



College of Engineering and Physics  
**Electrical Engineering Department**

---

EE562 - Digital Signal Processing I

Second Semester (212)

## Homework Assignment 2

Solved By: Mahmoud Yassin

ID: 202113650

Supervised by: Dr. Wail A. Mousa

P 2.6

(a)  $y(n) = x(n^2)$

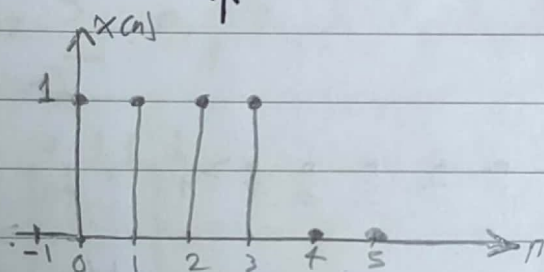
$x(n-k) \rightarrow y_1(n, k) = x((n-k)^2) = x(n^2 - 2nk + k^2)$

$y(n-k) = x(n^2 - k)$

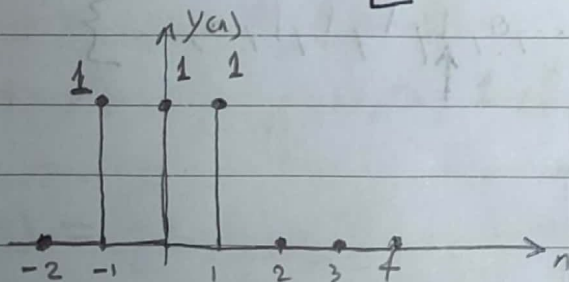
$\therefore y_1(n, k) \neq y(n-k)$

$\therefore$  The system is time variant

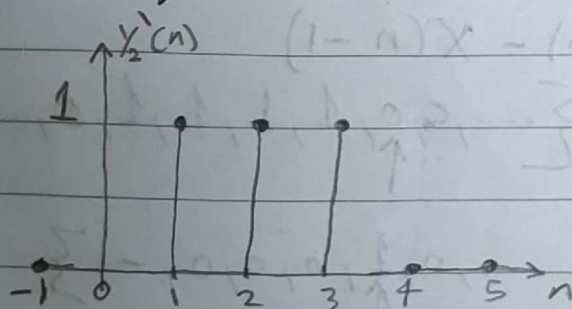
(b) 1  $x(n) = \{ \dots, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \dots \}$



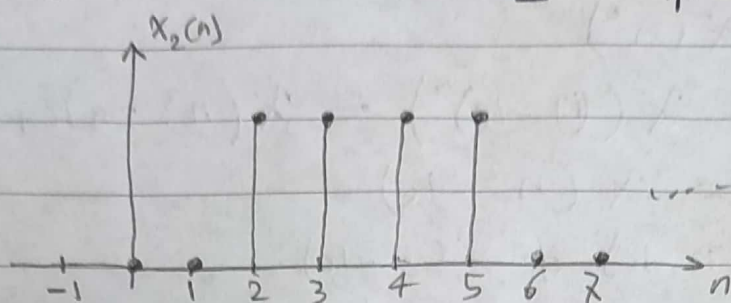
2  $y(n) = x(n^2) = \{ \dots, 0, 1, \underset{\uparrow}{1}, 1, 0, \dots \}$



