansion.

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(iii) Volumetric expansion:

$\Delta V \propto V_0$.

$\Delta V \propto \Delta T$

$\Delta V \propto V_0 \Delta T$.

$\Delta V = \gamma V_0 \Delta T$.

$\gamma = \dfrac{\Delta V}{V_0 \Delta T}$.

Final volume,

$\therefore V = V_0 + \Delta V$

$V = V_0 + \gamma V_0 \Delta T$

$V = V_0(1 + \gamma \Delta T)$.

γ = coefficient of volumetric expansion,

We know, $V = l^3$.

$\therefore l^3 = (l_0 + \Delta l)^3$.

$l^3 = l_0^3 + (\Delta l)^3 + 3l_0^2 \Delta l + 3(\Delta l)^2 l_0$.

$l^3 = l_0^3 + 3l_0^2 \Delta l$ $\quad$ ($\because$ Δl is small, $(\Delta l)^2$ & $(\Delta l)^3$ are too small & can be neglected)

$l^3 - l_0^3 = 3l_0^2 \Delta l$.

change in volume $= \Delta V = l^3 - l_0^3 = 3l_0^2 \Delta l$.

$\therefore \gamma = \dfrac{3l_0^2 \Delta l}{l_0^3 \Delta T}$ $\quad$ ($\because V_0 = l_0^3$)

$\gamma = 3\dfrac{\Delta l}{l_0 \Delta T} = 3\alpha$.

$\gamma = 3\alpha$

$\therefore \alpha : \beta : \gamma = 1 : 2 : 3$. $\quad$ ($\because \beta = 2\alpha$, $\gamma = 3\alpha$)

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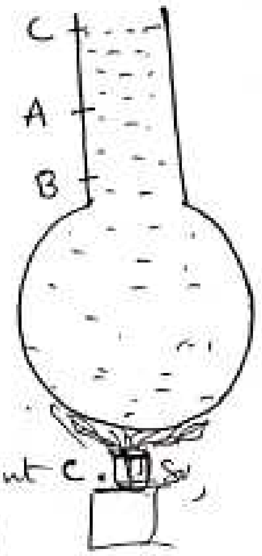
Thermal Expansion of liquids:

* Liquids do not have linear & superficial expansion but these only have volumetric expansion.
* Since liquids are always to be heated along with a vessel which contains them. So initially on heating the system (liquid + vessel), the level of liquid in vessel fall (as vessel expands more since it absorbs heat & liquid expands less) but later on, it starts rising due to faster expansion of liquid.



Let,

Point A initial level of liquid

V_{AB} = Expansion of vessel.

V_{BC} = Real Expansion of liquid.

But, practically we don't see the expansion of vessel and it appears to us that the liquid expands from point A to point C. So, (i.e: Expansion of vessel not take into account)

V_{AC} = Apparent Expansion of liquid.

From Diagram,

$$V_{BC} = V_{AC} + V_{AB} \longrightarrow (A)$$

Real Expansion of liquid = Apparent expansion + Expansion of vessel.

From volumetric expansion,

Real expansion of liquid $(V_{BC}) = \gamma_r V_0 \Delta T \longrightarrow (1)$

γ_r = coefficient of real expansion of liquid.

V_0 = Original volume of liquid.

ΔT = change in temperature.

Similarly,

Apparent Expansion of liquid $(V_{AC}) = \gamma_a V_0 \Delta T \longrightarrow$ (ii)

γ_a = Coefficient of apparent expansion of liquid.

Expansion of vessel $(V_{AB}) = \gamma_v V_0 \Delta T \longrightarrow$ (iii)

γ_v = coefficient of expansion of vessel.

Substitute, (i), (ii) & (iii) in A, we get.

$$\gamma_r V_0 \Delta T = \gamma_a V_0 \Delta T + \gamma_v V_0 \Delta T \longrightarrow (iv)$$

$$\therefore \boxed{\gamma_{real} = \gamma_{app} + \gamma_{vessel}}$$

From (iv), we can write,

$$\gamma_a V_0 \Delta T = \gamma_r V_0 \Delta T - \gamma_v V_0 \Delta T.$$

$$(\Delta V)_{apparent} = V_0 (\gamma_{real} - \gamma_{vessel}) \Delta T.$$

$$(\Delta V)_{apparent} = V_0 (\gamma_{real} - 3\alpha) \Delta T$$

$(\because \gamma_{vessel} = 3\alpha)$

α = linear expansion of vessel.

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Similarly,

Apparent expansion of liquid $(V_{AC}) = \gamma_a V_0 \Delta T \rightarrow (ii)$

γ_a = Coefficient of apparent expansion of liquid.

Expansion of vessel $(V_{AB}) = \gamma_v V_0 \Delta T \rightarrow (iii)$

γ_v = coefficient of expansion of vessel.

Substitute, (i), (ii) & (iii) in A, we get.

$$\gamma_r V_0 \Delta T = \gamma_a V_0 \Delta T + \gamma_v V_0 \Delta T \rightarrow (iv)$$

$$\therefore \boxed{\gamma_{real} = \gamma_{app} + \gamma_{vessel}}$$

From (iv), we can write,

$$\gamma_a V_0 \Delta T = \gamma_r V_0 \Delta T - \gamma_v V_0 \Delta T.$$

$$(\Delta V)_{apparent} = V_0 (\gamma_{real} - \gamma_{vessel}) \Delta T.$$

$$(\Delta V)_{apparent} = V_0 (\gamma_{real} - 3\alpha) \Delta T$$

$(\because \gamma_{vessel} = 3\alpha)$

α = linear expansion of vessel.

Thermal Expansion of gases:-

In case of gases, as the temperature is raised, molecular impacts become harder causing an increase in pressure. However, our aim is to find change in volume only with change in temperature. For this purpose, we need to keep the pressure constant by expansion.

Do we have a relation between volume, temperature & pressure?

Ideal gas equation:

$$PV = nRT$$

let P is constant,

$$P_0V_1 = nRT_1 \rightarrow (i)$$

$$P_0V_2 = nRT_2 \rightarrow (ii)$$

Subtract (i) from (ii).

$$P_0V_2 - P_0V_1 = nRT_2 - nRT_1$$

$$P_0(V_2 - V_1) = nR(T_2 - T_1)$$

$$P_0\Delta V = nR\Delta T$$

Divide with $V_0\Delta T$ both sides,

$$\frac{P_0\Delta V}{V\Delta T} = \frac{nR\Delta T}{V_0\Delta T}$$

$$P_0(\gamma_g) = \frac{nR}{V_0}$$

$$\left(\because \frac{\Delta V}{V\Delta T} = \gamma \right)$$

$$P_0V_0 = nRT_0$$

$$\frac{nR}{V_0} = \frac{P_0}{T_0}$$

$$P_0(\gamma_g) = \frac{P_0}{T_0}$$

$$\gamma_g = \frac{1}{T_0} \quad \text{(For ideal gas)}$$

This equation, shows the temperature dependence of γ_{gas}. It decreases with increasing temperature.

For a gas at RT & constant pressure,

$$\gamma_g = \frac{1}{300\,K} = 3300 \times 10^{-6}\,K^{-1}.$$ (Much larger than liquids & gases)

Material	α ($\times 10^{-6}\,K^{-1}$)	γ ($\times 10^{-6}\,K^{-1}$).	
Glass	4	13	Solids
Aluminium	24	70	
Copper	17	51	
Gold	14	42	
Alcohol	—	750	Liquids
Mercury	—	180	
Water	—	200	
GAS	—	3300.	Gas.

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Effect of temperature on density of liquids & Solids.

When a given mass of a solid or a liquid or a solid is heated, it expands i.e: its volume increases.

We know that,

$$\rho = \frac{M}{V}$$

$$\rho \propto \frac{1}{V} \quad \text{(As a given mass is fixed)} \quad \text{(i.e: M = constant)}.$$

$\therefore$ The density of solid would decrease on heating.

We know,

$$V = V_0(1+\gamma \Delta T)$$

$$\rho = \frac{M}{V} \Rightarrow V = \frac{M}{\rho}$$

$$\rho_0 = \frac{M}{V_0} \Rightarrow V_0 = \frac{M}{\rho_0}.$$

$$\frac{M}{\rho} = \frac{M}{\rho_0}(1+\gamma \Delta T)$$

$$\frac{1}{\rho} = \frac{1}{\rho_0}(1+\gamma \Delta T)$$

$$\rho = \frac{\rho_0}{(1+\gamma \Delta T)}$$

$$\rho = \rho_0(1+\gamma \Delta T)^{-1}$$

Binomial expansion,

$$\rho = \rho_0(1-\gamma \Delta T)$$

[Using $\because (1+x)^{-1} = 1-x+x^2-x^3+\ldots$ & neglecting higher order terms].

Clearly, $\rho < \rho_0$.

i.e: Density of a solid or liquid decreases with rise in temperature.

Anomalous behaviour of water:– (Anomalous expansion of water.)

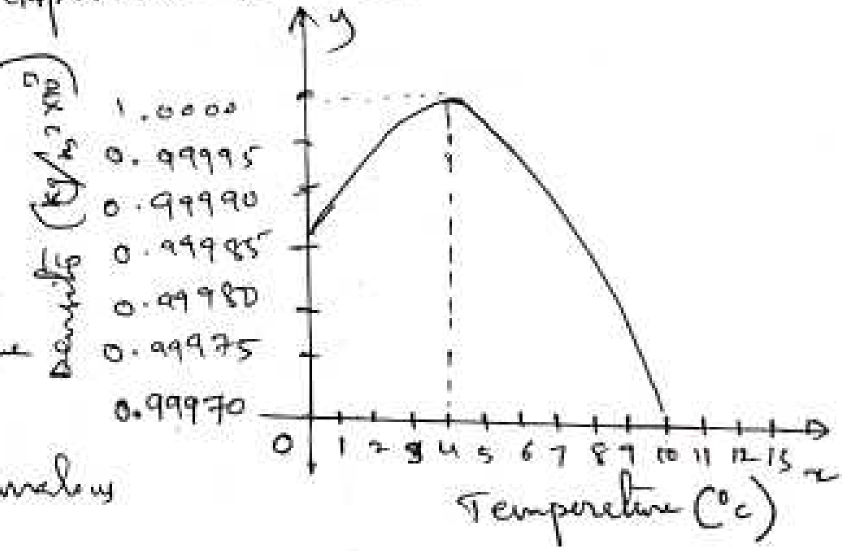
We know that the density of solids & liquids decreases with the rise in temperature.

or density of solid or liquid increases with reducing the temperature and cooling the substance.

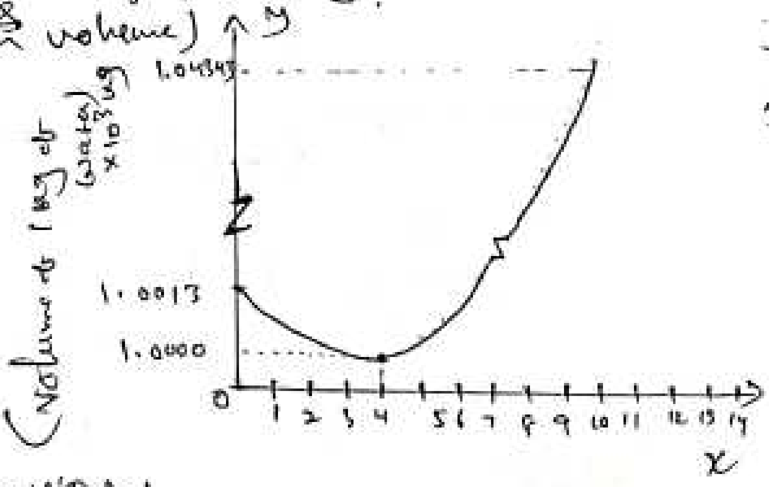
However, an anomalous behaviour is observed in case of water. That is the density of water increases as it is cooled from room temperature to 4°c.

With further decrease in temperature from 4°c to 0°c, the density of water decreases and its volume increases.

This is known as anomalous expansion of water.



∴ water has maximum density at 4°c.
(minimum & volume)



Advantages of this behaviour:

* In cold regions the temperature drops to less than zero °c, the top surface of water bodies gets freezed. As ice has lesser density and occupies top surface of water body and also prevent fall of temperature below the ice layer. The lower layers of water remain at 4°c. If water did not have

this property, lakes & ponds etc. would freeze from top to bottom which can destroy plants and animal life in underwater.

Reason:

For ice, the crystalline lattice is dominated by a regular array of hydrogen bonds which space the water molecules farther apart than they are in liquid water. This accounts for water's decrease in density upon freezing.

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this property, lakes & ponds etc.. would freeze from top to bottom which can destroy plants and animals life in under water.

Reason:

In the ice, the crystalline lattice is dominated by a regular array of hydrogen bonds which space the water molecules farther apart than they are in liquid water. This accounts for water's decrease in density upon freezing.

Specific heat or specific heat capacity

Let us suppose ΔQ is a small amount of heat energy required to raise the temperature of a certain mass (m) of a substance through a small range of temperature (ΔT).

It is found that,

$$\Delta Q \propto m. \qquad \text{(For a given } \Delta T\text{)}$$

also

$$\Delta Q \propto \Delta T \qquad \text{(For a given amount of mass)}$$

$$\therefore \Delta Q \propto m\Delta T.$$

$$\Delta Q = c\, m\Delta T$$

where 'c' called as specific heat.

$$c = \frac{\Delta Q}{m\Delta T}.$$

Definition: (Specific heat)

The amount of heat required to raise the temperature of unit mass of the substance through unit degree is called Specific heat.

Unit: $J\,Kg^{-1}K^{-1}$.

NOTE: specific heat depends on nature of the substance.

Molar specific heat:

The amount of energy needed to raise the temperature of one gram mole of a substance by $1^{\circ}C$ (or 1K) is called molar ~~heat capacity~~. specific heat.

Unit: $J\,mole^{-1}K^{-1}$.

We know, 1 mole = Molecular weight of the substance

i.e: number of moles in a given mass (m grams) of a substance,

$$n = \frac{m}{M}$$

where M is a molecular weight of substance.

$$\therefore m = nM.$$

We know, $c = \frac{\Delta Q}{m\Delta T} = \frac{\Delta Q}{nM(\Delta T)}$

$$cM = \frac{\Delta Q}{n\Delta T}.$$

$$\boxed{C = \frac{\Delta Q}{n\Delta T}} \Rightarrow \text{Molar specific heat of a substance.}$$

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Heat Capacity (Thermal Capacity)

The quantity of heat required to raise the temperature of the whole substance through $1^\circ C$ is called heat capacity.

