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# Wavelet-Based Edge Detection

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# 1. Literature Review

Throughout the course for Computer Vision and Image Processing many technologies from image pre-processing to image understanding were learnt and implemented. The basic technologies were:

- Histogram Equalization:
  It is a method that enhance contrast in an image by stretching its histogram. Equalization implies mapping one distribution (the given histogram) to another distribution (a wider and more uniform distribution of intensity values) so the intensity values are spreaded over the whole range.

- Fourier Transform:
  The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components. The output of the transformation represents the image in the frequency domain, while the input image is the spatial domain equivalent.

- Wavelet Transform:
  Wavelet Transform like Fourier Transform represents an image in frequency domain but unlike Fourier Transform it uses wavelets due to which we can both have frequency and temporal information. In this project we have used this technology for detecting edges in an image.

- Edge Detection:
  It is a technique for finding the boundaries of objects within images. It works by detecting discontinuities in brightness. Edge detection is used for image segmentation and data extraction.

- Image Segmentation:
  Segmenting an image is to divide an image into parts having strong correlation with objects or areas of real world reflected in the image. These regions may be defined with homogeneous properties in brightness, color, reflectivity, texture, etc.

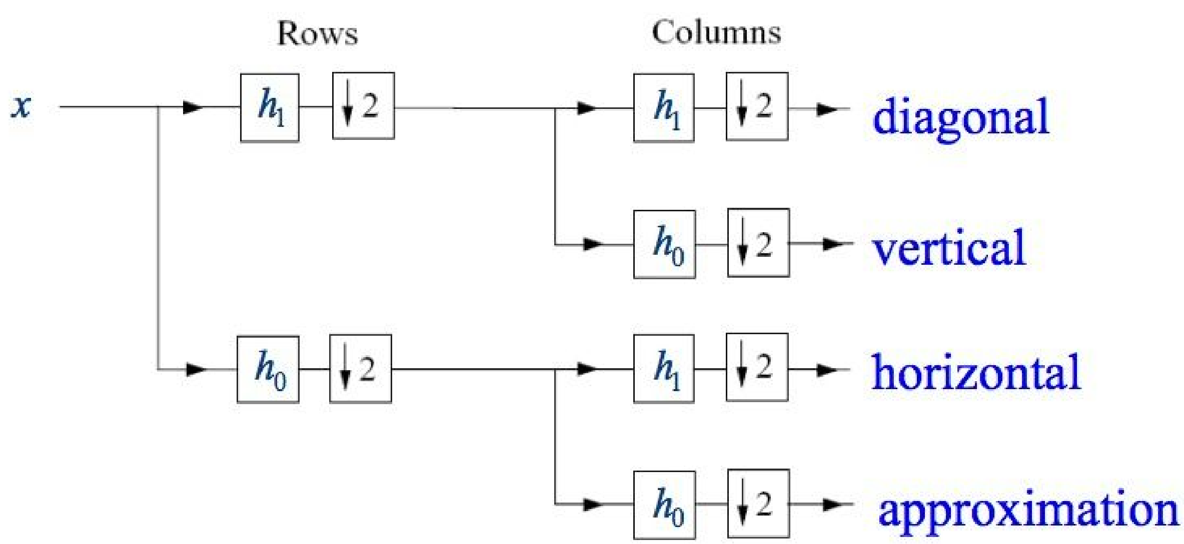
- Scale Multiplication:
  Multiplying wavelet transformed images at various levels. This process enhances strong edges in an image and suppress noise.

# 2. Introduction

The purpose of this project is detection of edges in gray-level images by wavelet transform and applying a scale multiplication scheme. Edge detection is an important step in image understanding. A good edge detection scheme should focus on three things: good detection, good localization and low spurious response. Many edge detection schemes have been developed so far, simplest of them is by using classical operators like Prewitt, Sobel, Roberts, etc. These are very efficient and easy to implement but they work only for high quality noiseless images. For noisy images more sophisticated techniques are developed one of which is edge detection by wavelet transform.

Signals and noise have different singularities mathematically (Mallat and Hwang, 1992). When we perform wavelet transform on an image repeatedly it is observed that the edge structures present observably while noise decreases rapidly along the scales. Multiplying the wavelet transforms at adjacent scales sharpen the edges while diluting the noise.

In this project I carried out wavelet transform at four different levels taking two different wavelets: Haar and Daubechies-2. Wavelet transform can be represented by




where $h_0$ is the low-pass analysis filter and $h_1$ is highpass analysis filter.

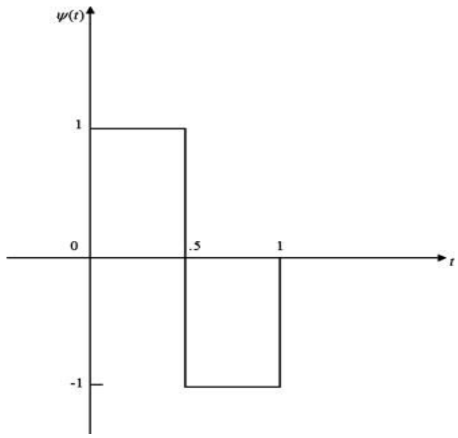
Haar wavelet is represented by:

$$\psi_{jk}(x) \equiv \psi\left(2^j x - k\right)$$

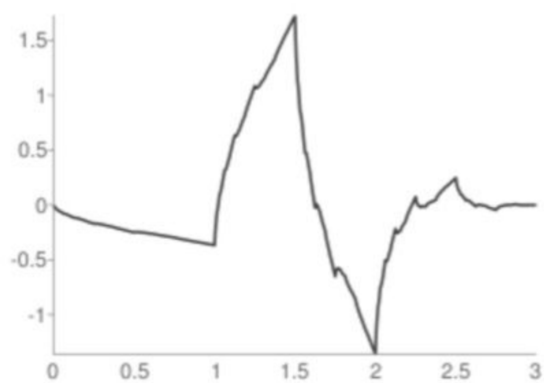
where,

$$\psi(x) \equiv \begin{cases} 1 & 0 \le x < \frac{1}{2} \\ -1 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

which looks like:




Similarly, Daubechies-2 wavelet is represented by:




Selection of these two wavelets was based on their ability to to represent polynomial behaviour or information in a signal. As we go for higher order wavelets their sensitivity to noise increase to capture more data from the signal. Therefore, for edge detection we needed simplest of the wavelets.

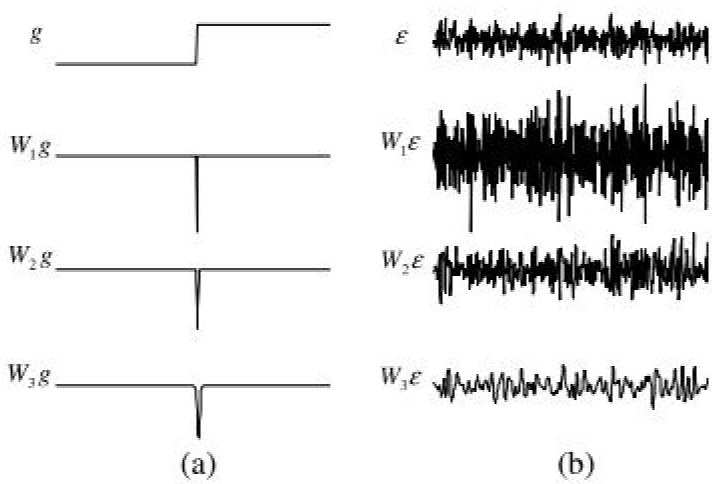
# 3. Approach

### Wavelet transform

A function is called a wavelet if its average is equal to zero. The wavelet transform of $f(x)$at scale $2^j$ and position $x$ is given by

$$W_j f(x) = f * \psi_j(x)$$

We performed wavelet transform of images at four contiguous scales. The DWT amplitudes would increase or keep invariant when increasing the scale $2^j$ . On the contrary, the regularity of white noise is less than 0, the transform amplitudes will decrease rapidly along the scales. In figure below the DWT of a step function and noise are illustrated. It can be observed that the DWT amplitudes of the step are large across scales, but those of noise decay rapidly.



