ELECTROSTATICS - I

⇒ Electric charges and their Conservation:

The total deficiency or excess of electrons in a body is called its charge or electric charge. In general, an object with excess of electrons is said to be negatively charged and that with deficiency of electrons is said to be positively charged.

Ex: when a glass rod is rubbed with silk some of the electrons transferred from glass rod to silk. Thus, we say that electron carries a negative charge and the additional electrons acquired by silk gives it a net negative charge. Glassrod was considered to be positive.

⇒ Properties of charge:

1. charge always be associated with mass
2. It can be transferred from one body to Another.
3. charge is conserved. It can neither be created nor be destroyed.
4. charge is quantised.
5. Like charges repel and unlike charges attract each other.

→ The Fundamental properties of charge are

i) charge is quantised

ii) charge is conserved.

Quantization of charge:

The amount of charge given to an object can have only certain values which are integral multiples of a certain minimum. This is known as quantization of charge. The minimum amount of charge that exists in free state is equal to the charge of electron (also an a proton).

$$e = 1.602 \times 10^{-19} \text{ colomb}$$

$\boxed{Q = \pm ne}$, where n is an integer.

An Object cannot have a fraction charge, like 2.3e, 4.6e... etc.

Conservation of charge:

In any isolated system, the net charge (algebraic sum of charges) remains constant. i.e., Charge is conserved.

Ex:

① when glassrod is rubbed with silk, negative charge appears on silk and an equal amount of +ve charge appears on glass rod. The net charge on glass-silk system remains zero before and after rubbing.

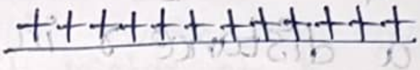
② In any electrical circuit the net charge remains constant.

⇒ Types of charge densities:

① **Point charge:** when linear size of charged body is much smaller than the distance under consideration. The size may be ignored and the charge body is called point charge.

② **Linear charge density:** charge per unit length of the object is called linear charge density. It is defined as

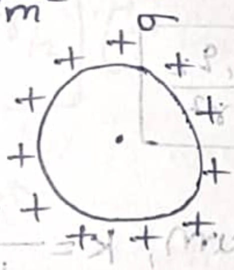
$$\lambda = \frac{dq}{dl}$$

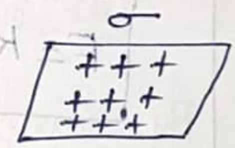


$$\boxed{\lambda = \frac{q}{l}} \ C/m$$

③ **Surface charge density:** The charge per unit surface of the body is called surface charge density.

$$\sigma = \frac{dq}{dA} = \frac{q}{A} \ C/m^2$$



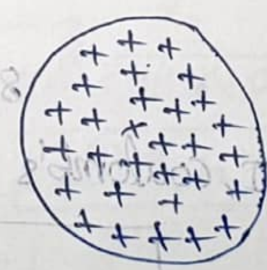


$$\boxed{\sigma = \frac{q}{A}} \ C/m^2$$

④ **Volume charge density:**

The charge per unit volume of the body is called volume charge density.

$$\rho = \frac{dq}{dV} = \frac{q}{V}$$



$$\therefore \boxed{\rho = \frac{q}{V}} \ C/m^3$$

Lecture - 13

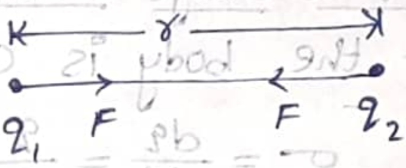
⇒ **Coulomb's law**

The force of attraction or repulsion between two point charges is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them and acts along the line joining the two charges.

q_1 and q_2 are two point charges seperated by a distance r in vaccum free space.

According to coulomb's law,

$F \propto q_1 q_2$; $F \propto \frac{1}{r^2}$



$$F \propto \frac{q_1 q_2}{r^2}$$

$$\boxed{F = K \frac{q_1 q_2}{r^2}}$$

For vaccum, $K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \; Nm^2C^{-2}$

ε_0 is called perimitivity of a free space and is equal to 8.854×10^{-12} farad/m

(or)

$8.854 \times 10^{-12} \; N^{-1} m^{-2} C^2$

In SI, coulomb's law is

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}$$

If the particles repel each other, the force on each particle is directed away from the other particle. If the particles attract each other, the force on each particle is directed toward the other particle.

To evaluate the magnitude of force b/w two charges coulomb's law is expressed as $\left.\right\}$ $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q_1||q_2|}{r^2}$

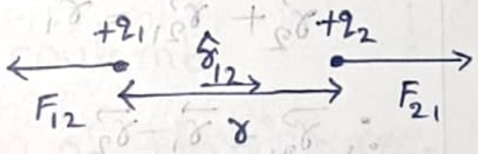
⇒ <u>Coulomb's law of vector form:</u>

If $\hat{r}_{12}$ is the unit vector of r from 1 towards 2, and charges are placed at seperation r, then force on charge q_2 due to charge q_1 is $\vec{F}_{12}$,

$$\boxed{\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\cdot\hat{r}_{12}} \quad [\because \text{ for } q_1 q_2 < 0]$$

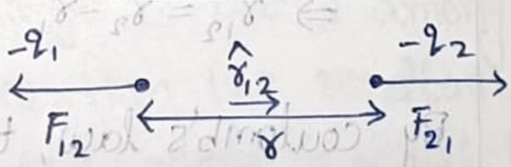
In vaccum, for $q_1 q_2 > 0$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\cdot\hat{r}_{21}$$



Fig(a) $- q_1 q_2 > 0$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\cdot\hat{r}_{12}$$

