Psychoakustik

4th lesson

Consonance/Dissonance

Pythagoras

- First concepts by Pythagoras (* around 570 BC, † after 510 BC).
  - Proportions of strings
  - Integer ratios
- Pythagorean tuning based on 3:2
- Pythagorean comma after 12 fifths

Physiological Studies

- First physiological studies since v. Helmholtz (1821 - 1894)
- Concept of roughness since Békésy (1899 - 1972)
- Studies by Terhardt and others show correlation of roughness and critical bandwidth.

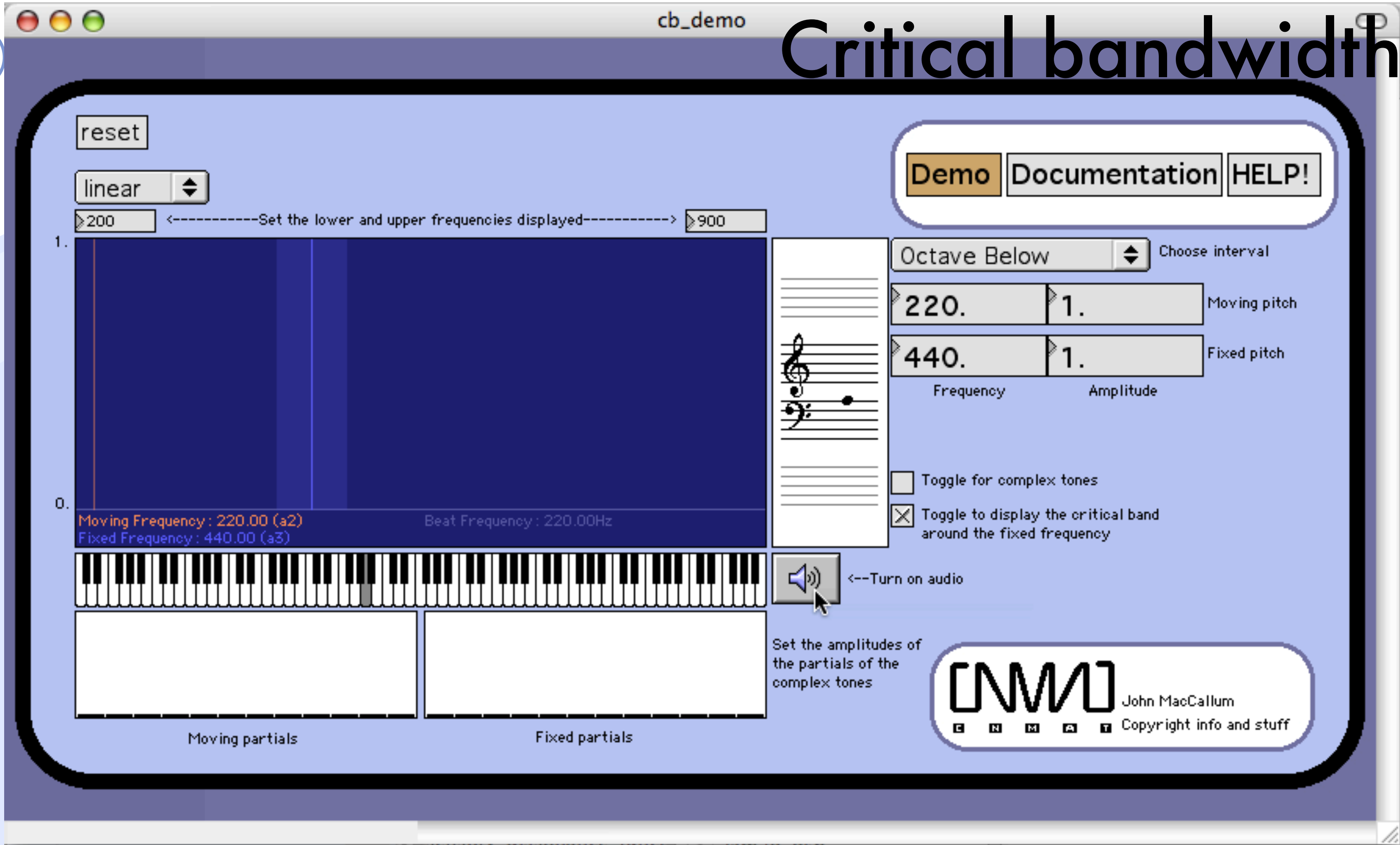
Definition of Dissonance

- In contrast to consonance, dissonance is a discordant harmony of
  - two or more notes that are perceived as rough or
  - tonally tense.
- The definition already reveals two different concepts (bottom-up, top-down)

