TU Delft
Delft University of Technology

Re-visiting Engineering Dynamics

ME 46055

Farbod Alijani

- I can always neglect reaction forces when writing D'Alambert principle because they cancel out!

If the system is holonomic and generalized coordinates are chosen in such a way that satisfy the constraint

- A system is said to be non-holonomic if it's pfafian form is not integrable.

A non-holonomic system is neither integrable nor an integration factor could be obtained to make it integrable

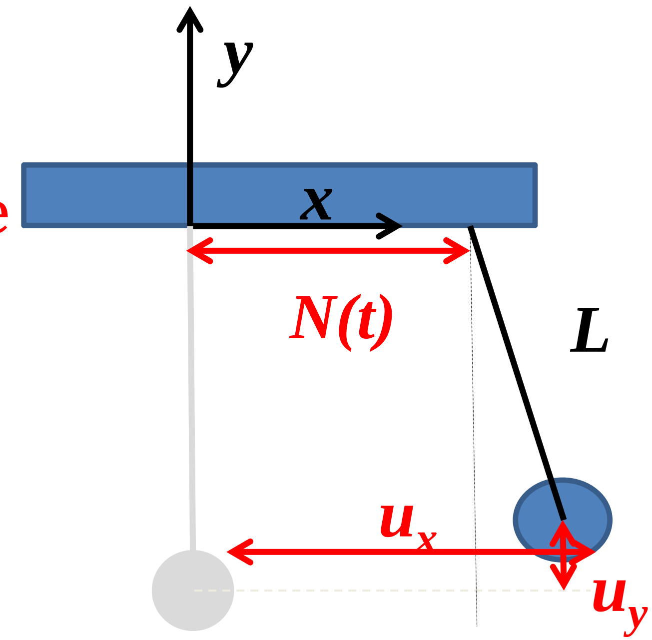
$$f_i\left(q_j,t\right)=0 \qquad j=1,\ldots,M$$

$$df_i=\sum_{j=1}^{M}\frac{\partial f_i}{\partial q_j}dq_j+\frac{\partial f_i}{\partial t}dt \;\Longrightarrow\; \sum_{j=1}^{M}a_{ij}\left(q_j,t\right)dq_j+b_i\left(q_j,t\right)dt=0$$

- This is a scleronomic system

Wrong, time is explicitly appearing in the constraint. So it is rheonomic

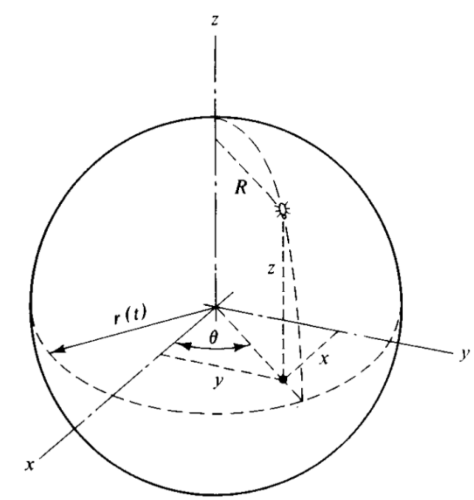
$$\left(u_x - N(\mathbf{t})\right)^2 + \left(u_y - L\right)^2 - L^2 = 0$$



- An insect walks along the surface of an expanding spherical balloon whose radius is r = ct. the position of insect on the balloon can be defined as a scleronomic constraint

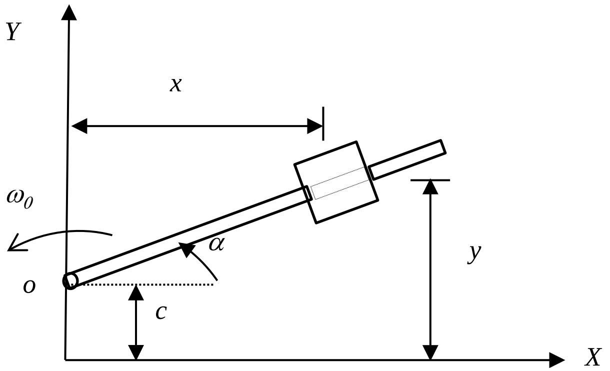
Wrong, this is a rheonomic constraint

$$x^2 + y^2 + z^2 = (ct)^2$$



- Figure below shows a bead that is free to slide along a rod which rotates in the XY plane with a constant angular velocity ω_0 about Z axis. The link is fixed at point o, and Z axis is perpendicular to XY plane and points outward from O. If the position of the bead is to be determined by x and y, this would be a non-holonomic constraint

