



College of Engineering and Physics
Electrical Engineering Department

EE562 - Digital Signal Processing I

Second Semester (212)

Homework Assignment 4

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3.11 Using long division, determine the inverse z -transform of

$$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

if (a) $x(n)$ is causal and (b) $x(n)$ is anticausal.

(a)

$$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}} = \frac{z^2 + 2z}{z^2 - 2z + 1} =$$

$$\begin{array}{r} 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} \\ \hline z^2 - 2z + 1 \sqrt{z^2 + 2z} \\ \hline z^2 - 2z + 1 \\ \hline 4z - 1 \\ 4z - 8 + 4z^{-1} \\ \hline z - 4z^{-1} \\ z - 1 + z^{-1} + z^{-2} \\ \hline + 10z^{-1} - 7z^{-2} \\ 10z^{-1} - 2z^{-2} + 10z^{-3} \\ \hline 13z^{-2} - 10z^{-3} \end{array}$$

$$\therefore X(z) = 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} + \dots$$

$$\boxed{\therefore x(n) = \{1, 4, 7, 10, \dots\}} \quad \times$$

(b)

3.11 Using long division, determine the inverse z -transform of

$$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

if (a) $x(n)$ is causal and (b) $x(n)$ is anticausal.

$$\begin{aligned} X(z) &= \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}} \\ &\xrightarrow{\text{z}^{-2} - 2z^{-1} + 1} \frac{2z + 5z^2 + 8z^3 + 11z^4}{2z^{-1} + 1} \\ &\quad \underline{-} \frac{2z^{-1} - 4 + 2z}{\underline{5 - 2z}} \\ &\quad \underline{-} \frac{5 - 10z + 5z^2}{\underline{8z - 5z^2}} \\ &\quad \underline{-} \frac{8z - 16z^2 + 8z^3}{\underline{11z^2 - 8z^3}} \end{aligned}$$

$$\therefore X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + \dots$$

$$\therefore x(n) = \left\{ \dots, 1, 8, 5, 2, 0 \right\} \quad \text{X} \cancel{2}$$

3.12 Determine the causal signal $x(n)$ having the z -transform

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{z^2}{(z-2)(z-1)^2} \Rightarrow \text{partial fraction}$$

$$\frac{X(z)}{z^2} = \frac{z^2}{(z-2)(z-1)^2} = \frac{A}{z-2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$z^2 = A(z-1)^2 + B(z-2)(z-1) + C(z-2)$$

$$\text{at } z=1$$

$$1 = C(1-2) \Rightarrow 1 = -C = \boxed{C = -1} \quad \#$$

$$\text{at } z=2$$

$$4 = A(2-1)^2 \Rightarrow \boxed{4 = A} \quad \#$$

$$\text{at } z=\infty$$

$$0 = A + 2B - 2C \Rightarrow \begin{cases} 2B = -A \\ B = -3 \end{cases} \quad \#$$

$$\frac{X(z)}{z^2} = \frac{4}{z-2} - \frac{3}{z-1} - \frac{1}{(z-1)^2} \Rightarrow X(z) = \frac{4z}{z-2} - \frac{3z}{z-1} - \frac{z}{(z-1)^2}$$

$$x(n) = 4(2^n u(n)) - 3u(n) - nu(n) = 2^2 z^n u(n) - 3u(n) - nu(n)$$

$$\boxed{x(n) = [2^{n+2} - 3 - n] u(n)} \quad \#$$

3.16 Determine the convolution of the following pairs of signals by means of the z -transform.

(a) $x_1(n) = (\frac{1}{4})^n u(n - 1)$, $x_2(n) = [1 + (\frac{1}{2})^n]u(n)$

(b) $x_1(n) = u(n)$, $x_2(n) = \delta(n) + (\frac{1}{2})^n u(n)$

(c) $x_1(n) = (\frac{1}{2})^n u(n)$, $x_2(n) = \cos \pi n u(n)$

(d) $x_1(n) = n u(n)$, $x_2(n) = 2^n u(n - 1)$

$$(b) x_1(n) = u(n) \rightarrow X_1(z) = \frac{1}{z-1} + \frac{1}{z} u(n)$$

$$X_1(z) = \frac{z}{z-1}, \quad X_2(z) = 1 + \frac{z}{z-0.5}$$

$$\therefore Y(z) = X_1(z) \cdot X_2(z) = \frac{z}{z-1} \cdot \left[1 + \frac{z}{z-0.5} \right]$$

$$\therefore Y(z) = \frac{z}{z-1} + \frac{z^2}{(z-1)(z-0.5)} \xrightarrow{\text{partial fraction}} Y_2(z)$$

$$\frac{Y_2(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

$$z = A(z-0.5) + B(z-1)$$

$$\Rightarrow \boxed{a + z = 0.5}$$

$$0.5 = B(0.5 - 1) \Rightarrow 0.5 = -0.5B \Rightarrow \boxed{B = -1}$$

$$\Rightarrow \boxed{a + z = 1}$$

$$1 = A \cdot 0.5 \Rightarrow \boxed{A = \frac{1}{0.5} = 2}$$

$$\therefore Y_2(z) = \frac{2z}{z-1} - \frac{z}{z-0.5}$$

$$\therefore Y(z) = \frac{z}{z-1} + \frac{2z}{z-1} - \frac{z}{z-0.5} \Rightarrow y(n) = u(n) + 2u(n) - (\frac{1}{2})^n u(n)$$