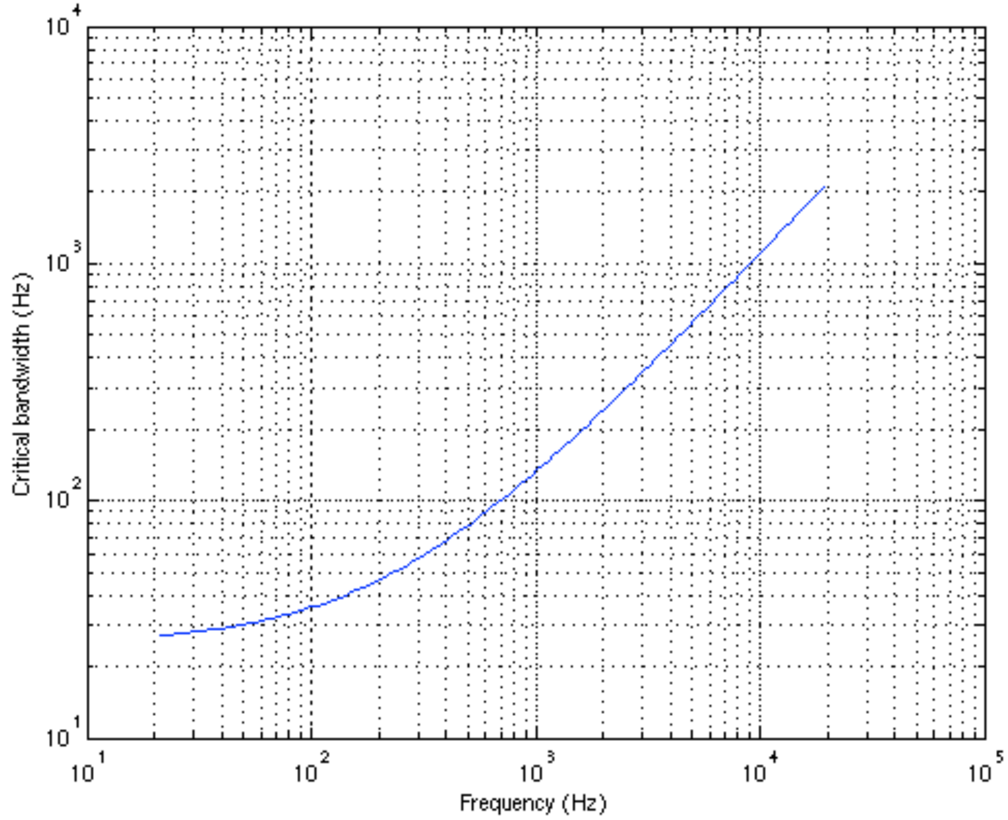
Basilar Membrane as Filter Bank

- Basilar membrane is organized tonotopically.
- Each hair cell has an optimal frequency response (http://en.wikipedia.org/wiki/ERB-scale).
- These are combined into banks of filters (critical bandwidth) within which it is difficult to distinguish two (sine) tones.
- The size of the critical bandwidth varies with the frequency (about octave in the low, 1/3 octave in the middle and again increasingly in the higher range).
- Bark: Constant area on the basilar membrane: 1.2 mm, or 140 hair cells: a total of 25 areas (Bark scale), related to Mel scale (doubling of pitch).

