DIPLÔME : Problem 8

Composition de :

INTERCALAIRE

Université Lille1 Sciences et Technologies

N° PLACE — INDIQUEZ VOTRE N° de place (en cas de perte, seul indice de recherche)

Si votre composition comporte plusieurs feuilles, numérotez-les : .... / ....

$f(K,l) = K^{\alpha} l^{1-\alpha}$

$f(\lambda K, \lambda l) = \lambda^{\alpha + (1-\alpha)} K^{\alpha} l^{1-\alpha}$

$f(\lambda K, \lambda l) = \lambda K^{\alpha} l^{1-\alpha}$

$= \underbrace{\lambda K^{\alpha} l^{1-\alpha}}$

Constant Returns to Scale (3)

$y = f(K, l)$ ← labor, ← capital equipment

↑ output ("focus")

By assumption $\alpha \in (0,1)$

$\Rightarrow \frac{\alpha - 1}{\alpha} < 0$. Set $\beta \equiv \frac{1-\alpha}{\alpha} > 0$ Then

and set $B = \tilde{y}^{1/\alpha} > 0$

$f(K,l) \doteq K^{\alpha} l^{1-\alpha}$

$f(K,l) \doteq \tilde{y}$ ← fix output

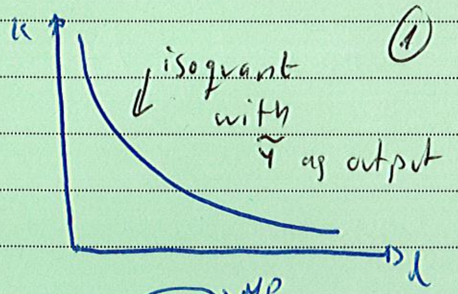
$\Rightarrow K^{\alpha} l^{1-\alpha} = \tilde{y} \Rightarrow K(l) = \left(\frac{\tilde{y}}{l^{1-\alpha}}\right)^{1/\alpha}$

$\Rightarrow K(l) = \tilde{y}^{1/\alpha} l^{\frac{\alpha-1}{\alpha}}$

$K(l) = B\, l^{-\beta} = \frac{B}{l^{\beta}}$

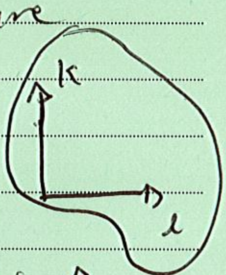
Go to wolphram alpha & type $K \to f(l) = \frac{1}{l^{1/2}}$ where $B = 1$, $\beta = 1/2 > 0$

(1)



Thus, l & K are compliments!

(K, l)-space is all possible (K, l) combinations, this



Marginal Product: $\frac{\partial y}{\partial l}$ ($\to MP_l$), $\frac{\partial y}{\partial K}$ ($\leftarrow MP_K$) (2)

Note! Always be careful choosing what factor is where:

$TRS = -\frac{MP_1}{MP_2} = -\frac{\frac{\partial y}{\partial x_1}}{\frac{\partial y}{\partial x_2}} = -\frac{dx_2}{dx_1}$

e.g.

$\frac{\partial (K^{\alpha} l^{1-\alpha})}{\partial l} = (1-\alpha) l^{1-\alpha-1} K^{\alpha} = (1-\alpha)\frac{K^{\alpha}}{l^{\alpha}}$

$\frac{\partial K^{\alpha} l^{1-\alpha}}{\partial K} = \alpha K^{\alpha-1} l^{1-\alpha} = \frac{\alpha K^{\alpha-1}}{l^{\alpha-1}}$

Technical Rate of Substitution $-\frac{MP_K}{MP_l}$

$\to TRS = -\frac{(1-\alpha)}{\alpha}\left(\frac{K^{\alpha}}{l^{\alpha}}\right)\left(\frac{l^{\alpha-1}}{K^{\alpha-1}}\right)$

(4) $TRS = \left(\frac{\alpha-1}{\alpha}\right)\frac{K}{l} = \frac{dl}{dK}$, not $\frac{dK}{dl}$ although both are correct answers

