SIR Model with Passive Immune Class M

Juwon Park Min Young Kim

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Introduction

In history, there are lots of diseases which threaten human's life, for example, Black death, SARS, MERS and COVID-19. Numerous people are dead from those serious epidemics, however, people always find out how to overcome those diseases by medicine, vaccine and passive immunity. From this, we are curious about how passive immunity can affect the pandemic period. This report will make an MSIR model with mathematical modeling assumption on the epidemic. The Varicella data set provided by Minnesota will be used to analyze how the passive immunity affects our model. We will set the various situations for the MSIR model, and investigate how those changes affect the result.

Background Data

Varicella, also known as chickenpox, is highly contagious. This disease is more common in preschool and school-age children. It spreaded by respiratory droplets and direct contact with skin lesions. Varicella was a serious epidemic before the vaccine was developed. There were about 4 million cases, 10,600 hospitalizations, and 100 to 150 deaths. However, the cases declined about 90% from 1995 to 2005. However, there is a difficulty finding exact data set we need. We will be going to use the data set from Minnesota in 2013.

Modeling Assumption and Weakness of Model

First, we need to set up modeling assumptions for our MSIR model. The real world data and data analysis might have some difference because there are numerous unexpected outside effects in the real world. Hence, we need to restrict our MSIR model on some conditions as below.

1. There should not be people who get other disease. It causes a population change.
2. Ignore change in population due to immigration and emigration. It causes a population change.
   - Population change by other disease, immigration, or emigration can impact every group of MSIR models, se we need to make our MSIR model with fixed population size.
3. Can never become susceptible again. We assume people get perfect immunity when recovered.
   - All population should move in the progress of M $\rightarrow$ S $\rightarrow$ I $\rightarrow$ R. If not, the MSIR model does not make sense.
4. Constant rate of infectivity
   - We are making the MSIR model with the one disease, so that disease must not change or be stronger during the MSIR model period.
5. The population is homogeneous.
6. No isolation, No vaccination.
7. Only people can spread disease (not animal).
   - Those changes (age, gender, isolation, vaccination, or animal spread) can affect the result of the MSIR model.

Also there is some weakness in our MSIR model. First, we assumed that the population is homogeneous, so there will be some error when we apply real world data. Second, the MSIR model would not be suitable if the population levels are small.

MSIR Epidemiology Model

In the MSIR model, we are looking for the change or impact of four groups of population under given time. At the start, we have four groups which are M, S, I and R. M is passively immune class, S is susceptible class, I is infectious class and R is recovery class. All populations have to flow from M to R as like the figure below. Unlike original SIR model, there will be births and deaths added to the compartments.

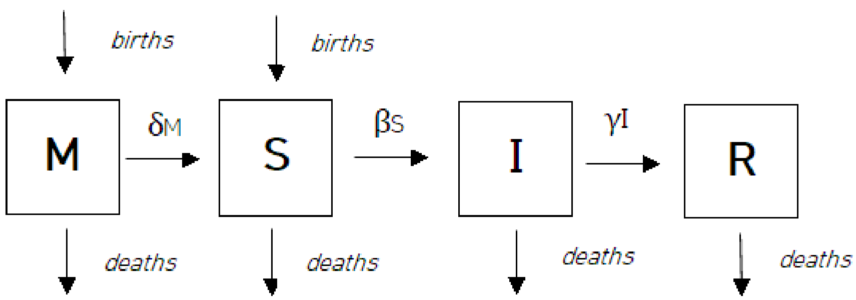


Figure 1: Transfer diagram for MSIR model

- The Passive immune compartment consists of those individuals who have never been infectious and have immunity of the disease. This will skip the infectious class and move on to recovery group. The passive immune class population size at time t is denoted by $M(t)$ with $M(t) \geq 0$.
- The susceptible compartment consists of those individuals who have never been infectious but capable of getting the disease and become infectious. This group would move into the infectious group if infected. The susceptible population size at time t is denoted by $S(t)$ with $S(t) \geq 0$.
- The infectious compartment consists of those individuals who are infectious and capable of transmitting the disease to the susceptible group. Infectious group will remain in this group for a given period and will be moved onto recovery group. The infectious population size at time t is denoted by $I(t)$ with $I(t) \geq 0$.
- The recovery compartment consist of those individuals who had the disease and are dead, or have recovered and are permanently immune from the disease. The is of recovery population at time t is denoted by $R(t)$ with $R(t) \geq 0$.
- All parameters, $\alpha, \mu, \gamma, \beta, \Omega$ defined below can be assumed to be positive.
  - Birth rate: α
  - Death rate: μ
  - Rate of non-passive immune people: Ω
  - Rate of infectivity: β
  - Recovery rate: γ