Critical bandwidth

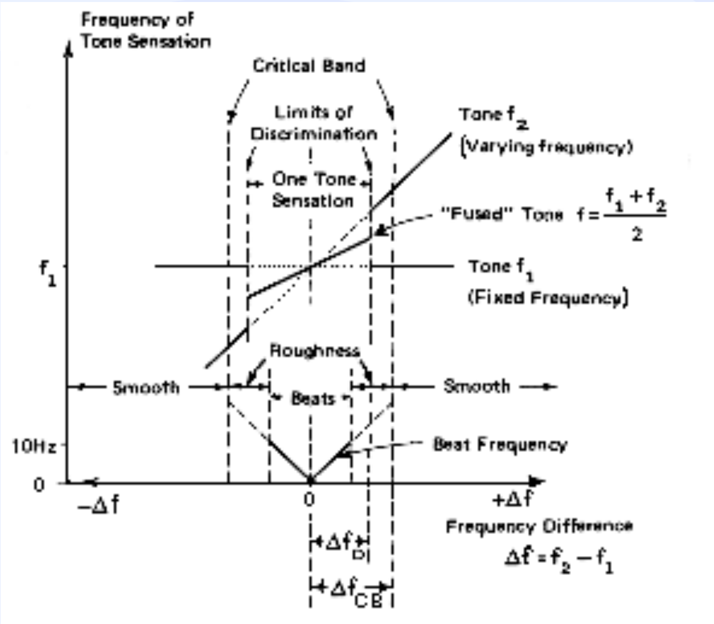


Critical bandwidth

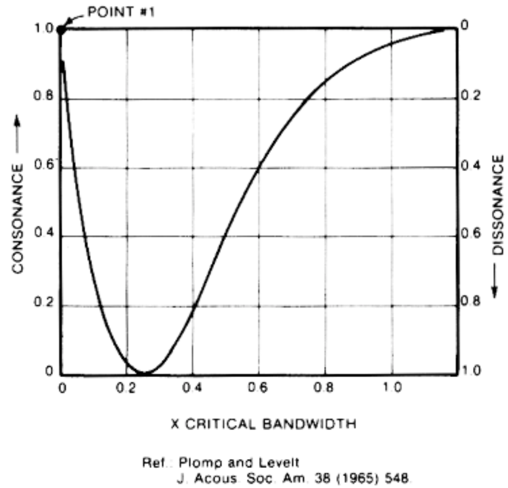


Critical bandwidth as a function of frequency

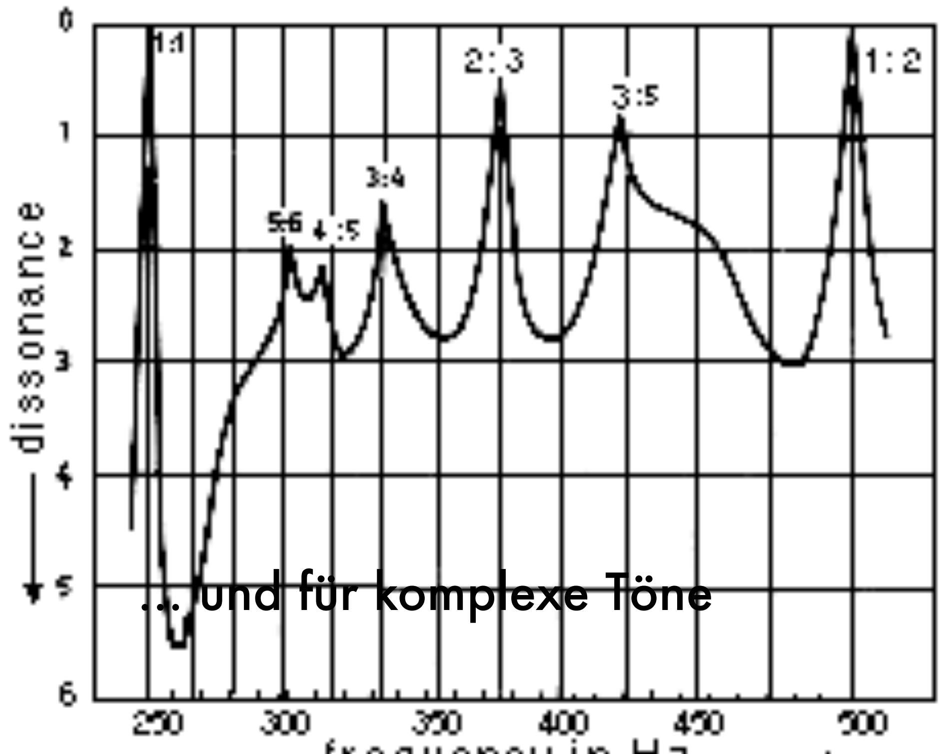
Konsonance/Dissonance



Perception of two tones within a critical bandwidth

