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MGSC 662 – Linear Programming – Assignment 1:
The Case of Filatoi Riuniti

Please note that all the code used for this assignment can be found in the file named, 'MGSC662_Assignment1_script.ipynb', which was included with my submission.Below, the questions are written in blue, my answers are in black.

(a) Formulate Filatoi Riuniti's purchasing problem for the coming month (March):

1. Write down the formula for the objective function of your model.

Objective Function – we want to minimize total cost

$$MIN\ f = \sum_{j=A}^{G} \sum_{i=1}^{4} X_{ij}\,(C_{ij} + T_{ij})$$

i = 1, 2, 3, 4
j = A, B, C, D, E, F, G
X_{ij} = The quantity of product i produced by supplier j
C_{ij} = Unit cost of product i produced by supplier j
T_{ij} = Transportation cost per kg of product i, produced by company j

2. Your model must have a capacity constraint for each local spinning mill. Write down the capacity constraints for all the mills as well as Filatoi Riuniti.

Capacity constraints:

Ambrosi Mill (j=A): $0.4X_{2A} + 0.375X_{3A} + 0.25X_{4A} \leq 2{,}500$
Bresciani Mill (j=B): $0.7X_{1B} + 0.5X_{2B} + 0.35X_{3B} + 0.25X_{4B} \leq 3{,}000$
Castri Mill (j=C): $0.675X_{1C} + 0.45X_{2C} + 0.4X_{3C} + 0.25X_{4C} \leq 2{,}500$
De Blasi Mill (j=D): $0.45X_{2D} + 0.35X_{3D} + 0.2X_{4D} \leq 2{,}600$
Estensi Mill (j=E): $0.65X_{1E} + 0.45X_{2E} + 0.4X_{3E} + 0.25X_{4E} \leq 2{,}500$
Filatoi Riuniti (j=F): $0.625X_{1F} + 0.5X_{2F} + 0.425X_{3F} + 0.425X_{4F} \leq 38{,}000$
Giuliani Mill (j=G): $0.7X_{1G} + 0.45X_{2G} + 0.35X_{3G} + 0.4X_{4G} \leq 2{,}500$

3. Filatoi Riuniti must meet demand for each of the four sizes of yarn. Your model must have a constraint for the demand for each of the four sizes of yarn. Write down the constraint for the demand for all types of yarns including extra fine, fine, medium, and coarse yarns.

Demand constraints:

Extra fine yarn (i=1): $X_{1B} + X_{1C} + X_{1E} + X_{1F} + X_{1G} \geq 25{,}000$

Fine yarn (i=2): $X_{2A} + X_{2B} + X_{2C} + X_{2D} + X_{2E} + X_{2F} + X_{2G} \geq 26{,}000$
Medium yarn (i=3): $X_{3A} + X_{3B} + X_{3C} + X_{3D} + X_{3E} + X_{3F} + X_{3G} \geq 28{,}000$
Coarse yarn (i=4): $X_{4A} + X_{4B} + X_{4C} + X_{4D} + X_{4E} + X_{4F} + X_{4G} \geq 28{,}000$

Note that in the complete formulation of this problem, there will also be a non-negativity constraint ($X_{ij} > 0$) since no supplier can produce a negative quantity. This non-negativity constraint will be captured in the formulation of the problem in Python by setting the lower bound of each decision variable equal to 0. Furthermore, there will be two constraints to capture the fact that the Ambrosi Mill and the De Blasi Mill cannot produce extra fine yarn. They will be as follows:

$$X_{1A} = 0$$
$$X_{1D} = 0$$

(b) Construct the code for your optimization model. First, open a Jupyter Notebook. Then, read the spreadsheet FILATOIR.XLS (or you may want to copy and paste the data into your code, as you wish). You can do it in any way you prefer. Finally, implement your model and optimize it using the packaged GUROBI. You will need to:

a. Create the objective function and the constraints, and

b. Optimize your model. Can you assume a linear model? What is the optimal supply strategy?

The model is in a linear relationship. The optimal solution is $1,382,544.

The optimal supply strategy is shown below:

Supplier	Product			
	Extra fine	Fine	Medium	Coarse
Ambrosi	0	6,250	0	0
Bresciani	4,286	0	0	0
Castri	3,704	0	0	0
De Blasi	0	0	2,040	0
Estensi	3,846	0	0	0
Filatoi Riuniti	13,164	19,750	18,817	28,000
Giuliani	0	0	7,143	0

(c) Filatoi Riuniti should obviously consider increasing its spinning machine capacity. They could slightly expand the production capacity of the existing machines by renting an upgrade. This would increase their spinning production capacity by 600 hours/month. The monthly rental cost is $1,500/month. Would you recommend that they rent the upgrade? (Try to answer this question without re-optimizing your model.)