We know, $\Delta Q = cm(\Delta T)$.

Here $\Delta T = 1^\circ C$,

$\therefore \Delta Q = cm$. (m is a given total mass).

Heat capacity $= \Delta Q =$ (Specific heat × mass)

unit: JK^{-1}

Latent heat (Hidden heat)

Definition: Latent heat of a substance is the amount of heat energy required to change the state of unit mass of the substance from solid to liquid or from liquid to gas/vapour without any change in temperature.

$Q = mL$ (L = latent heat)

$L = \frac{Q}{m}$. unit: $J\,kg^{-1}$.

NOTE:

- ✓ Solid changes into liquid at a constant temperature called as melting point of the solid.
- ✓ Liquid changes to gas at a constant temperature called as boiling point of the liquid.
- ✓ Temperature remains unaffected till the state of entire substance has changed.
- ✓ Thermometer ~~fails to detect~~ show neither rise nor fall in temperature even though there is a heat exchange. Hence it is called hidden heat.

Principle of calorimetry: (Law of mixtures)

When two bodies at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

$$\boxed{\text{Heat lost} = \text{Heat gained}}$$

(Principle of calorimetry).

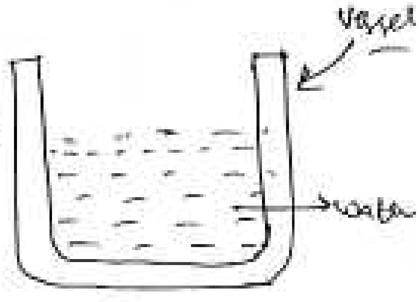
Principle of calorimetry represents the law of conservation of heat energy.

Determination of specific heat in laboratory:

Calorimetry is one of the methods for the determination of specific heat/latent heat of a substance.

It uses a calorimeter, which is a cylindrical vessel of copper provided with a stirrer and a lid.

The calorimeter & stirrer are cleaned, washed, dried and weighed. The calorimeter is then half filled with water & weighed to find the mass of water taken.



calorimeter

Steps:

* Note the initial temperature of water and calorimeter.
* Heat the substance whose specific heat to be determined, to a particular temperature.

9. * place the heated body in calorimeter & stir the mixture.

As the substance is at higher temperature, it loses heat to water and calorimeter. After some time, note temperature of the mixture.

Calculation

Here, take a solid, to measure the specific heat.

Let the mass of solid $= m_1$

mass of calorimeter & stirrer $= m_2$

mass of water $= m_3$.

specific heat of solid $= c_1$

specific heat of material of the calorimeter (& stirrer) $= c_2$

Specific heat of water $= c_3$.

Initial temperature of solid $= T_1$

Initial temperature of calorimeter, stirrer & water $= T_2$

Final temperature of mixture $= T$.

We have,

Heat lost by solid $= m_1 c_1 (T_1 - T)$.

Heat gained by calorimeter (& stirrer) $= m_2 c_2 (T - T_2)$

Heat gained by water = $m_3 c_3 (T - T_2)$.

Assuming no heat loss to surroundings, according to principle of calorimetry, we can have.

$$\cancel{m_1 c_1 (T_1 - T)} =$$

$$m_1 c_1 (T_1 - T) = m_2 c_2 (T - T_2) + m_3 c_3 (T - T_2)$$

$$c_1 = \frac{m_2 c_2 (T - T_2) + m_3 c_3 (T - T_2)}{m_1 (T_1 - T)}.$$

$$c_1 = \frac{[m_2 c_2 + m_3 c_3](T - T_2)}{m_1 (T_1 - T)}.$$

How to find specific heat of liquid using this method;

Specific heat capacity of a liquid can also be measured with the same method. Here a solid of known specific heat capacity c_1 is used & the experimental liquid is taken in the calorimeter in place of water. Here the solid should be denser than the liquid.

From the above equation, we will have;

$$m_1 c_1 (T_1 - T) - m_2 c_2 (T - T_2) = m_3 c_3 (T - T_2).$$

$$\frac{m_1 c_1 (T_1 - T) - m_2 c_2 (T - T_2)}{m_3 (T - T_2)} = c_3$$

$$c_3 = \frac{m_1 c_1 (T_1 - T)}{m_3 (T - T_2)} - \frac{m_2 c_2 (T - T_2)}{m_3 (T - T_2)}$$

$$\boxed{c_3 = \frac{m_1 c_1 (T_1 - T)}{m_3 (T - T_2)} - \frac{m_2 c_2}{m_3}.}$$

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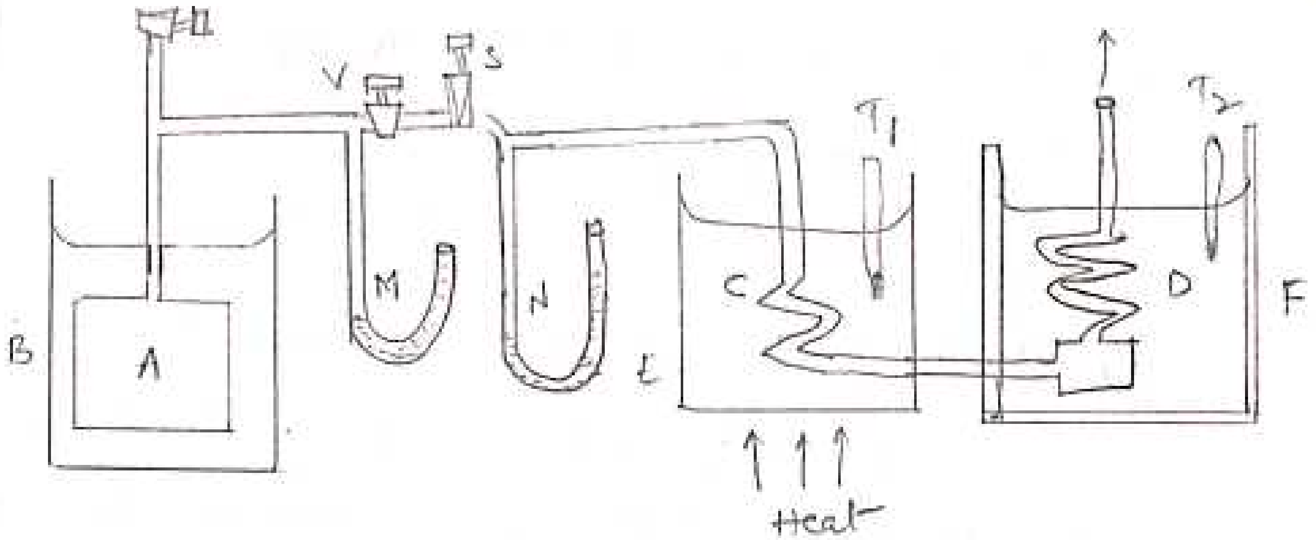
Determination of C_p of a gas.

C_p = specific heat of gas at constant pressure.



* Experimental gas is taken at a high pressure large tank A, which is immersed in water at constant temperature.
* The tank is connected to two copper coils C & D.
* valve V is to open or to close the tube for gas flow.
* Screw valve 'S' is to control rate of flow of gas.
* copper coil 'C' is immersed in hot oil bath E.
* copper coil D is immersed in a calorimeter F. containing water.
* Manometer M to measure pressure of gas in tank.
* Manometer N to measure pressure of gas flowing in calorimeter.

Steps:-

* oil bath is heated to temperature T_1.
* A measured mass ~~of~~ m of water is taken in the calorimeter.
* Initial temperature of water is measured by T_2.

* The difference in heights of two arms of the manometer M is noted.

Let water equivalent of the calorimeter together with the coil $= W$.

mass of water $= m$

temperature of oil bath $= T_1$

Initial temperature of water $= T_2$.

Final temperature of water $= T_3$.

& amount of the gas (in moles) passed through the water $= n$

The gas at T_1 enters coil D. In the beginning of the exp., the gas leaves the coil D at T_2. At the end of the exp, the gas leaves the coil D at T_3.

$\therefore$ The average temperature of gas leaving the coil

$$D = \frac{T_2+T_3}{2}.$$

$\therefore$ Heat lost by gas $= \Delta Q = n c_p \left[T_1 - \frac{(T_2+T_3)}{2}\right]$.

This heat is used to increase the temperature of calorimeter, the coil D and the water from T_2 to T_3.

Heat gained

$$\therefore \Delta Q = (W+m)\, c\, (T_3 - T_2)$$

where c = Specific heat of water.

From principle of calorimetry,

$$\therefore n c_p \left[T_1 - \frac{(T_2+T_3)}{2}\right] = (W+m)\, c\, (T_3-T_2)$$

$$c_p = \frac{(W+m)\, c\, (T_3-T_2)}{n\left[T_1 - \frac{(T_2+T_3)}{2}\right]}$$

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Determination of n:

Suppose the difference in the mercury levels in the manometer M is h & the atmospheric pressure is equal to a height H of mercury. The pressure in the tank is equal to height $H+h$. Note the difference in height h at beginning & at the end.

Then, Initial Pressure, P_1 & P_2 are determined.

Assuming gas to be ideal,

$\therefore$

$$P_1 V = n_1 RT$$

$$P_2 V = n_2 RT$$

$$(P_1 - P_2) V = (n_1 - n_2) RT$$

$$n = (n_1 - n_2) = \frac{(P_1 - P_2) V}{RT}$$

then we can find value of C_p if we know n. $\therefore$

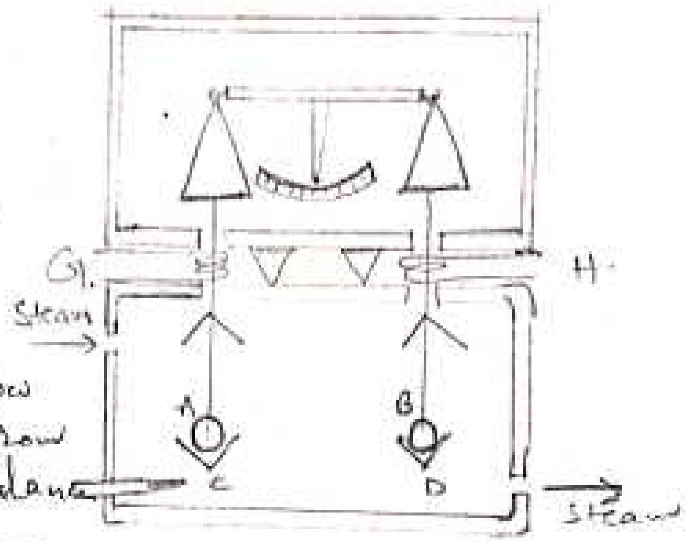
NOTE:

Water equivalent:

Water equivalent of a body is defined as mass of water which would absorb or evolve the same amount of heat as is done by the body in rising or falling through the same range of temperature. It is represented by ω.

Determination of Cv of a gas—

Fig. shows a schematic diagrams of Joly's differential steam calorimeter used to measure Cv of a gas.



Description:

* A & B are two hollow spheres suspended from pans of a sensitive balance
* spheres are enclosed in a steam chamber.
* Two pans C & D are fitted ~~upto~~ below the spheres.
* Umbrella like shields E, F are provided over the spheres
* Plaster of paris tubes are provided at the holes in the steam chamber through which suspension wires pass. These are electrically heated using G & H coils to avoid/ensure no drops are formed when steam sent into chamber so that the wires move freely to balance the pans.
* The shields E, F do not allow the drops to fall on spheres or on to pans.
* A thermometer is fitted in the steam chamber.

Experimental steps:

* Air pumped out of the spheres and spheres are balanced.
* The experimental gas is filled in one of the spheres, say B, and additional weight is placed on its balance pan so that spheres are again balanced. This gives mass of gas.
* The temperature of steam chamber noted. This gives initial temperature of gas.
* Steam passed through steam chamber.
* The steam condenses on the spheres and the water formed is collected in the pans C & D.

13.

* The temperature of the spheres rise. Also, the ~~temp~~ steam rises the temperature of the sphere as well as of the gas.
* When the temperature become steady, the spheres again balanced by putting extra weights. This extra weight gives the amount of steam needed to raise the temperature of the gas only.
* The final temperature in steady state is noted.

Suppose,

the mass of the gas taken $= m_1$
the mass of extra steam condensed $= m_2$
Initial temperature of the gas $= T_1$
Final temperature of the gas $= T_2$
specific latent heat of vapourisation of water $= L$

Number of moles of gas taken $= \frac{m_1}{M} = n$.
where M is molecular weight of gas.

$\therefore$ Heat loss by the steam $= m_2 L$

& Heat gained by the gas $= n C_V (T_2 - T_1)$

$$\therefore \quad m_2 L = n C_V (T_2 - T_1) = \frac{m_1}{M} C_V (T_2 - T_1)$$

$$\boxed{C_V = \frac{M m_2 L}{m_1 (T_2 - T_1)}}$$

NOTE: In this we have neglected the increase in the volumes of the spheres as temperature rises.

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Change of State:

a) <u>Melting</u>: Conversion of solid into liquid state at constant temperature is known as melting.

b) <u>Boiling</u>: Evaporation within the whole mass of the liquid at a constant temperature is called as Boiling.

c) <u>Sublimation</u>:
Direct conversion of solid into vapour state is called Sublimation.

d) <u>Condensation</u>: The process of conversion from gaseous or vapour state to liquid state is known as condensation

e) <u>Hoar frost</u>:
Direct conversion of vapour into solid is called hoar frost.
Ex: Formation of Snow by freezing of clouds.

f) <u>Regelation</u>:
Regelation is the melting of ice caused by pressure and its resolidification when its pressure is removed.

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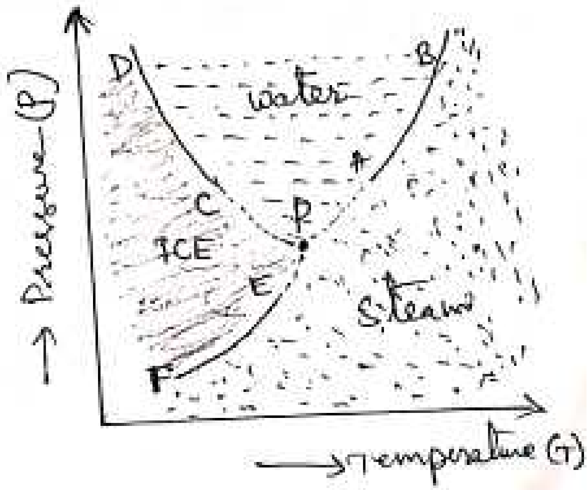
Pressure Temperature phase diagram:

In a pressure temperature phase diagram, each phase corresponds to a region or area. The boundary between any two areas is a curve on which the two phases co-exist in equilibrium.

