# Bulls and Cows Game with Entropy and Mutual Information

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# Introduction

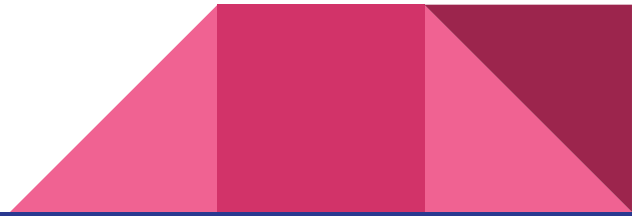
**Title**: Bulls and Cows Game: Using Entropy and Mutual Information for Optimal Strategy

**Objective:**

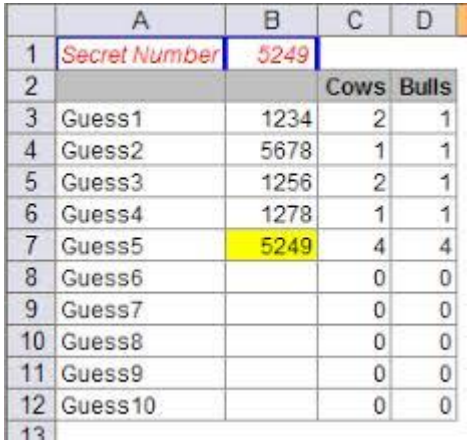
- To explain the Bulls and Cows game and how theoretical concepts like entropy and mutual information can be applied to enhance the game's guessing strategy.
- Demonstrate how these concepts can improve the speed and accuracy of guessing the secret code.

# Overview of the Bulls and Cows Game

- **Game Description:**
  - Bulls and Cows is a simple guessing game where the objective is to guess a 4-digit secret code.
  - The secret code contains 4 unique digits, and the player has to guess what those digits are.
  - The system gives feedback in the form of Bulls and Cows:
    - Bulls: The number of correct digits in the correct position.
    - Cows: The number of correct digits, but in the wrong position.
- **Example:**
  - Secret Code: 1592
  - Player Guess: 1235
  - Feedback: 1 Bull, 2 Cows (The number 1 is in the correct position, and 2 and 5 are correct digits but in the wrong places).



# Generating the Secret Code



| | A | B | C | D |
|---|---|---|---|---|
| 1 | Secret Number | 5249 | | |
| 2 | | | Cows | Bulls |
| 3 | Guess1 | 1234 | 2 | 1 |
| 4 | Guess2 | 5678 | 1 | 1 |
| 5 | Guess3 | 1256 | 2 | 1 |
| 6 | Guess4 | 1278 | 1 | 1 |
| 7 | Guess5 | 5249 | 4 | 4 |
| 8 | Guess6 | | 0 | 0 |
| 9 | Guess7 | | 0 | 0 |
| 10 | Guess8 | | 0 | 0 |
| 11 | Guess9 | | 0 | 0 |
| 12 | Guess10 | | 0 | 0 |
| 13 | | | | |

- **Function Used:** `generate_secret_code()`
  - This function randomly generates a secret code.
  - How it works:
    - A list of numbers (0-9) is created.
    - The numbers are shuffled randomly.
    - The first 4 digits are selected to form the code.
    - This ensures that the code has unique digits.
- **Why This is Important:**
  - The uniqueness of the digits makes the game challenging, as players have to guess not just the digits but also their correct positions.

# Bulls and Cows Calculation


**Function Used:** `calculate_bulls_and_cows(secret, guess)`

- This function calculates the number of bulls and cows for each guess.
- Bulls are the number of digits in the correct position.
- Cows are the number of correct digits but in the wrong position.

**Example:**

- Secret Code: 1592
- Guess: 1235
- Bulls: 1 (The digit 1 is in the correct position).
- Cows: 2 (The digits 2 and 5 are correct but not in the correct positions).

**Why this is Important:**

- These values guide the player to adjust their next guess based on which digits are correct and where they should be placed.