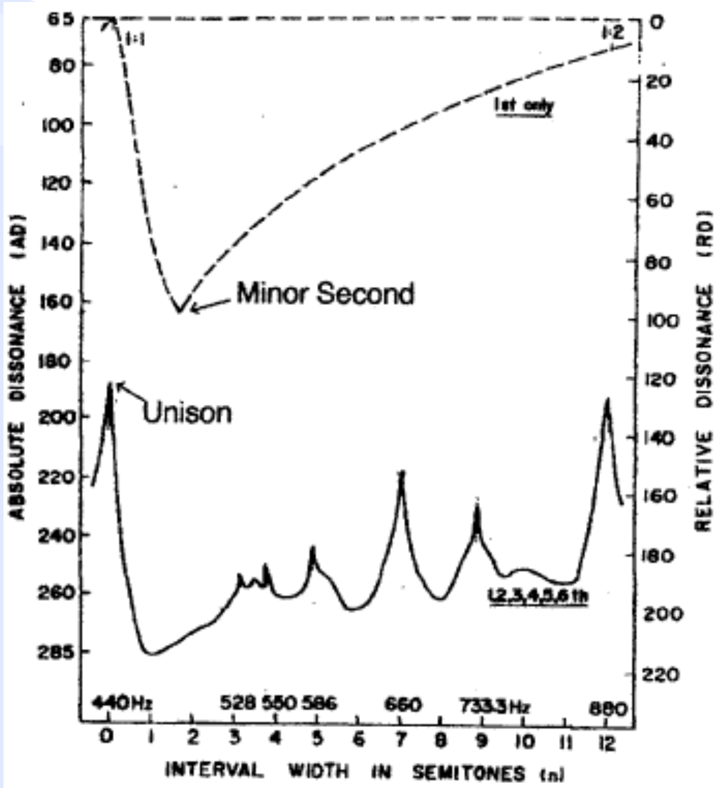


Calculation of maximum dissonance for simple tones according to Plomp and Levelt



... und für komplexe Töne

Kameoka & Kuriyagawa

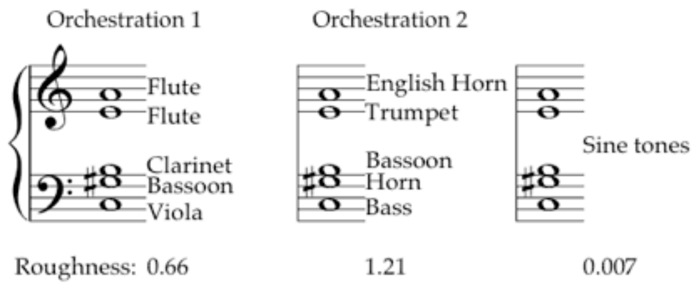


as well as similar results by Kameoka & Kuriyagawa (1969)

