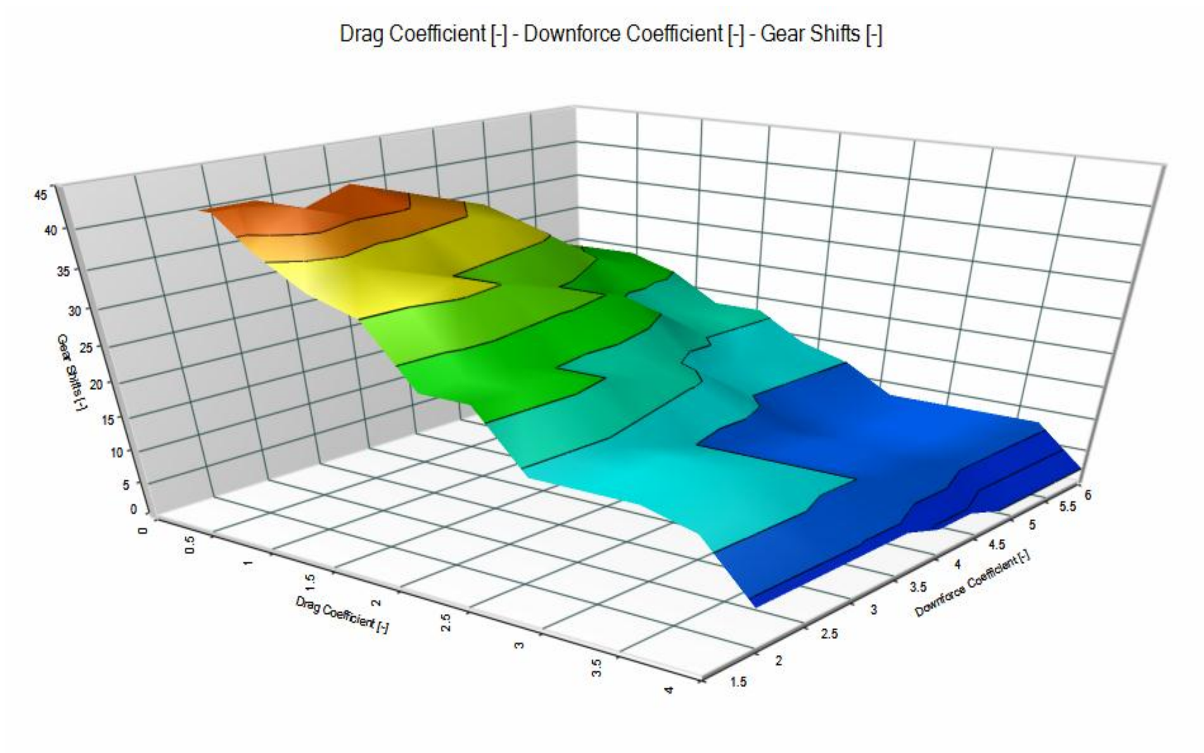
OptimumLap Vehicle Model

OptimumLap

Vehicle Dynamics Simulation. Simplified.



