Dynamics of a Single-Track Vehicle

(Engineering Dynamics Assignment)

Deadline: November 14, 2016 @ 12 pm

ME 46055

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1. Introduction

Single-Track ground vehicles (i.e bikes) are Multi-Degree-Of-Freedom (MDOF) systems whose vibration behavior is known as *ride* or *ride comfort*. Understanding vehicles vibrations is of paramount importance for improving the ride comfort simply because vibrations can influence passengers (e.g. biker). It is interesting to note that human bodies response to a vibration exposure is dependent on the frequency, amplitude, and duration of exposure. From an exposure point of view, the low frequency range of vibration is the most interesting. Exposure to vertical vibrations in the 5-10 Hz range generally causes resonance in the thoracic-abdominal system, at 20-30 Hz in the head-neck-shoulder system, and at 60-90 Hz in the eyeball.

A Mud bike is an interesting single-track vehicle that is very much used on the mountain slopes. It is a small bike with very large tires allowing bikers to roll down over rough terrain. The special design of mud bikes provides unexpected riding and handling qualities. In particular , its dynamics in the longitudinal plane exhibits low frequency modes which can make the ride uncomfortable if badly damped. Therefore, the goal of this assignment is to examine the applied methods of determining a single-track vehicle's equations of motion, natural frequencies, mode shapes, and damped motion.

Fig.1. Mud bikes

2. The Model

A simple model of a bike comprises of a rigid frame, a front suspension rigid in bending, and two wheels . Moreover, the biker would normally compose of a rigid body connected to the saddle via a torsional spring and arms that can be represented by a massless spring. A general view of how the bike and biker could be modeled are shown in Figure 2. The different names of the variables describing the position of the different components are described in the figure too. The subscript $_{tF}$ stands for front tire variables; $_{F}$ front rim variables ; $_{f}$ frame variables ; $_{b}$ body variables; $_{B}$ back rim variables ; and $_{tB}$ back tire variables.

