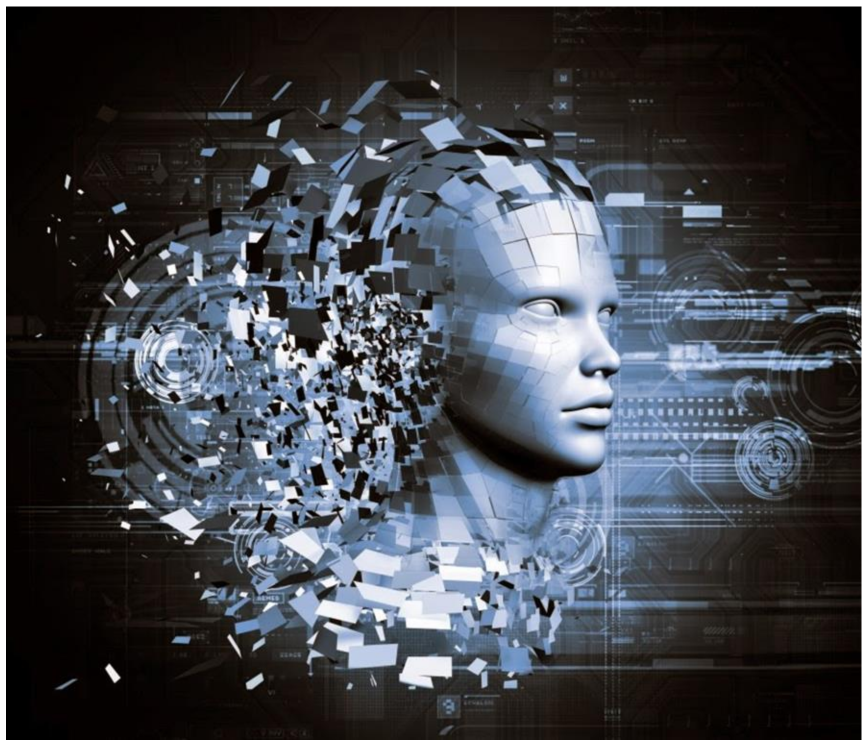
5/15/2018

Control System-I Lab - MCT-333L

Lab Reports



Submitted By:

Muhammad Waqar Nawaz

(2015-MC-167)

Submitted To:

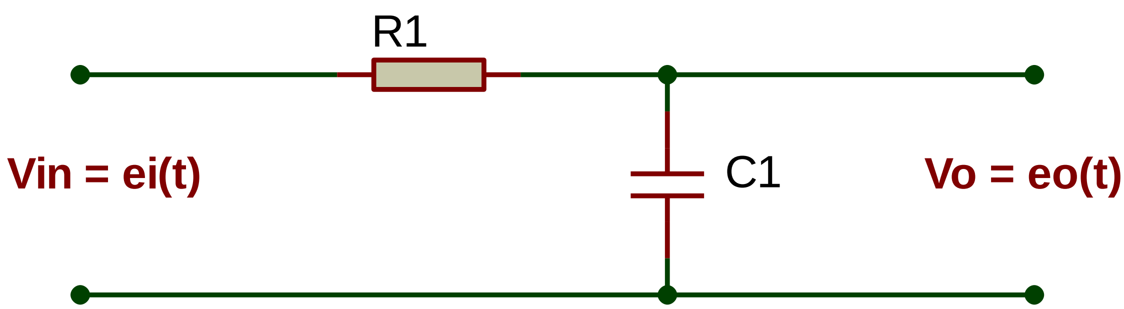
Engr. Abdullah Tahir

Lab # 1

First Order System

Objectives:

1. To model the following circuit and also select the components for maximum Ts = 0.8s and maximum Tr = 0.06s



Theory:

Modeling:

$$R\frac{dq}{dt}(t) + \frac{1}{C}q(t) = e_i(t)$$

❖ $q(t)=Ce_o(t)$

$$RCSE_o(s) + E_o(s) = E_i(s)$$

$$\frac{Eo(s)}{Ei(s)} = \frac{1}{RCS+1}$$

$$\frac{Eo(s)}{Ei(s)} = \frac{\frac{1}{RC}}{(S+\frac{1}{RC})}$$

Compare it with General T.F of 1st order equation T.F = $\frac{a}{S+a}$

$$a = 1/RC$$

$$Ts = 3.91/a$$

$$Tr = 2.195/a$$

Required:

$$a = 3.91/0.8$$

$$a = 1/RC = 4.8875$$

$$a = 2.195/0.06$$

$$a = 1/RC = 36.58$$

$$\mathbf{4.8875 < a < 36.58}$$

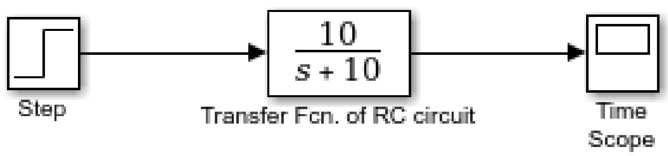
Let a = 10 then,

$$R = 1K(ohm)$$

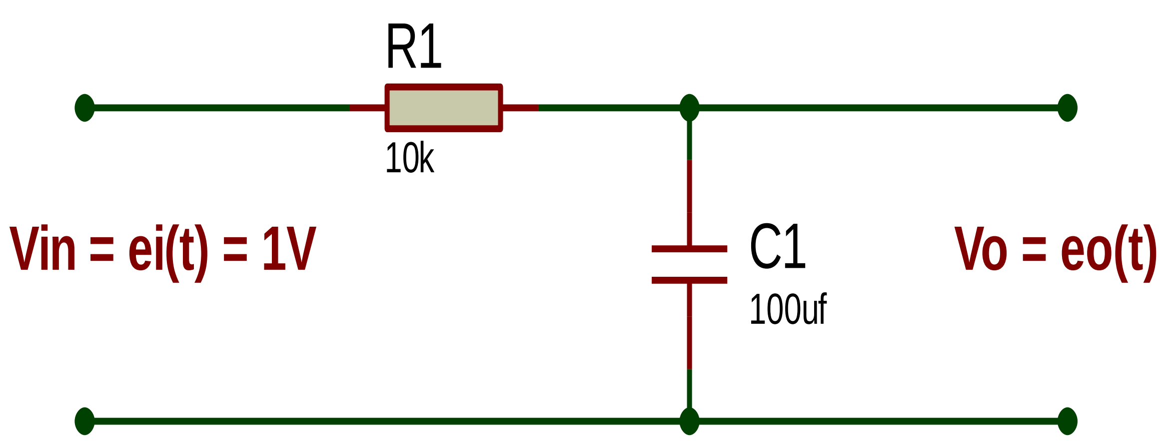
$$C = 100 \times 10^{-6}\ f$$

$$\text{Step input} = e_i(t) = 1V$$

Simulation:

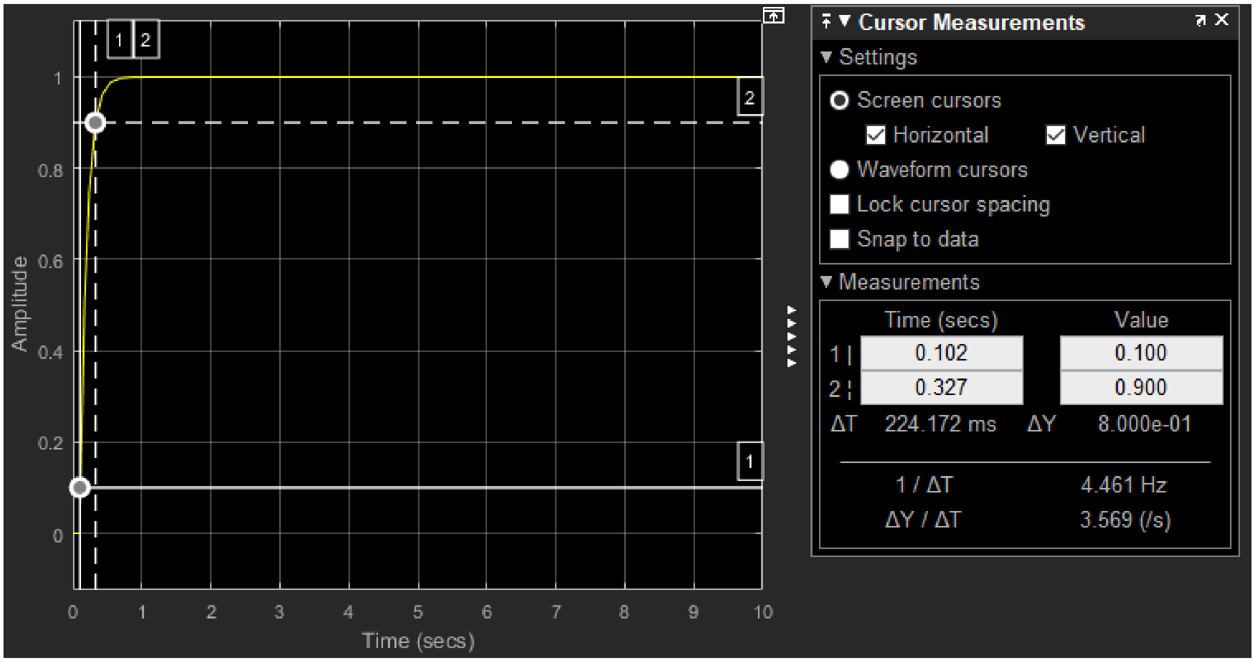


Hardware Schematic:



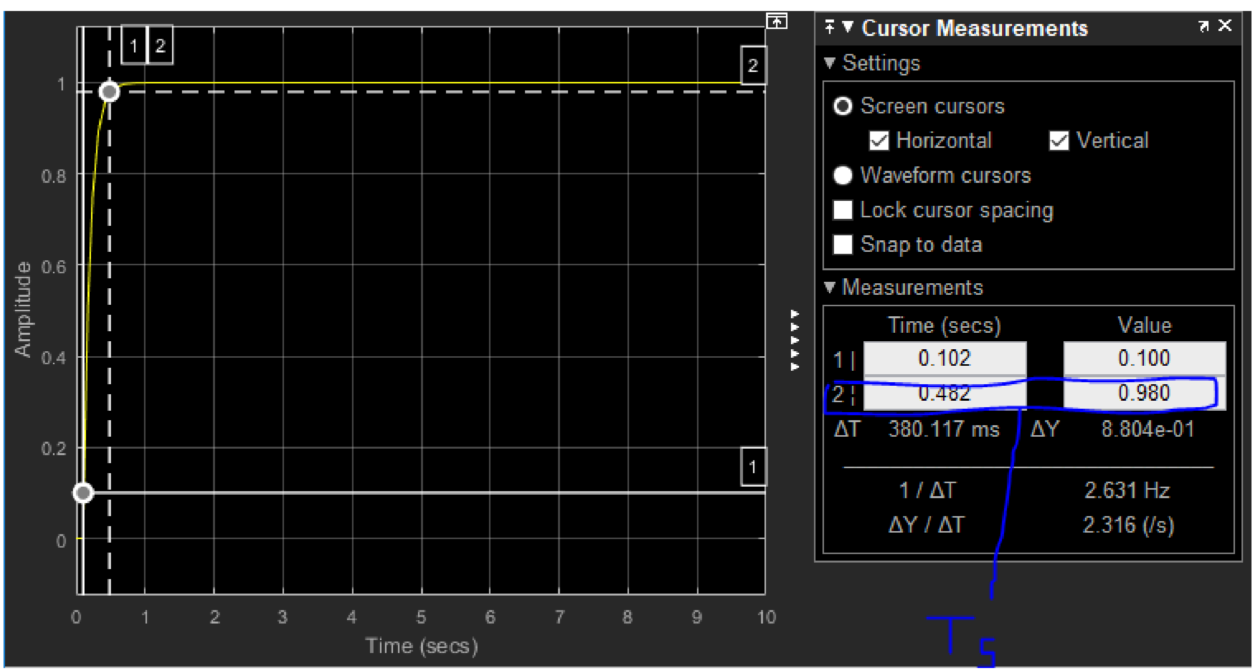
Result:

Rise Time (Tr):



Tr = 0.327 – 0.102 = 0.225s

Settling Time (Ts):



Ts = 0.482s

Comments:

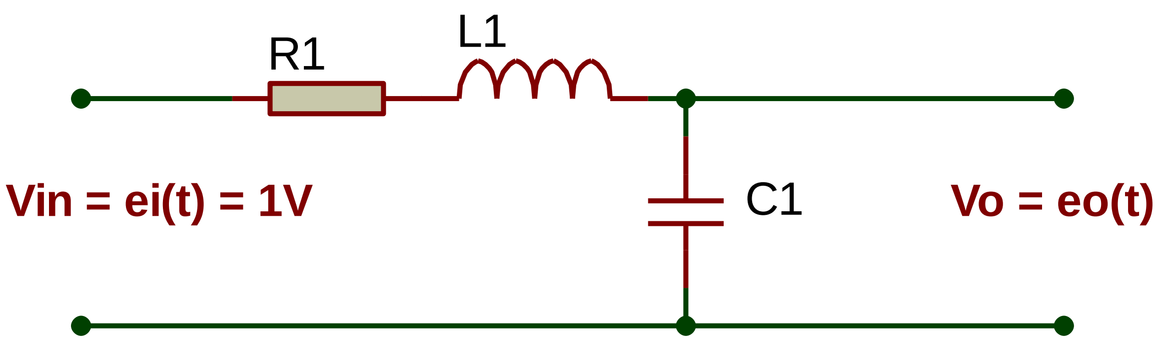
- System settling time less than 0.8s.

