- Consider designing an N-length linear-phase FIR filter with N being an even number.
- One can show that: $\boxed{H(\omega) = A(\omega)e^{j\phi(\omega)}}$

$A(\omega)$ | $\phi(\omega)$

$-\pi, \pi$

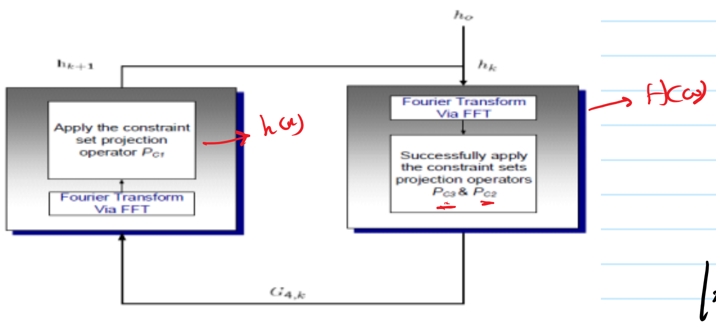
where

$$A(\omega) = \sum_{n=0}^{N/2-1} 2h[n]\cos[(n - \frac{N-1}{2})\omega].$$

$$\phi(\omega) = -\frac{N-1}{2}\omega \qquad (-\pi, \pi)$$

- We next define the set of constraint sets for such a problem. Our Hilbert space is $\mathbb{R}^L$, L>>N to insure a high-resolution Fourier transform without aliasing.

① $h(n)$ ideal
② $A(\omega)$, $\phi(\omega)$
③ $N =$ ___ ???? = order of filter
④ $$\mathbf{h}_{k+1} = P_{C_1}P_{C_2}P_{C_3}\mathbf{h}_k$$
⑤ P_{C_3}



$|A(\omega)|$

$|\omega| \in [\omega_s : \pi]$

P_{C_3}

$$P_{C_3}\mathbf{g} \leftrightarrow H^*(\omega) = \begin{cases} \frac{\delta_s G(\omega)}{|G(\omega)|}, \text{ for } |G(\omega)| > \delta_s, \omega \in \Omega_s \\ G(\omega), \text{ for } |G(\omega)| \le \delta_s, \omega \in \Omega_s \\ G(\omega), \text{ elsewhere.} \end{cases}$$

$\rightarrow \Omega_s = [\omega_s ; \pi] \cup [-\pi ; -\omega_s]$

① check $\omega \rightarrow |\omega| \in [\omega_s ; \pi]$

② check $|G(\omega)| > \delta_s \rightarrow \frac{\delta_s H(\omega)}{|H(\omega)|} = H^*_3(\omega)$

③ else $\boxed{H(\omega) = H^*_3(\omega)}$

P_{C_2} :- $\boxed{H_3(\omega) = G(\omega)} \rightarrow$

$\omega = \leftarrow \omega_p \rightarrow \leftarrow \omega_s \rightarrow$ [;] P_{C_2}, P_{C_3}

$$P_{C_2}\mathbf{g} \leftrightarrow H^*(\omega) = \begin{cases} (1+\delta_p)e^{j\phi(\omega)}, \text{ if cond. A} \\ (1-\delta_p)e^{j\phi(\omega)}, \text{ if cond. B} \\ |G(\omega)|\cos(\theta_{G(\omega)} - \phi(\omega))e^{j\phi(\omega)}, \text{ if cond. C} \\ G(\omega), \text{ if } \omega \in \Omega_p^c \end{cases}$$

where, cond. A is: $|G(\omega)|\cos(\theta_{G(\omega)} - \phi(\omega)) \ge (1+\delta_p)$ and $\omega \in \Omega_p$
cond. B is: $|G(\omega)|\cos(\theta_{G(\omega)} - \phi(\omega)) \le (1-\delta_p)$ and $\omega \in \Omega_p$
cond. C is: $(1-\delta_p) \le |G(\omega)|\cos(\theta_{G(\omega)} - \phi(\omega)) \le (1+\delta_p)$ and $\omega \in \Omega_p$

(1) check $\omega \in [-\omega_p ; \omega_p]$

① check $\omega \in [-\omega_p ; \omega_p]$

$|H(\omega)| \cos(\theta_{H(\omega)} - \phi(\omega))$

$\underbrace{H(\omega)} = A(\omega)\, e^{j\underline{\phi(\omega)}}$

$\boxed{H_2(\omega)} \longleftarrow \quad \Longrightarrow \underbrace{h_2(n)} = IFFT(H_2(\omega))$

$\# P_{C_1}$

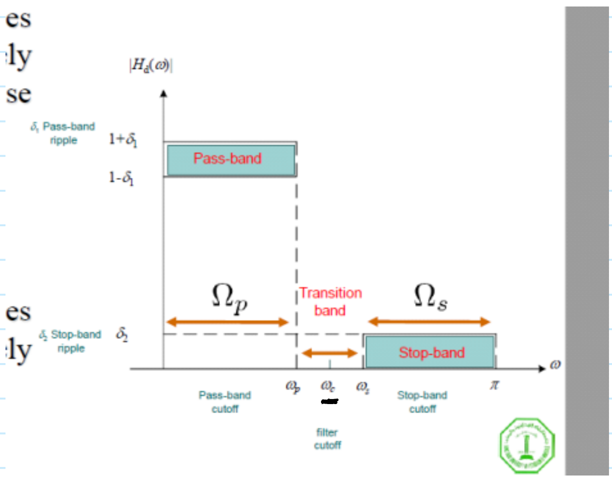
$h_2(n) = g(l)$ $\quad$ $n \rightarrow 0 \quad N-1$

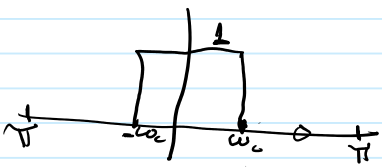
$$P_{C_1}\mathbf{g} = \begin{cases} \frac{g(l)+g(N-1-l)}{2}, & \text{for } l = 0, 1, \ldots, (N-1) \\ 0, & \text{elsewhere.} \end{cases}$$

$\boxed{h_{k+1}} \longleftarrow$

$\rightarrow \|\mathbf{h}_{k+1} - \mathbf{h}_k\| \le \epsilon \Longrightarrow$ norm? 2, 1

??





$N = ??$ $\qquad |H(\omega)| = 1 \quad , \quad -\omega_c : \omega_c$

$N = ??$

$\Theta = ??$

$|H_0(\omega)| = 1 \quad ; \quad -\omega_c : \omega_c$

$|H_0(\omega)| = 0 \quad ; \quad$ elsewhere

$$\begin{cases} |H(\omega)| = \_ \\ \Phi(\omega) = \_ \end{cases}$$

$H(\omega) = A(\omega)\, e^{j\Phi(\omega)} = |A(\omega)| \angle\Theta \cdot \angle\Phi(\omega)$

$= |A(\omega)| \angle\Theta + \Phi$

$A(\omega) = |A(\omega)| \angle\Theta$