Module- 1

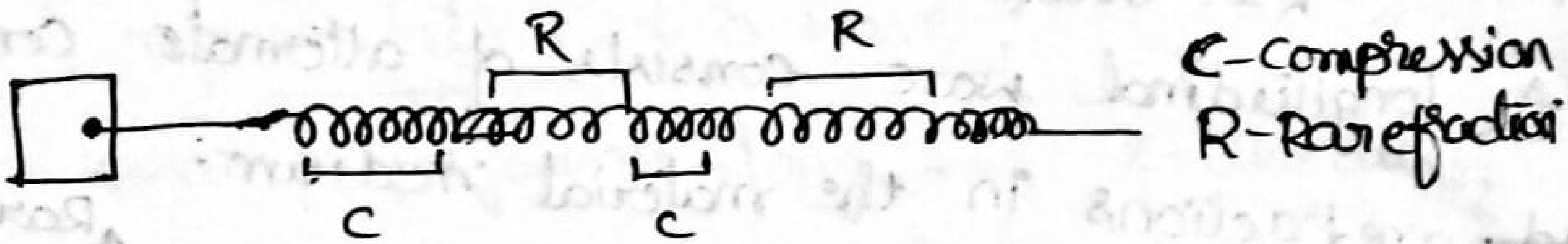
classification of Mechanical waves :-

Mechanical waves are two types
i) Longitudinal waves ii) Transverse waves

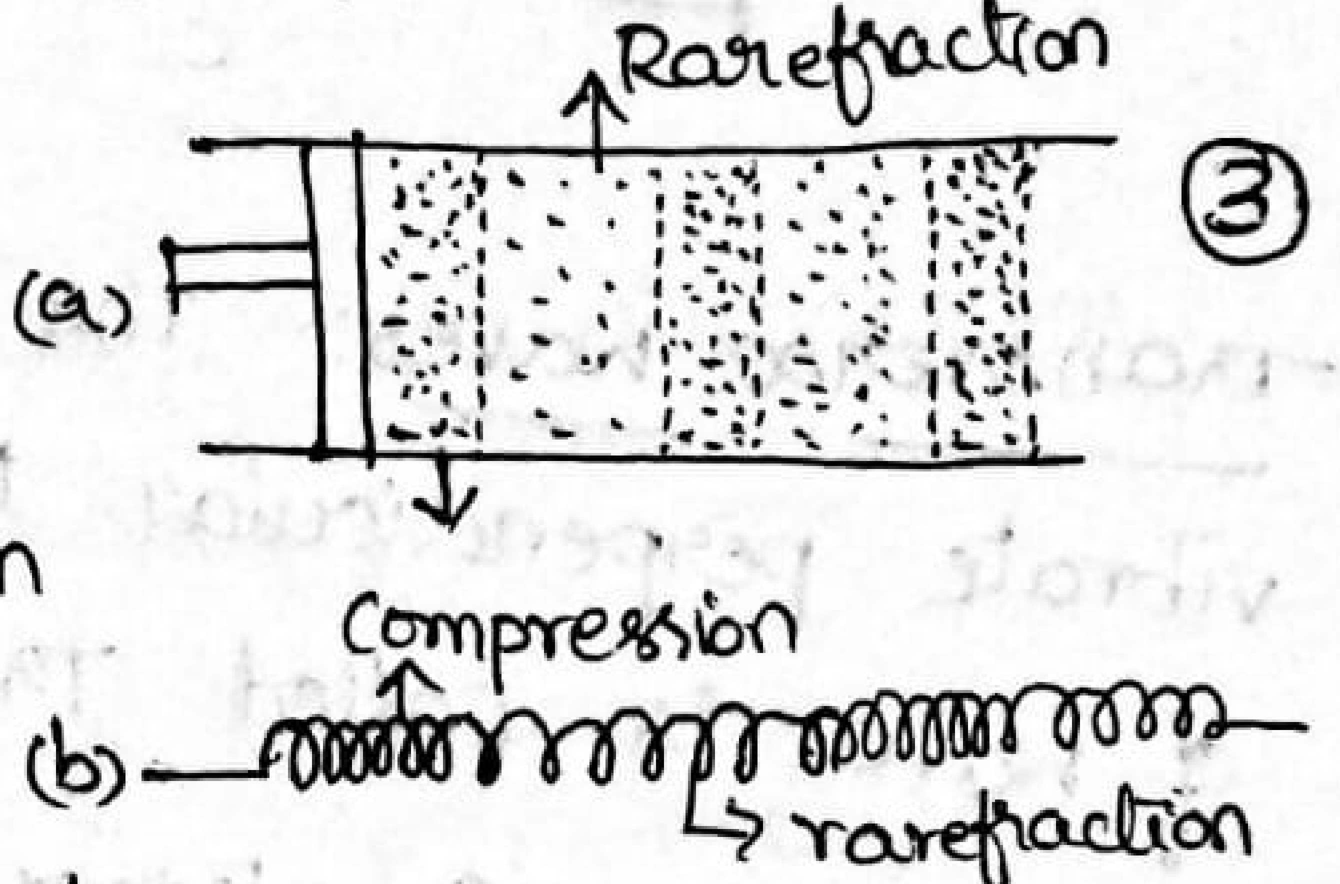
1) **Longitudinal Waves** :- The wave motion in which the particles of the material medium vibrate parallel to the direction of propagation of the wave is called Longitudinal wave. Ex: sound waves

A longitudinal

Explanation :- Let us consider a spring with one end fixed on a smooth horizontal table if the free end of the spring is moved to and fro along the direction of the spring, a wave with a back and forth displacement is generated and moves forward in the form of compressions and rarefactions. Such waves are called longitudinal waves



A longitudinal wave consists of alternate compressions and rarefactions in the material medium.