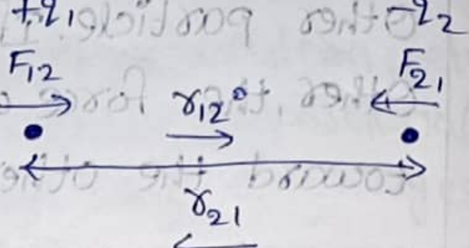


Fig(b) $- q_1 q_2 > 0$

In vaccum for $q_1 q_2 < 0$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \cdot \hat{r}_{12}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \cdot \hat{r}_{21}$$



Fig(c) $q_1 q_2 < 0$

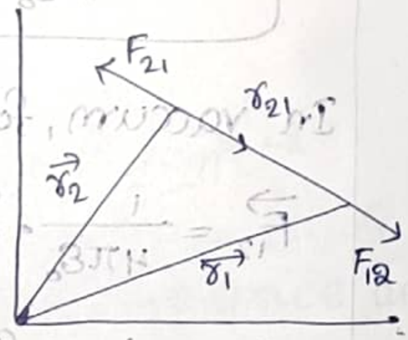
$$\therefore \boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

and $r_{12} = -r_{21}$

If we consider the signs of the charges (Positive and negative) then the vector form of Coulomb's law can be written as in the above form.

Let r_1 and r_2 be the position vectors of charges q_1 and q_2 situated at point A and Point B respectively w.r.t origin "O"

By triangle law of vector Addition



$$\vec{r}_2 + \vec{r}_{21} = \vec{r}_1$$

$$\therefore \vec{r}_{21} = \vec{r}_1 - \vec{r}_2$$

$$\Rightarrow \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

By coulomb's law, the force on

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{(r_2 - r_1)^2} (\widehat{r_2 - r_1})$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{(r_2 - r_1)^2} \cdot \frac{\bar{r}_2 - \bar{r}_1}{r_2 - r_1}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{(\bar{r}_2 - \bar{r}_1)^3} (\bar{r}_2 - \bar{r}_1)$$

(or)

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \cdot \bar{r}_{12}$$

llly $$F_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^3} \cdot \bar{r}_{21}$$

here, $\hat{r}_{12}$ is the unit vector from q_1 to q_2

SI unit of charge:

one coulomb is the charge transported in one second through the cross-section of a conductor carrying a current of one ampere.

Let $q_1 = q_2 = 1C$ and $r = 1m$

then $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1 \times 1}{1} = 9 \times 10^9 N$

If two identical charges seperated by 1m in vaccum repel each other with a force of $9 \times 10^9 N$, then each charge is 1 coulomb.

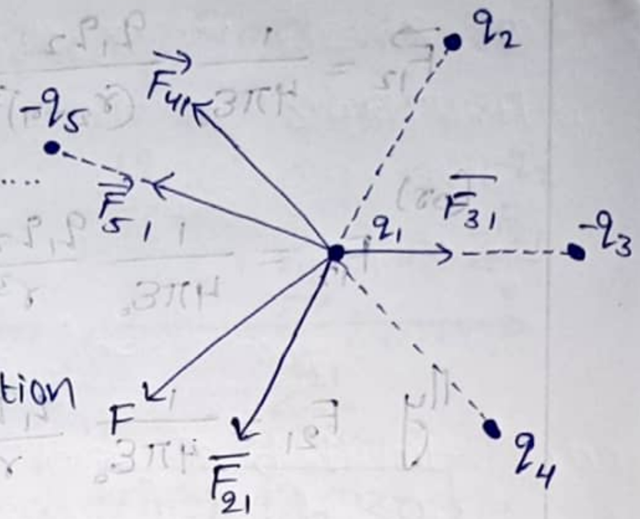
⇒ Principle of superposition of Electricforces:-

When a point charge is placed among the no. of point charges, then the resultant force acting on the point charge is equal to vector sum of the forces acting on it due to all other charges acting independently.

That is

$$\vec{F}_{res} = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} + \ldots\ldots$$

This is known as the principle of superposition of electronfaces.



Here q_1 the charge on which the net force is required as the recipient charge and the other charges be called the source charges.

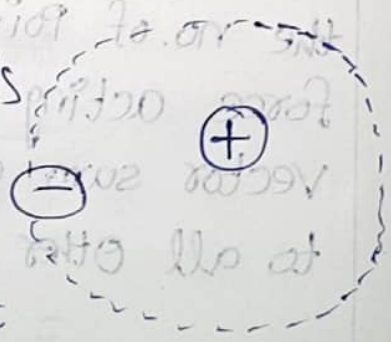
If the source and recipient are like charges, then mark the force at the recipient away from the source charge, if unlike charges then force at the recipient towards the source charge.

Lecture-14

⇒ Electric field due to a point charge

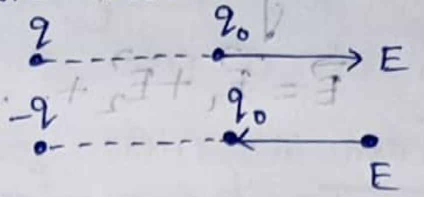
Electric field is defined as the space around the point electric charge which influences electric force on an unit test charge.

The electric field strength is usually called electric field at any point is the force experiences by unit test charge at that point.

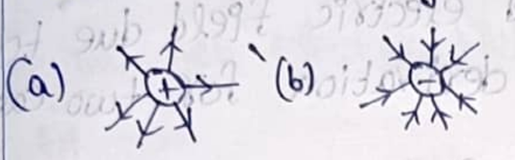


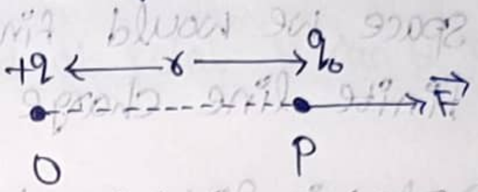
If $\vec{F}$ is the force acts on test charge q_0, then the electric field

$$\vec{E} = \frac{\vec{F}}{q_0} \text{ N/C}$$



The direction of electric field $\vec{E}$ is that of the force $\vec{F}$ acts on the positive test charge. The field due to negative charge is towards it.