$(r, w) \to$ prices of factors

$p \to$ output price

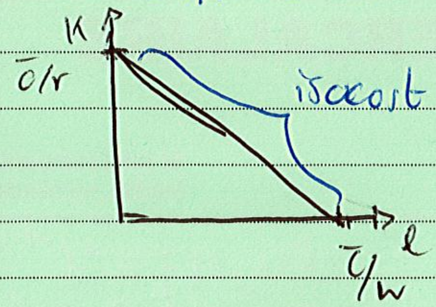
$\pi = py - wl - rK$

So the profit maximization problem is $\max \pi = \max py - wl - rK$

s.t. $y = f(K, l) = K^{\alpha} l^{1-\alpha}$

$C(y) = wl - rk$ s.t. $y = f(l,k)$

let $c(y) \stackrel{!}{=} \bar{c} \Rightarrow K(l) = \frac{\bar{c} - wl}{r}$



Short Run $K = \tilde{K}$

~~define h(K,l)~~ define

$h(l) \equiv f(K,l)\big|_{K=\tilde{K}}$

$h(l) = A\, l^{1-\alpha}$, $A = (\tilde{K})^{\alpha} > 0$

$1-\alpha > 0$

$\hookrightarrow h(l)$ is D.R.S. Why?

because $h(\lambda l) < \lambda h(l)$

(Show it!)

We ~~then~~ have to solve

$$\max_{l} \pi = \max py - wl - r\tilde{K} \quad \text{s.t. } y = h(l)$$

revenue, var cost, Fixed cost

Taking first order conditions

$$p\frac{\partial y}{\partial l} = w$$

Marginal product of labor = Marginal cost

"Value" since multiplying by the price gives value

This is the Optimality Condition to Satisfy

Why?

$\frac{\partial}{\partial l}[\pi] = \frac{\partial}{\partial l}(py) - \frac{\partial}{\partial l}(wl) \stackrel{!}{=} 0$

$\Rightarrow p\frac{\partial y}{\partial l} = w$

Solving: $(p)(1-\alpha)(\tilde{K}^{\alpha})\, l^{-\alpha} = w$

$$l = \left[\frac{w}{(p)(1-\alpha)(\tilde{K})^{\alpha}}\right]^{-1/\alpha} = \left[\frac{w}{(p)(1-\alpha)}\right]^{-1/\alpha}\tilde{K}$$

$$l^* = \left[\frac{(p)(1-\alpha)}{w}\right]^{1/\alpha}\tilde{K}$$

wage w

Comparative statics

- if you increase p, $(1-\alpha)$, $\tilde{K}$, then $l^* \uparrow$ (why??)
- if you increase w $l^* \downarrow$, (why??)

Now, the long Run (last question)

$\max_{l,k} p\cdot f(k,l) - wl - rk$

On the long run, the firm chooses (k,l) to maximize profits

$p\cdot\frac{\partial f}{\partial k} = r$

$p\cdot\frac{\partial f}{\partial l} = w$

Optimality Conditions

Marginal Value of factor = Marginal Cost of factor

$p\cdot\frac{\partial f}{\partial K} = (p)(\alpha)K^{\alpha-1}l^{1-\alpha} = r \Rightarrow \frac{\alpha p}{r}\, l^{1-\alpha} = K^{1-\alpha}$

$K^*(l) =$

$p\cdot\frac{\partial f}{\partial l} = p(1-\alpha)K^{\alpha}l^{-\alpha} = w \Rightarrow l(K) = \left[\frac{p(1-\alpha)}{w}\right]^{1/\alpha} K$

$K^{\alpha-1} = \frac{r}{p\alpha}\, l^{\alpha-1}$

$K(l) = \left(\frac{r}{p\alpha}\right)^{1/(\alpha-1)} l = \left(\frac{\alpha p}{r}\right)^{1/(1-\alpha)} l = \left(\frac{\alpha p}{r}\right)^{1/(1-\alpha)} K\left(\frac{p(1-\alpha)}{w}\right)^{1/\alpha}$

~~K^*~~