Compression :- A region of medium where the particles of the medium are closer to each other is known as compression.

Rarefaction :- A region of medium where the particles of the medium are far from each other is known as rarefaction.

Wavelength :- The distance between two successive compressions or rarefactions is called wavelength of a longitudinal wave.

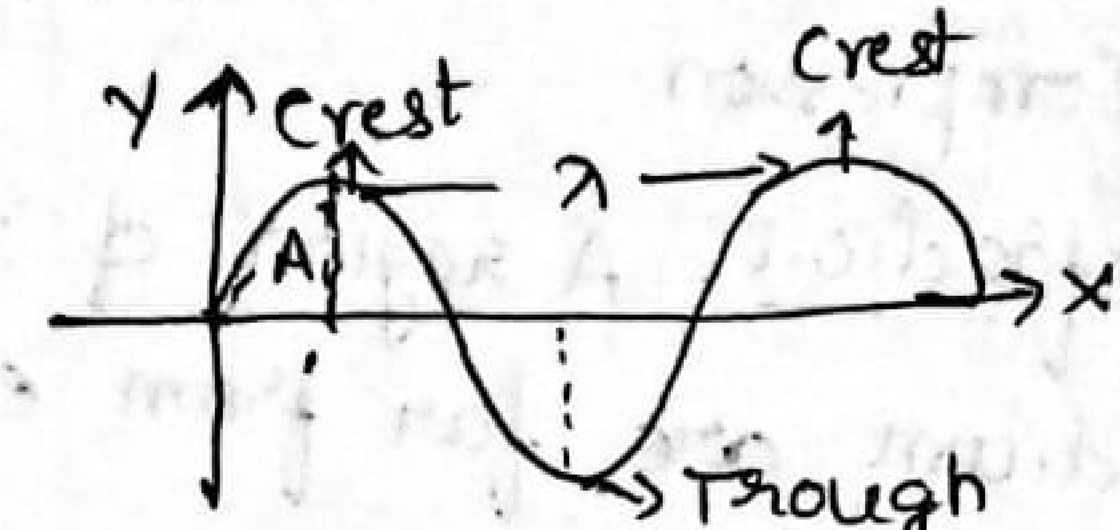
Amplitude :- The amplitude is the maximum displacement from equilibrium. It is a measure of how compressed or rarefied the medium becomes. Crowded compressions means the wave has a large amplitude.

Time period :- The time taken by the wave to move one wavelength is called time period.

Frequency :- The number of wavelengths that pass a fixed point each second.

Transverse waves:- The wave in which the particles
vibrate perpendicular to the direction of propagation of wave is called Transverse wave.

Ex:- Waves in strings.



A transverse wave consists of alternate crests and troughs.

Explanation:- 1) Whenever we through a pebble into the pond, we see the circular ripples formed on its surface which disappear gradually. The water moves up and down.

2) When a source fastened to a string executes S.H.M along vertical direction about its equilibrium position. When the string vibrates, the waves travel in the form of crests and troughs along a direction ⊥ar to

the direction of motion of particles of the string. That waves are called Transverse waves

Some examples are light waves, waves from a Guitar string etc

Crest:- The peak or highest point the medium rises is called Crest

(or)

The portion of the string displaced above its mean position is called crest

Trough:- The lowest point to which the medium sinks is called trough.

Wavelength:- The distance between two successive crests or troughs is called wavelength.

Amplitude:- The maximum value of displacement in a crest or trough is called Amplitude of the wave.

Speed of longitudinal wave in Air

Sound waves are compressional waves, which propogate through a compressive medium such as air. The Speed of such compressional waves depends upon the compressibility and the inertia of the Medium.

The compressible medium which has Bulk modulus "B" and density (ρ) [inertial property], then the speed of sound 'V' in the medium is given by