Since we are looking for the M,S,I,R change with the unit time, we have to make the system of differential equations for the numbers in the epidemiological classes and the population size as below.

$$\begin{aligned}
\frac{dM}{dt} &= -\Omega M(t) + \alpha M(t) - \mu M(t) \\
\frac{dS}{dt} &= -\beta S(t)I(t) + \Omega M(t) + \alpha S(t) - \mu S(t) \\
\frac{dI}{dt} &= \beta S(t)I(t) - \gamma I(t) - \mu I(t) \\
\frac{dR}{dt} &= \gamma I(t) + \mu I(t)
\end{aligned} \tag{1}$$

Where N is the size of initial population with $M(t)+S(t)+I(t)+R(t)=N$.

Simulation of the Solution

Using the systems of differential equations we will use ODE solver in Matlab to generate values and graph of M,S,I,R in each situation below.

1. When is the disease spreading most rapidly?
2. When is the disease decreasing most rapidly?
3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
4. What percentage of the population got infectious during the outbreak?

Based on these questions above, we will now change some variables in our MSIR model to investigate how those changes can affect our model.

First, we will set our values for MSIR model with the data set we found.

- Birth rate(α): 0.0116 [7]
- Death rate(μ): 0.008638 [8]
- Infectivity rate(β): $0.500e^{-06}$
- Recovery rate(γ): 1/7
- immune rate(Ω): 0.06

$M(0)=5499990,\ S(0)=0,\ I(0)=10,\ R(0)=0$

$N=M(0)\ +\ S(0)\ +\ I(0)\ +\ R(I)\ =\ 5500000$

Now we caluculate our MSIR model using the obove variables.

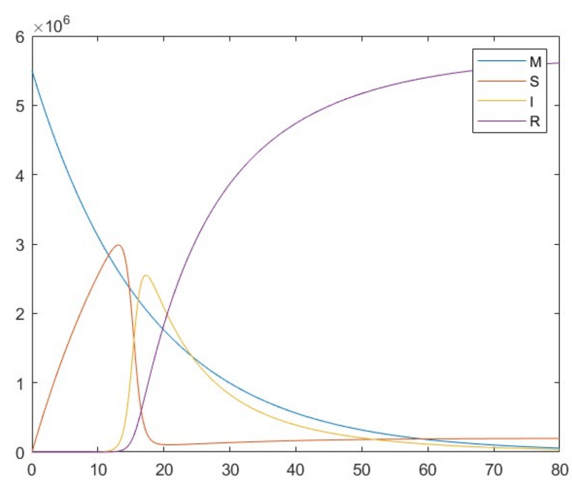


Figure 2: MSIR model for chicken pox

We can analyze our MSIR model using the following questions.

1. When is the disease spreading most rapidly?
   Using the values from the Figure 1 and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$, we can find the chicken pox spreading most rapidly on day 15.

2. When is the disease decreasing most rapidly?
   Using the values from the Figure 1 and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$, we can find the chicken pox spreading most rapidly on day 21.

3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
   Usind the values form the Figure 1, the epidemic reach its peak($2.5483e^{+06}$), when $t = 17.24$. Therefore, the epidemic reach its peak on day 17.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{2548295}{5500000} \times 100 = 46.33\% \tag{2}$$

   As we can see in the above equation, 46.33% of people are infected on peak day.

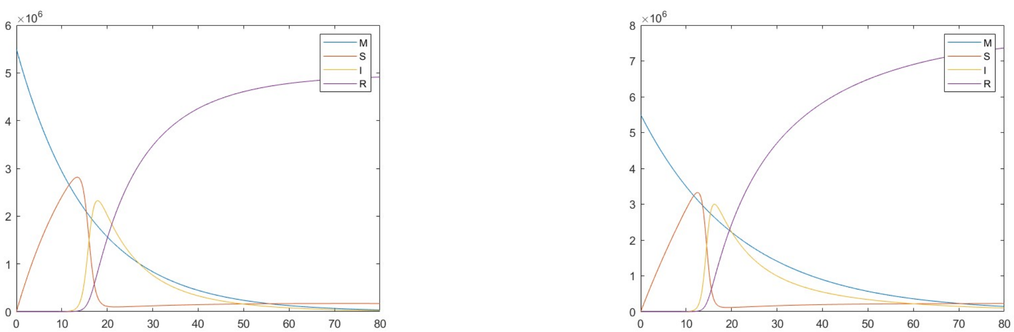
4. What percentage of the population got infectious during the outbreak?
   According to Figure 2, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point S = 105104 when t = 20.92.