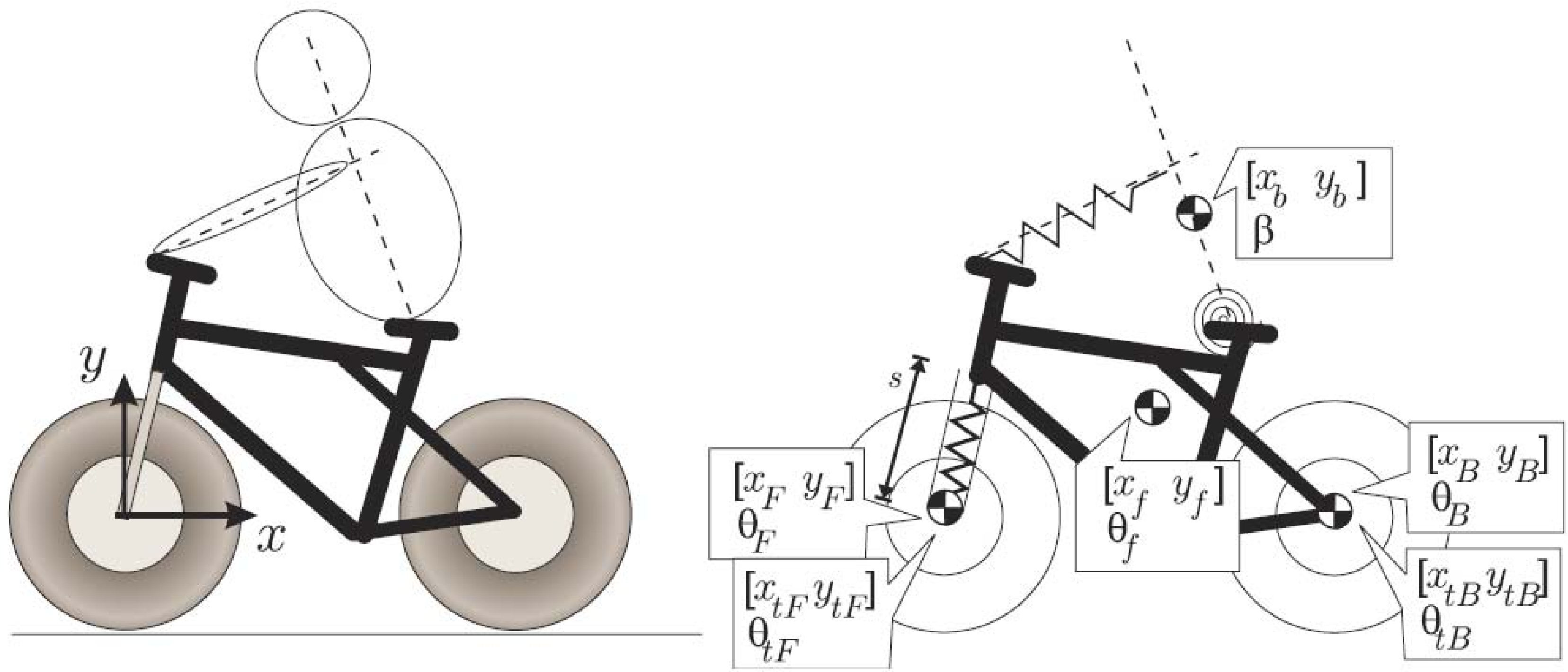

Fig.2. General scheme of the bike

The wheels of the bike are assumed to be two rigid rings , one representing the tire and the other representing the rim. The tire and the rim are assumed to be connected by a continuous flexible layer (distributed radial springs with zero natural length) representing the rubber of the tire (See Figure 3).

The wheel is assumed to roll without slipping. In general the contact mechanics of tire is not easy. Here a non-linear model is assumed that gives the following force: $f_{cont} = k_{cont0} y_{tF} + k_{cont2} y_{tF}^3$.

The same equation is satisfied for the rear tire.

For the rolling without slipping condition, in this simple flexible wheel mode, we consider that the rolling radius is equal to the nominal (non-deformed) radius of tire. Therefore, $\dot{x}_{tF} + r_{tire}\dot{\theta}_{tF} = 0$.

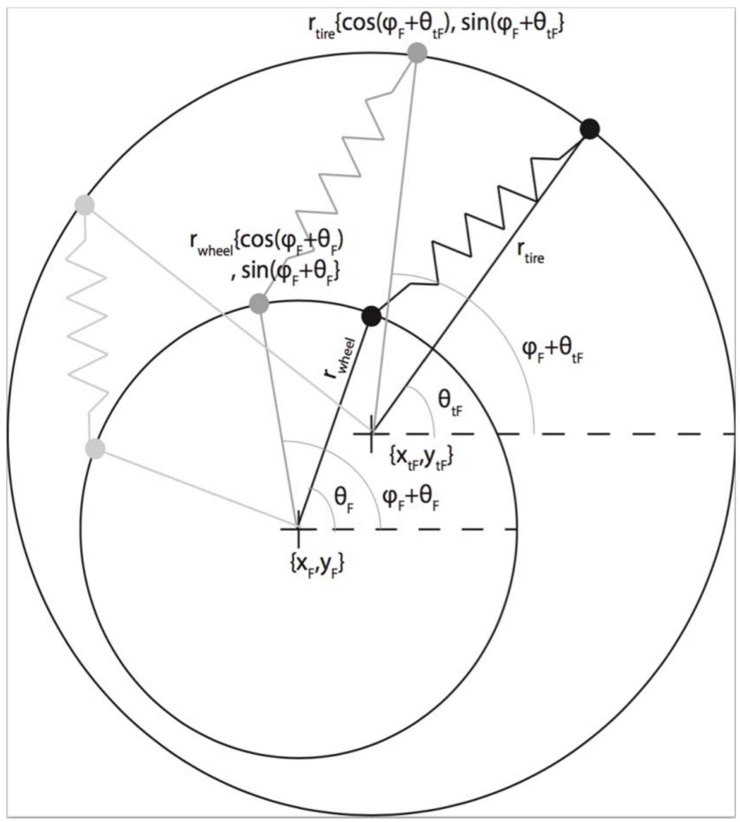

Fig.3. Tire model

3. Data

The necessary geometrical and mechanical data of the bike are given in Figure 4 and below. α_{fb} in the Figure represents natural angle between the frame and the body.