Wrong



$$\tan\alpha = \frac{y-c}{x} \;, \alpha = \omega_0 t$$

$$f = x\tan\omega_0 t - y + c = 0$$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{dt} + \frac{\partial f}{\partial y}\frac{\partial y}{dt} + \frac{\partial f}{\partial t} \Rightarrow df = \frac{x}{\cos^2\omega_0 t}dt - dy + \tan\omega_0 t\ dx$$

- If a system is holonomic , the variation of the constraint yield the direction of the constraint

True,

$$f\left(q_k, t\right) = 0$$

$$\delta f\left(q_k, t\right) = \sum_{k=1}^{N} \frac{\partial f}{\partial q_k} \delta q_k = 0$$

- The Lagrange multiplier indicates the intensity of an unknown reaction force.

True

$$R_k = \lambda \frac{\partial f}{\partial q_k}$$

- If I use kinematically admissible coordinates I still need to take into account Lagrange multipliers in my equations of motion.

Wrong, if generalized coordinates are kinematically admissible, then the constraint is identically satisfied.

- For holonomic systems, the principle of virtual work is looking at the dynamics only along a certain direction

True, it looks at dynamics only in the admissible direction which is orthogonal to the direction of reaction forces (Constraint)

- If I have a holonomic system whose motion can be defined by 3N coordinates, then I can define the motion of the system with n generalized coordinates where n=3N

Wrong, we have to take into account the number of constraints as well: n=3N-R

- If a system is non-holonomic then I can neglect reaction forces in writing Lagrange equations since they do not have virtual work!

Wrong, in non-holonomic case we will have reaction forces associated with each constraint that do virtual work--- This is indeed why we have Lagrange multipliers in Lagrange equations

- If a system is non-holonomic and I use n generalized coordinates to define the motion, I need to solve only the n differential equations that will be obtained from Lagrange equations

Wrong

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right)-\frac{\partial T}{\partial q_k}+\frac{\partial V}{\partial q_k}-\sum_{j=1}^{R}\lambda_j a_{jk}\left(q_k,t\right)=Q_k^{ncons} \quad , \quad k=1,\dots,M$$

$$f_j\left(\dot{q}_k,q_k,t\right)=0 \qquad\qquad j=1,\dots,R$$

- Generalized force is defined as a coefficient of an increment δq of generalized coordinate q which can only be a force component

  Wrong, if q is an angle of rotation then generalized force is a moment.

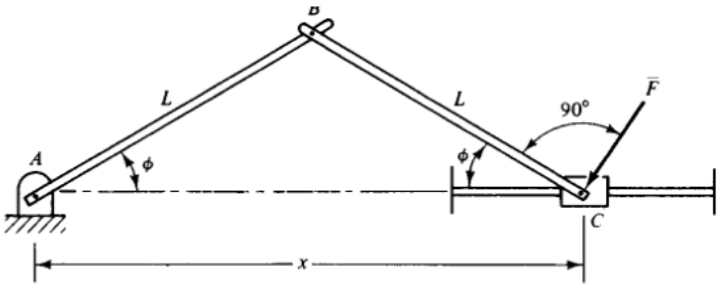
- For system below I can have generalized force only if I choose ϕ as the generalized coordinate

  Wrong,

$$\vec{r}_c = x\,i \quad , \quad \vec{r}_c = 2L\cos\phi\, i$$

$$\vec{F} = -F\sin\phi i - F\cos\phi j \quad ,$$

$$\cos\phi = \frac{x}{2L} \;,\; \sin\phi = \frac{1}{L}\left(L^2-\frac{x^2}{4}\right)^{1/2}$$



- The least action integral is nothing but the integral of the total energy of the system between two time instants.

