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Letters

Flocking of multiple autonomous agents with preserved network connectivity and heterogeneous nonlinear dynamics

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ABSTRACT

This paper investigates a flocking problem of multiple agents with heterogeneous nonlinear dynamics. In order to avoid fragmentation, we construct a potential function and a connectivity-preserving flocking algorithm to enable the multiple agents to move with the same velocity while preserving the connectivity of underlying networks with a mild assumption that the initial network is connected and the coupling strength of the initial network of the nonlinear velocity consensus term is larger than a threshold value. Furthermore the proposed flocking algorithm is extended to solve the problem of multi-agent systems with a nonlinear dynamical virtual leader. The result is that all agents' velocities asymptotically approach to the velocity of the virtual leader, and the distance between any two agents is asymptotically stabilized to avoid collisions among agents. Finally, some numerical simulations are presented to illustrate the effectiveness of the theoretical results.

1. Introduction

In nature, there exist many interesting collective behaviors such as flocking of birds, schooling of fish, swarming of bacteria and so forth [1]. Flocking is a mechanism in multi-agent systems to achieve velocity synchronization and regulation of relative inter-agent distances with agents obeying some basic rules. For decades, more and more researchers in physics, biology, computer science and control engineering devote themselves to studying flocking [2–4]. The application of multi-agent systems can be extended to many fields such as unmanned air vehicles (UAVs) and cooperative control of mobile robots [5–9].

Reynolds developed three heuristic rules in 1987 [2] that led to the appearance of the first computer animation of flocking. The Reynolds rules describe how an individual agent maneuvers itself based on the information of the positions and velocities of its nearby flockmates. A simple flocking model of multiple agents was then proposed by Vicsek et al. [3], where each agent updates its orientation by averaging its own direction and the neighbours' directions. Recently, Jadbabaie et al. [5] first rigorously proved the convergence of Vicsek's model. Based on the Vicsek model, Couzin et al. [4] introduced a three-dimensional model of multi-agent with three sensing zones, which can develop into some special phenomena such as torus and highly parallel movement.

Olfati-Saber provided a computational and theoretical framework in order to design and analyze scalable flocking [10]. In [10], including position and velocity feedback of the virtual leader to every agent in a group, stable flocking motion is achieved under some general initial conditions. In [11], flocking algorithm was investigated in both fixed and switching networks. In [12], the results of [10] are generalized to the situation where only a fraction of agents have information of the leader and the virtual leader has a time-varying velocity. The flocking problem with multiple virtual leaders was studied in [13]. A connectivity-preserving flocking algorithm for multi-agent systems based on position measurements is considered in [14].

Most previous works focus on linear systems especially systems with double-integrator dynamics [5–14]. However, in reality, autonomous agents might be governed by more complicated nonlinear dynamics. In fact, in synchronization of complex dynamical networks [15–17], nonlinear dynamics is commonly used. Consensus and flocking of multi-agent systems with some uniform nonlinear dynamics were investigated in [18,19], respectively. Using the neural network approximation and the robust control technique, a decentralized consensus algorithm was proposed for multi-agent systems with the uncertain nonlinear dynamics and external disturbances [20]. Furthermore, the authors of [21] extended the novel consensus algorithm in [20] to solve the

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leader-following problem with the leader having a time-varying state trajectory.

Many theoretical investigations in flocking of multiple agents tracking a (virtual) leader are formulated as a linear system or network with a fixed-coupling topology and uniform intrinsic agent dynamics. Therefore, a multi-agent system of flocking with heterogeneous nonlinear intrinsic dynamics is considered in this paper, where the network connectivity is preserved and the interconnection topology network based on the distance between agents varies with time. Furthermore, we extend the result to the multi-agent system with a nonlinear dynamical virtual leader. Similar to the flocking algorithms in [22], the signum function is used to construct our new flocking algorithm. Comparing with the results in [22], in which consensus tracking and swarm tracking are investigated for the agents with single integrator dynamics and double integrator dynamics, our model is more general, in which all the agents and the leader have heterogeneous nonlinear dynamics. To keep all agents moving at the same velocity and guarantee stabilization of the distance between agents for collision avoidance, a connectivity-preserving algorithm combined with an artificial potential function for multiple agents are presented under a mild assumption that the initial network is connected. Using the proposed flocking algorithm, the multiple agents move with the same velocity while preserving the network connectivity, and in addition all agents's velocities can asymptotically approach to the velocity of the virtual leader effectively in a multi-agent system with a nonlinear dynamic virtual leader.

The remainder of the paper is organized as follows. We first describe the model of flocking system in Section 2. Our new results on the flocking problem are established in Section 3. Then some numerical simulation examples are presented to validate the effectiveness of the theoretical results in Section 4. Conclusions are drawn in Section 5.

2. Model description

In our formulation, there are N agents, labeled as $1,2,3,\ldots,N$. We denote $p_i \in \mathbf{R}^n$ as the position vector of agent i and $v_i \in \mathbf{R}^n$ as the velocity vector of agent i. These agents move in an n-dimensional Euclidean space. The dynamic equations of each agent can be described as follows:

$$\dot{p}_i = v_i,$$

$$\dot{v}_i = f_i(v_i) + u_i, \tag{1}$$

where $f_i(v_i) \in \mathbf{R}^n$ is the intrinsic dynamic of agent i, $i = 1,2,\ldots,N$ and $u_i \in \mathbf{R}^n$ is the control input of agent i. In particular, the intrinsic dynamics of multiple agents are heterogeneous and nonlinear.

To avoid fragmentation, we set up a connectivity-preserving flocking algorithm. Assuming that all agents have the same influencing/sensing radius $r > 0$, let $\varepsilon \in (0,r]$ be an arbitrarily small constant [23]. $G(t) = (\mathcal{V},\mathcal{E}(t))$ is an undirected dynamic graph composed of a set of vertices $\mathcal{V} = 1,2,\ldots,N$, denoting the set of agents and a time-varying set of edges $\mathcal{E}(t) = \{(i,j) | i,j \in V\}$.

A symmetric indicator function $\sigma(i,j) \in \{0,1\}$ is used to describe when there is an edge between agent i and agent j at time t [23]. Algorithm 1 is described in Table 1. From Fig. 1, there is a hysteresis in the indicator function when adding new edges to the systems, which is important for convergence of the agents. In this paper, our major works is to design the control input u_i, $i = 1,2,\ldots,N$, to make the multi-agent system approach a stable flocking motion while preserving the connectivity of the dynamic network and avoiding collision between agents. To achieve the goal, the above-described control law u_i for agent i is designed as

Table 1
Algorithm 1: description of the indicator function.

