# Mathematical Foundations for Data Science

Bulls and Cows Game with Entropy

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# **Bulls and Cows Game Documentation**

#### Overview

The Bulls and Cows game is a number-based puzzle game where the player attempts to guess a 4-digit secret code. The code is generated with unique digits, and after each guess, the game provides feedback in the form of bulls and cows:

- Bulls: The number of digits that are correct in both value and position.
- Cows: The number of digits that are correct in value but incorrect in position.

The game continues until the player guesses the correct code, and the program calculates the entropy (a measure of uncertainty) and mutual information (a measure of how much information each guess provides) after every guess.

# **Game Flow**

#### 1. Secret Code Generation:

- A 4-digit secret code is randomly generated with unique digits.
- The code is not revealed to the player.

#### 2. Player's Guess:

- The player enters a guess consisting of 4 unique digits.
- The game checks if the guess is valid (4 digits, all unique).

#### 3. Feedback:

After each guess, the game calculates:

- Bulls: How many digits are in the correct position.
- Cows: How many digits are correct but in the wrong position.
- The game displays the number of bulls and cows.
- **4. Filtering Possible Codes:** After each guess, the program filters out the remaining possible secret codes based on the bulls and cows feedback, narrowing down the possible solutions.

#### 5. Game Completion:

- The game continues until the player correctly guesses the secret code (4 bulls).
- The number of guesses is displayed upon a correct guess.

#### 6. Entropy and Mutual Information:

- Entropy: Measures the uncertainty of the remaining possible codes after each guess. It is calculated based on the number of remaining possible codes.
- Mutual Information: Measures how much information each guess provides in reducing the uncertainty about the secret code.

## **Functions and Descriptions**

- generate secret code():
  - This function generates a random 4-digit secret code with unique digits.
  - It uses the random.shuffle() method to shuffle a list of digits and selects the first four digits to form the secret code.
- 2. calculate bulls and cows(secret, guess):
  - This function compares the secret code and the player's guess.
  - It returns two values: the number of bulls (correct digit and position) and cows (correct digit but wrong position).
- 3. calculate entropy (remaining codes):
  - This function calculates the Shannon entropy, which measures the uncertainty of the remaining possible codes.
  - It uses the formula for entropy, which is  $H(X) = -\Sigma p(x) \log_2 p(x)$  where p(x) is the probability of each possible outcome.
- 4. filter possible codes (possible codes, guess, bulls, cows):
  - This function filters the remaining possible codes based on the guess and the bulls and cows feedback.
  - It eliminates codes that do not match the feedback.
- calculate mutual information (possible codes, guesses):
  - This function calculates the mutual information between the guesses and the possible outcomes (bulls and cows).
  - The formula used for mutual information is I(X; Y) = H(Y) H(Y|X), where H(Y) is the entropy of the outcomes and H(Y|X) is the conditional entropy given the guesses.

# **Key Concepts**

#### Bulls and Cows:

• The game provides feedback in terms of bulls (correct digits in the right position) and cows (correct digits in the wrong position). These clues are essential for narrowing down the possible secret codes.

### • Entropy:

• Entropy is a measure of uncertainty or disorder. In this context, it is used to gauge how many possible secret codes remain after each guess. A high entropy value means there are many possible codes remaining, while a low entropy means the number of possibilities has been narrowed down.

#### Mutual Information:

• Mutual information measures how much information a guess provides in terms of reducing uncertainty about the secret code. It helps assess which guesses will be most useful in solving the puzzle.