$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 105104}{5500000} \times 100 = 98.08\% \tag{3}$$

   Therefore, we can say 98.08% of people are infected during the outbreak.

Now we will change some values of variable, and then compare with our original MSIR model to find out the effect of changing variable values.

1. Let other parameters constant and change birth rate (α) to see how the MSIR model changes using questions(Q.1 $\sim$ Q.4) that we made before. We will let birth rate (α) as 0.0058 (half birth rate) and 0.0232 (twice birth rate)



(a) Twice birth rate

(b) Half birth rate

1. When is the disease spreading most rapidly?
   Using the values from the graph and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\alpha = 0.0058$ (Half birth rate), we can find the chicken pox spreading most rapidly on day 16.
   When $\alpha = 0.0232$ (Twice birth rate), we can find the chicken pox spreading most rapidly on day 14.

2. When is the disease decreasing most rapidly?
   Using the values from the graph and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\alpha = 0.0058$ (Half birth rate), we can find the chicken pox decreasing most rapidly on day 21.
   When $\alpha = 0.0232$ (Twice birth rate), we can find the chicken pox decreasing most rapidly on day 23.

3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
   When $= 0.0058$ (Half birth rate), the epidemic reach its peak($2.3240e^{+06}$), when $t = 17.89$. Therefore, the epidemic reach its peak on day 17.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{2323981}{5500000} \times 100 = 42.25\% \tag{4}$$

   As we can see in the above equation, 42.25% of people are infected on peak day.

   When $= 0.0232$ (Twice birth rate), the epidemic reach its peak($3.0003e^{+06}$), when $t = 16.20$. Therefore, the epidemic reach its peak on day 16.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{3000253}{5500000} \times 100 = 54.55\% \tag{5}$$

   As we can see in the above equation, 54.55% of people are infected on peak day.

4. What percentage of the population got infectious during the outbreak?
   When $\alpha = 0.0058$ (Half birth rate), according to Figure 1, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 99541$ when $t = 21.96$.

$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 99541}{5500000} \times 100 = 98.19\% \tag{6}$$

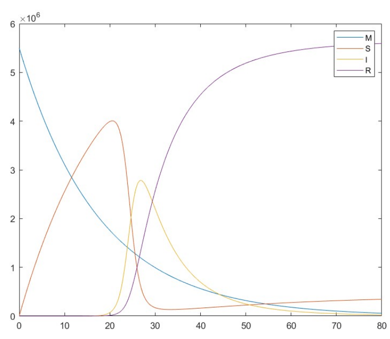
   Therefore, we can say 98.19% of people are infected during the outbreak.

   When $\alpha = 0.0232$ (Twice birth rate), according to Figure 2, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 115926$ when $t = 19.14$.

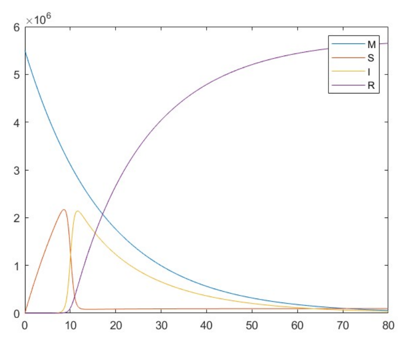
$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 115926}{5500000} \times 100 = 97.89\% \tag{7}$$

   Therefore, we can say 97.89% of people are infected during the outbreak.

2. Let other parameters constant and change infectivity rate (β) to see how the MSIR model changes using questions(Q.1 $\sim$ Q.4) that we made before. We will let infectivity rate (β) as $0.250e^{-06}$ (half infectivity rate) and $1.000e^{-06}$ (twice infectivity rate)



(a) Half infectivity rate



(b) Double infectivity rate

1. When is the disease spreading most rapidly?
   Using the values from the graph and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\beta = 0.250e^{-06}$ (half infectivity rate), we can find the chicken pox spreading most rapidly on day 24.
   When $\beta = 1.000e^{-06}$ (twice infectivity rate), we can find the chicken pox spreading most rapidly on day 10.