Wrong, least action is associated with the Lagrangian

L=T-V

- Among all variational paths that connect initial and final positions, the true path is the one that makes the action integral stationary.

True.

$$\delta \int_{t_1}^{t_2} \underbrace{\left(T - V\right)}_{L} dt = 0 \quad , \quad \delta q_k\left(t_1\right) = \delta q_k\left(t_2\right) = 0 \;\; k = 1, ..., n$$

- Hamilton principle if applied to continuous systems can only give the equations of motion

Wrong, Hamilton's principle will also give an idea of what would be the boundary conditions of a system.

- If a system is scleronomic the kinetic energy is only a function of generalized velocities

Wrong.

$$\dot{\mathbf{u}}_k = \frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \dot{\mathbf{q}} \quad ,$$

$$T = \frac{1}{2} \sum_{k=1}^{N} \sum_{i=1}^{3} m_k \dot{u}_{ik}^2 = \frac{1}{2} \sum_{k=1}^{N} m_k \, \dot{\mathbf{u}}_k^T \cdot \dot{\mathbf{u}}_k = \frac{1}{2} \sum_{k=1}^{N} m_k \, \dot{\mathbf{q}}^T \left(\frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \right)^T \cdot \frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \dot{\mathbf{q}}$$

- If a system is rheonomic then the kinetic energy is caused by the applied and relative motions only

Wrong.

$$T = \frac{1}{2} \sum_{k=1}^{N} m_k \, \dot{\mathbf{u}}_k^T \, \dot{\mathbf{u}}_k = \frac{1}{2} \sum_{k=1}^{N} m_k \left(\frac{\partial \mathbf{U}_k}{\partial t} + \frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \dot{\mathbf{q}} \right)^T \left(\frac{\partial \mathbf{U}_k}{\partial t} + \frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \dot{\mathbf{q}} \right) \quad ,$$

$$T_0 = \frac{1}{2} \sum_{k=1}^{N} m_k \left(\frac{\partial \mathbf{U}_k}{\partial t} \right)^T \left(\frac{\partial \mathbf{U}_k}{\partial t} \right)$$

$$T_2 = \frac{1}{2} \sum_{k=1}^{N} m_k \, \dot{\mathbf{q}}^T \left(\frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \right)^T \left(\frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \dot{\mathbf{q}} \right)$$

$$T_1 = \sum_{k=1}^{N} m_k \left(\frac{\partial \mathbf{U}_k}{\partial t} \right)^T \left(\frac{\partial \mathbf{U}_k}{\partial \mathbf{q}} \dot{\mathbf{q}} \right) \quad ,$$

- The equilibrium points of a scleronomic system depends also on the kinetic energy

Wrong.

$$\mathbf{f}_V\Big|_{\mathbf{q}=\mathbf{q}_{eq}} = \left(\frac{\partial V}{\partial \mathbf{q}}\right)_{\mathbf{q}=\mathbf{q}_{eq}} = 0$$

- The linearized stiffness matrix of a rheonomic system also includes contribution of the kinetic energy

True.

$$\mathbf{K} = \frac{\partial\left(\mathbf{f}_T+\mathbf{f}_V\right)}{\partial \mathbf{q}}\Bigg|_{\substack{\ddot{\mathbf{q}}=0\\ \dot{\mathbf{q}}=0\\ \mathbf{q}=\mathbf{q}_{eq}}} = \frac{\partial}{\partial \mathbf{q}}\left(\frac{d}{dt}\frac{\partial\left(T_2+T_1\right)}{\partial \dot{\mathbf{q}}} - \frac{\partial\left(T_2+T_1+T_0\right)}{\partial \mathbf{q}} + \frac{\partial V}{\partial \mathbf{q}}\right)\Bigg|_{\substack{\ddot{\mathbf{q}}=0\\ \dot{\mathbf{q}}=0\\ \mathbf{q}=\mathbf{q}_{eq}}}$$

$$= \left(-\frac{\partial^2 T_0}{\partial \mathbf{q}\,\partial \mathbf{q}} + \frac{\partial^2 V}{\partial \mathbf{q}\,\partial \mathbf{q}}\right)_{\mathbf{q}=\mathbf{q}_{eq}}$$