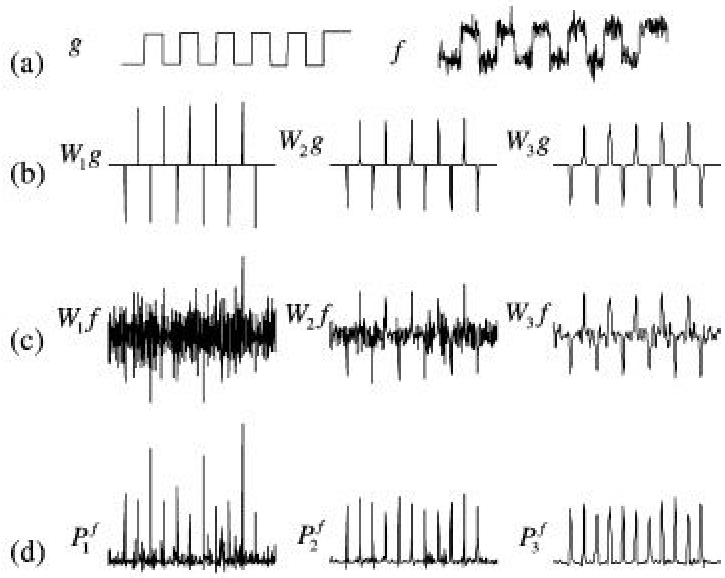

(a) The DWT of step g at the first three scales,
(b) the DWT of noise e at the first three scales.

### Scale Multiplication

After wavelet transform multiplication of adjacent scales is performed. The scale product function of $f(x)$ is defined as the correlation of two adjacent DWT scales $2^j$ and $2^{j+1}$

$$P_j^f(x) = W_j f(x) W_{j+1} f(x)$$

As shown above the peaks due to edges tend to propagate across scales. The production function will enhance the edge structures. But if $f(x)$ is noise, the average number of local maxima at scale $2^{j+1}$ is half of that at scales $2^j$ . Directly multiplying the DWT at adjacent scales will dilute the noise. With increasing scale the wavelet transform will become smoother rapidly. But if three or more adjacent scales were incorporated in the multiplication, edges would not be sharpened more but much edge dislocation would occur. So it is appropriate to analyze the multiplication using two scales.




(a) Blocks *g* and its noisy version *f*, (b) the DWT of *g* at the first three scales, (c) the DWT of noisy *f* at the first three scales, (d) the product function $P^f_j$ with $j$ = 1–3.

It is shown in the above figure that at the finest scale the wavelet coefficients $W_1f$ are almost dominated by noise. At the second and third scales, the noise diluted rapidly. It can also be seen that at the small scales the positions of the step edges are better localized. But some noise may be falsely considered as edges. At the large scales, the signal to noise ratio is improved and edges can be detected more correctly but with the decreasing of the accuracy of the edge location. In the above figure (d), the product $P^f_j$ , $j$ =1–3, are illustrated. Apparently the step edges are more observable in $P^f_j$ than in $W_jf$ .

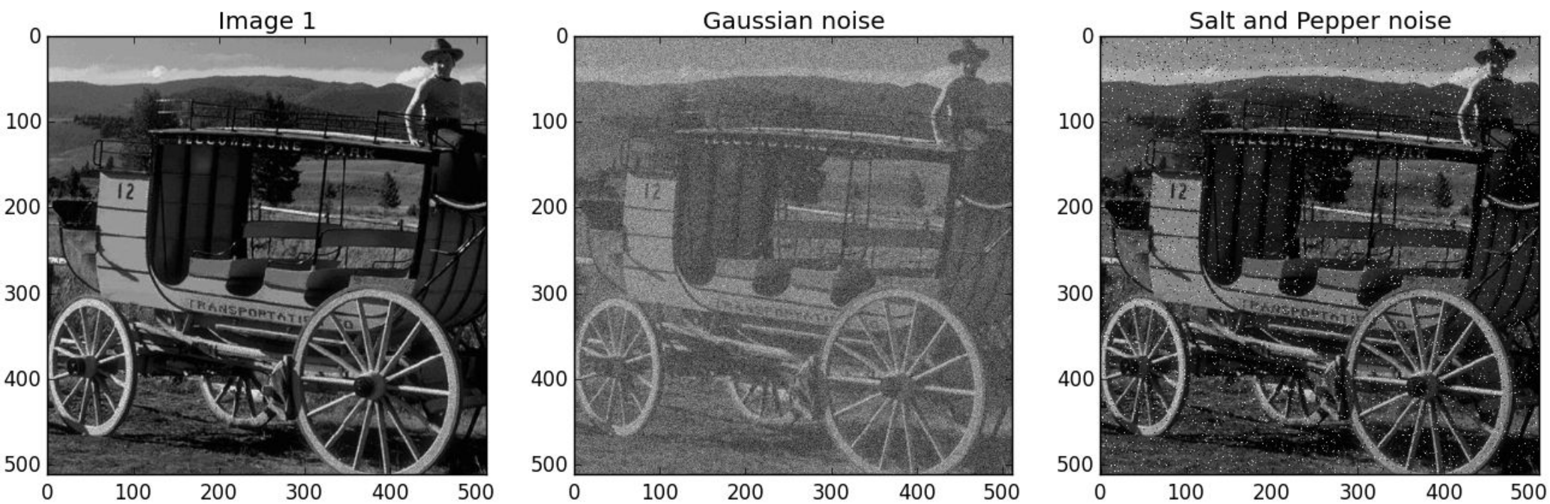
### Process

The above two process are applied on images with Gaussian Noise and Salt and Pepper Noise. Gaussian noise added with mean=20 and sigma=40 and Salt and Pepper noise added with 20% amount. The amount of noise decided to show its significance in edge detection process.

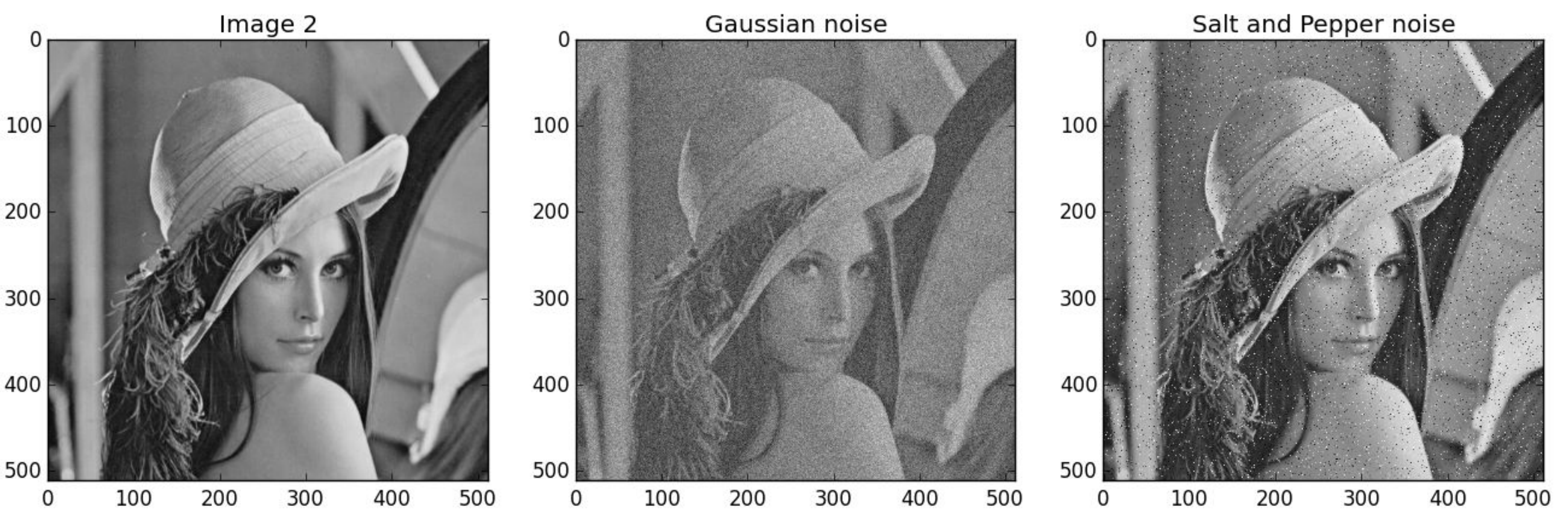
After addition of noise wavelet transform with dwt2() function of PyWavelets library. Scale multiplication of horizontal, vertical and diagonal details is performed respectively and the results are added together to form the final edge map.