Consider a point charge +q at a point 'O' in free space. Let $\vec{E}$ be the electric field at a point P distant 'r' from q. Let a test charge q_0 be placed at 'P'.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2}$$

Since electric field at a point is the force per unit positive charge,

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\frac{1}{4\pi\varepsilon_0} \frac{q \cancel{q_0}}{r^2}}{\cancel{q_0}}$$

$$\therefore \boxed{E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}} \quad \text{(or)} \quad E = K \cdot \frac{q}{r^2}$$

The field is directed away from the charge.

Electric field due to many charges can be obtained by the principle of superposition.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n$$

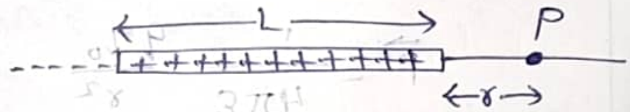
⇒ **Electric field due to line charge:-**

It means the charge is distributed along the one dimensional curve or line 'L' in space we would find electric field due to finite line charge derivation for two cases.

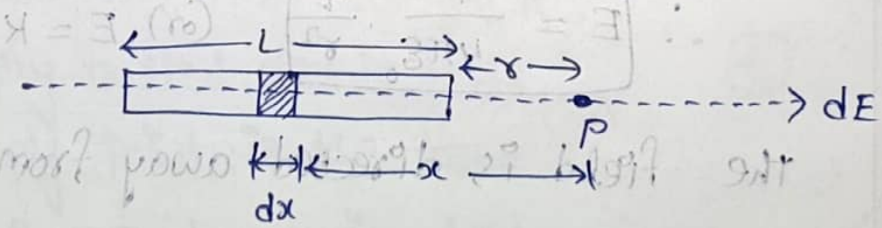
(1) Electric field due to finite line charge at equatorial point.

(2) Electric field due to a line of charge on axis.

At an axial point:-



Consider a rod of length L, uniformly charged with a charge 'Q'. To calucuate the electric field strength at a point "p" situated at a distance 'r' from one end of the rod. Consider an element of length dx on the rod as shown in figure.



charge on the elemental length

$$dq = \frac{Q}{L} dx \rightarrow (1)$$

we have

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \Rightarrow dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{x^2}$$

$$dE = \frac{1}{4\pi\varepsilon_0} \frac{Q}{L} \cdot \frac{1}{x^2} dx \quad [\text{from eq}(1)]$$

The net electric field at point 'p' can be given by

$$E_p = \int dE = \int_r^{r+L} \frac{Q}{4\pi\varepsilon_0 L} \cdot \frac{1}{x^2} dx$$

$$= \frac{Q}{4\pi\varepsilon_0 L} \int_r^{r+L} \frac{1}{x^2} dx$$

$$E_p = \frac{Q}{4\pi\varepsilon_0 L} \left[\frac{-1}{x}\right]_r^{r+L} = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{r} - \frac{1}{r+L}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{\cancel{r} + L - \cancel{r}}{r(r+L)}\right]$$

$$E_p = \frac{Q}{4\pi\varepsilon_0 \cancel{L}} \cdot \frac{\cancel{L}}{r(r+L)}$$

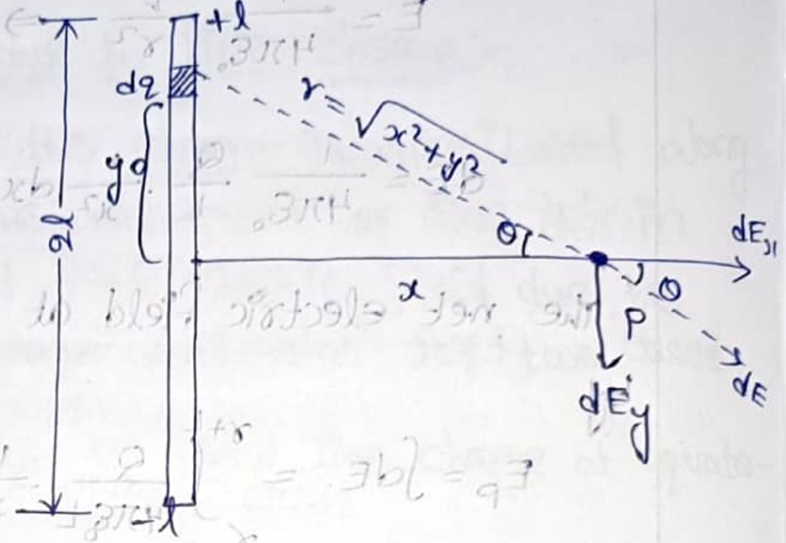
$$\boxed{\therefore E_p = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r(r+L)}}$$

At an equatorial point:

To find the electric field due to a rod at a point P situated at a distance 'x' from its centre on its equatorial line

$\boxed{dQ = \lambda dy}$

$\lambda = \frac{Q}{2l}$

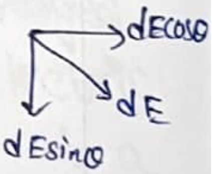


The total charge is uniformly distributed over the line segment.

$$r = \sqrt{x^2+y^2}$$

Magnitude of the field at point 'p' due to segment of height y is

$$dE = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2l} \frac{dy}{x^2+y^2}$$



From the fig $\cos\theta = \frac{x}{r} \Rightarrow \frac{x}{\sqrt{x^2+y^2}}$

$$\sin\theta = \frac{y}{r} \Rightarrow \frac{y}{\sqrt{x^2+y^2}}$$

$$\therefore dE_x \cos\theta = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{2l} \frac{dy}{x^2+y^2} \cos\theta$$

$$dE_x\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2l}\frac{x}{\sqrt{x^2+y^2}}\cdot\frac{1}{x^2+y^2}\,dy \quad \left[\because \cos\theta = \frac{x}{\sqrt{x^2+y^2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2l}\frac{x}{(x^2+y^2)^{3/2}}\,dy$$

$$E = \int dE_x\cos\theta = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2l}\int_{-l}^{+l}\frac{x}{(x^2+y^2)^{3/2}}\,dy$$