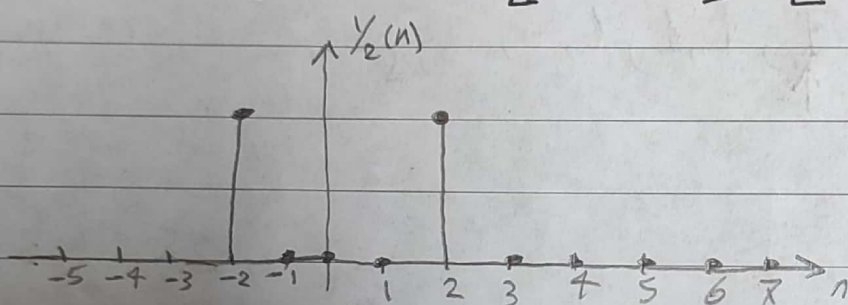
3  $y_2'(n) = y(n-2)$



$$[4] \quad x_2(n) = x(n-2) = \{ \dots, 0, 0, 1, 1, 1, 1, 0, \dots \}$$

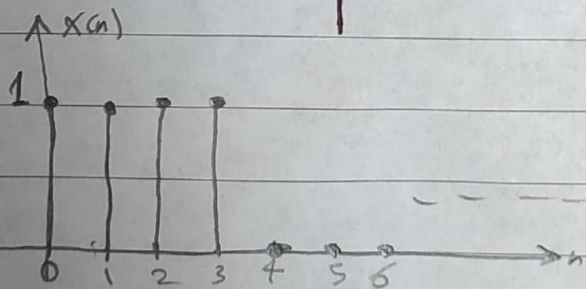


$$[5] \quad y_2(n) = \downarrow [x_2(n)] = \{ \dots, 0, 1, 0, 0, 0, 1, 0, \dots \}$$



$$[6] \quad y_2(n) \neq y(n-2), \text{ so the system is time varying.}$$

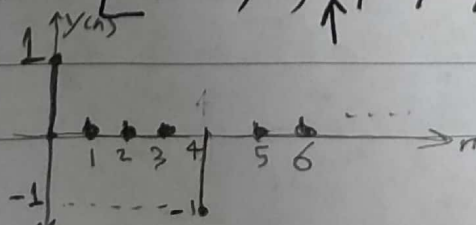
$$[C] \quad [1] \quad x(n) = \{ \dots, 0, 1, 1, 1, 1, 0, \dots \}$$



$$[2] \quad y(n) = x(n) - x(n-1]$$

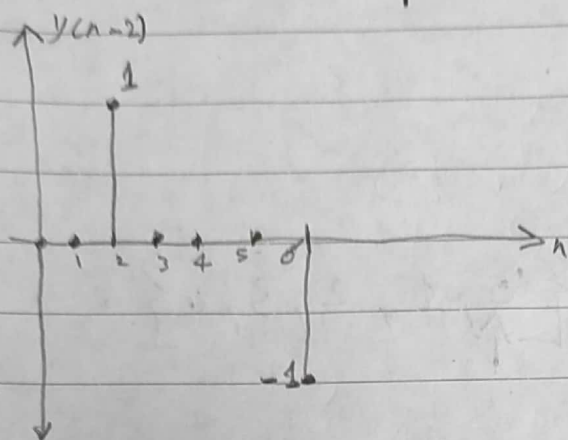
$$x(n-1] = \{ \dots, 0, 0, 1, 1, 1, 1, 0, \dots \}$$

$$y(n) = \{ \dots, 0, 1, 0, 0, 0, -1, \dots \}$$

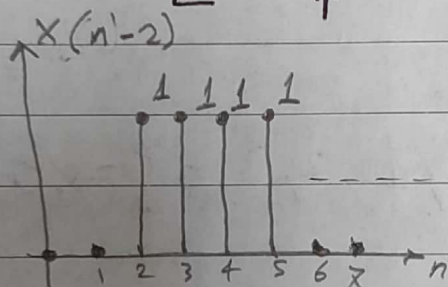




$$\boxed{3} \quad y(n-2) = \{ \dots, 0, 0, 0, 1, 0, 0, 0, -1 \}$$



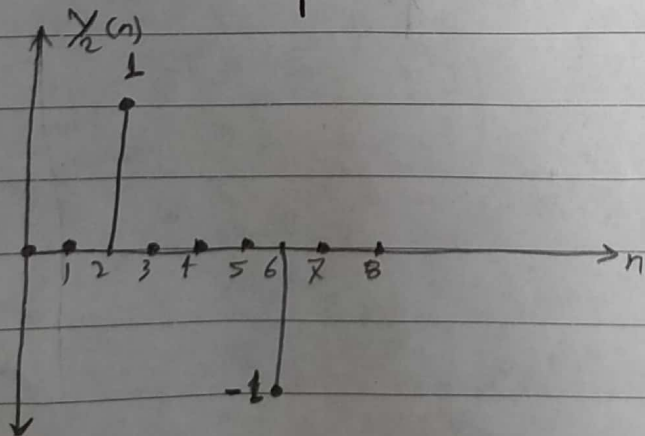
$$\boxed{4} \quad x(n-2) = \{ 0, 0, 0, 1, 1, 1, 1, 0, \dots \} = x_2(n)$$



