$$\boxed{3.2}$$

(a) $x(n) = (1+n)\, u(n)$

$$X(z) = \sum_n x(n)\, z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (1+n)\, z^{-n} = \underbrace{\sum_0^{\infty} z^{-n}}_{\textcircled{1}} + \underbrace{\sum_0^{\infty} n\, z^{-n}}_{\textcircled{2}}$$
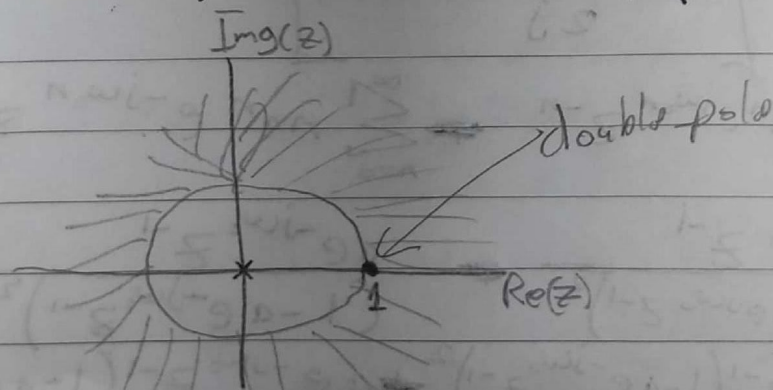
$$\textcircled{1} \longrightarrow \sum_0^{\infty} z^{-n} = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

$$\textcircled{2} \longrightarrow \sum_0^{\infty} n\, z^{-n} = \frac{z^{-1}}{(1-z^{-1})^2}$$

$$\therefore X(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{(1-z^{-1})^2}$$

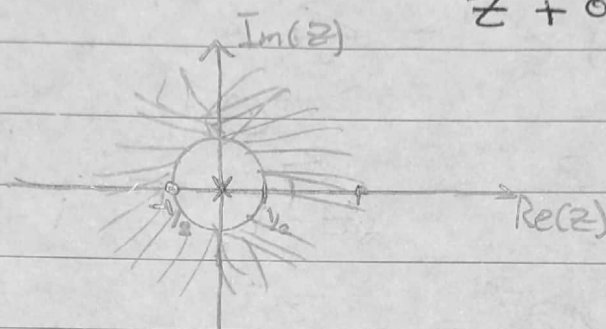$$= \frac{1-z^{-1}}{(1-z^{-1})^2} + \frac{z^{-1}}{(1-z^{-1})^2}$$

$$= \frac{1}{(1-z^{-1})^2} = \frac{z^2}{(z-1)(z-1)}, \quad |z| > 1$$

⑥ $x(n) = (-1)^n 2^{-n} u(n) = (-1)^n (2^{-1})^n u(n)$

$$= (-2^{-1})^n u(n)$$

$$= (-0.5)^n u(n)$$

$$\therefore X(z) = \sum_{n=0}^{\infty} (-0.5)^n z^{-n} = \frac{z}{z+0.5}, \quad |z| > \frac{1}{2}$$

ⓓ $x(n) = (na^n \sin \omega_0 n) u(n)$

$$\therefore \quad n a^n u(n) \longrightarrow \frac{a z^{-1}}{(1-az^{-1})^2}$$

$$\therefore X(z) = \sum_{n=0}^{\infty} n a^n \sin \omega_0 n \; z^{-n}$$

$$= \sum_{n=0}^{\infty} n a^n z^{-n} \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \right]$$

$$= \frac{1}{2j} \left[\sum_{n=0}^{\infty} n a^n e^{j\omega_0 n} z^{-n} - \sum_{n=0}^{\infty} n a^n e^{-j\omega_0 n} z^{-n} \right]$$

$$= \frac{1}{2j} \left[\frac{a e^{j\omega_0} z^{-1}}{(1-a e^{j\omega_0} z^{-1})^2} - \frac{a e^{-j\omega_0} z^{-1}}{(1-a e^{-j\omega_0} z^{-1})^2} \right]$$

$$= \frac{1}{2j} \left[\frac{a e^{j\omega_0} z^{-1}(1-a e^{-j\omega_0} z^{-1})^2 - a e^{-j\omega_0} z^{-1}(1-a e^{j\omega_0} z^{-1})^2}{(1-a e^{-j\omega_0} z^{-1} - a e^{j\omega_0} z^{-1} + a^2 z^{-2})^2} \right]$$

