Q5.1

Monday, March 21, 2022 10:00 PM

5.1 The following input–output pairs have been observed during the operation of various systems:

(a) $x(n) = (\frac{1}{2})^n \xrightarrow{\mathcal{T}_1} y(n) = (\frac{1}{8})^n$ Determine their frequency response if each of the above systems is LTI.

(a) $\because$ $H(\omega)$ exsist if the system is BIBO

$\because$ $n = -\infty \Rightarrow x(-\infty) = \infty$ $\therefore$ input is not Bounded
$y = (-\infty) = \infty$ output is not bounded

$\boxed{\therefore H(\omega) \text{ does not exsist}}$ #

(c) $x(n) = e^{j\pi/5} \xrightarrow{\mathcal{T}_3} y(n) = 3e^{j\pi/5}$

$\Rightarrow$ BIBO

$\Rightarrow Ae^{j\omega n} \longrightarrow \boxed{h(n)} \longrightarrow AH(\omega)e^{j\omega n}$

$\therefore H(\omega) = 3 \Rightarrow \boxed{H(\pi/5) = 3}$ #

Q5.3

Monday, March 21, 2022 10:32 PM

5.3 Consider an LTI system with impulse response $h(n) = (\frac{1}{2})^n u(n)$.

(a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

1. $x(n) = \cos \frac{3\pi n}{10}, \; -\infty < n < \infty$
2. $x(n) = \{\ldots, 1, 0, 0, \underset{\uparrow}{1}, 1, 1, 0, 1, 1, 1, 0, 1, \ldots\}$

(a) $H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{\infty} (\frac{1}{2})^n e^{-j\omega n} = \sum_{n=0}^{\infty} (\frac{1}{2} e^{-j\omega})^n$

$= \frac{1}{1 - 0.5 e^{-j\omega}} = \frac{1}{1 - 0.5(\cos\omega - j\sin\omega)}$

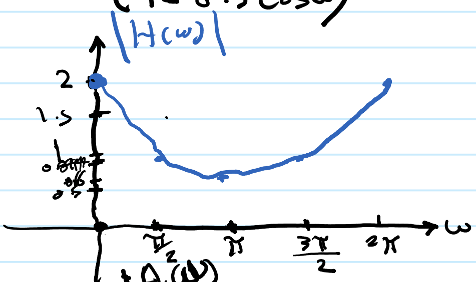
$|H(\omega)| = \frac{1}{\sqrt{(1 - 0.5\cos\omega)^2 + (0.5\sin\omega)^2}} = \frac{1}{\sqrt{1 - \cos\omega + 0.25\cos^2\omega + 0.25\sin^2\omega}}$

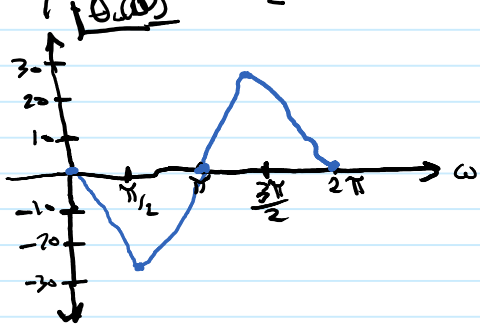
$= \frac{1}{\sqrt{1 - \cos\omega + 0.25}} = \boxed{\frac{1}{\sqrt{1.25 - \cos\omega}}}$ #

ω	|H(ω)|
0	2
90	0.8944
180	0.6666
270	0.8944
360	2

$\angle\theta(\omega) = -\tan^{-1}\left(\frac{0.5\sin\omega}{1 - 0.5\cos\omega}\right)$

ω	∠θ(ω)
0	0
90	-26.56
180	0
270	26.565
360	0





5.3 Consider an LTI system with impulse response $h(n) = (\frac{1}{2})^n u(n)$.

(a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

1. $x(n) = \cos \frac{3\pi n}{10}, \; -\infty < n < \infty$
2. $x(n) = \{\ldots, 1, 0, 0, \underset{\uparrow}{1}, 1, 1, 0, 1, 1, 1, 0, 1, \ldots\}$

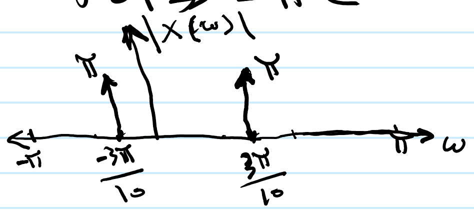
(b) (1) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n-\infty}^{\infty} \cos\frac{3\pi n}{10} e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \frac{1}{2}\left[e^{j\frac{3\pi n}{10}} + e^{-j\frac{3\pi n}{10}}\right] e^{-j\omega n}$

