P 2.6

(a) $y(n) = x(n^2)$

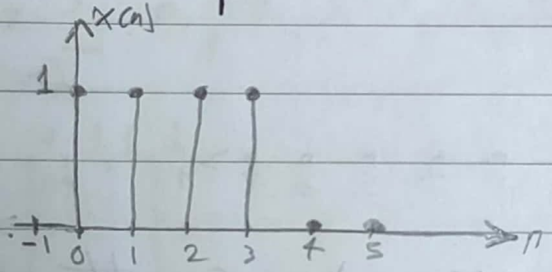
$x(n-k) \rightarrow y_1(n,k) = x((n-k)^2) = x(n^2 - 2nk + k^2)$

$y(n-k) = x(n^2 - k)$

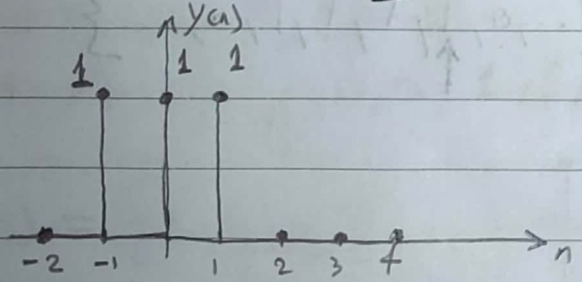
$\therefore y_1(n,k) \neq y(n-k)$

$\therefore$ The system is time Variant

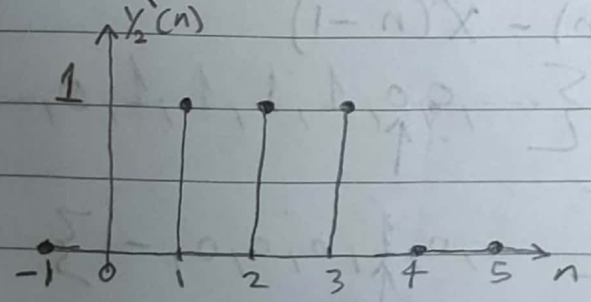
(b) 1 $x(n) = \{\ldots, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$



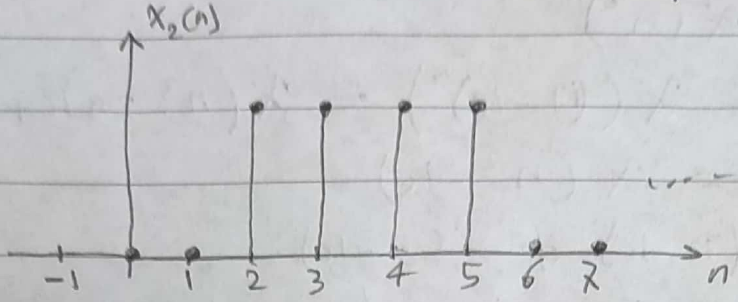
2 $y(n) = x(n^2) = \{\ldots, 0, 1, \underset{\uparrow}{1}, 1, 0, \ldots\}$



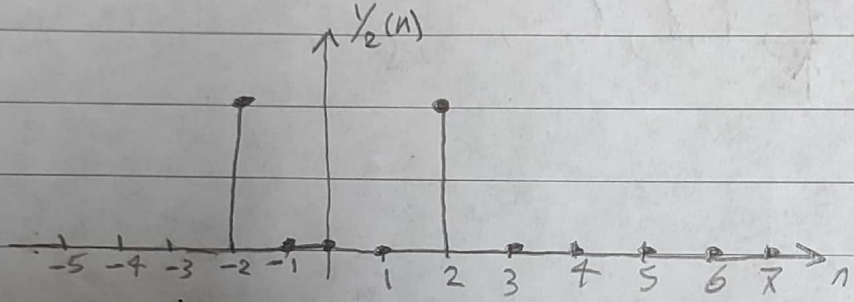
3 $y_2'(n) = y(n-2)$



$\boxed{4}$ $x_2(n) = x(n-2) = \{\ldots, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \ldots\}$