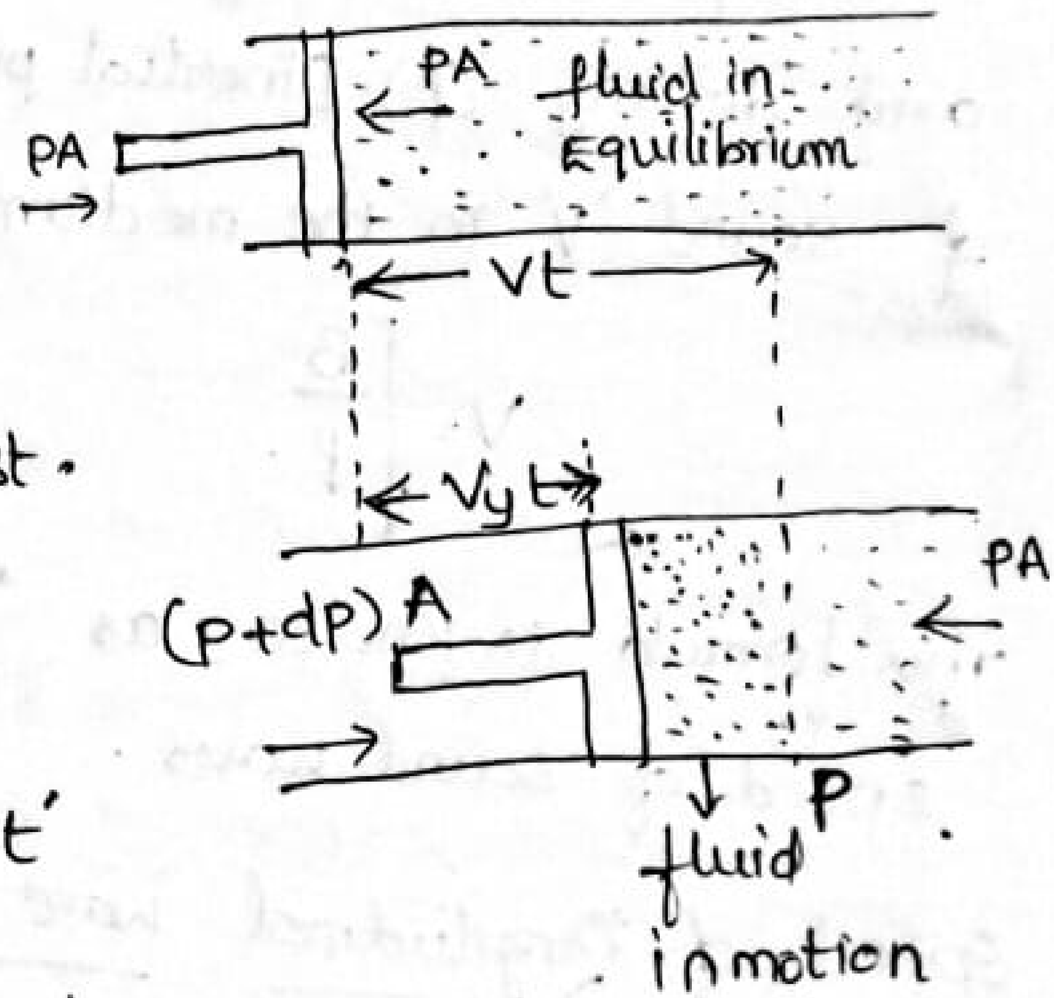
$$V = \sqrt{\frac{B}{\rho}}.$$

This formula is known as "Newton's formula" for the speed of sound waves.

speed of longitudinal wave in fluid :-

consider a fluid of density 'ρ' in a pipe with cross-sectional area "A". 'p' is the uniform pressure of fluid in equilibrium state.

Let the wave propagate along x-axis along length of pipe. V_y is the speed of piston towards right.



After a time "t" the fluid upto point 'p' is in motion and the fluid after point 'p' is still at rest.

In a time ~~time~~ 't'. the fluid in motion has moved through a distance 'vt'

when the piston is pushed to increase in pressure on the fluid is 'dp'

If the piston moves in time 't'. then $V_y t$ is the distance piston moved

original volume of the fluid is $= A \times vt$

decrease in volume of fluid is $= -A \times V_y t$

we know Bulk modulus, $B = \dfrac{dp}{-\left(\frac{dv}{V}\right)}$

$$B = \frac{dP}{-\left[\frac{-A V_y t}{A V t}\right]} = dP\left[\frac{V}{V_y}\right]$$

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$$dp = \frac{V_y}{V} B$$

The net force on fluid is $F_{net} = dp \times A$

$$F_{net} = \frac{V_y}{V} B \times A$$

$$\text{Impulse} = F_{net} \times t$$

$$F_{net} \times t = \frac{V_y}{V} B \times A \times t \rightarrow (1)$$

change in momentum $= \Delta P$

$$\Delta P = m \cdot \Delta V$$

$$= (\rho v)\, V_y$$

$$= \rho\, (A l)\, V_y$$

$$\Delta p = \rho\, (A V t)\, V_y \rightarrow (2)$$

From Impulse-Momentum theorem, The net force acts on a fluid mass is equal to the change in momentum of the flow per unit time in that direction

$$\Delta p = F_{net} \times t$$

$$\rho\, (A V t)\, V_y = \frac{V_y}{V} B A t$$

$$\rho V^2 = B \Rightarrow V^2 = \frac{B}{\rho}$$

$$\boxed{V = \sqrt{\frac{B}{\rho}}}$$

B - Bulk modulus of fluid. ρ - density of fluid.

speed of o

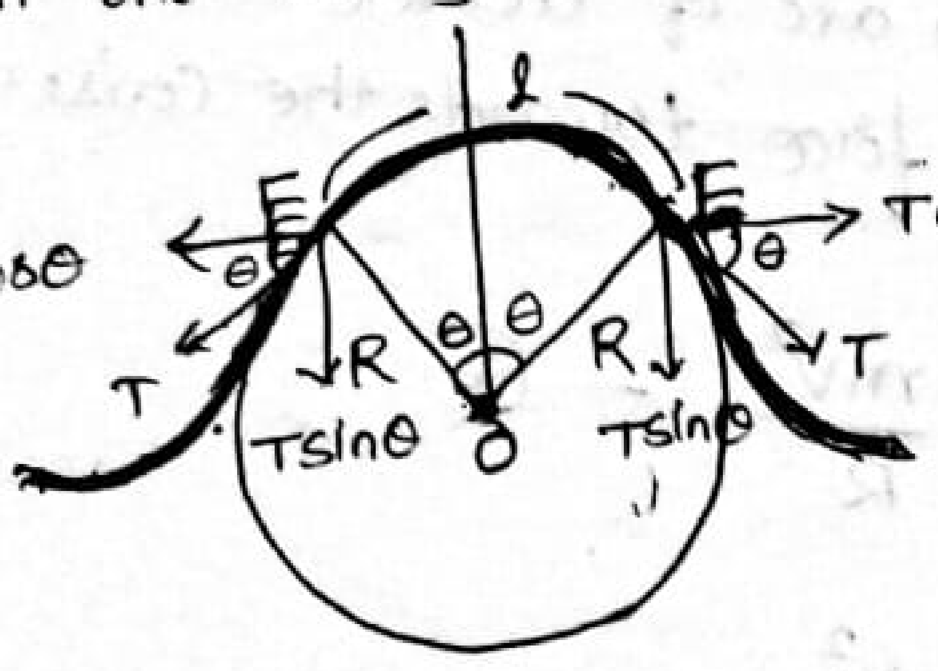
Module - 3

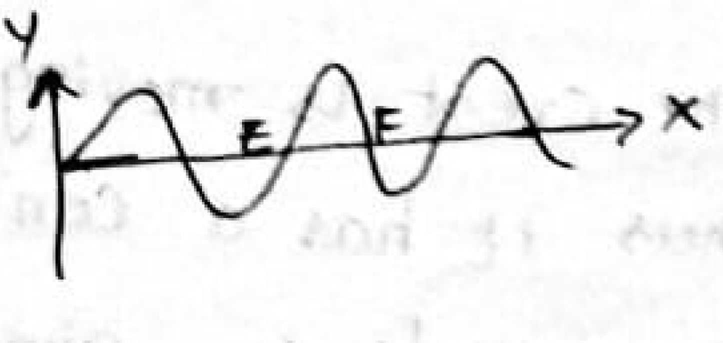
speed of a transverse wave on stretched string :-

The speed of transverse wave set up in the string depends on tension applied (T), mass per unit length (μ) of the string.