From fig. $\tan\theta = \frac{y}{x} \Rightarrow y = x\tan\theta \Rightarrow dy = x\sec^2\theta\, d\theta$

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{2l}\right)x\int_{-l}^{+l}\frac{1}{(x^2+x^2\tan^2\theta)^{3/2}}\,x\sec^2\theta\, d\theta$$

$$E = K\cdot\frac{Q}{2l}\cdot x\int_{-l}^{+l}\frac{1}{(x^2\sec^2\theta)^{3/2}}\,x\sec^2\theta\, d\theta \quad \left[\because K = \frac{1}{4\pi\varepsilon_0}\right]$$

$$= K\cdot\frac{Q}{2l}\cdot x\int_{-l}^{+l}\frac{1}{x^3\sec^3\theta}\cdot x\sec^2\theta\, d\theta$$

$$= K\cdot\frac{Q}{2l}\cdot\frac{1}{x}\left[\sin\theta\right]_{-l}^{+l}$$

$$= K\frac{Q}{2l}\frac{1}{x}\left[\frac{y}{\sqrt{x^2+y^2}}\right]_{-l}^{+l}$$

$$= K\cdot\frac{Q}{2l}\frac{1}{x}\left[\frac{l}{\sqrt{x^2+l^2}}+\frac{l}{\sqrt{x^2+l^2}}\right]$$

$$= K\cdot\frac{1}{x}\cdot\frac{Q}{\cancel{2l}}\cdot\frac{\cancel{2l}}{\sqrt{x^2+l^2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x}\cdot\frac{1}{\sqrt{x^2+l^2}}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 x}\cdot\frac{1}{\sqrt{x^2+l^2}}}$$

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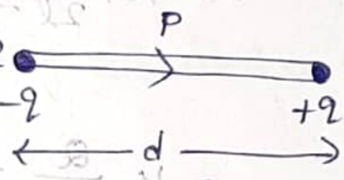
If $x << l$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 x} \cdot \frac{1}{(l^2)^{1/2}} \Rightarrow \frac{Q}{4\pi\varepsilon_0 x} \cdot \frac{1}{l}$$

$$\Rightarrow \left(\frac{Q}{2l}\right) \frac{1}{2\pi\varepsilon_0 x}$$

$$\boxed{\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 x}}$$

⇒ Electric dipole: A system of two equal and opposite point charges seperated by a distance is called an electric dipole.



The product of the magnitude of either charge of a dipole and the seperation between them is called electric dipole moment (p)

Dipole moment is a vector drawn from the negative charge to the positive charge of the dipolemoment is $\bar{p} = qd$, where $\hat{p}$ is a unit vector from $-q$ to $+q$.

The line joining the two charges of a dipole is called it axis. The S.I unit of electric dipolemoment is Cm (coulumb metre).

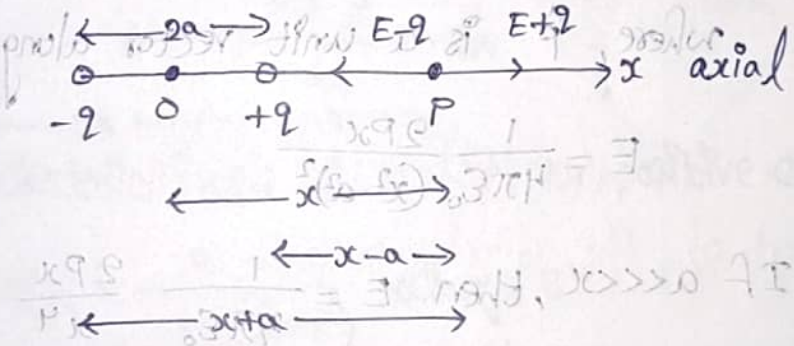
⇒ Electric field due to a Dipole:

Consider two equal and opposite charges $+q$ and $-q$ seperated by a distance $2a$ along the x-axis. The field at any point due to the dipole is the resultant of the fields due to the individual charges.

A line passing through the positive and negative charge of the dipole is called axial line

① Field at point on the axis

Let 'p' be a point on the axis of a dipole at a distance 'x' from its centre 'o'.



The electric field at 'p' due to the positive charge is

$$E_{+q} = \frac{q}{4\pi\varepsilon_0 (x-a)^2}$$

The electric field at 'p' due to negative charge is

$$E_{-q} = \frac{q}{4\pi\varepsilon_0 (x+a)^2}$$

The net Electric field at 'p' is $E = E_{+q} - E_{-q}$

$[\because E_{+q} > E_{-q}]$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(x+a)^2-(x-a)^2}{(x^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(x^2+a^2+2ax)-(x^2+a^2-2ax)}{(x^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4ax}{(x^2-a^2)^2}\right]$$

$$\therefore \bar{E} = \frac{1}{4\pi\varepsilon_0}\frac{2px}{(x^2-a^2)^2}\hat{p} \quad [\because p = q\times 2a]$$

where, $\hat{p}$ is a unit vector along dipolemoment $\bar{p}$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2px}{(x^2-a^2)^2}$$

