College of Engineering and Physics

Electrical Engineering Department

EE562 - Digital Signal Processing I

Second Semester (212)

# Homework Assignment 3


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$\boxed{3.2}$

(a) $x(n) = (1+n)\,u(n)$

$$X(z) = \sum_n x(n)\, z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (1+n)\, z^{-n} = \underbrace{\sum_0^{\infty} z^{-n}}_{①} + \underbrace{\sum_0^{\infty} n\, z^{-n}}_{②}$$

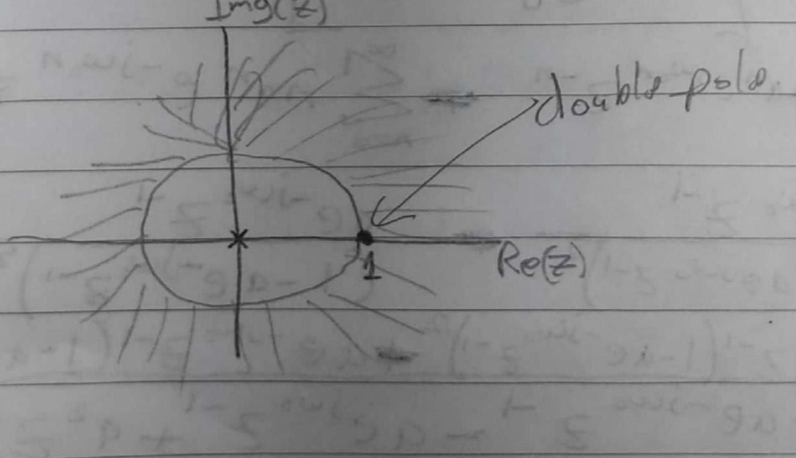
$$① \longrightarrow \sum_0^{\infty} z^{-n} = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

$$② \longrightarrow \sum_0^{\infty} n z^{-n} = \frac{z^{-1}}{(1-z^{-1})^2}$$

$$\therefore X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2}$$

$$= \frac{1-z^{-1}}{(1-z^{-1})^2} + \frac{z^{-1}}{(1-z^{-1})^2}$$

$$= \frac{1}{(1-z^{-1})^2} = \frac{z^2}{(z-1)(z-1)}, \quad |z| > 1$$

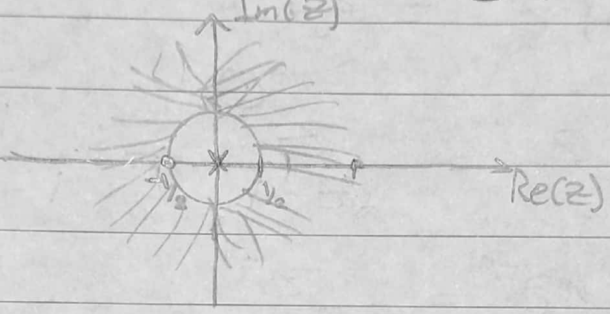


(c) $x(n) = (-1)^n 2^{-n} u(n) = (-1)^n (2^{-1})^n u(n)$

$$= (-2^{-1})^n u(n)$$

$$= (-0.5)^n u(n)$$

$$\therefore X(z) = \sum_{n=0}^{\infty} (-0.5)^n z^{-n} = \frac{z}{z+0.5}, \ |z| > 1/2$$




(d) $x(n) = (n a^n \sin \omega_0 n) u(n)$

$$\because n a^n u(n) \longrightarrow \frac{a z^{-1}}{(1 - a z^{-1})^2}$$

$$\therefore X(z) = \sum_{n=0}^{\infty} n a^n \sin \omega_0 n \, z^{-n}$$

$$= \sum_{n=0}^{\infty} n a^n z^{-n} \left[ \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \right]$$

$$= \frac{1}{2j} \left[ \sum_{n=0}^{\infty} n a^n e^{j\omega_0 n} z^{-n} - \sum_{n=0}^{\infty} n a^n e^{-j\omega_0 n} z^{-n} \right]$$

$$= \frac{1}{2j} \left[ \frac{a e^{j\omega_0} z^{-1}}{(1 - a e^{j\omega_0} z^{-1})^2} - \frac{a e^{-j\omega_0} z^{-1}}{(1 - a e^{-j\omega_0} z^{-1})^2} \right]$$

$$= \frac{1}{2j} \left[ \frac{a e^{j\omega_0} z^{-1} (1 - a e^{-j\omega_0} z^{-1})^2 - a e^{-j\omega_0} z^{-1} (1 - a e^{j\omega_0} z^{-1})^2}{(1 - a e^{-j\omega_0} z^{-1} - a e^{j\omega_0} z^{-1} + a^2 z^{-2})^2} \right]$$

$$= \frac{1}{2j}\left[\frac{\checkmark}{\left(1-2az^{-1}\left[\frac{e^{j\omega_0}+e^{-j\omega_0}}{2}\right]+a^2z^{-2}\right)^2}\right]$$