# 4. Outcomes

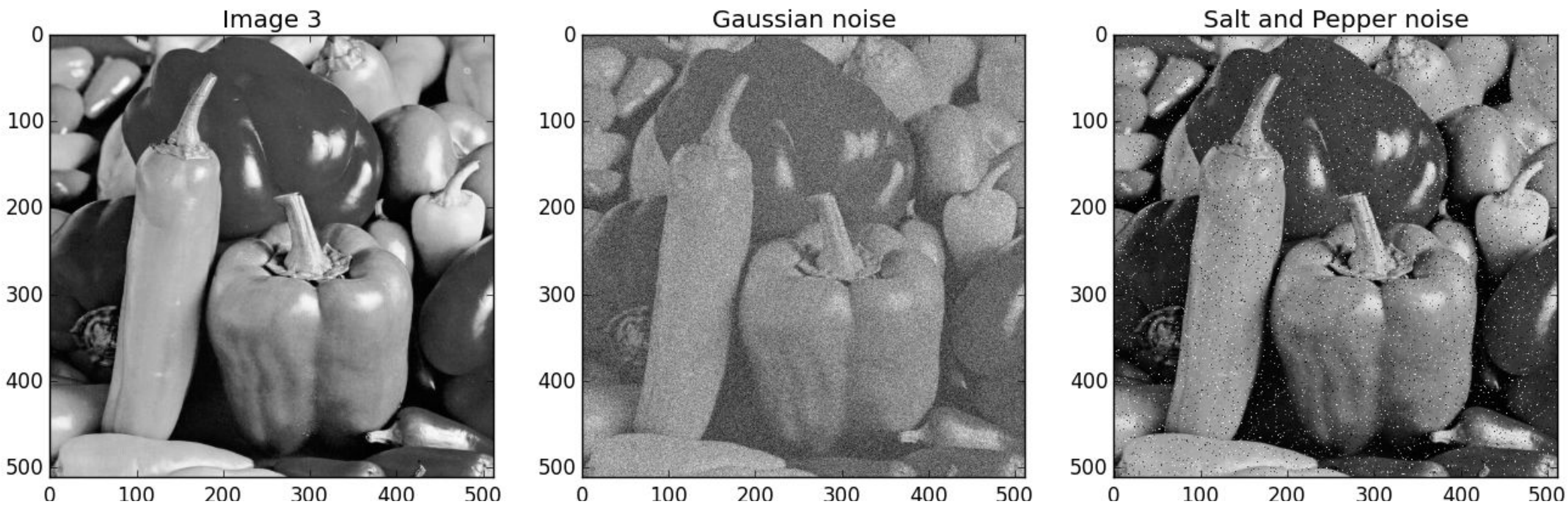
Adding noise to the three images:




Gaussian Noise(mean=20, sigma=40), S&P noise(40%)

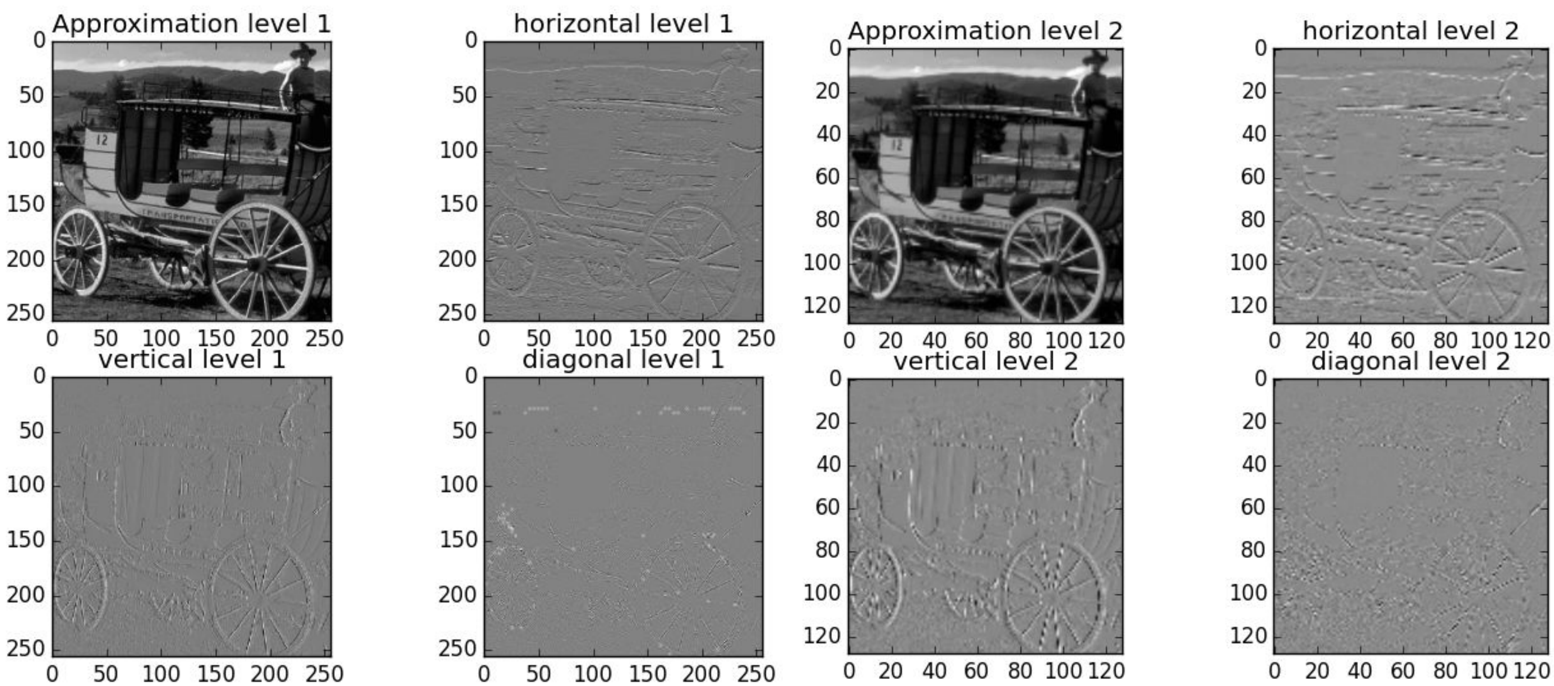


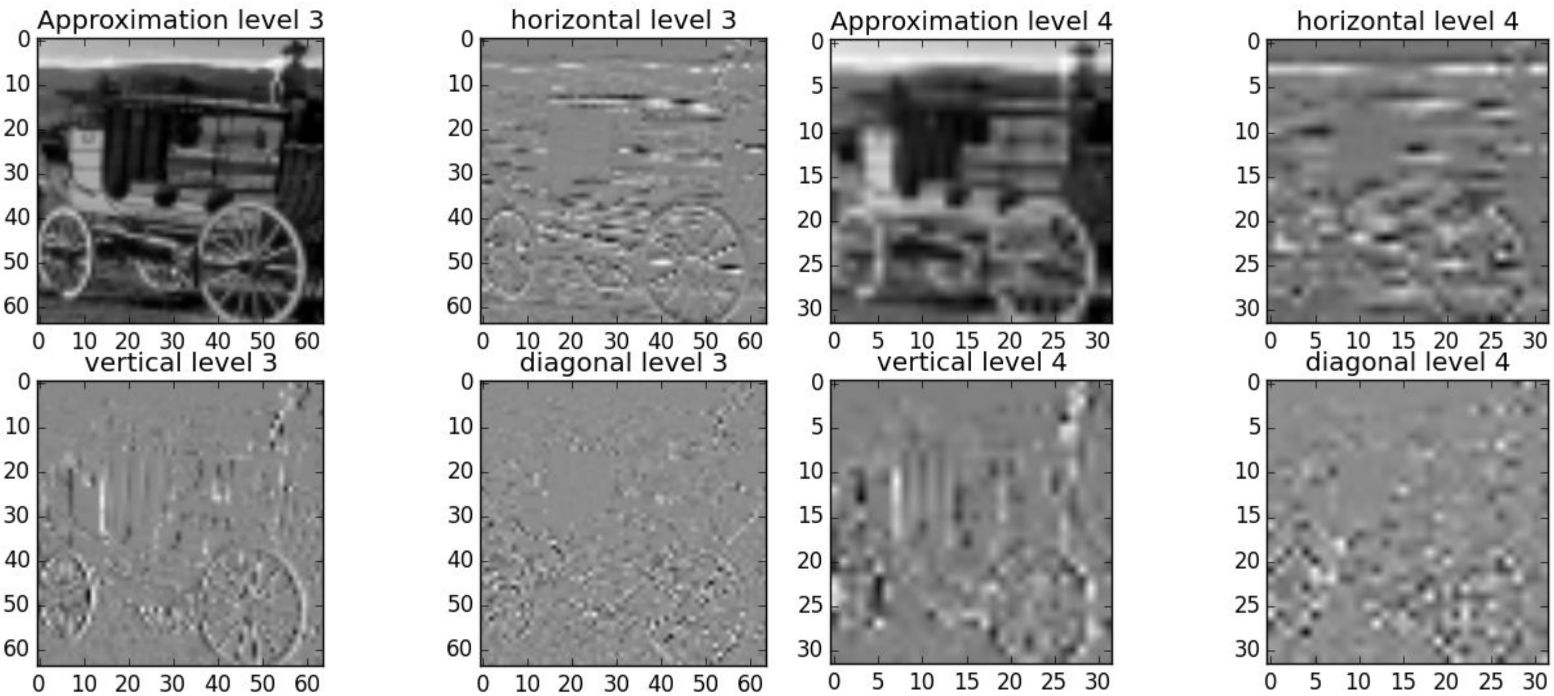

Gaussian Noise(mean=20, sigma=40), S&P noise(40%)