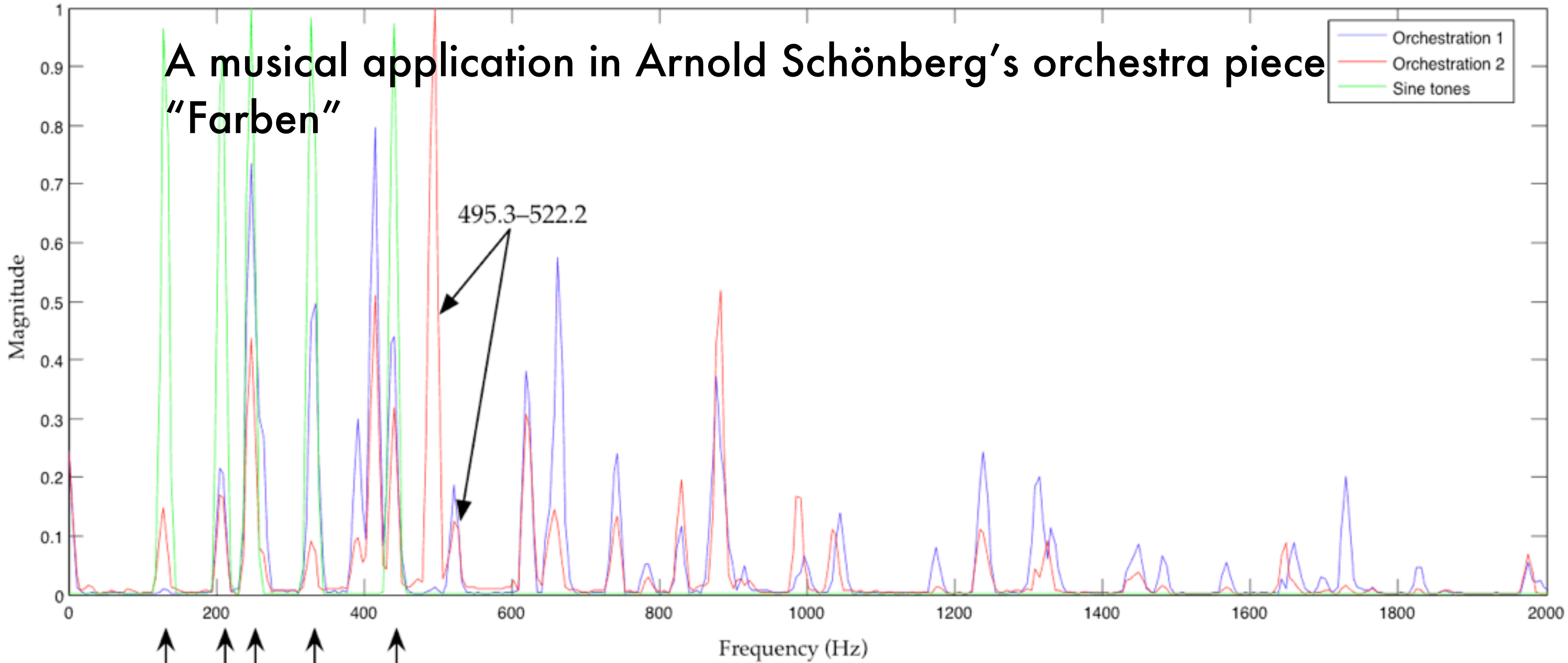
Musical Application

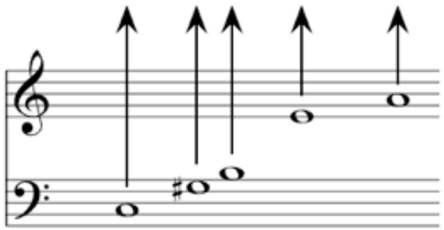
$$\rho = \sum_i \rho_i$$

Roughness values add up.



A musical application in Arnold Schönberg’s orchestra piece “Farben”





The “Farben” chord from Schoenberg’s *Fünf Orchesterstüke*, Op. 16 in its original orchestrations (blue and red) and sinusoids (green) showing differences in roughness estimates.

The Swiss mathematician Euler observed that the consonance (Gradus suavitatis = degree of sweetness) of an interval depends on the prime factorization of the numbers in the frequency ratio, with complex ratios being substituted by simpler ones.

- The Eulerian Gradus Suavitatis (short: G) is a function which evaluates the consonance of dyads - i.e. intervals.
- The interval of first i.e. highest degree is the unison: G (1/1) = 1
- The second-order interval is the octave: G (2/1) = G (1/2) = 2
- G is only defined for intervals represented by fractions. G can be determined calculated according to the following recipe:
  - First convert the interval (numerator / denominator) into the reduced fraction a / b.
  - Then determine the so-called product value a · b.
  - Now divide the product value into prime factors: a · b = p1 · p2 · p3 · .... · pn
  - Then calculate G as follows: G (numerator / denominator) = 1 + (p1 -1) + (p2 -1) + (p3 -1) + ... + (pn -1)

- FIRST EXAMPLE
- Numerator: denominator = 12/15 = 4/5
- Prime factorization: 4 · 5 = 2 · 2 · 5

- SECOND EXAMPLE
- Numerator: denominator = 2/18 = 1/9
- Prime factorization: 9 = 3 · 3

Der Komponist und Musiktheoretiker Clarence Barlow geht von Euler und Hindemith
The composer and music theorist Clarence Barlow starts from Euler and Hindemith and refines the Eulerian formula. He takes into account the cognitive-psychological properties of simple integers and their relationships. By introducing the term "polarity" Barlow is capable of explaining the “deviant” behavior of the fourth.

Indigestibility:

$$\xi(N) = 2 \sum_{r=1}^{\infty} \left\{ \frac{n_r (p_r - 1)^2}{p_r} \right\} \quad \text{where} \quad N = \prod_{r=1}^{\infty} p_r^{\,nr},\ p \text{ is a prime, and } n \text{ is a natural number.} \qquad (1)$$

Harmonic Consonance ("Harmonicity"):

$$h(P,Q) = \frac{sgn[\xi(P) - \xi(Q)]}{\xi(P) + \xi(Q) - 2\xi(hcf_{P,Q})} \qquad (2)$$

where $sgn(x) = -1$ when x is negative, otherwise $sgn(x) = +1$,

$hcf_{a,b}$ is the highest common factor of a and b, and $\xi(x)$ is indigestibility of x

N	□(N)
1	0.000000
2	1.000000
3	2.666667
4	2.000000
5	6.400000
6	3.666667
7	10.285714
8	3.000000
9	5.333333
10	7.400000
11	18.181818
12	4.666667
13	22.153846
14	11.285714
15	9.066667
16	4.000000

Unverdaulichkeit einer ganzen Zahl

Barlow's formulas yield a measure for the stability of musical intervals: "harmonic energy". It uses a principle from natural sciences, according to which a system is most stable when it has low energy. Strong intervals are marked in the lower graph by valleys with a certain depth and extent. The peaks between the valleys are called categorical boundaries. Generally, we distinguish 12 pitch categories or interval classes. The boundaries are not fixed, but are determined by the direction through which one approaches them (analogy to colors).