Assignment-04

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Download all python codes from

https://github.com/MVKKanth/Assignment4.git

and latex-tikz codes from

https://github.com/MVKKanth/Assignment4.git

Question taken from

https://github.com/gadepall/ncert/blob/main/linalg/linear_forms/gvv_ncert_linear_forms.pdf

1 Linear Forms Exercise 2.5(e)

Find points on the curve

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{25} \end{pmatrix} \mathbf{x} = 1 \tag{1.0.1}$$

at which the tangents are a)parallel to x-axis b)parallel to y-axis

2 Solution

Given curve,

$$\mathbf{x}^T \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{25} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

where,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{25} \end{pmatrix}, \mathbf{V}^{-1} = \begin{pmatrix} 4 & 0 \\ 0 & 25 \end{pmatrix} \mathbf{u} = 0, f = -1 \tag{2.0.2}$$

$$\because |\mathbf{V}| > 0 \tag{2.0.3}$$

given curve (2.0.1) is ellipse. For an ellipse, the point of contact for the tangent is

$$\mathbf{q} = \mathbf{V}^{-1}(\kappa\mathbf{n} - \mathbf{u}) \tag{2.0.4}$$
$$= \mathbf{V}^{-1}\kappa\mathbf{n} \qquad (\because \mathbf{u} = 0). \tag{2.0.5}$$

where,

$$\kappa = \pm\sqrt{\frac{\mathbf{u^T V^{-1} u} - f}{\mathbf{n^T V^{-1} n}}} \tag{2.0.6}$$
$$= \pm\sqrt{\frac{-f}{\mathbf{n^T V^{-1} n}}} \qquad (\because \mathbf{u} = 0) \tag{2.0.7}$$

1) For the tangents parallel to x-axis, then direction and normal vectors are,

$$\mathbf{m_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{n_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \kappa_1 = \pm\sqrt{\frac{-f}{\mathbf{n_1^T V^{-1} n_1}}} \tag{2.0.8}$$
$$= \pm\frac{1}{5} \tag{2.0.9}$$

$\therefore$ By Substituting $\kappa_1, \mathbf{n_1}, \mathbf{V}^{-1}$ in (2.0.5)

$$\mathbf{q} = \mathbf{V}^{-1}\kappa_1\mathbf{n_1} \tag{2.0.10}$$
$$= \begin{pmatrix} 0 \\ \pm 5 \end{pmatrix} \tag{2.0.11}$$

2) For the tangents parallel to the y-axis, the direction and normal vectors are

$$\mathbf{m_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \kappa_2 = \pm\sqrt{\frac{-f}{\mathbf{n_2^T V^{-1} n_2}}} \tag{2.0.12}$$
$$= \pm\frac{1}{2} \tag{2.0.13}$$

$\therefore$ substituting $\kappa_2, \mathbf{n_2}, \mathbf{V}^{-1}$ in (2.0.5)

$$\mathbf{q} = \mathbf{V}^{-1}\kappa_2\mathbf{n_2} \tag{2.0.14}$$
$$= \begin{pmatrix} 0 \\ \pm 2 \end{pmatrix} \tag{2.0.15}$$

The above results are verified in Fig. 2

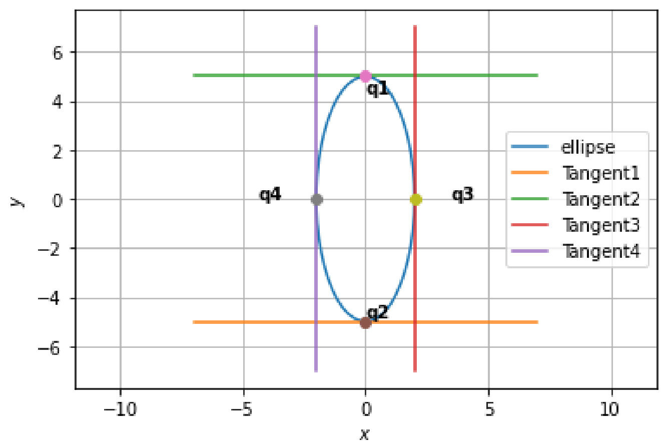

Fig. 2: Tangents to ELLIPSE.