2. When is the disease decreasing most rapidly?
Using the values from the graph and find the minimum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
When $\beta = 0.250e^{-06}$ (half infectivity rate), we can find the chicken pox decreasing most rapidly on day 31.
When $\beta = 1.000e^{-06}$ (twice infectivity rate), we can find the chicken pox decreasing most rapidly on day 18.

3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
When $\beta = 0.250e^{-06}$ (half infectivity rate), the epidemic reach its peak($2.785e^{+06}$), when $t = 26.91$. Therefore, the epidemic reach its peak on day 26.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{2784556}{5500000} \times 100 = 50.62\% \tag{8}$$

As we can see in the above equation, 50.62% of people are infected on peak day.

When $\beta = 1.000e^{-06}$ (twice infectivity rate), the epidemic reach its peak($2.138e^{+06}$), when $t = 11.61$. Therefore, the epidemic reach its peak on day 16.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{2137911}{5500000} \times 100 = 38.87\% \tag{9}$$

As we can see in the above equation, 38.87

4. What percentage of the population got infectious during the outbreak?
When $\beta = 0.250e^{-06}$ (half infectivity rate), according to Figure 1, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 132947$ when $t = 33.40$.

$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 132947}{5500000} \times 100 = 97.58\% \tag{10}$$

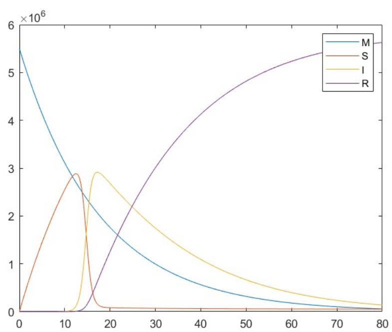
Therefore, we can say 97.58% of people are infected during the outbreak.

When $\beta = 1.000e^{-06}$ (twice infectivity rate), according to Figure 2, Mvalue is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 80048$ when $t = 14.07$.

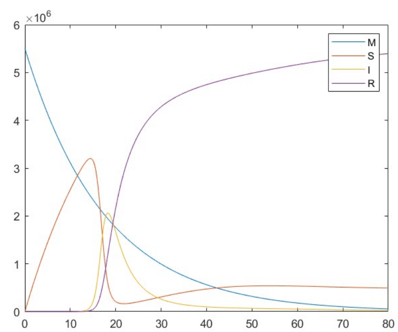
$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 80048}{5500000} \times 100 = 98.54\% \tag{11}$$

Therefore, we can say 98.54

3. Let other parameters constant and change recovery rate (γ) to see how the MSIR model changes using questions(Q.1 $\sim$ Q.4) that we made before. We will let recovery rate (γ) as 1/14 (half recovery rate) and 2/7 (twice recovery rate).



(a) Twice Recovery rate



(b) Half Recovery rate

1. When is the disease spreading most rapidly?
   Using the values from the graph and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\gamma = \frac{1}{14}$ (half recovery rate), we can find the chicken pox spreading most rapidly on day 15.
   When $\gamma = \frac{2}{7}$ (twice recovery rate), we can find the chicken pox spreading most rapidly on day 17.

2. When is the disease decreasing most rapidly?
   Using the values from the graph and find the minimum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\gamma = \frac{1}{14}$ (half recovery rate), we can find the chicken pox decreasing most rapidly on day 26.
   When $\gamma = \frac{2}{7}$ (twice recovery rate), we can find the chicken pox decreasing most rapidly on day 21.

3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
   When $\gamma = \frac{1}{14}$ (half recovery rate), the epidemic reach its peak($2.914e^{+06}$), when $t = 17.21$. Therefore, the epidemic reach its peak on day 17.

   $$\frac{Infected people}{Total number of people} \times 100 = \frac{2913704}{5500000} \times 100 = 52.97\% \tag{12}$$

   As we can see in the above equation, 52.97% of people are infected on peak day.

   When $\gamma = \frac{2}{7}$ (twice recovery rate), the epidemic reach its peak($2.138e^{+06}$), when $t = 18.18$. Therefore, the epidemic reach its peak on day 18.

   $$\frac{Infected people}{Total number of people} \times 100 = \frac{2061738}{5500000} \times 100 = 37.48\% \tag{13}$$

   As we can see in the above equation, 37.48% of people are infected on peak day.

