4.6 (a)

Monday, February 21, 2022 2:07 PM

Determine and sketch the magnitude and phase spectra of the following periodic signals.

(a) $x(n) = 4\sin\frac{\pi(n-2)}{3}$

$$x(n) = 4\sin\frac{\pi(n-2)}{3} = 4\sin\frac{2\pi(n-2)}{6} \rightarrow N$$

$$\because x(n) = \sum_{k=0}^{N-1} c_k e^{\frac{j2\pi kn}{N}} \Rightarrow \therefore c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}}$$

$$\therefore c_k = \frac{1}{6}\sum_{n=0}^{5} x(n) e^{-\frac{j2\pi kn}{6}} = \frac{4}{6}\sum_{n=0}^{5} \sin\left(\frac{2\pi(n-2)}{6}\right) e^{-\frac{j2\pi kn}{6}}$$

$$= \frac{4}{6}\left[\sin\left(\frac{-4\pi}{6}\right) + \sin\left(\frac{-2\pi}{6}\right)e^{-\frac{j2\pi k}{6}} + \sin(0)e^{-\frac{j4\pi k}{6}} + \sin\left(\frac{2\pi}{6}\right)e^{-\frac{j6\pi k}{6}} + \sin\left(\frac{4\pi}{6}\right)e^{-\frac{j8\pi k}{6}} + \sin\left(\frac{6\pi}{6}\right)e^{-\frac{j10\pi k}{6}}\right]$$

$$= \frac{4}{6}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}e^{\frac{-j\pi k}{3}} + \frac{\sqrt{3}}{2}e^{-j\pi k} + \frac{\sqrt{3}}{2}e^{-j\frac{4\pi k}{3}}\right]$$

$$= \boxed{\frac{1}{\sqrt{3}}\left[-1 - e^{\frac{-j\pi k}{3}} + e^{-j\pi k} + e^{\frac{-j4\pi k}{3}}\right]} \Leftarrow c_k$$

$$\Rightarrow c_0 = \frac{1}{\sqrt{3}}[-1-1+1+1] = \boxed{0} \#$$

$$\Rightarrow c_1 = \frac{1}{\sqrt{3}}\left[-1 - e^{\frac{-j\pi}{3}} + e^{-j\pi} + e^{\frac{-j4\pi}{3}}\right] = \frac{1}{\sqrt{3}}\left[-1 - \left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + (-1) + \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{1}{\sqrt{3}}[-3 + j\sqrt{3}] = -\sqrt{3} + j = \boxed{2\angle 150^\circ} \#$$

$$\Rightarrow c_2 = \frac{1}{\sqrt{3}}\left[-1 - e^{\frac{-j2\pi}{3}} + e^{-j2\pi} + e^{\frac{-j8\pi}{3}}\right] = \frac{1}{\sqrt{3}}\left[\cancel{-1} - \left(\cancel{\frac{-1}{2}} - j\frac{\sqrt{3}}{2}\right) + \cancel{1} + \left(\cancel{\frac{-1}{2}} - j\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{1}{\sqrt{3}}(0) = \boxed{0} \#$$

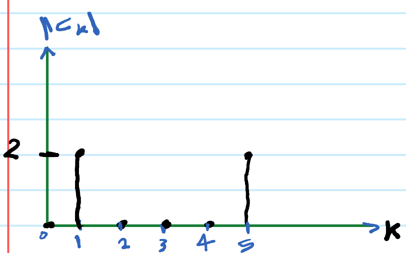
$$\Rightarrow c_3 = \frac{1}{\sqrt{3}}\left[-1 - e^{-j\pi} + e^{-j3\pi} + e^{-j4\pi}\right] = \frac{1}{\sqrt{3}}[-1+1-1+1] = \boxed{0} \#$$

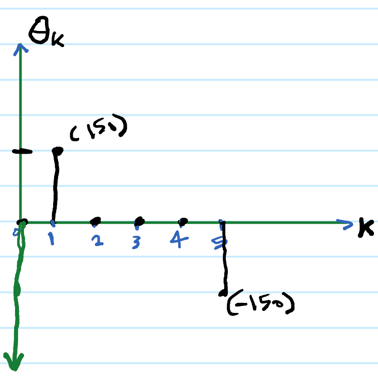
$$\Rightarrow c_4 = \frac{1}{\sqrt{3}}\left[-1 - e^{\frac{-j4\pi}{3}} + e^{-j4\pi} + e^{\frac{-j16\pi}{3}}\right] = \frac{1}{\sqrt{3}}\left[\cancel{-1} - \left(-0.5 + j\cancel{\frac{\sqrt{3}}{2}}\right) + \cancel{1} + \left(\frac{-1}{2} + j\cancel{\frac{\sqrt{3}}{2}}\right)\right]$$

$$= \boxed{0} \#$$

$$\Rightarrow c_5 = \frac{1}{\sqrt{3}}\left[-1 - e^{\frac{-j5\pi}{3}} + e^{-j5\pi} + e^{\frac{-j20\pi}{3}}\right] = \frac{1}{\sqrt{3}}\left[-1 - \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right) - 1 + \left(\frac{-1}{2} - \frac{\sqrt{3}}{2}j\right)\right]$$

$$= \frac{1}{\sqrt{3}}[-3 - \sqrt{3}j] = -\sqrt{3} - j = \boxed{2\angle -150^\circ} \#$$





4.9 (g)