Require	$\mathcal{E}(0) = \{(i,j) \mid \|p_i(0)-p_j(0)\| < r, i,j \in \mathcal{V}\}$ and $\sigma(i,j)[t^-] \in \{0,1\}$
1	**if** $\sigma(i,j)[t^-] = 1$ and $\|p_i(t)-p_j(t)\| \geq r$
2	**Then** $\sigma(i,j) = 0, i,j \in \mathcal{V}$
3	**else if** $\sigma(i,j)[t^-] = 0$ and $\|p_i(t)-p_j(t)\| \geq r-\varepsilon$
4	**Then** $\sigma(i,j) = 0, i,j \in \mathcal{V}$
5	**else if** $\sigma(i,j)[t^-] = 1$ and $\|p_i(t)-p_j(t)\| \leq r$
6	**Then** $\sigma(i,j) = 1, i,j \in \mathcal{V}$
7	**else if** $\sigma(i,j)[t^-] = 0$ and $\|p_i(t)-p_j(t)\| \leq r-\varepsilon$
8	**Then** $\sigma(i,j) = 1, i,j \in \mathcal{V}$
9	**end if**

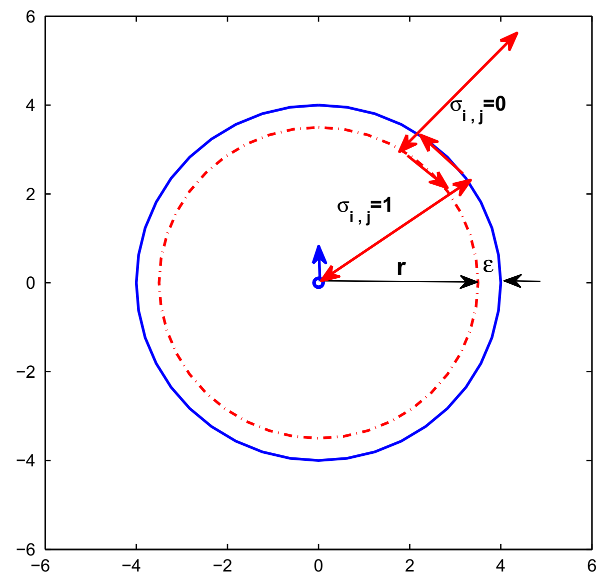

Fig. 1. A symmetric indicator function $\sigma(i,j)$ designed to describe whether there is a link between agent i and agent j at time t. $\sigma(i,j) = 1$ when there exists a link, otherwise $\sigma(i,j) = 0$.

$$u_i = \alpha_i + \beta_i, \tag{2}$$

where $\alpha_i \in \mathbf{R}^n$ is a gradient-based term to enforce the position of each agent to converge to a common value, used to avoid collision among nearby agents, and to keep the connectivity of the network. $\beta_i \in \mathbf{R}^n$ is a velocity consensus term, used to govern the velocity of agents to a common direction. However, sometimes, some certain purposes may be considered in a multi-agent system, for example, tracking a desired common time-varying velocity. In this case, the control law of following a dynamic virtual leader is modified by:

$$u_i = \alpha_i + \beta_i + \gamma_i, \tag{3}$$

where $\gamma_i \in \mathbf{R}^n$ is the navigational nonlinear feedback term that enforce the velocities of all agents can track the virtual leader.

3. Main result

3.1. Flocking of multiple agents without a virtual leader

In this subsection, the flocking problem of multi-agent system without a virtual leader is investigated. In order to derive the main results, the following assumption is needed.

Assumption 1. Suppose ℓ is a positive constant such that

$$\|f_i(v_i) - f_j(v_j)\| \le \ell, \quad \forall v_i, v_j \in \mathbf{R}^n i,\ j = 1,2,\ldots,N, \tag{4}$$

where $\|\cdot\|$ denotes the Euclidean norm in $\mathbf{R}^n$.

Denote the central position and velocity of all the agents in the system (1) by $\overline{p} = \sum_{i=1}^{N} p_i/N$ and $\overline{v} = \sum_{i=1}^{N} v_i/N$, respectively. The control law is governed by

$$u_i = -\sum_{j \in \mathcal{N}_i(t)} \nabla_{p_i} \psi(\|p_{ij}\|) - \rho \sum_{j \in \mathcal{N}_i(t)} a_{ij} \left\{ \mathrm{sgn}\left[\sum_{k \in \mathcal{N}_i(t)} a_{ik}(v_i - v_k) \right] - \mathrm{sgn}\left[\sum_{k \in \mathcal{N}_j(t)} a_{jk}(v_j - v_k) \right] \right\}, \tag{5}$$

where $\|p_{ij}\| = \|p_i - p_j\|$, ρ is the a positive constant, and $\mathcal{N}_i(t)$ denotes the neighborhood of agent i at time t, the explicit definition is

$$\mathcal{N}_i(t) = \{j \mid \sigma(i,j)[t] = 1,\ j \ne i,\ j = 1,2,\ldots,N\}. \tag{6}$$

The nonnegative potential $\psi(\|p_{ij}\|)$ is defined as a function of the distance $\|p_{ij}\|$ between agent i and agent j, differentiable with respect to $\|p_{ij}\| \in [0,r)$, satisfying

(1) $\psi(\|p_{ij}\|) \to \infty$ as $\|p_{ij}\| \to 0$ or $\|p_{ij}\| \to r$;
(2) $\psi(\|p_{ij}\|)$ attains its unique minimum when $\|p_{ij}\|$ equals a desired distance.

One example of such potential functions is as follows [23]:

$$\psi(\|p_{ij}\|) = \begin{cases} +\infty, & \|p_{ij}\| = 0, \\ \dfrac{r}{\|p_{ij}\|(r - \|p_{ij}\|)}, & \|p_{ij}\| \in (0,r), \\ +\infty, & \|p_{ij}\| = r. \end{cases} \tag{7}$$

The potential function is in the form of Fig. 2.

