physics642: Quantum Technologies, Problem Sheet 2

ST 2023

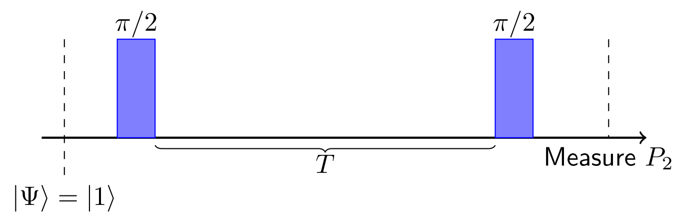
Problem 4: *Ramsey Spectroscopy*

Assume a Ramsey sequence, consisting of a short pulse with Rabi frequency Ω and length τ, a free evolution of length T, and a final pulse identical to the first one, with Rabi frequency Ω and length τ. The laser field is assumed to be detuned by Δ from the atomic resonance. The decay terms are assumed to be negligible, i.e. $\gamma = \gamma_\perp = 0$, which means you can use the solutions from the Schrödinger equation for $c_1(t)$ and $c_2(t)$.

a) Derive the population in the excited state after the sequence that was given in the lecture.

b) Determine the width of the central fringe.

c) Plot the dynamics on the Bloch sphere for $\Omega\tau \approx \pi/2$ and $\Delta T = \{0, \pi/4, \pi/2, \pi\}$



Problem 5: *Atomic Clocks*

Using Ramsey spectroscopy, a modern atomic clock measures the frequency of the cesium transition between the two hyperfine components $F = 3$ and $F = 4$ of the ground state with an accuracy of down to $\Delta\nu/\nu_0 \approx 10^{-16}$. Due to this precision we have to consider effects that disturb this transition. Calculate the magnitude of the following effects on the transition frequency and compare to the measurement precision:

a) Relativistic time dilation: In a cesium clock the atoms move with a velocity v, while the Ramsey cavity is stationary. Correspondingly, the relativistic time dilation leads to a shift of the measured frequency. Estimate the effect for a beam clock ($v = 300\,\mathrm{m/s}$) and a fountain clock ($v = 1\,\mathrm{m/s}$)

b) Gravitational red shift: According to general relativity, clocks run faster at higher altitude. Estimate this effect for a cesium clock.

c) Blackbody Radiation: The blackbody radiation induces an ac Stark shift of the transition (see e.g the advanced atomic, molecular and optical physics lecture), with the dominant contribution coming from dipole allowed transitions, the lowest one being the D-lines and $6^2S_{1/2} \leftrightarrow 6^2P_{3/2}$ at 852.3 nm and 894.6 nm, respectively. For "reasonable" temperatures one can treat the blackbody radiation as an effective dc-field (give an argument why). Then the shift of the clock transition is mostly due to the different polarizability of the $F = 3$ and $F = 4$ hyperfine state, with $\alpha(F = 4) = 2.023\mathrm{Hz/(kV/cm)}^2$ and $\Delta\alpha = \alpha(F = 4) - \alpha(F = 3) = 16/7 \times \alpha(F = 4)$. Using this, find the shift of the clock transition as a function of temperature, and find the value for room temperature.

Problem 6: *Bloch vs Rate Equations*

Coherent interaction is not quite intuitive, as in 'classical' spectroscopy one learns that only have the population can reach the excited state. The difference of course is that the classical rate equations ignore coherence, i.e. they are only valid in the limit of large dephasing or long times. To show that this statement is reasonable, we will look at the the Bloch equations with losses and compare them to the rate equations. Unfortunately, the Bloch equations can only be solved numerically, so you will again have to resort to a computer.
The classical rate equations for the excitation of a two-level atom are given by

$$\frac{dP_1}{dt} = -\frac{dP_2}{dt} = -BP_1 + (A+B)P_2$$

where P_1 (P_2) is the probability to find the atom in state 1 (2), A is the rate of spontaneous emission, and B is the rate of absorption and stimulated emission, i.e. it is the Einstein coefficient multiplied by the spectral density of the light field (thus no light means $B = 0$). These equations can be solved analytically, the solution is given by

$$\begin{aligned} P_1(t) &= P_1(0) - \frac{1}{A+2B}\left(1 - e^{-(A+2B)t}\right)(BP_1(0) - (A+B)P_2(0)) \\ P_2(t) &= P_2(0) + \frac{1}{A+2B}\left(1 - e^{-(A+2B)t}\right)(BP_1(0) - (A+B)P_2(0)). \end{aligned}$$

a) Use the steady state solutions of the rate equations and the Bloch equations (for $\gamma \neq \gamma_\perp$)) to derive a connection of A and B with γ, $\gamma_\perp$ and Ω.

b) Solve the Bloch equations numerically, using $\Delta = 0$, $\gamma = 0.1$ and $\Omega = 1$, and the population initially in the ground state. The decay rate $\gamma_\perp$ should take different values in the range $0.1 < \gamma_\perp < 10$.

c) Compare the solution of the Bloch equation to the solution of the rate equations, where the parameters A and B have been chosen according to your solution for the first subquestion.

d) Do a "'safety check"' of your solution of the Bloch equations. You can for instance look at the case $\gamma = \gamma_\perp = 0$, or the case where the atom is initially excited and $\Omega = 0$, and see if your result makes sense.

Please upload your solutions until Tuesday, April 30th , 6pm. The solutions will be discussed in the exercises on Thursday, May 2nd .