- Irrespective of the type of motion (scleronomic or rheonomic) the linearized mass matrix is always obtained from the part of kinetic energy that only depends on the generalized velocities

True.

$$\mathbf{M} = \frac{\partial^2 T}{\partial \dot{\mathbf{q}}\,\partial \dot{\mathbf{q}}}\Bigg|_{\mathbf{q}=\mathbf{q}_{eq}} = \frac{\partial^2 T_2}{\partial \dot{\mathbf{q}}\,\partial \dot{\mathbf{q}}}\Bigg|_{\mathbf{q}=\mathbf{q}_{eq}}$$

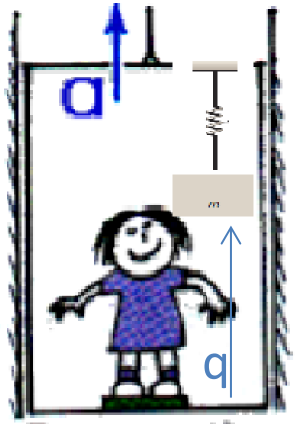
- If the elevator is accelerating, the equilibrium position of the spring-mass system depends on acceleration

True

$$T=\frac{1}{2}m(\dot{u})^2=\frac{1}{2}m(at+\dot{q})^2=\underbrace{\frac{1}{2}m\dot{q}^2}_{T_2}+\underbrace{mat\dot{q}}_{T_1}+\underbrace{\frac{1}{2}ma^2t^2}_{T_0}$$

$$\left(\frac{d}{dt}\left(\frac{\partial T_1}{\partial \dot{q}}\right)\right)_{q=q_{eq}}-\left(\frac{\partial T_0}{\partial q}\right)_{q=q_{eq}}+\left(\frac{\partial V}{\partial q}\right)_{q=q_{eq}}=0$$

$$ma+kq_{eq}=0\Rightarrow q_{eq}=-\frac{k}{ma}$$

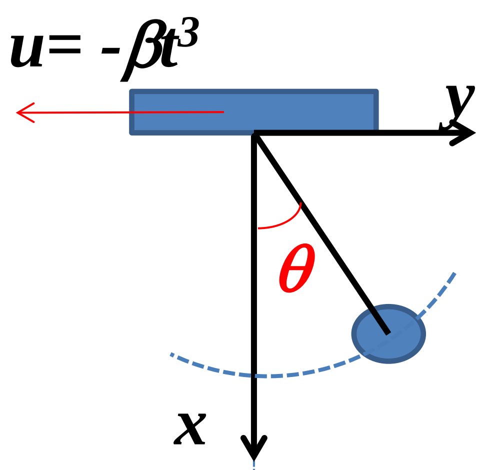


- Equilibrium position could be found for below system

Wrong.

$$T=\frac{1}{2}m\left(-l\dot{\theta}\sin\theta\right)^2+\frac{1}{2}m\left(l\dot{\theta}\cos\theta-3\beta t^2\right)^2=\underbrace{\frac{1}{2}ml^2\dot{\theta}^2}_{T_2}\underbrace{-3ml\beta t^2\dot{\theta}\cos\theta}_{T_1}+\underbrace{\frac{9}{2}m\beta^2t^4}_{T_0}$$

$$V=-mgl\cos\theta$$



$$\left(\frac{d}{dt}\left(\frac{\partial T_1}{\partial \dot{\theta}}\right)\right)_{\theta=\theta_{eq}}-\left(\frac{\partial T_0}{\partial \theta}\right)_{\theta=\theta_{eq}}+\left(\frac{\partial V}{\partial \theta}\right)_{\theta=\theta_{eq}}=0$$

$$\left(\frac{d}{dt}\left(-3ml\beta t^2\cos\theta\right)\right)_{\theta=\theta_{eq}}-0+\left(mgl\sin\theta\right)_{\theta=\theta_{eq}}=-6m\beta tl\cos\theta_{eq}-3m\beta t^2l\dot{\theta}\sin\theta_{eq}+mgl\sin\theta_{eq}\neq 0$$