Let us consider a string filled at one of its ends and tension be applied at the other end. when the string is plucked at a point, it begins to vibrate. Consider a transvere wave proceeding from left to right in the form of a pulse when the string is plucked at a point

we cannot send a wave along a string unless the string is under tension. we can associate the tension in the string with the stretching (elasticity) of the string





EF is the (crest) displaced position at an instant of time. It forms an arc of a circle with 'O' as center and R as the radius. it subtends an angle 2θ and has length 'l'.

$\therefore \boxed{l = R(\theta+\theta)} \Rightarrow \boxed{l = R(2\theta)}$

If 'm' is the mass of the string then linear density is, $\mu = \frac{m}{l} \Rightarrow \boxed{m = \mu l}$

A force with a magnitude equal to the tension in the string pulls tangentially on this element at each end. The horizontal components of Tension forces ($T\cos\theta$) cancel, but the vertical components add to form a radial restoring force.

∴ The results of the tensions acting at E and F is

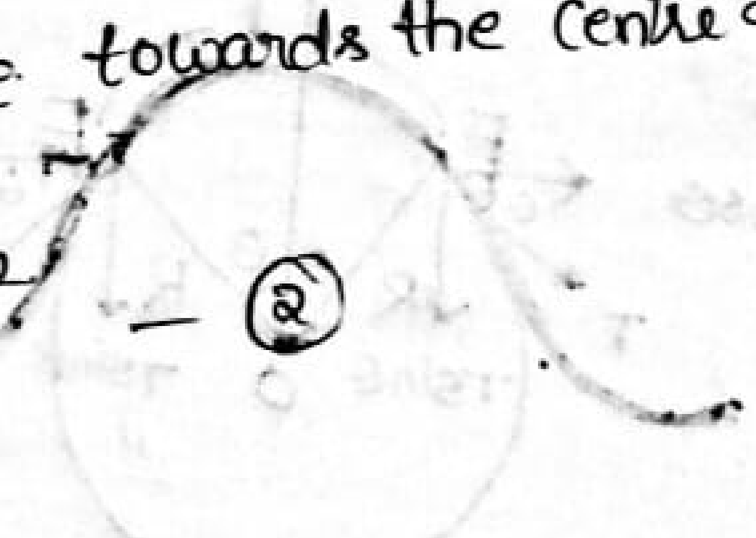
$$T\sin\theta + T\sin\theta = 2T\sin\theta$$

$$F = 2T\sin\theta$$

If θ is very small $\sin\theta \approx \theta$, $F = 2T\theta$

$$F = T(2\theta) \rightarrow ①$$

The crest is moving as an arc of a circle continuously. Thus it has a centripetal force towards the centre of that circle is given by



$$F = \frac{mv^2}{R} \text{ — } ②$$

from Eq^{n}s ① & ②

$$T(2\theta) = \frac{mv^2}{R}$$

$$T\left(\frac{l}{R}\right) = \frac{mv^2}{R} \quad [\because l = R \cdot 2\theta]$$

$$Tl = mv^2$$

$$Tl = \mu l v^2 \quad [\because m = \mu l]$$

$$\therefore \quad T = \mu v^2$$

$$v^2 = \frac{T}{\mu}$$

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

The speed of a transverse wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

Module-4

principle of superposition of waves :-

When two or more waves travel simultaneously in a medium the resultant displacement at any point is due to the algebric sum of the displacement due to individual wave.
This is the principle of superposition.

If y_1, y_2, y_3 are the displacement due to individual waves at a particular time and at particular position then the resultant displacement y is

$$y = y_1 + y_2 + y_3$$

for two waves $y = y_1 + y_2$.

