Supplementary Material for "Computation-Aware Link Repair for Large-Scale Damage in Distributed Cloud Networks"

The subproblem in the article can be written as

$$\mathcal{SP}: \min_{\boldsymbol{d},\boldsymbol{f},\boldsymbol{\ell}} \ \theta = \sum_{i\in\mathcal{N}} w_i \ell_i \tag{1a}$$

$$\text{s.t. } 0 \le \mathrm{d_i} \le \mathrm{d_i^m}, \ \forall \mathrm{i} \in \mathcal{N}, \tag{1b}$$

$$f_{ij} = -f_{ji}, \ \forall ij \in \mathcal{E}, \tag{1c}$$

$$|f_{ij}| \le f_{ij}^{\mathrm{m}}, \ \forall ij \in \mathcal{E}^1, \tag{1d}$$

$$\sum_{ij\in\mathcal{E}^0} f_{ij} + \sum_{ij\in\mathcal{E}^1} f_{ij} + d_i + \ell_i = r_i, \ \forall i \in \mathcal{N}, \tag{1e}$$

$$\ell_i \ge 0, \ \forall i \in \mathcal{N}, \tag{1f}$$

$$|f_{ij}| \le f_{ij}^{\mathrm{m}} x_{ij}^t, \ \forall ij \in \mathcal{E}^0. \tag{1g}$$

To begin, we introduce the Lagrange multiplier for the each constraint of subproblem:

1) For constraint (1b), we divide it into two parts and define the Lagrange multipliers c_i^1 and c_i^2.
2) For constraint (1c), we define the Lagrange multiplier c_{ij}^3.
3) For constraint (1d), we divide it into two parts and define the Lagrange multipliers c_{ij}^4 and c_{ij}^5.
4) For constraint (1e), we define the Lagrange multiplier c_i^6.
5) For constraint (1f), we define the Lagrange multiplier c_i^7.
6) For constraint (1g), we divide it into two parts and define the Lagrange multipliers c_{ij}^8 and c_{ij}^9.

Subsequently, the corresponding Lagrangian can be expressed as

$$\begin{aligned}
L(c_i^1, c_i^2, c_{ij}^3, c_{ij}^4, c_{ij}^5, c_i^6, c_i^7, c_{ij}^8, c_{ij}^9) = &\sum_{i\in\mathcal{N}} w_i \ell_i \\
&- \sum_{i\in\mathcal{N}} c_i^1 d_i + \sum_{i\in\mathcal{N}} c_i^2 (d_i - d_i^m) \\
&+ \sum_{ij\in\mathcal{E}} c_{ij}^3 (f_{ij} + f_{ji}) \\
&+ \sum_{ij\in\mathcal{E}^1} c_{ij}^4 (f_{ij} - f_{ij}^m) + \sum_{ij\in\mathcal{E}^1} c_{ij}^5 (-f_{ij} - f_{ij}^m)
\end{aligned}$$

$$+\sum_{i\in\mathcal{N}} c_i^6\left(\sum_{ij\in\mathcal{E}^0} f_{ij}+\sum_{ij\in\mathcal{E}^1} f_{ij}+d_i+\ell_i-r_i\right)$$
$$+\sum_{i\in\mathcal{N}} c_i^7(-\ell_i)$$
$$+\sum_{ij\in\mathcal{E}^0} c_{ij}^8(f_{ij}-f_{ij}^m x_{ij}^t)+\sum_{ij\in\mathcal{E}^0} c_{ij}^9(-f_{ij}-f_{ij}^m x_{ij}^t). \quad (2)$$