Lab # 2

Second Order System

Objectives:

1. To model the following circuit and also select the components for
    - Under-damped
    - Over damped
    - Critically damped
    - Un-damped



Theory:

Modeling:

$$L\,(d^2q/dt^2)\,(t)\;R\frac{dq}{dt}(t) + \frac{1}{C}q(t) = e_i(t)$$

- ❖ $q(t)=Ce_o(t)$

$$LCS^2E_o(s) + RCSE_o(s) + E_o(s) = E_i(s)$$

$$\frac{Eo(s)}{Ei(s)} = \frac{1}{(LCS+RC)S+1}$$

$$\frac{Eo(s)}{Ei(s)} = \frac{\frac{1}{LC}}{(\left(S+\frac{R}{L}\right)S+\frac{1}{LC})}$$

Compare it with General T.F of 2st order equation T.F = $\omega_n^2 / (s^2 + 2\rho\omega_n s + \omega_n^2)$

$$a = R/L = 2\rho\omega_n$$

$$\rho = a/(2\omega_n)$$

$$b = 1/(LC) = \omega_n^2$$

Required:

- Under-damped

$$0 < \rho < 1$$

R = 40 (ohm); C = 510x10-6F; L= 1H; ei= 1volt

$$\rho = 0.452$$

- Over damped

$$\rho < 0$$

R = 100 (ohm); C = 510x10-6F; L= 1H; ei= 1volt

$$\rho = 1.129$$

- Critically damped

$$\rho = 1$$

R = 885.6 (ohm); C = 510x10-6F; L= 1H; ei= 1volt

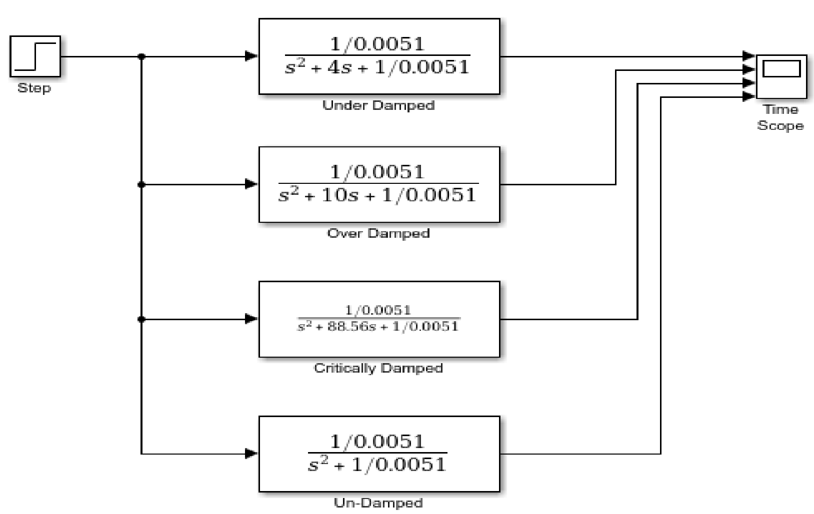
$$\rho \approx 1$$

- Un-damped

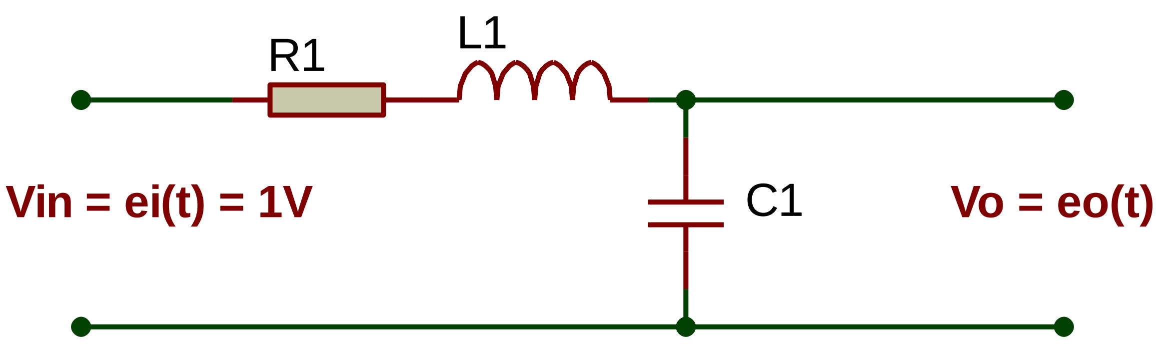
$$\rho = 0$$

R = 0 (ohm); C = 510x10-6F; L= 1H; ei= 1volt

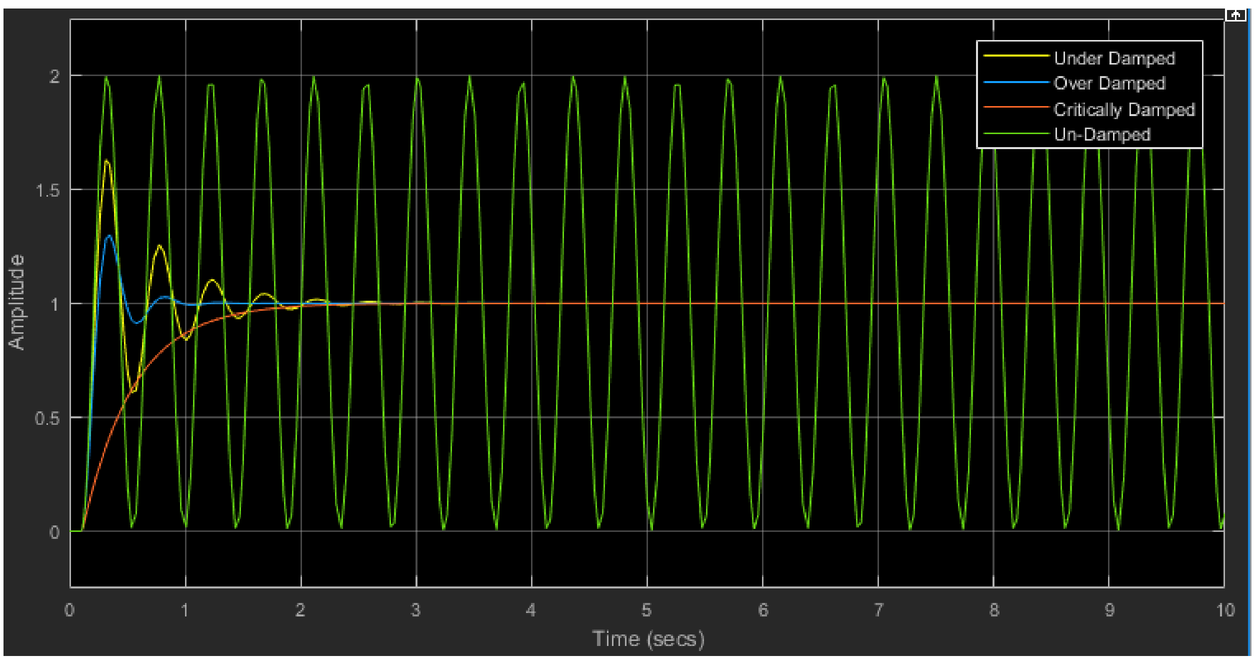
Simulation:



Hardware Schematic:



Result:



Comments:

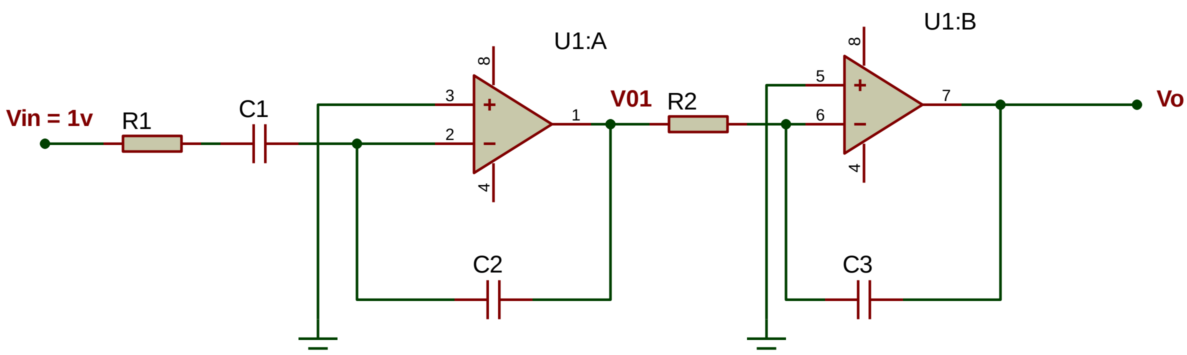
- In 1st order system value starts from zero of the graph but in Over damped system value starts after zero of the graph.

Lab # 3

Operational Amplifier

Objectives:

1. Design the following circuit for maximum OS% = 10%, maximum Tp = 0.2s and maximum Ts = 0.6s.



Theory:

Modeling:

$$\frac{Vo1}{Vin} = -\frac{Z2(s)}{Z1(s)}$$

$$Z_1(s) = (1/C_1S) + R_1 = (R_1C_1S + 1) / (C_1S)$$

$$Z_2(s) = (1/C_2S)$$

$$\frac{Vo1}{Vin} = -C_1 / (R_1C_2C_1S + C_2)$$

$$\frac{Vo}{Vo1} = -\frac{Z4(s)}{Z3(s)}$$

$$\frac{Vo}{Vo1} = -1 / (R_2C_3S)$$

$$\frac{Vo}{Vin} = ((1 / R_1R_2C_2C_3) / (S^2 + (1 / (R_1C_1))))$$

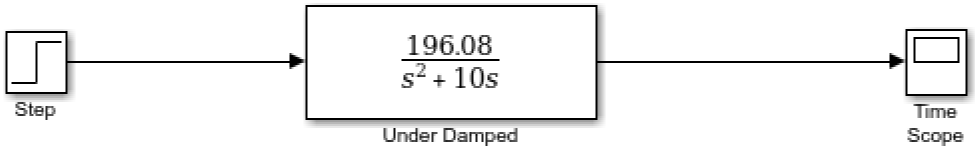
Required:

- Under-damped

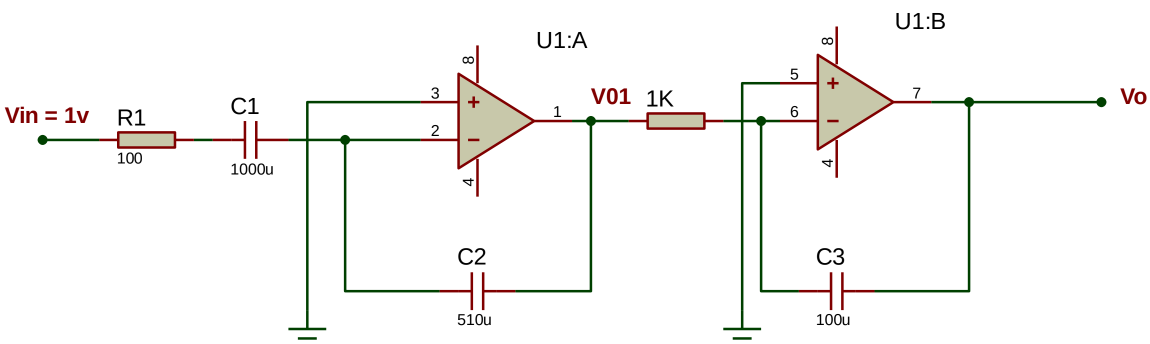
$$0 < \rho < 1$$

R_1 = 100 (ohm); R_2 = 1000 (ohm); C_2 = 510x10-6F; C_3= 100uF; C_1 = 1000uF; ei= 1volt

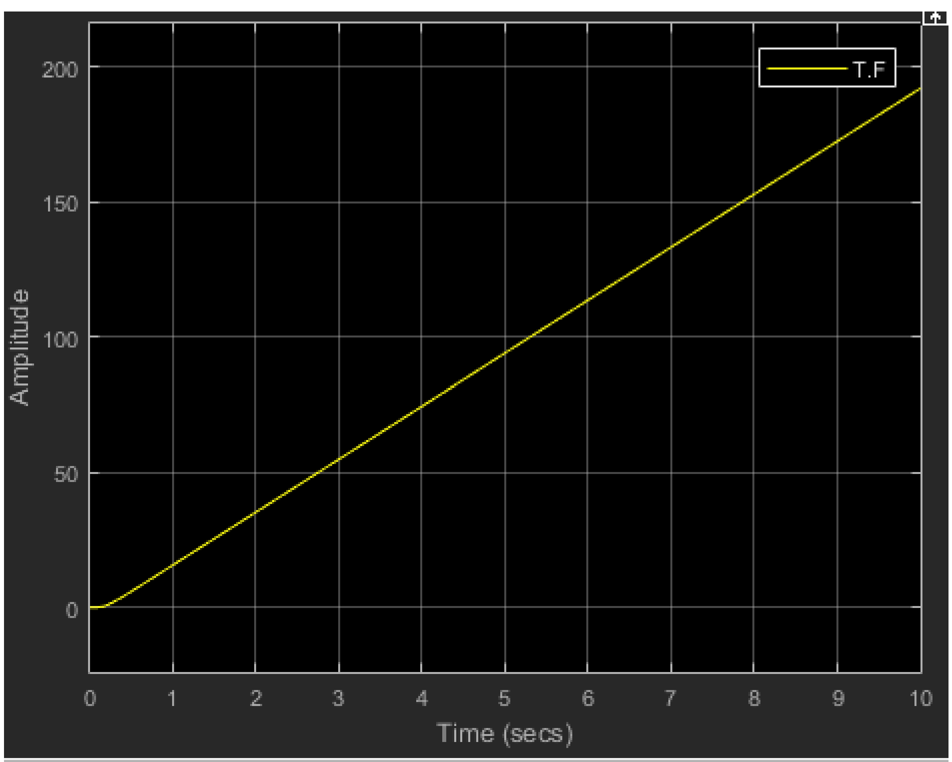
Simulation:



Hardware Schematic:

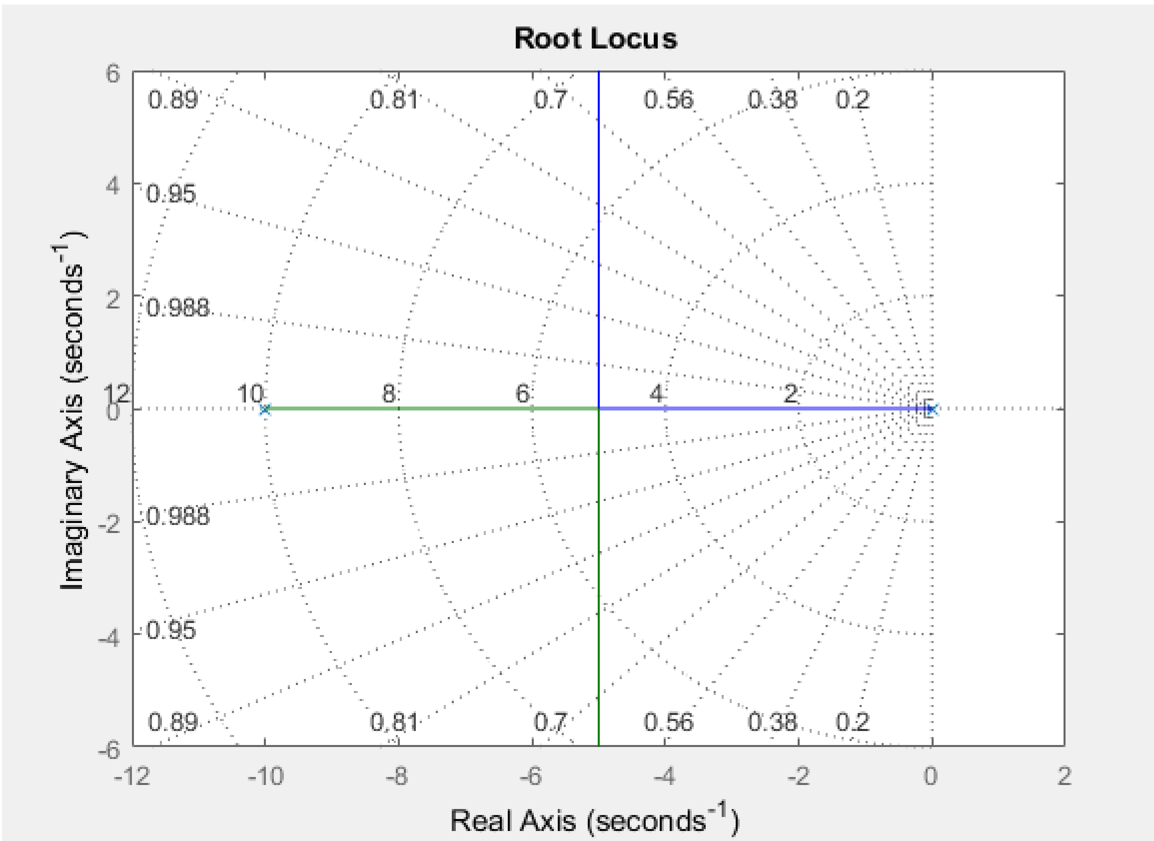


Result:



Comments:

- According to root locus diagram its %OS didn't reached 10%.



Lab # 4

Stability Design via Routh-Hurwitz

Objectives:

1. Find out the range of gain in which system is stable.

$$G(s) = \frac{0.25K(s+0.435)}{s^4+3.456s^3+3.457s^2+0.719s+0.0416}$$

Theory:

$$T(s) = \frac{G(s)}{1+G(s)}$$

$$= \frac{\frac{0.25K(s+0.435)}{s^4+3.456s^3+3.457s^2+0.719s+0.0416}}{1+\frac{0.25K(s+0.435)}{s^4+3.456s^3+3.457s^2+0.719s+0.0416}}$$

$$= \frac{0.25K(s+0.435)}{s^4+3.456s^3+3.457s^2+(0.719+0.25K)s+(0.0416+0.109K)}$$

s^4	1	3.457	$0.0416+0.109K_1$
s^3	3.456	$0.719+0.25K_1$	
s^2	$11.228-0.25K_1$	$0.144+0.377K_1$	
s^1	$\dfrac{-0.0625K_1^2+1.324K_1+7.575}{11.228-0.25K_1}$		
s^0	$0.144+0.377K_1$		

From s^2 the value of K, where over-all answer should be in +ve

$$11.228 - 0.25K = 0$$

$$K = 44.91$$

$$-\infty < K < 44.91$$

From s^1 the value of K, where over-all answer should be in +ve

Quadratic equation so, $K = -4.685$ and 25.87

For +ve numerator $-4.685 < K < 25.87$

From s^0 the value of K, where over-all answer should be in +ve

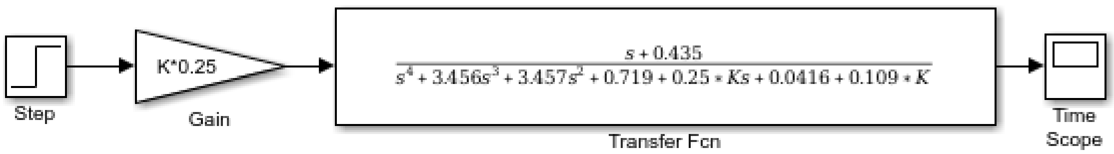
$$0.144 + 0.377K = 0$$

$$K = -0.3819$$

$$-0.3819 < K < \infty$$

Over-all System Stability lies b/w **– 0.3819 < *K* < 25.87**

Simulation:

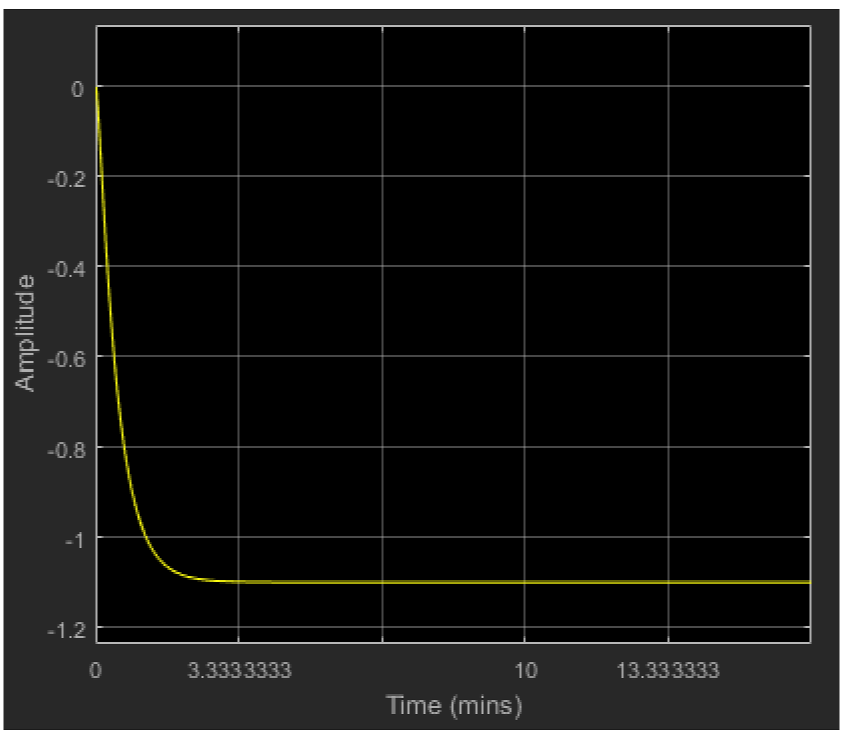


Hardware Schematic:

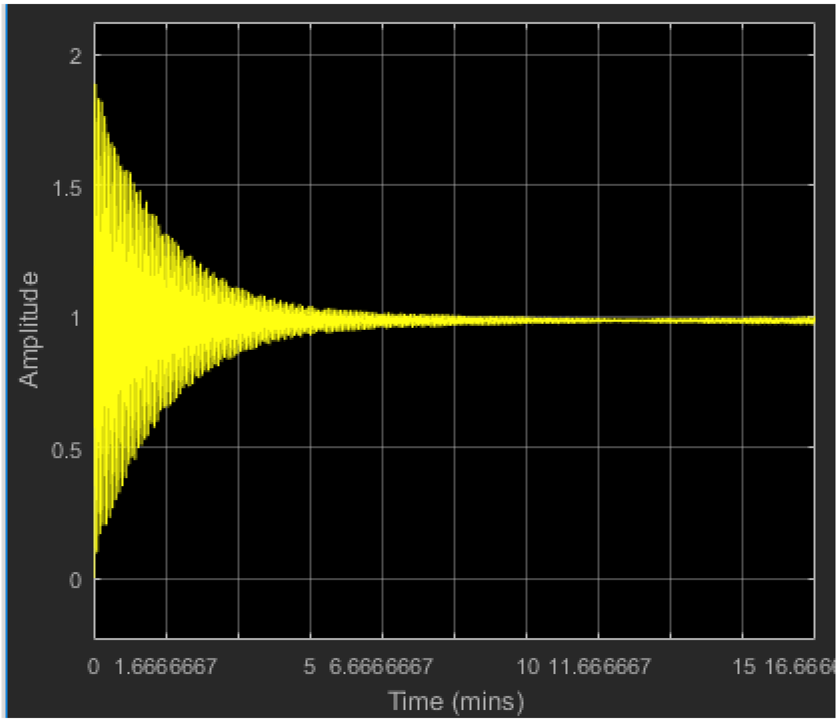
No hardware for this lab.

Result:

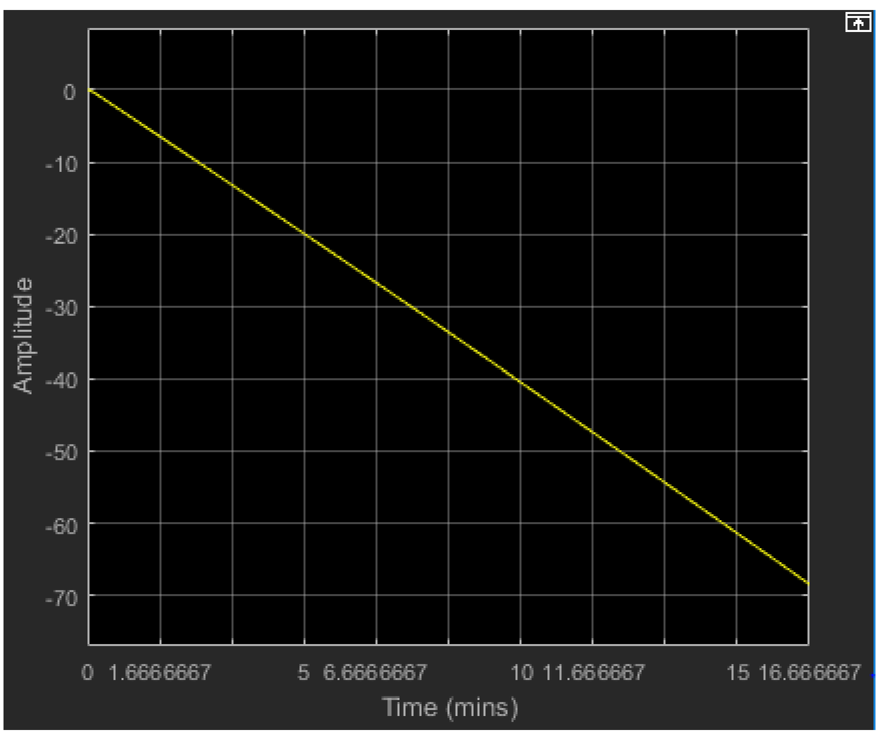
$K = -0.2$:



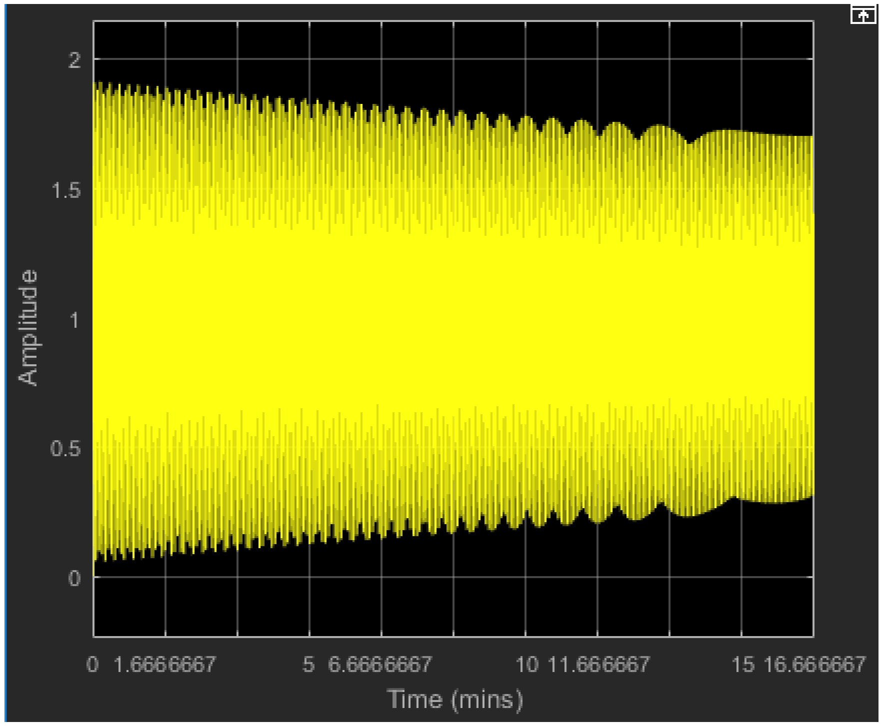
$K = 25$:


2
1.5
1
0.5
0
Amplitude
0 1.6666667
5 6.6666667
10 11.666667
15 16.666
Time (mins)

$K = -0.382$:


0
-10
-20
-30
-40
-50
-60
-70
Amplitude
0 1.6666667
5 6.6666667
10 11.666667
15 16.666667
Time (mins)

$K = 25.87$:



Comments:

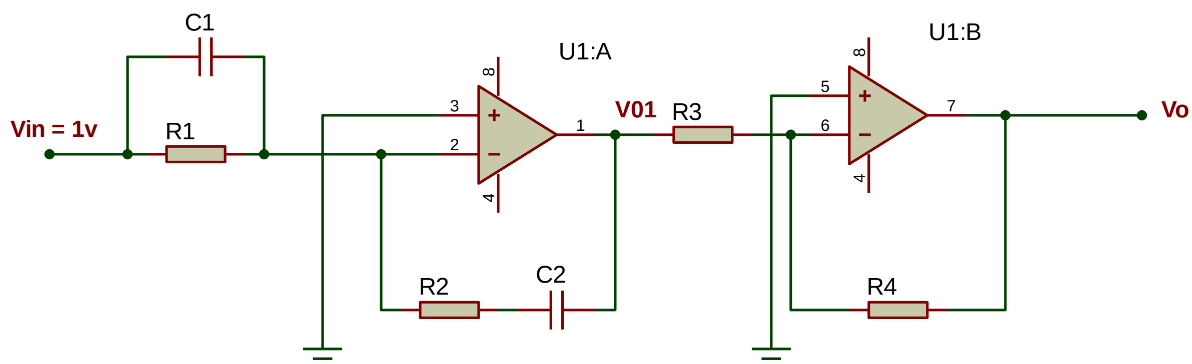
- System was stable when K value lie b/w –0.382 to 25.87.
- When K value is less than or equal to –0.382 and great than or equal to 25.87 then system became unstable.

Lab # 5

Operational Amplifier

Objectives:

1. Design the following circuit for maximum DC gain =20, Kp = 5, Ki = 0.9 & Kd = 1.6.



Theory:

Modeling:

$$\frac{Vo1}{Vin} = -\frac{Z2(s)}{Z1(s)}$$

$$1 / Z_1(s) = C_1S + 1 / R_1 = (1/C_2S)$$

$$Z_1(s) = R_1 / (1 + R_1C_1S)$$

$$Z_2(s) = (1 + R_2C_2S) / C_2S$$

$$\frac{Vo1}{Vin} = -(R_1R_2C_1C_2S^2 + (R_1C_1 + R_2C_2)S + 1) / (R_1C_2S)$$

$$\frac{Vo}{Vo1} = -\frac{Z4(s)}{Z3(s)}$$

$$\frac{Vo}{Vo1} = -R_4 / R_3$$

$$\frac{Vo}{Vin} = (R_1R_2R_4C_1C_2S^2 + (R_1C_1 + R_2C_2) R_4S + R_4) / (R_3R_1C_2S)$$

In form of Kp, Ki, Kd

$$T.F = Gain\ (Kp + Ki/S + KdS)$$

$$Dc\ gain = R_4$$

$Kp = (R_1C_1 + R_2C_2) / (R_3R_1C_2)$ eq. (a)

$Ki/S = 1 / (R_3R_1C_2S)$ eq. (b)

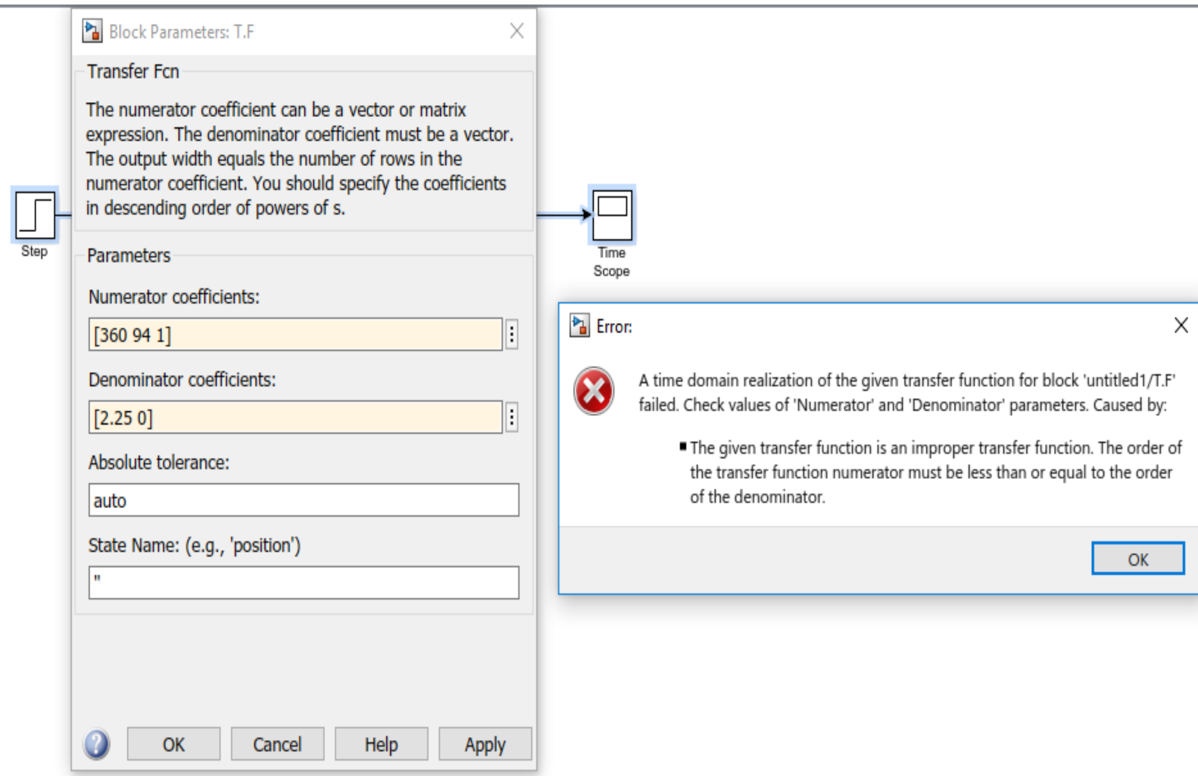
$KdS = (R_2C_1S) / R_3$ eq. (c)

Required:

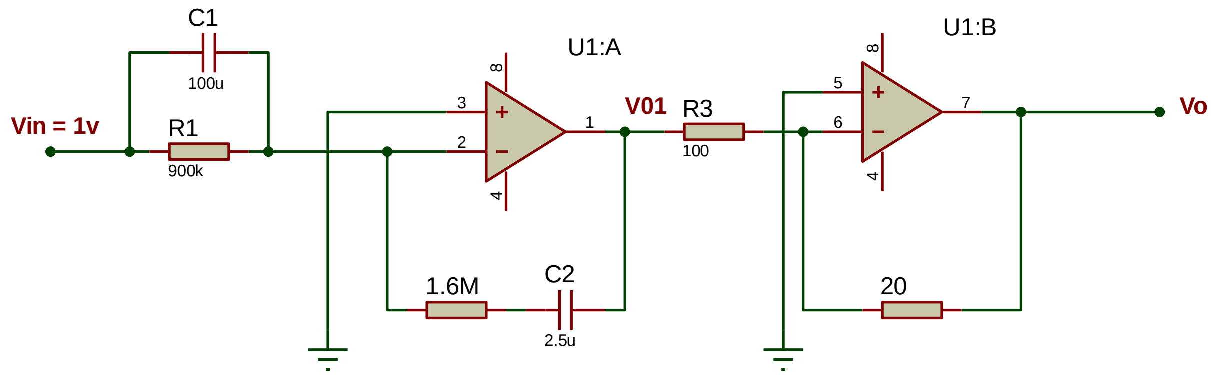
- Dc Gain = 20
- Kp =5
- Ki =0.9
- Kd = 1.6

R_1 = 10k (ohm); R_2 = 1.6M (ohm); R_3 = 100 (ohm); R_4 = 20 (ohm); C_1 = 100x10-6F; C_2= 2.5uF; ei= 1volt

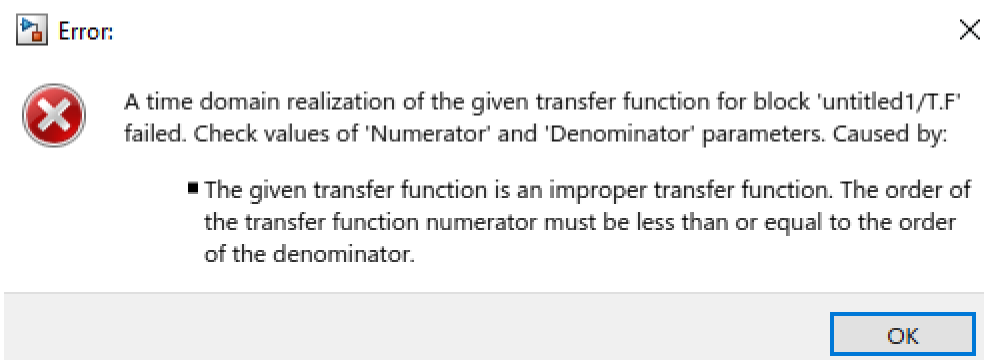
Simulation:



Hardware Schematic:



Result:



Comments:

- According to equation a, b and c different values are required for R_1, C_2 etc. This logic is wronged.

Lab # 6

Steady-State Error

Objectives:

1. Find the steady-state error for the inputs of 5u(t) (Step Input), 5tu(t) (Ramp Input).

$$G(s) = \frac{120(s+2)}{(s+3)(s+4)}$$

Theory:

SSE for Step Input:

$$e(\infty) = e_{step}(\infty) = 5/\left(1 + \lim_{s\to 0} G(s)\right)$$

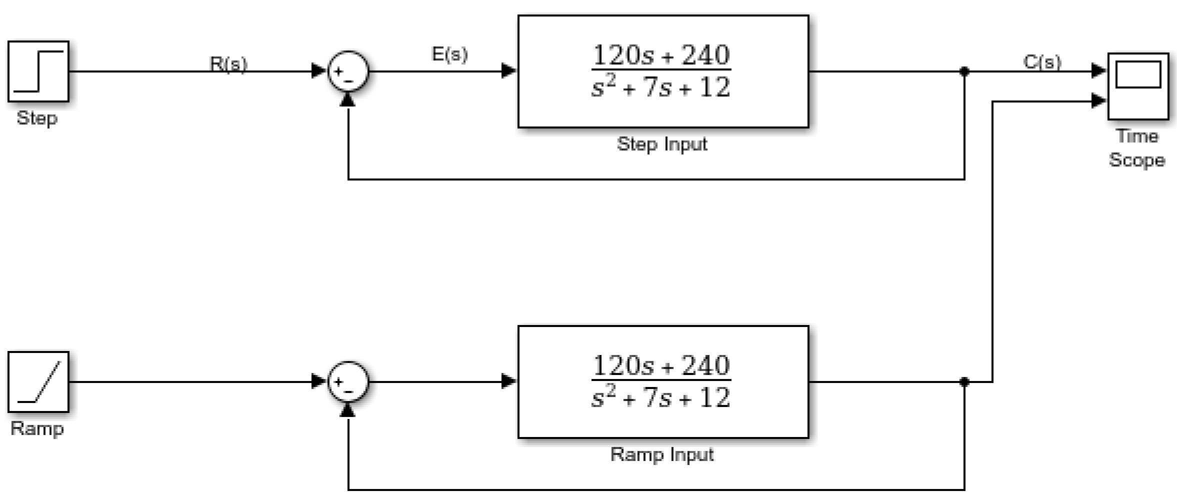
$$e_{step}(\infty) = e(\infty) = 5/21$$

SSE for Ramp Input:

$$e(\infty) = e_{ramp}(\infty) = 5/\left(\lim_{s\to 0} G(s)\right)$$

$$e_{step}(\infty) = e(\infty) = 5/0 = \infty$$

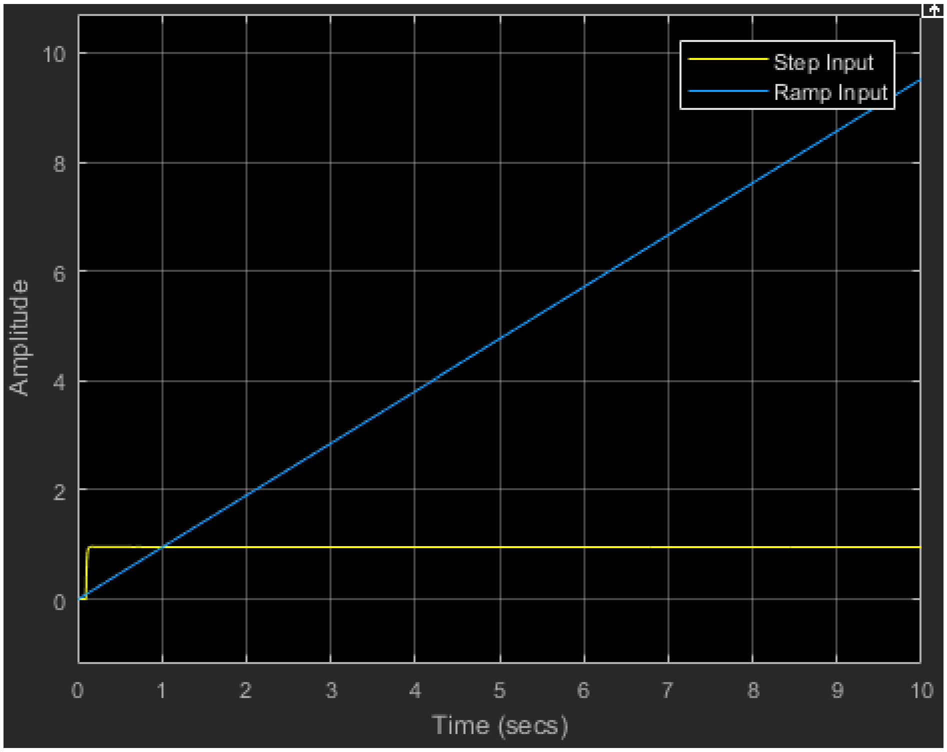
Simulation:



Hardware Schematic:

No hardware for this lab.