$$= \frac{1}{2j} \left[\frac{1}{\left(1 - 2az^{-1}\left[\frac{e^{j\omega_0} + e^{-j\omega_0}}{2}\right] + a^2 z^{-2}\right)^2} \right]$$

$$= \frac{1}{2j} \left[\frac{1}{\left(1 - 2az^{-1}\cos\omega_0 + a^2 z^{-2}\right)^2} \right]$$

$$= \frac{1}{2j} \frac{\left(ae^{j\omega_0}z^{-1} - 2a^2 z^{-2} + a^3 e^{-j\omega_0} z^{-3}\right) - \left[ae^{-j\omega_0}z^{-1} - 2a^2 z^{-2} + a^3 e^{j\omega_0} z^{-3}\right]}{}$$

$$= \frac{1}{2j} \frac{az^{-1}\left(e^{j\omega_0} - e^{-j\omega_0}\right) + a^3 z^{-3}\left(e^{-j\omega_0} - e^{j\omega_0}\right)}{}$$

$$= \frac{az^{-1}\sin\omega_0 - a^3 z^{-3}\sin\omega_0}{\left(1 - 2az^{-1}\cos\omega_0 + a^2 z^{-2}\right)^2}$$

$$= \boxed{\frac{\left[az^{-1} - a^3 z^{-3}\right]\sin\omega_0}{\left(1 - 2a\cos\omega_0 z^{-1} + a^2 z^{-2}\right)^2}} \quad \#$$

$\boxed{3.3}$

(a) $X_1(n) = \begin{cases} (1/3)^n, & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} & n < 0 \end{cases}$

$X_1(n) = \left(\frac{1}{3}\right)^n u(n) + \left(\frac{1}{2}\right)^{-n} u(-n-1)$

$\because a^n u(n) \longleftrightarrow \frac{z}{z-a}$

$\because -a^n u(-n-1) \longleftrightarrow \frac{z}{z-a}$

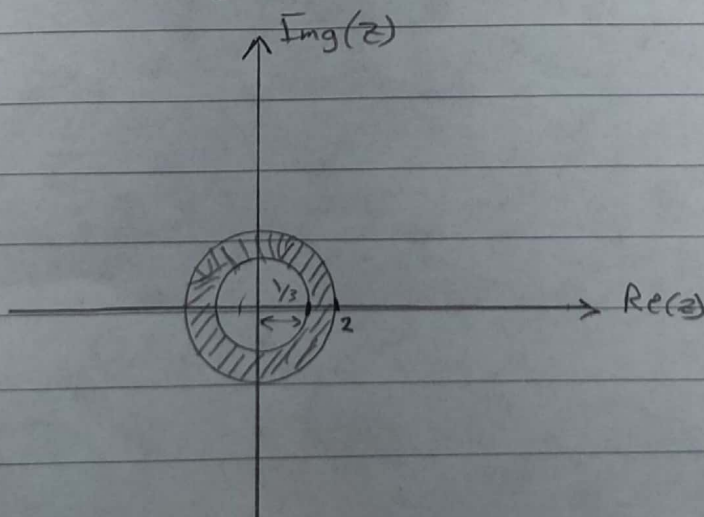$\therefore X_1(z) = \frac{z}{z-1/3} - \frac{z}{z-2}$

$\quad = \frac{z^2 - 2z - z^2 + \frac{1}{3}z}{(z-1/3)(z-2)}$

$\quad = \frac{-5/3\, z}{z^2 - \frac{1}{3}z - 2z + 2/3}$

$\quad = \boxed{\frac{-5/3\, z}{z^2 - \frac{7}{3}z + 2/3}} \quad \#$

$ROC: \frac{1}{3} < |z| < 2$

④ $x_4(n) = x_1(-n)$

$$X_4(z) = \sum_{n=-\infty}^{\infty} x_1(-n) \, z^{-n}$$

$$= \sum_{m=-\infty}^{\infty} x_1(m) \, z^{m} = X_1(z^{-1})$$

$$= \frac{-5/3 \, z^{-1}}{(z^{-1} - 1/3)(z^{-1} - 2)} = \boxed{\frac{-5/3 \, z}{(1 - 1/3 z)(1 - 2z)}} \, \#$$

Roc: $\dfrac{1}{2} < |z| < 3$