Q:1: $C(x,y) = e^x + \sin y - 4x - y$

Sol:

$C_x = e^x - 4$

$C_y = \cos y - 1$

So. $e^x - 4 = 0$, $\cos y - 1 = 0$

$e^x = 4$, $\cos y = 1$

$x = \ln 4$, $y = \cos^{-1}(1)$

$x = \ln 4$ $y = 0$

$(x,y) = (\ln 4, 0)$

$C_{xx} = e^x$, $C_{yy} = -\sin y$, $C_{xy} = 0$

$\therefore D = f_{xx}(x_0,y_0) \cdot f_{yy}(x_0,y_0) - f_{xy}^2(x_0,y_0)$

$D = (e^{\ln 4})(0) - 0$

$D = 0$

inconclusive

Q:2: $M(a,b) = 100a + 150b - 2a^2 - 3b^2 - ab$

Sol:

$M_a = 100 - 4a - b$ — (1)

$M_b = 150 - 6b - a$ — (2)

$100 - 4a - b = 0$

$-b = -100 + 4a$

$b = 100 - 4a$

$150 - 6(100 - 4a) - a = 0$

$150 - 600 + 24a - a = 0$

$-450 + 23a = 0$

$a = \frac{450}{\cancel{25}\ 23}$

Date________ Day________

$b = 100 - 4\left(\frac{450}{23}\right)$

$\Rightarrow b = \frac{2300 - 1800}{23} = \frac{500}{23}$.

$M_{aa} = -4$

$M_{bb} = -6$.

$M_{ab} = -1$.

$\Rightarrow D = (-4)(-6) - (-1)$

$= 24 + 1$

$= 25$.

$D > 0$ and $f_{xx} < 0$

relative maximum.

$x \longrightarrow \alpha$

Q:3:- $C(m, n) = 3m^2 + 2mn + 4n^2$. to

$g \Rightarrow m^2 + n^2 + mn = 400$.

Sol:-

$\langle 6m + 2n, 2m + 8n \rangle = \lambda \nabla g$

$= \lambda \langle 2m + n, 2n + m \rangle$

$6m + 2n = \lambda(2m + n)$, $2m + 8n = \lambda(2n + m)$

$\lambda = \frac{6m + 2n}{2m + n}$, $\lambda = \frac{2m + 8n}{2n + m}$.

$\frac{6m + 2n}{2m + n} = \frac{2m + 8n}{2n + m}$

$\frac{\cancel{2}(3m + n)}{2m + n} = \frac{\cancel{2}(m + 4n)}{2n + m}$

$$(3m+n)(2n+m) = (2m+n)(m+4n).$$

$$6mn + 3m^2 + 2n^2 + nm = 2m^2 + 8mn + mn + 4n^2$$

$$7mn + 3m^2 + 2n^2 - 2m^2 - 9mn - 4n^2 = 0.$$

$$\cancel{m^2 - 2n^2 - 2mn = 0}$$

$$\cancel{m^2 - 2mn + mn - 2n^2 = 0}$$

$$\cancel{m(m-2n) + n}$$

$$m^2 - 2n^2 - 2mn = 0$$

$$a = 1 \quad c = 2n^2, \quad b = -2n$$

$$m = \frac{+2n \pm \sqrt{(2n)^2 + 4(1)(2n^2)}}{2(1)}$$

$$= \frac{\pm 2n \pm \sqrt{4n^2 + 8n^2}}{2}$$

$$m = \frac{+2n \pm 2n\sqrt{3}}{2}$$

$$m = n \pm n\sqrt{3}$$

Put m in constraint.

$$(n+n\sqrt{3})^2 + \cancel{(n+n)}\, n^2 + ((n+n\sqrt{3})n) = 400$$

$$n^2 + 2\sqrt{3}n^2 + 3n^2 + n^2 + n^2 + \sqrt{3}n^2 = 400.$$

$$6n^2 + 3\sqrt{3}n^2 = 400.$$

$$\cancel{3n^2(2+\sqrt{3}) = 400}$$

$$11.196\, n^2 = 400$$

$$n^2 = \frac{400}{11.196}$$

$$n^2 = 35.727$$

$$n = \pm 5.977.$$

$$m = n(1+\sqrt{3}) \Rightarrow m = 5.97(1+\sqrt{3})$$

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$$m = 16.310$$

$(m, n) = (\pm 16.31, \pm 5.977)$

For $m = n - n\sqrt{3}$.

$m = n(1-\sqrt{3})$.

$(n - n\sqrt{3})^2 + n^2 + (n - n\sqrt{3})n = 400$.

$n^2 - 2\sqrt{3}n^2 + 3n^2 + n^2 + n^2 - \sqrt{3}n^2 = 400$

$6n^2 - 3\sqrt{3}n^2 = 400$

$n^2 = \frac{400}{0.803}$

$n^2 = 498.132$

$n = \pm 22.318$.

