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MGSC 662 – NLP – Assignment 3:

Please note that all the code used for this assignment can be found in the file named, 'MGSC662_Assignment3_script.ipynb', which was included with my submission.Below, the questions are written in blue, my answers are in black.

Problem 1: The Markowitz Problem

(a)
Formulate an NLP to devise the optimal portfolio that minimizes the portfolio risk subject to non-negative return. What are the optimal solution and value?
Hint: Notice that for the stock FTSE 100, for example, the rate of return from May 3 to June 1, 2016, is (5,996.80 – 6230.80)/6230.80 = -0.0424.

Formulation of the Nonlinear optimization problem:

Objective Function – We want to find the portfolio of the five indexes (FTSE 100, DAX , DJI, DJ Asian Titans 50, and Russell 2000) that minimizes risk while guaranteeing non-negative return.

$$MIN\ f = \sigma_R$$

Where σ_R is the standard deviation of portfolio R, which includes some collection of assets FTSE 100, DAX, DJI, DJ Asian Titans 50, and Russell 2000.

Let E(R) represent the expected return of portfolio R and let X_i represent the fraction between 0 and 1 invested in each asset i. i = 1, 2, 3, 4, or 5, for the five assets.

Subject to:

Constraint 1 (Budget):	$\sum_{i=1}^{5} X_i = 1$
Constraint 2 (Non-negativity):	$E(R) \geq 0$

Please refer to the MGSC662_Assignment3_script.ipynb file under the heading " P1 (a)" for the solution to the above NLP optimization problem using Python.

Note that the expected return, standard deviation, and correlation for the assets was calculated in the Excel file and then imported to the notebook file. When calculating the standard deviation for the assets, Excel's STDEV.S function was used (instead of STDEV.P) because the prices in the Excel are a sample of the prices for these assets during the entire time that they have been listed on the market. That being said, the optimal solution for this problem is not impacted meaningfully by using STDEV.P instead of STDEV.S to calculate standard deviation.

Optimal Solution:

The optimal solution is to invest in the assets as follows:
FTSE_100: 52.29%
DAX: 0.00%
DJIA: 47.71%
DJ Asian Titans 50: 0.00%
Russell 2000: 0.00%

Note that when Gurobi solves this problem the values for DAX, DJ Asian Titans, and Russell 2000 are slightly greater than 0. They are 2.9415e-12, 1.5207e-10, and 1.7794e-11 respectively. This is because Gurobi was told to treat the fraction of investment in each asset as a continuous variable whose value is between 0 and 1. Thus, it can find any fraction between 0 and 1. However, in practice we can treat these 3 allocations as 0 because it is not possible or feasible to invest such a small fraction in each of them. Therefore, our optimal solution is to not invest in these three assets.

For this allocation, the optimal value (minimal standard deviation or 'risk') is approximately **3.08**.

(b)
Assume the initial allocation is 20% in each index. Changing the position requires incurring transaction costs. Formulate an NLP to find the optimal portfolio to minimize risk subject to non-negative return. What are the optimal solution and value?

Formulation of the Nonlinear optimization problem:

Objective Function – We want to find the portfolio of the five indexes (FTSE 100, DAX, DJI, DJ Asian Titans 50, and Russell 2000) that minimizes risk while guaranteeing non-negative return and accounts for the fact that changing our position will incur a transaction cost.

$$MIN\ f = \sigma_R$$

Where σ_R is the standard deviation of portfolio R, which includes some collection of assets FTSE 100, DAX, DJI, DJ Asian Titans 50, and Russell 2000.

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Let E(R) represent the expected return of portfolio R and let X_i represent the fraction between 0 and 1 invested in each asset i. i = 1, 2, 3, 4, or 5, for the five assets.

Subject to:

Constraint 1 (Budget):	$\sum_{i=1}^{5} X_i = 1$
Constraint 2 (Non-negativity):	$E(R) \geq 0$
Constraint 3 (Transaction cost):	$(E(R) - 0.2)^2 \geq 0$

Please refer to the MGSC662_Assignment3_script.ipynb file under the heading " P1 (b)" for the solution to the above NLP optimization problem using Python.

<u>Optimal Solution:</u>

The optimal solution is to invest in the assets as follows:
FTSE_100: 31.11%
DAX: 7.84%
DJIA: 31.92%
DJ Asian Titans 50: 14.34%
Russell 2000: 14.79%

For this allocation, the optimal value (minimal standard deviation or 'risk') is approximately **0.0137**.

