Wednesday, February 16, 2022 9:52 AM

3.11 Using long division, determine the inverse z-transform of

$$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

if **(a)** $x(n)$ is causal and **(b)** $x(n)$ is anticausal.

a

$$x(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}} = \frac{z^2+2z}{z^2-2z+1} =$$

$$\begin{array}{r|l}
 & 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} \\
\hline
z^2-2z+1 & \cancel{z^2} + 2z \\
 & \cancel{z^2} - 2z + 1 \\
\hline
 & \cancel{4z} - 1 \\
 & \cancel{4z} - 8 + 4z^{-1} \\
\hline
 & \cancel{7} - 4z^{-1} \\
 & \cancel{7} - 14z^{-1} + 7z^{-2} \\
\hline
 & +10\cancel{z^{-1}} - 7z^{-2} \\
 & 10\cancel{z^{-1}} - 20z^{-2} + 10z^{-3} \\
\hline
 & 13z^{-2} - 10z^{-3}
\end{array}$$

$\therefore x(z) = 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} + \ldots\ldots$

$$\boxed{\therefore x(n) = \{\underset{\uparrow}{1}, 4, 7, 10, \ldots\}}$$ ⋇

(b)

3.11 Using long division, determine the inverse z-transform of

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$

if **(a)** $x(n)$ is causal and **(b)** $x(n)$ is anticausal.

$$x(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$

$$\begin{array}{r|l} & 2z+5z^2+8z^3+11z^4 \\ z^{-2}-2z^{-1}+1 & 2z^{-1}+1 \\ & \cancel{2z^{-1}}-4+2z \\ \hline & 5-2z \\ & 5-10z+5z^2 \\ \hline & \cancel{8z}-5z^2 \\ & \cancel{8z}-16z^2+8z^{-3} \\ \hline & 11z^2-8z^{-3} \end{array}$$

$$\therefore X(z) = 2z+5z^2+8z^3+11z^4+\cdots\cdots$$

$$\boxed{\therefore x(n) = \{\cdots\cdots, 11, 8, 5, 2, \underset{\uparrow}{0}\}}$$

#2

Wednesday, February 16, 2022 11:20 AM

3.12 Determine the causal signal $x(n)$ having the z-transform

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})2}$$

$$x(z) = \frac{z^3}{(z-2)(z-1)^2} \Rightarrow \text{partial fraction}$$

$$\frac{x(z)}{z} = \frac{z^2}{(z-2)(z-1)^2} = \frac{A}{z-2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$z^2 = A(z-1)^2 + B(z-2)(z-1) + C(z-2)$$

at $z=1$

$$1 = C(1-2) \Rightarrow 1 = -C = \boxed{C=-1} \#$$

at $z=2$

$$4 = A(2-1)^2 \Rightarrow \boxed{4 = A} \#$$

at $z=0$

$$0 = A + 2B - 2C \Rightarrow 2B = -A + 2C = -4 - 2 = -6$$

$$\boxed{B = -3} \#$$

$$\frac{X(z)}{z} = \frac{4}{z-2} - \frac{3}{z-1} - \frac{1}{(z-1)^2} \Rightarrow x(z) = \frac{4z}{z-2} - \frac{3z}{z-1} - \frac{z}{(z-1)^2}$$

$$x(n) = 4(2^n u(n)) - 3u(n) - nu(n) = 2^2 z^n u(n) - 3u(n) - nu(n)$$

$$\boxed{x(n) = \left[2^{n+2} - 3 - n\right] u(n)} \#$$

Wednesday, February 16, 2022 11:20 AM

3.16 Determine the convolution of the following pairs of signals by means of the z-transform.

(a) $x_1(n) = (\frac{1}{4})^n u(n-1)$, $x_2(n) = [1 + (\frac{1}{2})^n]u(n)$

(b) $x_1(n) = u(n)$, $x_2(n) = \delta(n) + (\frac{1}{2})^n u(n)$

(c) $x_1(n) = (\frac{1}{2})^n u(n)$, $x_2(n) = \cos \pi n u(n)$

(d) $x_1(n) = nu(n)$, $x_2(n) = 2^n u(n-1)$

(b) $x_1(n) = u(n)$, $x_2(n) = \delta(n) + \left(\frac{1}{2}\right)^n u(n)$

$X_1(z) = \frac{z}{z-1}$, $X_2(z) = 1 + \frac{z}{z-0.5}$

$\therefore Y(z) = X_1(z) \cdot X_2(z) = \frac{z}{z-1} \cdot \left[1 + \frac{z}{z-0.5}\right]$

$\therefore Y(z) = \frac{z}{z-1} + \underbrace{\frac{z^2}{(z-1)(z-0.5)}}_{\text{partial fraction}} \rightarrow Y_2(z)$

$\frac{Y_2(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$

$z = A(z-0.5) + B(z-1)$

$\Rightarrow \boxed{\text{at } z = 0.5}$

$0.5 = B(0.5-1) \Rightarrow 0.5 = -0.5B \Rightarrow \boxed{B = -1}$

$\boxed{\Rightarrow \text{at } z = 1}$

$1 = A\, 0.5 \Rightarrow \boxed{A = \frac{1}{0.5} = 2}$

$\therefore Y_2(z) = \frac{2z}{z-1} - \frac{z}{z-0.5}$

$\therefore Y(z) = \frac{z}{z-1} + \frac{2z}{z-1} - \frac{z}{z-0.5} \Rightarrow y(n) = u(n) + 2u(n) - \left(\frac{1}{2}\right)^n u(n)$