The dual problem we are seeking is the minimum value of L with respect to its variables $\boldsymbol{d},\boldsymbol{f},\boldsymbol{\ell}$, which can be written as

$$\max_{c_i^1,c_i^2,c_{ij}^3,c_{ij}^4,c_{ij}^5,c_i^6,c_i^7,c_{ij}^8,c_{ij}^9}\ \min_{\boldsymbol{d},\boldsymbol{f},\boldsymbol{\ell}}\ L(c_i^1,c_i^2,c_{ij}^3,c_{ij}^4,c_{ij}^5,c_i^6,c_i^7,c_{ij}^8,c_{ij}^9) \quad (3a)$$

$$\text{s.t. } \mathrm{c_i^1,c_i^2,c_{ij}^3,c_{ij}^4,c_{ij}^5,c_i^6,c_i^7,c_{ij}^8,c_{ij}^9} \geq 0. \quad (3b)$$

To achieve this, we derive the partial derivatives of the Lagrangian with respect to each variable and set them to zero. Note that for a given i, f_{ij} and f_{ji} are treated as two separate variables for differentiation, and cases with $ij\in\mathcal{E}^1$ and $ij\in\mathcal{E}^1$ need to be considered separately.

1) For $\boldsymbol{\ell}$:
$$\frac{\partial L}{\partial \ell_i}=w_i+c_i^6-c_i^7=0,\forall i\in\mathcal{N}. \quad (4)$$

2) For $\boldsymbol{d}$:
$$\frac{\partial L}{\partial d_i}=-c_i^1+c_i^2+c_i^6=0,\forall i\in\mathcal{N}. \quad (5)$$

3) For f_{ij} when $ij\in\mathcal{E}^1$:
$$\frac{\partial L}{\partial f_{ij}}=c_{ij}^3+c_{ij}^4-c_{ij}^5+c_i^6=0,\forall ij\in\mathcal{E}^1. \quad (6)$$

4) For f_{ij} when $ij\in\mathcal{E}^0$:
$$\frac{\partial L}{\partial f_{ij}}=c_{ij}^3+c_i^6+c_{ij}^8-c_{ij}^9=0,\forall ij\in\mathcal{E}^0. \quad (7)$$

5) For f_{ji}:
$$\frac{\partial L}{\partial f_{ji}}=c_{ij}^3=0,\forall ji\in\mathcal{E}. \quad (8)$$

Substituting the above results into $L(c_i^1,c_i^2,c_{ij}^3,c_{ij}^4,c_{ij}^5,c_i^6,c_i^7,c_{ij}^8,c_{ij}^9)$, we get:

$$\mathcal{DSP}:\ \max_{c_i^1,c_i^2,c_{ij}^3,c_{ij}^4,c_{ij}^5,c_i^6,c_i^7,c_{ij}^8,c_{ij}^9}\sum_{i\in\mathcal{N}}(-c_i^2 d_i^{\mathrm{m}}-c_i^6 r_i)+\sum_{ij\in\mathcal{E}^1}(-c_{ij}^4-c_{ij}^5)f_{ij}^{\mathrm{m}}+\sum_{ij\in\mathcal{E}^0}(-c_{ij}^8-c_{ij}^9)f_{ij}^{\mathrm{m}}x_{ij}^t \quad (9a)$$

$$\text{s.t.}\quad c_i^1,c_i^2,c_{ij}^3,c_{ij}^4,c_{ij}^5,c_i^6,c_i^7,c_{ij}^8,c_{ij}^9\geq 0, \quad (9b)$$

$$c_i^2 + c_i^6 = c_i^1, \forall i \in \mathcal{N}, \tag{9c}$$

$$w_i + c_i^6 = c_i^7, \forall i \in \mathcal{N}, \tag{9d}$$

$$c_{ij}^4 + c_i^6 = c_{ij}^5, \forall ij \in \mathcal{E}^1, \tag{9e}$$

$$c_i^6 + c_{ij}^8 = c_{ij}^9, \forall ij \in \mathcal{E}^0, \tag{9f}$$

The c_i^k and c_{ij}^k in the function of DSP correspond to the ρ_i^k and ρ_{ij}^k mentioned in our article. However, this derivation is part of the standard procedure for converting optimization problems into the dual forms, which appears frequently in much of mainstream literature. Since the derivation process and intermediate forms can be complex, we chose to omit it in our main text for the sake of brevity. The original form of DSP in the article is the result of our inference based on (9a)-(9f).