$\Rightarrow \because \quad 1 \xleftrightarrow{DTFT} \sum_{k=-\infty}^{\infty} 2\pi\, \delta(\omega + 2\pi k)$

$\therefore \quad e^{j\omega_0} \xleftrightarrow{DTFT} \sum_{k=-\infty}^{\infty} 2\pi\, \delta(\omega - \omega_0 + 2\pi k)$

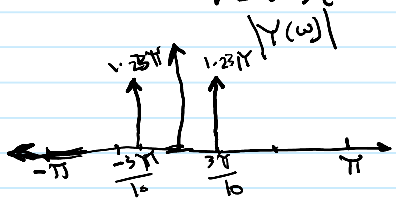
$\therefore X(\omega) = \frac{1}{2} \times 2\pi \left[\delta(\omega - \frac{3\pi}{10}) + \delta(\omega + \frac{3\pi}{10})\right]$

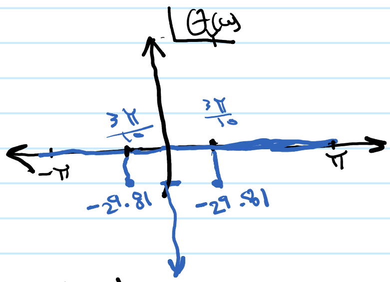
for $\Rightarrow -\pi < \omega < \pi$



$\Rightarrow Y(\omega) = X(\omega) \cdot H(\omega) = H(\frac{3\pi}{10}) \cdot \pi\left[\delta(\omega - \frac{3\pi}{10}) + \delta(\omega + \frac{3\pi}{16})\right]$

$\because H(\omega) = \frac{1}{1 - 0.5 e^{-j\omega}}, \quad H(\frac{3\pi}{10}) = 1.23 \underline{\angle -29.81}$





$y(n) = \left|H(\frac{3\pi}{10})\right| \cos\left(\frac{3\pi n}{10} + \angle\Theta(\frac{3\pi}{10})\right)$

$= 1.23 \cos\left(\frac{3\pi}{10} n - 29.81\right)$

5.3 Consider an LTI system with impulse response $h(n) = (\frac{1}{2})^n u(n)$.

(a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

1. $x(n) = \cos\frac{3\pi n}{10}, \quad -\infty < n < \infty$
2. $x(n) = \{\ldots, 1, 0, 0, \underset{\uparrow}{1}, 1, 1, 0, 1, 1, 1, 0, 1, \ldots\}$

(b) $\boxed{2}$ $x(n) = \delta(n+3) + \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-4) \ldots$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} =$$

$$= \cdots + e^{j3\omega} + 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j4\omega} + e^{-j5\omega} + e^{-j6\omega} + \cdots$$

$0\,1\,0\,0 \quad \underset{\uparrow}{1}\,1\,1\,0 \quad 1\,1\,1\,0 \quad 1$

$w_1(n) = \delta(n) + \delta(n-1) + \delta(n-2), \quad N_1 = 4$

$w_2(n) = \delta(n+3), \quad N_2 = 4 \Rightarrow N_1 = N_2 = N$

$$x(n) = \sum_{k<0} w_2(n-kN) + \sum_{k\geq 0} w_1(n-kN)$$

$$= W_2(n) * \sum_{k<0} \delta(n-kN) + W_1(n) * \sum_{k\geq 0} \delta(n-kN)$$

$$X(\omega) = W_2(\omega) \cdot \sum_{-\infty}^{-1} e^{-jN\omega k} + W_1(\omega) \cdot \sum_{k\geq 0} e^{-jkN\omega}$$

$$\sum_{-\infty}^{-1} e^{-jN\omega k} = \sum_{1}^{\infty} e^{jN\omega k} = \frac{e^{j4\omega}}{1 - e^{j4\omega}}$$

$$\sum_{0}^{\infty} e^{-jN\omega k} = \frac{1}{1 - e^{-j4\omega}}, \quad W_1(\omega) = 1 + e^{-j\omega} + e^{-j2\omega}$$

$$W_2(\omega) = e^{j3\omega}$$

$$\therefore X(\omega) = \frac{e^{j3\omega}\, e^{j4\omega}}{1 - e^{j4\omega}} + \frac{1 + e^{-j\omega} + e^{-j2\omega}}{1 - e^{-j4\omega}}$$

$$= \frac{e^{j7\omega} - e^{j3\omega} + 1 + e^{-j\omega} + e^{-j2\omega} - e^{j4\omega} - e^{j3\omega} - e^{j2\omega}}{1 - e^{j4\omega} - e^{-j4\omega} + 1}$$

$$= \frac{e^{j7\omega} + 1 + e^{-j\omega} - 2e^{j3\omega} + [e^{-j2\omega} - e^{j2\omega}] + e^{j4\omega}}{2 - [e^{j4\omega} + e^{-j4\omega}]}$$

$$= \frac{1 + e^{-j\omega} + e^{j7\omega} - 2e^{j3\omega} - 2j\sin 2\omega + e^{j4\omega}}{2 - 2\cos 4\omega}$$

$Y(\omega) = X(\omega) \cdot H(\omega)$

Q5.7

Saturday, March 26, 2022 2:24 AM

Consider the FIR filter

$$y(n) = x(n) + x(n-4)$$

(a) Compute and sketch its magnitude and phase response.