$\therefore y(n) = [3 - (0.5)^n]\, u(n)$

Wednesday, February 16, 2022 11:20 AM

3.16 Determine the convolution of the following pairs of signals by means of the z-transform.

(a) $x_1(n) = (\frac{1}{4})^n u(n-1), \quad x_2(n) = [1 + (\frac{1}{2})^n]u(n)$

(b) $x_1(n) = u(n), \quad x_2(n) = \delta(n) + (\frac{1}{2})^n u(n)$

(c) $x_1(n) = (\frac{1}{2})^n u(n), \quad x_2(n) = \cos \pi n u(n)$

(d) $x_1(n) = nu(n), \quad x_2(n) = 2^n u(n-1)$

(c) $x_1(n) = \left(\frac{1}{2}\right)^n u(n)$, $x_2(n) = \cos \pi n\, u(n)$

$X_1(z) = \frac{z}{z-0.5}$, $X_2(z) = \frac{z^2 - z\cos\pi}{z^2 - 2z\cos\pi + 1} = \frac{z^2+z}{z^2+2z+1} =$

$X_2(z) = \frac{z\cancel{(z+1)}}{(z+1)^{\cancel{2}}}$, $Y(z) = X_1(z) \cdot X_2(z) = \frac{z^2}{(z-0.5)(z+1)}$ ⇐ partial fraction

$\frac{Y(z)}{z} = \frac{z}{(z-0.5)(z+1)} = \frac{A}{z-0.5} + \frac{B}{z+1}$

$z = A(z+1) + B(z-0.5)$

$\Rightarrow$ at $z = -1$
$-1 = -1.5B \Rightarrow B = \frac{1}{1.5} = 2/3$

$\Rightarrow$ at $z = 0.5$
$0.5 = 1.5A \Rightarrow A = \frac{0.5}{1.5} = \frac{1}{3}$

$Y(z) = \frac{1}{3}\frac{z}{z-0.5} + \frac{2}{3}\frac{z}{z+1}$

$\therefore y(n) = \frac{1}{3}(0.5)^n u(n) + \frac{2}{3}(-1)^n u(n) = \boxed{\left[\frac{1}{3}(0.5)^n + \frac{2}{3}(-1)^n\right] u(n)}$ #

Wednesday, February 16, 2022 11:20 AM

3.37 Compute the response of the system

$$y(n) = 0.7y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2)$$

to the input $x(n) = nu(n)$. Is the system stable?

$$y(n) = 0.7\,y(n-1) - 0.12\,y(n-2) + x(n-1) + x(n-2)$$

$$y(n) - 0.7\,y(n-1) + 0.12\,y(n-2) = x(n-1) + x(n-2)$$

$$Y(z) - 0.7z^{-1}Y(z) + 0.12z^{-2}Y(z) = z^{-1}X(z) + z^{-2}X(z)$$

$$Y(z)\left[1 - 0.7z^{-1} + 0.12z^{-2}\right] = X(z) \cdot \left[z^{-1} + z^{-2}\right]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{z^{-1} + z^{-2}}{1 - 0.7z^{-1} + 0.12z^{-2}} = \frac{z+1}{z^2 - 0.7z + 0.12}$$

$$\therefore Y(z) = H(z) \cdot X(z)$$

$$\because x(n) = nu(n) \Rightarrow X(z) = \frac{z}{(z-1)^2}$$

$$\therefore Y(z) = \frac{z+1}{z^2 - 0.7z + 0.12} \cdot \frac{z}{(z-1)^2} = \frac{z(z+1)}{(z-1)^2(z-0.3)(z-0.4)} \Leftarrow \text{partial Fraction}$$

$$\frac{Y(z)}{z} = \frac{z+1}{(z-1)^2(z-0.3)(z-0.4)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{(z-0.3)} + \frac{D}{(z-0.4)}$$

$$z+1 = A(z-1)(z-0.3)(z-0.4) + B(z-0.3)(z-0.4) + C(z-1)^2(z-0.4) + D(z-1)^2(z-0.3)$$

$\Rightarrow$ at $z = 1$

$$2 = B(1-0.3)(1-0.4) = B \times 0.7 \times 0.6 = 2 \Rightarrow \boxed{B = \frac{2}{0.7 \times 0.6} = \frac{100}{21}}$$

$\Rightarrow$ at $z = 0.3$

$$1.3 = C(0.3-1)^2(0.3-0.4) = C(-0.7)^2 \cdot -0.1 = -0.049C = 1.3$$

$$\boxed{C = \frac{1.3}{-0.049} = \frac{-1300}{49}}$$

$\Rightarrow$ at $z = 0.4$

$$1.4 = D(0.4-1)^2(0.4-0.3) = D \times 0.36 \times 0.1 = 0.036D = 1.4$$

$$D = \frac{1.4}{0.036} = \frac{350}{9}$$

$\Rightarrow$ at $z = 0$

$$1 = A(-1)(-0.3)(-0.4) + B(-0.3)(-0.4) + C(-1)^2(-0.4) + D(-1)^2(-0.3)$$

$$1 = -0.12A + 0.12B - 0.4C - 0.3D$$

$$-0.12A = 1 - 0.12 \times \frac{100}{21} + 0.4 \times \frac{-1300}{49} + 0.3 \times \frac{350}{9} = \frac{218}{147}$$

$$A = \frac{218}{147} \div -0.12 = -\frac{5450}{441}$$

$$Y(z) = -\frac{5450}{441}\frac{z}{z-1} + \frac{100}{21}\frac{z}{(z-1)^2} - \frac{1300}{49}\frac{z}{z-0.3} + \frac{350}{9}\frac{z}{z-0.4}$$

$$\boxed{y(n) = -\frac{5450}{441}u(n) + \frac{100}{21}nu(n) - \frac{1300}{49}(0.3)^n u(n) + \frac{350}{9}(0.4)^n u(n)}$$

$\because$ poles of $H(z)$ is inside the unit circle $\Rightarrow \therefore$ System is stable