$$\boxed{5} \quad y_2(n) = x_2(n) - x_2(n-1)$$

$$x_2(n-1) = \{ \dots, 0, 0, 0, 1, 1, 1, 1, 0, \dots \}$$

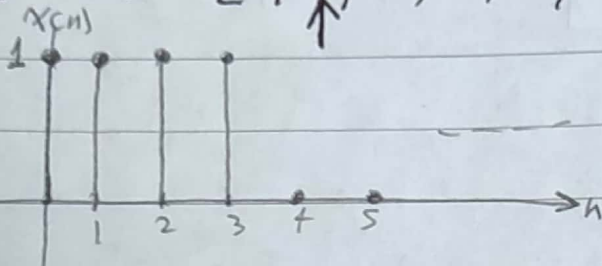
$$y_2(n) = \{ \dots, 0, 0, 1, 0, 0, 0, -1, 0, \dots \}$$



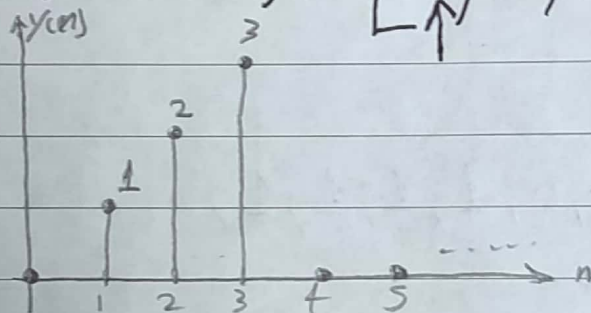
$$\boxed{6} \quad \therefore y_2(n) = y(n-2) \quad \therefore \text{system is time invariant}$$

(d)

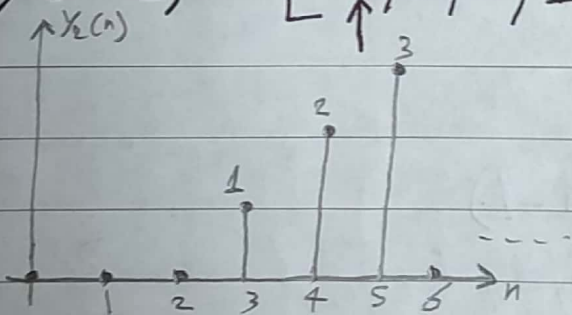
$$[1] X(n) = \{0, 1, 1, 1, 1, 0\}$$



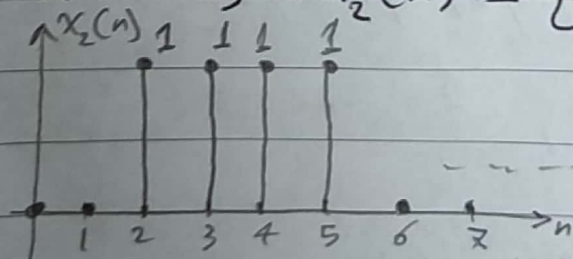
$$[2] Y(n) = nX(n) = \{0, 1, 2, 3, 0\}$$



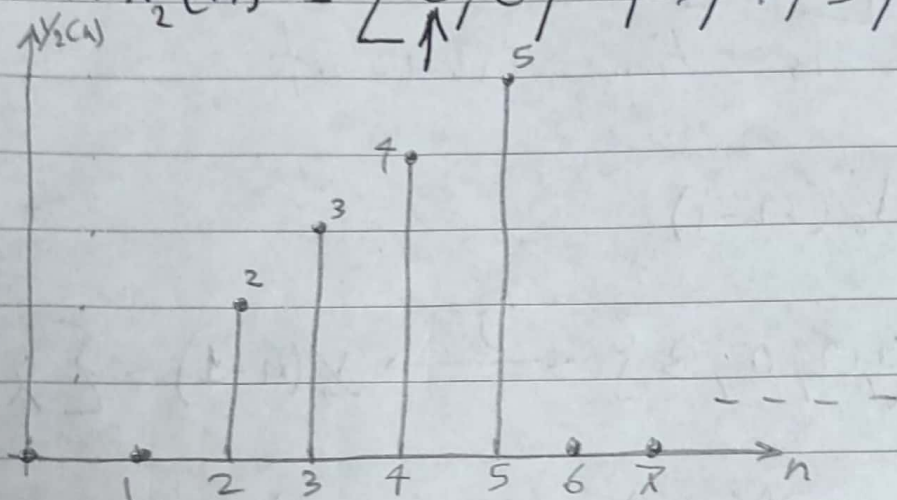
$$[3] Y(n-2) = \{0, 0, 0, 1, 2, 3, 0\} = X_2(n)$$