The adjacency matrix $A(t) = (a_{ij}(t))$ of system (1) is defined as $a_{ij}(t) = 1$ if $(i,j) \in \mathcal{E}(t)$, otherwise, $a_{ij}(t) = 0$. The Laplacian is defined as $L(t) = D(A(t)) - A(t)$, where the degree matrix $D(A(t))$ is a diagonal matrix with the ith diagonal element equal to $\sum_{j=1, j\ne i}^{N} a_{ij}(t)$. Denote $\lambda_1(\cdot)$ as the minimal eigenvalue of a symmetric matrix and the eigenvalues of $L(t)$ can be written as $\lambda_1(L(t)) \le \cdots \le \lambda_N(L(t))$. Then, $\lambda_1(L(t)) = 0$ and we can deduce its corresponding eigenvector is $1 = [1,1,\ldots,1]^T \in R^N$. Furthermore, if $G(t)$ is a connected graph, then $\lambda_2(L(t)) > 0$ [24]. For notational convenience, we denote

$$p = \begin{bmatrix} p_i \\ p_2 \\ \vdots \\ p_N \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}.$$

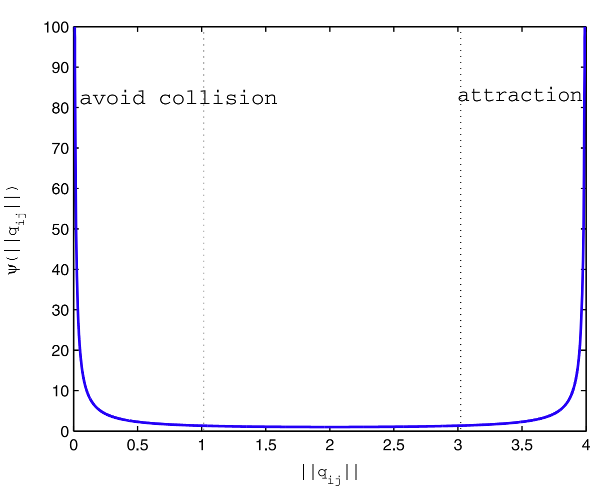

Fig. 2. The potential function of $\psi_{ij}(\|p_{ij}\|)$ with $r = 4$. The symmetric function respect to agent i and agent j. It preserves the distance $\|p_{ij}\| \to r/2$.

Lemma 1 (*Merris [25]*). *Suppose G be an undirected graph of order N, and G_1 be the undirected graph by adding some edge(s) into the graph G. Then $\lambda_i(L_1) \ge \lambda_i(L)$, for all $i = 1,2,\ldots,N$, where L_1 and L are the Laplacian matrices of G_1 and G, respectively.*

Denote the difference of position and velocity between agent i and the center of mass are given as $\hat{p}_i = p_i - \overline{p}$ and $\hat{v}_i = v_i - \overline{v_i}$, respectively. We define the sum of the total artificial potential energy and the total relative kinetic energy among agents and central of mass as

$$\hat{V}(\hat{p}, v, p, \overline{v}) = \frac{1}{2}\sum_{i=1}^{N}(U_i(p,\overline{p})) + \frac{1}{2}\sum_{i=1}^{N}(v_i - \overline{v})^T(v_i - \overline{v}), \tag{8}$$

where

$$U_i(p,\overline{p}) = \sum_{j \in \mathcal{N}_i(t)} \psi(\|\hat{p}_i - \hat{p}_j\|). \tag{9}$$

Theorem 1. *Consider a system of N autonomous agents with dynamic motion (1) driven by protocol (5). Suppose that the initial network $G(0)$ is connected, the initial energy $\hat{V}_{t_0}$ is finite, and $\rho(L(t_0)) - \ell N I > 0$. Then, the following results hold:*

(1) *$G(t)$ will remain connected all the time $t \ge 0$;*
(2) *all agents asymptotically approaches the same velocity and attain a relatively invariable distance;*
(3) *each agent's global potential $\sum_{j \in \mathcal{N}_i(t)} \nabla_{p_i} \psi(\|p_{ij}\|)$ is locally minimized for almost every final configuration;*
(4) *collisions among agents are avoided.*

Proof. We first prove part (1) of Theorem 1. As defined above, simple calculations are given as

$$\dot{\hat{p}}_i = \hat{v}_i,$$

$$\dot{\hat{v}}_i = f_i(v_i) - \frac{1}{N}\sum_{j=1}^{N} f_j(v_j) - \sum_{j \in \mathcal{N}_i(t)} \nabla_{\hat{p}_i} \psi(\|\hat{p}_{ij}\|) - \rho \sum_{j \in \mathcal{N}_i(t)} a_{ij} \left\{ \mathrm{sgn}\left[\sum_{k \in \mathcal{N}_i(t)} a_{ik}(\hat{v}_i - \hat{v}_k) \right] - \mathrm{sgn}\left[\sum_{k \in \mathcal{N}_j(t)} a_{jk}(\hat{v}_j - \hat{v}_k) \right] \right\}. \tag{10}$$

Moreover, the potential energy function (8) can be rewritten as

$$V(\hat{p}, \hat{v}) = \frac{1}{2}\sum_{i=1}^{N}(U_i(\hat{p}) + \hat{v}_i^T \hat{v}_i), \tag{11}$$

where

$$U_i(\hat{p}) = \sum_{j \in \mathcal{N}_i(t)} \psi(\|\hat{p}_i - \hat{p}_j\|), \tag{12}$$

and

$$\hat{p} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \vdots \\ \hat{p}_N \end{bmatrix}, \quad \hat{v} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_N \end{bmatrix},$$

where $V(\hat{p}, \hat{v})$ is a positive semi-definite function of $(\hat{p}, \hat{v})$.

Suppose that $G(t)$ switches at time t_k, $k=1,2,\ldots$, and remains fixed over each time interval $[t_{k-1},t_k]$. Without loss of generality, we assume that $t_0=0$ and the initial energy $\hat{V}(t_0)$ is finite. Considering the time derivative of $Q(t)$ on $[t_0,t_1)$ gives

$$\begin{aligned}\dot{\hat{V}} &= \sum_{i=1}^{N}\hat{v}_i^T\left[f_i(v_i)-\frac{1}{N}\sum_{j=1}^{N}f_j(v_j)\right]-\sum_{i=1}^{N}\hat{v}_i^T\rho\sum_{j\in\mathcal{N}_i(t)}a_{ij}\\ &\quad\times\left\{\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_i(t)}a_{ik}(\hat{v}_i-\hat{v}_k)\right]-\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_j(t)}a_{jk}(\hat{v}_j-\hat{v}_k)\right]\right\}\\ &= \sum_{i=1}^{N}\hat{v}_i^T\left[f_i(v_i)-\frac{1}{N}\sum_{j=1}^{N}f_j(v_j)\right]-\rho\hat{v}^TL(t_0)\,\mathrm{sgn}[L(t_0)\hat{v}]\\ &\le \|\ell I\hat{v}\|_1-\rho\|L(t_0)\hat{v}\|_1=(\|\ell I\|_1-\rho\|L(t_0)\|)\|\hat{v}\|_1\le 0,\end{aligned}\tag{13}$$

which implies that $\hat{V}(t)<\hat{V}(t_0)<\infty$, $\forall t\in[t_0,t_1)$. By the definition of the potential function, $\lim_{p_{ij}(t)\to r}\psi(\|p_{ij}(t)\|)=\infty$. Therefore, there is no distance of existing edges tend to r for $t\in[t_0,t_1)$, which implies that no edge will be lost before time t_1. Thus, new edges must be added in the interaction network at switching time t_1. Note that the hysteresis ensures that if a finite number of links are added to $G(t)$, then the associated potentials remain finite. Thus, $\hat{V}(t_1)$ is finite.