# Theoretical Background: Entropy

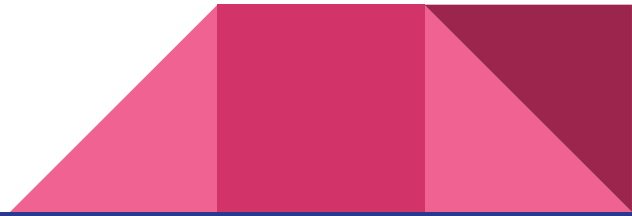
- What is Entropy?
  - Entropy is a concept from information theory that measures the uncertainty or randomness in a system.
  - In this game, entropy measures how uncertain we are about the secret code after each guess.
- Entropy Formula:
  - $H(Y)=-\sum_{x\in X} P(x)log2(p(x))$
  - higher entropy means more uncertainty (more possible codes left).
  - A lower entropy means less uncertainty (fewer codes left).
- Why is Entropy Important?
  - It helps the system understand how many possible secret codes remain after each guess.
  - It enables efficient decision-making, narrowing down the options faster.

## Entropy in Action

- Example of Entropy Calculation:
  - After several guesses, we may have 100 possible codes remaining.
  - The entropy is calculated as:
  - $H(Y) = \log_2(100) \approx 6.64 \text{ bits}$
  - This means there is 6.64 bits of uncertainty in the game.

# Filtering Possible Codes

- **Function Used:** `filter_possible_codes(possible_codes, guess, bulls, cows)`
  - After each guess, the system checks which codes still fit the bulls and cows feedback.
  - Codes that don't match the feedback are filtered out, leaving only the codes that are still possible.
- **Why This is Useful:**
  - This filtering reduces the number of possible secret codes, helping the player narrow down their guesses.
  - As the game progresses, the number of possible codes shrinks, and the game becomes easier.
- **Example:**
  - If a guess of "1234" results in 1 bull and 2 cows, all codes that don’t give the same feedback are eliminated from the list of possible codes.

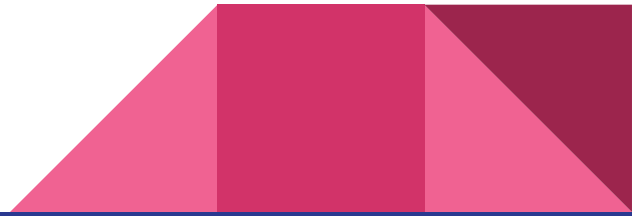


# Theoretical Background: Mutual Information

- **What is Mutual Information?**
  - Mutual Information (MI) measures how much knowledge about one variable reduces uncertainty about another.
  - In the game, MI measures how much knowing the outcome (bulls and cows) of a guess helps us learn about the secret code.
- **Formula:**
- $I(X;Y)=H(Y)-H(Y|X)$
  - H(Y): The entropy of possible outcomes (bulls and cows).
  - H(Y|X): The conditional entropy (uncertainty about outcomes after a guess).
- **Why is Mutual Information Important?**
  - It helps us choose guesses that give the most information about the secret code.
  - By maximizing mutual information, we can make guesses that eliminate the largest number of possible codes.

# Mutual Information in Practice

- **How Mutual Information Improves Guessing:**
  - By calculating the mutual information for each possible guess, we can choose the guess that reduces uncertainty the most.
  - This results in fewer guesses and faster game completion.
- **Example:**
  - For a given set of possible codes and guesses, calculating the MI helps us understand which guess maximizes the reduction in the number of remaining possible codes.



# Practical Application: How Entropy and Mutual Information Help in the Game

**Game Strategy Using Entropy:**

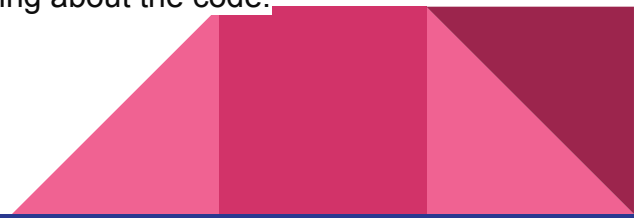
- After each guess, entropy is calculated to assess how many codes are still possible.
- The lower the entropy, the more certain we are about the secret code.

**Game Strategy Using Mutual Information:**

- Mutual information guides us to choose the next guess that gives us the most information, reducing the possible codes quickly.