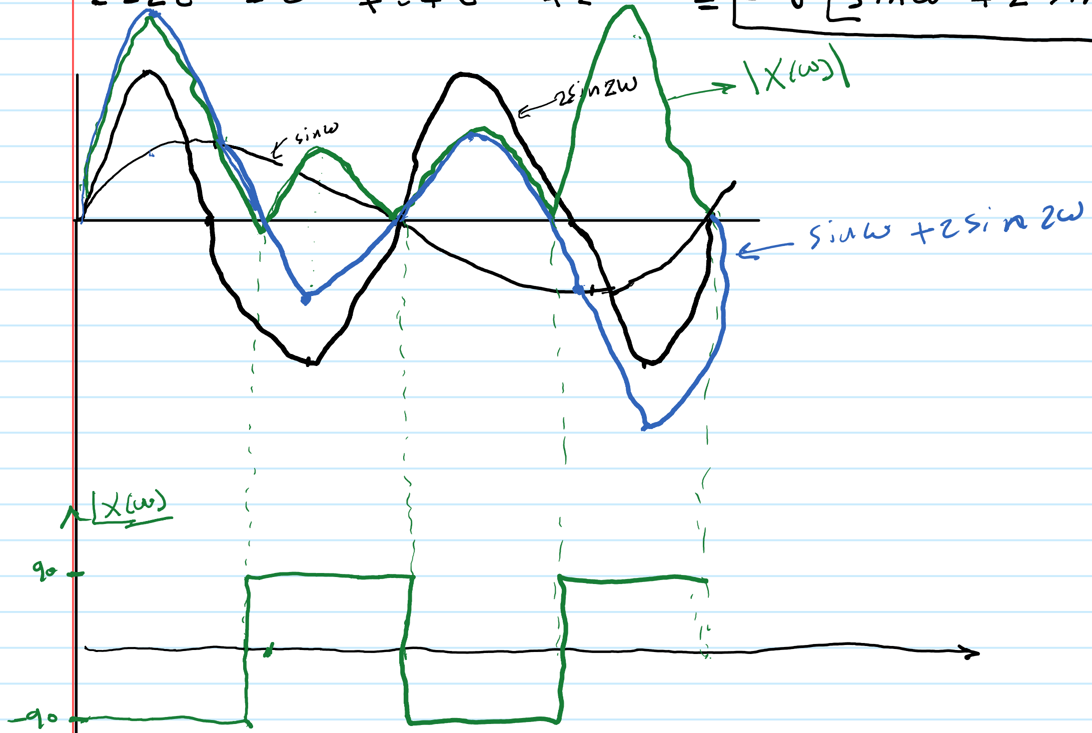
Monday, February 21, 2022 3:27 PM

4.9 Compute the Fourier transform of the following signals.

(g) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} = \sum_{n=-2}^{2} x(n)\, e^{-j\omega n}$$

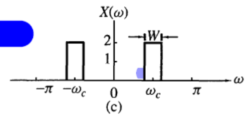
$$= -2e^{+2j\omega} - e^{j\omega} + 0 + e^{-j\omega} + 2e^{-j2\omega} = \boxed{-2j\left[\sin\omega + 2\sin 2\omega\right]}$$



4.12(c)

Monday, February 21, 2022 3:59 PM

Determine the signal $x(n)$ if its Fourier transform is as given in Fig. P4.12.



$$x(n) = \frac{1}{2\pi}\int_{\omega_c - \frac{W}{2}}^{\omega_c + \frac{W}{2}} 2\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\omega_c - \frac{W}{2}}^{-\omega_c + \frac{W}{2}} 2\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{2}{jn} e^{j\omega n}\right]_{\omega_c - \frac{W}{2}}^{\omega_c + \frac{W}{2}} + \frac{1}{2\pi}\left[\frac{2}{jn} e^{j\omega n}\right]_{-\omega_c - \frac{W}{2}}^{-\omega_c + \frac{W}{2}}$$

$$= \frac{1}{j\pi n} \cancel{*} \left[e^{jn(\omega_c + \frac{W}{2})} - e^{jn(\omega_c - \frac{W}{2})} + e^{j(-\omega_c + \frac{W}{2})n} - e^{jn(-\omega_c - \frac{W}{2})} \right]$$

$$\boxed{= \frac{2}{\pi n}\left[\sin\left[\left(\omega_c + \frac{W}{2}\right)n\right] - \sin\left[\left(\omega_c - \frac{W}{2}\right)n\right] \right]}$$

\#\#

4.14

Monday, February 21, 2022 4:10 PM

4.14 Consider the signal

$$x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$$

with Fourier transform $X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$:

(a) $X(0)$

(b) $\angle X(\omega)$

(c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

(d) $X(\pi)$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$

$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

(a) $X(0) = \sum_n x(n) = \boxed{-1}$ #

(b) $\because$ max value of $X(\omega)$ is < 0 $\Rightarrow$ $\therefore$ $\angle X(\omega)$ is π for all ω

4.14

Monday, February 21, 2022 4:38 PM

4.14 Consider the signal

$$x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$$

with Fourier transform $X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$:

(a) $X(0)$

(b) $\angle X(\omega)$

(c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

(d) $X(\pi)$

(e) $\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$

(c) $\because x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n}\, d\omega \Rightarrow x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, d\omega$

$\therefore \int_{-\pi}^{\pi} X(\omega)\, d\omega = 2\pi\, x(0) = 2\pi \cdot -3 = \boxed{-6\pi}$ #

(d) $X(\omega) = \sum_{n} x(n) e^{-j\omega n} \Rightarrow X(\pi) = \sum_{n} x(n) e^{-j\pi n} = \sum_{n} (-1)^n x(n) =$

$\therefore X(\pi) = -1 - 2 - 3 - 2 - 1 = \boxed{-9}$ #

(e) $\because$ parseval's theorem: $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$

$\therefore \int_{-\pi}^{\pi} |X(\omega)|^2 = 2\pi \sum_{n=-\infty}^{\infty} |x(n)|^2 = 2\pi[1+4+9+4+1] = \boxed{38\pi}$ #