From Lemma 1 and $\rho(L(t_0))-\ell NI>0$, we have

$$\rho(L(t_{k-1}))-\ell NI>0.$$

Similar to the above analysis, the time derivative of $V(t)$ in every $[t_{k-1},t_k)$ is

$$\begin{aligned}\dot{\hat{V}} &= \sum_{i=1}^{N}\hat{v}_i^T\left[f_i(v_i)-\frac{1}{N}\sum_{j=1}^{N}f_j(v_j)\right]-\sum_{i=1}^{N}\hat{v}_i^T\rho\sum_{j\in\mathcal{N}_i(t)}a_{ij}\\ &\quad\times\left\{\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_i(t)}a_{i,k}(\hat{v}_i-\hat{v}_k)\right]-\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_j(t)}a_{j,k}(\hat{v}_j-\hat{v}_k)\right]\right\}\\ &= \sum_{i=1}^{N}\hat{v}_i^T\left[f_i(v_i)-\frac{1}{N}\sum_{i=1}^{N}f_i(v_j)\right]-\rho\hat{v}^TL(t_{k-1})\,\mathrm{sgn}[L(t_{k-1})\hat{v}]\\ &\le \|\ell I\hat{v}\|_1-\rho\|L(t_{k-1})\hat{v}\|_1=(\|\ell I\|_1-\rho\|L(t_{k-1})\|)\|\hat{v}\|_1\le 0,\end{aligned}\tag{14}$$

which implies that $\hat{V}(t)<\hat{V}(t_{k-1})<\infty$, $\forall t\in[t_{k-1},t_k),k=1,2,\ldots$. Therefore, no distance of existing edges will tend to r for $t\in[t_{k-1},t_k)$, which also implies that no edges will be lost before time t_k and $V(t_k)$ is finite. Since $G(0)$ is connected and no edge in $E(0)$ can be lost, $G(t)$ will remain connected for all $t\ge 0$.

We now prove parts (2) and part (3) of Theorem 1. Assume that n_k new edges are added to the evolving network G at time t_k. Because $0<n_k\le(N-1)(N-2)/2\triangleq\overline{N}$ from (8) and (14), we have

$$\hat{V}(t_k)\le\hat{V}_0+(n_1+n_2+\cdots+n_k)\psi(\|r-\varepsilon\|)=\hat{V}_{\max}.$$

Due to the fact that there are at most M new edges added to $G(t)$, we know $k\le\overline{N}$ and $V_t\le V_{\max}$ for all $t\ge 0$. Then, the number of switching times k of system (1) is finite, namely, $G(t)$ finally becomes fixed. Thus, we only need to discuss on the time interval (t_k,∞). Since that all the lengths of edges are no longer than $\psi^{-1}(\hat{V}_{\max})$, then the set

$$\Omega=\{\hat{\tilde{p}}\in\mathcal{G},\tilde{v}\in R_{Nn}\,|\,\hat{V}(\hat{\tilde{p}},\hat{v})\le\hat{V}_{\max}\},\tag{15}$$

is positively invariant, where

$$\mathcal{G}=\{\hat{\tilde{p}}\in R^{N^2n}\,|\,\|p_{ij}\|\in[0,\psi^{-1}(\hat{V}_{\max})],\ \forall(i,j)\in\mathcal{E}(t)\},$$

and $\hat{\tilde{p}}=[p_{11}^T,p_{12}^T,\ldots,p_{1N}^T,\ldots,p_{N1}^T,p_{N2}^T,\ldots p_{NN}^T]$.

For $G(t)$ is connected for all $t\ge 0$ as mentioned above, for all i and j we have $\|\hat{p}_i\|-\|\hat{p}_j\|<(N-1)$. Due to the fact that $\hat{V}(t)\le\hat{V}_{\max}$, we have $\hat{v}_i^T\hat{v}_i<2\hat{V}_{\max}$, and so $\|\hat{v}_i\|\le\sqrt{2\hat{V}_{\max}}$. Therefore, the set Ω satisfying $\hat{V}_t\le\hat{V}_{\max}$ is closed, and furthermore, compact. As we know, system (1) with control input (5) is autonomous on the concerned time interval (t_k,∞). Hence, the LaSalle Invariance Principle can be applied to achieve that if we limit the initial conditions of the system in Ω, then the corresponding trajectories will converge to the largest invariant set inside the region

$$\Gamma=\{\hat{\tilde{p}}\in\mathcal{G},\hat{v}\in R_{Nn}\,|\,\dot{\hat{V}}=0\}.$$

From (14), $\dot{\hat{V}}=0$ if and only if $\hat{v}_1=\ldots=\hat{v}_N$, namely, the velocities of all agents will converge to the virtual leader's velocity in an asymptotical way. Since $\hat{v}_1=\cdots=\hat{v}_N$, one has

$$\dot{\hat{v}}_i=-\sum_{j\in\mathcal{N}_i(t)}\frac{\partial\psi(\|p_{ij}\|)}{\partial\|p_{ij}\|}\frac{1}{\|p_{ij}\|}(p_i-p_j)=0,\tag{16}$$

for all $i=1,2,\ldots,N$. Generally, every final configuration will locally minimize each agent's global potential, unless the initial configuration of the agents is close enough to the global minimum.

We now prove part (4) of Theorem 1. From (15), $\hat{V}(\hat{\tilde{p}},\hat{v})\le\hat{V}_{\max}$ for all $t\ge 0$. However, we have $\lim_{\|p_{ij}(t)\|\to r}\psi(\|p_{ij}(t)\|)=\infty$ from the definition of potential functions. Hence, collisions among agents can be avoided. □

3.2. Flocking of multiple agents with a virtual leader

In this subsection, flocking control in multi-agent system (1) is considered with a virtual leader. The motion description of the virtual leader is as follows:

$$\begin{aligned}\dot{p}_\gamma&=v_\gamma,\\ \dot{v}_\gamma&=f_\gamma(v_\gamma),\end{aligned}\tag{17}$$

where $p_\gamma\in\mathbf{R}^n$ and $v_\gamma\in\mathbf{R}^n$ are, respectively, the position vector and velocity vector of the virtual leader. $f_\gamma(v_\gamma)\in\mathbf{R}^n$ is the intrinsic dynamics of the leader, which is different from any agent. Here, we assume that the vector field f_i and $f_\gamma:\mathbf{R}^n\to\mathbf{R}^n$ satisfies the following assumption.