$$= \frac{1}{2j}\left[\frac{\checkmark}{\left(1-2az^{-1}\cos\omega_0+a^2z^{-2}\right)^2}\right]$$

$$= \frac{1}{2j}\,\frac{\left(ae^{j\omega_0}z^{-1}\cancel{-2a^2z^{-2}}+a^3e^{-j\omega_0}z^{-3}\right]-\left[ae^{-j\omega_0}z^{-1}\cancel{-2a^2z^{-2}}+a^3e^{j\omega_0}z^{-3}\right]}{\checkmark}$$

$$= \frac{1}{2j}\,\frac{az^{-1}\left(e^{j\omega_0}-e^{-j\omega_0}\right)+a^3z^{-3}\left(e^{-j\omega_0}-e^{j\omega_0}\right)}{\checkmark}$$

$$= \frac{az^{-1}\sin\omega_0 - a^3z^{-3}\sin\omega_0}{\left(1-2az^{-1}\cos\omega_0+a^2z^{-2}\right)^2}$$

$$= \boxed{\frac{\left[az^{-1}-a^3z^{-3}\right]\sin\omega_0}{\left(1-2a\cos\omega_0\, z^{-1}+a^2z^{-2}\right)^2}} \;\# = \frac{N(z)}{D(z)}$$

$\Rightarrow$ for $N(z)$:

zeros at:

$$\boxed{z_1 = 0,\quad z_{2,3} = \pm a} \;\#$$

for $D(z)$:

$$z_{1,2} = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$= \frac{2a\cos\omega_0 \pm \sqrt{4a^2\cos^2\omega_0 - 4a^2}}{2}$$

$$= a\cos\omega_0 \pm a\sqrt{-\underbrace{(1-\cos^2\omega_0)}_{\text{complex}}}$$

$$= a\cos\omega_0 \pm ja\sin\omega_0$$

$\therefore$ poles at:

$$z_1 = a\cos\omega_0 + ja\sin\omega_0$$

$$z_2 = a\cos\omega_0 - ja\sin\omega_0 \;\#$$

$\boxed{3.3}$

(a) $x_1(n) = \begin{cases} \left(\frac{1}{3}\right)^n , & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} & n < 0 \end{cases}$

$$x_1(n) = \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{2}\right)^{-n} u(-n-1)$$

$$\because \quad a^n u(n) \longleftrightarrow \frac{z}{z-a}$$

$$\because \quad -a^n u(-n-1) \longleftrightarrow \frac{z}{z-a}$$

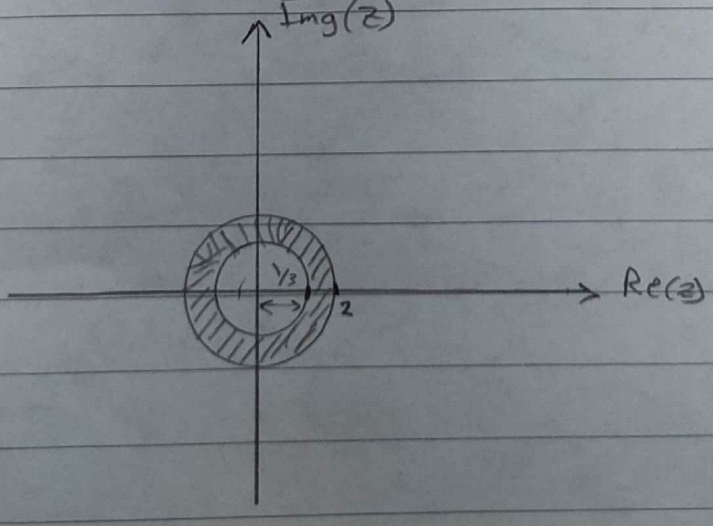
$$\therefore X_1(z) = \frac{z}{z-1/3} - \frac{z}{z-2}$$

$$= \frac{\cancel{z^2} - 2z - \cancel{z^2} + 1/3\, z}{(z-1/3)(z-2)}$$

$$= \frac{-5/3\, z}{z^2 - 1/3\, z - 2z + 2/3}$$

$$= \boxed{\frac{-5/3\, z}{z^2 - \frac{7}{3} z + 2/3}} \quad \#$$

ROC: $1/3 < |z| < 2$



(b) $x_4(n) = x_1(-n)$

$$X_4(z) = \sum_{n=-\infty}^{\infty} x_1(-n)\, z^{-n}$$

$$= \sum_{m=-\infty}^{\infty} x_1(m)\, z^{m} = X_1(z^{-1})$$

$$= \frac{-5/3\ z^{-1}}{(z^{-1}-1/3)(z^{-1}-2)} = \boxed{\frac{-5/3\ z}{(1-1/3 z)(1-2z)}}\ \#$$

ROC: $\frac{1}{2} < |z| < 3$