**Deggendorf Institute of Technology**

**Case Study Cyber Physical Production Systems using Additive Manufacturing**

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# Involute Gearing: Evoloid Drive With High Gear Ratio

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## Abstrakt

Evoloid-Antriebssysteme sind schrägverzahnte Zahnradpaare mit einer Zähnezahl von wen iger als 7 am Ritzel oder Mitnehmer. Ein Evoloid-Planetengetriebe besteht aus einem Sonnenrad mit einer Zähnezahl von eins und einem festen Hohlrad, Planetenrädern und einem Planetenträger, auf dem die Planetenräder drehbar angeordnet sind. Jedes der Sonnenräder, Planetenräder und das Hohlrad haben eine Evoloidverzahnung oder eine Schrägverzahnung. Dieses System hat eine hohe Tragfähigkeit und kann ein hohes Übersetzungsverhältnis in einer einzigen Stufe erreichen. In dieser Studie haben wir zunächst das mathematische Modell des schrägverzahnten Zahnrads analysiert, das durch Zahnstangenfräsen erzeugt wird. Das Ritzel hat weniger als 7 Zähne, so dass beim Zahneingriff das Problem der Unterschneidung auftritt. Um dieses Problem zu lösen, wird die Methode der Profilverschiebung beschrieben. Diese Methode hat den Nachteil, dass sie die Dicke der Hohlkehle vergrößert und den Zahnradzusatz verringert, wodurch sich das Zahnradübersetzungsverhältnis verringert. Daher werden auch einige andere Methoden angeboten. Schließlich planen wir die Modellierung eines vereinfachten Evolventenantriebs, der aus einem nicht genormten Ritzel mit einem genormten Schrägstirnrad besteht. Auf der Grundlage der geometrischen Theorie, die sich auf die Eingriffseigenschaften und das Erreichen eines hohen Übersetzungsverhältnisses konzentriert, wurde in diesem Bericht eine systematische Studie vorgestellt. Die aus dieser Untersuchung gewonnenen Informationen werden für die parametrische Modellierung des Evoloid-Getriebes in IceSL mit Hilfe des Lua-Skripts verwendet. Darüber hinaus wird ein Lehrvideo erstellt, um die funktionellen Eigenschaften zu erläutern.



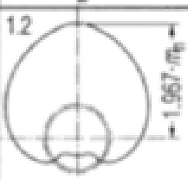
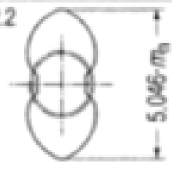
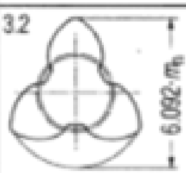
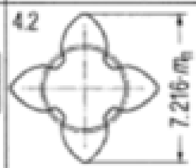
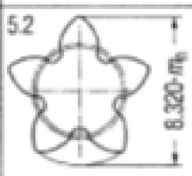
## Abstract

Evoloid drive systems are helical cut pairs of gears with number of teeth less than 7 at the pinion or driver. An evoloid planetary gear box includes a sun gear where number of teeth is one and a fixed ring gear, planetary gears, and a planet carrier where the planet gears are rotatably arranged. Each of the sun gear, planet gears and the ring gear have evoloid toothing or helical toothed. This system has high load capacity, and it can achieve a high gear ratio in a single stage. In this study, we have first analysed the mathematical model of the helical gear generated through rack cutter. The pinion gear has number of teeth less than 7 so, during meshing tooth undercutting problem occurs. To solve this problem, profile shift method is described. This method has a drawback that it increases the fillet thickness and decreases the gear addendum, which decreases the gear contact ratio. Hence, some other methods are also provided. In the end, we plan to model the simplified evoloid drive consisting of a nonstandard pinion with a standard helical gear. Based on their geometric theory focusing on meshing characteristics and achieving high gear ratio, a systematic study has been presented in this report. Information gathered from this research will be used for parametric modelling of the Evoloid gearbox in IceSL with the help of the Lua script. Moreover, an educational video will be compiled to elaborate on their functional characteristics.

# Contents

## Appendix

## Gear tooth combinations with profile shifts and addendum factors [1]

| Classifying criteria | | Main part | | | | Selection characteristics | | Appendix |
|---|---|---|---|---|---|---|---|---|
| Number of teeth | | Side view | Transverse profile | Pinion $z_1$ | Wheel $z_2$ | Transmission ratio | Efficiency | Reference |
| | No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 1 | 1.1  $\alpha_n = 20°$, $\beta = 30°$ | 1.2   | 1.3 $h_{aP1} = 0.383 m_n$ ($0.281 m_n$) $h_{fP1} = 1.1\, m_n$ $x_1 = +1.013$ (+1.031) | 1.4 $h_{aP2} = 1.1\, m_n$ $h_{fP2} \geq 0.38 m_n$ $x_1 = -0.6$ | 1.5 about until 1:48 | 1.6 about 88% | 1.7 See Ref. [6] |
| 2 | 2 | 2.1  $\alpha_n = 20°$, $\beta = 25°$ | 2.2   | 2.3 $h_{aP1} = 0.481 m_n$ ($0.431 m_n$) $h_{fP1} = 1.1\, m_n$ $x_1 = +0.947$ (+0.961) | 2.4 $h_{aP2} = 1.1\, m_n$ $h_{fP2} \geq 0.48 m_n$ $x_2 = -0.6$ | 2.5 about until 1:20 | 2.6 about 90% | |
| 3 | 3 | 3.1  $\alpha_n = 20°$, $\beta = 20°$ | 3.2   | 3.3 $h_{aP1} = 0.554 m_n$ $h_{fP1} = 1.1\, m_n$ $x_1 = +0.892$ | 3.4 $h_{aP2} = 1.1\, m_n$ $h_{fP2} \geq 0.55\, m_n$ $x_2 = -0.6$ | 3.5 about until 1:16 | 3.6 92% | |
| 4 | 4 | 4.1  $\alpha_n = 20°$, $\beta = 20°$ | 4.2   | 4.3 $h_{aP1} = 0.659 m_n$ $h_{fP1} = 1.1\, m_n$ $x_1 = +0.822$ | 4.4 $h_{aP2} = 1.1\, m_n$ $h_{fP2} \geq 0.66\, m_n$ $x_2 = -0.6$ | 4.5 about until 1:12 | 4.6 94% | |
| 5 | 5 | 5.1  $\alpha_n = 20°$, $\beta = 20°$ | 5.2   | 5.3 $h_{aP1} = 0.752\, m_n$ $h_{fP1} = 1.1\, m_n$ $x_1 = +0.753$ | 5.4 $h_{aP2} = 1.1\, m_n$ $h_{fP2} \geq 0.75\, m_n$ $x_2 = -0.6$ | 5.5 about until 1:10 | 5.6 96% | |