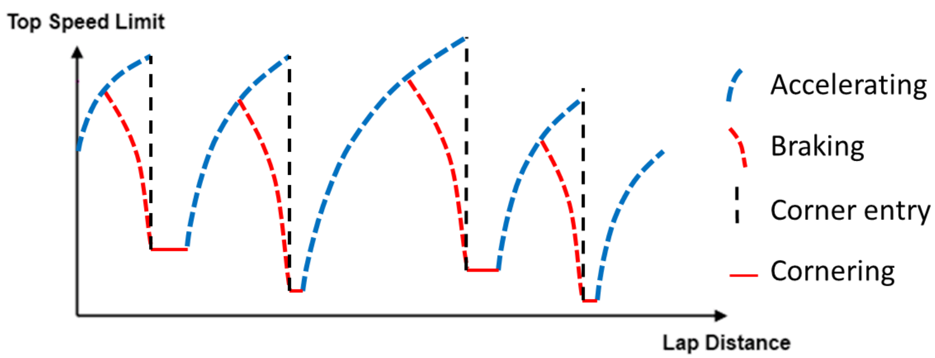
vehicle dynamics solutions

8801 E. Hampden Avenue
Suite 210
Denver, CO 80231 USA
Phone: +1-303-752-1562
engineering@optimumg.com
www.optimumg.com

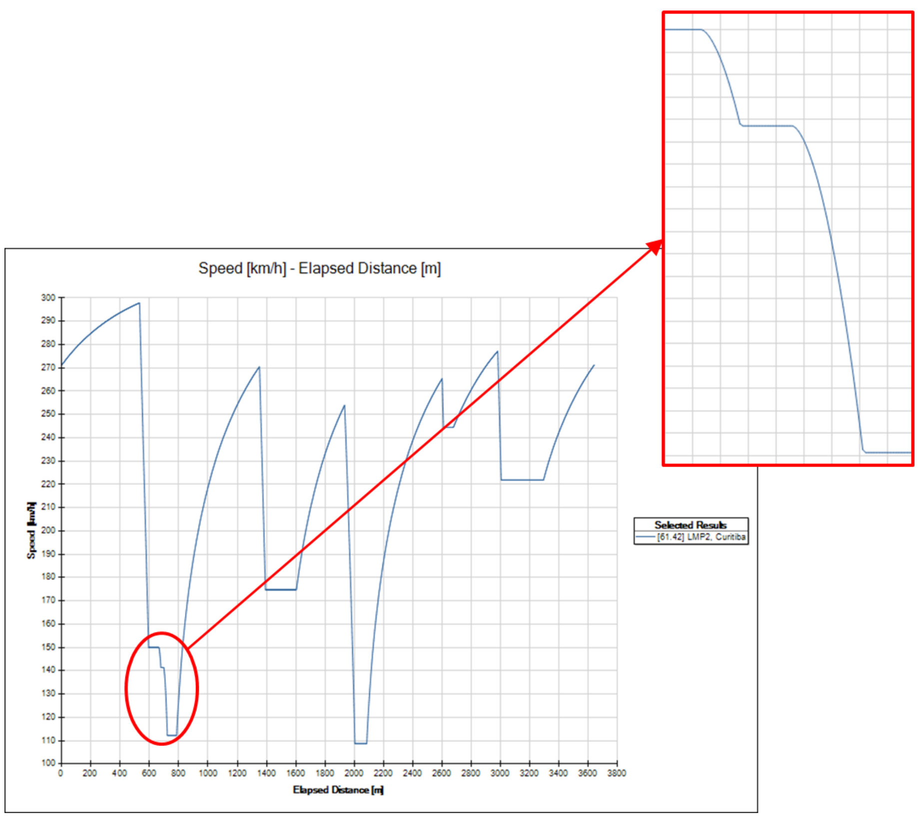
OptimumLap utilizes a quasi-steady-state point mass vehicle model. What this means to the user is that though the model is simplistic mathematically, it has the ability to be accurate due to the combined states that the vehicle can achieve. The vehicle is able to accelerate and corner simultaneously as well as decelerate and corner simultaneously.

OptimumLap does this process in three major steps:

1. Calculate the corner speeds
2. Calculate the speed accelerating out of the corners
3. Calculate the distance needed in order to decelerate the car for the corners



Once the combined states are applied we are able to achieve a smoother transition between track sections. Utilizing combined states ensures that there will be no unrealistic leaps in vehicle speed.



It is important to understand that OptimumLap utilizes both the vehicle and the driver at 100% of their capabilities! This means that the lap time calculated by OptimumLap will always be faster than what will actually happen on the track. It is crucial that you think in 'Delta' and less in 'Absolute', meaning utilizing a simplified model you are able to determine which parameters are most sensitive to lap time (IE – Increasing tire grip is more important than increasing engine power), but the values that you attain in OptimumLap will be different than the actual values used on the vehicle. With any software that you use, it is important to understand the limitations from the vehicle model. OptimumLap for example:

1. Doesn't account for weight transfer (lateral or longitudinal).
    a. No suspension affects.
    b. No Inertia
    c. Tire grip is a linear function.
    (IE – $Tractive\ Force = \mu_{Longitudinal} * Normal\ Load + Downforce$)
2. Doesn't utilize a real tire model.
    a. The effects of camber, slip ratio and slip angle not taken into account.
    b. The effects of temperature and pressures not taken into account.
    c. The effect of adding weight and over saturating the tire can be taken into account by utilizing the 'Tire Load Sensitivity' parameters in the vehicle. These can be turned on in the Options Menu.
3. Doesn't account for vehicle yaw.
    a. Since there is no CoG (Center of Gravity) location or wheelbase entered the vehicle doesn't have the capacity to oversteer or understeer.
4. Doesn't account for banking or grade on the track.
    a. No increase or decrease in traction due to centrifugal forces or added weight transfer.
    b. Doesn't take into account transient effects (IE - damping or inerter).

Given these limitations it is still possible to achieve a greater understanding of your vehicle and the sensitivities of vehicle parameters. You will be able to determine whether your time and money should be spent more on Aerodynamics, Vehicle Mass, Tire Grip, Gear Ratios or Engine Power.

The following is a brief overview of some of the equations used in OptimumLap. In this simple example, downforce is neglected.