Assumption 2. There exists a positive constant ℓ such that

$$\|f_i(v_i)-f_\gamma(v_\gamma)\|\le\ell,\quad\forall v_i\in\mathbf{R}^n,\ i=1,2,\ldots,N.\tag{18}$$

where $\|\cdot\|$ denotes the Euclidean norm in $\mathbf{R}^n$.

In this case the control input (3) for the flocking algorithm is given as follows:

$$\begin{aligned}u_i&=-\sum_{j\in\mathcal{N}_i(t)}\nabla_{p_i}\psi(\|p_{ij}\|)-h_ic_1(p_i-p_\gamma)\\ &\quad-\rho\sum_{j\in\mathcal{N}_i(t)}a_{ij}\left\{\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_i(t)}a_{ik}(v_i-v_k)+h_i(v_i-v_\gamma)\right]\right.\\ &\quad\left.-\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_j(t)}a_{jk}(v_j-v_k)+h_j(v_j-v_\gamma)\right]\right\},\end{aligned}\tag{19}$$

where if agent i can receive the information from the virtual leader, then $h_i=1$; otherwise, $h_i=0$. The positive constant c_1 is a weighting parameter on the position navigational feedback.

Lemma 2 (Su et al. [18]). *If G is a connected undirected graph, L is the symmetric Laplacian of the graph G and the matrix $H=\mathrm{diag}(h_1,h_2,\ldots,h_N)$ with at least one positive element, then all eigenvalues of the matrix $(L+H)$ are positive. Moreover, if G_1 is a graph generated by adding some edge(s) into the graph G, then $\lambda_1(L_1+H)\ge\lambda_1(L+H)>0$, where L_1 is the symmetric Laplacian of the graph G_1.*

We define the total relative artificial potential energy function among agents and the virtual leader as

$$V(p,v,p_\gamma,v_\gamma)=\frac{1}{2}\sum_{i=1}^{N}[U_i(p,p_\gamma)+(v_i-v_\gamma)^T(v_i-v_\gamma)], \tag{20}$$

and

$$U_i(p,p_\gamma)=\sum_{j\in\mathcal{N}_i(t)}\psi(\|p_i-p_j\|)+h_ic_1(p_i-p_\gamma)^T(p_i-p_\gamma). \tag{21}$$

Clearly, V is a positive semi-definite function. The main result is stated in the following theorem.

Theorem 2. *Consider a system of N autonomous agents with heterogeneous nonlinear dynamic motion (1) and a virtual leader moving with dynamics (17), where each agent is steered by protocol (19). Suppose that the initial network $G(0)$ is connected, the initial energy V_{t_0} is finite, and $\rho(L(t_0)+H(t_0))-\ell NI>0$. Then, the following results hold*:

(1) *$G(t)$ will remain connected all the time $t>0$;*
(2) *all agents asymptotically move with the same velocity and attain a relatively invariable distance;*
(3) *each agent's global potential $\sum_{j\in\mathcal{N}_i(t)}\nabla_{q_i}\psi(\|q_{ij}\|)$ is locally minimized for almost every final configuration;*
(4) *collisions between agents are avoided.*

Proof. Denote the position differential and the velocity differential as $p_{i\gamma}=p_i-p_\gamma$ and $v_{i\gamma}=v_i-v_\gamma$, respectively. Then, we have

$$\dot{p}_{i\gamma}=v_{i\gamma},$$

$$\begin{aligned}\dot{v}_{i\gamma}&=f_i(v_i)-f_\gamma(v_\gamma)-\sum_{j\in\mathcal{N}_i(t)}\nabla_{p_i}\psi(\|p_{i\gamma}-p_{j\gamma}\|)-h_ic_1p_{i\gamma}\\&\quad-\rho\sum_{j\in\mathcal{N}_i(t)}a_{ij}\left\{\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_i(t)}a_{ik}(v_{i\gamma}-v_{k\gamma})+h_i(v_{i\gamma})\right]\right.\\&\quad\left.-\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_j(t)}a_{jk}(v_{j\gamma}-v_{k\gamma})+h_j(v_{j\gamma})\right]\right\}.\end{aligned} \tag{22}$$

Moreover, the potential energy function (20) can be rewritten as

$$V(\tilde{p},\tilde{v})=\frac{1}{2}\sum_{i=1}^{N}(U_i(\tilde{p})+v_{i\gamma}^Tv_{i\gamma}), \tag{23}$$

where

$$U_i(\tilde{p})=\sum_{j\in\mathcal{N}_i(t)}\psi(\|p_{i\gamma}-p_{j\gamma}\|)+h_ic_1p_{i\gamma}^Tp_{i\gamma}, \tag{24}$$

and

$$\tilde{p}=\begin{bmatrix}p_{1\gamma}\\p_{2\gamma}\\\vdots\\p_{N\gamma}\end{bmatrix},\quad \tilde{v}=\begin{bmatrix}v_{1\gamma}\\v_{2\gamma}\\\vdots\\v_{N\gamma}\end{bmatrix}.$$

Here, $V(\tilde{p},\tilde{v})$ is a positive semi-definite function of $(\tilde{p},\tilde{v})$.

We assume that $G(t)$ switches at time t_k, $k=1,2,\ldots$. Therefore, $G(t)$ is a fixed graph on each time-interval $[t_{k-1},t_k)$. Note that, on $[t_0,t_1)$, V_{t_0} is finite and the time derivative of $V(t)$ satisfies

$$\begin{aligned}\dot{V}&=-\sum_{i=1}^{N}v_{i\gamma}^T[f_i(v_i)-f_\gamma(v_\gamma)]-\sum_{i=1}^{N}v_{i\gamma}^T\rho\sum_{j\in\mathcal{N}_i(t)}a_{ij}\\&\quad\times\left\{\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_i(t)}a_{ik}(v_{i\gamma}-v_{k\gamma})+h_i(v_{i\gamma})\right]\right.\\&\quad\left.-\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_j(t)}a_{jk}(v_{j\gamma}-v_{k\gamma})+h_j(v_{j\gamma})\right]\right\}\\&=-\sum_{i=1}^{N}v_{i\gamma}^T[f_i(v_i)-f_\gamma(v_\gamma)]-\rho\tilde{v}^T(L(t_0)+H(t_0))\cdot\mathrm{sgn}[(L(t_0)+H(t_0))\tilde{v}_\gamma]\\&\le\|\ell I\tilde{v}\|_1-\rho\|(L(t_0)+H(t_0))\tilde{v}\|_1\\&=(\|\ell I\|_1-\rho\|(L(t_0)+H(t_0))\|_1)\|\tilde{v}\|_1\le 0,\end{aligned} \tag{25}$$

which implies that

$$V(t)\le V_0<\infty,\quad \forall t\in[t_0,t_1).$$

Because of the definition of the potential function, we know $\lim_{\|p_{ij}(t)\|\to r}\psi(\|p_{ij}(t)\|)=\infty$. Therefore, the distance of existing links will not tend to r for $t\in[t_0,t_1)$. In other words, the existing links will not disconnect before time t_1. At the switching time t_1, new links can be added to the evolving network. Note that the hysteresis ensures that the potential remains finite if a finite number of edges are added to $G(t)$.