The figure show P-T diagram for water.



(a) **Vaporization line AB**:
This line represents the variation of boiling point of the substance (in liquid state) with its pressure.

* At any point on vapourization line, the substance co-exist in equilibrium in liquid & vapour phase.
* Vapourization line called as steam line incase of water.

(b) Fusion line: CD

This line represents the variation of melting point of the substance (in solid form) with pressure.

* At every point on fusion line, the temperature & pressure values are such that the substance co-exists in equilibrium in solid & liquid phases.
* Fusion line is also called as ice line in case of water.

(c) Sublimation line: EF.

This line represents the variation of pressure with temperature at which a solid changes directly into vapour state (without going into liquid phase).

* Sublimation line called as hoar frost line in case of water.

The three curves AB, CD, & EF when extrapolated come to meet at a point P, which is called as triple point of the substance.

Triple point :-

Triple point of a substance defined as a particular point on the P-T phase diagram representing a particular pressure & a particular temperature at which the substance can co-exist in the three states i.e: solid, liquid & vapour.

For water:

Triple point temperature = 273.16 °K. (or 0.01°C)
Triple point pressure = 0.46 cm of Hg column.

~~Conduction~~ Modes of heat transfer

Transfer of heat from one body to another body takes place through three mechanisms. These are,

(a) Conduction
(b) Convection
(c) Radiation.

(a) Conduction

The process in which the transfer of heat energy takes place due to exchange of energy between the atoms of the medium is called conduction.

NOTE:

* It is usually takes place in solids.
* The atoms of a medium are in constant state of vibration about their mean position. There is no actual motion of molecules or atoms from one place to another.

(b) Convection :

The process in which heat is transferred from one place to another place by actual movement of heated substance is called convection.

NOTE.

* Convection requires medium for heat transfer usually fluids.
* It can take place due to difference in densities of the fluids. (Ex. A fluid container heated through its bottom).
* If heated fluid is forced to move by a blower, fan or pump, the process called forced convection.

(c) Radiation:

The process of transfer of heat from one place to another place without heating the intervening medium/without any need of material medium is called radiation.

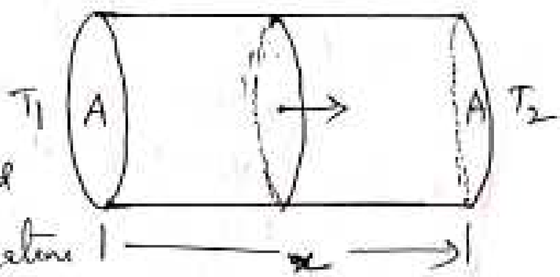
NOTE:

- When a body heated & placed in vaccum, it loses heat even when there is no medium surrounding it.
- Here the heat is lost through the process of radiation.

Thermal conductivity:

~~Heat~~ The ability of a material to conduct heat is measured by thermal conductivity of the material.

When one end of a metal rod is heated, heat flows by conduction from hot end to cold end. In this process, ~~a~~ part of heat received is spent in raising the temperature of the cross section. Another part of heat is conducted away to next cross section towards colder end.



Variable state: As the temperature of every crosssection of the rod goes on increasing, the rod is said to be in variable state.

NOTE: In variable state of the rod, both the properties of material i.e. specific heat & thermal conductivity play their roles one in raising temperature and another in transmission of heat respectively.

After some time a state is reached when temperature of every cross section of the rod becomes constant which is called as steady state.

Steady State:
The state of the rod in which, temperature of each part becomes constant and there is no further absorption of heat anywhere in the rod is called steady state.

NOTE:
* In steady state, temperature of each part of the rod is constant but not same.
* Temperature decreases as we move away from hot end of rod.
* Specific heat has no role to play in steady state of thermal conductivity alone is effective in the steady state.

Let us consider a ~~uniform~~ slab of uniform cross section A and length x. Let one face of slab is maintained at T_1 & another end at T_2. Also suppose, remaining surface is covered with a non-conducting material so that no heat is lost to the surroundings.

After some time a steady state is reached, and temperature at any point remains unchanged as time passes. In such case, the amount of heat crossing per unit time through any cross section is same. i.e: Equal.

If ΔQ amount of heat crosses through any cross section in time Δt, $\Delta Q/\Delta t$ is heat current.

It is found that, in steady state,

$$\frac{\Delta Q}{\Delta t} \propto A$$

$$\frac{\Delta Q}{\Delta t} \propto (T_1 - T_2)$$

$$\frac{\Delta Q}{\Delta t} \propto \frac{1}{x}$$

15.

Thus,

$$\frac{\Delta Q}{\Delta t} \propto A \cdot \frac{(T_1 - T_2)}{x}$$

$$\frac{\Delta Q}{\Delta t} = KA \frac{(T_1 - T_2)}{x}.$$

where K is called thermal conductivity of the material.

If the area of cross-section is not uniform or if steady state conditions are not reached, the equation can only be applied to thin layer of material, perpendicular to heat flow. then we can write,

$$\frac{\Delta Q}{\Delta t} = KA \frac{dT}{dx}$$

the quantity $\frac{dT}{dx}$ is called temperature gradient.

Unit: $J s^{-1} m^{-1} K^{-1}$ or $W m^{-1} K^{-1}$ or $W m^{-1} {}^\circ C^{-1}$.

Thermal resistance to conduction:

For example, keeping the meal hot in your tiffin-box, you are more interested in poor heat conductors, rather than good conductors. For this reason, the concept of thermal resistance R has been introduced.

We know,

$$\frac{\Delta Q}{\Delta t} = KA \frac{\Delta T}{\Delta x}.$$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta T}{\left(\frac{\Delta x}{KA}\right)}$$

This is analogous to ohm's law equation: $I = \frac{V}{R}$.

The quantity, $R = \frac{\Delta x}{KA}$. is called thermal resistance

NOTE:

Greater the thermal resistance, poorer will be the thermal conduction.

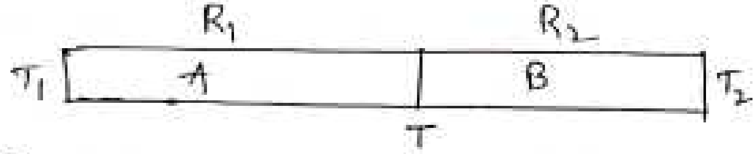
For a slab of cross section A, lateral thickness L & thermal conductivity K, we have

$$R = \frac{L}{KA}.$$

Series and parallel connections of rods:-

(a) Series connection:

Consider two rods of thermal resistance R_1 & R_2 joined one after other as shown in the figure.



Equivalent to 

Here $T_1 > T_2$.

At steady state, any heat that goes through the first rod also goes through the second rod. Thus same heat current passes through the two rods. This is called as series connection, and T is a temperature at junction.

$$\therefore \; i = \frac{\Delta Q}{\Delta t} = \frac{T_1 - T}{R_1} \quad \text{(Heat current through first rod)}$$

$$(T_1 - T) = iR_1 \longrightarrow (1)$$

Heat current through second rod is,

$$i = \frac{\Delta Q}{\Delta t} = \frac{T - T_2}{R_2}.$$

$$(T - T_2) = iR_2. \longrightarrow (2)$$

Adding (1) & (2).

$$(T_1 - T) + (T - T_2) = iR_1 + iR_2.$$

$$(T_1 - T_2) = i(R_1 + R_2).$$

$$i = \frac{T_1 - T_2}{(R_1 + R_2)}$$

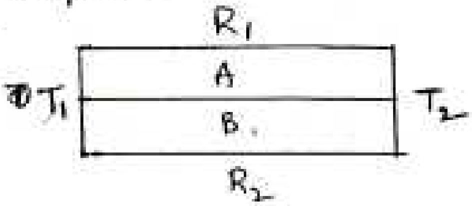
Thus, the two rods together equivalent to a single rod of thermal resistance $(R_1 + R_2)$.

If more than two rods are joined, then

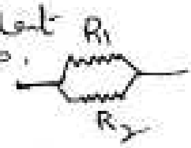
$$R = R_1 + R_2 + R_3 + R_4 + \dots\dots$$

(b) Parallel connection:

Let two rods are joined at their ends as shown in fig. the left ends of both the rods are kept at temperature T_1 & right ends at T_2. This is called as parallel connection.



Equivalent to.



Heat current through first rod,

$$i_1 = \frac{\Delta Q_1}{\Delta t} = \frac{T_1 - T_2}{R_1}.$$

Heat current through second rod,

$$i_2 = \frac{\Delta Q_2}{\Delta t} = \frac{T_1 - T_2}{R_2}.$$

the total current going through the left end is,

$$i = i_1 + i_2.$$

$$i = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

or $$i = \frac{(T_1 - T_2)}{R}.$$

where $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If more than two rods are connected in parallel, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots$$

Thermal Radiation:

Thermal radiations are emitted by a body on account of its temperature.

The energy emitted depends on
(a) Temperature of the body.
(b) Nature of radiating surface of the body.

NOTE:
* Thermal radiation has nearly all the properties possessed by light and these are also electromagnetic waves with the only difference of wavelength or frequency.
* The wavelength of thermal radiation is larger than that of visible light. Hence smaller energy compared to visible light.
* They obey inverse square law. i.e. their intensity varies inversely as the square of the distance from source.
* They require no medium for propagation.
* They show reflection, refraction, interference, diffraction and polarisation as do the light radiation.

Reflectance, Absorptance & Transmittance:

When thermal radiation fall on a body, they partly reflected, partly absorbed and partly transmitted.

(a) Reflecting power (Reflectance):
The ratio of amount of radiation reflected by the body to the total amount of radiation incident on the body is defined as reflecting power.

Represented by reflective coefficient $(r) = \frac{Q_r}{Q}$.

(b). Absorbing power (Absorptance):
The ratio of amount of radiation absorbed by the body to the total amount of radiation incident on the body is defined as absorbing power.

Represented by absorptive coefficient $(a) = \frac{Q_a}{Q}$.

(c) Transmittance (Transmitting power)

The ratio of amount of radiation transmitted by body to the total amount of radiation incident on the body is defined as transmitting power.

Represented by transmittive coefficient $t = \frac{Q_t}{Q}$.

From energy conservation,

we know, $Q = Q_r + Q_t + Q_a$.

$$1 = \frac{Q_r}{Q} + \frac{Q_t}{Q} + \frac{Q_a}{Q}.$$

$$\boxed{1 = r + t + a}$$

If $r = 1$, $a = 0$, $t = 0$ $\Rightarrow$ Perfect reflector

$a = 1$, $r = 0$, $t = 0$ $\Rightarrow$ perfect absorber (Ideal black body)

$t = 1$, $r = 0$, $a = 0$ $\Rightarrow$ Perfect transmitter.

Monochromatic emittance, Emissive power, Emissivity & monochromatic absorptance:

Every body at a temperature greater than 0K emits thermal radiations of all wavelengths within a certain range. The amount of thermal energy radiated in a given small wavelength interval is not same for all wavelengths. Hence we define following terms,

(a) Monochromatic absorptance or spectral absorbing power :-

Monochromatic absorptance related to a certain wavelength λ is defined as the ratio of the amount of heat energy absorbed in a certain time to the total heat energy ~~absorbed~~ incident upon it in the same time, both in the unit wavelength interval around the λ i.e. $(\lambda - \frac{1}{2})$ & $(\lambda + \frac{1}{2})$

Represented by a_λ

$$a_\lambda = \frac{Q_{a\lambda}}{Q_\lambda}.$$ (Dimensionless)

Total absorptive power, $a = \int_0^\infty a_\lambda d\lambda$.

(b) Monochromatic emittance or spectral emissive power:

spectral emissive power of a body corresponding to a particular wavelength λ at a particular temperature, is defined as amount of radiant energy emitted per unit time, per unit surface area of the body within unit wavelength interval around λ i.e. between $(\lambda - \frac{1}{2})$ & $(\lambda + \frac{1}{2})$

Represented by e_λ.

unit: $J s^{-1} m^{-2} \mathring{A}^{-1}$ or $W m^{-2} \mathring{A}^{-1}$.

Total emissive power: $e = \int_0^\infty e_\lambda d\lambda$.

Unit: $J s^{-1} m^{-2}$.

(c) Emissivity:— (ϵ)

Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of a body (e) to the total emissive power of a perfectly black body (E) at that temperature.

$$\epsilon = \frac{e}{E} \quad \Rightarrow \boxed{e = \epsilon E}$$

Prevost's theory of exchange:

According to Prevost, at every possible temperature (except zero kelvin temperature) there is a continuous heat energy exchange between a body and its surrounding & this exchange carry on for infinite time.

(a) When a cold body is placed in the hot surrounding:

The body radiates less energy and absorbs more energy from the surrounding therefore the temperature of body increases (Heating effect).

(b) When a hot body placed in cooler surrounding:

The body radiates more energy & absorbs less energy from surrounding. Therefore temperature of body decreases (Cooling effect).

(c) When temperature of body = temperature of surrounding.

The energy radiated per unit time by the body is equal to the energy absorbed per unit time by the body

∴ Temperature remains constant

and no heating & no cooling effect.

Ideal Black body:

A body surface which absorb all incident thermal radiations at low temperature, irrespective of their wave length and emits out all these absorbed radiations at high temperature is assumed to be an ideal black body surface.

or.

A perfect black body is that which absorbs completely the radiations of all wavelengths incident on it.

Kirchaff's law:

At a given temperature, for all bodies, the ratio of their spectral emissive power (e_λ) to spectral absorptive power (a_λ) is constant and this constant is equal to spectral emissive power (E_λ) of the ideal black body at the same temperature.

$$\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant.}$$

$$\left(\frac{e_\lambda}{a_\lambda}\right)_1 = \left(\frac{e_\lambda}{a_\lambda}\right)_2 = \text{constant}$$

$$\therefore \boxed{e_\lambda \propto a_\lambda}$$

⇒ Good absorbers are good emitters & bad absorbers are bad emitters.

Stefan's - Boltzman law:

The amount of heat energy (E) radiated per second by unit area of a perfect black body is directly proportional to the fourth power of absolute temperature (T) of the body.

$$E \propto T^4.$$

$$E = \sigma T^4$$

where σ is called as Stefan-Boltzman constant.

$\sigma = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}$.