**Important Links**

https://www.sciencedirect.com/science/article/abs/pii/S0890695505000210

https://patents.google.com/patent/DE102018107021A1/en

## Nomenclature

$\boldsymbol{a}_i$ = tool setting of rack cutter generating the involute gear($i$= 0, 1)
$\boldsymbol{e}$ = total profile shifting coefficient
$\boldsymbol{e}_i$ = profile shifting coefficient (i=1, 2, ρ, g)
$\boldsymbol{C}$ = clearance of the mating gears
$\boldsymbol{E}$ = center distance of the mating gears (or shaper and generated gear)
$\boldsymbol{E'}$= operating center distance of the mating gears
$[\boldsymbol{L}_{ij}]$= projection transformation matrix (from $S_j$ to $S_i$)
$\ell$ = surface parameter of rack cutter (in mm)
$[\boldsymbol{M}_{ij}]$ = coordinate transformation matrix (from $S_j$ to $S_i$)
$\boldsymbol{m}_{12}$ = gear ratio
$\boldsymbol{m}_n$ = normal module of the gear
$\boldsymbol{n}_i^{(j)}$ = unit normal vector of rack cutter surface j (j = 1, 2 and 3, which represent part 1, 2 and 3 of rack cutter normal section (Fig. 1), respectively represented in coordinate system i (i = c, a)
$\boldsymbol{n}_1^{(j)}$ =unit normal vector of generated gear surface j (j = 1 ∼ 3)
$\boldsymbol{R}_i^{(j)}$ = position vector of surface j (j = 1, 2 and 3, which represent part 1, 2 and 3 of rack cutter normal section , respectively, and their corresponding generated gear tooth surfaces) represented in coordinate system I ($i$ = 1, 2, c, a)
$\boldsymbol{r}$ = distance between gear rotational center and beginning point of modified region (in mm)
$\boldsymbol{r}_i$ = radius of pitch circle of pinion and gear (I = 1, 2) (in mm)
$\boldsymbol{r}_j$ = radius of centrode of pinion and gear (j = p, g) (in mm)
$\boldsymbol{r}_r$ = radius of the modified root fillet of shaper (in mm)
$\boldsymbol{r}_t$ = radius of the modified tip fillet of shaper (in mm)
$\boldsymbol{S}_i$ = coordinate system i (i = 1, 2, a, c, h)
$\boldsymbol{T}$ = tangent vector to a curve
$\boldsymbol{T}_i$ = number of teeth of pinion and gear (i =1, 2)
$\boldsymbol{x}_i^{(j)}$, $\boldsymbol{y}_i^{(j)}$, $\boldsymbol{z}_i^{(j)}$ position vector of modified fillet surface j (j =r, t where r represents root fillet and t indicates tip fillet) represented in coordinate system i (i= 1, 2)
$\boldsymbol{u}$ = surface parameter of rack cutter (in mm)
$\Sigma_a$ =normal section of rack cutter surface
$\Sigma_e$ = rack cutter surface
$\boldsymbol{\psi_{bs}}$ = operating pressure angle of helical gear
$\boldsymbol{\psi_n}$ = normal pressure angle of rack cutter (in degrees)
$\boldsymbol{\psi_s}$ = transverse pressure angle helical gear
$\boldsymbol{\rho_i}$ = radius of tip and root fillets of rack cutter (i=0, 1) (in mm)
$\boldsymbol{\theta_n}$ = variable parameter of root fillet of rack cutter (in degrees)
$\boldsymbol{\zeta_n}$ = variable parameter of tip fillet of rack cutter (in degrees)
$\boldsymbol{\phi_i}$ = rotational angle of pinion and gear (i =1, 2) (in degrees)
$\boldsymbol{\lambda}$ = lead angle of gear (in degrees)

# Chapter 1

# Introduction

Evoloid drives are high ratio gear boxes that have number of teeth smaller than five at the pinion and are helical toothed. Evoloid toothing can be used in all types of gears with involutes on the gears. Evoloid gearing offers special advantages for external helical gear, inner helical wheel, planetary rack, and pinion. In an evoloid planetary gear system includes a sun gear having one tooth, a fixed ring gear, planet gears, and a planet carrier on which planet gears are rotatably arranged. Each of the sun gear, planet gears, and the ring gear have evoloid toothing. This system enables high load capacity at high transmission ratios. The sun gear comprising only one tooth contributes to the high possible transmission ratios. This system enables high load capacity at high transmission ratios. The planet gears do not hit each other at high transmission ratio of 24:1 because of defined addendum modification coefficients and addendum coefficients of the individual gears of the planetary gearbox. In the case of straight gearing, if we reduce the number of teeth's less than seven then the tooth becomes pointy and undercut occurs and thus the uniform transmission is no longer possible. The solution to this problem is to replace the lack of coverage in straight gearing with jump coverage in helical gearing. The conversion to helical gearing with a very large angle of inclination up to 30 degrees in some cases up to 45 degrees offers many advantages. This angle of inclination produces a high transverse contact ratio of roughly 2 which produces good gearbox running properties. With small numbers of teeth and relatively large angle of inclination speaks of an evoloid toothing.[2]

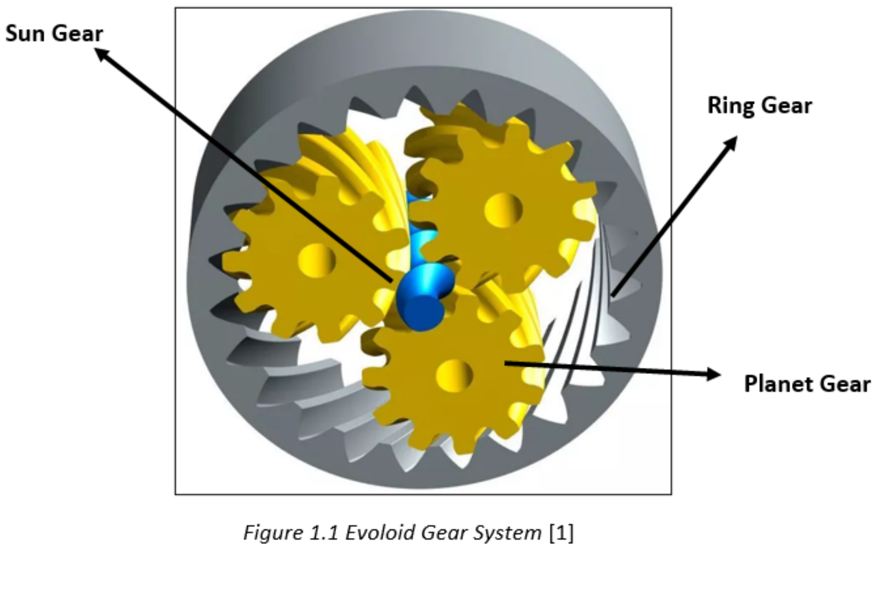



*Figure 1.1 Evoloid Gear System* [1]

Figure 1.1 shows an evoloid planetary gearbox with a transmission ratio of i=24:1. It includes a sun gear having one tooth, a ring gear, planet gears. Each of the sun gear, the planet gears, and the ring gear have evoloid toothing. The planetary gearbox enables high load capacity at high transmission ratios by using three circulating planetary gears which mate with the ring gear, the sun gear comprising only one tooth contributes to the high possible transmission ratios. Each of the three planetary gears mate with the ring gear via a first path of contact and with the sun gear via a second path of contact, and an operating angle of contact of the first path of contact coincides with an operating angle of contact of the second path of contact.

When the evoloid planetary gearbox is driven by the sun gear, the engagement of the mating teeth of the sun gear and the planet gears before the pitch point benefits the rotation of the planet gears through progressive (impacting) friction and the engagement of the mating teeth of the ring gear and the planet gears after the pitch point benefits the running of the planet gears on the ring gear through degressive (dragging) friction.

MAUL KONSTRUKTIONEN GMBH was the patent applicant for this invention under the U.S. patent application number 17/040112 for planetary gearbox having single-tooth sun gear having evoloid teething. Planetary gearboxes have many advantages compared with other types of gearboxes, in particular: a compact, heavy-duty design, supporting many teeth simultaneously, a relatively high transmission ratio in one stage and a multi-stage arrangement through the series connection of multiple planetary gearboxes

But at anything above a transmission ratio of roughly i=12:1, the planetary gearbox with three planet gears fails because the planet gears would strike one another and collide. The problem addressed by this invention by creating a generic planetary gearbox with evoloid toothing which facilitates a high load-bearing capacity, even at high transmission ratios including i=24:1 in particular. The three rotating planet gears which do not strike each other inside the gearbox because of its evoloid toothing, even at high transmission ratios, the sun gear comprising only one tooth contributes to the high possible transmission ratios. The gears inside the evoloid drive will not hit each other because of the modified addendum coefficients of the individual gears. To reduce the planet gear diameter, a sharp addendum reduction and, in addition, For the planet gears a negative addendum modification is proposed, the negative profile shift coefficient $e_p$ of each planet gear falls within the range.of -0.2 to -0.4. The addendum height coefficient $h_p$ of the planet gear falls within the range of 0.5 to 0.7. At the same time, the sun gear receives a large positive profile shift, The positive profile shift coefficient $e_s$ of the sun gear falls within the range of 1.4 to 1.6. The addendum height coefficient $h_s$ of the sun gear falls within the range of 0.1 to 0.2.The internally toothed, frame-fixed ring gear is subject to a negative addendum modification, as a result, part of the toothing is displaced outwardly. The negative profile shift coefficient $e_r$ of the ring gear falls within the range of -0.8 to -1.0. The addendum height factor $h_r$ of the ring gear falls within the range of 1.3 to 1.5.[2]

