Problem 0: Refresher on optimization

Let us do a quick refresher on optimization. Find the maximum of

1. $f_1(x) = -x^2 + 10$
2. $f_2(x) = -3x^2 + 2x + 1$

And on constrained optimization.

Maximize $z = f(x, y) = 49 - x - y$ such that the sum of the two independent variables is 10. Proceed both by substitution through the constraint and through the method of Lagrange.

Problem 1: The Housing Market

Let us retake the example of the housing market, more especifically the market for apartments of a specific kind, in a certain area, all of which command the same monthly price. Suppose that market demand, as a function of price, is given in functional form by:

$$D(p) = 100 - 2p$$

1. Graph this demand function for all positive prices that command a non-negative demand (Otherwise demand would not make sense!). What is the domain of the demand function?

2. Suppose that the amount of available apartments for rent in the short run is 50. Solve for the competitive equilibrium monthly price.

3. Suppose that the amount of apartments in the short run is given by Q_s, a positive number. Solve for the *inverse demand function*, i.e. find an expression for the monthly price of an apartment as a function of the amount of apartments available.

4. Now suppose that all landlords in this market get together. Among them is an economist that has gone out of his way to estimate the demand function described above and now perfectly knows it. He shares it with the rest of the landlords and tells them he has utilized this demand function to arrive at a *profit maximizing price.* If landlords have no expenses, what is this price?

5. Now suppose that a local politician imposes a *yearly* tax on landowners. How is that previous profit maximizing price affected? Denote the tax by τ. What is the new profit maximizing price? What is the highest tax the politician can impose so that landlords don't experience loses?

Problem 2: Taxi Drivers

Now let us take a look at taxi drivers. We will be working with a fictional taxi driver (analogously to spherical cows in physics), which is an abstraction of reality used to simplify analysis. This fictional taxi driver only has to choose between consumption of a single good c and the share of her/his time devoted to leisure l. Suppose we can represent his/her preferences among these two goods with the utility function $u(c, l) = \alpha ln(c) + \beta ln(l)$, where α, β are positive parameters that sum to one. Further, let us suppose that all his/her income comes from labor, which they provide for a wage of $w > 0$, and that the consumption good has a per-unit price of $p > 0$.

1. Set up the relevant budget constraint in terms of labor supply $n = 1 - l$. What is the slope of the budget constraint? What is the amount of consumption the taxi driver can get if he/she doesn't rest at all?

2. Set up the optimization problem faced by the taxi driver. What type of problem is this? Can you identify the relevant endogenous and exogenous variables? What are we interested in here, what is the point of this problem?

3. Solve the optimization problem using a substitution technique.

4. Solve the optimization problem using the method of Lagrange. What is the optimal allocation between consumption and leisure? Can you use this to explain what are the roles of α and β?

5. Now suppose that the utility function is represented by $u(c, l) = c^\alpha l^\beta$ and solve the associated optimization problem. How does the solution to the previous problem change? Why?

6. Now suppose that, besides the labor income, the taxi driver also receives an endowment from his/her mother of m as pocket money. How does the taxi driver's budget constraint change?

References

Varian, H. R. (2014). *Intermediate Microeconomics: A Modern Approach.* 9th ed. W. W. Norton & Company.