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MGSC 662 – MIP – Assignment 2:
International Industries, Inc.

Please note that all the code used for this assignment can be found in the file named, 'MGSC662_Assignment2_script.ipynb', which was included with my submission.Below, the questions are written in blue, my answers are in black.

(a)
Suppose that you are an associate of the firm GCG. Using the data provided in the spreadsheet INT-INDUSTRIES-DATA.XLS, construct a discrete optimization model of the problem faced by International Industries. Solve the problem on the computer.

Formulation of the discrete optimization problem:

Objective Function – We want to maximize the Net Present Value (NPV) of the International Industries portfolio of 50 divisions.

$$MAX\ f = \sum_{d=1}^{50} (Ni_d * I_d + Nm_d * M_d + Ns_d * S_d)$$

d = 1, …, 50; representing the 50 divisions
Ni_d = net present value for the decision to invest in each division d
Nm_d = net present value for the decision to maintain each division d
Ns_d = net present value the decision to sell each division d
I_d = binary decision to invest or not to invest in division d; $I_d \in \{0,1\}$
M_d = binary decision to maintain or not maintain division d; $M_d \in \{0,1\}$
S_d = binary decision to sell or not sell division d; $s_d \in \{0,1\}$

Subject to:
Investment and Cashflow Constraint: Total Investment ≤ Total Cashflow

$$\sum_{d=1}^{50} (Ii_d * I_d + Im_d * M_d + Is_d * S_d) \leq \sum_{d=1}^{50} (Ci_d * I_d + Cm_d * M_d + Cs_d * S_d)$$

Ii_d = investment required for decision to invest in each division d
Im_d = investment required for decision to maintain each division d
Is_d = investment required for decision to sell each division d
Ci_d = cash flow for next year for the decision to invest in division d
Cm_d = cash flow for next year for the decision to maintain division d
Cs_d = cash flow for next year for the decision to sell division d

Constraints from portfolio restrictions:

Constraint 1 (restriction a):	$S_1 = I_2$
Constraint 2 (restriction a):	$S_1 = S_3$
Constraint 3 (restriction a):	$S_1 = S_4$
Constraint 4 (restriction b):	$I_6 = I_7$
Constraint 5 (restriction c):	$S_3 + S_4 + S_5 + M_6 + S_7 + S_9 \leq 1$
Constraint 6 (restriction d):	$I_4 + M_6 + S_2 + S_6 + S_8 + S_{12} + I_{14} \leq 1$
Constraint 7 (restriction e):	$I_{24} \leq I_{28}$
Constraint 8 (restriction f):	$M_{30} \leq M_{32} + I_{32}$
Constraint 9 (must either invest, maintain, or sell)	$I_d + M_d + S_d = 1,\ d = 1, \ldots, 50$
Constraint 10 (binary decision):	$I_1, \ldots, I_{50}, M_1, \ldots, M_{50}, S_1, \ldots, S_{50} = 0 \text{ or } 1$

Please refer to the MGSC662_Assignment2_script.ipynb under the heading "(a)" for the solution to the above discrete optimization problem using Python.

Optimal Solution:

The optimal solution to the problem (the NPV of the optimal choices to either invest, maintain, or sell each division) is **$34,036 million**.

The optimal decisions (invest, maintain, sell) for each of the 50 divisions are shown in the below table:

Division	Decision	Division	Decision	Division	Decision
1	Maintain	18	Invest	35	Maintain
2	Sell	19	Invest	36	Sell
3	Invest	20	Sell	37	Invest
4	Maintain	21	Sell	38	Invest
5	Maintain	22	Invest	39	Maintain
6	Invest	23	Maintain	40	Invest
7	Invest	24	Sell	41	Sell
8	Maintain	25	Sell	42	Invest
9	Maintain	26	Invest	43	Maintain
10	Sell	27	Invest	44	Invest
11	Sell	28	Sell	45	Invest
12	Maintain	29	Invest	46	Maintain
13	Invest	30	Invest	47	Invest
14	Sell	31	Sell	48	Sell
15	Maintain	32	Sell	49	Maintain
16	Maintain	33	Sell	50	Invest
17	Invest	34	Sell		

(b)
What is the penalty for imposing restrictions (a) to (f) outlined above? That is, how much does the optimal NPV increase if these restrictions are removed?

To calculate the penalty for imposing restrictions (a) to (f) we need to reformulate the problem from part (a) without the constraints that were included because of restrictions (a) to (f). Please refer to the MGSC662_Assignment2_script.ipynb file under heading "(b)" for this new formulation of the problem.