An evoloid drive can transmit very high output torques with a very small installation space. With an evoloid planetary gear system it is possible to achieve transmission ratios of 6: 1, 12: 1, 18: 1, 24: 1 in a single stage. For a standard transmission ratio of the evoloid planetary gearbox i=24:1, the number of teeth on the planet gears is z=11 and the number of teeth on the ring gear is z=23. With good lubrication and the use of suitable materials, a planetary gearbox of this kind has an efficiency factor of over 94.Between the lowest transmission ratio i=6:1 and the standard ratio i=24:1, planetary gearboxes with the transmission ratio i=12:1 can be realized, wherein the number of teeth of the planetary gears is z=5 and the number of teeth of the ring gear is z=11, or with the transmission ratio i=18:1, wherein the number of teeth of the planetary gears is z=8 and the number of teeth of the ring gear is z=17. Evoloid planetary gears with sun gear Z=1 have many advantages over planetary gears of conventional design. Since there are fewer friction points, the efficiency increases by about 10-20%, the load capacity of the gearbox increases by approximately 30%, the cost compared to the conventional planetary gearbox drops by about 30-40%, Gearbox noise drops by up to 10 dB(A) and finally the installation space can be reduced up to 30%.[2]

This paper emphasizes primarily on the geometric characteristics of high ratio evoloid drive. With a mathematical approach, the geometric models of the evoloid drive system based on their meshing and high transmission ratio are investigated.As shown in Figure 1.2, for simplification of the project, our aim is to design an evoloid system but with one standard helical gear and one nonstandard with number of teeth less than 7. It is expected that this study should be helpful in the designing and parametric modelling of the evoloid drive in IceSL with the help of Lua script. Moreover, to compile an educational video elaborating the functional characteristics of the evoloid drive system.

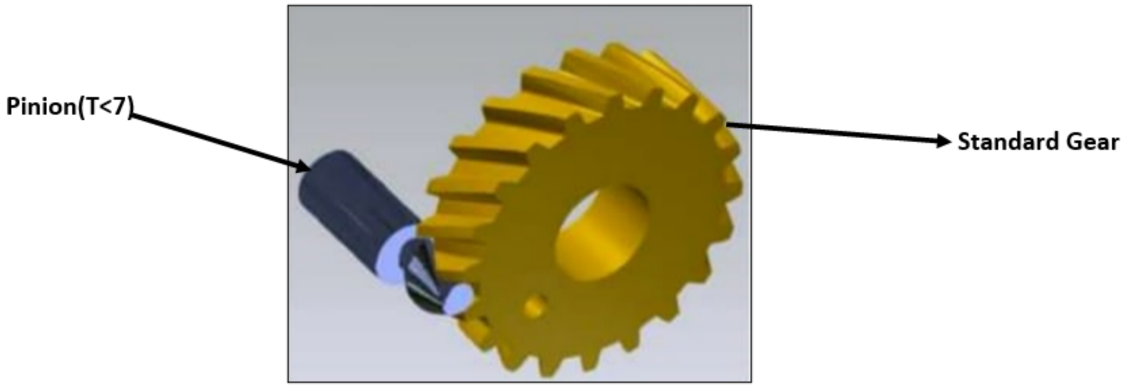



*Fig 1.2 Evoloid Drive(Simplified)* **[1]**

# Chapter 2

# Gear Geometry

## 2.1 Parameters and Terminologies

As our drive system includes a sun gear, three planet gears and a ring gear all of which are helical gears, so the first part will explain about the terminologies of helical gears. Figure 2.1 shows the basic terminologies utilized in helical gears.

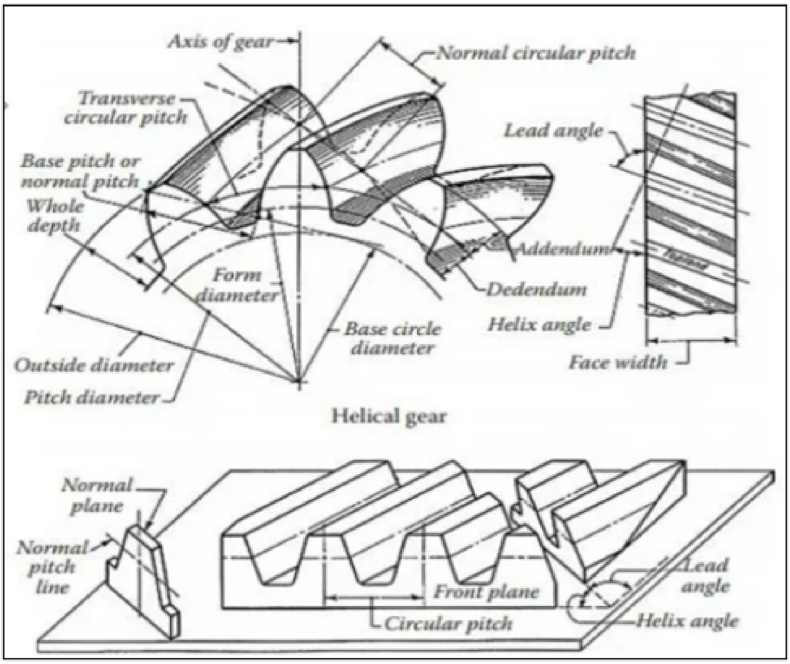



*Fig 2.1 Helical Gear Terminologies* [12]

## 2.2 Involute Curve

### 2.2.1 Rack Gear

It is the gear responsible for subtracting the material to execute the required involute gear. As you can see in fig 2.2, where[3]

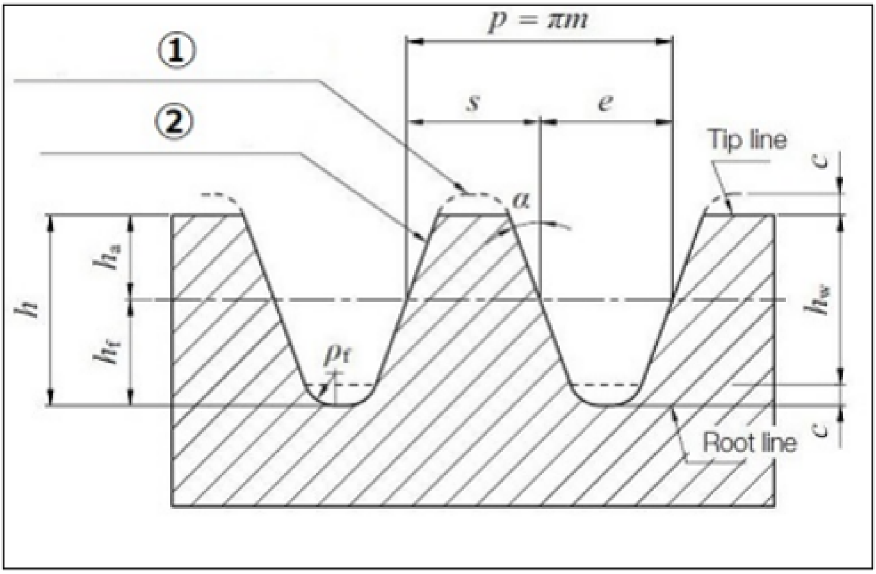


Fig 2.2 Standard Rack tooth profile [3]

Module m is unit size indicated in (mm) and calculated by dividing reference pitch with pi($P = m.\pi$)
Reference Pitch P is the distance between corresponding points on adjacent teeth and calculated by multiplying module with pi ($m = P/\pi$)
Pressure Angle a is the angle of a gear tooth leaning against a normal reference line (Standardized $\alpha = 20°$.
Addendum $h_a$ is the distance between reference line and tooth tip.
Dedendum $h_f$ is the distance between reference line and tooth root. - Tooth Depth $h$ is the distance between tooth tip and tooth root.
Working Depth $h_w$ is the depth of tooth meshed with the mating gear.
Tip & Root Clearance $c$ is the distance (clearance) between tooth root and the tooth tip of mating gear.
Dedendum Fillet Radius $\rho_f$ is the radius of curvature between tooth surface and the tooth root.

** To indicate tooth size, there are two other units as well as the Module, which are" Circular Pitch" (CP) and" Diametral Pitch" (D.P). Circular Pitch denotes the reference pitch P. Diametral Pitch (D.P.), the unit to denote the size of the gear tooth.