$$[4] X(n-2) = X_2(n) = \{0, 0, 1, 1, 1, 1, 0\}$$



5  $y_2(n) = n x_2(n) = \{0, 0, 2, 3, 4, 5, 0\}$



6  $\therefore y_2(n) \neq y(n-2), \therefore$  system is time variant

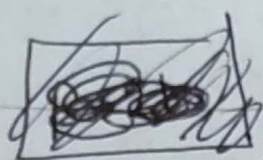
P2.10

~~$$a_1 y_1[n] + a_2 y_2[n] + a_3 y_3[n]$$~~

~~$$= a_1 \{0, 1, 2\} + a_2 \{0, 1, 0, 2\} + a_3 \{1, 2\}$$~~

~~$$y[a_1 x_1(n) + a_2 x_2(n) + a_3 x_3(n)]$$~~





P[2.10]

$\therefore$  system is time invariant

$$x_2(n-1) \Rightarrow y_2(n-1)$$

$$x_2(n-1) = \{ \underset{\uparrow}{0}, 0, 0, 3 \} \xrightarrow{\quad} y_2(n-1) = \{ \underset{\uparrow}{0}, 0, 1, 0, 2 \}$$

$\rightarrow$  we can say  $3x_3(n) = x_2(n-1)$

but,  $3y_3(n) = \{ 3, 6, 3 \}$

so  $3y_3(n) \neq y_2(n-1)$

$\therefore$  system is non linear.

P 2.11

$\therefore$  system is linear

$$x_1(n) + x_2(n) \xrightarrow{\quad} y_1(n) + y_2(n)$$

$$\{ 0, \underset{\uparrow}{1}, 0 \} \xrightarrow{\quad} \{ 0, \underset{\uparrow}{3}, -1, 2, 1 \} \quad (1)$$

now suppose system is time invariant and make shift

$$\{ 0, \underset{\uparrow}{0}, 1 \} \xrightarrow{\quad} \{ 0, \underset{\uparrow}{0}, 3, -1, 2, 1 \} \quad (2)$$

$\therefore$  system is linear  $\therefore (1) + (2)$

$$\{ 0, 1, 1 \} \xrightarrow{\quad} \{ 0, \underset{\uparrow}{3}, 2, 1, 3, 1 \}$$

but:

$$x_3(n) = \{ 0, 1, 1 \} \xrightarrow{\quad} y_3(n) = \{ \underset{\uparrow}{1}, 2, 1 \}$$

$\therefore$  system is time varying

P2.18

$$x(n) = \{ \underset{\uparrow}{0}, 1/3, 2/3, 1, 4/3, 5/3, 2 \}$$

$$h(n) = \{ 1, 1, \underset{\uparrow}{1}, 1, 1 \}$$

[1]

	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

$$\therefore y(n) = x(n) * h(n) =$$

$$= \{ 0, 1/3, \underset{\uparrow}{1}, 2, 10/3, 5, 20/3, 6, 5, 11/3, 2 \}$$

[2]

$$x(n) = \frac{1}{3} n [u(n) - u(n-8)]$$

$$h(n) = u(n+2) - u(n-3)$$

$$y(n) = x(n) * h(n)$$

$$x(n) = \frac{1}{3} \delta(n-1) + \frac{2}{3} \delta(n-2) + \delta(n-3) + \frac{4}{3} \delta(n-4) + \frac{5}{3} \delta(n-5) + 2 \delta(n-6)$$

$$h(n) = \delta(n+2) + \delta(n-2) + \delta(n) + \delta(n-1) + \delta(n-2)$$

$$y(n) = \frac{1}{3} \delta(n+1) + \delta(n) + 2 \delta(n-1) + \frac{10}{3} \delta(n-2) + 5 \delta(n-3) + \frac{20}{3} \delta(n-4) + 6 \delta(n-5) + 5 \delta(n-6) + \frac{11}{3} \delta(n-7) + 2 \delta(n-8)$$