i) waves in phase:-

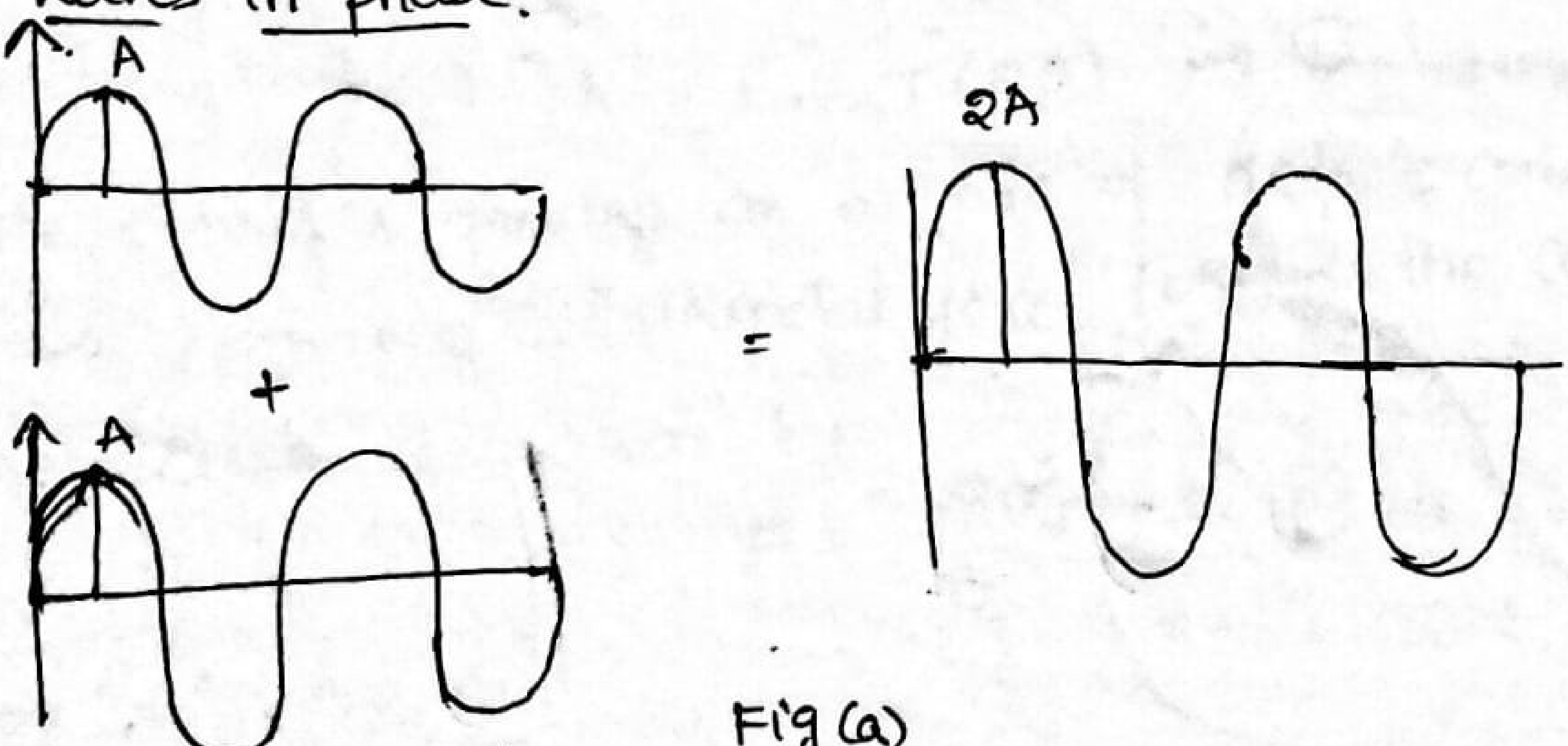


Fig (a)

ii) waves out of phase:-

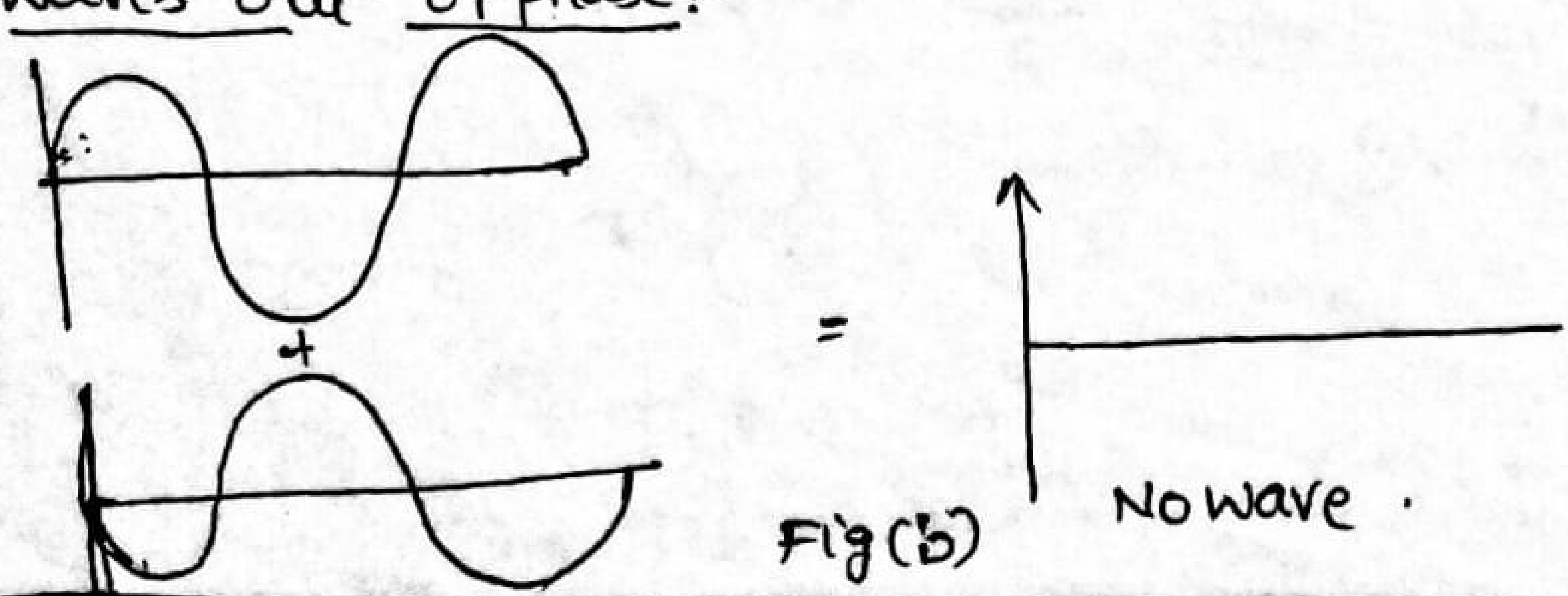


Fig (b)

consider two waves travelling in the same direction and overlap one another, then the resultant displacement is maximum as shown in Fig (a).