### 2.2.2 Involute Tooth Parameters

An involute tooth is formed by two involutes unwound from the base circle db,outside circle diameter da and fillet as depicted in Fig 2.3. The profile angle in the intersection point of the two involutes (tip angle) is [**4**]
V = $\mathrm{acos}(d_b/d_\Delta)$

The profile angle on the outside diameter da is
$\alpha_a = \mathrm{acos}(d_b/d_a)$
$d_a = d_b/\cos(\alpha_a)$

The Base pitch is
$p_b = \pi \cdot d_b/z$
where z is the number of teeth.

The proportional base tooth thickness is
$m_b = S_b/p_b = z \cdot \mathrm{inv(v)}/\pi$
$\mathrm{inv}(v) = \pi \cdot m_b/z$
where Sb is the base thickness.

The proportional top land thickness is
$m_a = S_a/p_b = z \cdot (\mathrm{inv(v)} - \mathrm{inv}(\alpha_a))/(\pi \cdot \cos(\alpha_a)) \cos(\alpha_a) + \mathrm{inv}(\alpha_a)/(\pi \cdot m_a) = m_b/m_a$
$S_a = p_b m_a$
$\mathrm{inv}(v) = (\pi \cdot m_a \cos(\alpha_a) + z\mathrm{inv}(\alpha_a))/z$
Where Sa is the top land thickness.

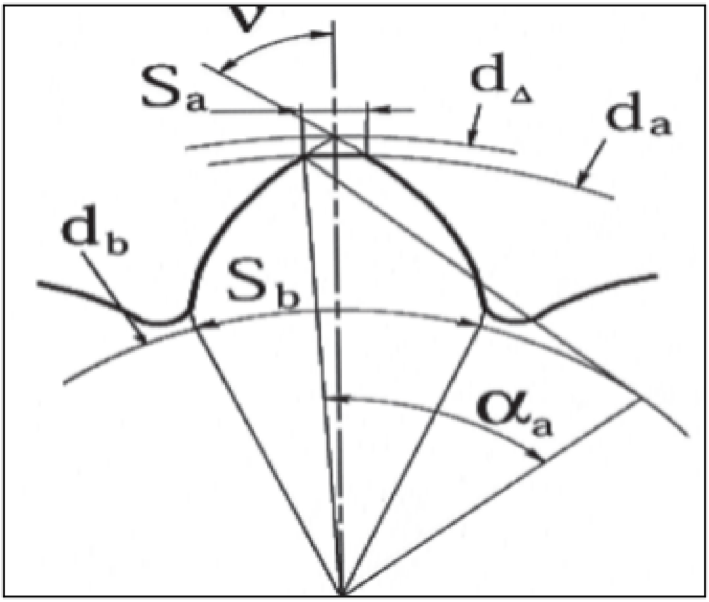



*Fig 2.3 Involute Tooth Profile* **[4]**

### 2.2.3 Involute Gear Mesh Parameters

Figure 2.4 shows the zone of tooth action of the pinion and the gear in close mesh (backlash is zero). The close mesh condition is [4]
$p_w = S_{w1} + S_{w2}$
where the operating circular pitch is,
$p_w = \pi \cdot d_{w1}/z_1 = \pi \cdot d_{w2}/z_2$

The pinion and gear operating pitch diameters are respectively
$d_{w1} = d_{b1}/\cos(\alpha_w), d_{w2} = d_{b2}/\cos(\alpha_w)$
$S_{w1} = (\mathrm{inv}(v_1) - \mathrm{inv}(\alpha_w)) \cdot d_{b1}/\cos(\alpha_w)$
$S_{w2} = (\mathrm{inv}(v_2) - \mathrm{inv}(\alpha_w)) \cdot (d_{b2}/\cos(\alpha_w)$
are the pinion and the gear operating tooth thicknesses respectively

The operating pressure angle is
$\mathrm{inv}(\alpha_w) = (\mathrm{inv}(v_1) + u \cdot \mathrm{inv}(v_2) - \pi/z_1)/(1+u)$ where u is the gear ratio u = z2/z1

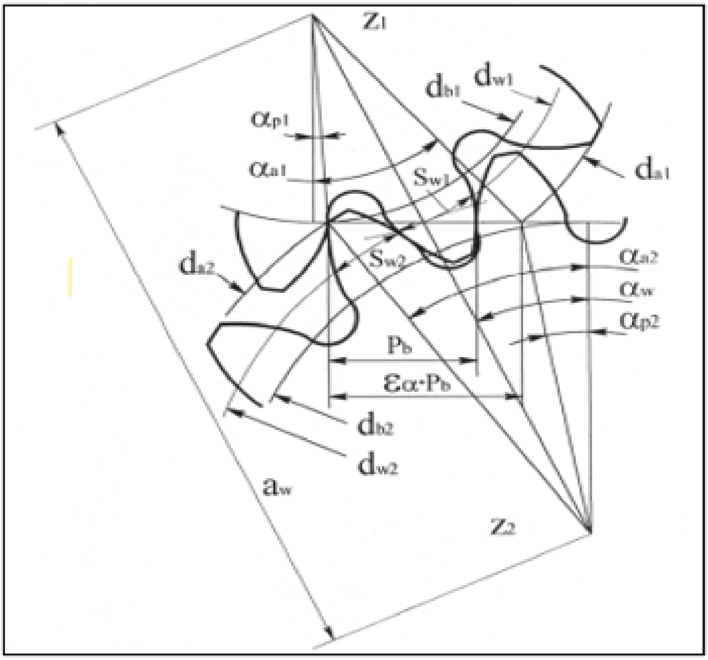



*Fig 2.4 Meshing of Gears* **[4]**

# Chapter 3

# Model Design

## 3.1 Rack cutter profile

Fig 3.1 shows the normal section of rack cutter $\sum a$ used to produce involute helical gears. Part 1 of the generating rack cutter's tip fillet is an arc of radius $\rho_0$, which creates the gear's root fillet; part 2 of the tool profile is a linear line $\mathbf{M_0M_2}$, which generates the gear's involute profile; and part 3 of the generating rack cutter's bottom fillet is an arc of radius $\rho_1$, which creates the gear's tip fillet. The following equations can be used to express the position vector of the straight line $\mathbf{M_0M_2}$ of the rack cutter normal section in the coordinate system $S_a(X_a, Y_a, Z_a)$.[7]

$$\begin{pmatrix} \ell\cos\psi_n - a_0 \\ \pm\ (\ell\sin\psi_n - a_o\tan\psi_n - b_o) \\ 0 \end{pmatrix}, 0 \le \ell \le a_0 + a_1/\cos\psi_n \tag{3.1}$$

Likewise, the tip fillet of the normal section of the generating rack cutter has the following equation:

$$\begin{pmatrix} -a_0 + \rho_0\sin\psi_n - \rho_0\sin\zeta_n \\ \pm\ (-a_0\tan\psi_n - b_o - \rho_0\cos\psi_n + \rho_0\cos\zeta_n) \\ 0 \end{pmatrix}, \psi_n < \zeta_n < \frac{\pi}{2} \tag{3.2}$$

The equations of the bottom fillet are as follows.

$$\begin{pmatrix} a_1 - \rho_1\sin\psi_n + \rho_1\sin\theta_n \\ \pm\ (a_1\tan\psi_n - b_o + \rho_1\cos\psi_n - \rho_1\cos\theta_n) \\ 0 \end{pmatrix}, \psi_n < \theta_n < \frac{\pi}{2} \tag{3.3}$$

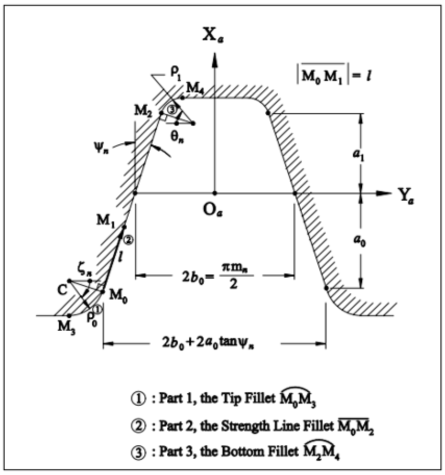



*Fig 3.1 Normal Profile of Rack Cutter* [7]

In Eqs. (3.1)–(3.3), the upper indications denote the left-side of the rack cutter normal section, whereas the lower signs denote the right-side of the rack cutter normal section. The design parameters of the rack cutter surface, $\ell$, $\zeta_n$, and $\theta_n$, correspondingly, dictate the position of points on the linear line, tip fillet, and bottom fillet.