p 2.33

$$(a) \quad y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n]$$

$$y(0) = 0.6y(-1) - 0.08y(-2) + \delta(0)$$

$$\boxed{y(0) = 1}$$

$$y(1) = 0.6y(0) - 0.08y(-1) + \delta(1)$$

$$\boxed{y(1) = 0.6}$$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$$

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda_1 = 0.4, \quad \lambda_2 = 0.2$$

$$\therefore y_h(n) = C_1(0.4)^n + C_2(0.2)^n$$

$$y(0) = C_1 + C_2 = 1$$

$$y(1) = 0.4C_1 + 0.2C_2 = 0.6$$

$$C_1 = 1 - C_2 \rightarrow 0.4(1 - C_2) + 0.2C_2 = 0.6$$

$$0.4 - 0.4C_2 + 0.2C_2 = 0.6$$

$$-0.2C_2 = 0.2 \Rightarrow \boxed{C_2 = -1}$$

$$\therefore C_1 = 1 - C_2 = 1 - (-1) = \boxed{2}$$

$$\therefore h(n) = \boxed{[2(0.4)^n - (0.2)^n] U(n)}$$

$$S(n) = \sum_{k=0}^n h(n-k), n \geq 0$$

$$= \sum_{k=0}^n \left[ 2(0.4)^{n-k} - (0.2)^{n-k} \right]$$



(5)

Q 2.35

$$(a) h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$$

$$(b) h_3(n) * h_4(n) = (n+1-2)u(n-2) \\ = (n-1)u(n-2)$$

$$h_2(n) - h_3(n) * h_4(n) = (n+1)u(n-2) - (n-1)u(n-2)$$

$$= \delta(n) + 2\delta(n-1) + (n+1)u(n-2) - (n-1)u(n-2)$$

$$= \delta(n) + 2\delta(n-1) + (n+1-n+1)u(n-2)$$

$$= \delta(n) + 2\delta(n-1) + 2u(n-2)$$

$$= \delta(n) + 2u(n-1) = \delta(n) + \delta(n) - \delta(n) + 2u(n-1)$$

$$= 2\delta(n) + 2u(n-1) - \delta(n) = \boxed{2u(n) - \delta(n)}$$

$$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$$

$$h_1(n) * [h_2(n) - h_3(n) * h_4(n)] = h(n)$$

$$h(n) = \left[ \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) \right] * [2u(n) - \delta(n)]$$

$$= u(n) - \frac{1}{2}\delta(n) + \frac{1}{2}u(n-1) - \frac{1}{4}\delta(n-1) + u(n-2) - \frac{1}{2}\delta(n-2)$$

$$= \delta(n) + \delta(n-1) + u(n-2) - \frac{1}{2}\delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{2}u(n-2) - \frac{1}{4}\delta(n-1)$$

$$+ u(n-2) - \frac{1}{2}\delta(n-2) = \left[ \frac{5}{2}u(n-2) + \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) - \frac{1}{2}\delta(n-2) \right]$$

$$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$$



$$(c) \quad h(n) = \frac{1}{2} \delta(n) + \frac{5}{4} \delta(n-1) + 2 \delta(n-2) + \frac{5}{2} \delta(n-3)$$

$$x(n) = \delta(n+2) + 3 \delta(n-1) - 4 \delta(n-3)$$

$$\therefore y(n) = x(n) * h(n) = \frac{1}{2} \delta(n+2) + \frac{5}{4} \delta(n+1) + 2 \delta(n) + \frac{5}{2} \delta(n-1)$$

$$+ \frac{3}{2} \delta(n-1) + \frac{15}{4} \delta(n-2) + 6 \delta(n-3) + \frac{15}{2} \delta(n-4)$$

$$+ 2 \delta(n-3) + 5 \delta(n-4) + 6 \delta(n-5) + 10 \delta(n-6)$$

$$y(n) = \frac{1}{2} \delta(n+2) + \frac{5}{4} \delta(n+1) + 2 \delta(n) + \frac{5}{2} \delta(n-1) + \frac{3}{2} \delta(n-1) + \frac{15}{4} \delta(n-2) + 6 \delta(n-3) + \frac{15}{2} \delta(n-4)$$

$$+ 2 \delta(n-3) + 5 \delta(n-4) + 6 \delta(n-5) + 10 \delta(n-6)$$

P. 2.46

$$(b) \quad y(n] = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$$