If the two waves travelling in the opposite direction, then the displacement becomes minimum or zeroas shown in Fig(b).

Module-4

Wave Interference

The pattern resulting from the superposition of two waves is called wave Interference.

when two or more waves overlap with each other in a same or opposite directions with the same frequency then the resultant wave has maximum or minimum amplitude. This phenomenon is called interference.

Interference is two types

1. constructive Interference
2. destructive Interference.

~~constructive Interference~~ The points of maximum Intensity in the regions of superposition of waves are said to be in constructive interference whereas the points of minimum Intensity are said to be in destructive Interference.

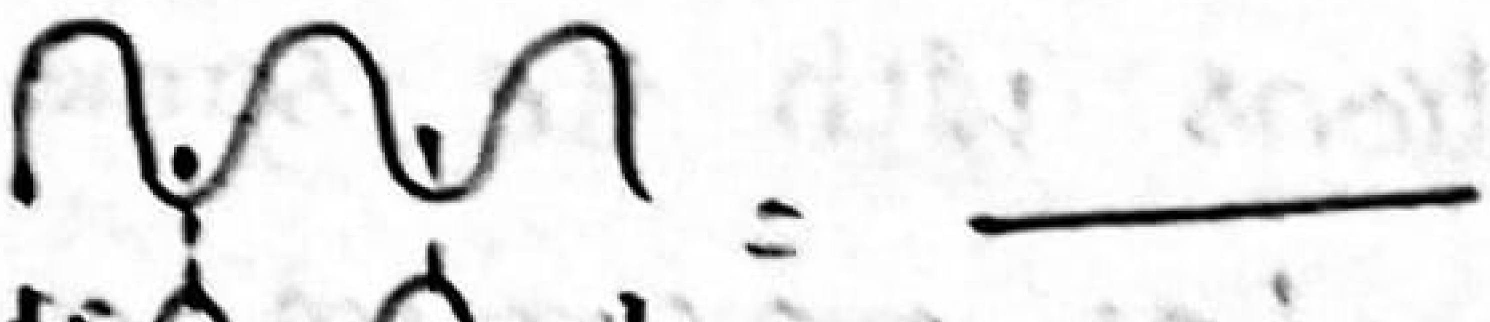
Constructive Interference :-

When crest of one wave falls on the crest of another wave, the resultant amplitude is the sum of two waves and Intensity is increased. It is known as constructive Interference.

$A = a_1 + a_2$

Destructive Interference :-

When crest of one wave falls on the trough of another wave, the resultant amplitude is the difference of the amplitudes of two waves and Intensity is decreased. It is known as destructive Interference.



No wave

$A = a_1 - a_2$

Mathematical

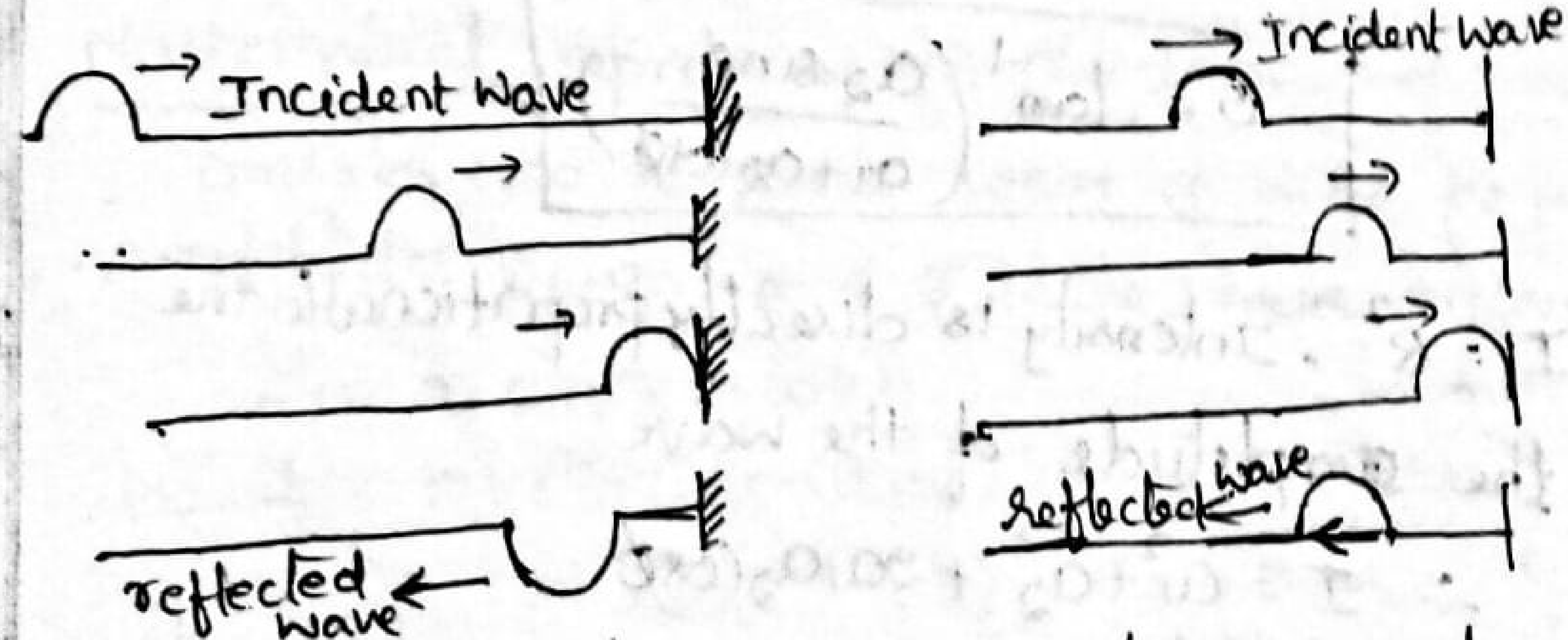
Standing waves in a string :-

Reflection of waves :-

If a pulse or wave meets a rigid boundary, they get reflected e.g echo. If the boundary is not completely rigid, a part of the incident wave is reflected and a part is transmitted into the second medium.