$m = n(1-\sqrt{3})$.

$m = 22.318(-0.732)$

$m = \pm 16.337$.

$(m, n) = (\pm 16.337, \pm 22.318)$

Function is max. at $(\pm 16.337, \pm 22.318)$ and min at $(\pm 16.31, \pm 5.977)$.

Date______________ Day______________

Q:4. $E(p,q,r) = p^2 + 4q^2 + 2r^2$.
to $p^2 + q^2 + r^2 = 250,000$

Sol:

$\nabla E(p,q,r) = \lambda \nabla g(p,q,r)$

$E_p = 2p + 0 + 0$.

$E_q = 8q$, $E_r = 4r$.

$g_p = 2p$, $g_q = 2q$, $g_r = 2r$.

$\langle 2p, 8q, 4r \rangle = \lambda \langle 2p, 2q, 2r \rangle$

$\cancel{2p} = \lambda \cancel{2p} \Rightarrow \lambda = 1$, $p \neq 0$.

$4\cancel{8q} = \lambda \cancel{2q} \Rightarrow \lambda = 4$, $q \neq 0$.

$2\cancel{4r} = \lambda \cancel{2r} \Rightarrow \lambda = 2$, $r \neq 0$.

Case: 1 $p \neq 0$ q & $r = 0$.

$\Rightarrow p^2 + 0^2 + 0^2 = 250,000$

$\sqrt{p^2} = \sqrt{250,000}$

$p = \pm 500$

Case: 2 $q \neq 0$ p & $r = 0$

$\Rightarrow 0^2 + q^2 + 0^2 = 250,000$

$\sqrt{q^2} = \sqrt{250000}$

$q = \pm 500$.

Case: 3 $r \neq 0$ p & $q = 0$.

$0^2 + 0^2 + r^2 = \sqrt{250000}$

$r = \pm 500$

$(p,q,r) = (\pm 500, 0, 0), (0, \pm 500, 0), (0, 0, \pm 500)$

$E(p,q,r)_1 = 250,000$

$E(p,q,r)_2 = 1000000$ Max at $(0, \pm 500, 0)$

$E(p,q,r)_3 = 500,000$

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Date______________ Day______________

Q:5 $\theta_0 = 1$, $\theta_0 = 1$ $\alpha = 0.1$

$$J(\theta_1, \theta_2) = \sin(\theta_1) + \theta_2^2 + \theta_1\theta_2.$$

Sol:

$$\nabla J(\theta_1, \theta_2) = \begin{bmatrix} \cos(\theta_1) + \theta_2 \\ 2\theta_2 + \theta_1 \end{bmatrix}.$$

$$\because \cancel{\theta = \theta_1}\ x_1 = x_0 - \alpha f(x_0)$$

$$\theta_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 0.1 \begin{bmatrix} \cos(1) + 1 \\ 2(1) + 1 \end{bmatrix}$$

$$\theta_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 0.1 \begin{bmatrix} 1.54 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.154 \\ 0.3 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1' \\ \theta_2' \end{bmatrix}. = \begin{bmatrix} 0.846 \\ 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.846 \\ 0.7 \end{bmatrix} - 0.1 \begin{bmatrix} \cos(0.846) + 1 \\ 2(0.7) + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.846 \\ 0.7 \end{bmatrix} - \begin{bmatrix} 0.166 \\ 0.24 \end{bmatrix}$$

$$= \begin{bmatrix} 0.680 \\ 0.46 \end{bmatrix}$$

Date ______ Day ______

Q6: $R(f,g,h) = f^2 + 2g^2 + 3h^2 + fg - 5f - 7g - 9h + 30$

$\alpha = 0.05 \qquad (f_0, g_0, h_0) = (1,1,1).$

Sol:

$$\nabla R(f,g,h) = \begin{bmatrix} 2f + g - 5 \\ 4g + f - 7 \\ 6h - 9 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 0.05 \begin{bmatrix} 2(1) + (1) - 5 \\ 4(1) + (1) - 7 \\ 6(1) - 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 0.05 \begin{bmatrix} -2 \\ -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.1 \\ 1.15 \end{bmatrix}.$$

$$R_2 = \begin{bmatrix} 1.1 \\ 1.1 \\ 1.15 \end{bmatrix} - 0.05 \begin{bmatrix} 2(1.1) + (1.1) - 5 \\ 4(1.1) + (1.1) - 7 \\ 6(1.15) - 9 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1.185 \\ 1.175 \\ 1.225 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 1.185 \\ 1.175 \\ 1.225 \end{bmatrix} - 0.05 \begin{bmatrix} 2(1.185) + (1.175) - 5 \\ 4(1.175) + (1.185) - 7 \\ 2(1.225) - 9 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.225 \\ 1.230 \\ 1.3075 \end{bmatrix}$$

Date______ Day______

Q. 7. $J(\theta_1, \theta_2, \theta_3) = e^{\theta_1} + \theta_2^2 + 4\theta_3^2 + \theta_1\theta_2$

sol.

