This problem set is based on Chapter 12 Varian 2014

Problem 3: Discrete Probabilities

Suppose the following options are each a discrete probability distribution. Which of them is not well-defined? Why?

- $p_1 = 0.25,\ p_2 = 0.75$
- $p_1 = 0.5,\ p_2 = p_3 = 0.25$
- $p_1 = 0.8,\ p_2 = 0.3$
- $\{p \in \mathrm{R}^n | \sum_{i=1}^{i=n} p_i = 1\}$

Graph the last entry in the previous list using $n = 2$. How would the graph look like for $n = 3$? Graph it too.

Problem 4: Using Discrete Probabilities

Suppose you are someone who enjoys throwing money away and decide to bet on a lottery that pays \$100 with a probability of 0.25 and 0 with a probability of 0.75.

- Is this probability well-defined? Why?
- What is the expected payoff of the lottery?
- Now change the probabilities. Assume that the lottery pays the \$100 with probability p_1 and 0 with a probability of p_2. State the relationship between p_1 and p_2. Hint: It's trivial, comes from the definition of a probability distribution...
- State a formula that gives the lottery's expected payoff in terms of the *underlying* probability distribution (Hint: We are looking for a function that, given the payoff structure described, takes the *probabilities of each payoff* and gives you the *expected payoff*).

Now suppose that you are risk averse:

- For which values of α does the utility function $u(c) = c^\alpha$ correctly represent your relationship with uncertainty (i.e. for which values of α is that utility function risk averse)?
- What about risk-neutral? And what about risk loving?
- Can you use the parameter alpha to define what kind of utility function $u(c)$ is? How? Provide a summary of your results.

Problem 5: Budget Constraints, Utility maximization and Uncertainty

Let us retake the example you saw in class but with more context. Suppose that you are a fisherman whose monthly income depends on the weather, and further suppose that a storm is potentially coming to town. Taking **everything** into account (i.e. costs, market irregularities, demand, price, etc.) suppose that you know that, at this time of the year, **conditional** on there being no storm, you will for sure make \$100 this month. But, if the storm does happen, you will only make \$50 this month. **Suppose the probability of the storm taking place is 50%**:

- What is the **expected income** the fisherman faces?
- Suppose the fisherman is risk-averse and has an utility function over state-contingent consumption of the form $u(c) = c^{\alpha}$. What are the possible values for alpha?
- Pick a suitable value for α and show that indeed this utility function is risk-averse (use a graph).
- Let c_s represent consumption in case the storm happens and c_{ns} represent consumption in case the storm does not happen. What is the marginal rate of substitution between consumption in each scenario?

Suppose that there is a **fair insurance company** that offers insurance for up to any amount of money. They charge γ for each dollar of coverage you hire, i.e. if you want to get insurance for K amount of money you have to pay γK.

- Derive the **state-contingent** budget constraint using c_{ns} as the dependant variable. (Hint: Solve for $c_{ns}(c_s)$)
- What is the slope of the state-contingent budget constraint?
- What is the optimality condition for the optimal allocation of c_{ns} and c_s? Show it graphically and analytically. Also, relate the result to the fact that the insurance company is **fair**. Why do we require profits to be exactly zero?
- Now suppose that the insurance is not fair. In particular, suppose that the insurance company makes positive profits. How is the optimal allocation you just solved for affected?

References

Varian, H. R. (2014). *Intermediate Microeconomics: A Modern Approach*. 9th ed. W. W. Norton & Company.