A body which is not a ~~the~~ perfect black body, emits less radiation than given by the above equation. It is however proportional to T^4. Hence the energy emitted by such a body per unit time by unit area is given by

$$\boxed{E = \epsilon \sigma T^4}$$

($\because \epsilon$ = emissivity or relative emittance.)

where ϵ is a constant for a given surface having value between 0 and 1.

Consider a body of emissivity ϵ kept in thermal equilibrium in a room temperature T_0. The energy of radiation absorbed by it per unit time by unit area should be equal to energy emitted by it per unit time by unit area. This is because temperature remains constant. Thus the energy of radiation absorbed per unit time by unit area is

$$E_0 = \epsilon \sigma T_0^4$$

Now, suppose the temperature of the body is changed to $T > T_0$. Then the energy of thermal radiation emitted by the body per unit time by unit area is

$$E = \epsilon \sigma T^4.$$

We already know, that the energy absorbed per unit time by unit area is $E_0 = \epsilon \sigma T_0^4$.

$\therefore$ Net loss of thermal energy $= E - E_0$.

$$E - E_0 = \epsilon \sigma T^4 - \epsilon \sigma T_0^4$$

$$\boxed{\Delta E = E - E_0 = \epsilon \sigma (T^4 - T_0^4)}$$

$$\Delta E = \epsilon \sigma (T^4 - T_0^4).$$

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Newton's law of cooling:

Rate of cooling $\left(\frac{dT}{dt}\right)$ is directly proportional to excess of temperature of the body over that of surroundings.

$$\boxed{-\frac{dT}{dt} \propto (T - T_0)} \quad \text{when } T - T_0 \not> 35°c.$$

where T = temperature of the body,
T_0 = temperature of surroundings.
$T - T_0$ = excess temperature $(T > T_0)$.

If the temperature of body is decreased by dT ~~in time~~ ~~dT~~ in time dt, then rate of fall of temperature

$$\boxed{-\frac{dT}{dt} = K(T - T_0)}$$

- This is known as Newton's law of cooling. The constant K depends on the nature of the surface involved & surrounding conditions.
- Negative sign indicates that the rate of cooling is decreasing with time.

Limitations of Newton's law of cooling:

→ Temperature difference should not exceed $35°c$, $(T - T_0) \not> 35°c$
→ Loss of heat should only be by radiation.
→ This law is an extended form of Stefan-Boltzman's law

Derivation of Newton's law from Stenfan-Boltzman law

We know the net rate of loss of thermal energy from the body due to radiation,

$$\Delta E = \epsilon\sigma(T^4 - T_0^4).$$

If the temperature difference is small, we can write

$$T = T_0 + \Delta T \Rightarrow \Delta T = (T - T_0).$$

$$\therefore \Delta E = \epsilon\sigma\left[(T_0 + \Delta T)^4 - T_0^4\right].$$

$$= \epsilon\sigma\left[T_0^4\left(1 + \frac{\Delta T}{T_0}\right)^4 - T_0^4\right]$$

$$= \epsilon\sigma T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

If $x \ll 1$, then $(1+x)^n = 1 + nx$ using Binomial expansion.

Here $\Delta T \ll T_0 \Rightarrow \frac{\Delta T}{T_0} \ll 1$, $\therefore \left(1 + \frac{\Delta T}{T_0}\right)^4 = \left(1 + 4\frac{\Delta T}{T_0}\right)$

$$\therefore \Delta E = \epsilon\sigma T_0^4\left[1 + 4\frac{\Delta T}{T_0} - 1\right].$$

$$= \epsilon\sigma T_0^4\left[4\frac{\Delta T}{T_0}\right]$$

$$= 4\epsilon\sigma T_0^3\,\Delta T.$$

$$\Delta E = 4\epsilon\sigma T_0^3\,(T - T_0).$$

ΔE is change in thermal energy of a body in a given time dt, this thermal energy change causes temperature change in body in a given time. Rt this temperature change is dT for a body of mass m & specific heat c.

$\therefore$ For a thermal energy $(\Delta E = \Delta Q = mc\,dT)$.

$$\therefore mc\,dT = 4\epsilon\sigma T_0^3\,(T - T_0).$$

$$dT = \frac{4\epsilon\sigma T_0^3\,(T - T_0)}{mc}.$$

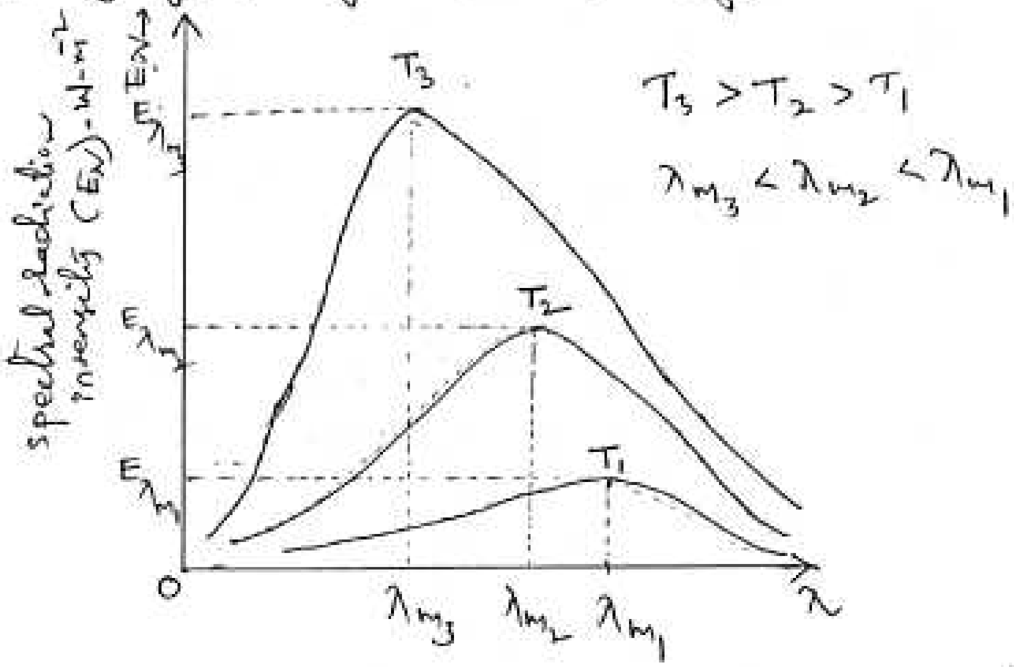
This temperature change in a given time results in rate of change of temperature i.e: rate of cooling.

$$\frac{dT}{dt} = \frac{4\epsilon\sigma T_0^3}{mc}(T - T_0).$$

$$\boxed{\frac{dT}{dt} = K(T - T_0)} \quad \text{where} \quad \boxed{K = \frac{4\epsilon\sigma T_0^3}{mc}}$$

Energy distribution curve of a Black body radiation:

Practically given by: Lumers & Pringshem.



In the figure, each curve represents the variation of monochromatic emittance (E_λ) of the black body ~~radiation emitted~~ with the wavelength (λ) of the radiation emitted.

Different curves have been shown for different temperatures of the black body. We observe that.

(a) At a given temperature of the black body.

(1). The energy emitted is not distributed uniformly among all wavelengths.

(2) The energy emitted is maximum for a particular wavelength (λ_m). The emitted energy falls on either side of λ_m.

(b) With rise in temperature of black body:

(1) The total energy emitted increases rapidly for any given wavelength.

(2) λ_m decreases with rise in temperature.

$$\boxed{\lambda_m \propto \frac{1}{T}}$$

Wein's displacement law:

The wavelength corresponding to maximum emission of radiation decreases with increasing temperature,

$$\boxed{\lambda_m \propto \frac{1}{T}}$$

This is known as wein's displacement law,

$$\lambda_m = \frac{b}{T}$$

$$\boxed{\lambda_m T = b}$$

where b is called wein's constant

$$b = 2.89 \times 10^{-3}\, mK.$$

1) Applications of thermal expansions of solids:

(a) Opening a tight metal lid on a glass container after running hot water over the lid.

Reason: High temperature water causes metal lid to expand

NOTE: Glass has low coefficient of expansion compared to metal lid.

(b) cooling of iron sheet around a wooden wheel on hot summer days to avoid thermal expansion of iron sheet.

(c) Gap in railway tracks to allow space for expansion & to avoid bending of rails.

(d) Cracking of a glass by heating one end at desired position.

(e) A small gap is left between two slabs of a concrete road to allow space for thermal expansion.

2) Application of thermal expansion of liquids:

(a) The behaviour of ~~gasl~~ gasoline on a hot day when it is filled in tank of a already warm car. when it comes from its underground tank at the gas station, the gasoline is relatively cool and if the car's tank is filled - the gasoline might very well expand in volume faster than the fuel tank, overflowing onto the pavement if the ~~car~~ car is not driven after filling.

(b) Mercury thermometer is designed based on thermal expansion of mercury.

(3) Applications of expansion of gases:

(a) Gas thermometers.

(b) Hot air ballons.

UNIT 6: Oscillations

Periodic and non-periodic motion:

Motion in physics can be classified as repetitive (periodic motion) and non-repetitive (non-periodic motion).

1. Periodic motion
   Any motion which repeats at regular intervals of time is called "periodic motion".

   Examples: Hands in pendulum clock, swing of a cradle, the revolution of the Earth around the Sun, waxing and waning of Moon, etc.

2. Non-Periodic motion
   Any motion which does not repeat itself after a regular interval of time is known as non-periodic motion.
   Example: Occurrence of Earth quake, eruption of volcano, etc.

- Classify the following motions as periodic and non-periodic motions?

a. Motion of Halley's Comet -Periodic motion

b. Motion of clouds-Non-periodic motion

c. Moon revolving around the Earth-Periodic motion

Oscillatory Motion

A periodic motion taking place to and fro or back and forth about a fixed point is called oscillatory motion, e.g. motion of a simple pendulum, motion of a loaded spring etc.

Note: **All the oscillatory motions are periodic, whereas all periodic motions need not be oscillatory.**

The oscillatory motion of simple pendulum, heartbeat, etc., is periodic motion.
But the periodic motions like motion of Earth around the Sun, waxing - waning of Moon etc., do not have to and fro motion (i.e.) oscillatory motion.

Harmonic Oscillation: The oscillation which can be expressed in terms of single harmonic function, i.e. sine or cosine function is called harmonic oscillation.

SIMPLE HARMONIC MOTION:

Simple harmonic motion is a special type of oscillatory motion in which the acceleration or force on the particle is directly proportional to its displacement from a fixed point and is always directed towards that fixed point.

CONDITIONS FOR SHM

The following are the conditions to be obeyed for a particle to execute SHM.

1) motion of the particle must be periodic
2) Particle must move to and fro about a fixed point called mean position.
3) Acceleration of the particle must be directly proportional to the displacement from mean position.
4) Acceleration of the particle must always be directed towards the mean position.
5) acceleration of the particle and its displacement must always be opposite to each other in direction.

Example of simple harmonic Motion

There are two types of simple harmonic motion

- ❖ Linear simple harmonic motion

a) Oscillations of a liquid column in a U – tube.
b) Oscillations of a swing with small amplitude
c) Oscillations of a loaded spring
d) Oscillations of a simple pendulum for small amplitude
e) Vibrations of the prongs of an excited tuning fork
f) Vibrations of particle of a medium when a mechanical wave propagates in that medium
g) Oscillations of a floating object.

b) Angular simple harmonic motion

a) oscillations of a balance wheel in a watch
b) oscillations of a torsional pendulum
c) Oscillations of a freely suspended magnet in a uniform magnetic field.

Which of the following examples represent simple harmonic motion and which represent periodic but not simple harmonic motion?
a) The rotation of earth about its axis.
b) Motion of an oscillating mercury column in a U tube;
c) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most position.

Solution:
a) It is periodic but not S.H.M. because it is not to and fro motion about a fixed point.
b) It is S.H.M.
c) It is S.H.M.

Some Terms Related to SHM

(i) Time Period:

Time taken by the body to complete one oscillation is known as time period. It is denoted by T.

(ii) Frequency:

The number of oscillations completed by the body in one second is called frequency. It is denoted by n.

Frequency = 1 /Time period

Its SI unit is $\boldsymbol{hertz}$ $(\boldsymbol{Hz})$ or $\boldsymbol{s^{-1}}$.

(iii) Angular Frequency :

The product of frequency with factor 2π, is called angular frequency. It is denoted by $\boldsymbol{\omega}$.

Angular frequency $\boldsymbol{(\omega) = 2\pi n}$ Its SI unit is radian per second.

(iv) Displacement:

A physical quantity which represents change in position with respect to mean position or equilibrium position is called displacement. It is denoted by $\boldsymbol{x \text{ or } y}$.

(v) Amplitude:

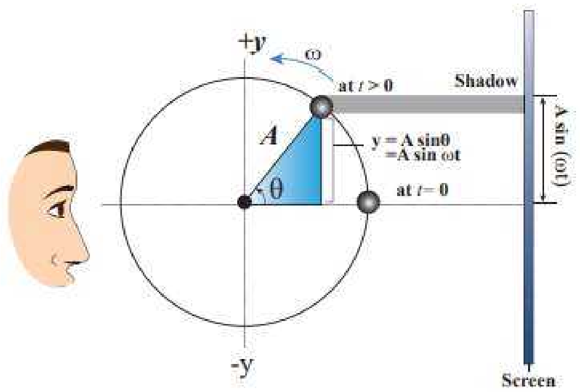
The maximum displacement in any direction from mean position is called amplitude. It is denoted by **a**.

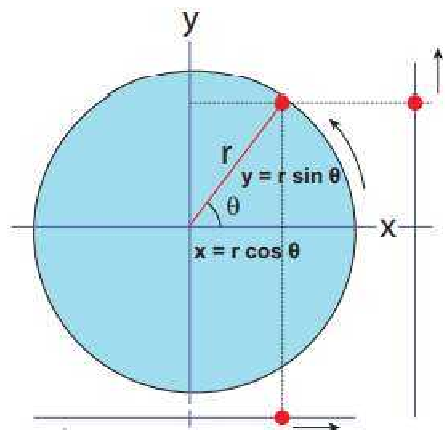
(vi) Phase:

A physical quantity which express the position and direction of motion of an oscillating particle is called phase. It is denoted by $\boldsymbol{\phi}$

<u>The projection of uniform circular motion on a diameter of SHM</u>

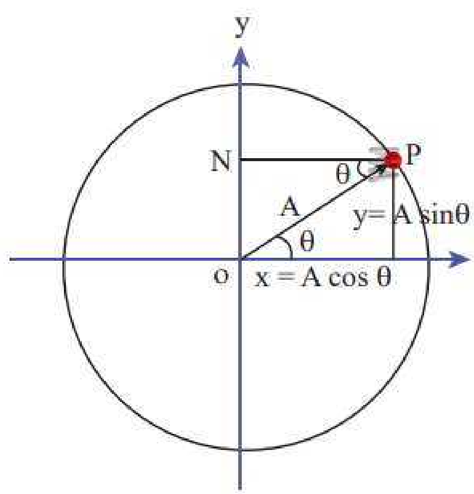
Consider a particle of mass $\boldsymbol{m}$ moving with uniform speed $\boldsymbol{v}$ along the circumference of a circle whose radius is $\boldsymbol{r}$ in anti-clockwise direction as shown in the fig. n. This circle is called "reference circle" and the particle is called "reference particle".





