General

- The tutorials take place thursdays 14:15 - 16:00 in place of the lectures.
- Hand in is usually on Tuesday until 6pm via ecampus (upload section).
- Discuss the problems! Don't try to do it all on your own, there is much more gain in discussing physics. And don't copy the solutions without thinking, as you will have to work on it for the exam anyway ...
- You can hand in the solutions in groups of two.
- To participate in the final exam you will need 50% of the sheet points.
- We are happy to help, so if you have questions to the sheets you can always ask, e.g. after the lecture. Of course this also means you should start working early on the sheets.

Remarks to this sheet

This sheet is supposed to remind you of things you in principle know from earlier lectures, i.e. question 1 is just applied quantum mechanics, question 2 is EDV, and question 3 is atomic physics. Some of the exercises will be numerical solutions of something, which is everyday life for an experimentalist. Accordingly, from time to time there will be exercises where you have to numerically address a problem. Don't be scared by this, take this as an opportunity to get familiar with some numerical tools.

Problem 1: *Fidelity*

We will mostly look at particles which only have two (relevant) states, which could be for instance ground and excited state of an atom, left- and right circular polarization of a photon, or spin up / spin down for an electron. Let's assume we have two normalized basis states $|0\rangle$ and $|1\rangle$, and use that the two are orthogonal, i.e. $\langle 0|1\rangle = 0$. Any pure state can be described as the superposition of these basis states, $|\Psi\rangle = a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$, i.e. they lie on a sphere, the *Bloch sphere*[1]. Mixed states can be described by a 2×2 density matrix, they lie in the Bloch sphere, only pure states are on the surface.

In many experiments the fidelity between two states is given to show the success of the experiment. The idea is quite simple: Assume the theory tells you that the expected result is state $|\Psi\rangle$, but your measurement gives you state $|\Phi\rangle$, how close is your result to the ideal case? This is expressed in the fidelity.

a) For pure states the fidelity between two states $|\Psi\rangle$ and $|\Phi\rangle$ is given as $F = |\langle\Psi|\Phi\rangle|^2$. Assume that in each experimental run the measured state Φ is random, or in other words equally distributed on the Bloch sphere. What is the average fidelity?

b) For the fidelity between a pure and a mixed state $\hat{\rho}$ one defines the fidelity as $F = \langle\Psi|\hat{\rho}|\Psi\rangle$. What do you get for the average fidelity if the measured state $\hat{\rho}$ is random, i.e. equally distributed within the Bloch sphere? Keep in mind the properties of the density matrix, the state described by $\hat{\rho}$ still has to be a physical state.

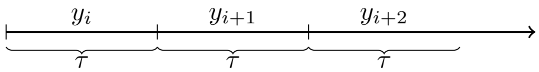
[1] If you are not familiar with the Bloch sphere, the Wikipedia article is quite ok.

Problem 2: *Allan Variance*

The non-overlapping Allan standard deviation $\sigma_y(\tau)$ is defined via

$$\sigma_y^2(\tau) = \frac{1}{2(K-1)} \sum_{i=1}^{K} (y_{i+1}(\tau) - y_i(\tau))^2$$

where $y_i(\tau)$ is the mean value of the data points in the i-th interval of length τ, and K is the number of intervals.



On ecampus you will find datasets where some value (doesn't matter what) was measured every second using different techniques.

a) Plot the data.

b) Plot a histogram of the data.

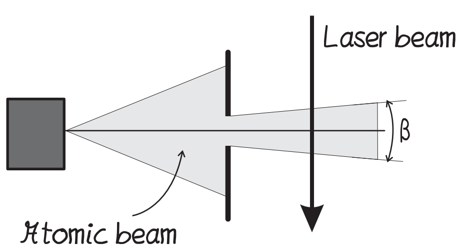
c) Calculate and plot the Allan standard deviation for the datasets.

d) Make some educated guess what the features in the Allan deviation correspond to. Which dataset would you deem to come from the best measurement device?

Note: The datasets have a length 0f 10,000, so use a computer.
You can use any software for numerics that you like and are familiar with, but obviously we can't support every platform. We are usually working with *Python* or *Mathematica*, so these two should be safe.
For*python*, there is a jupyter hub environment directly in the ecampus course, so no local installation is required. You can just use the provided platform.
Mathematica is available for physics students, see `http://mathematica.physik.uni-bonn.de/`.
Remark: When you start your master thesis it will be very unlikely that you can avoid using numerical tools, so this exercise is a preparation for real life.

Problem 3: *Atomic beam spectroscopy*

The figure below shows the principle of atomic beam spectroscopy, which is often used in high resolution spectroscopy. Atoms emitted from an oven with a large velocity component along the direction of the laser beam are removed in order to reduce the DOPPLER width. Calculate the DOPPLER width $\Delta\nu$ of the prominent sodium D line assuming the oven is at $T = 400\,$K, slightly above the melting temperature of sodium. How small do you have to choose the opening angle β to reduce the width along the laser beam to $\Delta\nu^* = 100$ MHz and 1 MHz, respectively?



Please upload your solutions until Tuesday, April 16th, 2pm. The solutions will be discussed in the exercises on Thursday, April 18th.