MAP 2302
Lecture Notes

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Chapter 1

Preface

Welcome to my LaTex written notes for MAP-2302, or Differential Equations. I wanted to attempt to be able to write notes for mathematical classes since MAC2302 or Calculus 3 after seeing the beautiful blogs and notes by Gilles Castel, so much of what I create and design inside these notes are based on his work. Check out his website here: https://castel.dev.

Chapter 2

Unit 1

2.1 Lecture 1: What are differential equations?

Remember that differential equations are equations defined by equations with derivatives. One of the simplest examples, with variables x and y, are:

$$dy = dx$$

Where the initial function, through integration can be found as:

$$y = x$$

The differential equation learned from Calc 1 is the one that describes a population, where:

$$\frac{dP}{dt} = kP$$

Where the derivative, or rate of change, is dependent on the *current population*.

2.2 Lecture 2

Example 2.1 (Excplicit Solution)
Now let's look at an explicit solution.
Verify that $\phi = 3\sin 2x + e^{-x}$ is a solution of $y'' + 4y = 5e^{-x}$.

$$\begin{aligned} \phi' &= 6\sin 2x - e^{-x} \\ \phi'' &= -12\sin 2x + e^{-x} \end{aligned}$$

Now we plug in

$$\begin{aligned} -12\sin 2x + e^{-x} + 12\sin 2x + 4e^{-x} &= 5e^{-x} \\ 5e^{-x} &= 5e^{-x} \checkmark \end{aligned}$$

Theorem 2.1 Existence and Uniqueness
If $f(x, y)$ and $\frac{\partial f}{\partial y}(x, y)$ are continuous about the point (x_0, y_0) then the Initial Value Problem $y' = f(x, y)$ and $y(x_0) = y_0$ has a unique solution in a neighborhood of the point (x_0, y_0).

Example 2.2
$y' = xy^{\frac{1}{2}} = f(x, y)$.

Clearly $f(x,y)$ is continuous about $(0,0)$ but $\frac{\partial f}{\partial y} = \frac{x}{2y^{\frac{1}{2}}}$ is not continuous at $(0,0)$. So the theorem cannot say that there is a unique solution.
Lets say $y_1 = 0$ for every x:

$$y_1' = 0 \qquad xy^{\frac{1}{2}} = 0$$

$$y' = xy^{\frac{1}{2}} \checkmark$$

Lets now try $y_2 = \frac{x^4}{16}$

$$y_2' = \frac{x^3}{4} xy^{\frac{1}{2}} = x(\frac{x^4}{16})^{\frac{1}{2}} = \frac{x^3}{4}$$

$$y' = xy^{\frac{1}{2}} \checkmark$$

2.3 Lecture 3

Let's say $y' = f(x,y)$. Try to get an idea of how the solution curves. Now, remember the interpretation of y': is it the slope the tangent line. Now let's plot plenty of small tangent lines along some graph for an equation of f(x,y).

This is defined as a **Direction Field**.

Definition 2.1: Direction Field

A field of vectors or slopes that represent a function at any given set of points.

Example 2.3 ($y' = x^2 = f(x,y)$)

Example 2.4 ($y' = \frac{x}{y}$)

Definition 2.2: Isoclines

Isoclines are curves of *equal* slope. Isoclines do not intersect unless $f(x,y)$ is not defined at the point. Isoclines are used to develop vector fields or directional fields.

We can use isoclines to create a direction field. To do so we set the derivative or y' to m and solve for y.

Example 2.5 ($y' = \frac{x}{y} = m$; $y = \frac{1}{m}x$)

Example 2.6 ($y' = -\frac{x}{y} = m$; $y = -\frac{1}{m}x$)

Chapter 3

Unit 2

3.1 Introduction to the uses of the first DE's

Example 3.1 (Gravity)