① $f_1(x) = -x^2 + 10 \Rightarrow$ We want to find the max

Note Take a derivative at x! & set it to $\emptyset$

"$\forall$" means "for all"

$f_1'(x) = -2x \overset{!}{=} 0$

$\Rightarrow \underline{x^* = 0}$ Is this a max?

for $x < x^*$,

Hence, $f_1'(x) = -2x$, with $x < x^* = 0$

$-2(x)$, $x < 0 \Rightarrow \underline{-x > 0}$

$2(-x) > 0$

$\Rightarrow f_1'(x) > 0$ for all $x < 0$

The derivative is positive $\Rightarrow$

Similar analysis shows

$f_1'(x) < 0$ ~~$\forall x$~~ for all $x > 0$

This shows that $x = 0$ is indeed the max point & thus $\max_x f_1(x) = 10$

②

$f_2(x) = -3x^2 + 2x + 1 \xrightarrow{\text{derivative}} f_2'(x) = -6x + 2 \overset{!}{=} 0$ set to $\emptyset$

$\Rightarrow -6x + 2 = 0$

$\Rightarrow 2 = 6x \Rightarrow x^* = 2/6 = 1/3$ Is this a max? Show it!!

$f_2(1/3) = -3(1/3)^2 + 2/3 + 1 = 4/3$

③ Constrained optimization!

Extra problem (You should know this!)

$\max_{x,y} f(x,y) = 2x + y$ — function to be maximized

$\underbrace{x,y}_{\text{control variables}}$ s.t. $\underbrace{x^2 + y^2 = 1}_{\text{constraint}}$

No solution by substitution! (why?)

Using Lagrange multipliers

Built the lagrangian (Why? See the Khan Academy article! (about Lagrange Multipliers

$\mathcal{L}(x, y, \lambda) = 2x + y - \lambda[x^2 + y^2 - 1]$

(Answer is $x = +\frac{2}{\sqrt{5}}$, $y = +1/\sqrt{5}$ but try to understand why!)

$\frac{\partial \mathcal{L}}{\partial x} = 2 - 2\lambda x \overset{!}{=} 0 \Rightarrow \lambda 2x = 2$, $\lambda = \frac{1}{x}$

$\frac{\partial \mathcal{L}}{\partial y} = 1 - 2\lambda y \overset{!}{=} 0 \Rightarrow 2\lambda y = 1$, $\lambda = 1/2y = 1/x$

$\Rightarrow \boxed{x = 2y}$

$\frac{\partial \mathcal{L}}{\partial \lambda} = -[x^2 + y^2 - 1] = 1 - x^2 - y^2 \overset{!}{=} 0$

$1 - (2y)^2 - y^2 = 0$

$1 - 4y^2 - y^2 = 0$

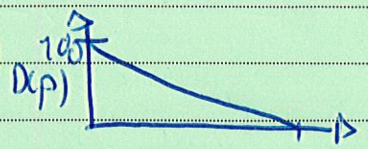
$1 - 5y^2 = 0$

$y^2 = 1/5$

$y = \pm\sqrt{1/5} = \pm\frac{1}{\sqrt{5}} \Rightarrow x = \pm\frac{2}{\sqrt{5}}$

We have 4 possible combinations! which is the correct?

(P1) 1)



$D(p) = 100 - 2p \quad p \in [0, 50]$

② $D(p) \stackrel{!}{=} 50$

$\Rightarrow \boxed{p^*_{CE} = 25}$

③ $D(p) \stackrel{!}{=} Q_S$

$\boxed{P_{CE}(Q_S) = 50 - \frac{1}{2} Q_S}$

④ revenue: $p \cdot D(p)$

profit = revenue − expenses
$\underbrace{\text{expenses}}_{=0}$

$\Rightarrow$ profit = revenue

$\Rightarrow \underbrace{\pi(p)}_{\text{profit}} = \underbrace{p \cdot D(p)}_{\text{revenue}}$ $\nearrow$ $\max_p \pi(p) = \max_p p D(p)$

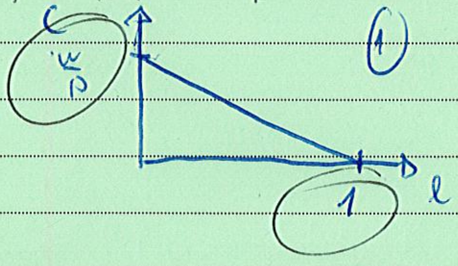
$\Rightarrow \boxed{p^* = 25}$ why? maximize.

⑤ $\pi(p) = p \cdot D(p) - \frac{\tau}{12}$, $p^*_p = 25$, what happens to profits?

(P2) $U(c, l) = \alpha \ln(c) + \beta \ln(l)$

B.C. $p \cdot c \leq I = w(\underbrace{1-l}_{n = 1-l}) \Rightarrow p \cdot c + wl \leq w$

Suppose equality, (why?) $pc + wl = w$



① $\Rightarrow l(c) = 1 - \frac{p}{w} c$

② $U(c, l) = \alpha \ln(c) + \beta \ln(l)$

$\max_{c,l} U(c, l)$ s.t. $p.c + wl = w$

③ By substitution $\max_c u(c, l(c)) = \max_c \alpha \ln(c) + \beta \ln l(c)$,

$\frac{\alpha}{c} + \frac{\beta}{l(c)} l'(c) = \frac{\alpha}{c} + \frac{\beta}{1 - \frac{p}{w} c} \left(-\frac{p}{w}\right) = \frac{\alpha}{c} + \beta \left[\frac{1}{1 - \frac{p}{w} c}\right] \frac{-p}{w} \stackrel{!}{=} 0$

$\frac{\alpha}{c} - \frac{\beta p}{w - pc} = 0$ solve for c $\boxed{c^* = \frac{w}{p}\left(\frac{\alpha}{\alpha + \beta}\right)}$

$\boxed{l(c^*) = \frac{\beta}{\beta + \alpha}}$

By Lagrange $\mathcal{L} = \alpha \ln(c) + \beta \ln(l) - \lambda(w - pc - wl)$

$\frac{\partial \mathcal{L}}{\partial c} = \frac{\partial \mathcal{L}}{\partial l} = \frac{\partial \mathcal{L}}{\partial \lambda} \stackrel{!}{=} 0$ yields the same answer! Do it!