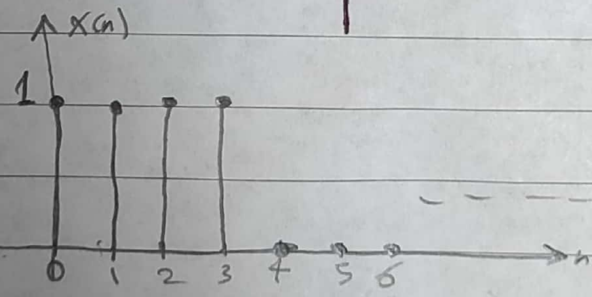


$\boxed{5}$ $y_2(n) = T[x_2(n)] = \{\ldots, 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0, \ldots\}$



$\boxed{6}$ $y_2(n) \neq y(n-2)$, so the system is time varyi

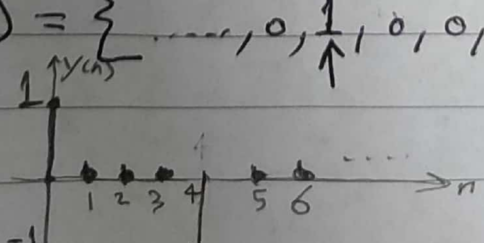
Ⓒ $\boxed{1}$ $x(n) = \{\ldots, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$



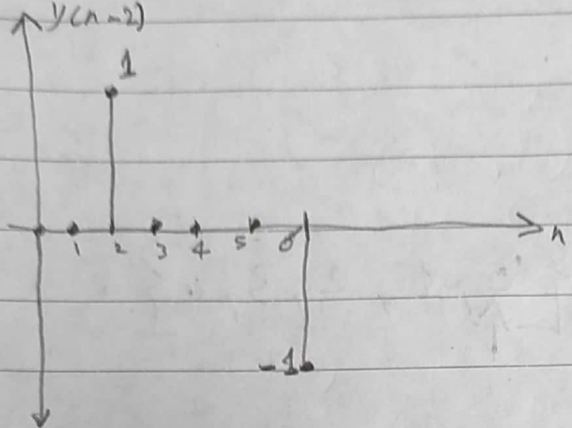
$\boxed{2}$ $y(n) = x(n) - x(n-1)$

$x(n-1) = \{\ldots, 0, \underset{\uparrow}{0}, 1, 1, 1, 1, 0, \ldots\}$

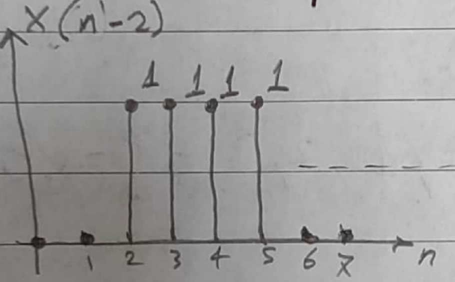
$y(n) = \{\ldots, 0, \underset{\uparrow}{1}, 0, 0, 0, -1\}$



3 $y(n-2) = \{\ldots,0,\underset{\uparrow}{0},0,1,0,0,0,-1\}$



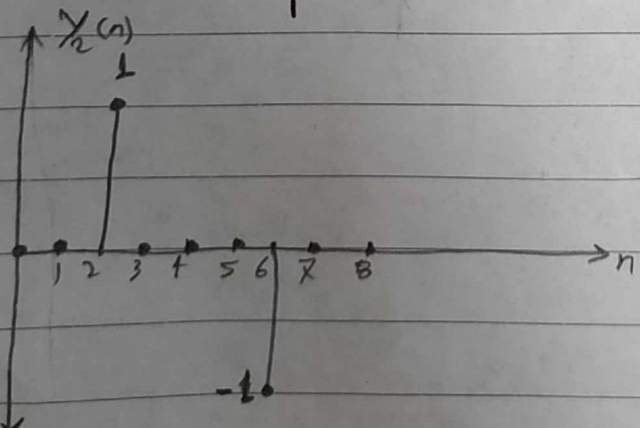
4 $x(n-2) = \{0,\underset{\uparrow}{0},0,1,1,1,1,0,\ldots\} = x_2(n)$