The incident and refracted waves obey snell's law of refraction and the incident and reflected waves obey the laws of reflection.

The reflection of a wave or a pulse, can happen from two types of surfaces, it can either be a fixed wall (rigid boundary) or a ring (free end).



(a) Fixed end (b) Free end

In Fig(a) the incident wave hits the rigid boundary then it gets reflected. when the pulse reaches the rigid end, it exerts a force on the wall and the wall also exerts an equal and opposite force on the string, so the reflected wave takes a phase difference of π or 180°, then it is inverted.

If boundary point is completely free to move, the reflected pulse has the same phase and amplitude as the incident pulse which is shown in Fig(b)

Let the incident travelling wave is, $y(x,t) = a\sin(kx-\omega t)$

At the rigid boundary, the reflected wave is,

$$y(x,t) = a\sin(kx+\omega t+\pi) = -a\sin(kx+\omega t)$$

$$\therefore \boxed{y = -a\sin(kx+\omega t)}$$

At the free boundary, the reflected wave is

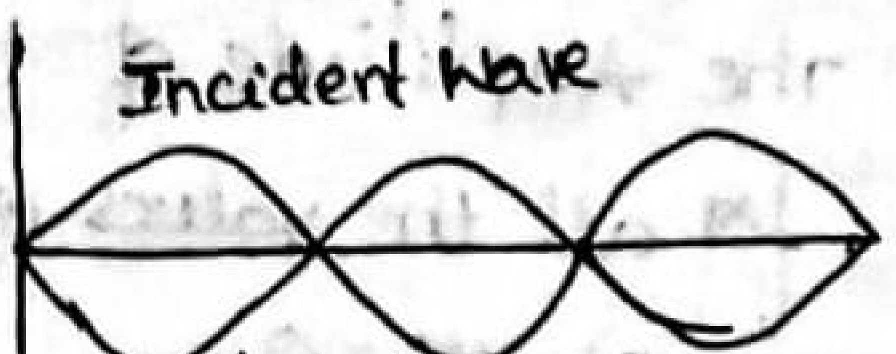
$$\boxed{y(x,t) = a\sin(kx+\omega t)}$$

Standing waves on a string:-

Two waves of same amplitude and frequency travelling in opposite direction overlap one another to form a standing wave.

(or)

Standing waves are formed by the superposition of two waves of equal amplitude and frequency travelling through the medium in the opposite direction. These waves are localised and not progressive, hence the name stationary waves.



Let us consider two waves of the same amplitude and period(T) and wavelength travelling with the same speed in opposite direction.

$Y_1 = A\sin(kx-\omega t)$ [wave along +ve x-axis]

$Y_2 = A\sin(kx+\omega t)$ [wave along -ve x-axis] considering the principle of superposition the resultant can be calculated as

$$Y = Y_1 + Y_2$$

$$Y = A\sin(Kx - \omega t) + A\sin(Kx + \omega t)$$

$$= A\left[\sin(Kx - \omega t) + \sin(Kx + \omega t)\right]$$

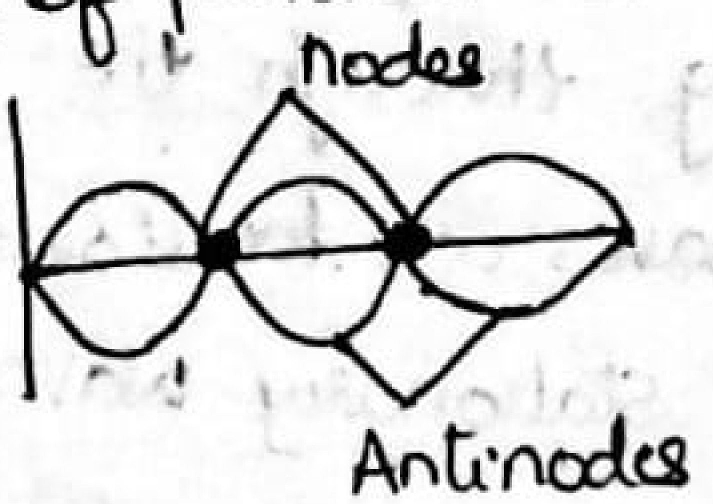
$$= A \cdot 2\sin\left[\frac{(Kx - \omega t) + (Kx + \omega t)}{2}\right] \cdot \cos\left[\frac{(Kx - \omega t) - (Kx + \omega t)}{2}\right].$$

$$Y = 2A\sin(Kx)\cos(\omega t)$$

Here the term $2A\sin Kx$ is the amplitude of the resultant wave. It can be concluded that the amplitude of the particles executing SHM depends upon the location of the particles.

The points where the displacement of particles is zero is called nodes.

The points where the displacement of particles is Maximum is called Antinodes.



<u>nodes</u> :- The Amplitude of the wave is zero for all the values of Kx that give $\sin Kx = 0$

$$Kx = n\pi \Rightarrow x = \frac{n\pi}{K} = \frac{n\pi}{2\pi/\lambda} = \frac{n\lambda}{2}$$

$$\therefore \boxed{x = \frac{n\lambda}{2}}$$, where $n = 0, 1, 2, 3 \ldots$

$$\therefore x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$$

These points of zero displacements of the particles are called the nodes.