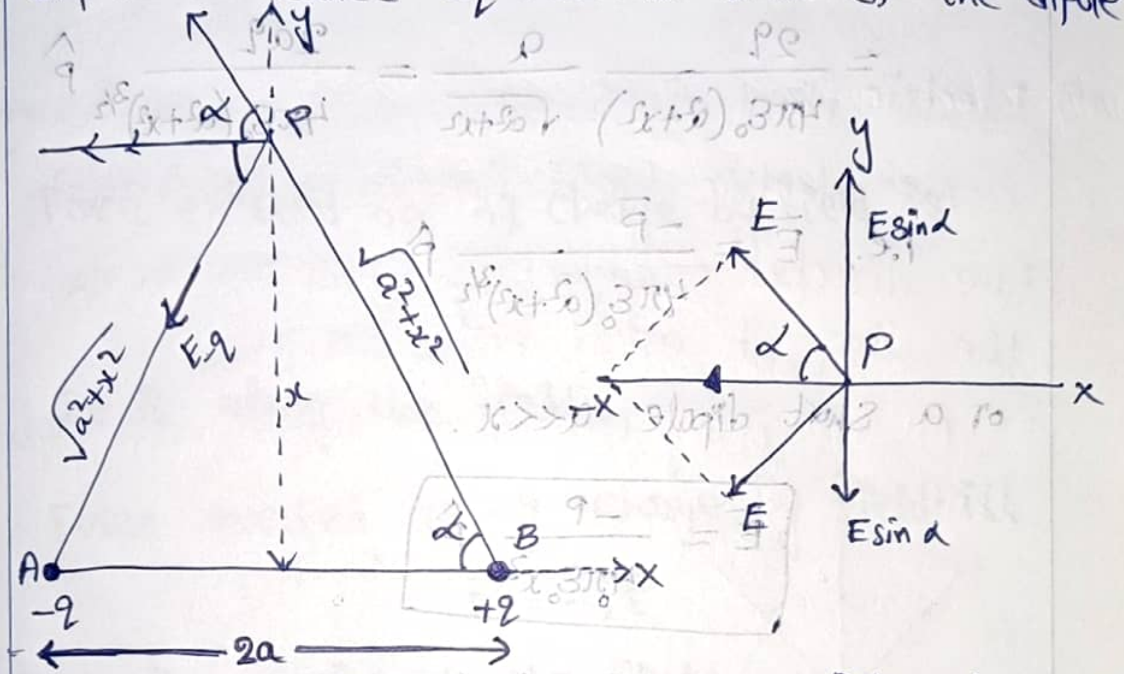
If $a <<< x$, then $E = \frac{1}{4\pi\varepsilon_0}\frac{2px}{x^4}$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{x^3} \Rightarrow \boxed{E = \frac{2pk}{x^3}}$$

The field at a point due to a short dipole varies inversely as the cube of its distance from the dipole.

(2) Field at a point on Equitorial line:

Consider a point 'p' at a distance x from the centre of the dipole and lying in the equatorial plane. A line $\perp^{ar}$ to axial line and passing through the Midpoint of electric dipole is called equitorial line of the dipole.



The electric field at P due to positive charge +q is

$$E_{+q} = \frac{q}{4\pi\varepsilon_0(a^2+x^2)} \text{ along BP}$$

The electric field at P due to negative charge -q is

$$E_{-q} = \frac{q}{4\pi\varepsilon_0(a^2+x^2)} \text{ along PA}$$

$$\cos\alpha = \frac{a}{\sqrt{a^2+x^2}}$$

. Also, the components along the y-direction cancel each other while the components along the x-direction add up.

The resultant field is

$$\vec{E}_{eq} = -\left[E_{+q}\cos\alpha + E_{-q}\cos\alpha\right]\hat{p}$$ Parallel to the negative x-direction

$$E = -2E_{+q}\cos\alpha\,\hat{p}$$

$$= \frac{2q}{4\pi\varepsilon_0(a^2+x^2)}\frac{a}{\sqrt{a^2+x^2}} = \frac{2aq}{4\pi\varepsilon_0(a^2+x^2)^{3/2}}\hat{p}$$

i.e., $$\vec{E} = \frac{-\bar{p}}{4\pi\varepsilon_0(a^2+x^2)^{3/2}}\hat{p}$$

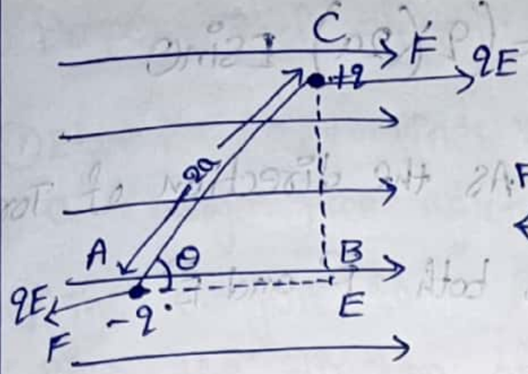
or a short dipole $a<<x$.

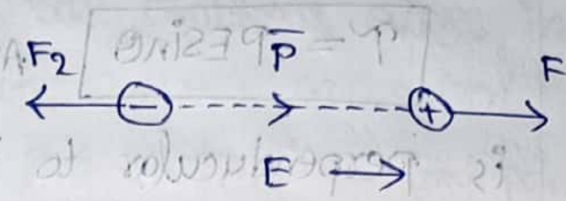
$$\boxed{\vec{E} = \frac{-p}{4\pi\varepsilon_0 x^3}\hat{p}}$$

The direction of electric field is opposite to that of the dipolemoment.

⇒ <u>Torque on a dipole in a uniform Electric field</u>

Consider an electric dipole consisting of charges $+q$ and $-q$ and of length $2a$ placed in a uniform electric field $\vec{E}$, making an angle θ with the direction of field as shown in figure.





Force exerted on +q charge by field is

$$\vec{E} = +q\vec{E}$$

It is along the field.

Force exerted on -q charge by field is.

$$\vec{E} = -q\vec{E}$$

It is opposite to the field.

The net Force $\vec{F}_{total} = +q\vec{E} - q\vec{E} = 0$

Hence the net translating force on a dipole in a uniform electric field is zero. But the two equal and opposite forces act at different points of the dipole.

They form a couple which exerts a torque.