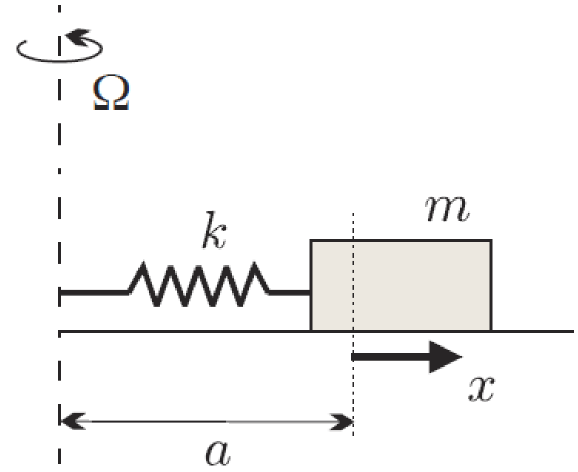
- If the system rotates with constant velocity Ω , then there will be a gyroscopic force acting on the system

Wrong.

$$u_1 = (a+x)\cos\Omega t \quad , \quad u_2 = (a+x)\sin\Omega t$$

$$T = \frac{1}{2}m\dot{u}_1^2 + \frac{1}{2}m\dot{u}_2^2$$

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m(a+x)^2\Omega^2$$



- In the previous case if Ω=cost , then it is possible to find an equilibrium position

Wrong. Only rheonomic systems that are subjected to uniformly accelerated translation or rotate with constant angular velocity will have equilibrium.

$$\left(\frac{d}{dt}\left(\frac{\partial T_1}{\partial \dot{x}}\right)\right)_{x=x_{eq}} - \left(\frac{\partial T_0}{\partial x}\right)_{x=x_{eq}} + \left(\frac{\partial V}{\partial x}\right)_{x=x_{eq}} = 0$$

$$kx_{eq} - m\left(a+x_{eq}\right)(\cos t)^2 \neq 0$$

- Similar to damping, gyroscopic terms dissipate energy from the system

Wrong. This is fundamentally different from damping. The instantaneous power related to gyroscopic force $G\dot{q}$ is zero (Why??):

$$\dot{\mathbf{q}}^T \mathbf{G} \dot{\mathbf{q}} = 0$$

- For scleronomic systems with no damping T+V=E is only true if the system is linear

Wrong. This is a direct outcome of Lagrange equations and it is true irrespective of the system being linear or nonlinear

- A conservative system is stable if and only if its stiffness matrix is positive definite

True.

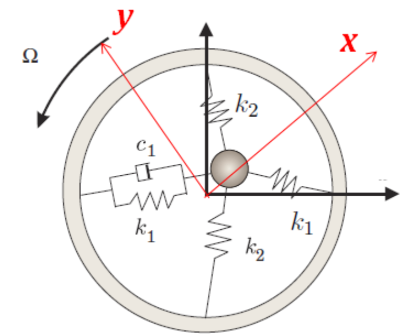
$$V(\tilde{\mathbf{q}}) = \frac{1}{2}\sum_{s=1}^{n}\sum_{r=1}^{n} k_{sr}\tilde{q}_s\tilde{q}_r = \frac{1}{2}\tilde{\mathbf{q}}^T \mathbf{K} \tilde{\mathbf{q}} > 0$$

- In scleronomic systems damping always stabilizes the system

Wrong. There are certain cases where you can have negative damping (Flutter)

- In system below, gyroscopic forces were the main reason of destabilizing the system.