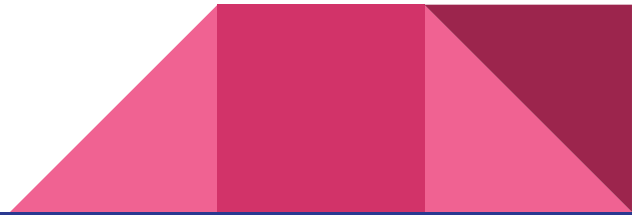
**Example:**

- After a guess with feedback of 2 bulls and 1 cow, entropy helps us filter down to fewer possible codes.
- Mutual information helps us select the next guess that will maximize our learning about the code.



# Game Flow and Interaction

1. **Start the Game:** The system generates a secret 4-digit code.
2. **Player Makes a Guess:** Player inputs a guess.
3. **Calculate Bulls and Cows:** The system provides feedback based on the guess.
4. **Filter Possible Codes:** Filter out invalid codes based on feedback.
5. **Calculate Entropy:** Evaluate how uncertain we are about the remaining codes.
6. **Calculate Mutual Information:** Evaluate which next guess provides the most useful information.
7. **Repeat:** Continue until the player guesses the code.

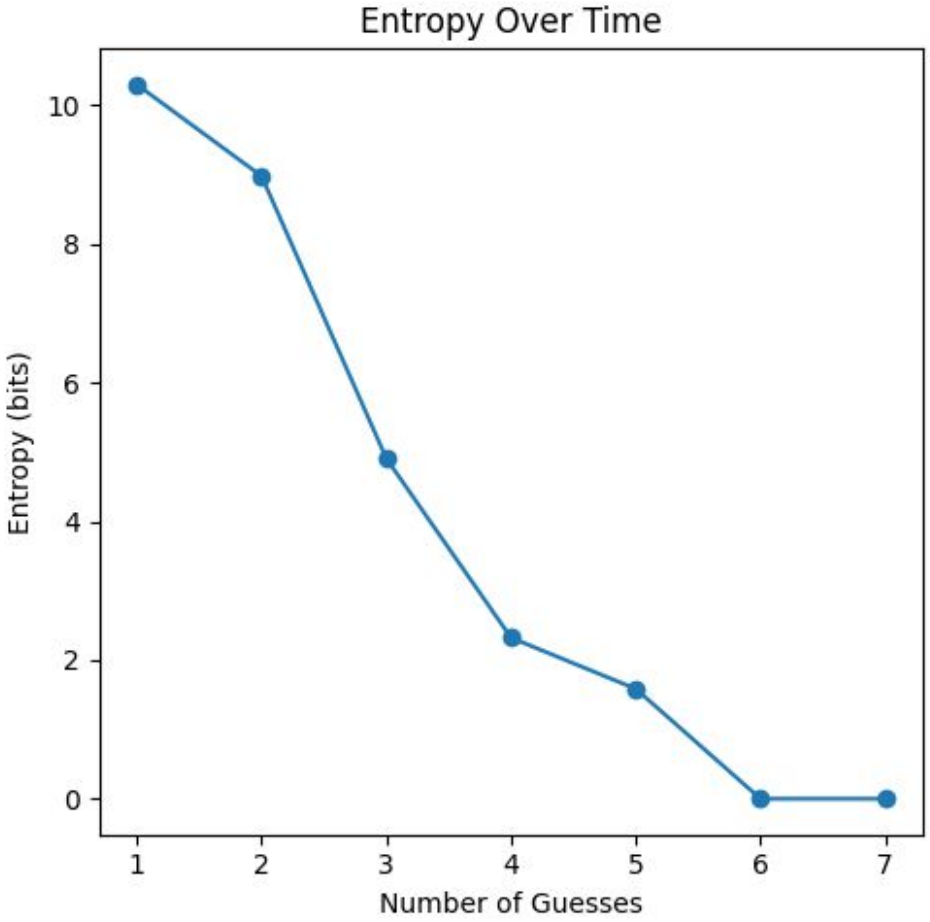


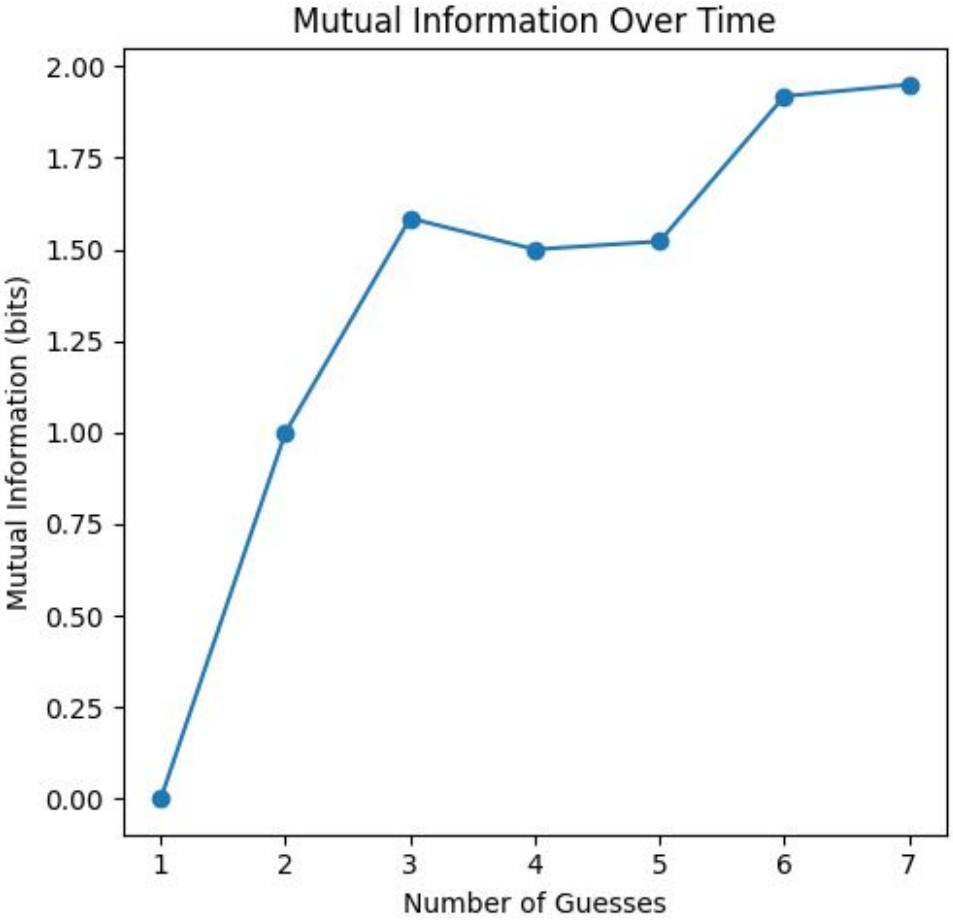
## Practical Results from Applying Entropy and Mutual Information

- **Efficiency:** The game solving process is faster because we use entropy and mutual information to guide our guesses.
- **Accuracy:** The concepts help us eliminate incorrect guesses and narrow down the possibilities quicker.

**Visual Aid:**

1. **Entropy Graph:**
   - Starts with a high value at the beginning of the game (indicating maximum uncertainty).
   - The line decreases over time as guesses help eliminate possible codes.
   - The graph becomes flatter towards the end as fewer possible solutions exist.
2. **Mutual Information Graph:**
   - Starts with a low value (since the guesses give little information initially).
   - The line increases as guesses narrow down the possibilities, each providing more information.
   - The graph becomes steeper towards the end as each guess helps significantly reduce the remaining possibilities.


Entropy Over Time
Entropy (bits)
0
2
4
6
8
10
1
2
3
4
5
6
7
Number of Guesses


Mutual Information Over Time
Mutual Information (bits)
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
1
2
3
4
5
6
7
Number of Guesses

# Conclusion

- **Summary:**
  - Bulls and Cows is a fun and challenging game that benefits from applying information theory concepts.
  - By using entropy, we reduce uncertainty about the secret code.
  - Mutual information helps us choose the most informative guesses, speeding up the game-solving process.
- **Key Takeaways:**
  - Entropy and mutual information are powerful tools in games and real-world scenarios, such as data compression, cryptography, and machine learning.