If we generate the sensitivity analysis report for the model from (b), we can see that the allowable increase for Filatoi Riuniti's capacity usage is 867, which is greater than 600. Therefore, we do not need to re-solve the model because the proposed increase of 600 hours/month is within the allowable range. We can calculate the value of the extra 600 hours/month as 600*2.11765 (the shadow price) = $1270.59 /month. Thus, we conclude that we should not recommend the upgrade because the value ($1270.56 /month) is less than the cost to rent the upgrade ($1,500/month).

(d) Alternatively, Filatoi Riuniti could increase its spinning machine capacity by renting another spinning machine for the production of only medium size yarn, for a monthly rental cost of $3,000. The machine has a production capacity of 300 hours per month (the machine would run at the same rate of 0.425 hours/Kg). Suppose that the estimated production cost of running this machine is less than for Filatoi Riuniti's existing machines and is estimated to be $5.70/Kg (as opposed to $11.40/Kg for their existing machines according to Table 2). Would you recommend that Filatoi Riuniti rent the machine? (Try to answer this question without re-optimizing your model.)

Key information from the question:

- Rental cost of spinning machine for *medium* yarn = $3,000/month
- Production capacity = 300 hours/month
- Machine hours required for production = 0.425 hours/Kg
- Cost of production = $5.70/Kg

If we look at the sensitivity analysis report generated for the model from (b), we can see that the allowable increase for the demand for medium yarn is 5,388.4 Kg/month. The extra production capacity from the rental machine would be 300hours/month/0.425hours.kg = 706 Kg/month. Therefore, we do not need to re-solve the problem because the increase in medium yarn production from the rental machine is within the allowable increase for demand for medium yarn. Next, we can calculate the value of the production of 706 Kg more medium yarn per month as $5.70/Kg*706kg = $4,024.20/month. Since $4,024.20/month (revenue) > $3,000/month (rental cost), we conclude that we should recommend renting the additional machine to increase Filatoi Riuniti's production capacity of medium yarn.

(e) A new client is interested in purchasing up to 6,000 Kg/month of medium size yarn. What is the minimum price that Filatoi Riuniti should quote to this new client? Would it be a fixed price per Kg? Which additional question(s) might you ask this client? (In answering this question, assume that Filatoi Riuniti has not decided to expand its spinning machine capacity, and that Filatoi Riuniti does not want to change the prices that they currently charge their existing clients.)

Key information from the question:

- New client wants to purchase up to 6,000 Kg/month of medium size yarn.
- Assume Filatoi Riuniti has not decided to expand its spinning machine capacity
- Assume Filatoi Riuniti does not want to change the prices that they currently charge their existing clients.

i. What is the minimum price that Filatoi Riuniti should quote to this new client?

First, we re-solve the model with an additional demand of 6,000 Kg/month such that demand for medium yarn increases from 28,000 Kg/month to 34,000 Kg/month (see Python script for the new formulation of the problem and solution). We find that the optimal solution is $1,457,238. We find the minimum price that Filatoi Riuniti should quote to this new client by finding the difference between this new optimal solution for total cost and the optimal solution for total cost from part (b). Then, we divide this by maximum amount that the new client will purchase (i.e., 6,000 Kg/month). The calculation is as follows:

($1,457,238 - $1,382,544)/6,000 Kg/month = $74,694/6,000 Kg/month = $12.45/month.

Therefore, the minimum price that Filatoi Riuniti should quote to this new client is $12.45/month.

ii. Would it be a fixed price per Kg?

No. This price will change depending on the quantity of medium yarn and length of time (in months) that the new client wants to purchase medium yarn for. Above, we calculated the minimum price, but this price increase if the client wanted to order less than 6,000 Kg/month, which is possible because we only know that the client is, "interested in purchasing up to 6,000 Kg/month of medium size yarn."

iii. Which additional question(s) might you ask this client?

We should ask the client what the exact quantity of their monthly order will be. Will it be 6,000 Kg/month, or will it be less than 6,000 Kg/month? We should also ask the client if they would like to purchase the same quantity of medium yarn per month? Finally, we should ask the client how long (in months) they would like to purchase medium yarn? The quality and the length of the contract are essential pieces of information for Filatoi Riuniti because they determine what price the new client should be charged and for how long Filatoi Riuniti will have additional revenue from the new client.

(f) You estimate that the production capacity of one of your local mills, De Blasi, could vary within a 20% range of the figures shown in Table 2. Would your recommendations change in the extreme cases? Why or why not? (Try to answer this question without re-optimizing your model.)