(b) Compute its response to the input

$$x(n) = \cos\frac{\pi}{2}n + \cos\frac{\pi}{4}n, \qquad -\infty < n < \infty$$

(c) Explain the results obtained in part (b) in terms of the magnitude and phase responses obtained in part (a).

(a) $y(n) = x(n) + x(n-4)$

$$Y(\omega) = X(\omega) + X(\omega)e^{-j4\omega} = X(\omega)\left[1 + e^{-j4\omega}\right]$$

$$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 + e^{-4j\omega}$$

$$= 1 + \cos 4\omega - j\sin 4\omega$$

$$\therefore |H(\omega)| = \sqrt{(1+\cos 4\omega)^2 + (\sin 4\omega)^2}$$

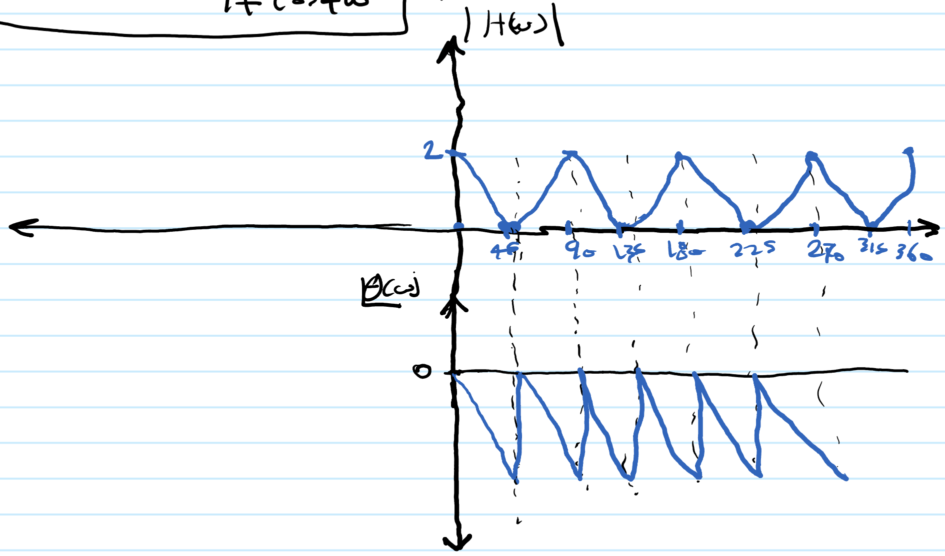
$$= \sqrt{1 + 2\cos 4\omega + \cos^2 4\omega + \sin^2 4\omega}$$

$$= \sqrt{1 + 2\cos 4\omega + 1} = \boxed{\sqrt{2 + 2\cos 4\omega}}$$ #

ω	$H(\omega)$
0	2
90	2
180	2
270	2
360	2

ω	$H(\omega)$
45	0
135	0
225	0
315	0

$$\boxed{\theta(\omega) = \tan^{-1}\frac{-\sin 4\omega}{1 + \cos 4\omega}}$$ #



Saturday, March 26, 2022 2:41 AM

Consider the FIR filter

$$y(n) = x(n) + x(n-4)$$

(a) Compute and sketch its magnitude and phase response.

(b) Compute its response to the input

$$x(n) = \cos\frac{\pi}{2}n + \cos\frac{\pi}{4}n, \qquad -\infty < n < \infty$$

(c) Explain the results obtained in part (b) in terms of the magnitude and phase responses obtained in part (a).

(b) $y(n) = |H(\frac{\pi}{2})| \cos[\frac{\pi}{2}n + \theta(\frac{\pi}{2})] + |H(\frac{\pi}{4})| \cos[\frac{\pi}{4}n + \theta(\frac{\pi}{4})]$

$H(\frac{\pi}{2}) = 2$, $H(\frac{\pi}{4}) = 0$ $\Rightarrow H(\omega) = 1 + e^{-j4\omega}$

$\boxed{\therefore y(n) = 2\cos(\frac{\pi}{2}n)}$ #

(c) the filter response for $\omega = \frac{\pi}{4}$ equals zero

that's why the filter dose not passs $\cos\frac{\pi}{4}n$