If $\boldsymbol{\omega}$ is the angular velocity of the particle and $\boldsymbol{\theta}$ the angular displacement of the particle at any instant of time $\boldsymbol{t}$, then $\boldsymbol{\theta} = \boldsymbol{\omega t}$.

Let us first project the position of a particle moving on a circle, on to its vertical diameter or on to a line parallel to vertical diameter as shown in Figure. Similarly, we can do it for horizontal axis or a line parallel to horizontal axis.



Displacement:

The distance travelled by the vibrating particle at any instant of time **t** from its mean position is known as displacement.

Let **P** be the position of the particle on a circle of radius **A** at some instant of time t as shown in Figure.

Then its displacement **y** at that instant of time **t** can be derived as follows in ΔOPN.

$$\sin\theta = \frac{ON}{OP} \Rightarrow ON = OP\sin\theta$$

$\boldsymbol{But\ \theta = \omega t, ON = y\ and\ OP = A}$

$$y = A \sin \omega t$$

The displacement **y** takes maximum value (which is equal to **A**) when$\boldsymbol{\sin \omega t = 1}$.

This maximum displacement from the mean position is known as amplitude $(\boldsymbol{A})$ of the vibrating particle.

Velocity:

The rate of change of displacement is velocity.

$$v = \frac{dy}{dt} = \frac{d}{dt}(\mathrm{A} \sin \omega t)$$

For circular motion (of constant radius), amplitude A is a constant and further, for uniform circular motion, angular velocity $\boldsymbol{\omega}$ is a constant. Therefore,

$$v = \frac{dy}{dt} = \mathrm{A}\ \omega \cos \omega t$$

Using trigonometry identity, $sin^2\ \omega t\ +\ cos^2\ \omega t\ = 1$

$$\Rightarrow cos\ \omega t\ =\ \sqrt{1\ - sin^2\ \omega t}$$

$$v = A\omega\sqrt{1 - \sin^2 \omega t}$$

We have

$$\sin \omega t = \frac{y}{A}$$

$$v = A\omega\sqrt{1-\left(\frac{y}{A}\right)^2}$$

$$v = \omega\sqrt{A^2 - y^2}$$

From the above equation, when the displacement $y = 0$, the velocity $v = \omega A$ (maximum) and

For the maximum displacement $y = A$, the velocity $v = 0$ (minimum).

As displacement increases from zero to maximum, the velocity decreases from maximum to zero.

<u>Acceleration</u>

The rate of change of velocity is acceleration.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(A\omega \cos \omega t\right)$$

$$a = -\omega^2 A \sin \omega t = -\omega^2 y$$

$$\therefore \quad a = \frac{d^2 y}{dt^2} = -\omega^2 y$$

The negative sign indicates that **a** and **y** are in opposite directions.

We observe that at the mean position the acceleration of the particle is zero.

At the extreme position (y = ±A), the acceleration is maximum $\mp A\omega^2$ acting in the opposite direction.

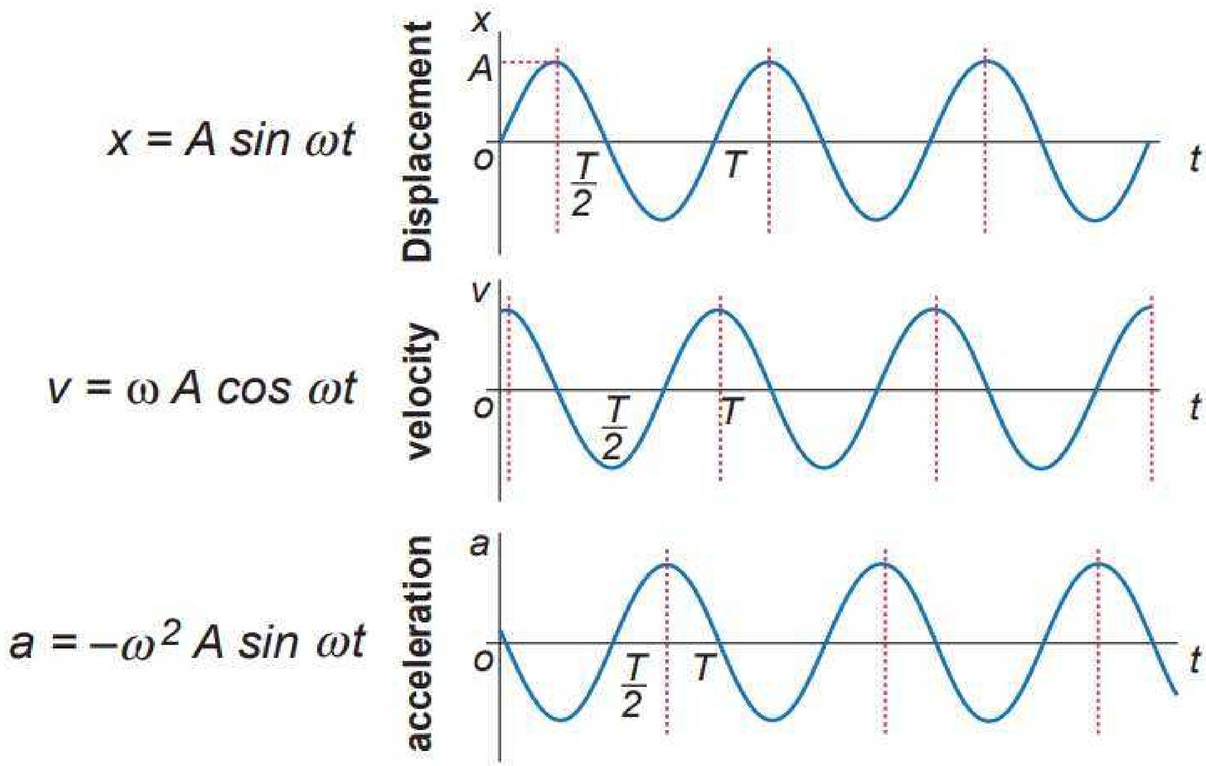
Since 'ω^2 ' is a constant, the above expression can be written as $a \, \alpha - y$ i.e., the acceleration of the particle is directly proportional to the displacement 'y' in magnitude and opposite in direction, always directed towards the fixed point "O".

Hence, the projection of a particle in uniform circular motion on to a diameter is simple harmonic.

Displacement, velocity and acceleration at different instant of time.

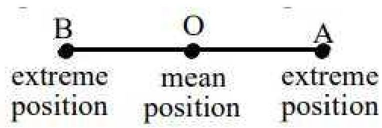
Time	0	$\frac{T}{4}$	$\frac{2T}{4}$	$\frac{3T}{4}$	$\frac{4T}{4}=T$
ωt	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Displacement $y = A \sin \omega t$	0	A	0	$-A$	0
Velocity $v = A\omega \cos \omega t$	$A\omega$	0	$-A\omega$	0	$A\omega$
Acceleration $a = -A\omega^2 \sin \omega t$	0	$-A\omega^2$	0	$A\omega^2$	0

Variation of displacement, velocity and acceleration at different instant of time



Time period, frequency, phase, phase difference and epoch in SHM.

The time period is defined as the time taken by a particle to complete one oscillation. It is denoted by T.



a) Time taken to go from O to A, then to B and then back to O again is one time period. (or)

b) Time taken to from A to B and back to A again is also one time period.

For one complete revolution, the time taken is t = T, therefore

$$\omega T = 2\pi \Rightarrow T = \frac{2\pi}{\omega}$$

Frequency:

The number of oscillations completed by the body in one second is called frequency. It is denoted by n.

Frequency = 1 /Time period
Its SI unit is $\boldsymbol{hertz}$ $(\boldsymbol{Hz})$ or$\boldsymbol{s^{-1}}$.

Phase:
A physical quantity which express the position and direction of motion of an oscillating particle is called phase. It is denoted by $\boldsymbol{\phi}$
It expresses the position and direction of motion of the particle at that instant with respect to its mean position.

$$y = A \sin(\omega t + \varphi_0)$$

Where $\boldsymbol{\omega t + \varphi_0 = \varphi}$ is called the phase of the vibrating particle. At time t = 0 s (initial time), the phase $\boldsymbol{\varphi = \varphi_0}$ is called epoch (initial phase) where $\boldsymbol{\varphi_0}$ is called the angle of epoch.

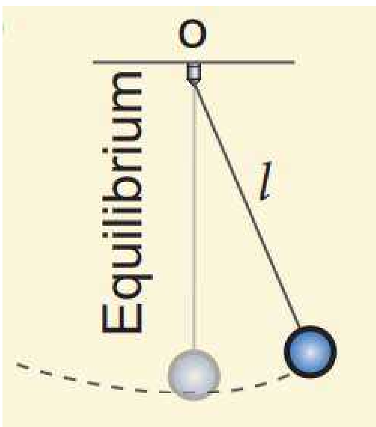
SIMPLE PENDULUM:

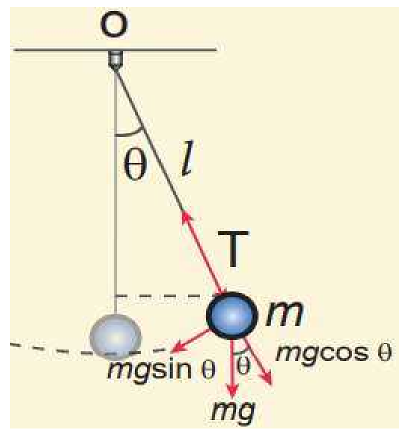
A heavy point sized mass suspended by a massless inextensible, torsionless string is called an ideal "simple pendulum".

The distance between the point of suspension of the pendulum and the centre of gravity of the sphere is known as the "length of the pendulum **(*l*).**

EXPRESSION FOR THE TIME PERIOD OF A SIMPLE PENDULUM:

Consider a simple pendulum of length "**l**" carrying a bob of mass 'm'. Let the bob be given a small angular displacement and released. The bob oscillates to and fro about the equilibrium position in the vertical plane describing an arc of a circle.





At an instant, let the pendulum is making an angle 'θ ' with the vertical and.

The length of the arc at this instant is given by $\boldsymbol{x = l\theta}$ --------------------(1)

The various forces acting on the bob are

(i) The gravitational force acting on the body ($\boldsymbol{F = mg}$) which acts vertically downwards.

(ii) The tension in the string $\boldsymbol{T}$ - which acts along the string to the point of suspension.

Resolving the gravitational force into its components $\boldsymbol{mgCos\theta}$ and $\boldsymbol{mg\,Sin\theta}$ as shown in the figure.

The tension in the string **T** balances one of the components of weight $mgCos\theta$.

$\boldsymbol{T = mgCos\theta}$ ------- (2)

The only unbalanced force $\boldsymbol{mg\,Sin\theta}$ acting on the bob is given by $\boldsymbol{F = -mg\,Sin\theta}$ ------- (3)

The negative sign indicates that this is a restoring force trying to pull the bob back to its mean position in a direction opposite to displacement.

If “a” is the acceleration of the bob at the given instant,

$$\boldsymbol{ma \ = \ -mg\ Sin\theta}$$

$$\boldsymbol{a \ = \ -g\ Sin\theta}$$

When θ is very small $\boldsymbol{sin\theta \ \approx \ \theta}$

$$\boldsymbol{a \ = \ -g\ \theta}$$

Using Eq. (1) in the above equation

$$\boldsymbol{a = -g\ \frac{x}{l}}$$

$$\boldsymbol{a = -\frac{g}{l}\ x} \qquad \text{------------------------------ } \mathbf{(4)}$$

Since '$\boldsymbol{g}$' and '$\boldsymbol{l}$' are constants $\boldsymbol{a\ \alpha - \ x}$

Therefore acceleration “a” is directly proportional to the displacement “x” and the negative sign indicates the direction of acceleration is opposite to the displacement.

Hence the oscillations of the simple pendulum are “simple harmonic”.

Comparing Eq (4) with the S.H.M equation

$\boldsymbol{a = -\omega^2 x}$ **And** $\boldsymbol{a = -\frac{g}{l}\ x}$

we get, $\omega^2 = \frac{g}{l}$

$$\omega = \sqrt{\frac{g}{l}}$$

The time period of oscillation of the simple pendulum is given by

$$T \ = \frac{2\pi}{\omega}$$

$$\boldsymbol{T \ = 2\pi\sqrt{\frac{l}{g}}}$$

Laws of simple pendulum

(i) Law of length:

For a given value of acceleration due to gravity, the time period of a simple pendulum is directly proportional to the square root of length of the pendulum.

$$T \propto \sqrt{l}$$

(ii) Law of acceleration:

For a fixed length, the time period of a simple pendulum is inversely proportional to square root of acceleration due to gravity.

$$T \propto \frac{1}{\sqrt{g}}$$

Independent of the following factors

(i) Mass of the bob: The time period of oscillation is independent of mass of the simple pendulum.

(ii) Amplitude of the oscillations: For a pendulum with small angle approximation (angular displacement is very small), the time period is independent of amplitude of the oscillation.

RESTORING FORCE:

"Whenever a particle leaves the mean position, the force acting on the particle which always tends to bring back the particle to the mean position is called restoring force".

The acceleration of a particle in S.H.M at a displacement 'x' from the mean position is given by

$$\boldsymbol{F = ma}$$

$$\mathrm{F} = -\boldsymbol{m\omega^2 x}$$

$$\boldsymbol{F = -Kx}$$

Where $\boldsymbol{K = m\omega^2}$ is called force constant

Since **K** is a constant, $\boldsymbol{F\alpha - x}$

Thus restoring force acting on a particle in SHM is directly proportional to displacement of the particle from mean position.

The negative sign indicates that restoring force is always directed towards mean position (opposite to the direction of displacement).

