



College of Engineering and Physics
Electrical Engineering Department

EE562 - Digital Signal Processing I

Second Semester (212)

Homework Assignment 5

Solved By: Mahmoud Yassin

ID: 202113650

Supervised by: Dr. Wail A. Mousa

4.6 (a)

Monday, February 21, 2022 2:07 PM

Determine and sketch the magnitude and phase spectra of the following periodic signals.

$$(a) x(n) = 4 \sin \frac{\pi(n-2)}{3}$$

$$\begin{aligned} x(n) &= 4 \sin \frac{\pi(n-2)}{3} = 4 \sin \frac{2\pi(n-2)}{6} \xrightarrow{N} \\ \therefore x(n) &= \sum_{k=0}^{N-1} c_k e^{\frac{j2\pi kn}{N}} \Rightarrow c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \\ \therefore c_k &= \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-\frac{j2\pi kn}{6}} = \frac{4}{6} \sum_{n=0}^{5} \sin\left(\frac{\pi(n-2)}{3}\right) e^{-\frac{j2\pi kn}{6}} \\ &= \frac{4}{6} \left[\sin\left(-\frac{4\pi}{6}\right) + \sin\left(-\frac{2\pi}{6}\right) e^{-\frac{j2\pi k}{6}} + \sin(0) e^{-\frac{j4\pi k}{6}} + \sin\left(\frac{2\pi}{6}\right) e^{-\frac{j6\pi k}{6}} \right. \\ &\quad \left. + \sin\left(\frac{4\pi}{6}\right) e^{-\frac{j8\pi k}{6}} + \sin\left(\frac{6\pi}{6}\right) e^{-\frac{j10\pi k}{6}} \right] \\ &= \frac{4}{6} \left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} e^{\frac{j2\pi k}{3}} + \frac{\sqrt{3}}{2} e^{-j\pi k} + \frac{\sqrt{3}}{2} e^{-\frac{j4\pi k}{3}} \right] \\ &= \boxed{\frac{1}{\sqrt{3}} \left[-1 - e^{\frac{-j2\pi k}{3}} + e^{-j\pi k} + e^{-\frac{j4\pi k}{3}} \right]} \end{aligned}$$

$$\Rightarrow c_0 = \frac{1}{\sqrt{3}} [-1 - 1 + 1 + 1] = \boxed{0} \#$$

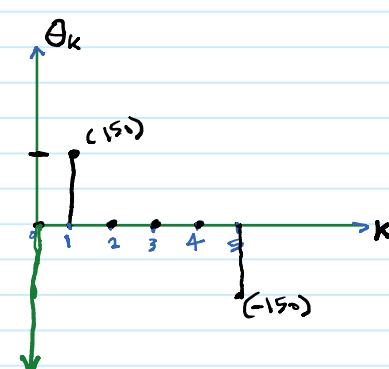
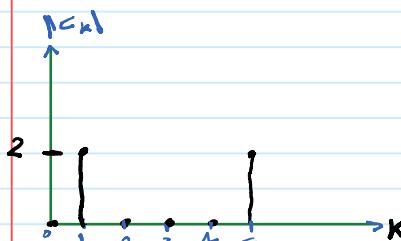
$$\Rightarrow c_1 = \frac{1}{\sqrt{3}} \left[-1 - e^{-\frac{j2\pi}{3}} + e^{-j\pi} + e^{-\frac{j4\pi}{3}} \right] = \frac{1}{\sqrt{3}} \left[-1 - \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) + (-1) + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right] \\ = \frac{1}{\sqrt{3}} [-3 + j\sqrt{3}] = -\sqrt{3} + j = \boxed{2 \angle 150^\circ} \#$$

$$\Rightarrow c_2 = \frac{1}{\sqrt{3}} \left[-1 - e^{-\frac{j4\pi}{3}} + e^{-j2\pi} + e^{-\frac{j8\pi}{3}} \right] = \frac{1}{\sqrt{3}} \left[-1 - \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) + 1 + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right]$$

$$\Rightarrow c_3 = \frac{1}{\sqrt{3}} \left[-1 - e^{-j\pi} + e^{-j3\pi} + e^{-j5\pi} \right] = \frac{1}{\sqrt{3}} [-1 + 1 - 1 + 1] = \boxed{0} \#$$

$$\Rightarrow c_4 = \frac{1}{\sqrt{3}} \left[-1 - e^{-\frac{j6\pi}{3}} + e^{-j4\pi} + e^{-\frac{j10\pi}{3}} \right] = \frac{1}{\sqrt{3}} \left[-1 - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) + 1 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right]$$

$$\Rightarrow c_5 = \frac{1}{\sqrt{3}} \left[-1 - e^{-\frac{j14\pi}{3}} + e^{-j5\pi} + e^{-\frac{j20\pi}{3}} \right] = \frac{1}{\sqrt{3}} \left[-1 - \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) - 1 + \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) \right] \\ = \frac{1}{\sqrt{3}} [-3 - \sqrt{3}j] = -\sqrt{3} - j = \boxed{2 \angle -150^\circ} \#$$



