This problem set is based on Varian 2014

## Problem 8: Technology, Costs and Profit Maximization

Let us now inmerse ourselves in the taco market. We focus on the supply-side. Suppose that there are enough taquerias to support a competitive market, and suppose that they all use the same technology, captured by a Cobb-Douglas production function that takes as input the amount of available *capital equipment "k"* and amount of *labor "l"* and outputs tacos produced. We will make use of the idea of a *representative taqueria*. The production function is:

$$y = f(k, l) = k^{\alpha} l^{1 - \alpha}$$

Where  $\alpha \in (0,1)$ .

- Fix output at  $\tilde{y}$ . Draw the isoquant graph. (Hint: on (k, l) space).
- What is the marginal product of capital? And of labor?
- What type of production function is this? (Constant returns to scale, decreasing, increasing?), why?
- What is the technical rate of substitution?
- Suppose that r, w are the prices of 1 unit of capital and labor, respectively, while a single taco sells at price p. Set up the profit maximization problem of the firm, but do not solve it yet. What are the endogenous variables? i.e., what are we solving for? How realistic is this model of firm behaviour? Discuss.
- Suppose total costs are equal to  $\bar{c}$ . Draw the isocost line.
- Suppose that capital is fixed at  $\tilde{k}$ . How is the firm's profit maximization problem affected? What are we now solving for?
- Solve for the optimal labor demand. What is the optimality condition that must be satisfied for profits to reach a maximum?
- Do some comparative statics: What happens to output if p moves? If w moves? If r moves?
- Now suppose that the taqueria is deciding how much capital to acquire in order to maximize profits. How does this change the taqueria's problem?
- Try to find the optimal factor allocation. What do you notice? What can you do about it?

## References

Varian, H. R. (2014). Intermediate Microeconomics: A Modern Approach. 9th ed. W. W. Norton & Company.