Problem 2: The CYCOM Corporation Problem

(a)
Construct and solve a nonlinear optimization model to determine the number of engineers to assign to each project that will maximize the expected contribution to profit of the six projects minus the cost of assigning engineers to the projects and the start-up costs of the projects. What is the optimal solution? (Rounded to one decimal place)

The **expected profit** if the projects are successful is given by the following equation:

EP = 1750p1 +700p2 + 1300p3 + 800p4 + 1450p5 + 1300p6

Where p_1, p_2, p_3, p_4, p_5, and p_6, represent the probability of projects 1, 2, 3, 4, 5, and 6 being successful.

The **expected start-up cost** is given by the following equation:

ES = 325(1-p1) + 200(1-p2) + 490(1-p3) + 125(1-p4) + 710(1-p5) + 240(1-p6)

Formulation of the Nonlinear optimization problem:

Objective Function – We want to determine the number of engineers to assign to each project that will maximize the expected contribution to profit of the six projects, minus the cost of assigning engineers to the projects and the start-up costs of the projects.

$$MAX\ f = EP - ES - 150 * [\sum_{i=1}^{6}(x_i)]$$

i = project number, i = 1, 2, 3, 4, 5, or 6.
p_i = probability of project i being successful.
x_i = number of engineers assigned to project i.

Subject to:

Constraint 1 (Engineer availability): $x_1 + ,\dots, + x_6 \leq 25$

Constraint 2 (Non-negativity): $x_1 ,\dots, x_6 \geq 0$

Please refer to the MGSC662_Assignment3_script.ipynb file under the heading " P2 (a)" for the solution to the above NLP optimization problem using Python.

Optimal Solution:

The optimal solution is to allocate the engineers on the projects as follows:
$x_1 = 2.8$
$x_2 = 1.2$
$x_3 = 3.0$
$x_4 = 1.5$
$x_5 = 3.4$
$x_6 = 2.6$

Recall the interpretation of these values: 2.8 engineers on project 1 means that it is optimal to assign 2 engineers to project 1 full-time, and 1 engineer to project 1 80% of the time. This logic is the same for each x_i.

This optimal allocation of engineers yields an expected contribution to profit of **$1,396,484.34.**

(b)

Construct and solve an optimization model that minimizes the standard deviation of the

contribution to profit subject to the constraint that the expected contribution to profit (minus the cost of engineers) is at least $1.1 million. What is the optimal solution? (Rounded to one decimal place) Hint: Assume that the success or not of one project is statistically independent of the success or not of any other project.

Objective Function – We want to determine the number of engineers to assign to each project that will minimize the standard deviation of the contribution to profit subject to the constraint that the expected contribution to profit (minus the cost of engineers) is at least $1.1 million.

Consider a random variable X takes the value of A with probability p and the value B with probability (1-p). The variance of X is calculated as $Var(X) = (A-B)^2p(1-p)$. In the context of our problem, A is the contribution to profit from a project if it is successful, and B is the start-up cost for each project (note that B values are negative since they represent costs). For example, for project 1, $Var(X) = (1750+325)^2p_1(1-p_1)$.

Using this formula, and the assumption we are given that that the success or not of one project is statistically independent of the success or not of another project, we can write the equation for the variance of the contribution to profit as follows:

$$VP = (1750 + 325)^2p_1(1-p_1) + (700 + 200)^2p_2(1-p_2) + (1300 + 490)^2p_3(1-p_3)$$
$$+ (800 + 125)^2p_4(1-p_4) + (1450 + 710)^2p_5(1-p_5) + (1300 + 240)^2p_6(1-p_6)$$

Standard deviation is equal to the square root of variance; thus, our minimization problem is formulated as follows:

$$MIN\ f = \sqrt{VP}$$

i = project number, i = 1, 2, 3, 4, 5, or 6.
p_i = probability of project i being successful.
x_i = number of engineers assigned to project i.

Subject to:

Constraint 1 (Minimum contribution): $EP – ES – 150*(x_1 + \ldots + x_6) \geq 1100$
Constraint 2 (Engineer availability): $x_1 + \ldots + x_6 \leq 25$
Constraint 3 (Non-negativity): $x_1 ,\ldots, x_6 \geq 0$

Gurobi cannot handle square roots in the objective function. Therefore, to implement this minimization problem we must remove the square root from our objective function. To do so, we can simply minimize the VP function, and then take the square root of the objective value to find the standard deviation of the contribution to profit.

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Please refer to the MGSC662_Assignment3_script.ipynb file under the heading " P2 (b)" for the solution to the above NLP optimization problem using Python.

Optimal Solution:

The optimal solution is to allocate the engineers on the projects as follows:
$x_1 = 5.0$
$x_2 = 1.6$
$x_3 = 4.2$
$x_4 = 1.5$
$x_5 = 6.0$
$x_6 = 3.0$

This optimal allocation of engineers yields a standard deviation of **$1,800,000.36**
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