



College of Engineering and Physics
Electrical Engineering Department

EE562 - Digital Signal Processing I

Second Semester (212)

Homework Assignment 6

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Q5.1

Monday, March 21, 2022 10:00 PM

- 5.1 The following input-output pairs have been observed during the operation of various systems:

(a) $x(n) = \left(\frac{1}{2}\right)^n \xrightarrow{\mathcal{T}_1} y(n) = \left(\frac{1}{8}\right)^n$ Determine their frequency response if each of the above systems is LTI.

① $\therefore H(\omega)$ exists if the system is BIBO

$\because n = -\infty \Rightarrow x(-\infty) = \infty$ $y(-\infty) = \infty$ \therefore input is not bounded
output is not bounded

$\therefore H(\omega)$ does not exist \neq

(c) $x(n) = e^{j\pi/5} \xrightarrow{\mathcal{T}_3} y(n) = 3e^{j\pi/5}$

\Rightarrow BIBO

$\Rightarrow A e^{j\omega n} \xrightarrow{h(n)} A H(\omega) e^{j\omega n}$

$\therefore H(\omega) = 3 \Rightarrow H(\pi/5) = 3 \neq$

Q5.3

Monday, March 21, 2022 10:32 PM

5.3 Consider an LTI system with impulse response $h(n) = (\frac{1}{2})^n u(n)$.

(a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

$$1. \quad x(n) = \cos \frac{3\pi n}{10}, -\infty < n < \infty$$

$$2. \quad x(n) = \{\dots, 1, 0, 0, \underset{\uparrow}{1}, 1, 1, 0, 1, 1, 1, 0, 1, \dots\}$$

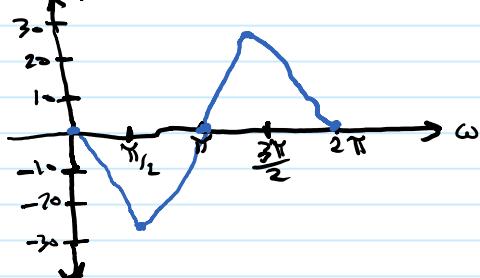
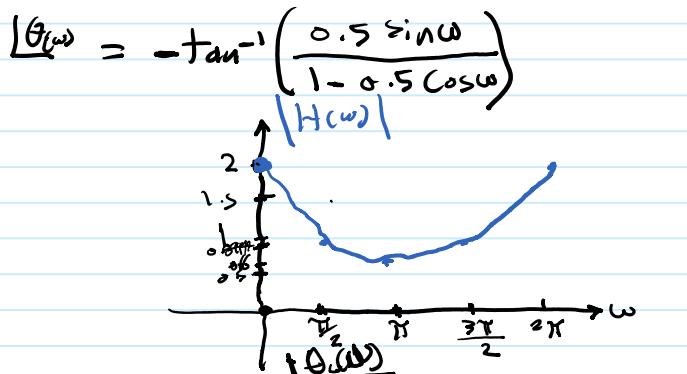
$$\textcircled{a} \quad H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n$$

$$= \frac{1}{1 - 0.5 e^{-j\omega}} = \frac{1}{1 - 0.5(\cos \omega - j \sin \omega)}$$

$$|H(\omega)| = \sqrt{(1 - 0.5 \cos \omega)^2 + (0.5 \sin \omega)^2} = \sqrt{1 - \cos \omega + 0.25(\cos^2 \omega + 0.25 \sin^2 \omega)} = \sqrt{1 - \cos \omega + 0.25} = \boxed{\sqrt{1.25 - \cos \omega}}$$

ω	$ H(\omega) $
0	2
90°	0.8944
180°	0.6666
270°	0.8944
360°	2

ω	$\angle H(\omega)$
0	0
90°	-26.56
180°	0
270°	26.56
360°	0



Q5.b

Tuesday, March 22, 2022 12:11 AM

5.3 Consider an LTI system with impulse response $h(n) = (\frac{1}{2})^n u(n)$.

- (a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.
- (b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

$$1. \quad x(n) = \cos \frac{3\pi n}{10}, -\infty < n < \infty$$

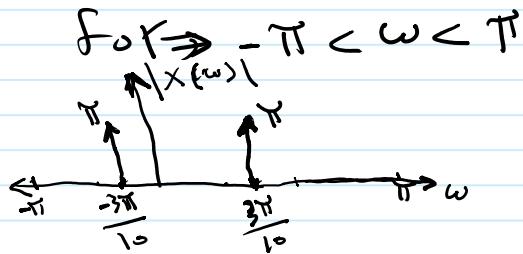
$$2. \quad x(n) = \{\dots, 1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, \dots\}$$