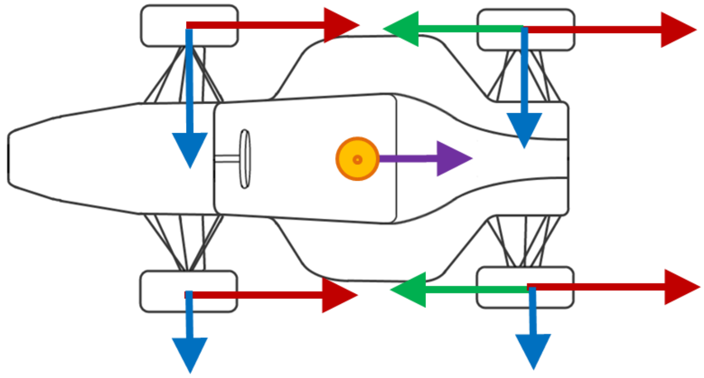
Driving Force
- **Tire longitudinal μ**
- **Normal load**
- **Engine Driving Force**

Braking Force
- **Tire longitudinal μ**
- **Normal load**

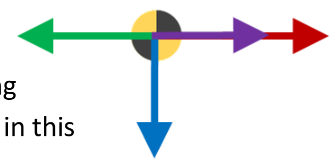
Cornering Force
- **Tire lateral μ**
- **Normal load**

Drag Force
- **Varies with speed**

Lift Force (downforce)
- **Varies with speed**
- **Changes Normal Load**



Let's do a small simplification…



Downforce and Rolling Resistance neglected in this example.

OptimumLap first breaks the track up into segments. It then determines which segments the vehicle is in the braking state, cornering state or accelerating state. Let's take a look at each of these vehicle states.

- ## Braking segment

$$\boxed{F_t = Normal\ load \times \mu_x}$$

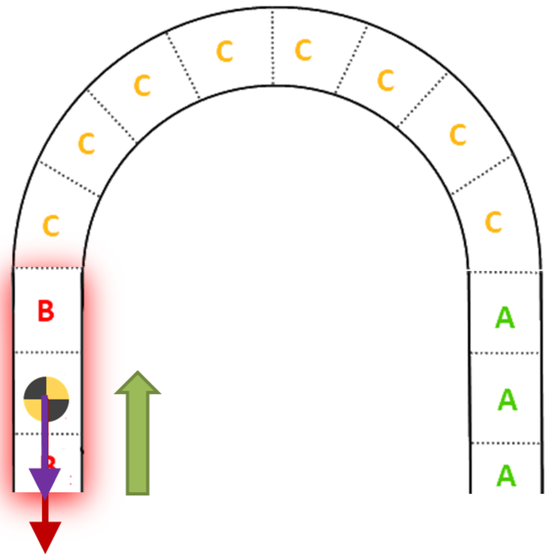
$$\boxed{F_d = \frac{1}{2} \times \rho \times C_d \times A \times v^2}$$

$$\boxed{\textbf{Newton 2}^{\textbf{nd}}\textbf{ law:} \quad \sum F_{ext} = m \times a}$$

$$\boxed{a = \frac{F_t + F_d}{m}}$$

$$\boxed{v = -a \times t + v_0}$$

With:
ρ = Air density [kg/m^3 or lb/ft^3]
A = Front area [m^2 or ft^2]
V = Vehicle speed [m/s or ft/s]
C_d = Drag coefficients
μ_x = Longitudinal friction coefficient



Downforce and Rolling Resistance neglected in this example.

To determine the tractive force under braking (F_t) we must know the Longitudinal friction coefficient and the Normal load. The Normal load under braking utilizes 100% of the vehicle weight.

The drag force will also be helping to slow the vehicle down. To know this value we need to know the density of the air, the drag coefficient of the vehicle, its frontal area and the speed at which the vehicle is traveling.

Then from Newton's 2nd Law we can determine the available deceleration the vehicle can attain. The equation for the velocity at the end of the first segment becomes the following when everything is substituted together:

$$v = -\left(\frac{F_t + F_d}{m}\right) t + v_o$$

The time will be dependent on the segment size, for example if we have a segment size of 1 m and the vehicle has started at a speed of 3 m/s:

$$t = \frac{x - x_o}{v_o} = \frac{1}{3} = 0.33\ s$$

- Corner segment

$$\boxed{F_y = Normal\ load \times \mu_y}$$

$$\boxed{a = \frac{v^2}{R}}$$

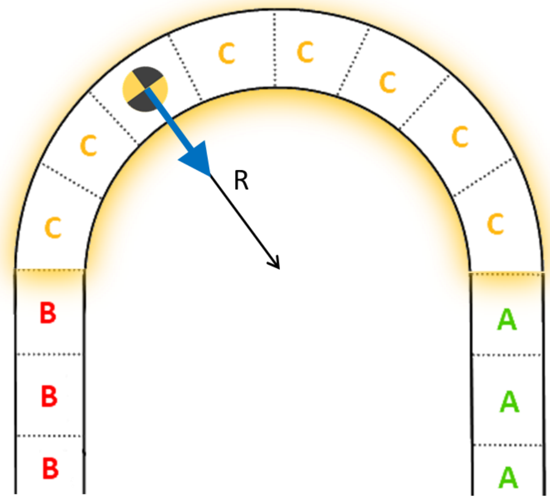
$$\boxed{\textbf{Newton 2}^{\textbf{nd}}\textbf{ law:} \quad \sum F_{ext} = m \times a}$$

$$\rightarrow \sum F_{ext} = F_y = m \times a$$

$$\rightarrow a = \frac{F_y}{m}$$

$$\boxed{v = \sqrt{\frac{F_y \times \mathrm{R}}{m}}}$$

With:
m = vehicle mass [kg or lb]
μ_y = Lateral friction coefficient



Downforce and Rolling Resistance neglected in this example.

To know the lateral force that the vehicle can apply to the ground we need to know the lateral coefficient of friction (μ_y) and the normal load, which similarly to the tractive force under braking, the Normal load will take into account the entire weight of the vehicle.

When calculating the acceleration in a corner we have to remember the equation for centrifugal force:

$$a = \frac{v^2}{R}$$

Then when we apply Newton's 2nd Law and combine it with the equation for the centrifugal force we obtain:

$$a = \frac{v^2}{R} = \frac{F_y}{m} \rightarrow \boldsymbol{v} = \sqrt{\frac{\boldsymbol{F_y * R}}{\boldsymbol{m}}}$$

- ## Accelerating segment

$$F_s = Engine\ Driving\ Force$$
As long as it is smaller than Normal Load x μ_x

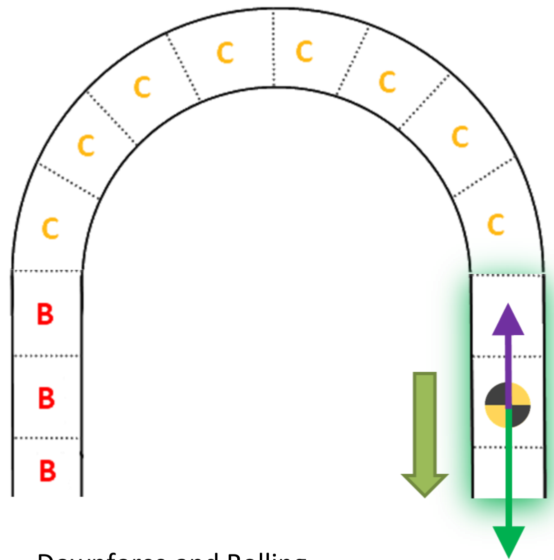
$$F_d = \frac{1}{2} \times \rho \times C_d \times A \times v^2$$

Newton 2nd law: $\sum F_{ext} = m \times a$

$$a = \frac{F_s - F_d}{m}$$

$$v = a \times t + v_0$$

With:
ρ = Air density [kg/m³ or lb/ft³]
A = Front area [m² or ft²]
V = Vehicle speed [m/s or ft/s]
C_d = Drag coefficients
μ_x = Longitudinal friction coefficient



Downforce and Rolling Resistance neglected in this example.

To determine the tractive force under acceleration (F_s) we must know the Longitudinal friction coefficient and the Normal load. The Normal load will depend on the driven type of the vehicle, if it is 2WD then it is assumed to be 50% of the weight of the vehicle and for AWD it is assumed to be 100% of the weight of the vehicle.

The drag force will also be working against the forward acceleration of the vehicle. To know this value we need to know the density of the air, the drag coefficient of the vehicle, its frontal area and the speed at which the vehicle is traveling.

Then from Newton's 2nd Law we can determine the available acceleration the vehicle can attain. The equation for the velocity at the end of the first segment becomes the following when everything is substituted together:

$$v = \left(\frac{F_t - F_d}{m}\right) t + v_o$$

The time will be dependent on the segment size, for example if we have a segment size of 1 m and the vehicle has started at a speed of 3 m/s:

$$t = \frac{x - x_o}{v_o} = \frac{1}{3} = 0.33\ s$$

To make this a quasi-steady state solver, the vehicle is allowed to accelerate in a corner until it reaches its maximum allowed corner speed. Similarly the vehicle is allowed to decelerate in a corner as well.

What we have noticed using the methods described above are very close (within 10%) correlations between logged data and OptimumLap results as shown in the following figure. Of course, the closeness of the data relies on having an accurate vehicle model as well as an accurate track representation.