4. What percentage of the population got infectious during the outbreak?
   When $\gamma = \frac{1}{14}$ (half recovery rate), according to Figure 1, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 76802$ when $t = 22.49$.

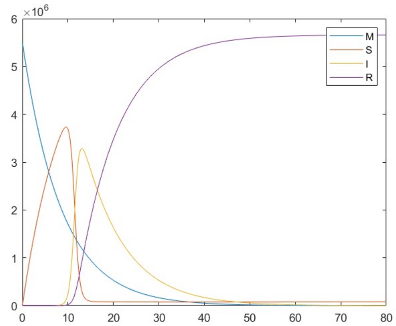
   $$\frac{Infected people - nonInfected people}{Total number of people} \times 100 = \frac{5500000 - 76802}{5500000} \times 100 = 98.60\% \tag{14}$$

   Therefore, we can say 98.60% of people are infected during the outbreak.

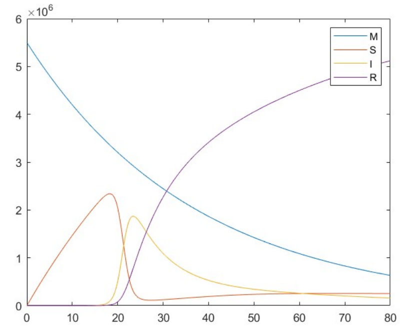
   When $\gamma = \frac{2}{7}$ (twice recovery rate), according to Figure 2, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 165541$ when $t = 21.73$.

   $$\frac{Infected people - nonInfected people}{Total number of people} \times 100 = \frac{5500000 - 165541}{5500000} \times 100 = 96.99\% \tag{15}$$

4. Let other parameters constant and change passive immunne rate (Ω) to see how the MSIR model changes using questions(Q.1 $\sim$ Q.4) that we made before. We will let passive immunne rate (Ω) as 0.03(half immune rate) and 0.12 (twice immune rate).



(a) Twice Immune rate



(b) Half Immune rate

1. When is the disease spreading most rapidly?
   Using the values from the graph and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\Omega = 0.03$(half immune rate), we can find the chicken pox spreading most rapidly on day 21.
   When $\Omega = 0.12$(twice immune rate), we can find the chicken pox spreading most rapidly on day 11.

2. When is the disease decreasing most rapidly?
   Using the values from the graph and find the minimum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$
   When $\Omega = 0.03$(half immune rate), we can find the chicken pox decreasing most rapidly on day 30.
   When $\Omega = 0.12$(twice immune rate), we can find the chicken pox decreasing most rapidly on day 18.

3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
   When $\Omega = 0.03$(half immune rate), the epidemic reach its peak($1.868e^{+06}$), when $t = 23.39$. Therefore, the epidemic reach its peak on day 23.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{1868400}{5500000} \times 100 = 33.97\% \tag{16}$$

   As we can see in the above equation, 33.97% of people are infected on peak day.

   When $\Omega = 0.12$(twice immune rate), the epidemic reach its peak($3.284e^{+06}$), when $t = 13.14$. Therefore, the epidemic reach its peak on day 18.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{3284471}{5500000} \times 100 = 59.72\% \tag{17}$$

   As we can see in the above equation, 57.72% of people are infected on peak day.

4. What percentage of the population got infectious during the outbreak?
   When $\Omega = 0.03$ (half immune rate), according to Figure 1, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 113126$ when $t = 27.16$.

$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 113126}{5500000} \times 100 = 97.94\% \tag{18}$$

   Therefore, we can say 97.94% of people are infected during the outbreak.

   When $\Omega = 0.12$(twice immune rate), according to Figure 2, M value is getting closer to 0 while t is increasing. At the same time, S value get to the lowest point $S = 75253$ when $t = 43.46$.

$$\frac{Infectedpeople - nonInfectedpeople}{Totalnumberofpeople} \times 100 = \frac{5500000 - 75253}{5500000} \times 100 = 98.63\% \tag{19}$$

Therefore, we can say 98.63% of people are infected during the outbreak.