5 $y_2(n) = x_2(n) - x_2(n-1)$

$x_2(n-1) = \{\ldots,\underset{\uparrow}{0},0,0,1,1,1,1,0,\ldots\}$

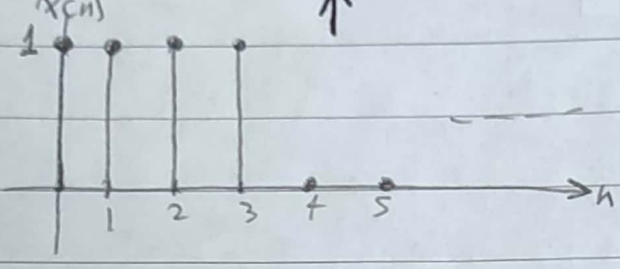
$y_2(n) = \{\ldots,\underset{\uparrow}{0},0,1,0,0,0,-1,0,\ldots\}$



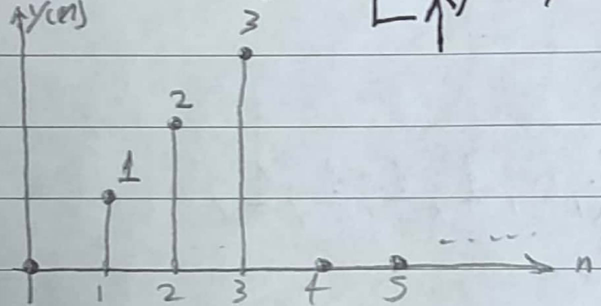
6 $\therefore y_2(n) = y(n-2) \therefore$ system is time invariant

d

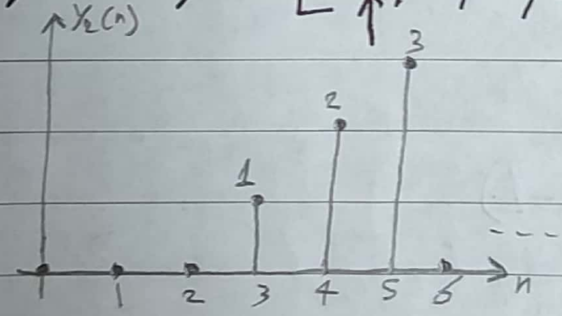
1. $x(n) = \{0, \underset{\uparrow}{1}, 1, 1, 1, 0\}$



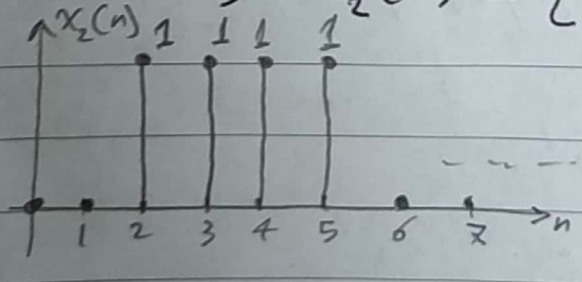
2. $y(n) = n\,x(n) = \{\underset{\uparrow}{0}, 1, 2, 3, 0\}$



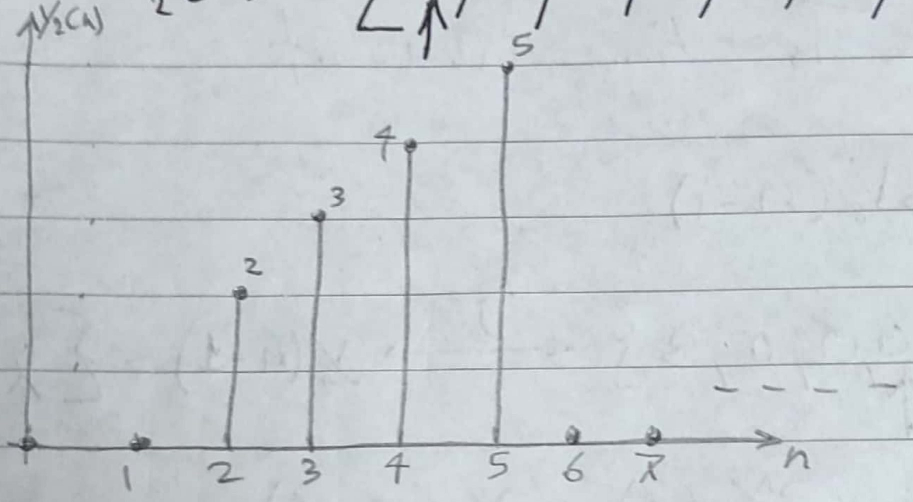
3. $y(n-2) = \{\underset{\uparrow}{0}, 0, 0, 1, 2, 3, 0\} = y_2(n)$