For a particle in SHM if mass 'm' and force constant 'K' are known then it's time period is calculated as follows.

$\boldsymbol{K = m\omega^2}$

$$\omega^2 = \frac{\boldsymbol{k}}{\boldsymbol{m}}$$

$$\boldsymbol{\omega} = \sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}}}$$

We have $T = \frac{2\pi}{\omega}$

$$\boxed{T = \boldsymbol{2\pi}\sqrt{\frac{\boldsymbol{m}}{\boldsymbol{k}}}}$$

POTENTIAL ENERGY OF A SIMPLE HARMONIC OSCILLATOR

Consider a particle of mass 'm' executing SHM along a straight line with amplitude 'A'. When the displacement of a particle executing simple harmonic oscillations increases, the restoring force also increases. The restoring force is in a direction opposite to the displacement. Therefore work is done in producing the displacement, against the restoring force.

If 'F' is the magnitude of restoring force acting on the particle at the displacement 'x' then

For the simple harmonic motion, the force and the displacement are related by Hooke's law

In one dimensional case $\boldsymbol{F = -k\,x}$

$$\mathbf{F} = -\boldsymbol{m\omega^2 x}$$

Where $\boldsymbol{k = m\omega^2}$

When the particle undergoes a displacement 'dx',

The work done against the restoring force is $\boldsymbol{dW = -F\,dx}$

$$= -\,(-\boldsymbol{Kx})\,\boldsymbol{dx} = \boldsymbol{K\,x\,dx}$$

Total work done in producing a displacement '$\boldsymbol{x}$' is obtained by integrating the above expression between the limits $\boldsymbol{x = 0\ to\ x}$

$$\boldsymbol{W = \int_0^x dw}$$

$$\boldsymbol{W = \int_0^x K\,x\,dx}$$

$$\boldsymbol{W = \frac{1}{2}k\,x^2}$$

This work done is associated with the particle as potential energy (P.E)

$$\mathbf{PE} = \boldsymbol{\frac{1}{2}k\,x^2}$$

$$\mathbf{PE} = \boldsymbol{\frac{1}{2}m\omega^2\,x^2}$$

At mean position ($x = 0$) P.E. of the particle becomes minimum which is given by $\boldsymbol{PE_{min}} = 0$

At Extreme position ($x = A$), P.E. of the particle becomes maximum which is given by

$$\mathbf{PE}_{max} = \boldsymbol{\frac{1}{2}m\omega^2}\,A^2$$

Variation of PE with time:

For the particle executing simple harmonic motion

$$x = A \sin \omega t$$

$$\mathrm{PE} = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$

KINETIC ENERGY OF SIMPLE HARMONIC OSCILLATOR

Consider the particle of mass 'm' executing SHM along a straight line with an amplitude 'A'.

The velocity of the particle in simple harmonic motion at any position is given by

$$v = \omega\sqrt{A^2 - x^2}$$

∴ The kinetic energy of the particle is given by $KE = \frac{1}{2} m v^2$

$$KE = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

At mean position ($x = 0$) KE of the particle is maximum which is given by

$$KE_{max} = \frac{1}{2} m\omega^2 A^2$$

At extreme position ($x = A$), KE of the particle is minimum which is given by min $KE_{min} = 0$

Variation of KE with time:

$$KE = \frac{1}{2} m v^2$$

For the particle executing simple harmonic motion

$$v = \frac{dy}{dt} = A\, \omega \cos \omega t$$

$$KE = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

TOTAL ENERGY OF SIMPLE HARMONIC OSCILLATOR

The total energy of a simple oscillator at any instant of time is equal to the sum of its PE and KE at that instant

$$T.E = P.E. + K.E$$

$$T.E = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t + \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$$

$$T.E = \frac{1}{2} m\omega^2 A^2 = \text{constant}$$

Total Energy of a particle in SHM at a displacement 'x' from the mean position is $T.E = P.E. + K.E$

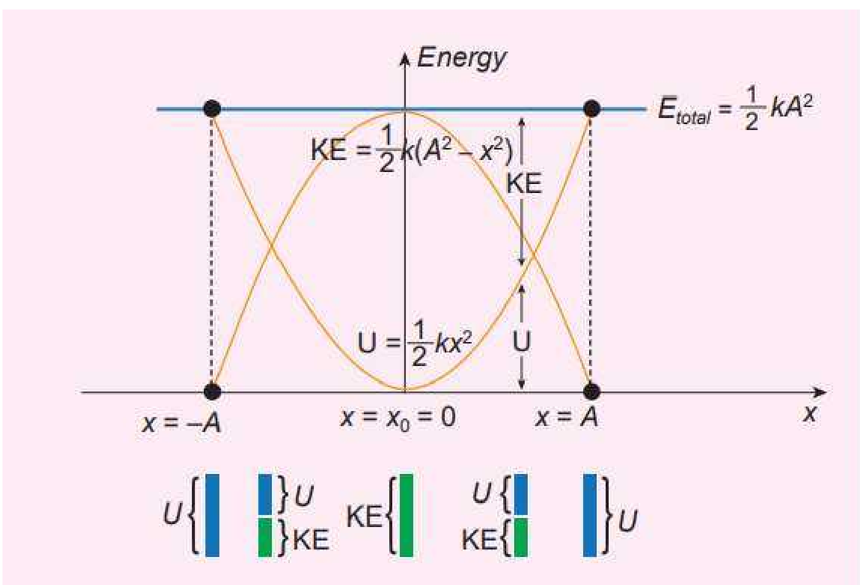
$$T.E = \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 (A^2 - x^2)$$

Hence, cancelling x^2 term, $T.E = \frac{1}{2} m\omega^2 A^2 = \text{constant}$

From the above equations we can conclude that the total energy of simple harmonic oscillator is constant and it is independent of time and position.

The energy is totally in the kinetic form at the mean position and is totally in the potential form at the extreme position.

As particle moves from mean position to extreme position kinetic energy is converted into potential energy and from extreme position to mean position potential energy is converted into kinetic energy in the same way.



Note:

Conservation of energy both the kinetic energy and potential energy are periodic functions, and repeat their values after a time period. But total energy is constant for all the values of x or t.

The kinetic energy and the potential energy for a simple harmonic motion are always positive.

Note that kinetic energy cannot take negative value because it is proportional to the square of velocity. The measurement of any physical quantity must be a real number. Therefore, if kinetic energy is negative then the numerical value of velocity becomes an imaginary number, which is physically not acceptable. At equilibrium, it is purely kinetic energy and at extreme positions it is purely potential energy.

Unit - V Lecture - 47 (1)

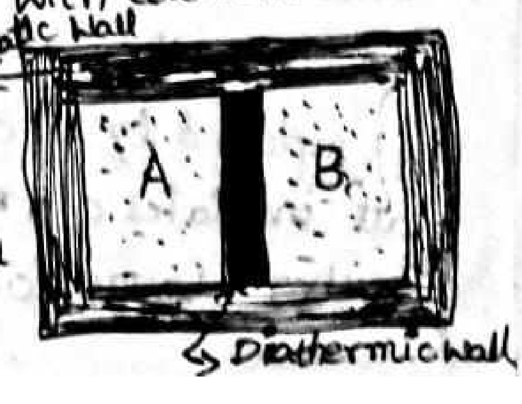
Thermal Equilibrium:- A thermodynamic System is said to be in thermodynamic Equilibrium when macroscopic variables (like pressure, volume, temperature, mass etc) that characterise the system do not change with time.

Two systems are said to be in thermal equilibrium with each other if they are at the same Temperature.

Consider the two gases A and B occupying two different Containers separated by ~~adiabatic~~ ~~wall~~ diathermic wall. On account of flow of heat energy, the macroscopic variables of the two gases change spontaneously, either pressure or volume or both may change until get thermal equilibrium with each other. There is no more energy flow from one to another.

Hence, the temperature of two gases become equal and the gases are said to be in thermal equilibrium with each other.

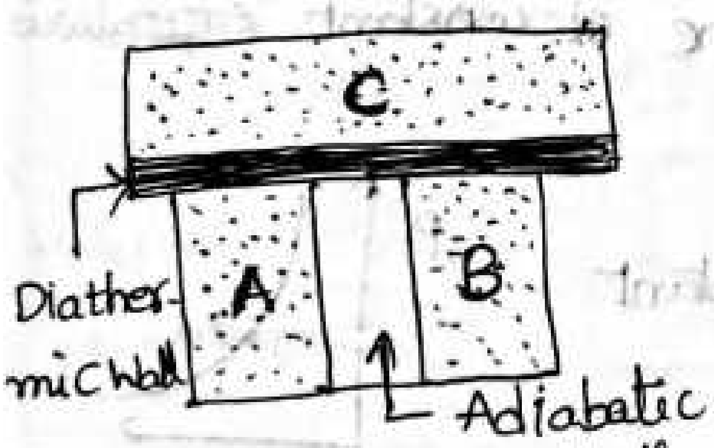
" Two Systems are said to be in thermal equilibrium, if there is no net flow of heat between them when they are brought into thermal Contact".



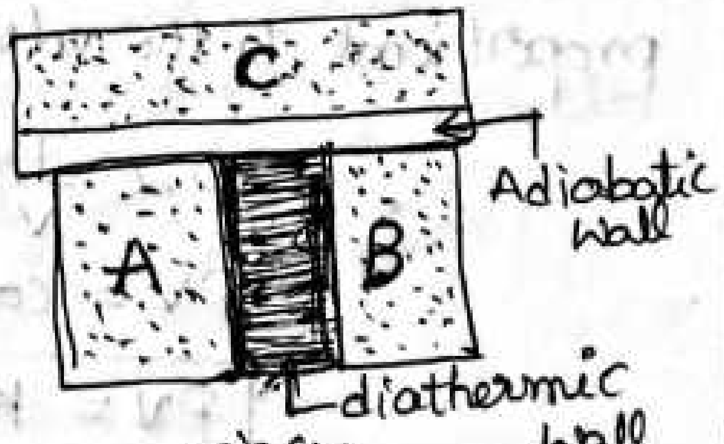
indicating thereby that system A was also in thermal equilibrium with B. This condition is shown in Fig(b).

Both the experiments prove the zeroth law of thermodynamics.

Zeroth law of thermodynamics implies that temperature is a physical quantity which has the same value for all systems which are in thermal equilibrium with each other.



Fig(a)



Fig(b)

Zeroth Law of Thermodynamics :- (2)

This law was formulated by RH Fowler in 1931. The zeroth law of thermodynamics states that "If two systems A and B are separately in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other."

Let us consider two systems A and B separated by a fixed adiabatic wall. The two systems A and B are in contact with a third system C through diathermic wall. The macroscopic variables of A and B will vary until both A and B come in thermal equilibrium with the third system C.

This shows that two systems A and B are separately in thermal equilibrium with a third system C. i.e

$T_A = T_C$ and $T_B = T_C$. This implies that $T_A = T_B$ i.e the systems A and B are also in thermal equilibrium.

Let the adiabatic wall between A and B is replaced by a diathermic wall and adiabatic wall insulates C from A and B. It show that no further change occurs in system A & B

Lecture -48 (4)

Gas Laws :-

1. Boyle's Law [P-V relationship]

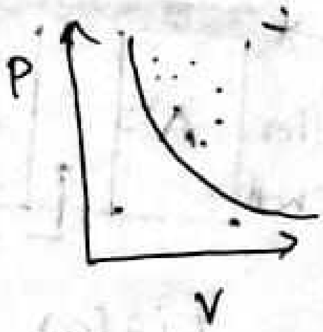
Boyles Law tells us about the relationship between the volume of a gas and its pressure at a constant temperature.

The law states that pressure is inversely proportional to the volume at constant temperature :

$$P \propto \frac{1}{V}$$

$$PV = \text{constant}$$

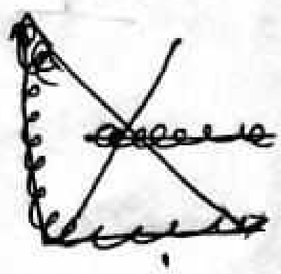
$$\boxed{PV = K}$$



P = pressure in N/m^2

V = volume in litres (dm^3)

K = Constant



$$\boxed{P_1V_1 = P_2V_2}$$

For a fixed mass of gas kept at constant temperature the pressure of the gas is inversely propational to its ~~pres~~ volume.

2. Charle's law :- (T-V relation ship)

If the pressure remaining constant, the volume of the given mass of a gas is directly propotional to its absolute temperature.

$$V \propto T \quad \text{or} \quad \frac{V}{T} = \text{constant}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad [\text{If } m \text{ and } p \text{ are constant}]$$

3. Gay-Lussac's law (P-T relationship)

According to this law at constant volume the pressure of the gas is directly propotional to its absolute temperature.

$$P \propto T \quad \text{(or)}$$

$$\frac{P}{T} = \text{constant}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Ideal gas Equation :-

→ A gas which follows assumptions of kinetic theory of gases to a good extent is called an ideal gas.

→ Any gas at high temperature and low pressure behaves very close to an ideal gas

→ An ideal gas follows the Equation

$$\boxed{PV = nRT}$$

where P = pressure
V = volume
n - no of moles
T - temperature
R - universal gas constant $8.31\ J\,mol^{-1}\,K^{-1}$

This Equation is known as ideal gas equation.

on combining the below laws we get the relation which is known as ideal gas law.

From Boyle's law. $P \propto \frac{1}{V}$ or $V \propto \frac{1}{P}$

From charle's law $V \propto T$

From Avogadro Law $V \propto n$

From the above Equations

$$V \propto \frac{nT}{P}$$

4. **Avogadro Law** (V-n relationship)

According to this law equal volume of all gases under the same conditions of T and P contain equal no of molecules

(or)

Volume of gas is directly proportional to the amount of gas at a constant temperature and pressure.

$$V \propto n$$

$$\boxed{\frac{V_1}{n_1} = \frac{V_2}{n_2}}$$

$$V = Kn$$

$$V = K\frac{m}{M} \quad \left[\because n = \frac{m}{M}\right]$$

$$M = K\left(\frac{m}{V}\right)$$

$$M = Kd \quad \left[\because \text{density } d = \frac{m}{V}\right]$$

$$\therefore \boxed{M \propto d}$$

M - molar mass of gas.

n - number of moles of gas

from Eq^n ①

$$PV = nRT$$

$$R = \frac{PV}{nT}$$

$$\boxed{\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}}$$

i, If T and n are constant

$$\boxed{P_1V_1 = P_2V_2}$$

(ii) If P and n are constant

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$

(iii) If P and T are constant

$$\boxed{\frac{V_1}{n_1} = \frac{V_2}{n_2}}$$

$$V = \frac{RnT}{P}$$

$$\boxed{PV = nRT} \quad \text{--- (1)}$$

here - R is known as universal gasconstant.

$$PV = nRT$$

$$\boxed{PV = \frac{m}{M} RT}$$

m - Total mass of gas.
M - molecular mass of gas

If $n = 1$ then Eq^n (1) becomes

$$PV = RT$$

$$\Rightarrow \boxed{R = \frac{PV}{T}}$$

$$R = \frac{P_1 V_1}{T_1}, \quad R = \frac{P_2 V_2}{T_2}$$

$$\Rightarrow \boxed{\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

Lecture - 49

Heat, Work and Internal energy :-

Heat:- Energy that is transferred between a system and its surroundings whenever there is temperature difference between the system and surroundings is called heat.