Torque = Either force × perpendicular distance b/w the two forces

$\tau = (qE) \times (BC)$ But $BC = 2a \sin\theta$

$$\tau = qE \times 2a\sin\theta = (q \times 2a)\, E\sin\theta$$

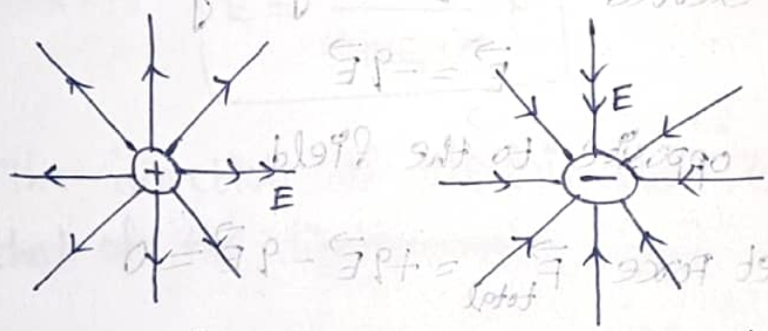
$\boxed{\tau = PE\sin\theta}$ As the direction of Torque $\vec{\tau}$ is perpendicular to both $\vec{P}$ and $\vec{E}$.

$$\text{so, } \bar{\tau} = \bar{P} \times \bar{E}$$

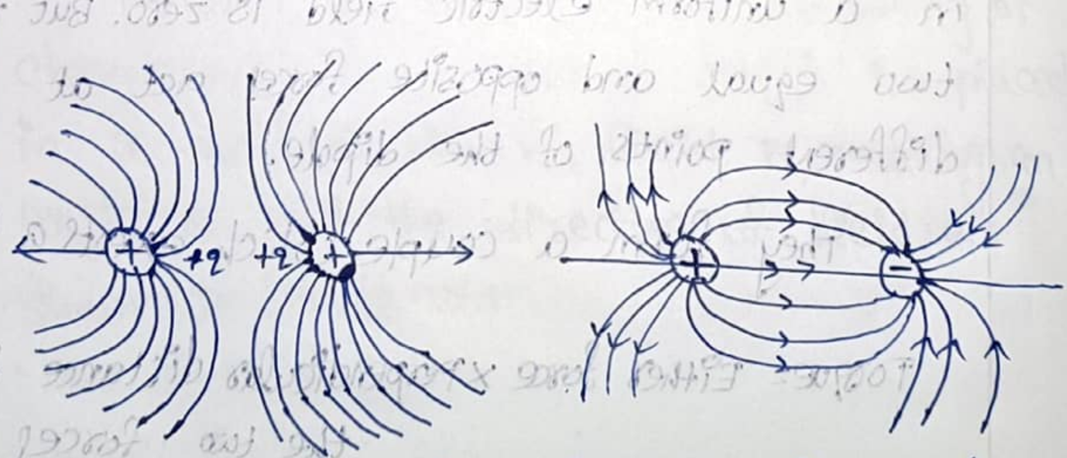
⇒ **Electric field lines:-**

An electric field line is a line such that the direction of electric field is tangential to the line at every point on it.

An imaginary test charge would move along the line.



(a) Positive charge (b) Negative charge



(c) A pair of Equal Positive charges.

(d) A pair of equal Positive & Negative charge.

Properties of electric field lines:-

1. Electric field lines are continous curves in a charge-free region.
2. The tangent to a line of field to any point gives the direction of $\vec{E}$ at that point.
3. These lines leave or enter the charged surface normally.
4. Electric field lines directed away from positive to Negative electric charge.
5. Field lines never cross each other because if they do so then at the point of intersection there will be two directions of Electric Field.
6. Electric lines per unit area is proportional to electric intensity.

   Hence, the lines of field are closely spaced where intensity is large and are widely seperated where intensity is small.
7. A uniform electric field is represented by a set of parallel uniformly spaced lines. Non-uniformly spaced lines and curved lines represent a non-uniform field.

Electric Flux:

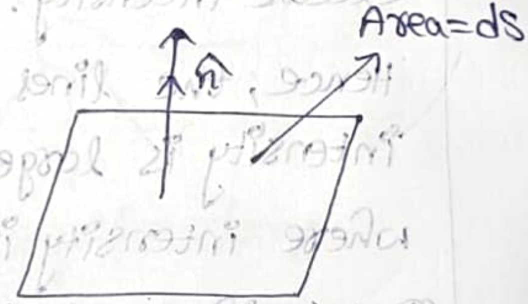
The electric flux through a surface held inside an electric field represents the total number of electric field lines crossing the surface in a direction normal to the surface.

Electric flux is a scalar quantity & is denoted by ϕ. The area of surface is treated as a vector quantity i.e., $\vec{ds}$ such that the arrow representing the area vector $\vec{ds}$ is perpendicular to area element ds.

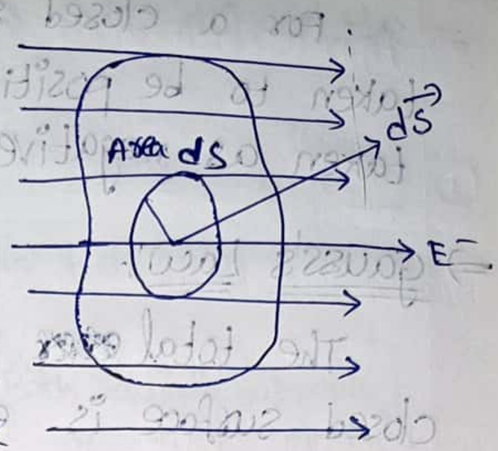
$$\vec{ds} = ds.\hat{n}$$

Here, $\hat{n}$ is a unit vector along normal to the area element ds.

Suppose that a surface having an area S is placed inside an electric field of intensity $\vec{E}$. To find the electric flux through the surface of area S, consider a small element ds and is represented by vector $\vec{ds}$ which is directed along normal to the area element ds.