From Lemma 2 and $\rho(L(t_0)+H(t_0))-\ell NI>0$, we have

$$\rho(L(t_{k-1})+H(t_{k-1}))-\ell NI>0.$$

Similar to the above analysis, on every $[t_{k-1},t_k)$, the time derivative of $V(t)$ satisfies

$$\begin{aligned}\dot{V}&=-\sum_{i=1}^{N}v_{i\gamma}^T[f_i(v_i)-f_\gamma(v_\gamma)]-\sum_{i=1}^{N}v_{i\gamma}^T\rho\sum_{j\in\mathcal{N}_i(t)}a_{ij}\\&\quad\times\left\{\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_i(t)}a_{ik}(v_{i\gamma}-v_{k\gamma})+h_i(v_{i\gamma})\right]\right.\\&\quad\left.-\mathrm{sgn}\left[\sum_{k\in\mathcal{N}_j(t)}a_{jk}(v_{j\gamma}-v_{k\gamma})+h_j(v_{j\gamma})\right]\right\}\\&=-\sum_{i=1}^{N}v_{i\gamma}^T[f_i(v_i)-f_\gamma(v_\gamma)]-\rho\tilde{v}^T(L(t_{k-1})\\&\quad+H(t_{k-1}))\,\mathrm{sgn}[(L(t_{k-1})+H(t_{k-1}))\tilde{v}_\gamma]\\&\le\|\ell I\tilde{v}\|_1-\rho\|(L(t_{k-1})+H(t_{k-1}))\tilde{v}\|_1\\&=(\|\ell I\|_1-\rho\|(L(t_{k-1})+H(t_{k-1}))\|_1)\|\tilde{v}\|_1\le 0,\end{aligned} \tag{26}$$

which implies that

$$V(t)\le V_{t_{k-1}}<\infty,\quad \forall t\in[t_{k-1},t_k),\ k=1,2,\ldots.$$

Therefore, when $t\in[t_{k-1},t_k)$, there will be no distance of existing edges tending to r. Therefore, no edge will be lost before t_k, and V is finite. Since $G(0)$ is connected and no edge in $\mathcal{E}(0)$ is lost, $G(t)$ must remain connected for all the time $t\ge 0$. Then we prove parts (2)–(4) of Theorem 2.

Theorem 2 can be proven by following the proof of Theorem 1. Similar to the proof of part (2) of Theorem 1, we can see that in steady state,

$$v_1=v_2=\cdots=v_n=v_\gamma,$$

which implies that $u=0$. Similar to the proof of part (3) of Theorem 1, we have

$$v_i=\sum_{j\in\mathcal{N}(t)}\frac{\partial\psi(\|p_{ij}\|)}{\partial\|p_{ij}\|}\frac{1}{\|p_{ij}\|}(p_i-p_j)-h_ic_1(p_i-p_j)=0. \tag{27}$$

Finally, we prove part (4) of Theorem 2. Similar to the analysis of Theorem 1, we recall that $V(\widehat{v},\tilde{p})\le V_{\max}$ for all $t\ge 0$. The results then follow by the same argument as that used in the proof of part (4) of Theorem 1. □

4. Simulation study

In this section, some numerical examples are presented to demonstrate the effectiveness of the theoretical analysis.

4.1. Simulation without a virtual leader

Consider a multi-agent system under the control protocol in (5) with 10 agents in a two-dimensional Euclidean space. For simplicity of presentation, the intrinsic dynamic of each agent is governed by $f_i(v_i)=[3\cos(i\cdot v_{i1}+v_{i2}),\ 3\sin(v_{i1}+i\cdot v_{i2})]^T$, $i=1,2,\ldots,10$. The potential function is designed as in (8) with the influencing/sensing radius $r=4$ and $\varepsilon=0.1$. Initial positions and initial velocities of agents are chosen randomly from the plane $[0,8]\times[0,8]$ and $[0,3]\times[0,3]$ shown in Fig. 3(a), the dotted lines represent the neighboring relations, and the solid lines with arrows represent the velocity vectors. We choose $\rho=10$. Then the multiple autonomous agents move with the same velocity as shown in Fig. 3(b), from $t=0$ s to $t=20$ s. The relative velocity values between agent 1 and agent i ($i=2,3,\ldots,10$) are shown in Fig. 4.

4.2. Simulation with a virtual leader

Consider a multi-agent system with a virtual leader under the control protocol in (19), add the intrinsic function of the virtual leader as $f_\gamma(v_\gamma)=[3\sin(v_{\gamma1}+v_{\gamma2}),\ 3\cos(v_{\gamma1}+v_{\gamma2})]^T$, $i=1,2,\ldots,10$

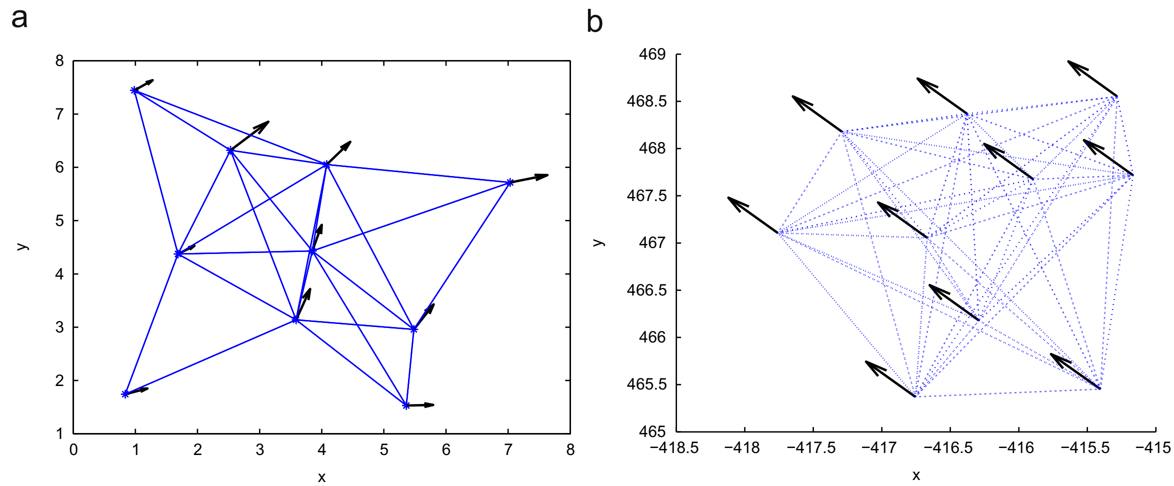

Fig. 3. Initial and final states of multiple agents ($n=10$) with velocity vectors. (a) The initial states and (b) the final states.