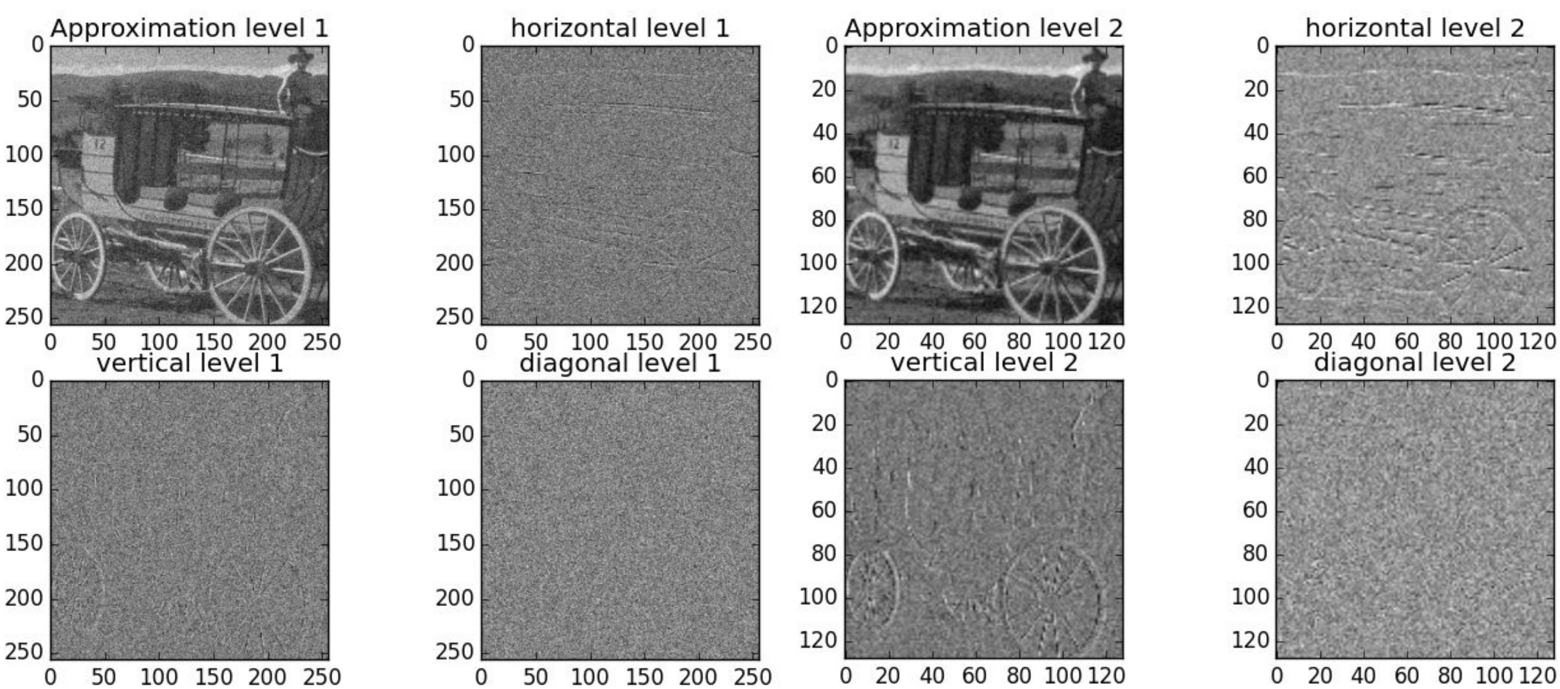

Gaussian Noise(mean=20, sigma=40), S&P noise(40%)

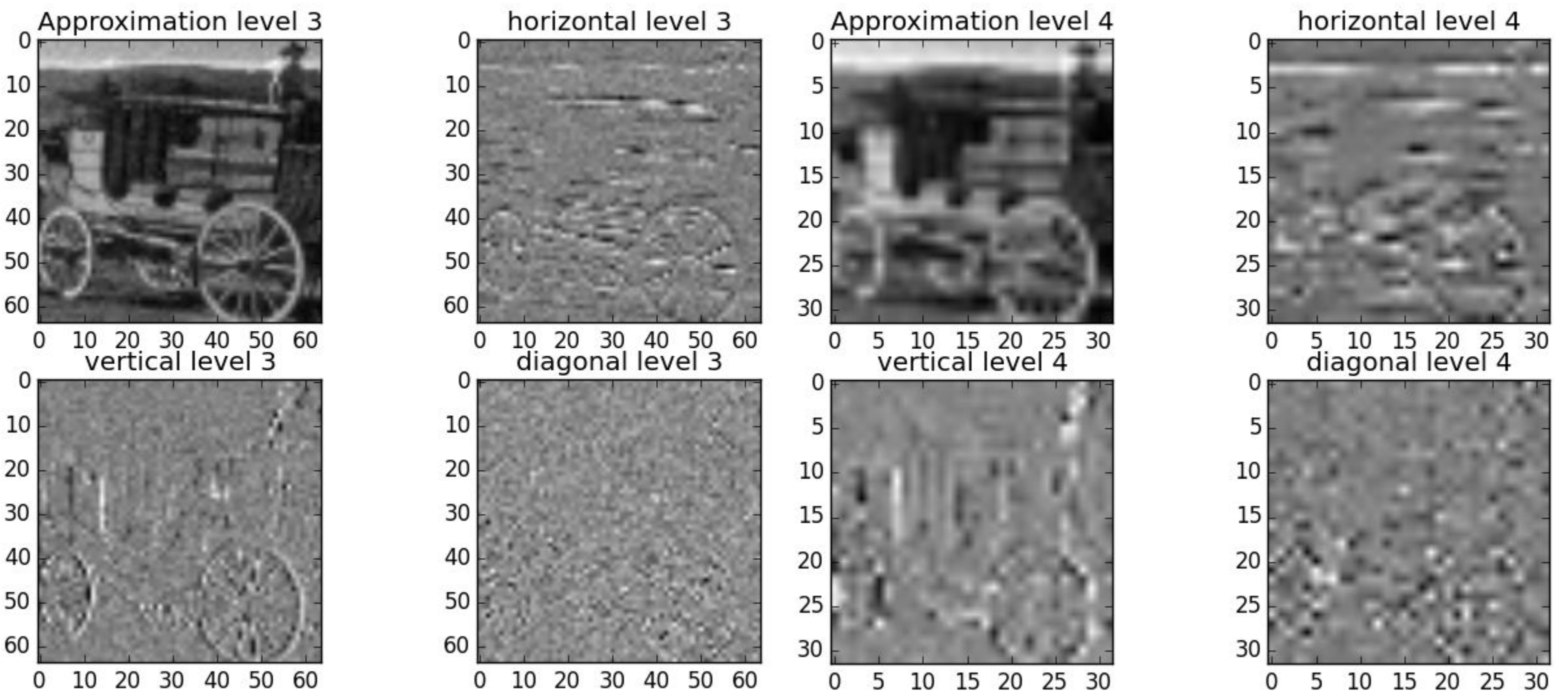
Wavelet transform of the original image:






Wavelet transform of the noisy image:






Scale multiplications at level 1&2, level 2&3 and level 3&4.