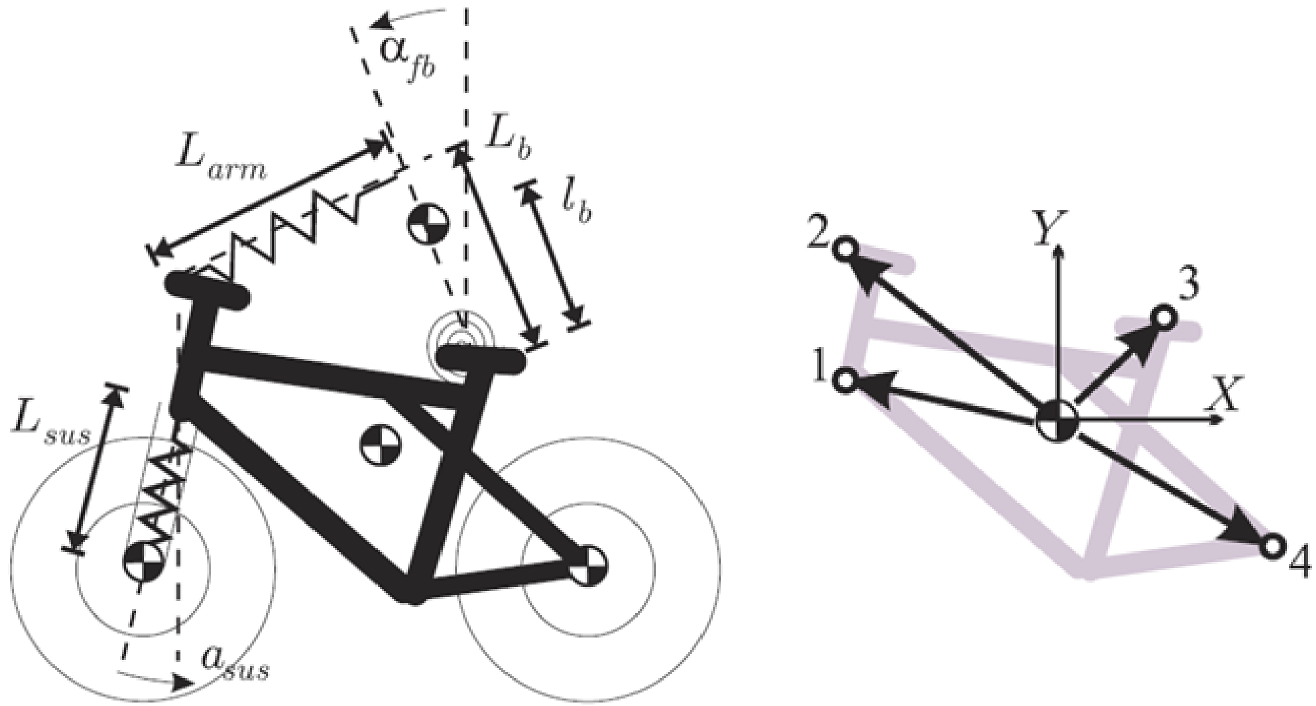

Fig. 3. Geometric data of the bike

$g = 9.81 m/s$, $m_{wheel} = 2.1 Kg, m_{tire} = 1 Kg, m_f = 8 Kg$, $m_b = 70 Kg$, $r_{wheel} = 0.15m$, $r_{tire} = 0.25m, I_{wheel} = m_{wheel} r_{wheel}^2$, $I_{tire} = m_{tire} r_{tire}^2$

$r_f = 0.5m$, $r_b = 0.34m$, $I_f = m_f r_f^2$, $I_b = m_b r_b^2$, $\alpha_{fb} = 35^\circ, L_{sus} = 0.5m$, $L_{arm} = 0.45m$, $a_{sus} = 15^\circ$, $L_b = 0.5m$, $l_b = 0.3m$

$X_1 = -0.32m$, $X_2 = -0.31m$, $X_3 = 0.1m$, $X_4 = 0.4m, Y_1 = 0.28m$, $Y_2 = 0.35m$, $Y_3 = 0.28m$, $Y_4 = -0.25m$