For simulation of rack cutter surface to produce helical gears, the normal section of rack cutter $\sum a$ connected to the coordinate system $S_a$, is translated along the line $\mathbf{O_a O_c}$ as indicated in Fig. 3.2. Thus, one more design parameter of the rack cutter surface is u $=|\boldsymbol{O_a O_c}|$. Employing the following homogeneous coordinate transformation matrix equation, results the rack cutter surface $\sum c$. $\lambda$ refers to the lead angle of the helical gear. [7]

$$\mathbf{R}_{\boldsymbol{c}}^{(\boldsymbol{i})} = \left[\ \mathrm{M}_{ca}\ \right]\mathbf{R}_{\boldsymbol{a}}^{(\boldsymbol{i})} \tag{3.4}$$

Where,

$$\left[\ \mathrm{M}_{ca}\ \right] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\lambda & \cos\lambda & \mathrm{u}\cos\lambda \\ 0 & -\cos\lambda & \sin\lambda & \mathrm{u}\sin\lambda \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $i$=1, 2 and 3. Substituting Eq. (3.1) into Eq. (3.4), gives us the position vector of the rack cutter surface $\sum c$ followed out by the straight line $\mathbf{M_0 M_2}$in coordinate system $S_c$, as

$$\mathbf{R}_{\boldsymbol{c}}^{(2)} \begin{pmatrix} \ell\cos\psi_n - a_0 \\ \pm\ (\ell\sin\psi_n \ - a_o\tan\psi_n - b_o)\sin\lambda\ + \mathrm{u}\cos\lambda \\ (\ell\sin\psi_n \ - a_o\tan\psi_n - b_o)\cos\lambda\ + \mathrm{u}\sin\lambda \end{pmatrix} \tag{3.5}$$

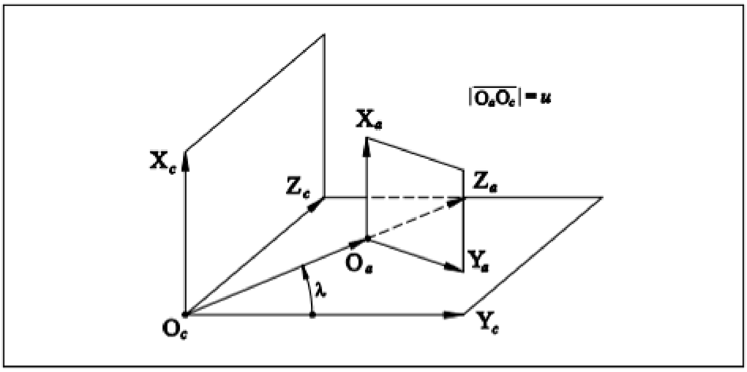



*Fig 3.2 Rack cutter surface $\sum c$ formation* [7]

The unit normal vectors of the rack cutter surface characterized in coordinate system $S_c$ are

$$\mathbf{n}_{\boldsymbol{c}}^{(\boldsymbol{i})} = \frac{\dfrac{\partial\mathbf{R}_c^{(i)}}{\partial\ell} \times \dfrac{\partial\mathbf{R}_c^{(i)}}{\partial\mathrm{u}}}{\left|\dfrac{\partial\mathbf{R}_c^{(i)}}{\partial\ell} \times \dfrac{\partial\mathbf{R}_c^{(i)}}{\partial\mathrm{u}}\right|} \tag{3.6}$$

## 3.2 Tooth surface

The connection between rack cutter $\sum c$ and generated gear of the gear generating mechanism is depicted in Figure 3.3. The coordinate systems $S_c\,(X_c, Y_c, Z_c)$. $S_1\,(X_1, Y_1, Z_1)$.), and $S_h\,(X_h, Y_h, Z_h)$ are related to the rack cutter, generated gear, and gear housing, accordingly, to derive equations for the gear tooth surface. The produced gear surface can be determined using gear theory by simultaneously taking the locus of the imaginary rack cutter expressed in coordinate system S1 and the meshing equation of the cutter and generated gear. So, the mathematical model is: [7]

$$\mathbf{R}_1^{(i)} = \left[\ \mathrm{M}_{1c}\ \right]\mathbf{R}_c^{(i)} \tag{3.7}$$

$$\frac{\mathrm{X}_c^{(i)} - \mathrm{x}_c^{(i)}}{\mathrm{n}_{cx}^{(i)}} = \frac{\mathrm{Y}_c^{(i)} - \mathrm{y}_c^{(i)}}{\mathrm{n}_{cy}^{(i)}} = \frac{\mathrm{Z}_c^{(i)} - \mathrm{z}_c^{(i)}}{\mathrm{n}_{cz}^{(i)}} \tag{3.8}$$

where,

$$\left[\ \mathrm{M}_{1c}\ \right] = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 & r_1(\cos\phi_1 + \phi_1\sin\lambda) \\ \sin\phi_1 & \cos\phi_1 & 0 & r_1(\sin\phi_1 - \phi_1\cos\lambda) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

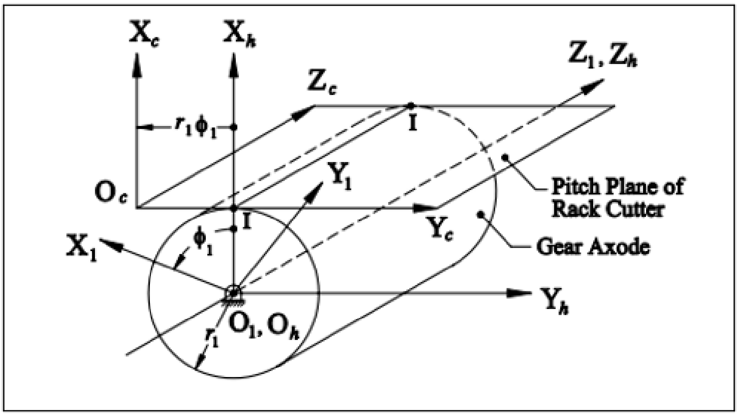



*Fig 3.3 Coordinate relationship between Rack and generated gear* [7]

Using the above equations produces the equation of generated helical gear with involute tooth helical surface:[7]

$$\mathrm{x}_1^{(2)} = (\ell\cos\psi_n - a_0 + r_1)\cos\phi_1 \mp (a_0 - \ell\cos\psi_n)\cot\psi_n\ \sin\lambda\ \sin\phi_1$$

$$\mathrm{y}_1^{(2)} = (\ell\cos\psi_n - a_0 + r_1)\sin\phi_1 \pm (a_0 - \ell\cos\psi_n)\cot\psi_n\ \sin\lambda\ \cos\phi_1 \tag{3.9}$$

$$\mathrm{z}_1^{(2)} = \pm(a_0 - \ell\cos\psi_n)\cot\psi_n\ \tan\lambda\ \sin\lambda \pm \left(\frac{a_0\tan\psi_n + b_0 - \ell\sin\psi_n}{\cos\lambda}\right) + r_1\phi_1\tan\lambda$$

The following matrix equation gives the unit normal of the tooth surface

$$\mathbf{n}_1^{(i)} = \left[\ \mathrm{L}_{1c}\ \right]\mathbf{n}_c^{(i)} \tag{3.10}$$

Where,

$$\left[\ \mathrm{L}_{1c}\ \right] = \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A point to be noted here is that we have only written the equations of part 2 that of the involute* profile.

## 3.3 Tooth Undercutting Analysis

Under specific conditions, such as a low number of teeth, tiny pressure angles, and negatively shifted profiles, undercutting of tooth occurs at the produced gear tooth surfaces. When teeth are undercut near the gear fillets, the tooth thickness is lowered, and the gear bending moment capability is reduced.