Antinodes :- The amplitude will have a maximum value of $2A$ for all values of kx that give $|\sin kx| = 1$

$$kx = \left(n+\tfrac{1}{2}\right)\pi, \quad n = 0,1,2,3 \ldots$$

For antinodes, $x = (2n+1)\frac{\lambda}{4}$, $n = 0,1,2 \ldots$

$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4} \ldots$

Normal modes in a string :-

Nodes are produced at the fixed ends. Consider a string of length 'L' rigidly held at two ends as shown in Fig. Let a standing wave is generated in the string with different modes of vibration.

Expression for node $x = \frac{n\lambda}{2}$, where $n = 0,1,2, \ldots$

The distance between two successive nodes is $\lambda/2$ and 'L' is the length of the string then,

$$L = \frac{n\lambda}{2}, \quad n = 0,1,2 \ldots$$

i) For $n = 1$, $L = \frac{\lambda}{2}$

The string will vibrate in one segment.

ii) For $n = 2$, $L = \frac{2\lambda}{2} = \lambda$

for $n = 2$, the string will vibrate in two segments

iii) For $n = 3$, $L = \frac{3\lambda}{2}$

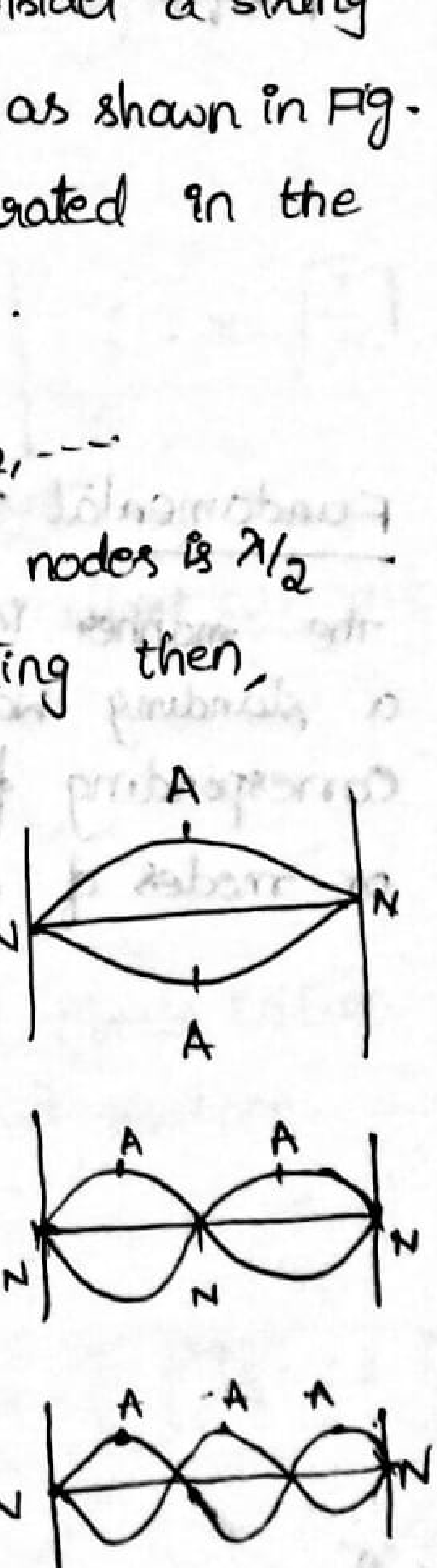


Fig: Different modes of v

For 'n' modes.

$$L = \frac{n\lambda}{2}$$

$$\therefore \boxed{\lambda_n = \frac{2L}{n}} \qquad n = 1, 2, 3, \ldots$$

$\lambda_n \rightarrow$ corresponds to series of possible wavelengths.

and corresponding frequencies are

Frequency of the vibration, $\therefore f = \frac{v}{\lambda}$

$$f = \frac{v}{\frac{2L}{n}} = \frac{n}{2L} \cdot v = \frac{nv}{2L}$$

$$\boxed{f = \frac{nv}{2L}}$$

Module-9

Fundamental mode and harmonics :-

When a body is vibrated, it vibrates with more than one frequencies. All the frequencies are an integral multiple of some least frequency.

The least frequency is called first harmonic or fundamental frequency.

i) For $n=1$, mode of oscillation is known as Fundamental mode in which string will vibrate in one segment

As $L = \frac{n\lambda}{2}$ for first harmonic $n=1$, then

$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

frequency $f = \frac{nv}{2L}$ $\Rightarrow$ $\boxed{f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}}$ $\left[\because v = \sqrt{\frac{T}{\mu}}\right]$

for $n=1$, $\boxed{f_1 = \frac{v}{2L}}$

This is called fundamental frequency or first harmonic.

$$f_1 = \frac{v}{2L} \Rightarrow \frac{v}{\lambda_1}$$

$$\therefore \boxed{f_1 = \frac{v}{\lambda_1}}$$

ii) The mode of vibration in which string will vibrate in two segments is called second harmonic or first overtone.

put $n=2$, then $L = \frac{2\lambda}{2} = \lambda_2 \Rightarrow \boxed{\lambda_2 = L}$

and $f_2 = \frac{2v}{2L} = 2\left(\frac{v}{2L}\right) \rightarrow$ $\boxed{f_2 = 2f_1}$

$$\therefore f_2 = \frac{2v}{2L} \text{ or } 2f_1$$

$$f_2 = \frac{v}{L} \Rightarrow \boxed{f_2 = \frac{v}{\lambda_2}}$$

iii) for $n=3$, $L = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2L}{3}$

$$\boxed{f_3 = \frac{3v}{2L}}$$ is called third harmonic or second overtone.

$\therefore f_3 = \frac{v}{\lambda_3}$

$\Rightarrow f_1 = \frac{v}{2L}$

$f_2 = \frac{2v}{2L} = 2f_1$

$f_3 = \frac{3v}{2L} = 3f_1$ and so on.

These frequencies are called harmonics or overtones of a stretched string fixed at both ends as shown in the figure below.