PS #2 Problem 3 The one that doesn't sum to 1

Problem 4 →

Prob of \$100 Prob of \$0

$P_{100} = 0.25$, $P_0 = 0.75$ → $E_p(\ell) = (P_{100})100 + (P_0)0$

$P_{100} + P_0 = 1$ ✓ well-defined $E_p(\ell) = (0.25)(100) + 0$

Lottery Expected Payoff → $E_p(\ell) = 25$

$P_2 = 1 - P_1$ comes from definition of probability

$E_p(\ell) = (100)p_1 + (0)(1-p_1)$

$E_p(\ell) = 100p_1$ → $u(c) = c^\alpha$

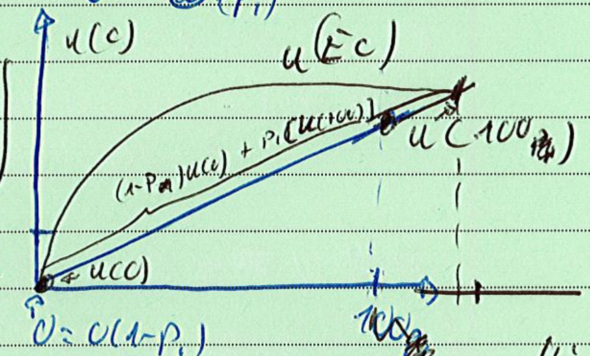
Line segment connecting $u(100p_1)$ to $u(p_1)$

How do we know this is true?

Jensen's inequality!

if $u(c)$ - concave

Then $u[E(c)] \geq E[u(c)]$



Hence, the agent is risk averse if & only if ("iff") $u(c)$ - concave (strictly! we want a strict inequality).

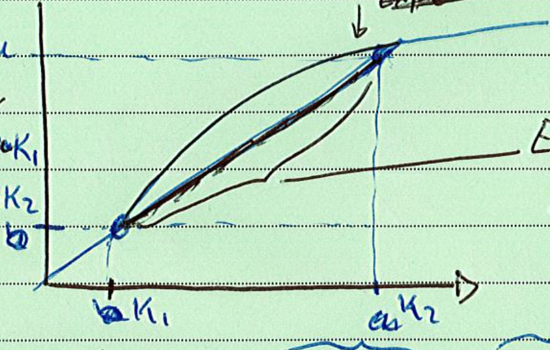
Try to grasp this idea. α → gives the shape of $u(c) = c^\alpha$

Again

Suppose a new lottery $\begin{cases} P_1 \to K_1 \\ P_2 \to K_2 \end{cases}$

$P_2 = 1 - P_1$

utility of expected payoff $u[E(\ell)]$, lottery



Expected utility line if $P_1 = 0$, $E[u(\ell)]$

Suppose $a > b$

↳ Expected utility ⇒ $u(\underset{K_1}{a})P_1 + u(\underset{K_2}{b})P_2 = u(K_1)P_1 + u(K_2)(1-P_1)$

$u(c) = c^\alpha \begin{cases} \text{concave if } \alpha < 1 \text{ (Risk averse)} \\ \text{Convex if } \alpha > 1 \text{ (Risk loving)} \\ \text{Both} \longrightarrow \text{(Risk Neutral)} \end{cases}$

Depending on the shape

$\underbrace{E[u(\ell)]}_{\text{expected utility}} \gtreqless \underbrace{u[E(\ell)]}_{\text{utility of expected payoff}}$

$\gtreqless$ means can be bigger, equal or smaller

Problem 5

Storm → 50 € No Storm → 100 €

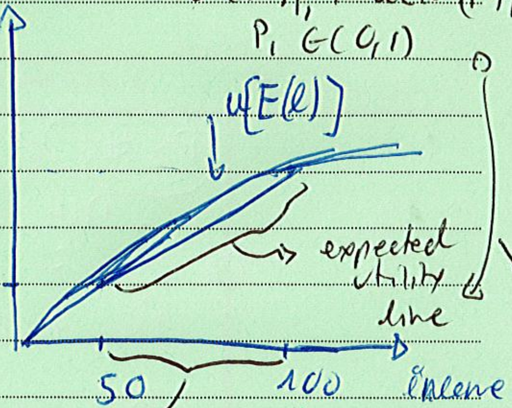
$P_s = 1/2$, $P_{ns} = 1/2$

$E_p(l) = (50)\frac{1}{2} + (100)1/2 = 75$

Recall the expected utility is $u(50)p_1 + u(100)(1-p_1)$, $p_1 \in (0,1)$

$u(c) = c^\alpha$, $0 < \alpha < 1$

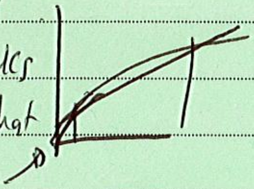
$u[E(l)] = u[100p_1 + 50(1-p_1)]$



if $0 < \alpha < 1$, $u(c)$ is concave & thus Jensen's Inequality holds & looks like that

$u(100)$

$\left.\begin{matrix} p_1 u(50) \\ +(1-p_1)u(100) \end{matrix}\right\}$ $u(50)$



Expected Payoff line $50p_1 + 100(1-p_1)$

The expected utility line has all the points corresponding each to a specific probability p_1

let us set $\alpha = 1/2$

Eu if $\alpha = 1/2$:

$E[u(l)] = u(50)1/2 + u(100)1/2$

$= (50^{1/2})1/2 + (100)^{1/2}1/2 = \frac{1}{2}(\sqrt{50} + 10) = \frac{\sqrt{50}}{2} + 5$