Result:



Comments:

- For ramp Input SSE of T.F approaches to infinity.

Lab # 7

Design a Lag Compensator

Objectives:

1. Improve SSE by a factor of 10 if system is operating with damping ratio of 0.174.

$$G(s) = \frac{1}{(s+1)(s+2)(s+10)}$$

Theory:

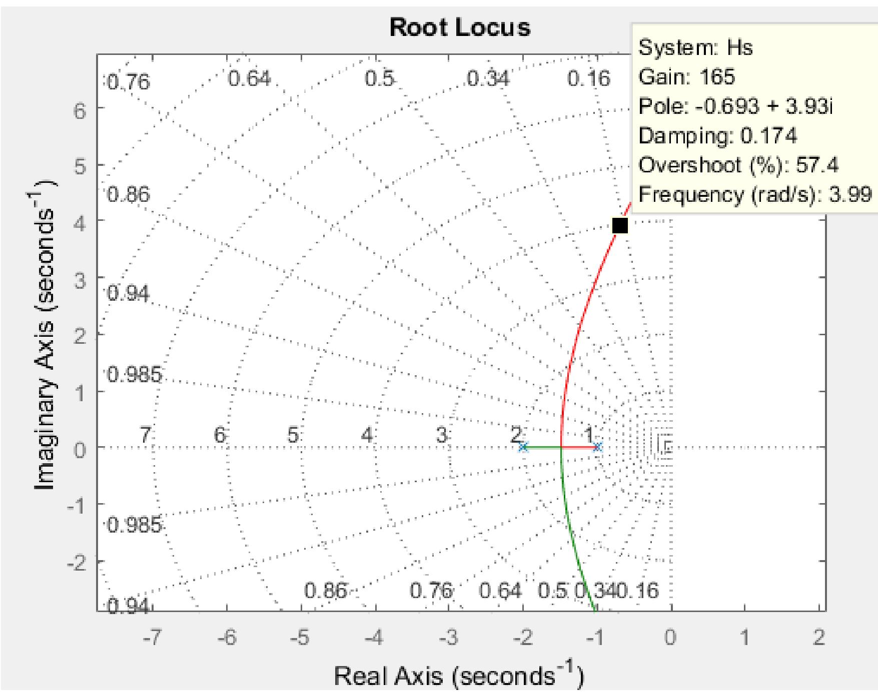
Root Locus:

>> num=[1];

>> den=[1 13 32 20];

>> Hs=tf(num,den)

>> rlocus(Hs)



K_p = 165 / ((1)(10)(2))

K_p = 8.25

$$SSE = e(\infty) = 1 / (1 + K_p)$$

$$e(\infty) = 0.108$$

Required SSE:

$$SSE_{req.} = e(\infty) / 10 = 0.0108$$

$$K_p = 91.6$$

Compensator:

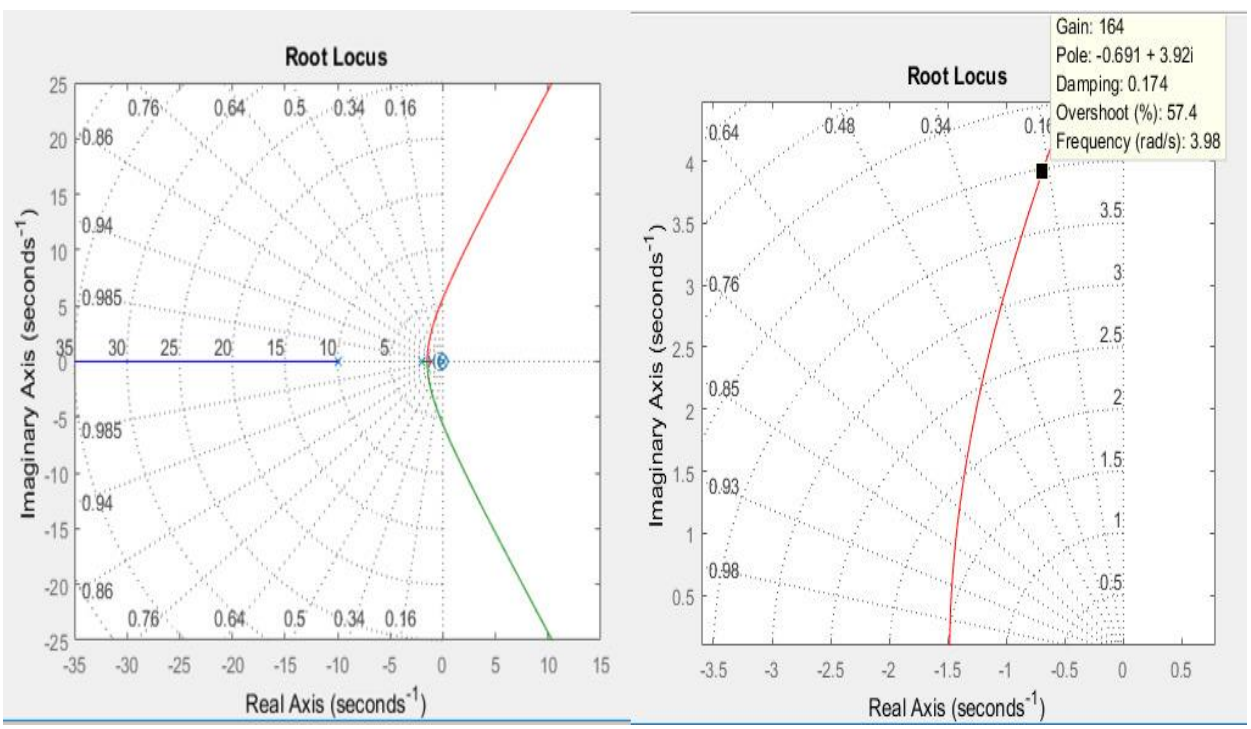
$$K_{comp.} / K_{uncomp.} = z_c / p_c = 91.6/8.25 = 11.1$$

Let p_c 0.001

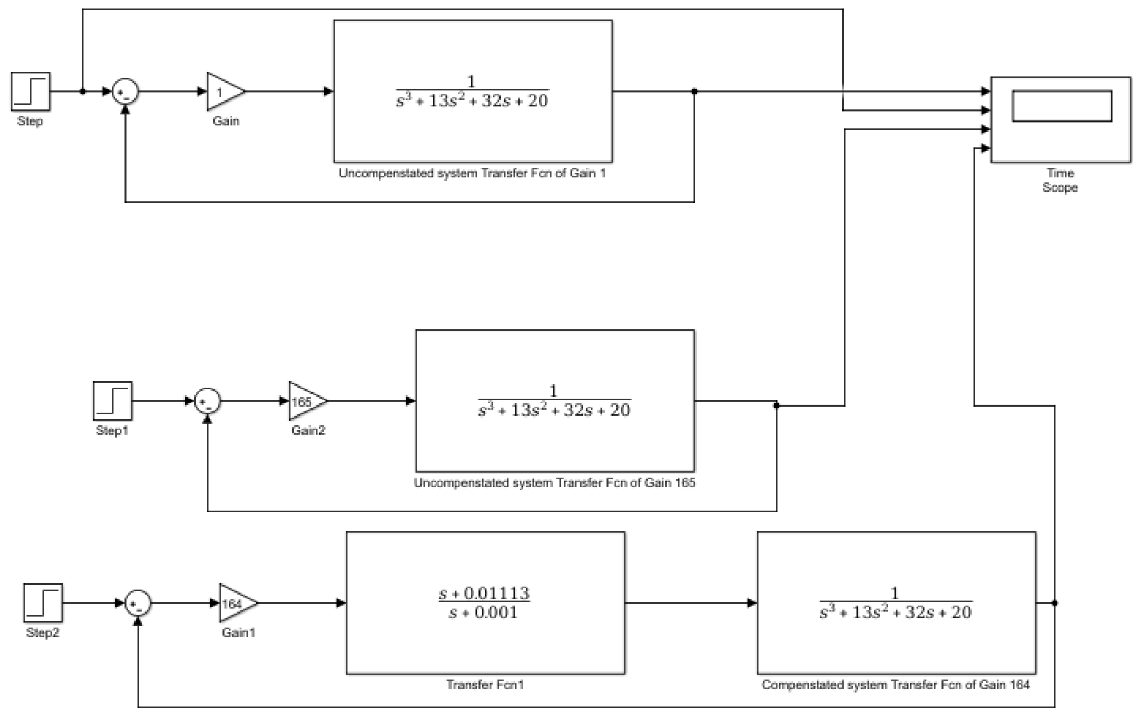
$$Z_c = (0.001)(11.1) = 0.0111$$

Root Locus:

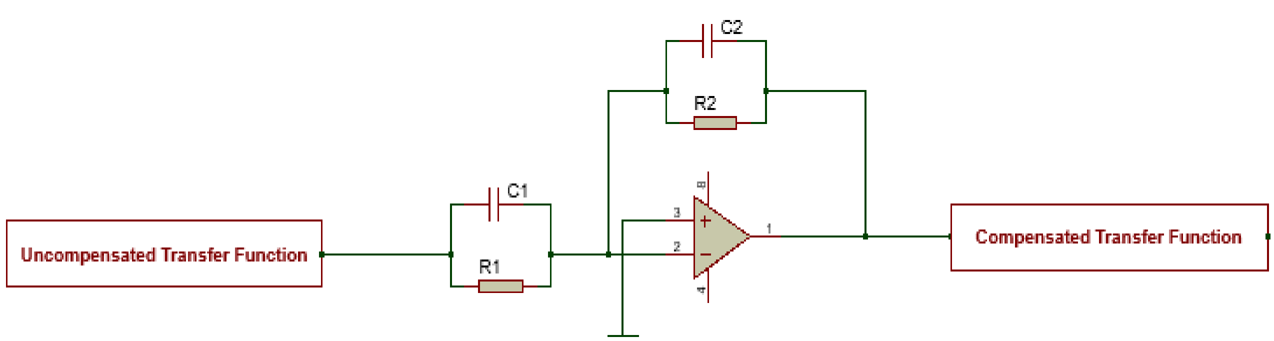
```
>> num=[1 0.011];

>> den=[ 1 13.001 32.013 20.032 0.0200];

>> Hs=tf(num,den)

>> rlocus(Hs)
```



Simulation:



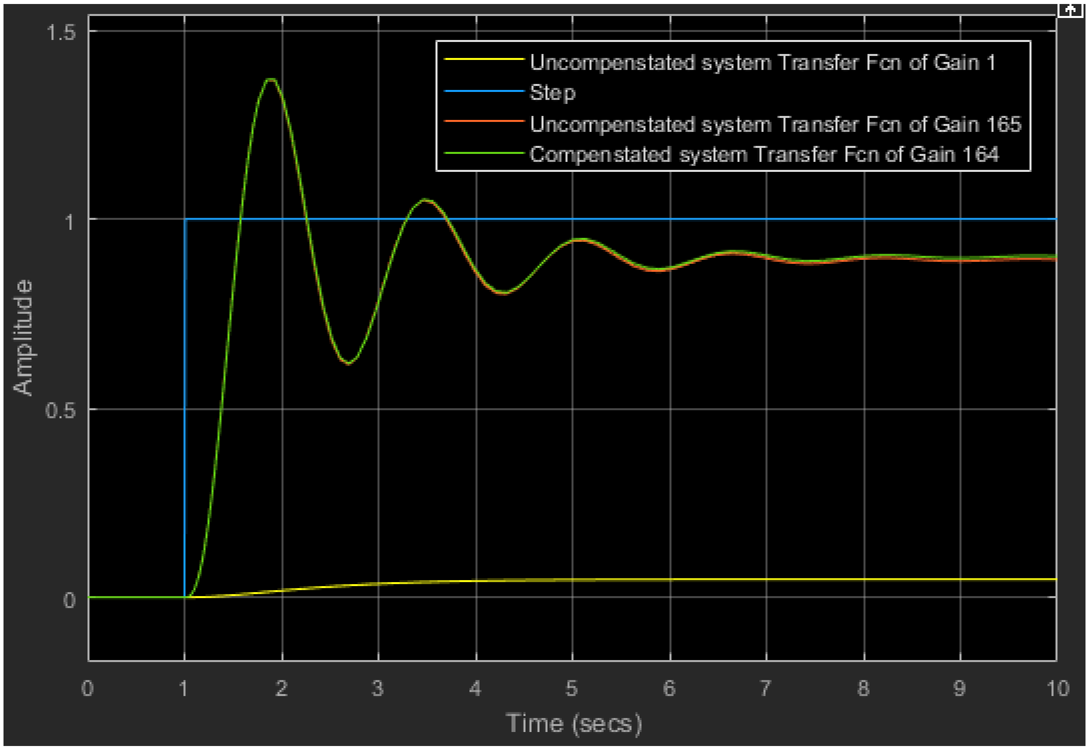
Hardware Schematic:



$$R_1C_1 < R_2C_2$$

$$\text{Gain} = K = -\,C_1 / C_2$$

Result:



Comments:

- Root locus diagram remain unchanged in Lag Compensation.
- Lag compensator didn't effect on transient response of any transfer function.
- It is used to improve SSE.