$k_{cont0} = 1.4\times10^4\ N/m$, $k_{cont2} = 1.8\times10^9\ N/m^3$, $k_{sus} = 10^5\ N/m$, $c_{cont} = 2\times10^2\ Ns/m$, $c_{sus} = 5\times10^3\ Ns/m$, $k_{tire} = 5.1\times10^3\ N/m^2$

$k_{arm} = 0.9\times10^5\ N/m$, $k_{pel} = 8\times10^3\ N/m$, $c_{arm} = 10^3\ Ns/m$

Part 1: kinematics, Energies and Equations of Motion

- ✓ Consider the different constraints on the system and find the number of degrees of freedom. Define the kinematic relations between the different variables with respect to the frame and choose the appropriate set of generalized coordinates. Are the constraints holonomic or non-holonomic?
- ✓ Build the expressions for the total kinetic and potential energy (including gravity). You shall obtain the kinetic energies associated with front and rear tires, wheels, frame and biker. Moreover, the following elastic potential energies shall be calculated: front suspension, biker arm, saddle-biker point, contact of the front tire, contact of the rear tire, stiffness between the tire and the wheel.
- ✓ In the absence of damping , find the dynamics equations of motion. You have the option of choosing either Lagrange equations or Hamilton's principle.

Part 2: Equilibrium, Linearization and Stability

- ✓ Find the equilibrium position of the bike under gravity forces. You shall be using Newton-Raphson technique. Assume that the rotation of the front tire is fixed. Then verify that the solution found is also an equilibrium position when the front tire is not fixed (why are we doing this? -Explain in the report)
- ✓ Find the linearized stiffness matrix around equilibrium. Would the bike position around the obtained equilibrium be stable?

Part 3: Vibrations

- ✓ Find the linearized mass matrix around the equilibrium, and determine the undamped frequencies and mode shapes. Also, plot the mode shapes.
- ✓ Using the orthogonality condition of the eigenmodes, decouple the equations of motion.
- ✓ Add to the equations of motion the generalized forces corresponding to the damping forces of the suspension, the arms and the tire contacts (discuss how you have obtained the generalized forces). Compute then the linearized damping matrix around the equilibrium position (with gravity). Evaluate the modal damping of the modes.
- ✓ Consider a situation in which the bike goes over a step. In this situation there will be a time lag between the front and rear wheel excitations. Suppose that the bike is moving with velocity v and hits a step of height Y. By measuring time from the moment the front wheel hits the step, it is possible to calculate the time t_0 at which the rear wheel reaches the step:

$$t_0 = \frac{a_1 + a_2}{v}$$

Therefore, the excitation of the bike would be a lagged step input:

$$y_{tF} = \begin{cases} Y & t > 0 \\ 0 & t \le 0 \end{cases} \quad , \quad y_{tB} = \begin{cases} YH\left(t - t_0\right) & t > 0 \\ 0 & t \le 0 \end{cases}$$

Where $H(t\text{-}t_0)$ is the heavyside function.

Find the response of every chosen generalized coordinate when the bike passes a step of height Y=0.1m with velocities 1m/s, 5m/s and 10 m/s. Compare responses and discuss.

Work and Report

- It is expected that the assignment is programmed using a symbolic toolbox (e.g. Matlab, Mathematica) in teams of TWO and that you describe in the report the steps required to programme the problem. The report is intended for you to outline your understanding of the problem, of its physics, the methodology and the obtained results. Imagine it is intended for a client that is an engineer but not a specialist in dynamics. The client has 20 minutes to read the report and to be convinced that you understand what you did, that you programmed the right steps and that you could properly outline the major results obtained from the analysis.
- Hence it should include at least the following parts:

✓ Brief description of the problem.

✓ Explanation of the way the problem has been modelled and how it has been programmed. For instance how the final number of degrees of freedom is determined, how kinetic and potential energies are computed, how the equations have been implemented, how the solution of the non-linear equilibrium is found...

✓ Discuss for each analysis step the results found.

✓ Make sure all the mathematical terms in your formulas are defined, and proper formatting is used.

✓ Include your code in an appendix.

✓ (OPTIONAL: BONUS) Create a user friendly Graphical User Interface (GUI) in which receives bike/biker data as input and gives frequencies, damping ratios, mode shapes and response to step function as output.