$dEU = 0 = \underline{p_1} MU(c_s)\, dc_s + \underline{(1-p_1)} MU(c_{ns})\, dc_{ns}$

Solving (use algebra) $\boxed{\dfrac{dc_{ns}}{dc_s} = -\dfrac{p_1 MU(c_s)}{(1-p_1)MU(c_{ns})}}$

Taking a total differential; Suppose $F(x,y)$ is a function of x & y

$dF(x,y) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy$

Now, Budget Constraint: Storm: $C_s = 50 € \underbrace{-\gamma K + K}_{-(\gamma-1)K}$

No Storm: $C_{ns} = 100 € - \gamma K$

Add the insurance policy:

Solve for K: K ~~= ...~~

No Storm: $K = \frac{100 - C_{ns}}{\gamma}$ $\Big\}$ Setting equal ⇒ $\frac{100 - C_{ns}}{\gamma} = \frac{C_s - 50}{1-\gamma}$

Storm $K = \frac{50 - C_s}{\gamma - 1}$

$\Rightarrow C_{ns} = \left[\frac{100}{\gamma} + \frac{50 - C_s}{1-\gamma}\right]\gamma = \boxed{100 + \left(\frac{\gamma}{1-\gamma}\right)[50 - C_s]}$

DIPLÔME : Problem 5 Continues

Composition de :

INTERCALAIRE

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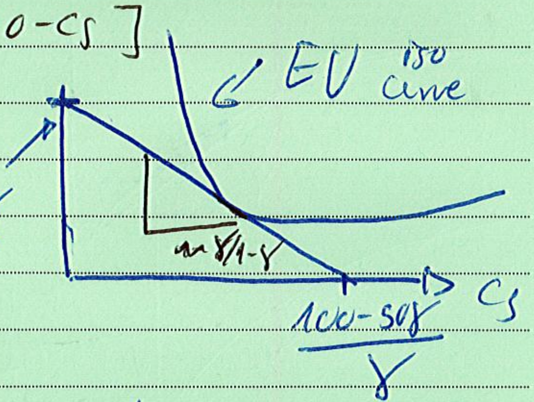
N° PLACE ← INDIQUEZ VOTRE N° de place (en cas de perte, seul indice de recherche)

Si votre composition comporte plusieurs feuilles, numérotez-les : .... / ....

B.C. $\Rightarrow C_{ns} = 100 + \frac{\gamma}{1-\gamma}[50 - C_s]$

$\Rightarrow C_{ns} = \frac{100-50\gamma}{1-\gamma} - \frac{\gamma}{1-\gamma} C_s$

The slope of the BC is $(-)\left[\frac{\gamma}{1-\gamma}\right]$



The optimality Condition is that the EU isocurve must touch at exactly 1 point the B.C. $\Rightarrow$ Their derivative match!

$$\frac{dC_{ns}}{dC_s} = \frac{-P_1}{1-P_1}\frac{MU(C_s)}{MU(C_{ns})} \stackrel{!}{=} -\frac{\gamma}{1-\gamma}$$

Fair insurance $\Rightarrow$ no profits:

- Revenue of Insurance Company $= \gamma K$
- Payoff $= (P_s)K$ — Probability of storm

Profits $= \gamma K - P_s K$
$= (\gamma - P_s)K \stackrel{!}{=} 0$ ← setting equal to 0 because fair insurance

$\Rightarrow \boxed{\gamma = P_s} \Rightarrow MU(C_s) = MU(C_{ns})$

$C_{ns} = 100 - \gamma K$
$C_s = 50 + (1-\gamma)K$
if $C_{ns} = C_s$
$\Rightarrow 100 - \gamma K = 50 + (1-\gamma)K$
$100 - 50 = (1-\gamma K)+K$
$\Rightarrow \underbrace{K = 50}$

The Fisherman is insured for the whole risk! (100 − 50 = 50)
↑ revenue if no storm; ↑ revenue if storm; storm "risk"

Non-fair Policy

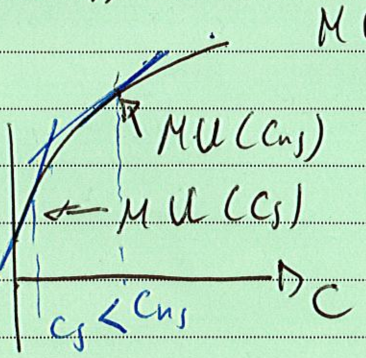
$(\gamma - P_s)K > 0 \Rightarrow \gamma > P_s$

Using our optimality Condition above

$$\frac{P_1}{1-P_1}\frac{MU(C_s)}{MU(C_{ns})} = \frac{\gamma}{1-\gamma} \Rightarrow MU(C_s) = \left(\frac{\gamma}{1-\gamma}\right)\frac{(1-P_1)MU(C_{ns})}{P_1}$$

$$MU(C_s) = \underbrace{\frac{\gamma}{P_1}}_{>1}\underbrace{\frac{1-P_1}{1-\gamma}}_{>1} MU(C_{ns}) > MU(C_{ns})$$

Recall utility is concave
Higher Marginal $\Rightarrow$ lower consumption



$\Rightarrow C_{ns} > C_s$

$\Rightarrow 100 - \gamma K >$

$C_{ns} > C_s$
$100 - \gamma K > 50 + (1-\gamma)K$
$\Rightarrow 100 - 50 > K$
$\underline{K < 50}$

Less insurance when non-fair policy

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