PROBABILITY RULES – FULL BREAKDOWN

💥 ADDITION RULE OF PROBABILITY

This rule helps calculate the probability that **at least one of multiple events** occurs. Depending on whether events can happen together, the formula changes.

1. Mutually Exclusive Events

These are events that **cannot occur at the same time**. If one happens, the other can't. **Formula:** $P(A \cup B) = P(A) + P(B)$

Events A and B **cannot occur together**.

Example: Draw 1 card from a deck. - P(King) = 4/52 - P(Queen) = 4/52

Since they can't happen together: P(King or Queen) = 4/52 + 4/52 = 8/52 = 2/13 ≈ 0.1538

This means there is about a 15.38% chance of drawing either a King or a Queen.

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2. Not Mutually Exclusive Events

These events **can happen at the same time**, so we must subtract the overlap. **Formula:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example: Roll a die: - A = even number = {2, 4, 6} → P(A) = 3/6 - B = number > 3 = {4, 5, 6} → P(B) = 3/6 - A ∩ B = {4, 6} → P(A ∩ B) = 2/6

So: P(A or B) = 3/6 + 3/6 - 2/6 = 4/6 = 2/3 ≈ 0.6667

There is about a 66.67% chance of rolling either an even number or a number greater than 3.

💢 MULTIPLICATION RULE OF PROBABILITY

Used when you're calculating the chance of **two or more events happening together (AND)**. The method depends on whether the events affect each other.

1. Independent Events

Events where the outcome of one **does not affect** the outcome of the other. **Formula:** $P(A \cap B) = P(A) \times P(B)$

Example: Flip coin + roll die: - P(Heads) = 1/2 - P(6) = 1/6

P(Heads and 6) = 1/2 × 1/6 = 1/12 ≈ 0.0833

There is an 8.33% chance of flipping heads and rolling a six in a single try.

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2. Dependent Events

Events where the outcome of one **affects** the outcome of the other. **Formula:** $P(A \cap B) = P(A) \times P(B|A)$

Example: Draw 2 cards without replacement: - P(Ace first) = 4/52 - P(Ace second | Ace first) = 3/51

$P(\text{Both Aces}) = 4/52 \times 3/51 = 12/2652 = 1/221 \approx 0.0045$

There's about a 0.45% chance of drawing two Aces in a row without replacement.

BAYES' THEOREM

Bayes' Theorem is a powerful tool for **updating probabilities** after getting new evidence. It flips conditional probabilities.

Formula: $P(A|B) = [P(B|A) \times P(A)] / P(B)$

You update your belief about A after seeing B.

Example: Medical Test

- $P(\text{Disease}) = 1\% = 0.01$
- $P(\text{Pos}|\text{Disease}) = 0.99$
- $P(\text{Pos}|\neg\text{Disease}) = 0.01$

Want: $P(\text{Disease}|\text{Pos})$

First, compute P(Pos): $P(\text{Pos}) = 0.99 \times 0.01 + 0.01 \times 0.99 = 0.0099 + 0.0099 = 0.0198$

Then: $P(\text{Disease}|\text{Pos}) = (0.99 \times 0.01) / 0.0198 = 0.0099 / 0.0198 = 0.5$

So even though the test is 99% accurate, if the disease is rare, a positive result only gives you a 50% chance of actually having it. This shows the power of Bayes in avoiding false assumptions.

USE CASES:

Bayes and probability rules are used across AI, healthcare, law, and everyday decision-making. - Spam filters - Medical diagnostics - Machine Learning (Naive Bayes) - Law and evidence analysis

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CODE:

```
def bayes_theorem(prior_A, likelihood_B_given_A, likelihood_B_given_not_A):
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```
    """
    Calculate P(A|B) using Bayes' Theorem.

    Parameters:
    - prior_A: P(A) → prior probability of A
    - likelihood_B_given_A: P(B|A) → probability of B given A
    - likelihood_B_given_not_A: P(B|¬A) → probability of B given not A

    Returns:
    - posterior: P(A|B) → probability of A given B
    """
    prior_not_A = 1 - prior_A
    total_B = (likelihood_B_given_A * prior_A) + (likelihood_B_given_not_A * prior_not_A)
    posterior = (likelihood_B_given_A * prior_A) / total_B
    return posterior

# Example usage
p_disease = 0.01                  # P(Disease)
p_pos_given_disease = 0.99        # P(Pos | Disease)
p_pos_given_not_disease = 0.01    # P(Pos | No Disease)

posterior = bayes_theorem(p_disease, p_pos_given_disease, p_pos_given_not_disease)
print(f"P(Disease | Positive Test) = {posterior:.4f}")
```