Lab # 8

Design a Lead Compensator

Objectives:

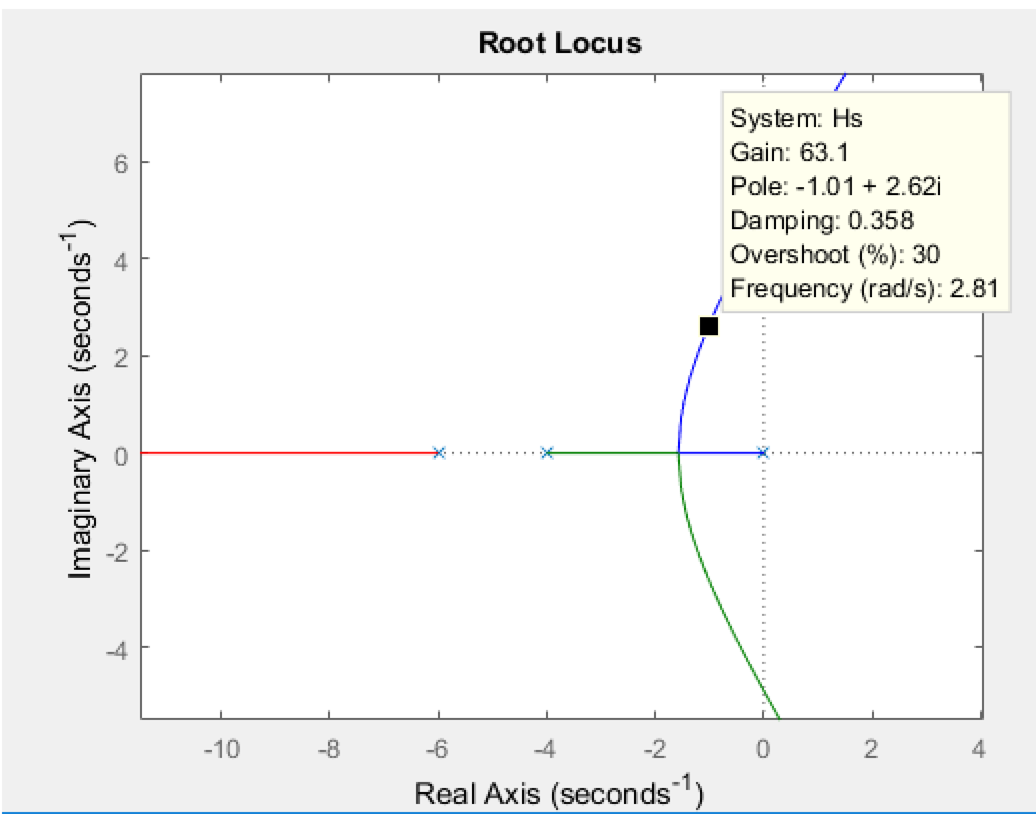
1. Improve transient response by a factor of 2 if system is operating at 30% overshoot.

$$G(s) = \frac{k}{s(s+4)(s+6)}$$

Theory:

Root Locus:

```
>> num=[1];

>> den=[1 10 24 0];

>> Hs=tf(num,den)

>> rlocus(Hs)
```



$$ʒ = \cos\theta$$

$$\theta = \cos^{-1}(ʒ)$$

$$\theta = 110.98$$

$$\text{Pole} = -1.01 + 2.62j$$

$$T_s = 4/1.01 = 3.96$$

Required T_s:

$$(\text{Transient Resposne})_{req.} = T_s / 2 = 1.98$$

$$\text{Real part} = \sigma_d = 2.02$$

$$\text{Img. Part} = -2.02\ (\tan(110.98) = 5.3$$

Compensator:

Let $z_c = 5$

$$p_c = 42.96$$

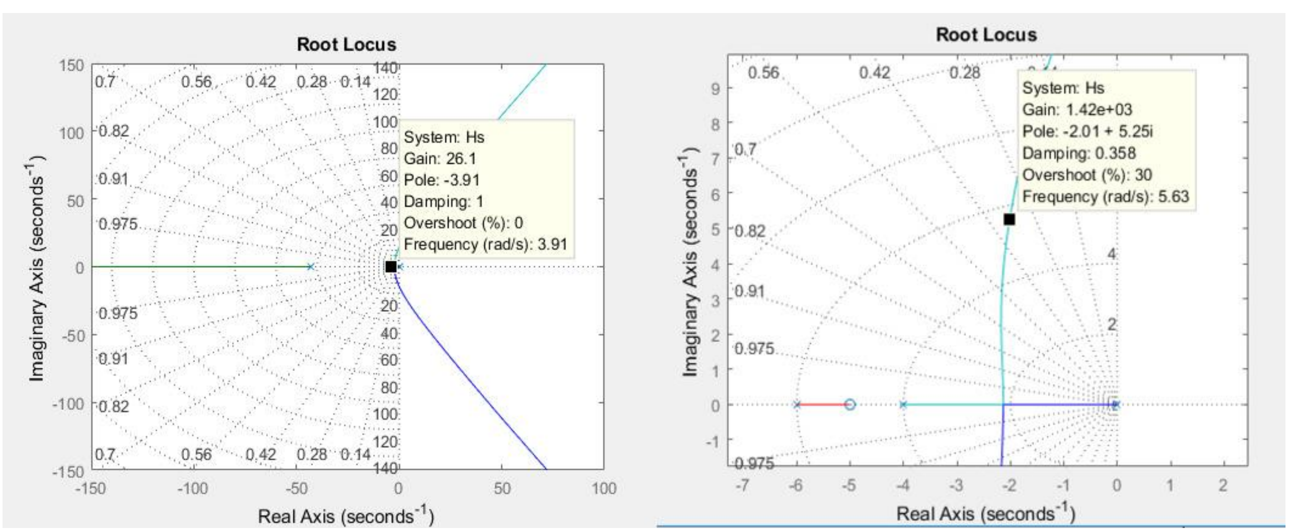
Root Locus:

>> num=[1 5];

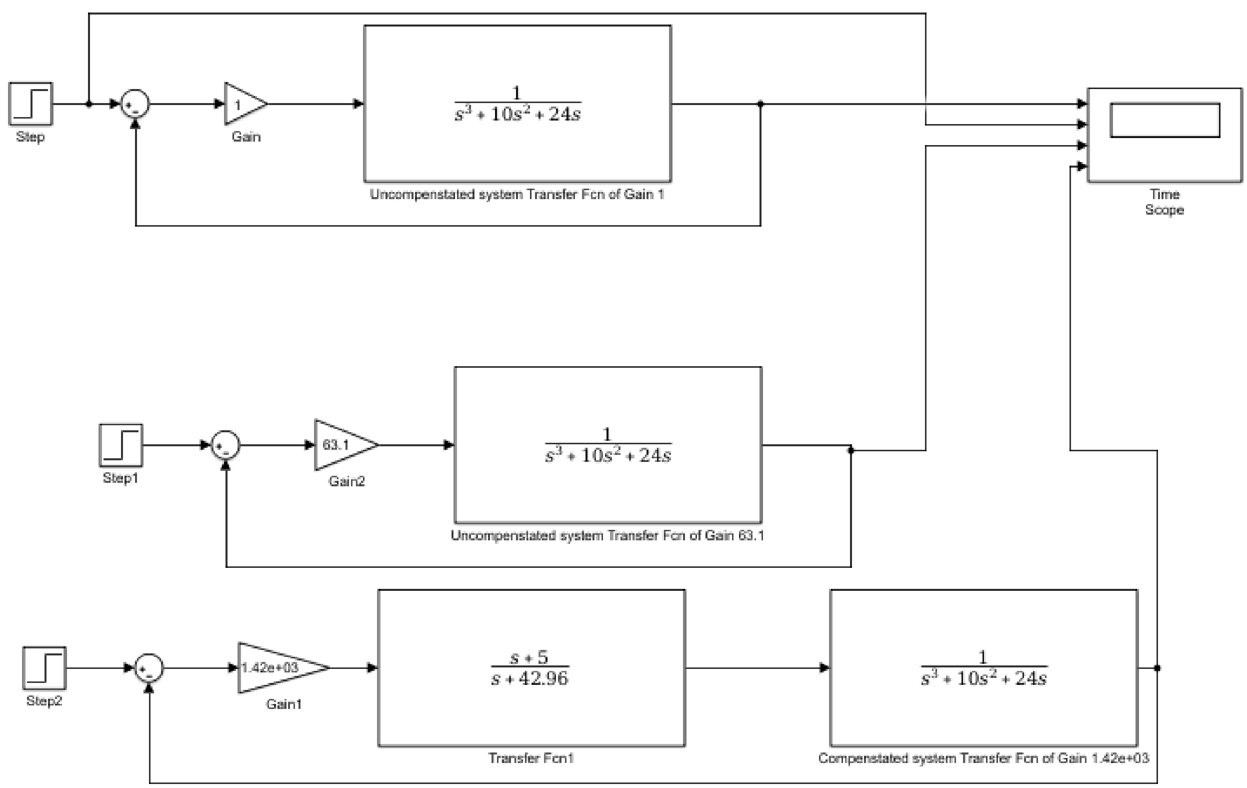
>> den=[1 52.96 453.6 1031.04 0];

>> Hs=tf(num,den)

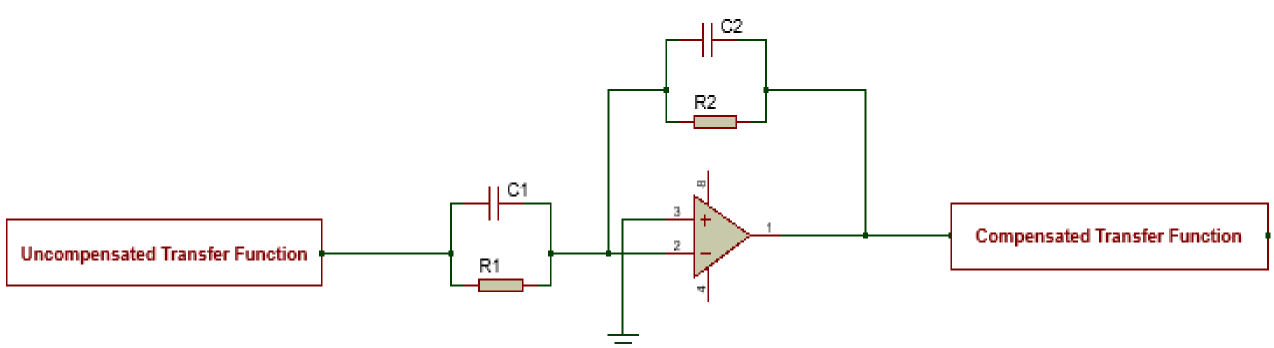
>> rlocus(Hs)



Simulation:



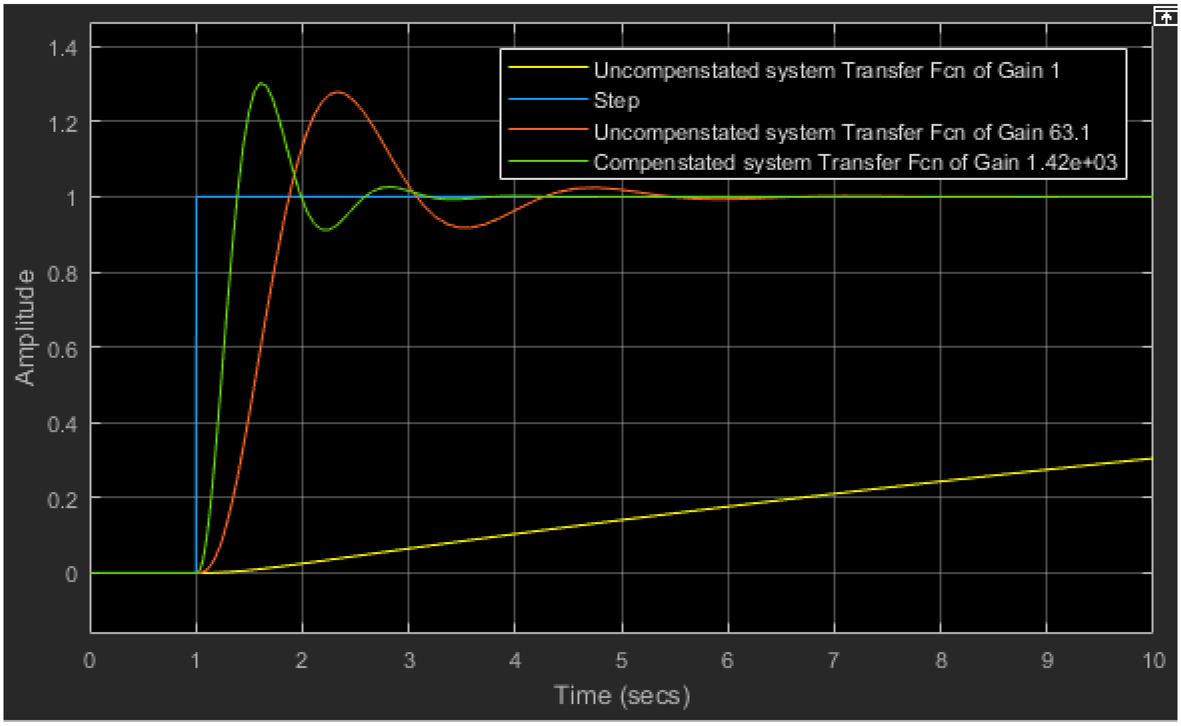
Hardware Schematic:



$$R_1C_1 > R_2C_2$$

$$\text{Gain} = K = -C_1 / C_2$$

Result:



Comments:

- Root locus diagram changed in Lead Compensation.
- Lead compensator didn’t effect on SSE of any transfer function.
- It is used to improve transient response of any transfer function.