$J_{\theta_1} = e^{\theta_1} + \theta_2$

$J_{\theta_2} = 2\theta_2 + \theta_1$

$J_{\theta_3} = 8\theta_3$

$\theta_3 = 0$

$2\theta_2 + \theta_1 = 0$

$\theta_1 = 2\theta_2$

$e^{\theta_1} + \theta_2 = 0$

$e^{\theta_1} \neq \frac{\theta_1}{2}$ Not possible.

No critical points.

$$H = \begin{bmatrix} e^{\theta_1} & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$D_1 = e^{\theta_1} > 0$ & $D_2 = 8(2e^{\theta_1} - 1)$

$2e^{\theta_1} - 1 > 0$

$e^{\theta_1} > 0.5$

Positive for $\theta_1 > \ln(0.5) = -0.693$

x ——— x

Date ______ Day ______

Q-8 $T(x,y) = xy - x^2$.

Sol:

$= \iint_{[D]} xy - x^2 \, dA$.

$= \int_1^3 \int_0^1 (xy - x^2) \, dx\,dy$

$= \int_1^3 \left(\frac{x^2 y}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1 \right) dy$.

$= \int_1^3 \left(\frac{y}{2} - 0 - \frac{1}{3} + 0 \right) dy$

$\frac{y^2}{4}\Big|_1^3 - \frac{y}{3}\Big|_1^3$

$9/4 - 1/4 - 1 + 1/3$.

$2 - 1 + 1/3$.

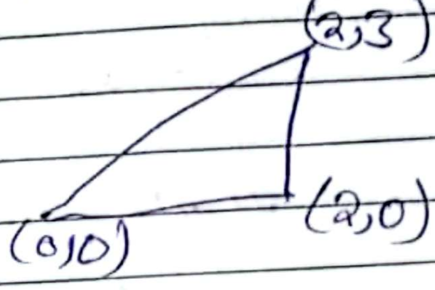
$4/3$

Date________ Day________

Q.4.

Sol:

Vertices = $(0,0), (2,0), (2,3)$



$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow m = \frac{3-0}{2-0} = 3/2.$

$\because y = mx.$

$\Rightarrow y = 3/2\, x.$

$\because z = 10 - x^2 - y^2.$

$\Rightarrow \int_0^2 \int_0^{3/2 x} (10 - x^2 - y^2)\, dy\, dx$

$= \int_0^2 \left[10y - x^2 y - \frac{y^3}{3}\right]_0^{3/2 x} dx.$

$= \int_0^2 \left[5x - \frac{3}{2}x^3 - \frac{9}{8}x^3\right] dx.$

$= \left[\frac{5x^2}{2} - 3/2\, x^3 - 9/8\, x^3\right]_0^2$

$= (10 - 12 - 9) - 0$

$= \cancel{6} - 11$

Date ____________ Day ____________

Q:-10 (a) $f(x,y,z) = x^2 + y^2 + z^2$. $P(1, 2, 5)$.

(b) $f_1(x,y,z) = x^2 + y^2 + z^2 - 9 = 0$

$f_2(x,y,z) = x + 2y - 3z + 4 = 0$

$P(2, -1, 1)$

(a) Soln

$\nabla f_x = 2x = 2$. at $(1,2,5)$

$\nabla f_y = 2y = 4$ at $(1,2,5)$.

$\nabla f_z = -1 = -1$ at $(1,2,5)$

$\nabla f = f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f(x_0,y_0,z_0)(z-z_0) = 0$.

$\Rightarrow 2(x-1) + 4(y-2) - 1(z-5) = 0$.

$\Rightarrow 2x + 4y - z = 5$.

For normal.

$x = x + \nabla f_x t$, $y = y_0 + \nabla f_y t$, $z = z_0 + \nabla f_z t$

$x = 1 + 2t$, $y = 2 + 4t$, $z = 5 - t$.

(b) Soln

$\theta = \cos^{-1}\left(\frac{N_1 \cdot N_2}{|N_1| \cdot |N_2|}\right)$ at $(2,-1,1)$

$N_1 = \nabla f_1 = \langle 2x, 2y, 2z \rangle = \langle 4, -2, 2 \rangle$

$N_2 = \nabla f_2 = \langle 1, 2, -3 \rangle$

$|N_1| = \sqrt{24}$, $|N_2| = \sqrt{14}$

Date______________ Day______________

$$\theta_{\phi} = \cos^{-1}\left(\frac{(4i-2j+2k)\cdot(i+2j-3k)}{\sqrt{24}\cdot\sqrt{14}}\right).$$

$$\theta = \cos^{-1}\left(\frac{\cancel{4}-\cancel{4}-6}{2\sqrt{2}\cdot\sqrt{7}\cdot}\right)$$

$$\theta = 109.106\cdot$$

$$\alpha \longrightarrow$$