(b) $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \cos \frac{3\pi n}{10} e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \frac{1}{2} [e^{j\frac{3\pi n}{10}} + e^{-j\frac{3\pi n}{10}}] e^{-j\omega n}$

$$\Rightarrow \therefore \begin{array}{c} 1 \\ \xleftarrow{\text{DTFT}} \end{array} \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega + 2\pi k)$$

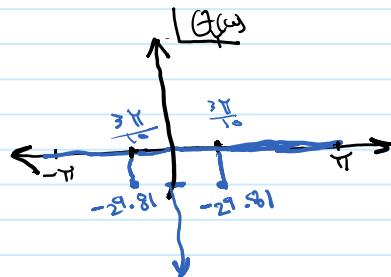
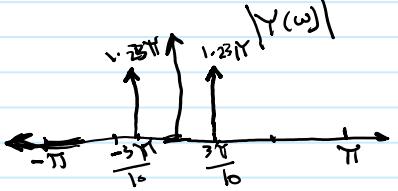
$$\therefore \begin{array}{c} e^{j\omega_0} \\ \xleftarrow{\text{DTFT}} \end{array} \sum_{k=-\infty}^{\infty} 2\pi \delta(\omega - \omega_0 + 2\pi k)$$

$$\therefore X(\omega) = \frac{1}{2} * 2\pi \left[\delta\left(\omega - \frac{3\pi}{10}\right) + \delta\left(\omega + \frac{3\pi}{10}\right) \right]$$



$$\Rightarrow Y(\omega) = X(\omega) \cdot H(\omega) = H\left(\frac{3\pi}{10}\right) \cdot \left[\delta\left(\omega - \frac{3\pi}{10}\right) + \delta\left(\omega + \frac{3\pi}{10}\right) \right]$$

$$\therefore H(\omega) = \frac{1}{1 - 0.5 e^{-j\omega}}, \quad H\left(\frac{3\pi}{10}\right) = 1.23 \underline{-29.81}$$



$$\begin{aligned} y(n) &= |H\left(\frac{3\pi}{10}\right)| \cos\left(\frac{3\pi}{10}n + \underline{\theta\left(\frac{3\pi}{10}\right)}\right) \\ &= 1.23 \cos\left(\frac{3\pi}{10}n - 29.81\right) \end{aligned}$$

5.3 Consider an LTI system with impulse response $h(n) = (\frac{1}{2})^n u(n)$.

(a) Determine and sketch the magnitude and phase response $|H(\omega)|$ and $\angle H(\omega)$, respectively.

(b) Determine and sketch the magnitude and phase spectra for the input and output signals for the following inputs:

$$1. \quad x(n) = \cos \frac{3\pi n}{10}, -\infty < n < \infty$$

$$2. \quad x(n) = \{\dots, 1, 0, 0, 1, \underset{\uparrow}{1}, 1, 1, 0, 1, 1, 1, 0, 1, \dots\}$$

$$\begin{aligned} \textcircled{b} \textcircled{2} \quad x(n) &= \delta(n+3) + \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-4) \dots \\ x(\omega) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \\ &= \dots + e^{j3\omega} + 1 + e^{j\omega} + e^{-j2\omega} + e^{-j4\omega} + e^{j5\omega} + e^{-j6\omega} + \dots \\ &\quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{aligned}$$

$\textcircled{0} \textcircled{1} \textcircled{0} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{0} \textcircled{1} \textcircled{1} \textcircled{1} \textcircled{0} \textcircled{1}$

$$\omega_1(n) = \delta(n) + \delta(n-1) + \delta(n-2), N_1 = 4$$

$$\omega_2(n) = \delta(n+3), N_2 = 4 \Rightarrow N_1 = N_2 = N$$

$$\begin{aligned} x(n) &= \sum_{k \geq 0} \omega_2(n - kN) + \sum_{k \geq 0} \omega_1(n - kN) \\ &= W_2(n) * \sum_{k \geq 0} \delta(n - kN) + W_1(n) * \sum_{k \geq 0} \delta(n - kN) \\ X(\omega) &= W_2(\omega) \cdot \sum_{k=-\infty}^{\infty} e^{jN\omega k} + W_1(\omega) \cdot \sum_{k \geq 0} e^{jN\omega k} \\ \sum_{k=-\infty}^{\infty} e^{jN\omega k} &= \sum_{k=1}^{\infty} e^{jN\omega k} = \frac{e^{j4\omega}}{1 - e^{j4\omega}} \\ \sum_{k=0}^{\infty} e^{jN\omega k} &= \frac{1}{1 - e^{-j4\omega}}, W_1(\omega) = 1 + e^{-j\omega} + e^{-j2\omega} \end{aligned}$$