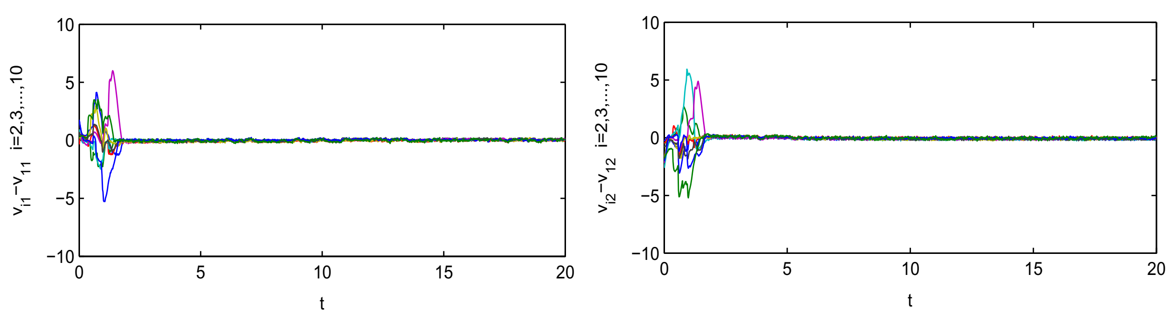

Fig. 4. Relative velocity states between agent 1 and agent i ($i=2,3,\ldots,10$).

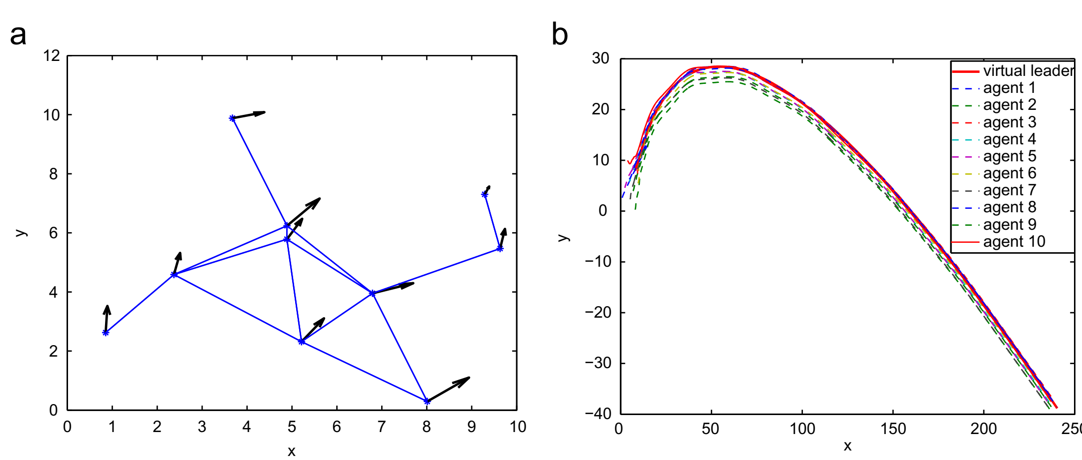

Fig. 5. States of multiple agents ($n=10$) and a virtual leader with velocity vector. (a) The initial states and (b) the path from $t=0$ to $T=20$.