A condition for tooth undercutting is the generation of a singular point on the tooth surface and its surface tangent T = 0.
Therefore, an adequate condition for the existence of a singularity of the generated tooth surface is: [7]

$$G(\ell, \mathrm{u}, \phi_1) = \Delta_1^2 + \Delta_2^2 + \Delta_3^2 = 0 \tag{3.11}$$

Eq. (3.11) generates the condition of tooth undercutting as follows:

$$\ell = \frac{1}{\cos\psi_n}\left(a_0 - \frac{r_1 \sin^2\psi_n}{B}\right) \tag{3.12}$$

where,

$$B = sin^2\psi_n + cos^2\psi_n sin^2\lambda$$

According to the known standards, to manufacture a standard gear [with pressure angle $\alpha$ = 20?] and avoid undercut, a minimum number of teeth [z = 17] is required. For certain applications, such standard number of teeth will not do the right job, as it needs to keep a certain gear ratio. And to reach such reduction in teeth number, methods of avoiding undercutting should take place.

Usually, modified tool settings are applied to prevent tooth undercutting of the generated gear. As shown in Fig 3.4 and according to Eq. (3.12), the rack cutter will not undercut the gear tooth if the following inequality is assured. [7]

$$\frac{e_p m_n}{\cos\psi_n} \geq |\ell| = \frac{1}{\cos\psi_n}\left(a_0 - \frac{r_1 \sin^2\psi_n}{B}\right) \tag{3.13}$$

And hence, the normal shifting coefficient of the rack cutter $e_p$ is given:

$$e_p \geq \left|\frac{a_0}{\mathrm{m_n}} - \frac{T_1}{2\sin\lambda}\frac{sin^2\psi_n}{B}\right| \tag{3.14}$$

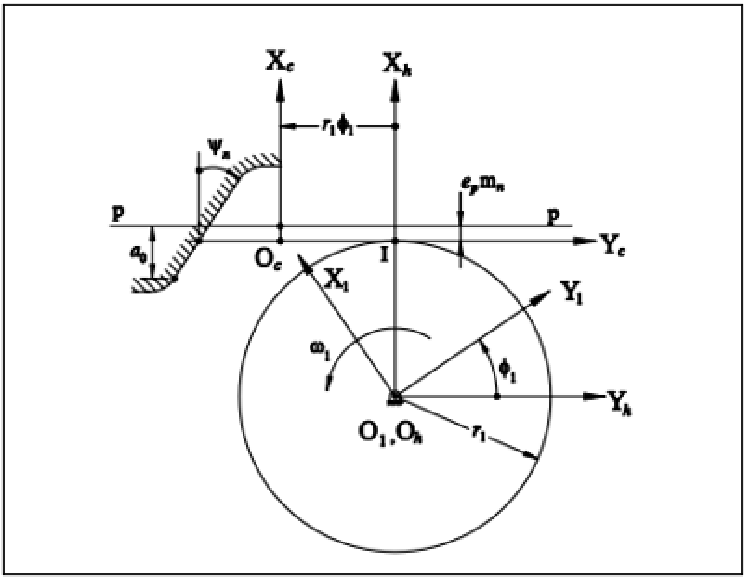



*Fig 3.4 Nonstandard gear generation* [7]

## 3.4 Alternative method using Shaper

The kinematic relationship between the shaper and the resulting gear is shown in Figure 3.5 . $S_1(X_1, Y_1, Z_1)$. and $S_2(X_2, Y_2, Z_2)$coordinate systems are permanently coupled to the shaper cutter and the gear, correspondingly. The below transformation matrix equation can be used to translate the shaper cutter's position vector from coordinate system $S_1$ to $S_2$.[7]

$$\mathbf{R}_2^{(i)} = \begin{bmatrix} \mathrm{M}_{21} \end{bmatrix} \mathbf{R}_i^{(i)} \tag{3.15}$$

Where,

$$\begin{bmatrix} \mathrm{M}_{21} \end{bmatrix} = \begin{bmatrix} \cos(\phi_1 + \phi_2) & \sin(\phi_1 + \phi_2) & 0 & -E\cos\phi_2 \\ -\sin(\phi_1 + \phi_2) & \cos(\phi_1 + \phi_2) & 0 & E\cos\phi_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the same process as defined above of using the equation of meshing, the following equations results in the involute profile of the generated gear tooth:[7]

$$\mathrm{x}_2^{(2)} = (\ell\cos\psi_n - a_0 + r_1)\cos\phi_2 \pm (a_0 - \ell\cos\psi_n)\cot\psi_n \sin\lambda \sin\phi_2 - E\cos\phi_2$$

$$\mathrm{y}_2^{(2)} = -(\ell\cos\psi_n - a_0 + r_1)\sin\phi_2 \pm (a_0 - \ell\cos\psi_n)\cot\psi_n \sin\lambda \cos\phi_2 + E\sin\phi_2 \tag{3.16}$$

$$\mathrm{z}_2^{(2)} = \pm(a_0 - \ell\cos\psi_n)\cot\psi_n \tan\lambda \sin\lambda \pm \left(\frac{a_0 \tan\psi_n + b_0 - \ell\sin\psi_n}{\cos\lambda}\right)\} + r_1 m_{12}\phi_2 \tan\lambda$$

$$m_{12} = \frac{T_2}{T_1} = \frac{\phi_1}{\phi_2}$$

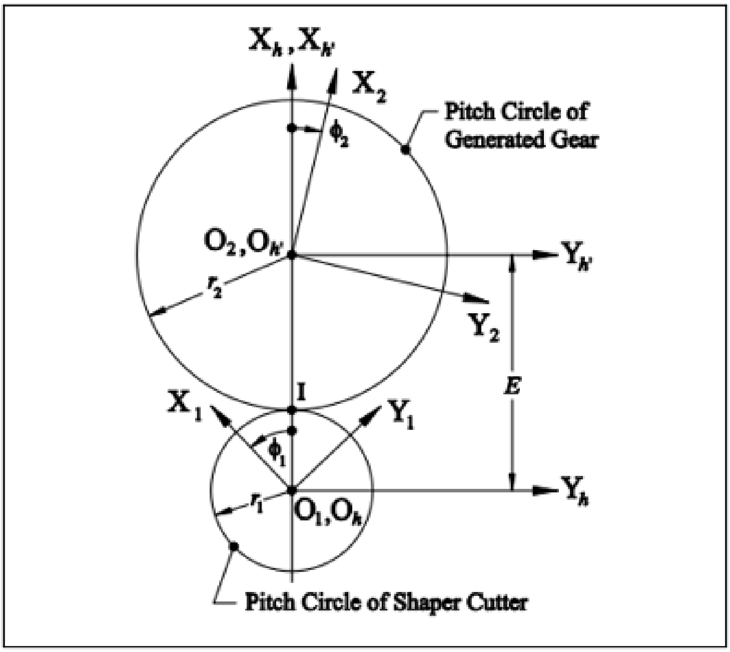



*Fig 3.5 Kinematic relationship* [7]

## 3.5 Design of Non-standard gear using Shaper

The profile shifting coefficient changes the center distance of gear pairs. The profile-shifting coefficients for the pinion and gear are $e_p$ and $e_g$, respectively, the total profile-shifting coefficient of the gear pairs is $e_1 = e_p + e_g$.
As a result, the operating center distance [7]

$$E^{|}=\frac{(r_1+r_2)\cos\psi_s}{\cos\psi_{bs}} \tag{3.17}$$

Where $\psi_s$ is the transverse pressure angle, and the operating pressure angle is $\psi_{bs}$ which is given by the equation:

$$inv\psi_{bs} = 2\tan\psi_s\left(\frac{e_p+e_g}{T_1+T_2}\right) + inv\psi_s \tag{3.18}$$

The involute function can also be stated as:

$$inv\psi_{bs} = \tan\psi_{bs} - \psi_{bs} \tag{3.19}$$

Figure 3.6 depicts the pinion and gear's profile shifting relation during their production. The pinion is made with a rack cutter with a positive profile shifting coefficient $e_p = e_1$, and the mating gear is made with a negative profile shifting coefficient $e_2$ utilizing the pinion as the shaper (i.e., the same pinion-type shaper).