Image 1 with Gaussian noise using haar wavelet:

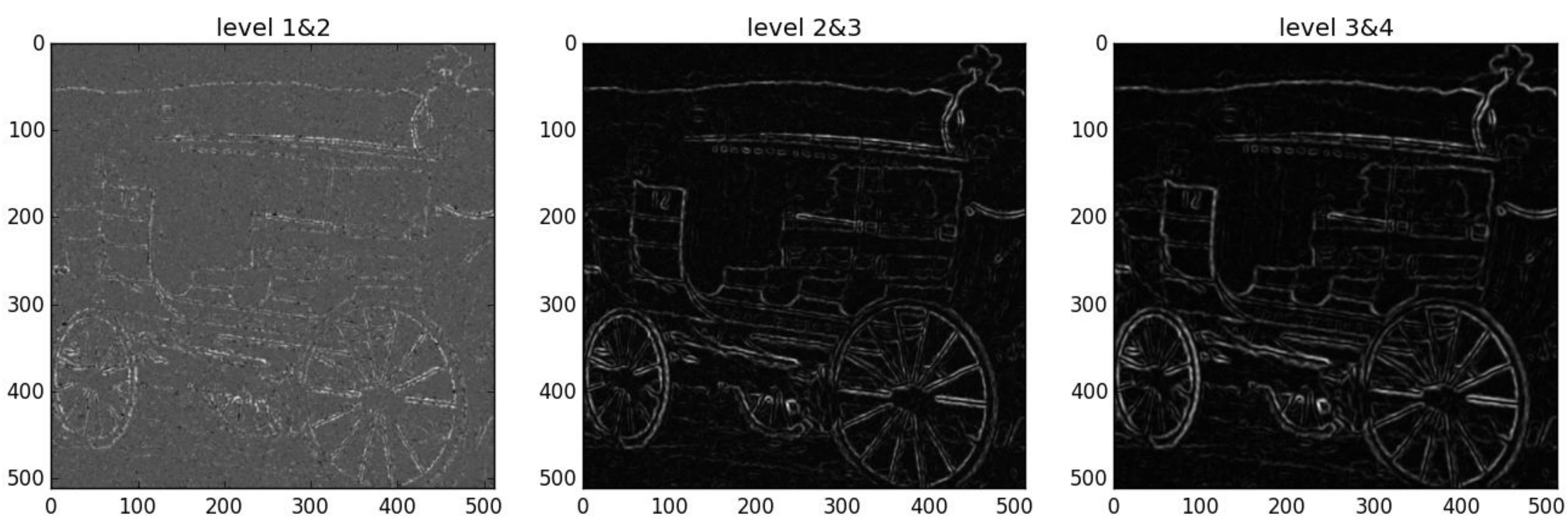


Image 1 with Gaussian noise using daubechies-2 wavelet:

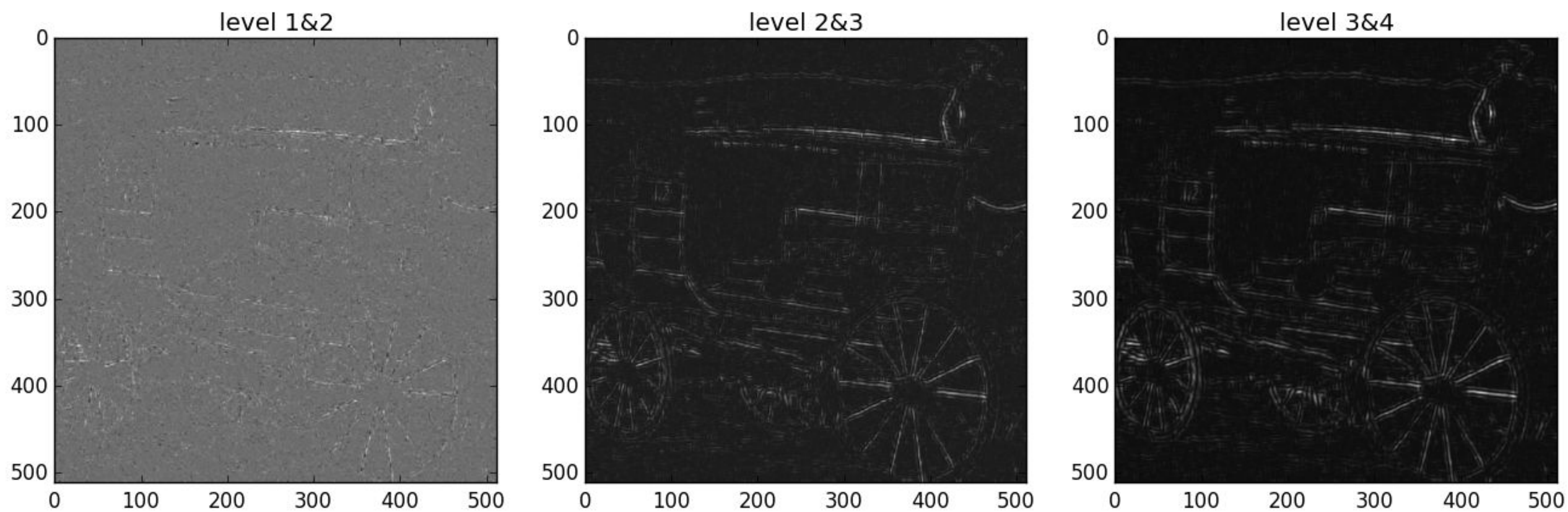



Image 1 with Salt and pepper noise using haar wavelet:

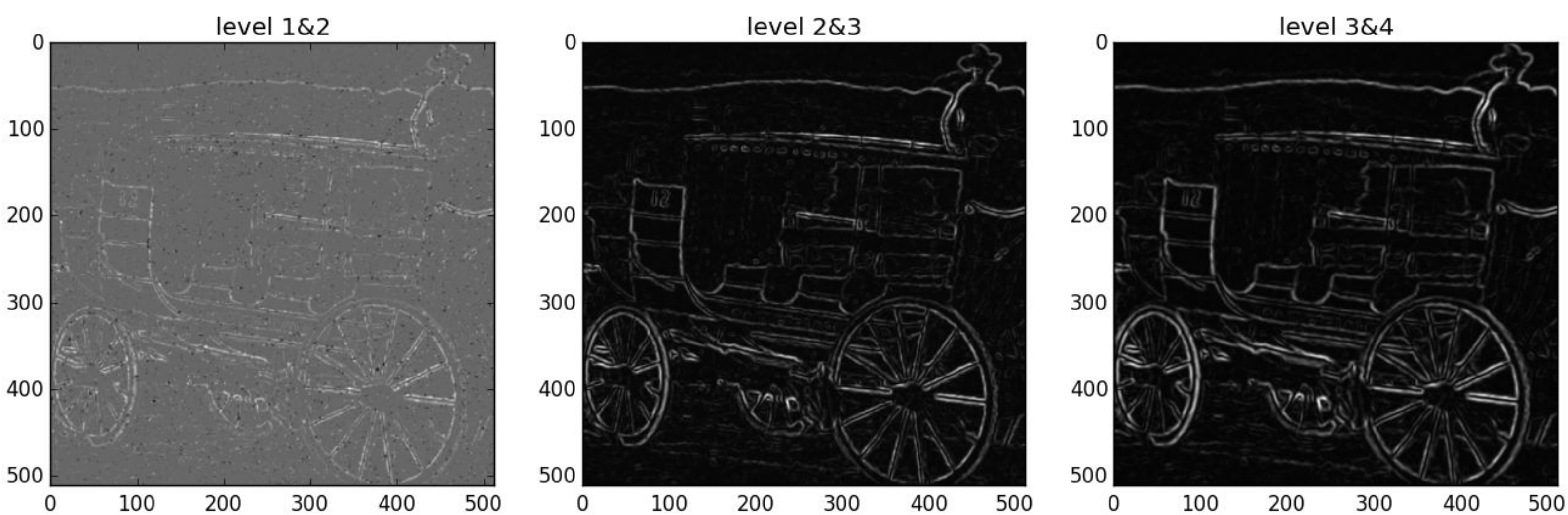


Image 1 with Salt and pepper noise using daubechies-2 wavelet:

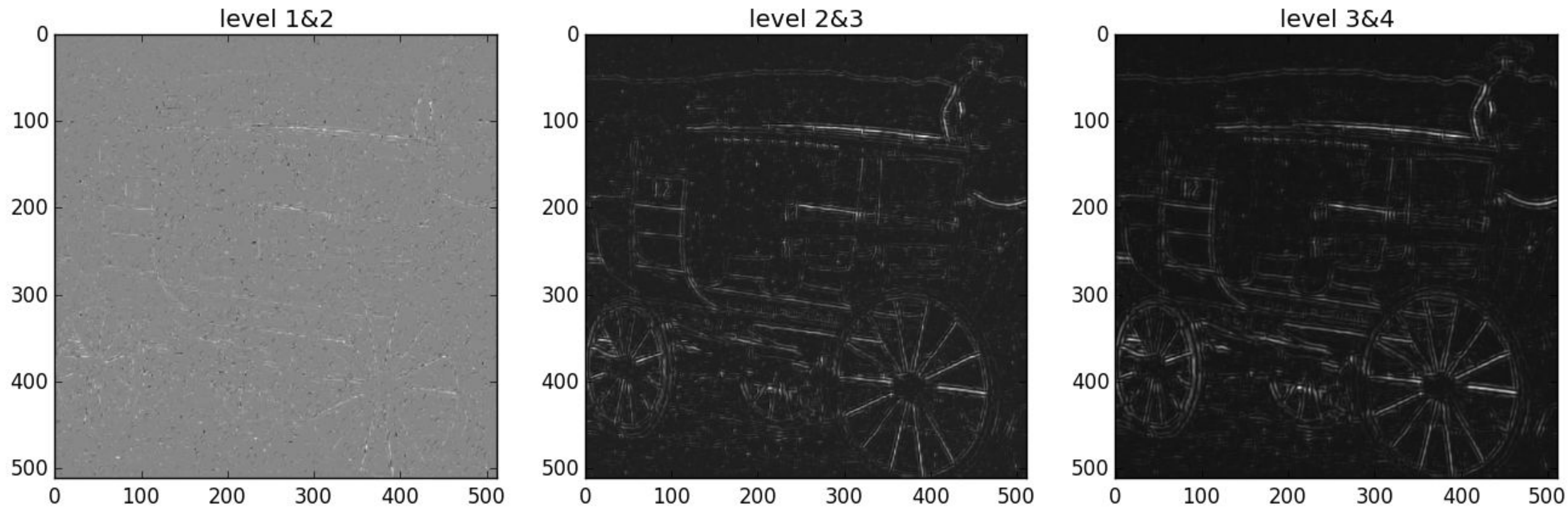



Image 2 with Gaussian noise using haar wavelet:

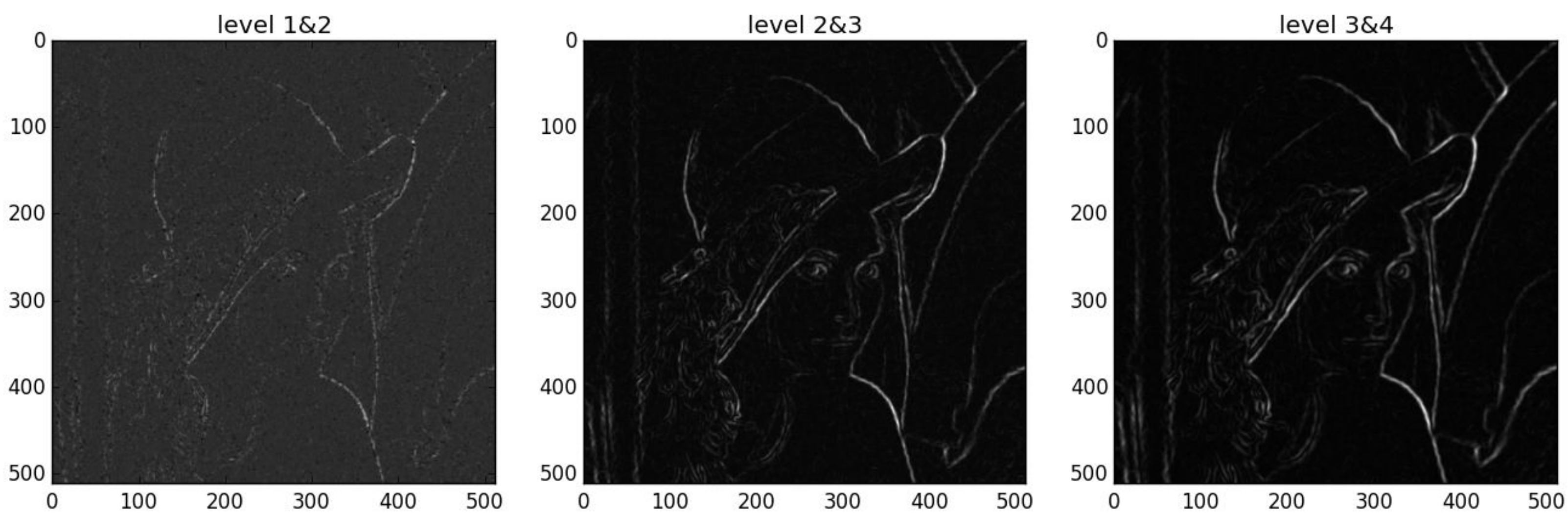


Image 2 with Gaussian noise using Daubechies-2 wavelet:

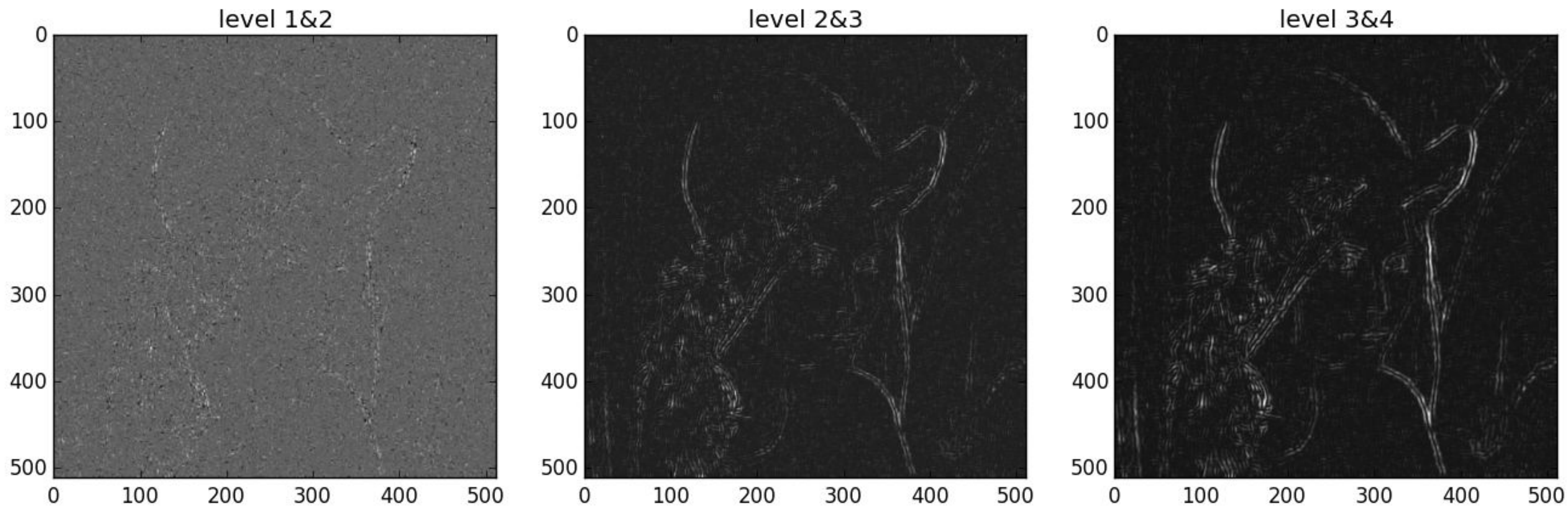



Image 2 with Salt and pepper noise using haar wavelet:

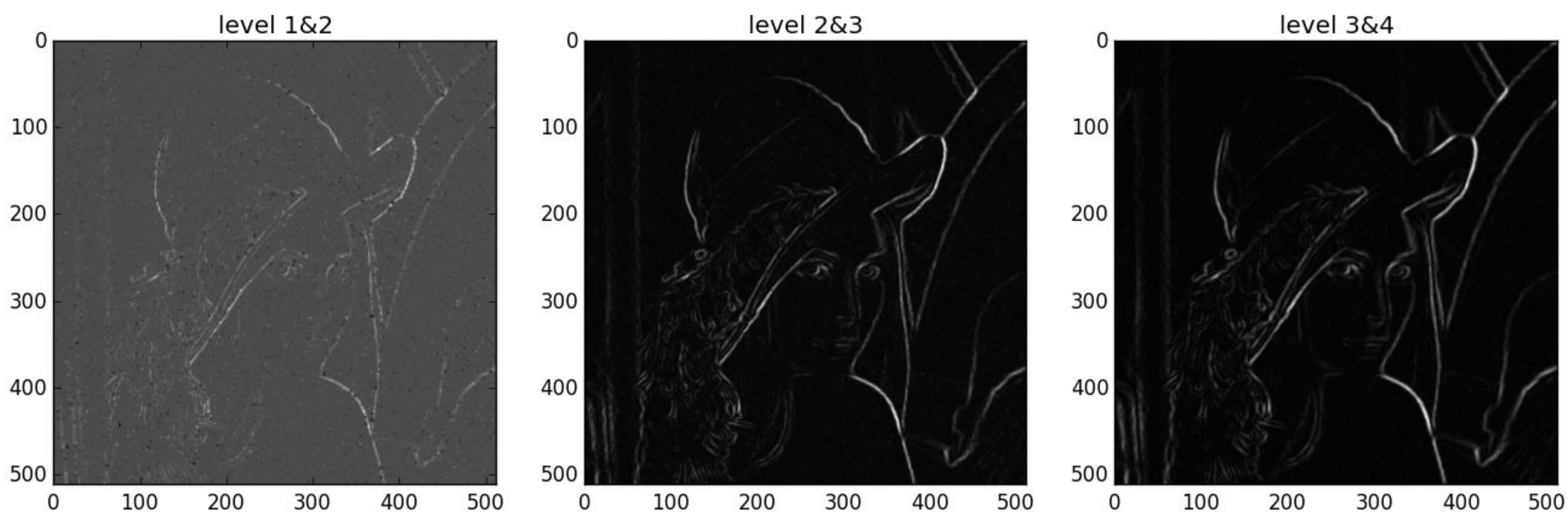


Image 2 with Salt and pepper noise using daubechies-2 wavelet:

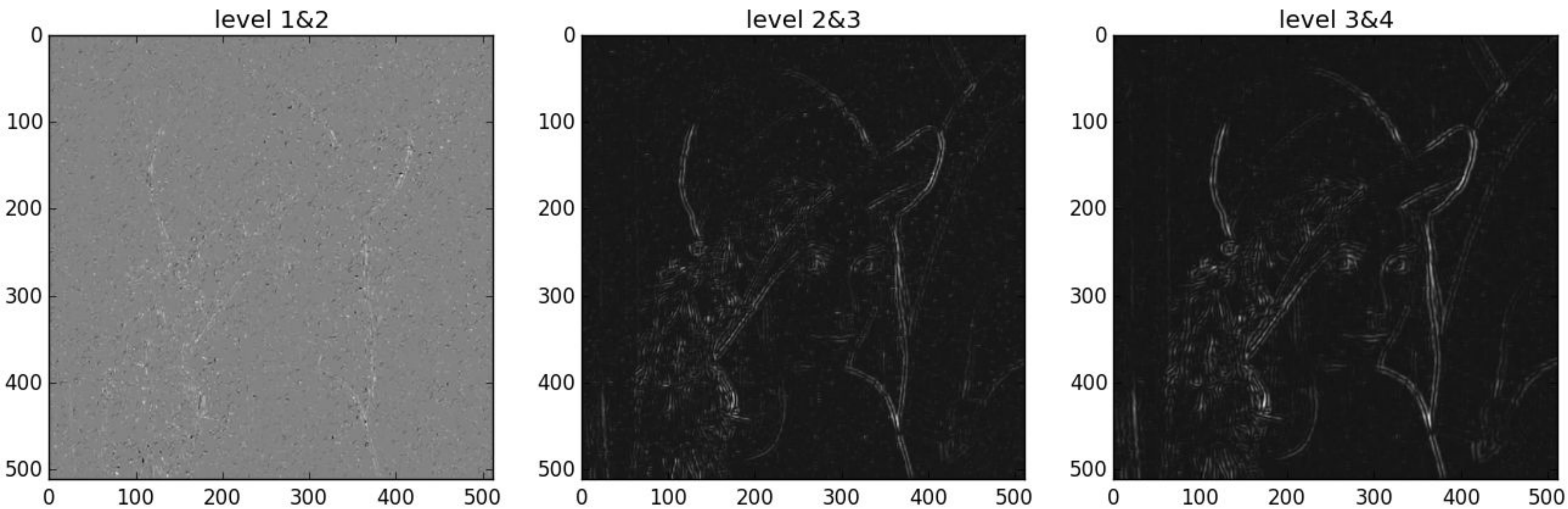



Image 3 with Gaussian noise using haar wavelet:

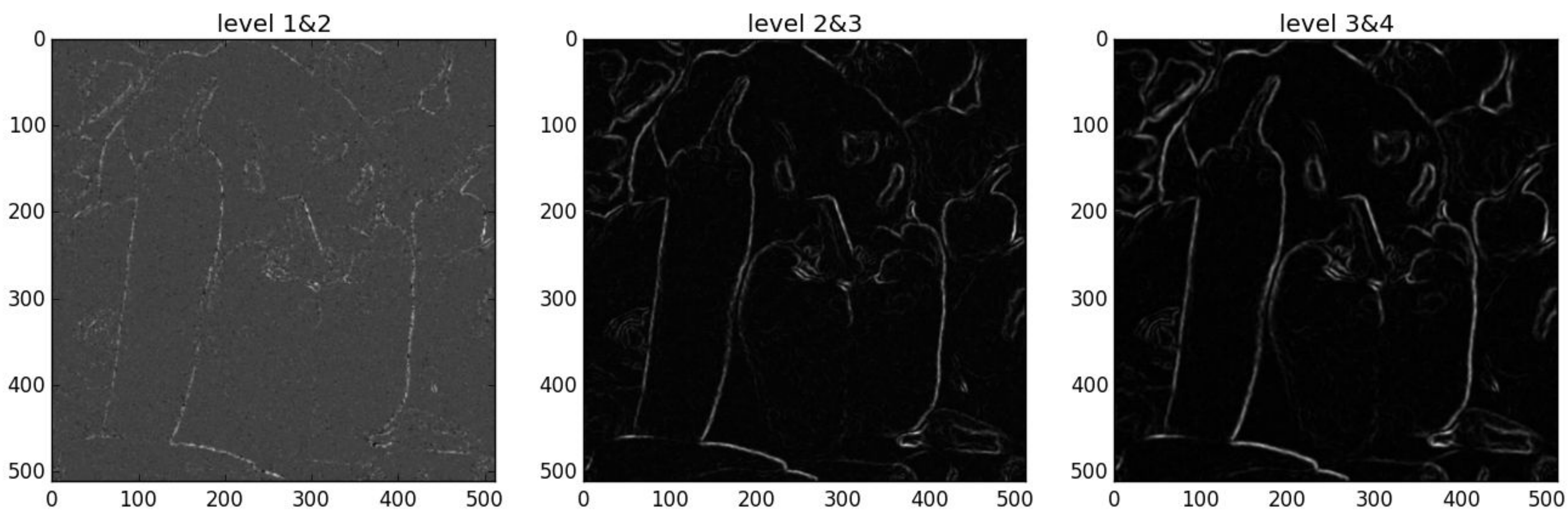


Image 3 with Gaussian noise using daubechies-2 wavelet:

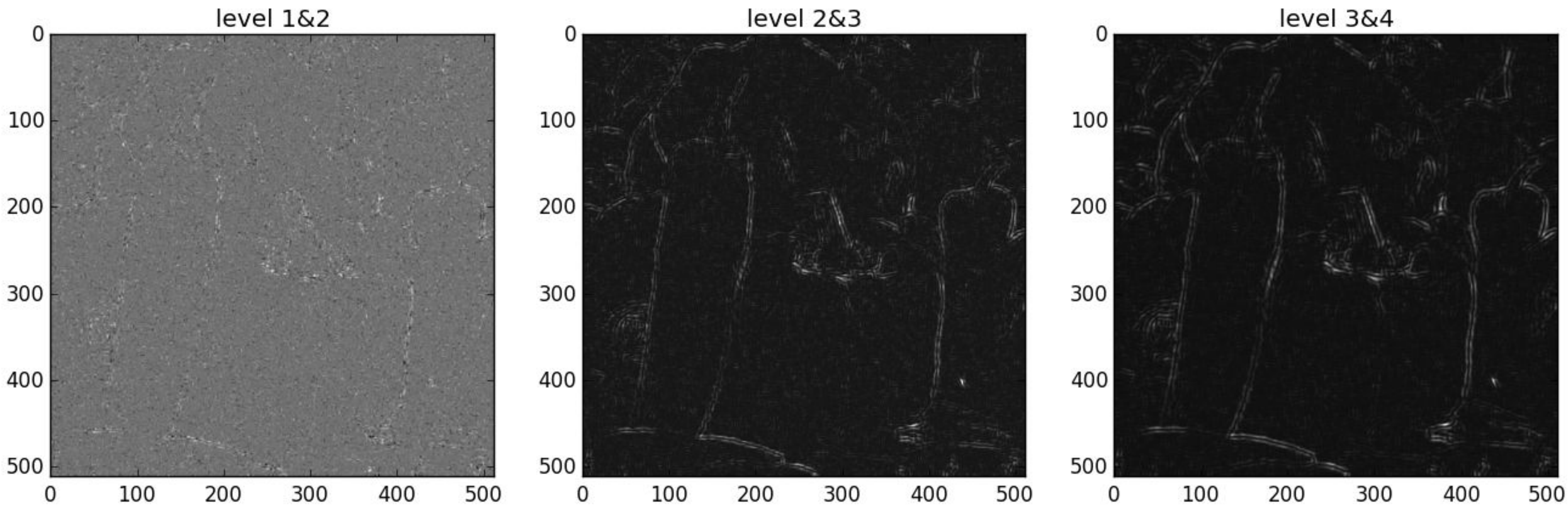



Image 3 with Salt and Pepper noise using haar wavelet:

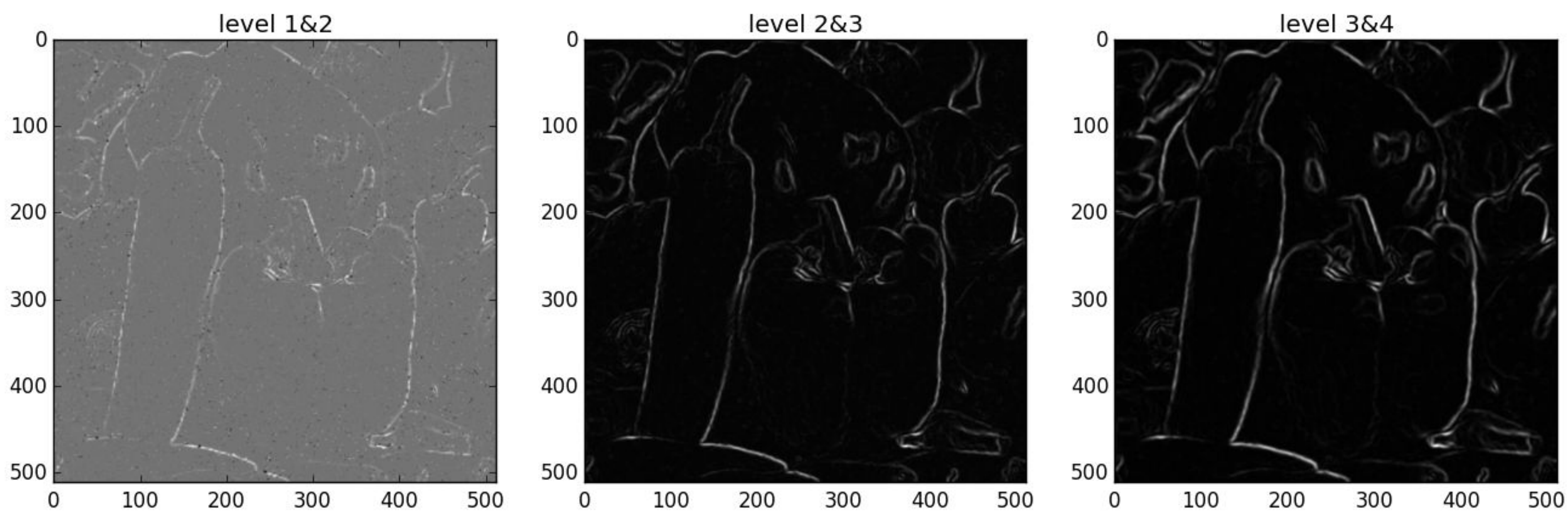
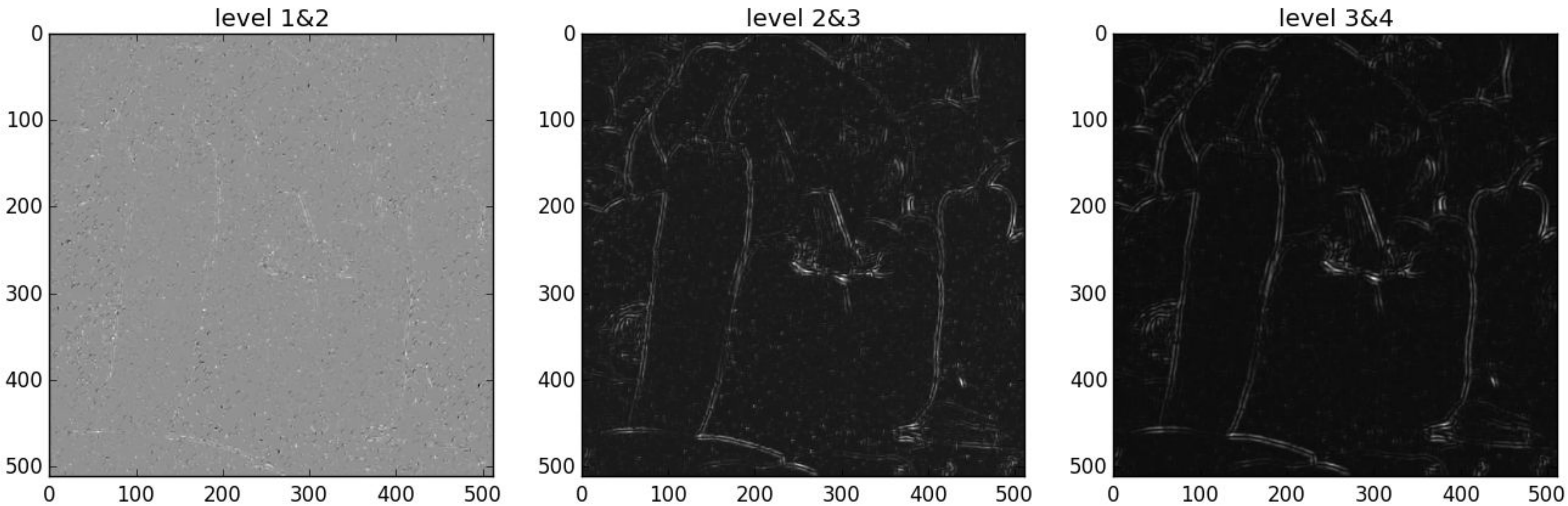


Image 3 with Salt and Pepper noise using daubechies-2 wavelet:




As we can see from the above results using haar wavelet gives clearer results but with some dislocation of edges. Clearly it shows only sharp edges. We can find fewer dislocation in case of daubechies-2 wavelet but we have some false edges as well because it is more sensitive to noise. If we compare the results from other edge detectors these results are much better.

# 5. Software and Program Development

Development of the project is done in Python using Enthought Canopy IDE. Python is a very powerful language with many libraries available for code development. Some of the libraries and their inbuilt functions used to develop this project are:

- pywt:
  This library contains various functions for wavelet transform. In this project dwt2() is used as it gives four results in the form of approximation and details and it is used for 2D images.
- cv2:
  This library contains various image processing functions. Here we used multiply() for scale multiplication.
- matplotlib.pyplot:
  This library is used for plotting of results.
- PIL.Image:

It is used for reading images.

One major difficulty faced in the project while coding was scale multiplication of wavelet transform result from daubechies-2 wavelet were not compatible. Wavelet transform reduces the size of image to half but when using daubechies wavelet it was reduced to half plus 1. So when expanding the 512 image was becoming 514 therefore not being compatible for multiplication. This was corrected by subtracting one pixel layer. This is different for different wavelets.

# 6. Summary

In this project, we performed a wavelet based edge detection by scale multiplication. Since there exist high spatial similarities in wavelet subbands, we defined a scale product function as the multiplication of two adjacent scales of wavelet coefficients to amplify edge structures while diluting noise. Then the edges are determined as the local maxima of the product to avoid the ill-posed edge synthesis process in most multiscale detection schemes.

In this course we learned various image processing steps. We learned about Fourier transform and wavelet transform and why they are important in image processing. Convolution was performed mathematically as well as while coding. We also performed Histogram equalization and experimented with many low contrast images. Different edge detection method like by simple filters, zero crossing in DoG and LoG and wavelet transform and their comparisons were performed by coding. Simple segmentation scheme was also implemented. Lastly some mathematical morphology functions was learnt in this course.

# 7. References


1. L. Zhang and P. Bao, “Edge detection by scale multiplication in wavelet domain,” Pattern Recognition Letters, Vol. 23, No. 14, pp. 1771-1784, December 2002
2. Milan Sonka, Roger Boyle, and Vaclav Hlavac, “Image Processing: Analysis and Machine Vision”
3. Nicholas Daras, “Applications of Mathematics and Informatics in Military Science”
4. www.pybytes.com for information regarding various wavelet families and methods for wavelet transform
5. www.stackoverflow.com for various small doubts