1 The quantitative rules

This chapter focuses on the deduction of quantitative rules for inference which follow these desiderata:

1. Representation of degrees of plausibility by real numbers
2. Qualtitative correspondence with common sense
3. Consistency

- $F(x, y)$, i.e. a Boolean function of propositions x and y (i'm fuzzy on this), must be a continuous monotonic increasing function of both x and y.
  - monotonic increasing function: a function which never decreases
  - $F_1(x, y) \equiv \frac{\delta F}{\delta x} \geq 0$
  - $F_2(x, y) \equiv \frac{\delta F}{\delta y} \geq 0$
  - equivalency only when x represents an impossibility
- Consistency requires that propositions be true regardless of association:
  - $F[F(x, y), z] = F[x, F(x, y)]$
  - This functional equation is a big deal in mathematics. Called "The Associativity Equation"
- Product rule:

  $w(AB|C) = w(A|BC)w(B|C) = w(B|AC)w(A|C)$
  - $w(x)$ must be continuous monotonic
- Sum rule (super long derivation which I didn't quite follow):

  $p(A|B) + p(\bar{A}|B) = 1$, or more generally,

  $p(A + B|C) = p(A|C) + p(B|C) - p(AB|C)$
- given several propositions A_i which are mutually exclusive, it can be shown that,

  $p(A_1 + \ldots + A_m|B) = \sum_{i=1}^{m} p(A_i|B)$ where $1 \leq m \leq n$
- if these propositions A_i are not only mutually exclusive but also exhaustive,

  $\sum_{i=1}^{m} p(A_i|B) = 1$
- we are given two sets of mutually exclusive propositions, $\{A_1, \ldots, A_n\}$ and $\{A'_1, \ldots, A'_n\}$, with the only differences between the two sets being the subscripts 1 and 2 being swapped in the prime set
  - If the given information B is the same between the two sets of propositions,

    $p(A_1|B)_I = p(A'_2|B)_{II}$

    and

    $p(A_2|B)_I = p(A'_1|B)_{II}$
  - If the information B is indifferent between propositions A_1 and A_2, and since we know that equivalent states of knowledge must be represented by equivalent plausibility, we can say

    $p(A_i|B)_I = p(A'_i|B)_{II}$
  - Therefore, $P(A_1|B)_I = p(A_2|B)_I$
  - this is a "baby" version of the group invariance principle for assigning plausibilites
  - If the information B is indifferent between all propositions A_i, it can be shown that

    $p(A_i|B)_I = \frac{1}{n}$ where $1 \leq i \leq n$)
    - this is called the "principle of indifference"

- the information given can determine numerical values of the quantities $p(x) = p(A_i|B)$, not the numerical values of the plausibilities $x = A_i|B$
- the plausibility $x \equiv A|B$ is an arbitrary monotonic function of p, defined in $(0 \leq p \leq 1)$
  - these functions p are called "probabilities"

1.1 EXAMPLE: Bernoulli urn

- Prior information:
  - Ten balls of identical size and weight are in an urn
  - three balls (4, 5, and 6) are black, the rest are white
  - what is the probability that we draw a black one?
- Propositions:
  - $A_i \equiv$ the i^{th} ball drawn
- the probability to choose a particular ball is,

  $p(A_i|B) = \frac{1}{10}$
- the probability to choose a black ball is,

  $p(\text{black}|B) = p(A_4 + A_5 + A_6|B)$
- and since A_4, A_5, and A_6 are mutually exclusive,

  $p(\text{black}|B) = \frac{3}{10}$