Under this new formulation of the problem, the optimal NPV increases to **$39,625 million**. The previous optimal solution was $34,036 million. Therefore, the penalty for imposing these restrictions is equal to $39,625 - $34,036 = **$5,589 million**.

(c)
Propose a methodology to generate the second best and the third best solutions and find the second best and third best solution to your discrete optimization model (the model you achieved in part (a)).

From part (a), we know that the optimal NPV is $34,036. To find the second-best and third-best optimal solutions, we can systematically impose new constraints that put an upper bound on the optimal solution. If we set this upper bound just below the optimal solution, we can find the second-best solution. Then, if we set this upper bound just below the second-best solution, we can find the third-best solution.

<u>Second-best solution:</u>

To find the second-best solution, we can impose a constraint saying that the total NPV must be less than or equal to $34,035 million. The constraint looks like this:

$$\sum_{d=1}^{50} (Ni_d * I_d + Nm_d * M_d + Ns_d * S_d) \leq 34{,}035$$

Please refer to MGSC662_Assignment2_script.ipynb file under heading "(c) – second best solution" for this new formulation of the problem that includes this new constraint.

Note that Gurobi can only handle constraints of type "==", ">=" and "<=" (source). As a result, we must implement the above constraint using "<=" instead of implementing a constraint that requires that the total NPV be less than (<) 34,036 million. This limitation means that we do not account for the fact that the second-best solution could be between 34,035 million and 34,036 million; however, in the context of this problem, this oversight is not consequential because the NPV cannot increase by less than $1 million because the all the NPVs in this problem have an absolute value greater than $1 million (this same logic will also hold when calculating the third-best solution).

When we resolve the problem with the new constraint, we find that the second-best solution is **$34,006 million**. The optimal decisions (invest, maintain, sell) for each of the 50 divisions in the second-best solution are shown in the below table. The only decision that is different than the optimal solution is to maintain division 37, rather than invest in it (this change is highlighted in blue).

Division	Decision		Division	Decision		Division	Decision
1	Maintain		18	Invest		35	Maintain
2	Sell		19	Invest		36	Sell
3	Invest		20	Sell		37	Maintain
4	Maintain		21	Sell		38	Invest
5	Maintain		22	Invest		39	Maintain
6	Invest		23	Maintain		40	Invest
7	Invest		24	Sell		41	Sell
8	Maintain		25	Sell		42	Invest
9	Maintain		26	Invest		43	Maintain
10	Sell		27	Invest		44	Invest
11	Sell		28	Sell		45	Invest
12	Maintain		29	Invest		46	Maintain
13	Invest		30	Invest		47	Invest
14	Sell		31	Sell		48	Sell
15	Maintain		32	Sell		49	Maintain
16	Maintain		33	Sell		50	Invest
17	Invest		34	Sell			

<u>Third-best solution:</u>

To find the third-best solution, we can impose a constraint saying that the total NPV must be less than or equal to $34,005 million. The constraint looks like this:

$$\sum_{d=1}^{50} (Ni_d * I_d + Nm_d * M_d + Ns_d * S_d) \leq 34{,}005$$

Please refer to MGSC662_Assignment2_script.ipynb file under heading "(c) – third best solution" for this new formulation of the problem that includes this new constraint.

When we resolve the problem with the new constraint, we find that the third-best solution is **$33,999 million**. The optimal decisions (invest, maintain, sell) for each of the 50 divisions in the third-best solution are shown in the below table. The only decision that is different than the optimal solution and second-best solution is to sell division 49, rather than maintain it (this change is highlighted in purple).

Division	Decision		Division	Decision		Division	Decision

1	Maintain	18	Invest	35	Maintain
2	Sell	19	Invest	36	Sell
3	Invest	20	Sell	37	Maintain
4	Maintain	21	Sell	38	Invest
5	Maintain	22	Invest	39	Maintain
6	Invest	23	Maintain	40	Invest
7	Invest	24	Sell	41	Sell
8	Maintain	25	Sell	42	Invest
9	Maintain	26	Invest	43	Maintain
10	Sell	27	Invest	44	Invest
11	Sell	28	Sell	45	Invest
12	Maintain	29	Invest	46	Maintain
13	Invest	30	Invest	47	Invest
14	Sell	31	Sell	48	Sell
15	Maintain	32	Sell	49	Sell
16	Maintain	33	Sell	50	Invest
17	Invest	34	Sell		