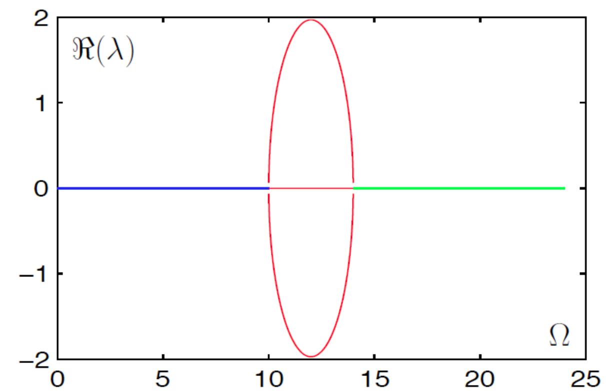
$$\mathbf{G}=\begin{bmatrix} 0 & -2m\Omega \\ 2m\Omega & 0 \end{bmatrix}\;,\;\mathbf{K}^*=\begin{bmatrix} -m\Omega^2+2k_1 & 0 \\ 0 & -m\Omega^2+2k_2 \end{bmatrix}$$



Wrong, Gyrsocopic terms were stabilizing and centrifugal terms were destabilizing .

- A rheonomic system is stable if and only if its stiffness matrix is positive definite.

Wrong. The gyroscopic terms can stabilize the system even if the stiffness matrix is negative definite

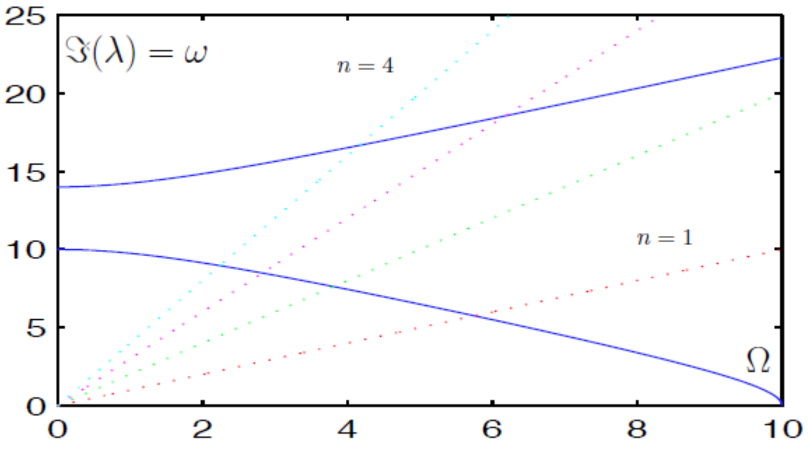


- Irrespective of a system being rheonomic or scleronomic , if an eigenvalue analysis is performed and a positive real eigenvalue is obtained then the system is unstable.

True, if λ is real and positive there exists an exponential motion that grows in time. $\left(\lambda^2 \mathbf{M} + \lambda \mathbf{C} + \mathbf{K}\right)\mathbf{x} = 0$

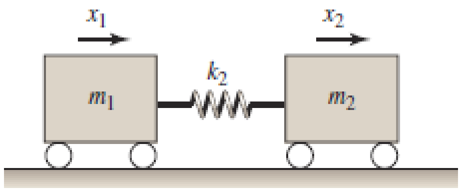
- If below figure is the Campbell diagram of a rotating shaft, the optimum operating conditions are points where harmonics are intersecting the imaginary part of the eigenvalue

Wrong, rotation systems are subjected to external excitations that have a frequency equal to *n* times the rotation speed. The intersection points are resonance.