MSIR model result and analysis

birth rate (α)	Chicken pox spreading most rapidly on	Chicken pox decreasing most rapidly on	Epidemic reach its peak and % of infectious	Percentage of the populations got infectious
$\alpha = 0.0058$	day 16	day 14	day 17.89 42.25 %	98.19%
$\alpha = 0.0116$	day 15	day 21	day 17.24 46.43 %	98.08%
$\alpha = 0.0232$	day 21	day 23	day 16.20 54.55%	97.89%

Figure 7: MSIR model result with change of birth rate

As we can see in the table 7, high birth rate affect on the infectious rate on peak day. This makes sense because when population increase, the probability of infection will be also increase.

infectivity rate (β)	Chicken pox spreading most rapidly on	Chicken pox decreasing most rapidly on	Epidemic reach its peak and % of infectious	Percentage of the populations got infectious
β = 0.2500e-06	day 24	day 31	day 26.91 50.62 %	97.58%
β = 0.5000e-06	day 15	day 21	day 17.24 46.43 %	98.08%
β = 1.000e-06	day 10	day 18	day 11.61 38.87%	98.54%

Figure 8: MSIR model result with change of infectivity rate

As we can see in the table 8, higher infectivity rate make the spread of chicken pox faster. Also, the total infected population is increase.

recovery rate (γ)	Chicken pox spreading most rapidly on	Chicken pox decreasing most rapidly on	Epidemic reach its peak and % of infectious	Percentage of the populations got infectious
$\gamma = \frac{1}{14}$	day 15	day 26	day 17.21 52.97 %	98.60%
$\gamma = \frac{1}{7}$	day 15	day 21	day 17.24 46.43 %	98.08%
$\gamma = \frac{2}{7}$	day 17	day 21	day 18.18 37.48%	96.99%

Figure 9: MSIR model result with change of recovery rate

As we can see in the table 9, high recovery rate decrease the percent of infectious on peak day and also decrease the total population got infected during the epidemic.

passive immune rate (δ)	Chicken pox spreading most rapidly on	Chicken pox decreasing most rapidly on	Epidemic reach its peak and % of infectious	Percentage of the populations got infectious
δ = 0.03	day 21	day 30	day 23.39 33.97 %	97.94%
δ = 0.06	day 15	day 21	day 17.24 46.43 %	98.08%
δ = 0.12	day 11	day 18	day 13.14 59.72%	98.63%

Figure 10: MSIR model result with change of passive immune rate

As we can see in the table 10, low passive immune rate decrease the percent of infectious on peak day and also decrease the total population got infected during the epidemic.

Non-Dimensionalization

$$\begin{aligned}
\frac{dM}{dt} &= -\Omega M(t) + \alpha M(t) - \mu M(t) \\
\frac{dS}{dt} &= -\beta S(t)I(t) + \Omega M(t) + \alpha S(t) - \mu S(t) \\
\frac{dI}{dt} &= \beta S(t)I(t) - \gamma I(t) - \mu I(t) \\
\frac{dR}{dt} &= \gamma I(t) + \mu I(t)
\end{aligned} \tag{20}$$

In this section, we will non-dimensionalize our MSIR model. Since, our MSIR model has a unit of $\frac{Population}{Time}$ therefore, we will make our parameter dimensionless by changing q to other variables and be independent of the units of time and population.

Let our non-dimensional variables

$$\begin{aligned}
&\tau = \gamma t,\ \mathbf{M} = \frac{M}{N},\ \mathbf{S} = \frac{S}{N},\ \mathbf{I} = \frac{I}{N},\ \mathbf{R} = \frac{R}{N} \\
&P = \frac{\beta N}{\gamma},\ \Theta = \frac{\Omega N}{\gamma},\ Y = \frac{N}{\gamma},\ Z = \frac{\mu N}{\gamma}
\end{aligned} \tag{21}$$

We can derive non-dimensionalize MSIR model based on our new variables

$$\begin{aligned}
\frac{dM}{dt} &= -\Theta\mathbf{M} + Y\mathbf{M} - Z\mathbf{M} \\
\frac{dS}{dt} &= -P\mathbf{SI} + \Theta\mathbf{M} + Y\mathbf{S} - Z\mathbf{S} \\
\frac{dI}{dt} &= P\mathbf{SI} - \mathbf{R} - Z\mathbf{I} \\
\frac{dR}{dt} &= \mathbf{I} + Z\mathbf{I}
\end{aligned} \tag{22}$$

Original SIR Epidemiology Model

This is the basic original SIR epidemiology model.