The pinion and gear centrode radii are affected by the change in center distance, but not the gear ratio $m_{12}$. The new pinion and gear pitch radii are as follows: [7]

$$r_p=\frac{E^{|}}{1+m_{12}} \tag{3.20}$$

$$r_g=\frac{E^{|}}{1+1/m_{12}} \tag{3.21}$$

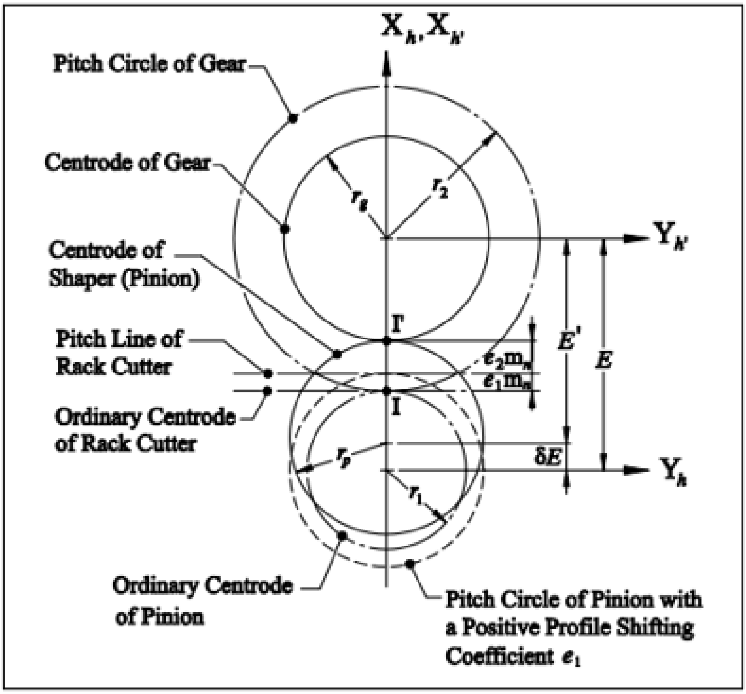



*Fig 3.6 Visualization of Profile Shifting* [7]

# Chapter 4

# Working Principle of Evoloid Drive (High Gear Ratio)

In actuator technology, building technology, medical technology and automotive engineering, transmissions are often used to generate the highest possible reduction ratio. This higher ratio can be used to slow down a car or operate the windows on a building in the event of a power failure that are electrically operated. Here, transmission designs are done with a pinion tooth number less than 5 which can then be designed for an actuation of the output. The pinion is called evoloid gear and is helical toothed.

## 4.1 Evoloid Gearbox to reduce the speed

Gearboxes are currently installed in industry with a conventional gear ratio of 4:1, so to achieve a gearing ratio of 1000:1 at least 5 steps and thus 10 gears and 6 axes are required. The evoloid toothing allows us to design the pinion with only one tooth and to achieve a ratio of i=60:1 with the same number of teeth in the hollow wheel (60), designed in the opposite way to pinion Evoloid. This halves the number of required gear stages at i=1000:1 and the required installation space.[**6**]

## 4.2 Evoloid Gearbox to increase the speed

Evoloid Gearbox to increase the speed With a conventional gear ratio of 1:4,to achieve 1:200, we need 4 stages with a total of 8 gears and 5 axes. With the ratio of i=1:60 achievable with Evoloid, where we design the pinion with only one tooth, we reduce to 2 gear stages and get by with 4 gears and 2 axles.

An Evoloid planetary gearbox has the following components, a sun gear having one tooth, a ring gear, planet gears, and a planet carrier on which the pant gears are rotatably arranged by means of needle bearings. Each of the sun gear, the planet gears, and the ring gear have evoloid toothing.

The planetary gearbox enables high load capacity at high transmission ratios by using three circulating planetary gears in the ring gear, which is frame fixed. The planet gears do not hit each other at a high transmission ratio of 24:1 because of the addendum modification coefficients and addendum coefficients of the individual gears of the planetary gearbox. Each of the three planetary gears mate with the ring gear via a first path of contact and with the sun gear via a second path of contact, and an operating angle of contact of the first path of contact coincides with an operating angle of contact of the second path of contact. The planet gears are rotatably mounted on the planet carrier via needle bearings. Angle of inclination of the evoloid toothing of each of the sun gear, the three planet gears, and the ring gear falls within a range of 30.degree. to 40.degree.

The evoloid planetary gearbox is driven via the sun gear and the output of the planetary gearbox is the planet carrier wherein the sun gear is driven via the drive shaft to obtain higher torque. For the planet gear, a negative addendum modification is proposed. At the same time, the sun gear receives a positive addendum modification. The internally toothed, frame-fixed ring gear is subject to a negative addendum modification. The drive of the Evoloid gearbox customarily takes place via the sun gear and the output via the planet carrier, a reversal of the output is entirely possible. [**6**]

The planet carrier is rotatably mounted in the frame-fixed ring gear, On the outer casing of the ring gear three holders are arranged over the circumference offset by 120 degrees in respect of one another, said holders each having passages for receiving screws, to mechanically connect to one another. The sun gear sits on a drive shaft, the drive-side portion thereof extends through the drive-side planet carrier disc and is connected in a non-rotatable manner to a motor. The shaft portion projecting beyond the sun gear on the output side is mounted in a rotatable manner in a central bore of the planet carrier disc on the output side.

The ring gear has a circumferential web on both front sides of the inner toothing, on which web the planet carrier disc on the drive side, or the output side is rotatably mounted. In an axial direction of the planetary gearbox, the planet carrier discs are each secured between one of the two circumferential webs and a cover. The fastening of the two covers to the frame-fixed ring gear takes place via the holders fastened to the outer casing of the ring gear, the passages of which are aligned with corresponding passages on holders of the two covers. The passages serve to receive screws which connect the two covers to the ring gear. The output may take place, for example, through a pin inserted in a non-rotatable manner into the central bore in the planet carrier disc.[**6**]

## 4.3 Use Case

Evoloid Pinions can be used to increase greatly the transformation ratios per gearwheel pairing. this allows the number of gear stage to be reduced and thus also to reduce the total size of a gearbox. Fig 4.1 shows an application. using a hand drill as an example. The usual two stage gear unit of conventional machine has been replaced by a single stage gear unit whose pinion has $z = 3$ teeth.

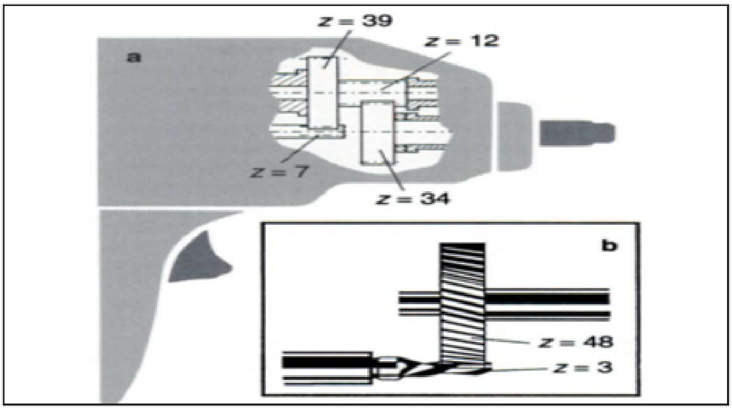


*Fig 4.1 Hand Drill* **[8]**

A gearbox with two gearwheel stage and the usual involute gear wheels with i= (34/7) (34/12) = 15.8, B) gearbox with evoloid gear teeth has one gearbox stage with a transmission ratio I = 48/3 = 16

# Chapter 5

# Alternate solutions

## 5.1 Stackable planetary gearbox with high gear ratio

Here stackable means we can take two or more gearboxes and insert them into each other, increasing the overall output ratio, i.e., 4:1, 16:1, 64:1, etc. It has several stages to have a variable reduction ratio depending on the number of revolutions. A single stage planetary gearbox consists of 3 fundamental elements sun gear, planet gear and ring gear. The sun is the cog at the centre of the system. Planets are the gears that are around the sun, and they can be in variable numbers. The ring is the outer element and "encloses" the otherwise elements. Depending on which components are relatively input, output and stationery we will have different reduction ratios. There are other constraints to consider, and they concern the number of teeth that the various components must have. Suppose that S, R, P are the number of teeth of the sun, planet, and ring, respectively.

The reduction ratio is given by the following formula:
$Ratio = 1 + R/S$

## 5.2 Eccentrically cycloidal drive with high ratio

The heart of a cycloidal drive is the cycloidal disc, whose geometry plays a central role in the kinematic of the gearbox. The profile of such a disc can be traced back to a cycloid. That's why the gearbox is called a cycloidal drive. Since cycloidal drives are used to reduce the speed, they are also referred to as cycloidal speed reducers.