Lab # 9

Design a Lead-Lag Compensator

Objectives:

1. Improve transient response and SSE by a factor of 2 and 10 respectively if system is operating at 20% overshoot.

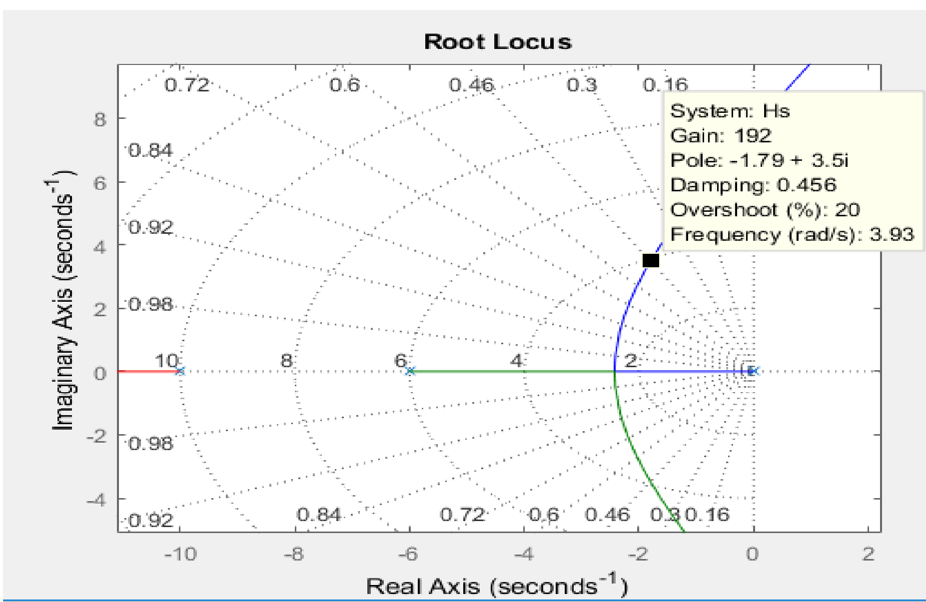
$$G(s) = \frac{k}{s(s+10)(s+6)}$$

Theory:

Lead Compensation

Root Locus:

```
>> num=[1];
>> den=[1 16 60 0];
>> Hs=tf(num,den)
>> rlocus(Hs)
```



$$ʒ = \cos\theta$$

$$\theta = \cos^{-1}(0.456)$$

$$\theta = 62.87$$

$$\text{Pole} = -1.79 + 3.5j$$

$$T_s = 4/1.79 = 2.23$$

Required T_s:

$$(\text{Transient Resposne})_{req.} = T_s / 2 = 1.117$$

$$\text{Real part} = \sigma_d = 3.58$$

$$\text{Img. Part} = 3.58\ (\tan(62.87)) = 6.987$$

$$S = -3.58 + 6.987j$$

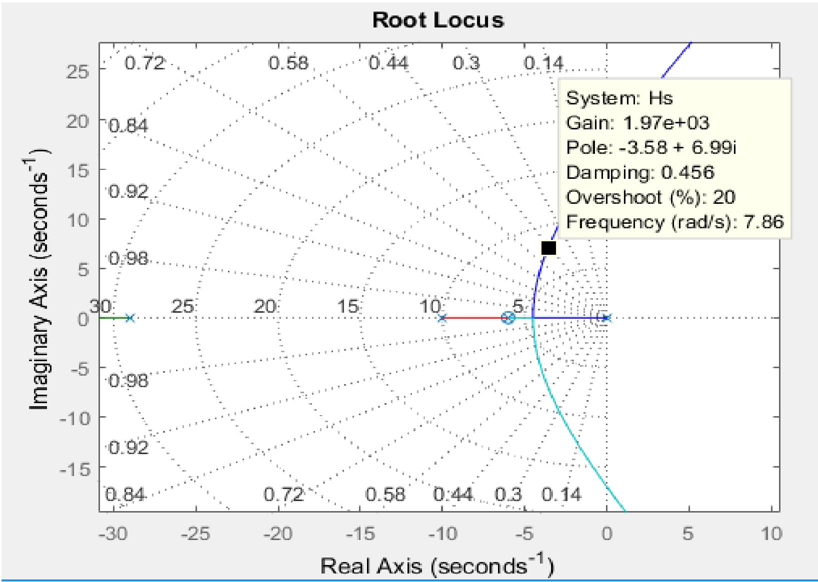
Compensator:

Let $z_c = -6$

$$p_c = -29.03$$

Root Locus:

```
>> num=[1 6];
>> den=[1 45.03 524.485 1741.8 0];
>> Hs=tf(num,den)
>> rlocus(Hs)
```



Lead Compensation

$$SSE = e(\infty) = 1 / K_v$$

$$K_v = 192 / 6(10) = 3.2$$

$$SSE = e(\infty) = 1 / 3.2 = 0.3125$$

Required SSE:

$$\mathbf{SSE_{req.} = e(\infty) / 10 = 0.03125 \text{ (overall)}}$$

$$K_v \text{ (lead compensator)} = 1977(6)/((29.03)(10)(6)) = 6.8$$

$$SSE = e(\infty) = 1/6.8 = 0.147$$

$$\frac{K(lead)}{K(uncomp)} = \frac{6.8}{3.2} = 2.12$$

Compensator:

$$\mathbf{SSE_{req.} = 10 / 2.12 = 4.716 \text{ (After Lead Compensation)}}$$

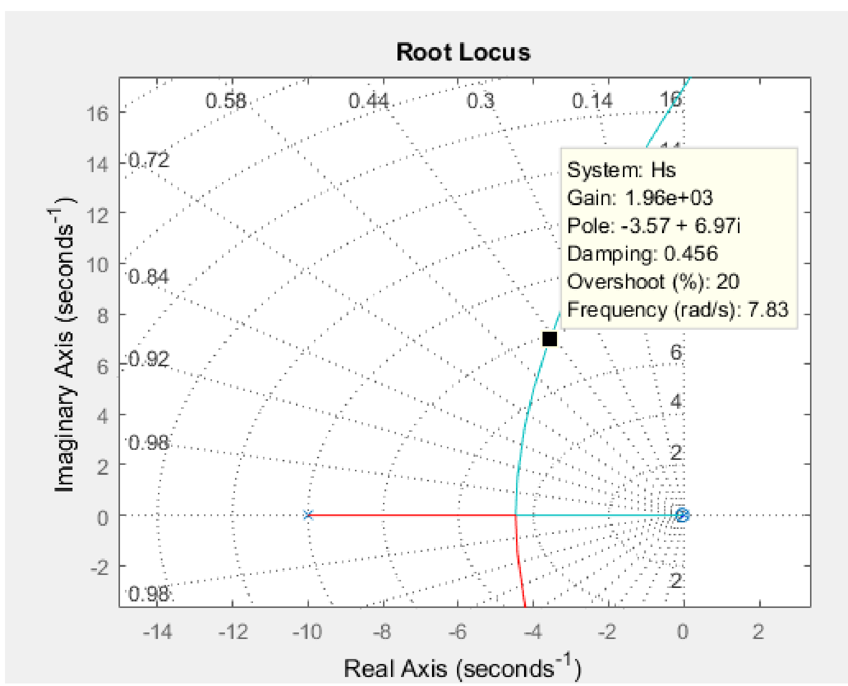
$$z_c / p_c = 4.716$$

Let $p_c = 0.01$

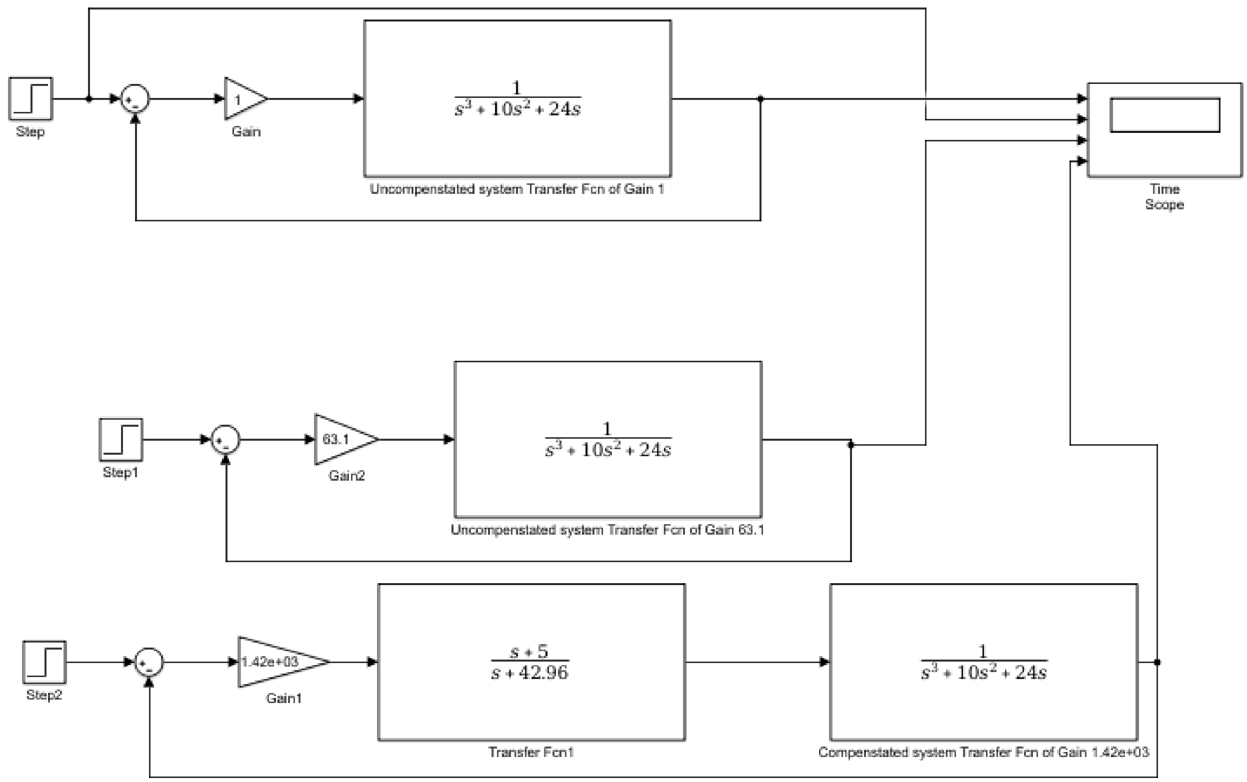
$$z_c = 0.04716$$

Root Locus:

```
>> num=[1 0.04716];
>> den=[1 39.04 290.69 2.903 0];
>> Hs=tf(num,den)
>> rlocus(Hs)
```

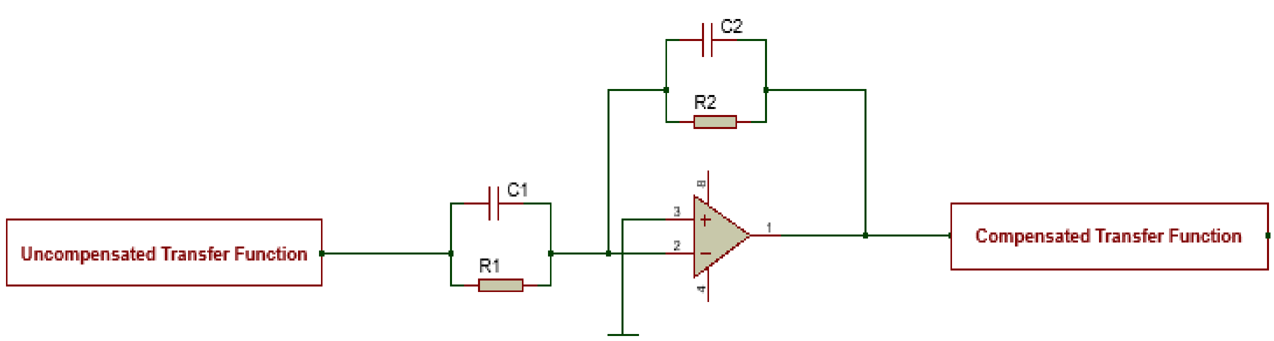


Simulation:



Hardware Schematic:

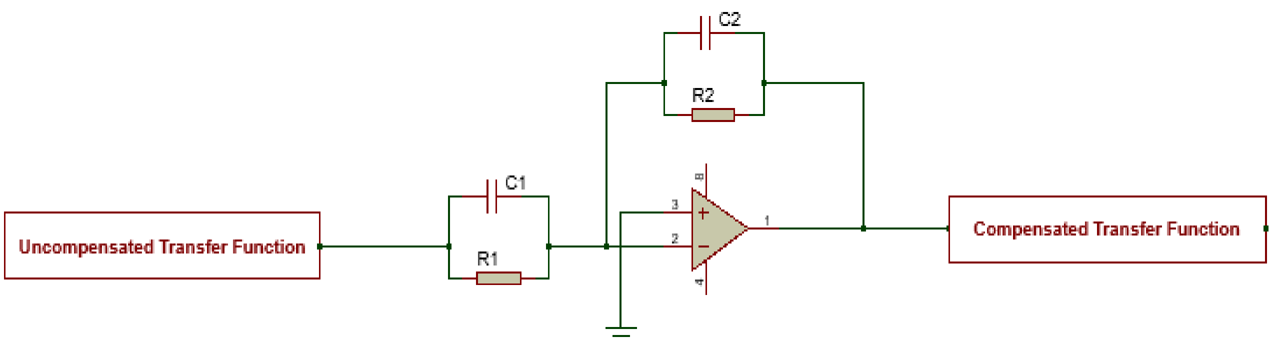
Lead Compensation:



$$R_1C_1 > R_2C_2$$

$$\text{Gain} = K = -C_1 / C_2$$

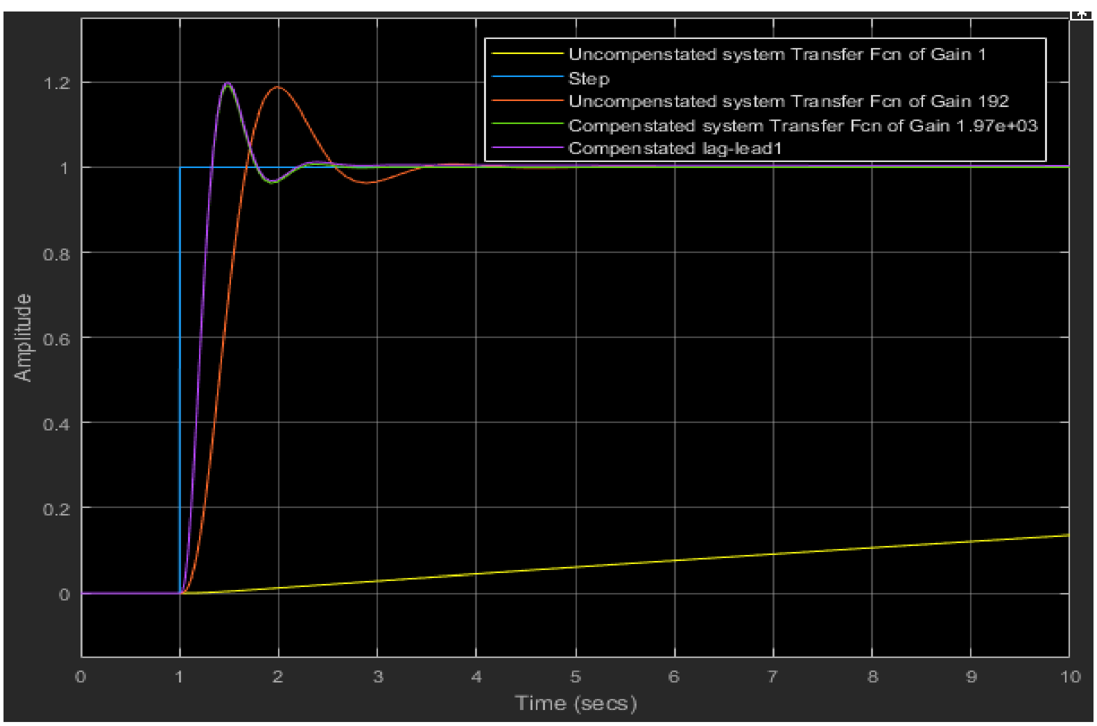
Lag Compensation:



$$R_1C_1 < R_2C_2$$

$$\text{Gain} = K = -C_1 / C_2$$

Result:



Comments:

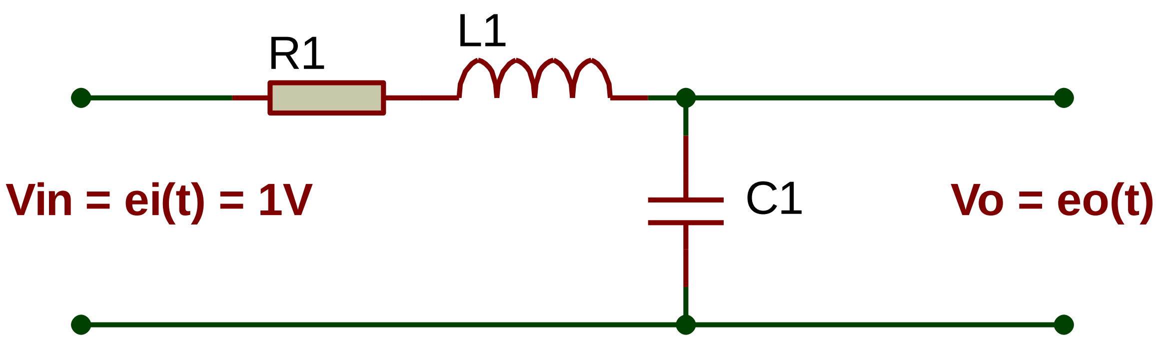
- Root locus diagram changed in Lead Compensation.
- Lead compensator didn’t effect on SSE of any transfer function.
- It is used to improve transient response of any transfer function.
- Root locus diagram remain unchanged in Lag Compensation.
- Lag compensator didn’t effect on transient response of any transfer function.
- It is used to improve SSE.
- By lead-lag compensation we improved both transient response and SSE.

Lab # 10

PID Controller

Objectives:

1. To model the following circuit and also use the PID controller for Under-Damped system



Theory:

Modeling:

$$L\,(d^2q/dt^2)\,(t)\;R\frac{dq}{dt}(t) + \frac{1}{C}q(t) = e_i(t)$$

❖ $q(t)=Ce_o(t)$

$$LCS^2E_o(s) + RCSE_o(s) + E_o(s) = E_i(s)$$

$$\frac{\mathrm{Eo(s)}}{\mathrm{Ei(s)}} = \frac{1}{(LCS+RC)S+1}$$

$$\frac{\mathrm{Eo(s)}}{\mathrm{Ei(s)}} = \frac{\frac{1}{LC}}{\left(\left(S+\frac{R}{L}\right)s+\frac{1}{LC}\right)}$$

Compare it with General T.F of 2st order equation T.F = $\omega_n^2 / (s^2 + 2\rho\omega_n s + \omega_n^2)$

$$a = R/L = 2\rho\omega_n$$

$$\rho = a/(2\omega_n)$$

$$b = 1/(LC) = \omega_n^2$$

$$0 < \rho < 1$$

R = 40 (ohm); C = 510x10-6F; L= 1H; ei= 1volt

$$\rho = 0.452$$
$$\omega_n = 44.28 \text{ rad/s}$$

$$\text{Tp} = \frac{\pi}{\omega\sqrt{(1-\rho^2)}} = 0.0795 \text{ s}$$

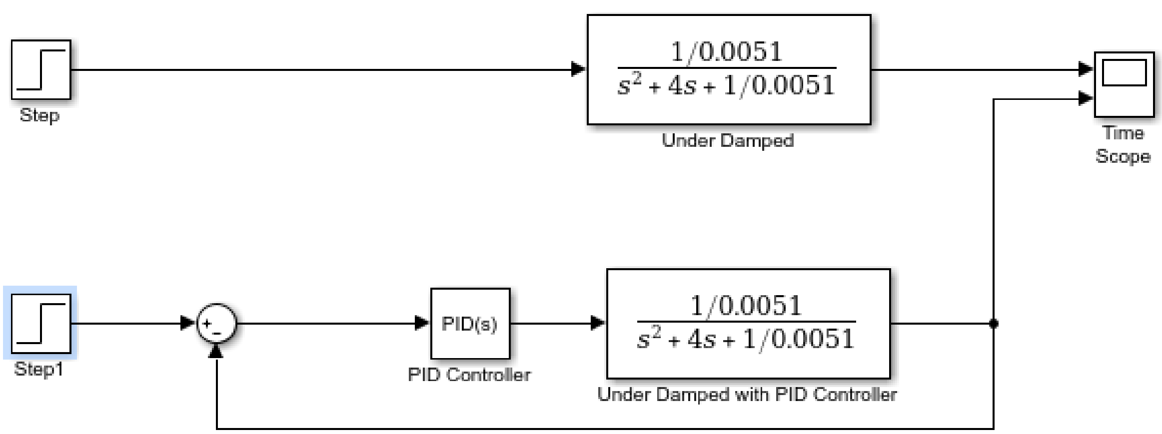
$$\%\text{OS} = e^{-\frac{\rho\pi}{\sqrt{1-\rho^2}}} \times 100 = 20.4\%$$

Required:

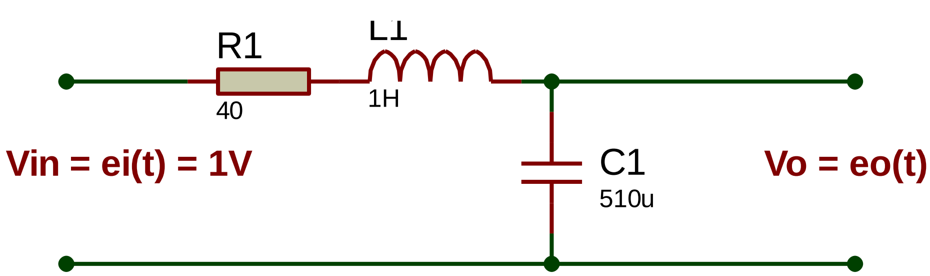
$$\text{Tp} = \frac{Tp}{5.3} = 0.015 \text{ s}$$

$$\%\text{OS} = e^{-\frac{\rho\pi}{\sqrt{1-\rho^2}}} \times 100 = 20.4\%$$

Simulation:

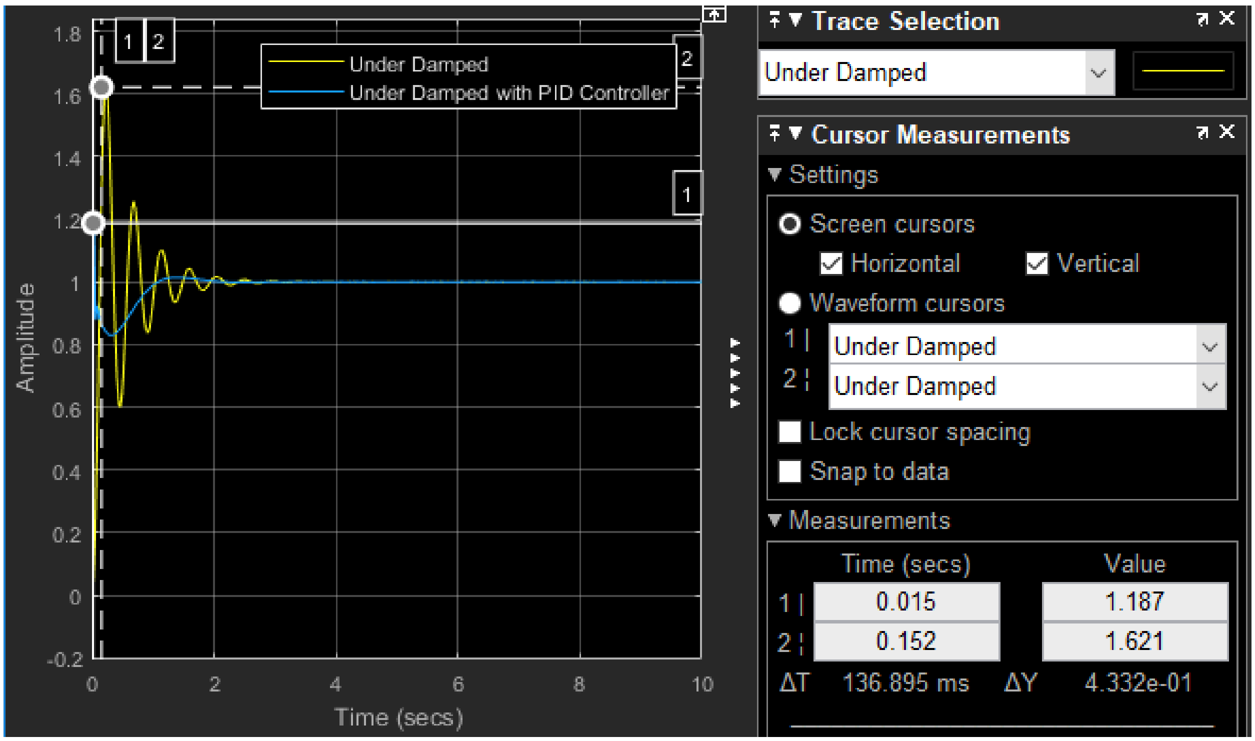


Hardware Schematic:



Result:

at Kp = 2.3, Ki = 10 & Kd = 0.7



Comments:

- By PID controller we can control the transient response as well as SSE, Tr, Ts etc.