

MAC 2313  
Lecture Notes

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# Chapter 1

## Preface

### 1.1 Examples

**Theorem 1.1** Theorem

Hello World!

**Definition 1.1:** Definition Test

Definition Example

**Lenma 1.1** Lenma Test

Lenma Example

**Proof 1.1** test

hello world

**Exercise 1.1** Exercise Test

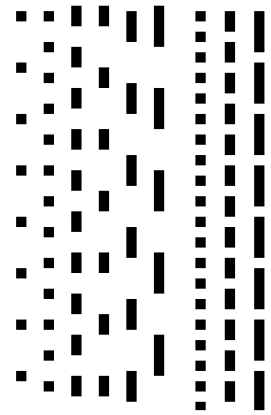
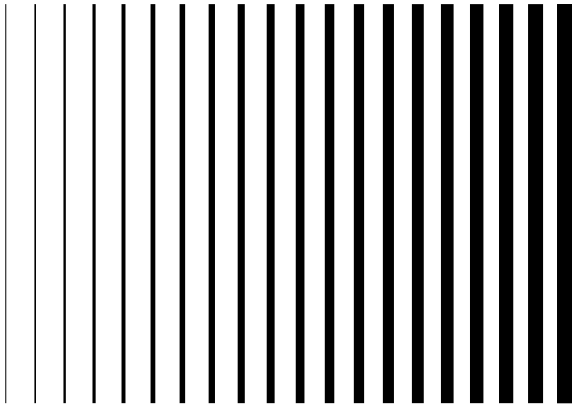
Ex Example

**Example 1.1** (Example Test)

Hello World Example!

**Definition 1.2:** Limit Test

$$\lim_{x \rightarrow \infty} \frac{1}{x}$$



# Chapter 2

## Unit 3

### 2.1 Lecture 1

#### 2.1.1 Double Integrals Intro

##### Definition 2.1: Double Integrals

The double summation of multivariable integrals is defined as:

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \Delta x \Delta y f(x_i, y_j) = \iint_R f(x, y) dA$$

Where R is the independent x and y ranges or  $\Delta x$  and  $\Delta y$  and A is the area defined by those ranges. R does not have to be rectangular and can be defined by the area of functions.

Double Integrals can be easily defined as twice iterated integrals.

**Lemma 2.1** Simple dA definition under the x and y coordinate system

$$dA = dx dy \quad \text{OR} \quad dA = dy dx$$

##### Definition 2.2: Volume of a box with height c

$$\iint_R c dA = c \iint_R dA = cA(R)$$

##### Theorem 2.1 Fubini's Theorem

Let  $a \leq x \leq b$  and  $g_1(x) \leq y \leq g_2(x)$

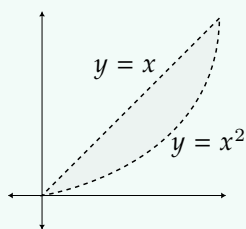
Let  $c \leq x \leq d$  and  $h_1(y) \leq x \leq h_2(y)$

Then for a continuous function  $f(x, y)$  or R

$$\begin{aligned} \iint_R f(x, y) dA &= \int_a^b \left[ \int_{g_1(x)}^{g_2(x)} f(x, y) dy \right] dx \\ &= \int_c^d \left[ \int_{h_1(y)}^{h_2(y)} f(x, y) dx \right] dy \end{aligned}$$

Alike partial derivatives, we can compare this to a partial integral.

**Example 2.1** (Example with figure given)



$$\iint_R f(x, y) dA = \int_0^1 \left[ \int_{x^2}^x f(x, y) dy \right] dx \quad (2.1)$$

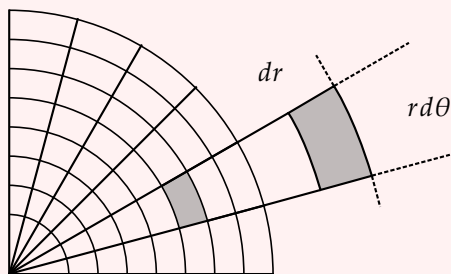
$$= \int_0^1 \left[ \int_y^{\sqrt{y}} f(x, y) dx \right] dy \quad (2.2)$$

### Definition 2.3: Average Area

$$\text{Average Area} = \frac{1}{A(R)} \iint_R f(x, y) dA$$

## 2.1.2 Polar Coordinates

### Definition 2.1: Polar Coordinates



$$\begin{aligned} x &= r \cos \theta & x^2 + y^2 &= r^2 \\ y &= r \sin \theta & \frac{y}{x} &= \tan \theta \end{aligned}$$