4. $x(n-2) = x_2(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0\}$



$\boxed{5}$ $y_2(n) = n\,x_2(n) = \{\underset{\uparrow}{0}, 0, 2, 3, 4, 5, 0\}$



$\boxed{6}$ $\because y_2(n) \neq y(n-2)$, $\therefore$ system is time variant

$\boxed{P2.10}$

$\cancel{a_1 T[x_1(n)] + a_2 T[x_2(n)] + a_3 T[x_3(n)]}$

$\cancel{\neq a_1\{\underset{\uparrow}{0}, 1, 2\} + a_2\{0, 1, 0, 2\} + a_3\{1, \underset{\uparrow}{2}}$

$\cancel{T[a_1 x_1(n) + a_2 x_2(n) + a_3 x_3(n)]}$

P[2.10]

∴ system is time invariant

$x_2(n-1) \Rightarrow y_2(n-1)$

$x_2(n-1) = \{0,0,0,3\} \xleftrightarrow{T} y_2(n-1) = \{0,0,1,0,2\}$

→ we can say $3x_3(n) = x_2(n-1)$

but, $3y_3(n) = \{3,6,3\}$

so $3y_3(n) \neq y_2(n-1)$

$\boxed{\text{∴ system is non linear.}}$

$\boxed{\text{P2.11}}$ ∴ system is linear

$x_1(n) + x_2(n) \xleftrightarrow{T} y_1(n) + y_2(n)$

$\{0,1,0\} \xleftrightarrow{T} \{0,3,-1,2,1\}$ ①

now suppose system is time invariant and make shift

$\{0,0,1\} \xleftrightarrow{T} \{0,0,3,-1,2,1\}$ ②

∵ system is linear ∴ ① + ②

$\{0,1,1\} \xleftrightarrow{T} \{0,3,2,1,3,1\}$

but:

$x_3(n) = \{0,1,1\} \xleftrightarrow{T} y_3(n) = \{1,2,1\}$

$\boxed{\text{∴ system is time varying}}$

P2.18

$$x(n) = \{\underset{\uparrow}{0}, 1/3, 2/3, 1, 4/3, 5/3, 2\}$$

$$h(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$$

1

	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

$$\therefore y(n) = x(n) * h(n) =$$
$$= \{0, 1/3, \underset{\uparrow}{1}, 2, \tfrac{10}{3}, 5, \tfrac{20}{3}, 6, 5, \tfrac{11}{3}, 2\}$$

2

$$x(n) = \frac{1}{3} n\,[u(n) - u(n-7)]$$
$$h(n) = u(n+2) - u(n-3)$$
$$y(n) = x(n) * h(n)$$
$$x(n) = \frac{1}{3}\delta(n-1) + 2/3\,\delta(n-2) + \delta(n-3) + 4/3\,\delta(n-4) + \frac{5}{3}\delta(n-5) + 2\,\delta(n-6)$$
$$h(n) = \delta(n+2) + \delta(n-2) + \delta(n) + \delta(n-1) + \delta(n-2)$$
$$y(n) = \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) + \frac{20}{3}\delta(n-4) + 6\delta(n-5) + 5\delta(n-6) + \frac{11}{3}\delta(n-7) + 2\delta(n-8)$$

$\boxed{P\,2.33}$

(a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

$y(0) = 0.6\,\underbrace{y(-1)}_{0} - 0.08\,\underbrace{y(-2)}_{0} + \underbrace{\delta(0)}_{1}$