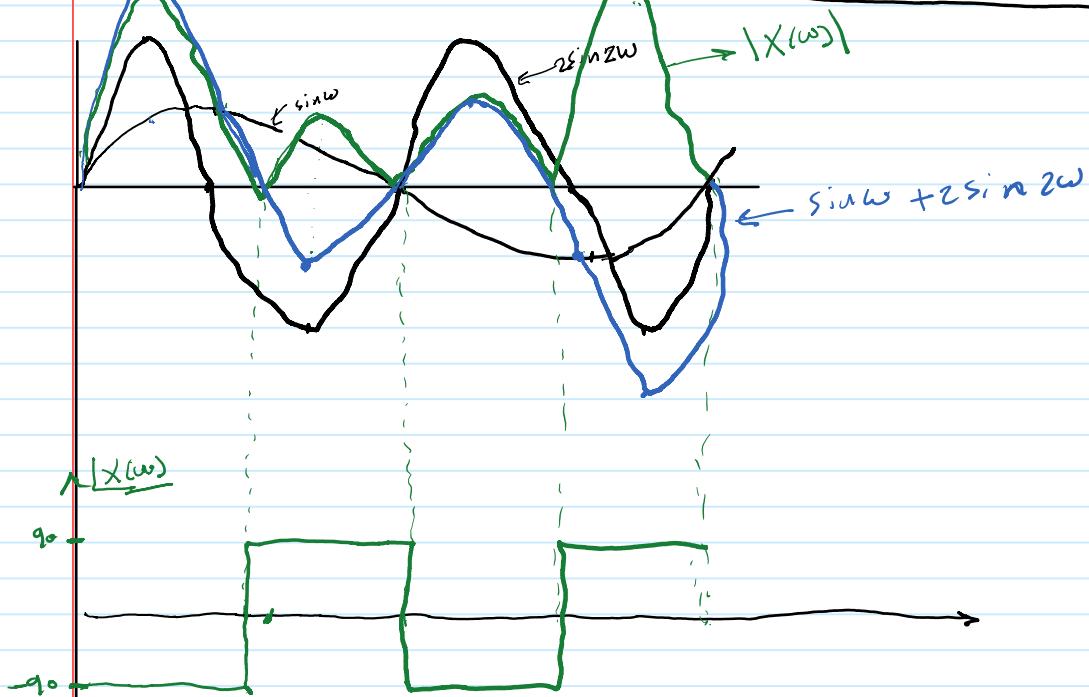
4.9 (g)

Monday, February 21, 2022 3:27 PM

4.9 Compute the Fourier transform of the following signals.

$$(g) \quad x(n) = \{-2, -1, 0, 1, 2\}$$

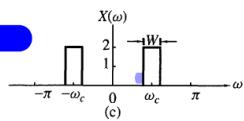
$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-2}^{2} x(n) e^{-j\omega n} \\ = -2e^{+j\omega} - e^{j\omega} + \dots + e^{-j\omega} + 2e^{-j2\omega} = -2j [\sin\omega + 2\sin 2\omega]$$



4.12(c)

Monday, February 21, 2022 3:59 PM

Determine the signal $x(n)$ if its Fourier transform is as given in Fig. P4.12.



$$\begin{aligned}
 X(n) &= \frac{1}{2\pi} \int_{-\omega_c - \frac{W}{2}}^{\omega_c + \frac{W}{2}} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{-\omega_c - \frac{W}{2}}^{-\omega_c + \frac{W}{2}} 2e^{j\omega n} d\omega \\
 &= \frac{1}{2\pi} \left[\frac{2}{jn} e^{jn(\omega_c + \frac{W}{2})} \right]_{\omega_c - \frac{W}{2}}^{\omega_c + \frac{W}{2}} + \frac{1}{2\pi} \left[\frac{2}{jn} e^{jn(\omega_c - \frac{W}{2})} \right]_{-\omega_c - \frac{W}{2}}^{-\omega_c + \frac{W}{2}} \\
 &= \frac{1}{jn} \left[e^{jn(\omega_c + \frac{W}{2})} - e^{jn(\omega_c - \frac{W}{2})} + e^{j(-\omega_c + \frac{W}{2})n} - e^{j(-\omega_c - \frac{W}{2})n} \right] \\
 &= \frac{2}{\pi n} \left[\sin\left[(\omega_c + \frac{W}{2})n\right] - \sin\left[(\omega_c - \frac{W}{2})n\right] \right]
 \end{aligned}$$

4.14

Monday, February 21, 2022 4:10 PM

- 4.14 Consider the signal

$$x(n) = \{-1, 2, -3, 2, -1\}$$

with Fourier transform $X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$:

- (a) $X(0)$
- (b) $\angle X(\omega)$
- (c) $\int_{-\pi}^{\pi} X(\omega) d\omega$
- (d) $X(\pi)$
- (e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

a) $X(\omega) = \sum_n x(n) = \boxed{-1} \neq$

b) $\because \max \text{ value of } X(\omega) \left. \right\} \Rightarrow \therefore X(\omega) \text{ is } \pi \text{ for all } \omega$
 $\text{is } < 0 \left. \right\}$

4.14

Monday, February 21, 2022 4:38 PM

4.14 Consider the signal

$$x(n) = \{-1, 2, -3, 2, -1\}$$

with Fourier transform $X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$:

- (a) $X(0)$
- (b) $\angle X(\omega)$
- (c) $\int_{-\pi}^{\pi} X(\omega) d\omega$
- (d) $X(\pi)$
- (e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

① $\therefore X(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega \Rightarrow X(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) d\omega$

$$\therefore \int_{-\pi}^{\pi} X(\omega) d\omega = 2\pi X(\omega) = 2\pi \star -3 = \boxed{-6\pi} \times$$

② $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \Rightarrow X(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\pi n} = \sum_{n=-\infty}^{\infty} (-1)^n x(n) =$
 $\therefore X(\pi) = -1 - 2 - 3 - 2 - 1 = \boxed{-9} \times$

③ $\therefore \text{Parseval's theorem: } \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

$$\therefore \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x(n)|^2 = 2\pi [1 + 4 + 9 + 4 + 1] = \boxed{38\pi} \times$$