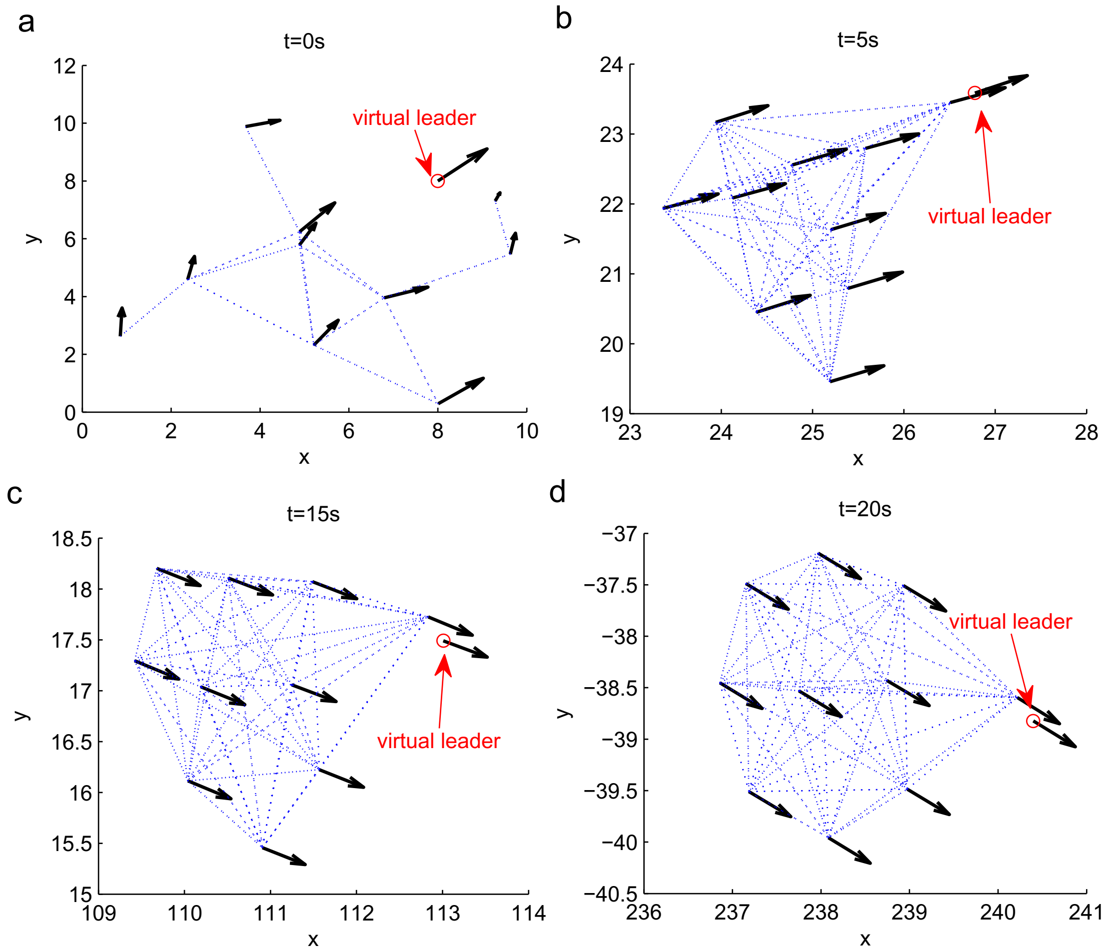

Fig. 6. Connectivity-preserving flocking of $n=10$ autonomous agents at the time of $t=0$ s, 10 s, 15 s, 20 s. The virtual leader is marked by a circle. The lines among agents are drawn when the distance is less than the influencing radius ($r=4$).

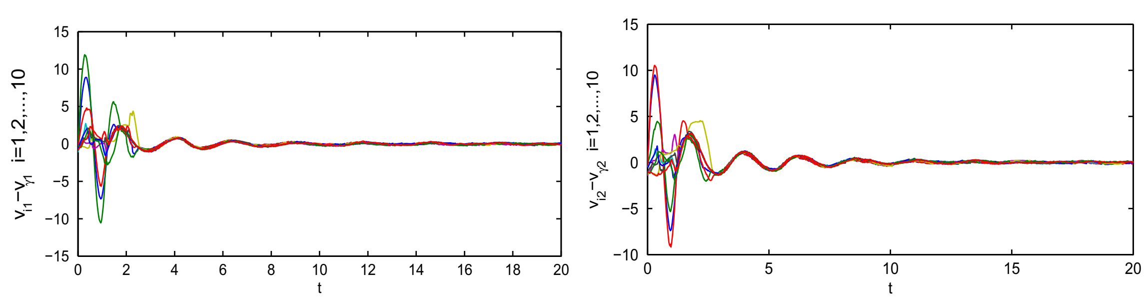

Fig. 7. Relative velocity states between agent i, $i=1,2,\ldots,10$ and the virtual leader.

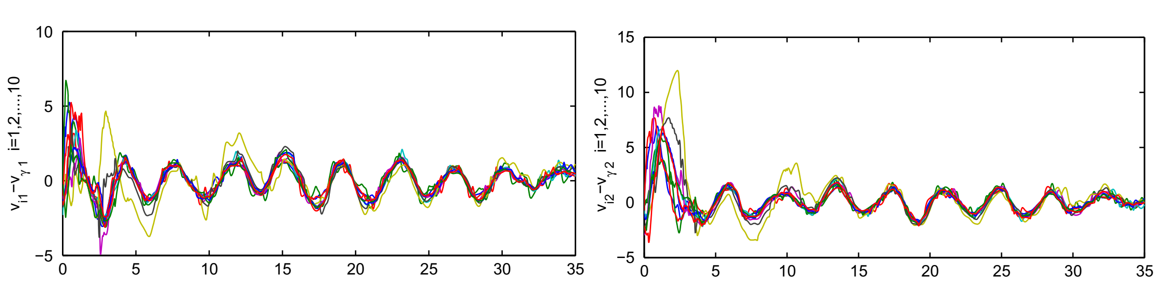

Fig. 8. Relative velocity states between agent i, $i=1,2,\ldots,10$ and the virtual leader with input (28).

and the intrinsic function of the agents designed as before. Set coupling strength $c_1 = 20$, $\rho = 10$ and $h_i = 1$ for $i = 1,2,3$, otherwise $h_i = 0$. Clearly only the first three agents can receive information from the leader. Furthermore, the potential function is also designed as in (20) with the influencing/sensing radius $r=4$ and $\varepsilon = 0.1$. The initial state of multiple agents are shown in Fig. 5(a), where the dotted lines represent the neighboring relations, and the solid lines with arrows represent the velocity vectors. Choose the initial position and velocity of the virtual leader as $p_\gamma(0) = [10,10]^T$ and $v_\gamma(0) = [3,3]^T$. From Theorem 2, the velocity of each agent approaches the velocity of the leader as shown in Fig. 5(b), and the distance between any two agents asymptotically stabilizes without any collision among agents as shown in Fig. 6(a)–(d). The relative velocity state between each agent and the virtual leader are shown in Fig. 7.

If we directly use the linear algorithm with linear velocity consensus term, the control inputs can be simply written as follows:

$$u_i = -\sum_{j\in\mathcal{N}(t)} \nabla_{p_i}\psi(\|p_{ij}\|) - h_i c_i (v_i - v_\gamma) - \sum_{j\in\mathcal{N}(t)} w_{ij}(v_i - v_j). \tag{28}$$

Under the same initial state of the 10 agents and the virtual leader. Simulation results for the control inputs (28) are shown in Fig. 8. It is easy to see that the velocities of all the agents fluctuate around the velocity of virtual leader by the control inputs (28). This is due to the fact that all the agents and virtual have heterogeneous nonlinear intrinsic dynamics. It is clear that the numerical simulation verifies the theoretical analysis in this paper very well.

5. Conclusion

In summary, we have investigated a flocking problem of multiple agents, where each agent has heterogeneous nonlinear dynamics. An artificial potential function and connectivity-preserving algorithm have been proposed for guaranteeing not to lose the existing edges, which shows that all agents can move with the velocity and preserving the network connectivity without collision. Furthermore, the result can be extended to a multi-agent system with a virtual leader. The velocities of the agents within the group asymptotically approach to the velocity of the virtual leader, and asymptotically stabilize the distance between any two agents for collision avoidance among all the agents. From the numerical examples, the theoretical results are verified effectively. Stimulated by the neural network approximation and the robust control technique in [20,21], future work will be to further investigate the flocking problem of multiple agents with uncertain nonlinear dynamics and external disturbances.

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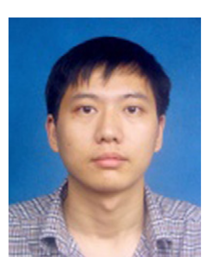

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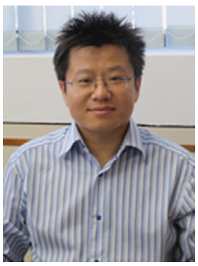

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