$\boxed{y(0) = 1}$

$y(1) = 0.6\,\underbrace{y(0)}_{1} - 0.08\,\underbrace{y(-1)}_{0} + \underbrace{\delta(1)}_{0}$

$\boxed{y(1) = 0.6}$

$y(n) - 0.6\,y(n-1) + 0.08\,y(n-2) = x(n)$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda_1 = 0.4 \quad , \lambda_2 = 0.2$

$\therefore y_h(n) = C_1 (0.4)^n + C_2 (0.2)^n$

$y(0) = C_1 + C_2 = 1$

$y(1) = 0.4C_1 + 0.2C_2 = 0.6$

$C_1 = 1 - C_2 \rightarrow 0.4(1 - C_2) + 0.2C_2 = 0$

$0.4 - 0.4C_2 + 0.2C_2 = 0.6$

$-0.2C_2 = 0.2 \Rightarrow \boxed{C_2 = -1}$

$\therefore C_1 = 1 - C_2 = 1 - (-1) = \boxed{2}$

$$\boxed{\therefore h(n) = \left[2(0.4)^n - (0.2)^n\right] u(n)}$$

$$s(n) = \sum_{k=0}^{n} h(n-k) \quad , n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\,(0.4)^{n-k} - (0.2)^{n-k} \right]$$

2.35

(a) $h(n) = h_1(n) * \left[h_2(n) - h_3(n) * h_4(n) \right]$

(b) $h_3(n) * h_4(n) = (n+1-2)\, u(n-2)$
$= (n-1)\, u(n-2)$

$h_2(n) - h_3(n) * h_4(n) = (n+1)\, u(n) - (n-1)\, u(n-2)$
$=$

$= \delta(n) + 2\delta(n-1) + (n+1)\, u(n-2) - (n-1)\, u(n-2)$

$= \delta(n) + 2\delta(n-1) + (\cancel{n}+1-\cancel{n}+1)\, u(n-2)$

$= \delta(n) + 2\delta(n-1) + 2u(n-2)$

$= \delta(n) + 2u(n-1) = \delta(n) + \delta(n) - \delta(n) + 2u(n-1)$

$= 2\delta(n) + 2u(n-1) - \delta(n) = \boxed{2u(n) - \delta(n)}$

$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$

$h_1(n) * \left[h_2(n) - h_3(n) * h_4(n) \right] = h(n)$

$h(n) = \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) \right] * \left[2u(n) - \delta(n) \right]$

$= u(n) - \frac{1}{2}\delta(n) + \frac{1}{2}u(n-1) - \frac{1}{4}\delta(n-1) + u(n-2) - \frac{1}{2}\delta(n-2)$

$= \delta(n) + \delta(n-1) + u(n-2) - \frac{1}{2}\delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{2}u(n-2) - \frac{1}{4}\delta(n-1)$

$+ u(n-2) - \frac{1}{2}\delta(n-2) = \boxed{\frac{5}{2}u(n-2) + \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) - \frac{1}{2}\delta(n-2)}$

$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

(c) $h(n) = \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

$\therefore y(n) = x(n) * h(n) = \frac{1}{2}\delta(n+2) + \frac{5}{4}\delta(n+1) + 2\delta(n) + \frac{5}{2}u(n-1)$
$+ \frac{3}{2}\delta(n-1) + \frac{15}{4}\delta(n-2) + 6\delta(n-3) + \frac{15}{2}u(n-4)$
$- 2\delta(n-3) - 5\delta(n-4) - 6\delta(n-5) - 10\,u(n-6)$

$y(n) = -6\delta(n-5) - 5\delta(n-4) + 4\delta(n-3) + \frac{15}{4}\delta(n-2)$
$+ \frac{3}{2}\delta(n-1) + 2\delta(n) + \frac{5}{4}\delta(n+1) + \frac{1}{2}\delta(n+2)$
$+ \frac{5}{2}u(n-1) + \frac{15}{2}u(n-4) - 10\,u(n-6)$

$\boxed{P.2.46}$

(b) $y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$