The amount of heat given to a body to raise its temperature is depends on mass, nature of substance and change in its temperature

$$\Delta Q \propto m \, \Delta T$$

$$\boxed{\Delta Q = mc \, \Delta T} \Rightarrow \Delta Q = mc \, (T_1 - T_2)$$

where m = mass of body

c = Specific heat capacity

ΔT = Change in temperature

Q = Heat

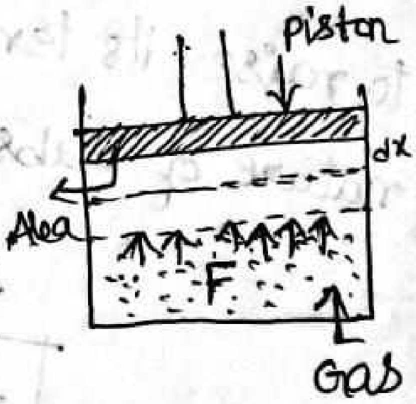
Work:- Work is said to be done if a body or System moves through a certain distance in the direction of the applied force.

when the piston is pushed outward an infinitesimal distance dx, then the workdone by the gas

$$dW = F \cdot dx = pA\,dx = p\,dV \qquad [\because A\,dx = dV]$$

for a finite volume change from V_i to V_f.

$$\text{workdone, } W = \int_{V_i}^{V_f} dW$$



$$\boxed{W = \int_{V_i}^{V_f} p\,dV}$$

here, p could be variable or constant

If system expands, $V_f > V_i$ i.e W = positive

If system contracts, $V_f < V_i$, i.e W = negative.

workdone is path dependent.

Internal Energy :-

If we consider a bulk system consisting of a large number of molecules, then internal energy of the system is the sum of the kinetic energies and potential energies of these molecules.

This energy is possessed by a system due to its molecular motion and molecular configuration. The internal energy is denoted by 'U'

$$U = U_K + U_P$$

where U_K and U_P represents the kinetic and potential energies of the molecules of the system.

Internal energy depends only on the state of the system.

There are four ways to change the internal energy of a system.

i) $Q > 0$ (+ve) heat transfer to the system

ii) $Q < 0$ (-ve) heat transfer from the system

iii) $W > 0$ (+ve) is workdone by the system

iv) $W < 0$ (-ve) is workdone on the system

First law of Thermodynamics :-

The first law of Thermodynamics is simply the general law of conservation of energy applied to any system.

According to this law "The total heat energy change in any system is the sum of the internal energy change and the workdone."

when a certain quantity of heat dQ is subjected to a system, a part of it is used in increasing the internal energy by "du" and a part is used in performing external work "dW", hence

$$dQ = dU + dW \rightarrow ① \quad (\text{or}) \quad \Delta Q = \Delta U + \Delta W$$

where dQ = heat supplied to the system by the surroundings

dU = change in internal energy of the system

dW = workdone by the system on the surroundings

du depends only on the initial and final states.

If work is done on the system dW is negative. When heat is given to the system or heat is absorbed, dQ is positive, If heat is given by the system or heat is evolved, dQ is negative.

we can write, $dU = dQ - dW$ [$\because$ from Eqⁿ ①]

In case of Isolated system, there is no interaction with the surroundings. No work is done by or on the system

i.e $dQ = 0$ and $dW = 0$

$\therefore dU = 0$ or $\boxed{U = \text{constant}}$

The internal energy of an isolated system is constant.

When the heat supplied is completely converted into work without changing the temperature of the system, the internal energy remains constant i.e $dU = 0$

from Eqⁿ ①

$\therefore \boxed{dQ = dW}$

from Eqⁿ ① $dQ = dU + dW$

we know $dW = Pdv$

then $\boxed{dQ = dU + Pdv}$

This is called first law of Thermodynamics.

Relation between P, V and T in isothermal process

In an ideal gas, molecules have no volume and do not interact. According to the ideal gas law, pressure varies linearly with temperature and quantity and inversly with volume.

In an isothermal process, changes may occur in pressure or volume; but temperature remains constant

$$PV = nRT$$

In this equation R is known as universal gas constant that has the same value for all gases namely, $R = 8.31\ J/mol\,K$.

for $n = 1 \Rightarrow PV = RT$

$$\Rightarrow R = \frac{PV}{T}$$

at constant temperature $\boxed{PV = \text{Constant}}$

or

$$\frac{P_1V_1}{T} = \frac{P_2V_2}{T}$$

$$\boxed{P_1V_1 = P_2V_2}$$

on Integrating, $\int \frac{dT}{T} + (\gamma-1)\int \frac{dv}{V} = \text{constant}$

$$\log T + (\gamma-1)\log V = \text{cons}$$

$$\log T + \log V^{\gamma-1} = \text{const}$$

$$\log T V^{\gamma-1} = \text{const}$$

$$TV^{\gamma-1} = \text{antilog}(\text{const})$$

$$\boxed{TV^{\gamma-1} = K}$$ [K = another constant]

$$\therefore TV^{\gamma-1} = \text{constant} \quad -① $$

This is the Relation between 'T' and 'V'

(ii) we know $PV = RT$

$$T = \frac{PV}{R} \quad - ②$$

Substitute Eq^n ② in Eq^n ①

$$\frac{PV}{R} V^{\gamma-1} = \text{const}$$

$$\frac{PV^{\gamma}}{R} = \text{const}$$

$$\boxed{PV^{\gamma} = \text{constant}} \quad — ③$$

it is the relation between P and V during an adiabatic process.

Relation between P, V and T in adiabatic process

Let us consider an ideal gas in adiabatic process. During the adiabatic process, by the first law of thermodynamics

$$dQ = du + dw$$

In adiabatic process, no heat is allowed to exchange between system and surrounding.

therefore $dQ = 0$

$$du + dw = 0$$

$$du + PdV = 0$$

$$C_V dT + PdV = 0 \quad [\because du = C_V dT]$$

$$C_V dT + \frac{RT}{V} dV = 0 \quad \left[\because PV = RT \Rightarrow P = \frac{RT}{V}\right]$$

$$C_V dT + RT \frac{dV}{V} = 0$$

dividing the above equation with C_V & T

$$\frac{C_V dT}{C_V T} + \frac{RT}{C_V T} \frac{dV}{V} = 0$$

$$\frac{dT}{T} + \frac{R}{C_V} \frac{dV}{V} = 0$$

$$\frac{dT}{T} + (\gamma - 1) \frac{dV}{V} = 0 \quad \left[\because \gamma = \frac{C_P}{C_V}\right]$$

iii) $PV = RT \Rightarrow V = \frac{RT}{P} \rightarrow (4)$

substitute Eqⁿ (4) in Eqⁿ (3)

$$PV^{\gamma} = \text{constant}$$

$$P\left[\frac{RT}{P}\right]^{\gamma} = \text{const}$$

$$P\, R^{\gamma}\, T^{\gamma}\, P^{-\gamma} = \text{const}$$

$$\boxed{P^{1-\gamma}\, T^{\gamma} = \text{constant}}$$

This is the relation between P and T for an adiabatic process.

-: Lecture - 50 :-

thermodynamic processes :-

when state of a system changes or the state variables changes with time, then this process is known as thermodynamic process.

During such process Energy may transferred to system or taken out.

quasi-static process :-

A quasi-static process is a thermodynamic process that happens slowly enough for the system to remain in internal equilibrium.

In this process, the change in pressure or volume or temperature of the system is very small.

To take a gas from the state (P, T) to another state (P', T') via a quasi-static process, we change the external pressure by a very small amount, allow the system to equalise its pressure with that of the surroundings and continue the process infinitely slowly until the system achieves the pressure P'. similarly we achieves the temperature T'.

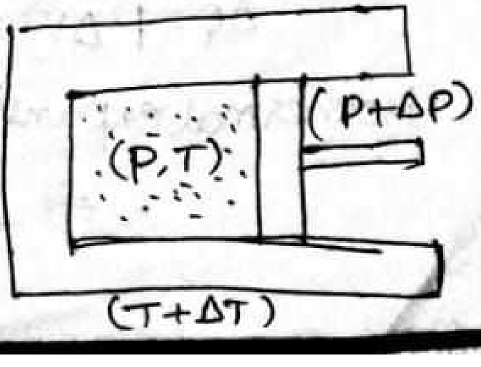


Fig: changes in temperature & pressure

Some important proces are

1. Isothermal process :(T-constant)
2. Isobaric process : (P. constant)
3. Isochoric process (V. constant)
4. Adiabatic process (no heat transfer)
5. cyclic and non-cyclic process.

1. Isothermal process :-

A change in pressure and volume of a gas without any change in its temperature, is called an isothermal change.

In such a change, there is a free exchange of heat between the gas and its surroundings

If a gas expands at constant temperature it is called isothermal expansion. Similarly if a gas is compressed at constant temperature it is called isothermal compression.

For isothermal process, $\boxed{\Delta Q = \Delta W}$

i.e $\Delta Q = \Delta U + P\Delta V$ [$\because$ from 1st law of thermodynamics]

$\Delta Q = P\Delta V = \Delta W$ [$\because \Delta U = 0$]

In isothermal expansion $\Delta V > 0$

$\Rightarrow \Delta Q = P\Delta V > 0$

4. Adiabatic process :-

A Thermodynamic process in which there is no heat exchange between the system and surroundings takes place.

i.e $dQ = 0$ (Heat neither given nor rejected)

In adiabatic process, the system is insulated from the surroundings and heat absorbed or released is zero. Since there is no heat exchange with the surroundings.

→ when expansion happens temperature falls

→ when gas is compressed, temperature rises.

In this process pressure and volume changes. adiabatic expansion or compression is always followed by a change in temperature

$\therefore dQ = 0$

Hence the first law of thermodynamics can be written as

$$\boxed{dU + dW = 0}$$

In isothermal compression $\Delta V < 0$

$\Delta Q = P\Delta V < 0$

In isothermal compression, work is done on the gas and heat is realeased by gas equal to work done on the gas.

In this process $PV =$ constant

2. Isobaric process :- If pressure of a gaseous system remains constant during a thermodynamic process, it is called an isobaric process.

There are changes in internal energy (du) and changes in ~~vol~~ system volume (ΔV)

In this process $\frac{V}{T}$ = constant, $\boxed{\Delta Q = \Delta U + \Delta W}$.

3. Isochoric process

In this process the volume of the closed system remains constant.

In Isochoric process there is a change in the internal energy. workdone will be zero.

In this $\frac{P}{T}$ = constant.

$\Delta W = 0$

$$\boxed{\Delta Q = \Delta U}$$

Workdone in Isothermal process :—

Let us consider one mole of an ideal gas in isothermal process.

Since the temperature remains constant in this process the relation between pressure and volume of a gas is given by

$pV =$ Constant, where p is pressure
V is volume.

Let the gas ~~to~~ be allowed to expand isothermally. Let its initial volume and pressure be V_1 & P_1 respectively.
Let its final volume and pressure be V_2 and P_2 respectively.

The workdone is given by

$$W = \int dw$$

$$= \int_{V_1}^{V_2} p\,dV$$

from ideal gas Equation $pV = nRT$

$$pV = RT \quad [\because n = 1]$$

$$P = \frac{RT}{V}$$

$$\therefore \quad W = \int_{V_1}^{V_2} \frac{RT}{V}\, dV$$

$$= RT \int_{V_1}^{V_2} \frac{1}{V}\, dV = RT\,[\ln V]_{V_1}^{V_2}$$

$$= RT\,[\ln V_2 - \ln V_1]$$

$$W = RT \ln\left(\frac{V_2}{V_1}\right)$$

$$\boxed{W = 2.303\ RT \log_{10}\left(\frac{V_2}{V_1}\right)}$$

For 'n' moles of the gas workdone is

$$W = 2.303\ nRT \log_{10}\left(\frac{V_2}{V_1}\right)$$

for constant temperature $P_1V_1 = P_2V_2$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\boxed{W = 2.303\ RT \log_{10}\left(\frac{P_1}{P_2}\right)}$$

As temperature of ideal gas remains constant

$$\Delta U = 0$$

So, by first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = \Delta W$$

$$W = \frac{K}{1-\gamma}\left[\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[\frac{P_f V_f^{\gamma}}{V_f^{\gamma-1}} - \frac{P_i V_i^{\gamma}}{V_i^{\gamma-1}}\right]$$

$$= \frac{1}{1-\gamma}\left[P_f V_f - P_i V_i\right]$$

$$W = \frac{P_i V_i - P_f V_f}{\gamma-1}$$

$PV = RT$, $P_i V_i = RT_i$, $P_f V_f = RT_f$

$$W = \frac{RT_i - RT_f}{\gamma - 1}$$

$$\boxed{W = \frac{R(T_i - T_f)}{\gamma-1}}$$

If work is done by the system, the internal energy and so the temperature of the system falls. i.e $W > 0$, $T_f < T_i$. Conversely, If work is done on the system, the internal energy and so the temperature of system increases. i.e $W < 0$, $T_f > T_i$.

Workdone in an Adiabatic process : (24)

Consider an ideal gas in adiabatic process. In an adiabatic process, there is no exchange of heat between system and surroundings. i.e $dQ = 0$

$$dQ = du + dw$$

$$du + dw = 0$$

$$du = -dW$$

For an adiabatic process $PV^{\gamma} = \text{constant}$

also workdone in an adiabatic process

$$W = \int_{V_i}^{V_f} P\,dV$$

$$= \int_{V_i}^{V_f} \frac{K}{V^{\gamma}}\,dV \qquad [\because PV^{\gamma} = K]$$

$$= K \int_{V_i}^{V_f} \frac{1}{V^{\gamma}}\,dV$$

$$= K \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_i}^{V_f}$$

$$= \frac{K}{1-\gamma}\left[\frac{1}{V^{\gamma-1}}\right]_{V_i}^{V_f}$$

$$= \frac{K}{1-\gamma}\left[\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right]$$

Lecture-53

Second law of Thermodynamics :-

- The second law of thermodynamics gives more information about the thermodynamic processes.
- Second law may be defined as

  "Heat cannot flow itself from colder body to a hotter body"
- The second law of Thermodynamics gives a fundamental limitation to the efficiency of a heat engine and the coefficient of performance of a refrigerator.