If we look at the sensitivity analysis generated for the model from (b), we can see that the capacity constraint for De Blasi is non-binding. We know that it is not binding because the shadow price is equal to 0. Furthermore, we can see from the sensitivity analysis that the slack for this non-binding constraint is 1885.96. A 20% change (up or down) in the right-hand side of the De Blasi capacity constraint will have a magnitude of 2,600*0.2=520. Any change to the right-hand side of a non-binding constraint that is less than its slack or surplus, will cause no change in the optimal solution. Therefore, since the De Blasi capacity constraint is non-binding and 520 < 1885.96, we conclude that even in the extreme cases where De Blasi's production is 20% higher or 20% lower, the solution to the model will not change, and thus neither would our recommendation.

(g) Suppose that you present your proposed outsourcing plan to the owners of the Ambrosi mill. They complain to you that their mill cannot easily produce fine size yarn; in fact, they presently can only produce medium and coarse size yarn, and they would incur substantial one-time set-up costs to ramp up for the production of fine size yarn. However, the optimal solution of the model indicates that it would be in Filatoi Riuniti's interests for the Ambrosi mill to produce fine size yarn. The owners want to maintain good business relations with Filatoi Riuniti, but they do not want to bear the full cost of ramping up for production of fine yarn. The contracts that Filatoi Riuniti currently has with its customers will not expire for at least another 12 months. Up to what amount would you be willing to share the one-time set-up costs for production of fine yarn with the owners of the Ambrosi mill?

We need to re-solve the model with the new constraint that the production capacity of fine yarn at the Ambrosi mill is equal to zero. The maximum amount that the owners of Filatoi Riuniti should be willing to pay for the one-time set-up costs for production of fine yarn will be equal to the difference between the optimal solution from part (b) and the optimal solution when the new constraint is added to the model (see Python script for the new formulation of the problem and solution). When the new constraint is added, we find that the new optimal solution is $1,384,912. Therefore, the maximum amount that the owners of Filatoi Riuniti should be willing to pay can be calculated as follows:

($1,384,912 - $1,382,544) = $2,368

Therefore, the maximum amount the owners of Filatoi Riuniti should be willing to pay for the one-time set-up costs for production of fine yarn is $2,368.

(h) Suppose that you find out that one of the local mills, Giuliani, has the possibility of running an overtime shift (which would double their capacity) by paying its workers only 13% more the normal wage (it is a family-owned business). You know that the workers' salaries contribute to approximately 50% of the prices that the Giuliani mill charges Filatoi Riuniti for spinning yarn. The transportation cost component of the objective function would not change, of course. Modify the model in order to take into account this possibility and re-optimize. Does the optimal solution change? Why? [Helpful modeling hint: Think of the "overtime" part of this mill as a new mill with higher product costs.]

To capture change that the overtime shift at the Giuliani mill would make, we can re-solve the model with a "new" mill which represents the production of yarn at the Giuliani mill during overtime.

<u>Information from question about the "new" mill:</u>

Production capacity = 2,500*2 = 5,000, thus 'new' mill's production capacity = 2,500
Workers' wage is 13% higher
Workers' wage/production cost = 50%

From the above information we can determine the production cost for each type of yarn at the new mill. For each type of yarn, the production cost is calculated as follows:

$$(C_i*0.5)*(1+0.13) + (C_i*0.5)$$
$$C_i = \text{production cost of product i}$$

The table below shows the production cost for each type of yarn at the new mill:

Supplier	Product			
	Extra fine	Fine	Medium	Coarse
New mill	$21.03	$14.80	$11.45	$10.01

When we re-solve the model with the new mill included in the formulation (see Python script for the new formulation of the problem and solution), we find that the new optimal solution is $1,382,340, and the optimal supply strategy is the following:

Supplier	Product			
	Extra fine	Fine	Medium	Coarse
Ambrosi	0	6,250	0	0
Bresciani	4,286	0	0	0
Castri	3,704	0	0	0
De Blasi	0	0	0	0
Estensi	3,846	0	0	0
Filatoi Riuniti	13,164	19,750	18,817	28,000
Giuliani	0	0	7,143	0
New mill	0	0	2,040	0

Therefore, we conclude that the optimal solution changes because with the option of overtime production at the Giuliani mill makes it more efficient to produce the medium yarn at the Giuliani mill during overtime than at the De Blasi firm during regular hours. This is because the cost of producing and transporting medium yarn at the 'new' mill (i.e., during overtime at the Giuliani mill) is less than the cost of producing and transporting medium yarn at the De Blasi mill.

- Cost of production and transportation of 1 Kg at De Blasi = $11.25 + $1.05 = $12.30
- Cost of production and transportation of 1 Kg at 'new' mill = $11.45 + $0.75 = $12.20

Since $12.20 < $12.30, it becomes more efficient to produce the medium yarn at Giuliani during overtime.