Suppose that electric field $\vec{E}$ makes an angle θ with the area vector $\vec{ds}$ then component of electric field along the normal to the area element ds, i.e., along the area vector $\vec{ds}$ is given by $E_n = E\cos\theta$



Hence, electric flux crossing the area element $d\phi$ is

$$d\phi = E_n ds = (E\cos\theta) ds$$

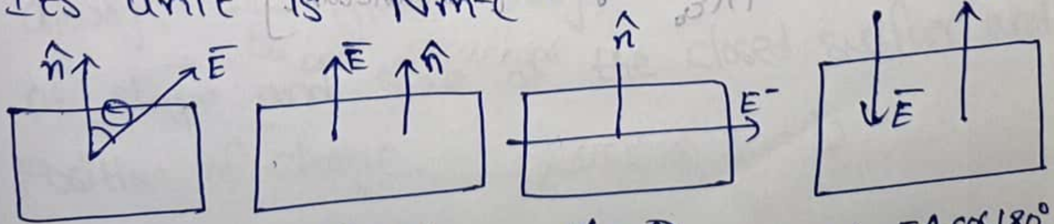
$$d\phi = \vec{E}\cdot\vec{dS}$$

The electric flux through the whole surface "S" can be found by Integrating the above over the whole surfaces.

$$\therefore \phi = \oint_S \vec{E}\cdot\vec{dS} = \oint_S E_n ds$$

The electric flux through a closed surface in an electric field is the surface integral of the electric field over that surface.

Its unit is Nm^2C^{-1}



$\phi = ES\cos\theta$

$\phi = ES$

$\phi = 0$
$[\cos 90^\circ = 0]$
[No Flux)

$\phi = EA\cos 180^\circ$
$\phi = -EA$

· For a closed surface, outward flux is taken to be positive while inward flux is taken as negative.

⇒ **Gauss's Law**:

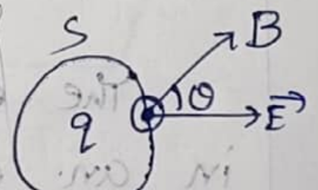
The total ~~[illegible]~~ electric flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by that surface.

Here, ε_0 is permitivity of free space.

$$\phi = \oint E.ds = \frac{q}{\varepsilon_0}$$

$$\boxed{\therefore \phi = \frac{q}{\varepsilon_0}}$$

Proof:- Consider a closed surface S in an electric field. Let "q" be the total charge in the volume enclosed by the gaussian surface of a radius 'r'.



The electric field at any point on the surface is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad [\because E = \text{constant}]$$

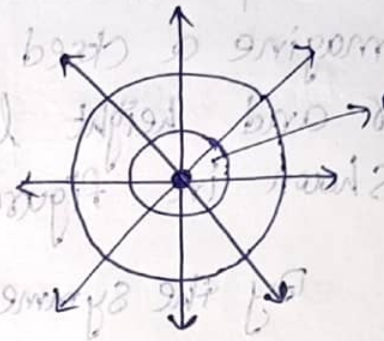
consider an element of area "ds" on the surface of the sphere.

Flux through that element,

$$d\phi = \vec{E}\cdot d\vec{S} \text{ (or) } d\phi = E\,ds\cos\theta$$

If $\theta = 0°$ $\Rightarrow d\phi = E\,ds$ [Both radially outwards]

$$d\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$$



$\therefore$ Total flux through the sphere is $\phi = \int d\phi$.

$$\phi = \int \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}ds$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\int ds = \frac{1}{\cancel{4\pi}\varepsilon_0}\cdot\frac{q}{\cancel{r^2}}(\cancel{4\pi r^2})$$

$$= \frac{q}{\varepsilon_0}$$

$$\boxed{\therefore \phi = \frac{q}{\varepsilon_0}}$$

$\therefore$ The total flux through a closed surface is always $\frac{1}{\varepsilon_0}$ times charge enclosed irrespective of shape and size of the closed surface and position of charge.

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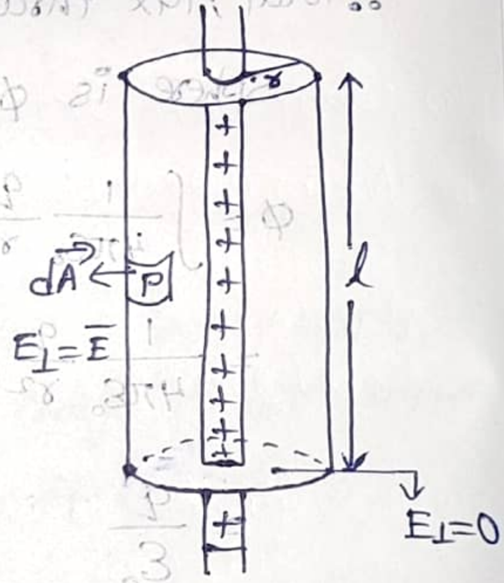
⇒ Applications of Gauss Law:-

① Field due to an infinitely long straight uniformly charged wire

Consider a uniformly charged infinitely long electric wire. To find the electric field at a point "p" at a distance r from the wire, imagine a closed cylindrical surface of radius r and height l as Gaussian surface as shown in figure.



By the symmetry, the electric field near the wire should be radially outwards and perpendicular to the wire.

If E is the electric field at P, the electric flux across the curved surface is

$\phi = E \times$ Area of curved surface of a cylinder of radius r and length l.

$$\phi = E \times 2\pi r l$$

There is no flux across the circular portions of the surface as E parallel to them. Hence, total flux across the closed surface,

$$\phi = E(2\pi r)l \quad \text{--- (1)}$$

The total charge enclosed with cylindrical surface is $q = \lambda l$.