Following are the two statements of second law of Thermodynamics.

Kelvin-planck's statement :-

"It is impossible to for any device as heat engine that operates on a cycle to receive heat from a single reservoir and produce net amount of work".

(or)

No process is possible whose sole result is the absorption of heat from ~~th~~ reservoir and the complete conversion of the heat into work.

By repeating the same cycle over and over again, work is continuously obtained

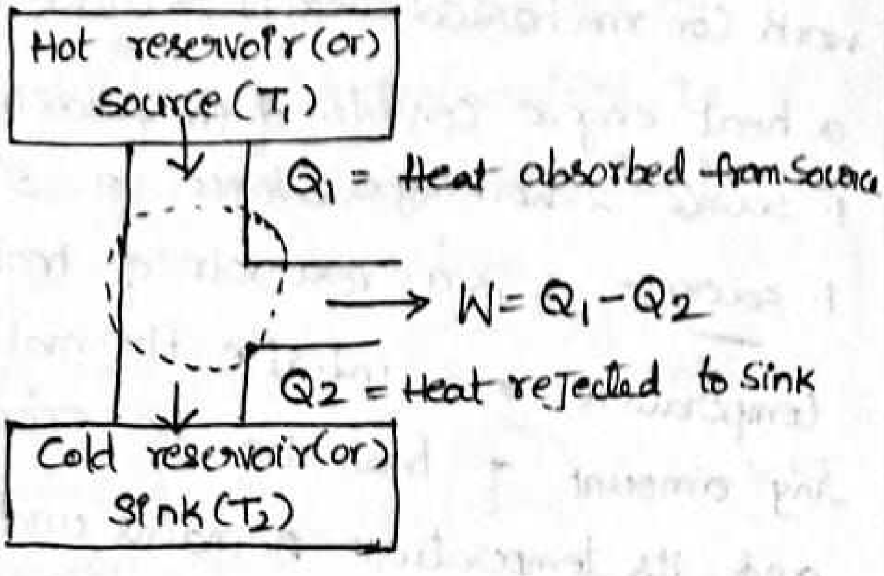


Fig: Working of heat Engine.

Thermal Efficiency :-

The ratio of the workdone by the heat engine and the heat absorbed by the working substance is called the efficiency of the heat engine.

$$\eta_{th} = \frac{W}{Q_1}$$

$$\eta_{th} = \frac{Q_1 - Q_2}{Q_1} \quad [\because W = Q_1 - Q_2]$$

$$\boxed{\text{Thermal efficiency}, \ \eta_{th} = 1 - \frac{Q_2}{Q_1}}$$

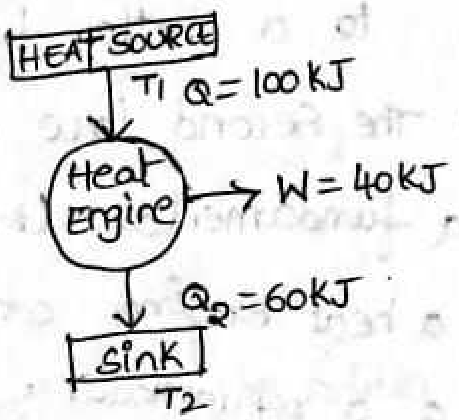
For ideal heat engine $Q_2 = 0$, so $\eta_{th} = 1$.

i.e 100% (practically not possible)

thermal efficiency is always less than 1 or less than 100%.

This means that there can be "no heat engine or refrigerator with 100% efficiency.

only part of total heat absorbed by heat engine from a high temperature is converted to work, the remaining heat must be rejected at a low temperature.



clausices statement :-

"It is impossible for a self acting machine, operating in a cycle, unaided by any external energy to transfer heat from a cold body to a hot body."

In other words heat can not flow itself from a colder body to a hotter body.

- kelvin-plank statement is applied to heat engine.
- clausius statement is applied to 'heat pump' and Refrigeration.

Any device that violates the first law of thermodynamics is called a perpetual motion machine of the first kind (PMM1) and the device that violates the second law is called a perpetual motion machine of the second kind (PMM2).

Heat engine :-

A device used to convert heat energy into work (or mechanical energy) is called a heat engine.

a heat engine consists of the following parts.

1. source 2. working substance 3. sink.

1. source:- It is a reservoir of heat at high temperature and infinite thermal capacity. Any amount of heat can be extracted from it and its temperature remains unchanged.

2. working substance:- A material which absorbs heat energy from the source and converts it into mechanical energy by rejecting some of the heat to sink is called working substance.

Ex: steam, petrol etc.

3. sink:- It is a reservoir of heat at low temperature and infinite thermal capacity. Any amount of heat can be given to the sink and its temperature remains unchanged.

Working:- The working substance absorbs heat Q_1 from the source, does an amount of work W, rejects the remaining amount of heat to the sink and comes back to its original state and there occurs no change in its internal energy.

Lecture - 54

Refrigerators :- (1)

The refrigerator is Just the reverse of heat engine. The transfer of heat from a low temperature region to a high temperature one requires special devices called refrigerators.

In Refrigerators the working substance is liquid Ammonia.

The main purpose of the Refrigerator is to cool the substance.

1. The working substance in the evaporator absorbs the heat from the sink (contents of refrigerator) at lower temperature
2. Low pressure vapor refrigerant is compressed and rising its pressure and pushes it into the condenser coils.
3. The high pressure ammonia gas condenses into ammonia liquid with low temperature by the condenser.

Heat pumps

(4)

A heat pump is a device that transfers heat energy from a heat source to a heat sink.

The objective of a heat pump is to supply heat to a warm medium.

In heat pump out system is at high temperature.

In this the evaporator coils consisting the working substance are outside, where they take heat from cold air which is later thrown inside the room for heating it.

The condenser is located inside the room and it acts as the heating device.

Coefficient of performance :-

③

The performance of a refrigerator is expressed by means of coefficient of performance β.

It is defined as the ratio of the heat extracted from the cold body to the work needed to transfer it to the hot body.

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

$$\therefore \quad \beta = \frac{Q_2}{Q_1 - Q_2}$$

A perfect refrigerator is one which transfers heat from cold to hot body without doing work.

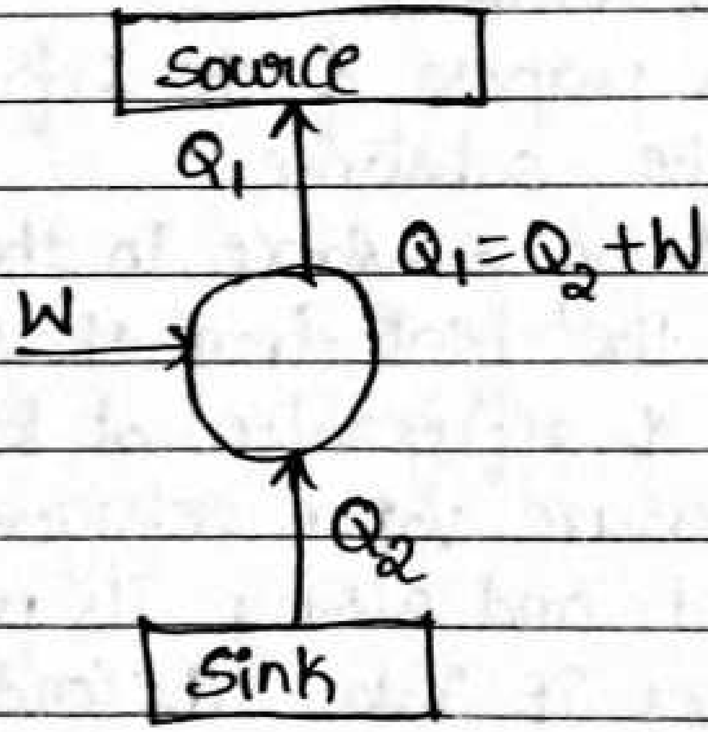
i.e. $W = 0$

$\Rightarrow \beta = \infty$

Through the Condenser the heat is thrown to the atmosphere. (2)

So. In the refrigerator the evaporator performs the main function of cooling and condenser performs the function of delivering the heat to the atmosphere.

Thus, it transfers heat from a cold to a hot body.



released.

EXAMPLE |8| Work Done in Isothermal Process

Three moles of an ideal gas kept at constant temperature of 300 K are compressed from a volume of 4 L to 1 L. Calculate the work done in the process. Take R as 8.31 J/mol-K.

Sol. Given, $\mu = 3$, $T = 300$ K, $V_i = 4$ L, $V_f = 1$ L,

$R = 8.31$ J/mol-K, $W = ?$

Work done in isothermal process is given by

$$W = 2.303\mu RT \log \frac{V_f}{V_i}$$

$$= 2.303 \times 3 \times 8.31 \times 300 \log \frac{1}{4} = -1.037 \times 10^4 \text{ J}$$

EXAMPLE |9| Compressing the Cylinder

A cylinder containing one gram molecule of the gas was compressed adiabatically until its temperature lose from 27°C to 97°C. Calculate the work done and heat produced in the gas. Take γ as 1.5

Sol. Given, $T_i = 27°C = 27 + 273 = 300\ K$

$$T_f = 97°C = 97 + 273 = 370\ K, \gamma = 1.5$$

Work done in adiabatic compression is given by

$$W = \frac{R}{1-\gamma}(T_i - T_f) = \frac{8.31}{1-1.5}(300-370) = 1163.4\ J$$

Heat produced, $H = \dfrac{W}{J} = \dfrac{1163.4}{4.2} = 277\ cal$

Work Done in an Isochoric Process

5. A system is provided with 200 cal of heat and the work done by the system on the surroundings is 40 J. Then, its internal energy

9. **An electric heater supplies heat to a system at a rate of 100 W. If the system performs work at a rate of 75 J/s. At what rate, is the internal energy increasing?** **[NCERT]**

Sol. Heat energy supplied per second by the heater

$$\Delta Q = 100 \text{ W} = 100 \text{ J/s}$$

Work done by the system $(\Delta W) = +75$ J/s

Rate of change in internal energy $(\Delta U) = ?$

According to first law of thermodynamics,

$$\Delta U = \Delta Q - \Delta W$$

$$= 100 - 75 = 25 \text{ J/s}$$

$$= 25 \text{ W}$$

28. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B *via* a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take, 1 cal = 4.19 J). **[NCERT]**

According to first law of thermodynamics, if ΔQ heat energy is given (or taken) to a thermodynamic system which is partially utilised in doing work (ΔW) and remaining part increases (or decreases) the internal energy of the system.

Sol. Given, work done $(W) = -22.3$ J

Work done is taken negative as work is done on the system.

In an adiabatic change, $\Delta Q = 0$

Using first law of thermodynamics,

$$\Delta U = \Delta Q - W = 0 - (-22.3) = 22.3 \text{ J}$$ [1]

For another process between states A and B,

Heat absorbed $(\Delta Q) = +9.35$ cal

$$= +(9.35 \times 4.19) \text{ J} = +39.18 \text{ J}$$ [1]

Change in internal energy between two states via different paths are equal.

$$\therefore \quad \Delta U = 22.3 \text{ J}$$

$\therefore$ From first law of thermodynamics,

$$\Delta U = \Delta Q - W$$

or

$$W = \Delta Q - \Delta U$$

$$= 39.18 - 22.3 = 16.88 \text{ J} \approx 16.9 \text{ J}$$ [1]

21. **In a refrigerator, one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power and heat is transferred from – 3°C to 27°C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.** **[NCERT Exemplar]**

Sol. Given, $T_1 = -3°C = -3 + 273 = 270\text{ K}$

$$T_2 = 27°C = 27 + 273 = 300\text{ K}$$

Efficiency, $\eta = 1 - \dfrac{T_1}{T_2} = 1 - \dfrac{270}{300} = \dfrac{1}{10}\%$ [1]

or $\dfrac{W}{Q} = 0.5\,\eta = \dfrac{1}{20}$

or $Q = 20\text{ W} = 20\text{ kJ per second}$ [1]

EXAMPLE |2| Refrigerator Machine

By using a refrigerator machine, 1 g of water at 0°C is to be freezed. If the temperature of the surrounding is 27°C. Calculate

(i) least amount of work done.

(ii) the heat which is passed to the surroundings in the process.

Sol. (i) Given, $T_1 = 27° \text{C} = 27 + 273 = 300 \text{ K}$

$$\Rightarrow \quad T_2 = 0° \text{C} = 0 + 273$$

$$= 273 \text{ K}$$

As we know, to freeze one gram of water at 0°C, 80 cal of heat must be transferred from water at 0°C to the surrounding at 27°C.

$$\therefore \quad Q_2 = 80 \text{ cal}$$

The coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2} \Rightarrow \frac{80}{W} = \frac{273}{300 - 273}$$

$$\Rightarrow \quad = 7.91 \text{ cal}$$

(ii) The heat transferred to the surrounding can be calculated by using first law of thermodynamics.

$$Q_1 = Q_2 + W = 80 + 7.91 = 87.91 \text{ cal}$$

11. **Find the values of two molar specific heats of nitrogen. Given, $\gamma = 1.41$ and $R = 8.31\,\text{J mol}^{-1}\text{K}^{-1}$.**

Sol. Given, $R = 8.31\,\text{J mol}^{-1}\text{K}^{-1}$ and $\gamma = 1.41$

We know, $C_V = \dfrac{R}{(\gamma - 1)} = \dfrac{8.31}{(1.41 - 1)} = 20.3\,\text{J mol}^{-1}\text{K}^{-1}$

$$\because \quad \frac{C_p}{C_V} = \gamma \Rightarrow C_p = C_V \cdot \gamma$$

$$= 20.3 \times 1.41 = 28.623\,\text{J/mol-K}$$

Sol. (c) Given, $dQ = +200$ cal $= 200 \times 4.2 = 840$ J, $dW = +40$ J

From first law of thermodynamics,

$$dQ = dU + dW$$

$$dU = dQ - dW = 840 - 40 = 800 \text{ J}$$

So, the internal energy of the system increases by 800 J.