First, set up the original SIR model using the same variable values which used in MSIR model.

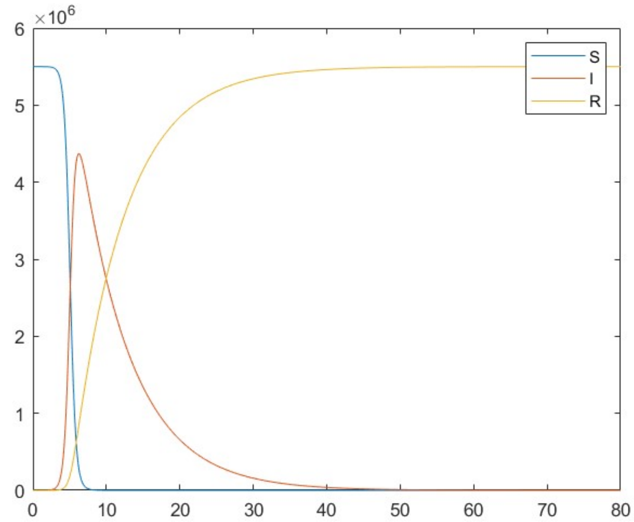


Figure 11: original SIR model

1. When is the disease spreading most rapidly?
   Using the values from the Figure 11 and find the maximum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$, we can find the chicken pox spreading most rapidly on day 5.

2. When is the disease decreasing most rapidly?
   Using the values from the Figure 11 and find the minimum $\frac{dI}{dt}$ value with $\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu S(t)$, we can find the chicken pox decreasing most rapidly on day 12.

3. On what day did the epidemic reach its peak and what percentage of people were infectious on that day?
   Using the values form the Figure 11, the epidemic reach its peak($4.37e^{+06}$), when $t = 6.223$. Therefore, the epidemic reach its peak on day 6.

$$\frac{Infectedpeople}{Totalnumberofpeople} \times 100 = \frac{4369535}{5500000} \times 100 = 79.44\% \tag{23}$$

   As we can see in the above equation, 79.44% of people are infected on peak day.

4. What percentage of the population got infectious during the outbreak?
   According to Figure 11, S value is getting closer to 0 while t is increasing. Therefore, we can say 100% of population are infected during the outbreak.

	Chicken pox spreading most rapidly on	Chicken pox decreasing most rapidly on	Epidemic reach its peak and % of infectious	Percentage of the populations got infectious
Original SIR model	day 5	day 12	day 6.22 79.44 %	100%
Original MSIR model	day 15	day 21	day 17.24 46.43 %	98.08%

Figure 12: Compare Original SIR model with Origianl MSIR model

As we can see in the figure 12, the Chicken pox spread faster in Origianl SIR model and more population are infected in Original SIR model. From this, we have the following result. If we have passive immune class during the outbreak, we can suppress the spreading of disease and also we can protect some people from the disease.

Conclusion and Future Work

Chicken pox(varicella) was very contagious. It was a tough epidemic for young children before the vaccine was developed. In this paper, we set up various MSIR models using the chicken pox data in Minnesota. We can find out that in the MSIR model, the passive immune class suppress the spreading of disease. Also, we need low birth rate(α), low infectivity rate(β), hight recovery rate(γ), and low passive immune rate(Ω) to suppress an epidemic. This result shows how important getting vaccine and having passive immune is. We can use this analyzed data for the future work. We can try SIR model with the vaccination or SIR model with isolation, and find out if having passive immune is powerful, or either doing vaccination or isolation is powerful for suppressing the spread of disease. After that we can try MSIR model with vaccination or MSIR model with isolation, and see how well it suppress the spread of disease.

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Code

Here is the code for generating graphs of MSIR model

```
function dx = sirproject(t, x)
  dx = [0; 0; 0; 0];
  beta = 0.5e-6;
  gamma = 1/7;
  omega = 0.06;
  birth = 0.0116;
  death = 0.008638;
  N = 5500000;

  %x(1) = M
  %x(2) = S
  %x(3) = I
  %x(4) = R
  dx(1) = - omega*x(1) + birth*x(1) - death*x(1);
  dx(2) = - beta*x(2)*x(3) + omega*x(1) + birth*x(2) - death*x(2);
  dx(3) = beta*x(2)*x(3) - gamma*x(3) - death*x(3);
  dx(4) = gamma*x(3) + death*x(3);
end
```