- This system has a rigid body mode (railway car system)

True,



$$\mathbf{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \quad , \quad \mathbf{K} = \begin{pmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix}$$

$$\mathbf{det(K} - \omega^2 \mathbf{M)} = 0 \quad , \quad \omega_1 = 0 \quad , \quad \omega_2 = \sqrt{\frac{k_2}{m_2}} \sqrt{1 + \frac{m_2}{m_1}}$$

$$\mathbf{x}_{(1)} = \begin{Bmatrix} 1 \\ X_1 \end{Bmatrix} \quad , \quad X_1 = \frac{k_2}{k_2 - m_1 \omega_1^2} = 1$$

- The motion of a conservative system is always synchronous

True, the generalized coordinates of a MDOF system have the same time dependence function yielding real eigenvalues and eigenmodes

- Irrespective of a system being damped or undamped, the work produced by elastic forces (of mode r)on a displacement described by mode s is always zero.

True, this is the orthogonality condition

- Orthogonality and normalization of eigenmodes are the same.

Wrong, orthogonality is a condition while normalization is when you normalize modes (scale modes) for instance with respect to modal mass.

- In free vibrations, eigenmodes give information about the motion in time.

Wrong, eigenmodes are time independent. The motion is defined by initial conditions

- I can express the motion of free conservative systems in terms of eigenmodes

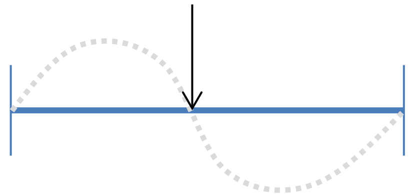
True, this is how we obtain also normal coordinates

- The ingredients I need to calculate FRF (Frequency Response Functions) is expensive to calculate.

True, this is why we use modal superposition

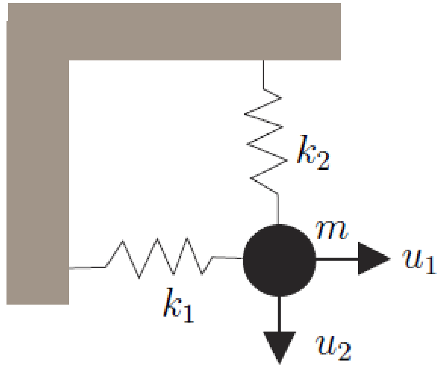
$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{s}\cos\omega t \qquad \mathbf{q} = \mathbf{x}\cos\omega t \qquad \mathbf{x} = \sum_{i=1}^{m} \xi_i \mathbf{u}_{(i)} + \sum_{s=m+1}^{n} \eta_s \mathbf{x}_{(s)}$$

- In below system, if the force is at the middle and has a frequency equal to the natural frequency of the beam it would cause resonance



Wrong, the force does not activate the mode since its modal participation factor is zero

- This system has two unique eigenmodes if $k_1 = k_2 = k$



No, this system has multiplicity 2. Any combination of modes is a mode too.

- The response of a system to arbitrary excitation can be interpreted as a response to a train of impulses

True, this is why we have convolution integral:

$$\eta_r^{(p)} = \int_0^t \phi_r(t) h(t-\tau) d\tau$$

- If a system has light damping and mode complexity is very small then the system exhibits normal modes

True, if damping is small and frequencies are well-seperated.

- I can always truncate the response of harmonically excited systems by a certain number of eigenmodes if modes that I include in the approximation have frequencies much higher than the excitation frequency.

Not only that, but also modes that we include should be participating in the response. In other words, we have to check both spectral and quasi-static convergence.

- It is not possible at all to uncouple equations of motion of a damped system.

Wrong, using the concept of proportional damping we can uncouple damped systems: $\mathbf{C} = a\mathbf{M} + b\mathbf{K}$

- The resonance frequencies of a lightly damped system are exactly the same as undamped one:

Wrong,

$$\omega_{dr} = \omega_r \sqrt{1-\varepsilon_r^2}$$

- At anti-resonance the system does not move:

Wrong, only a certain degree of freedom does not move.

- This is the frequency response curve of 3DOF system, and each peak gives information about system frequencies and damping.

True,

$$a_{kk}(\omega^2) = \sum_{s=1}^{n} \frac{1}{\left(\omega_{0s}^2 - \omega^2 + 2i\varepsilon_s \omega \omega_{0s}\right)} \frac{x_{k(s)}^2}{\mu_s}$$