$$W_2(\omega) = e^{j3\omega}$$

$$\begin{aligned} \therefore X(\omega) &= \frac{e^{j3\omega} e^{j4\omega}}{1 - e^{j4\omega}} + \frac{1 + e^{-j\omega} + e^{-j2\omega}}{1 - e^{-j4\omega}} \\ &= \frac{e^{j7\omega} - e^{j3\omega} + 1 + e^{-j\omega} + e^{-j2\omega} - e^{j4\omega} - e^{j3\omega} - e^{j2\omega}}{1 - e^{j4\omega} - e^{-j4\omega} + 1} \\ &= \frac{e^{j7\omega} + 1 + e^{-j\omega} - 2e^{j3\omega} + [e^{-j2\omega} - e^{j2\omega}] + e^{j4\omega}}{2 - [e^{j4\omega} + e^{-j4\omega}]} \\ &= \frac{1 + e^{-j\omega} + e^{j7\omega} - 2e^{j3\omega} - 2j \sin 2\omega + j4\omega}{2 - 2 \cos 4\omega} \end{aligned}$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

Q5.7

Saturday, March 26, 2022 2:24 AM

Consider the FIR filter

$$y(n) = x(n) + x(n-4)$$

(a) Compute and sketch its magnitude and phase response.

(b) Compute its response to the input

$$x(n) = \cos \frac{\pi}{2}n + \cos \frac{\pi}{4}n, \quad -\infty < n < \infty$$

(c) Explain the results obtained in part (b) in terms of the magnitude and phase responses obtained in part (a).

$$\textcircled{a} \quad X(\omega) = X(\omega) + X(\omega - 4\omega)$$

$$Y(\omega) = X(\omega) + X(\omega) e^{-j4\omega} = X(\omega) [1 + e^{-j4\omega}]$$

$$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)} = 1 + e^{-j4\omega}$$

$$= 1 + \cos 4\omega - j \sin 4\omega$$

$$\therefore |H(\omega)| = \sqrt{(1 + \cos 4\omega)^2 + (\sin 4\omega)^2}$$

$$= \sqrt{1 + 2 \cos 4\omega + (-j^2)^2 + \sin^2 4\omega}$$

$$= \sqrt{1 + 2 \cos 4\omega + 1}$$

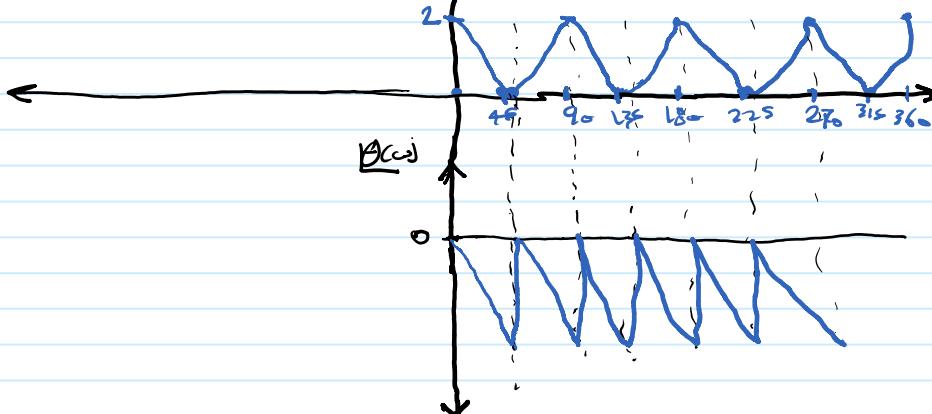
$$= \boxed{\sqrt{2 + 2 \cos 4\omega}} \quad \text{#}$$

ω	$H(\omega)$	ω	$H(\omega)$
0	2	45	0
90	2	135	0
180	2	225	0
270	2	315	0
360	2		

$$\Theta(\omega) = \tan^{-1} \frac{-\sin 4\omega}{1 + \cos 4\omega}$$

$$\#$$

$$|H(\omega)|$$



Consider the FIR filter

$$y(n) = x(n) + x(n - 4)$$

(a) Compute and sketch its magnitude and phase response.

(b) Compute its response to the input

$$x(n) = \cos \frac{\pi}{2}n + \cos \frac{\pi}{4}n, \quad -\infty < n < \infty$$

(c) Explain the results obtained in part (b) in terms of the magnitude and phase responses obtained in part (a).

$$(b) y(n) = |H(\frac{\pi}{2})| \cos \left[\frac{\pi}{2}n + \theta(\frac{\pi}{2}) \right] + |H(\frac{\pi}{4})| \cos \left[\frac{\pi}{4}n + \theta(\frac{\pi}{4}) \right]$$

$$H(\frac{\pi}{2}) = 2, \quad H(\frac{\pi}{4}) = 0 \Rightarrow H(\omega) = 1 + e^{-j4\omega}$$

$$\boxed{\therefore y(n) = 2 \cos \left(\frac{\pi}{2}n \right)} \#$$

(c) the filter response for $\omega = \frac{\pi}{4}$ equals zero
that's why the filter does not pass $\cos \frac{\pi}{4}n$