An eccentric shaft (drive shaft) first drives a cycloidal disk. Fixed ring pins are arranged in a circle around the eccentric shaft, in which the cycloidal disc engages. Due to the eccentric motion, the cycloidal disc is driven around these pins so that the cycloidal disc rotates around its axis of symmetry. There are holes in the cycloidal disc which, unlike the eccentric shaft, now rotate clockwise. Roller pins of a pin disc engage in these holes. In this way, the cycloidal disc drives the pin disc, to which the centrally mounted output shaft is attached, and which is coaxial with the input shaft.

Figure 5.2 shows the structure of a cycloidal drive. Due to the symmetrical load distribution, two cycloid discs are often used in practice, which are then offset by 180°. This ensures that the unbalance forces compensate each other, resulting in smoother operation at high speeds. The double design of the cycloidal discs also allows very high torques to be transmitted.

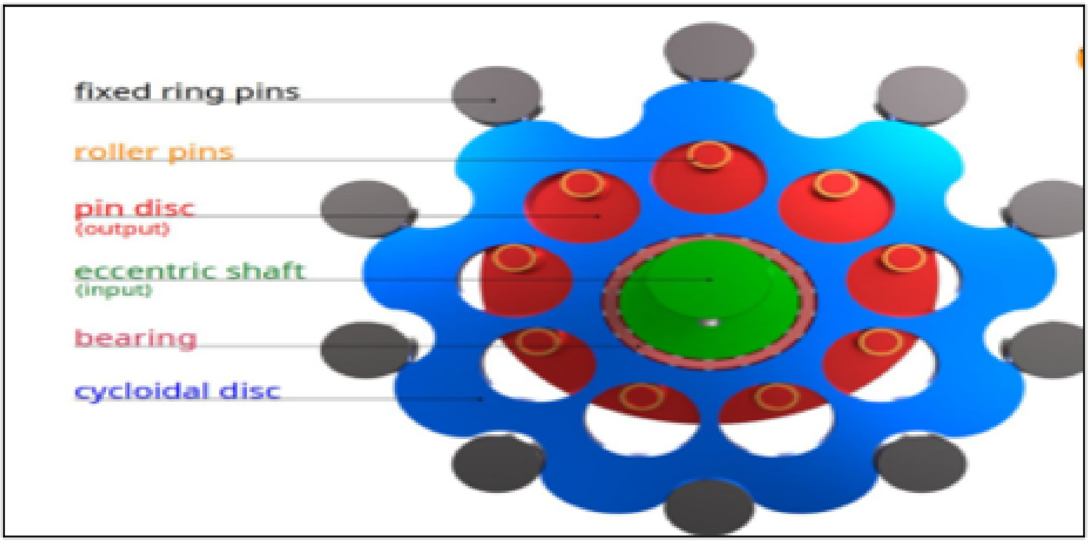



*Fig 5.2 Cycloidal Drive* **[6]**

# Chapter 6

# Discussion

## 6.1 Advantages

Evolid toothing allows for construction of highly gear reducing planetary gears. These planetary gears can take much more load capacity than planetary gears with a spur toothing. Higher gear reductions become possible.We get relatively high transmission ratio in one stage with a very small installation space. Also,If we use three planet gears with the evoloid gearbox we have the following advantages,an improved distribution of load over a greater number of planet gears, a uniform transmission of force in the planet carrier via the axes of the planet gears lying at the corner points of an equilateral triangle, and centering of the sun gear relative to the ring gear through the forces exerted by the planet gears.The evoloid gears will not strike one another even high transmission ratios because of the evoloid toothing. The angle of inclination within a range of 30.degree to 40.degree produces a high transverse contact ratio of roughly 2 which produces good gearbox running properties.

Evoloid planetary gears have many advantages over a conventional two-stage planetary gearbox

- Since there are fewer friction points, the efficiency increases by about 10-20
- The load capacity of the gearbox increased by approx. 30
- The cost compared to the conventional planetary gearbox drops by about 30-40
- Gearbox noise drops by up to 10 dB(A).
- The installation space can be reduced by up to 30%.[2]

## 6.2 Disadvantages

To eliminate tooth undercutting, the traditional approach of applying positive profile shifted cutting is frequently employed in industry to build helical gears with low numbers of teeth. Positively profile-shifted cutting, on the other hand, causes the thickness of gear fillets to increase as the number of teeth decreases. As the thickness of the fillet increases, it decreases the gear addendum. This results in reduction of gear contact ratio.

## 6.3 Solutions

As discussed above about the reduction of gear contact ratio, there also exists several solutions to this problem. One of the solutions is the modification of the root fillet surfaces of the pinion gear. Figure 6.1 shows the geometry of the tooth undercutting and the modification of the root fillet. For further equations and information, one can visit the provided links in the appendix.

To further achieve better contact ratio, there is also a solution to modify the tip fillet surface of the shaper which generates the gear. The equations and information for this method is also presented in links provided in the appendix.

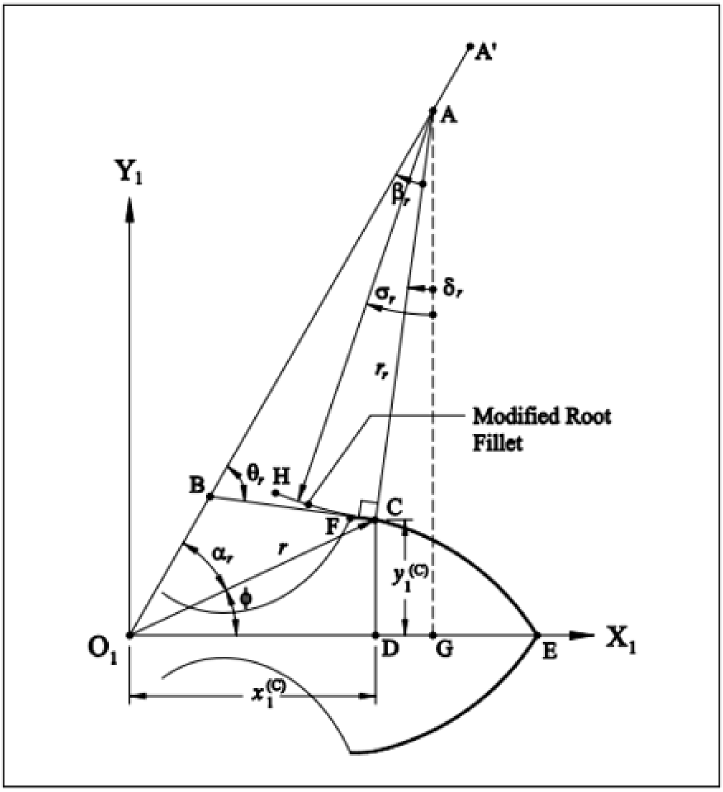



*Fig 6.1 Geometry of tooth Undercut and modified tooth fillet* **[5]**

# Chapter 7

# Conclusion

With all the information gathered through this paper, one is not only able to understand the geometry of an evoloid drive and how it works, but also able to design his own. Throughout this study, we investigated how the evoloid drive achieves the high gear ratio performance based on the geometrical and mathematical characteristics of it with only a small number of teeth at pinion less than five. Furthermore, a detailed examination of the evoloid drive geometry including helical gear generation using a rack profile, tooth undercutting analysis, second method of gear generation using shapers, involute gear meshing parameters and high gear ratio calculations has been conducted. The following findings were identified from this study.

A mathematical model of the modified helical gear pair with a small number of teeth has been inspected. Additionally, examined the tooth-profile shifting and basic geometry modification to avoid undercut and to achieve a high contact ratio.

The condition of tooth undercutting for the involute profile gears has been investigated using the developed mathematical models. Also, the mathematical depictions of the modified root fillet and tip fillet surfaces of the gear profile are derived along with formulas of basic involute gear mesh parameters.

Furthermore, alternative solutions of evoloid drive were also presented, and different modifications of teeth profile were also mentioned to improve gear contact ratio.

The developed mathematical models of the modified helical gear pair with a small number of teeth at pinion can be helpful to facilitate the design and manufacture of evoloid drive. This report is addressed as a primary resource for the subsequent modelling and analysis purpose. Moreover, the study provides insights for future works to explore and delve deeper into the geometric approaches of the evoloid drive along with further improvement in gear meshing.

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