According to Gauss's theorem, -

$$\phi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \quad \text{--- (2)}$$

From eq^ns (1) & (2)

$$E(2\pi r)l = \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

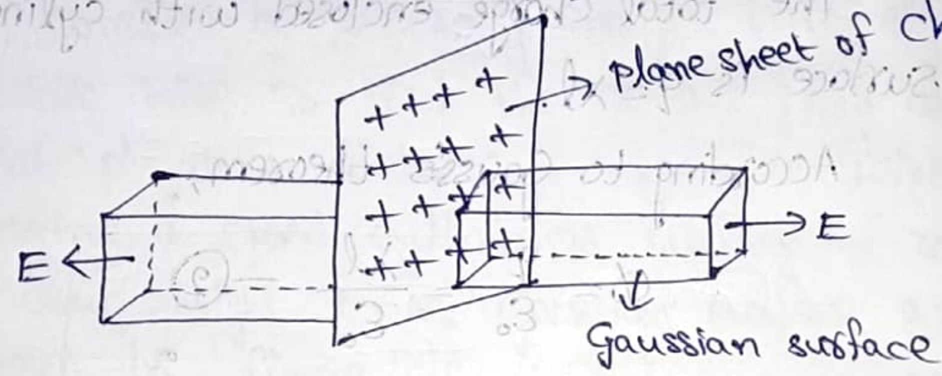
(2) <u>Uniformly charged infinite plane sheet</u>:-

Consider an infinite sheet of positive charge with uniform surface charge density 'σ'.

Consider a gaussian surface in the form of a cylinder as shown. The axis of the cylinder is perpendicular to the plane and its each end has an area ds.

Since $\vec{E}$ is parallel to the curved surface. The flux through the curved is zero.

The Electric lines of force are perpendicular to the flat end surfaces 1&2 and parallel to the remaining surfaces of the cylinder or ~~parallel to piped~~ parallelopiped.



so the flux due to electric field passes only the flat ends of the cylinder are considered the flux through each end is EA

∴ Total flux through Gaussian surface is

$$\phi = EA + EA = 2EA \rightarrow (1)$$

w.r.t, charge enclosed by the Gauss's law surface $q = \sigma A$

By gauss's law $\phi = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$ — (2)

From (1) & (2)

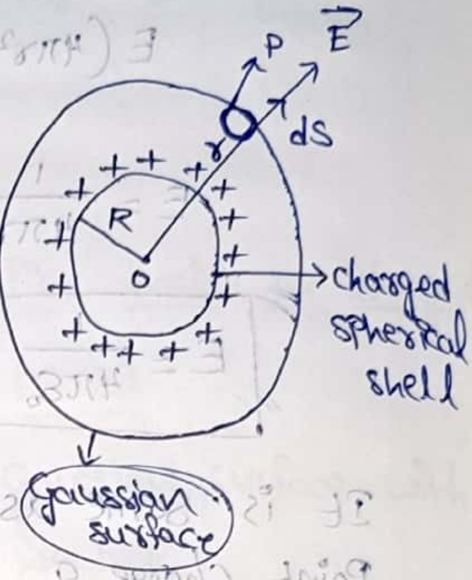
$$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{\therefore E = \frac{\sigma}{2\varepsilon_0}}$$

The field is uniform and does not depend on the distance from the ~~plane~~ sheet

③ Uniformly charged thin spherical shell

Consider a thin spherical shell of radius R and centre O. Let +q be the charge on the spherical shell. Let us find electric field at point 'p' distant r from the centre of the spherical shell.



case-(i): when point 'p' lies outside the spherical shell

Let 'p' be any point on the Gaussian surface. The electric field due to charged spherical shell is radical and spherically symmetric. At every point on the surface has same electric field and is normal to the surface.

The total flux through the Gaussian surface is given by

$$\phi = \oint_S E\,ds = E \oint_S ds = E\,4\pi r^2 \quad \text{--- (1)}$$

$[\because \theta = 0^\circ]$

For spherical surface.

From Gauss law,

$$\phi = \frac{q}{\varepsilon_0} \quad -(2)$$

From (1) & (2)

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi r^2}\frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}} \quad (\text{for } r>R)$$

It is same as that at distance r from a Point charge q.

In vector form $\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{r}}$ Here, $\hat{r}$ is a unit vector along $\overline{OP}$

Thus field outside a charged spherical shell can be obtained by assuming the entire charge to be concentrated at the centre of sphere.

<u>case-ii: when point P lies on the surface of spherical shell</u>

(1) The gaussian surface through point 'P' will just enclose the charged spherical shell.

Acc. to gauss's theorem,

$$E\, 4\pi R^2 = \frac{q}{\varepsilon_0}$$

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$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} \quad [\text{for } r = R]$$

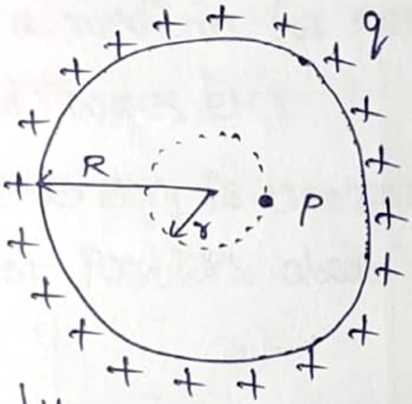
$$q = 4\pi R^2 \sigma \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{4\pi R^2 \sigma}{R^2}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

case-(iii): when point "p" lies inside the charged spherical shell ($r<R$)

consider a concentric Gaussian surface with radius $r<R$ as shown. we have

$$\oint_S \bar{E}\,ds = E.\,4\pi r^2$$



As 'p' is inside the shell, there is no charge enclosed by the Gaussian surface as charge residues only on the outer surface of the shell.

$$\oint_S E\,ds = \frac{q_{enclosed}}{\varepsilon_0} = 0$$

$$\boxed{\Rightarrow E = 0 \text{ inside the charged shell}}$$