Introductory Macroeconomics for Engineers

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IE 1

TD Macro 1

Exercise 1

Express the following equations as log-linear functions, i.e., take logs and simplify:

(a) $Y = zK^{\alpha}N^{1-\alpha}$.

(b) $Z = ce^{rt^{\beta}K}$.

Exercise 2

Calculate the first and second derivatives of the following functions:

(a) $f(c) = \ln(c)$.

(b) $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$.

(c) $h(w) = (-6w^3 + 17w - 4)^{\beta} - \ln(\theta w^{\beta})$.

Exercise 3

Calculate all the first, second, and cross derivatives of the following functions:

(a) $F(K, N) = \theta K^{\alpha} N^{1-\alpha}$.

(b) $F(K, N) = \ln\theta + \alpha \ln K + (1-\alpha)\ln N$.

Exercise 4

Solve the following constrained maximization problem.

$$\max_{x,w} U = \alpha \ln(x) + \beta \ln(w)$$

subject to

$$p_x x + p_w w \leq y$$
$$\alpha + \beta = 1.$$

Exercise 5

Consider the function $f(x) = \ln(1 + x)$. Calculate the first-order Taylor expansion of $f(x)$ around the point $x = 0$. Show that a growth rate can be approximated by the first-order Taylor expansion of the logarithm function around the point 1.

Note on First Order Taylor Expansion

The first-order Taylor expansion of a function $f(x)$ around a point a provides a linear approximation of $f(x)$ near a. It is given by:

$$f(x)\big|_{x=a} \approx f(a) + f'(a)(x - a),$$

where $f'(a)$ is the derivative of f at a.

Exercise 6

Suppose an economy produces steel, wheat, and oil. Here are the economic activities of each industry:

- The steel industry produces \$100,000 in revenue, spends \$4,000 on oil, \$10,000 on wheat, and pays workers \$80,000.
- The wheat industry produces \$150,000 in revenue, spends \$20,000 on oil, \$10,000 on steel, and pays workers \$90,000.
- The oil industry produces \$200,000 in revenue, spends \$40,000 on wheat, \$30,000 on steel, and pays workers \$100,000.

There is no government, and there are neither exports nor imports. None of the industries accumulate or deaccumulate inventories.

1. Calculate the GDP of this economy using the production method.
2. Calculate the GDP using the income method.

Exercise 7

Excel exercise: Download file data_td_1.xlsx from the moodle.

1. Generate a series for the natural logarithm of realGDP and realGDP per capita for both countries.
2. Use the series for the natural logarithm of gdp to calculate the growth rate of real gdp and real gdp per capita for both countries.
3. Compute the average growth rate of real gdp and real gdp per capita for both countries.
4. Plot the evolution of realGDP, realGDP per capita, and the growth rate of realGDP for both countries. How does the French economy compare to the economy of the United States?

Exercise 8

Consider the following Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

with total factor productivity represented by A_t. Derive the expression for growth rates using the logarithmic form of the production function. Show that the growth rate of output (g_{Y_t}) can be decomposed into the weighted sum of the growth rates of capital (g_{K_t}), labor (g_{L_t}), and total factor productivity (g_{A_t}):

$$g_{Y_t} = \alpha g_{K_t} + (1-\alpha) g_{L_t} + g_{A_t}.$$

Exercise 9

Maximize the profits of a representative competitive firm that produces the economy's total output using constant returns to scale Cobb-Douglas technologies, paying wages w_t and renting capital at rate r_t. Demonstrate that the weights for capital and labor growth rates, from the previous exercise, are the capital's and labor's shares of income. Hint: The firm's profit maximization problem is:

$$\max_{K_t, L_t} \Pi_t = A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t K_t.$$

Exercise 10

Download the data_td_2.xlsx file from Moodle. Follow these steps to compute the labor income share, capital income share, and the Solow residual, which is defined as the growth rate of productivity.

1. Calculate labor and capital income shares as the ratio of total labor income and total capital income to total output, respectively.
2. Calculate the growth rates of output, labor, and capital using the definition of the growth rate.
3. Compute the Solow residual as the difference between the growth rate of output and the weighted sum of the growth rates of labor and capital.
4. Now do the same using logarithms to compute the growth rates.
5. Plot both solow residuals.