$$\therefore y(n) = [3 - (\frac{1}{2})^n] u(n)$$

3.16 Determine the convolution of the following pairs of signals by means of the z -transform.

- (a) $x_1(n) = (\frac{1}{4})^n u(n-1)$, $x_2(n) = [1 + (\frac{1}{2})^n]u(n)$
- (b) $x_1(n) = u(n)$, $x_2(n) = \delta(n) + (\frac{1}{2})^n u(n)$
- (c) $x_1(n) = (\frac{1}{2})^n u(n)$, $x_2(n) = \cos \pi n u(n)$
- (d) $x_1(n) = nu(n)$, $x_2(n) = 2^n u(n-1)$

$$\textcircled{C} \quad x_1(n) = \left(\frac{1}{2}\right)^n u(n), \quad x_2(n) = \cos \pi n u(n)$$

$$X_1(z) = \frac{z}{z - 0.5}, \quad X_2(z) = \frac{z^2 - z \cos \pi}{z^2 - 2z \cos \pi + 1} = \frac{z^2 + z}{z^2 + 2z + 1} =$$

$$X_2(z) = \frac{z(z+1)}{(z+1)^2} \rightarrow Y(z) = X_1(z) * X_2(z) = \frac{z^2}{(z - 0.5)(z+1)} \quad \text{partial fraction}$$

$$\frac{Y(z)}{z} = \frac{z}{(z - 0.5)(z+1)} = \frac{A}{z - 0.5} + \frac{B}{z+1}$$

$$z = A(z+1) + B(z - 0.5)$$

$$\begin{aligned} \Rightarrow a + z &= -1 \\ -1 &= -0.5B \Rightarrow B = \frac{1}{0.5} = 2/3 \end{aligned} \quad \left| \begin{aligned} \Rightarrow a + z &= 0.5 \\ 0.5 &= 1 - 0.5A \Rightarrow A = \frac{0.5}{1.5} = \frac{1}{3} \end{aligned} \right.$$

$$Y(z) = \frac{1}{3} \frac{z}{z - 0.5} + \frac{2}{3} \frac{z}{z+1}$$

$$\therefore Y(n) = \frac{1}{3}(0.5)^n u(n) + \frac{2}{3}(-1)^n u(n) = \boxed{\left[\frac{1}{3}(0.5)^n + \frac{2}{3}(-1)^n \right] u(n)}$$

3.37 Compute the response of the system

$$y(n) = 0.7y(n-1) - 0.12y(n-2) + x(n-1) + x(n-2)$$

to the input $x(n) = nu(n)$. Is the system stable?

$$Y(n) = 0.7Y(n-1) - 0.12Y(n-2) + X(n-1) + X(n-2)$$

$$Y(n) - 0.7Y(n-1) + 0.12Y(n-2) = X(n-1) + X(n-2)$$

$$Y(z) - 0.7z^{-1}Y(z) + 0.12z^{-2}Y(z) = z^{-1}X(z) + z^{-2}X(z)$$

$$Y(z) \left[1 - 0.7z^{-1} + 0.12z^{-2} \right] = X(z) \cdot \left[z^{-1} + z^{-2} \right]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{z^{-1} + z^{-2}}{1 - 0.7z^{-1} + 0.12z^{-2}} = \frac{z+1}{z^2 - 0.7z - 0.12}$$

$$\therefore Y(z) = H(z) \cdot X(z)$$

$$\because X(n) = nu(n) \implies X(z) = \frac{z}{(z-1)^2}$$

$$\therefore Y(z) = \frac{z+1}{z^2 - 0.7z - 0.12} \cdot \frac{z}{(z-1)^2} = \frac{z(z+1)}{(z-1)^2(z-0.3)(z-0.4)} \quad \text{Partial Fraction}$$

$$\frac{Y(z)}{z} = \frac{z+1}{(z-1)^2(z-0.3)(z-0.4)} = \frac{A}{(z-1)} + \frac{B}{(z-1)^2} + \frac{C}{(z-0.3)} + \frac{D}{(z-0.4)}$$

$$z+1 = A(z-1)(z-0.3)(z-0.4) + B(z-0.3)(z-0.4) + C(z-1)^2(z-0.4) + D(z-1)^2(z-0.3)$$

$$\Rightarrow \text{at } z=1$$

$$2 = B(1-0.3)(1-0.4) = B \times 0.7 \times 0.6 = 2 \implies B = \frac{2}{0.7 \times 0.6} = \frac{100}{21}$$

$$\Rightarrow \text{at } z=0.3$$

$$1.3 = C(0.3-1)^2(0.3-0.4) = C(-0.7)^2 \times -0.1 = -0.049C = 1.3$$

$$C = \frac{1.3}{-0.049} = \frac{-1300}{49}$$

$$\Rightarrow \text{at } z=0.4$$

$$1.4 = D(0.4-1)^2(0.4-0.3) = D \times 0.36 \times 0.1 = 0.036D = 1.4$$

$$D = \frac{1.4}{0.036} = \frac{350}{9}$$

$$\Rightarrow \text{at } z=0$$

$$1 = A(-1)(-0.3)(-0.4) + B(-0.3)(-0.4) + C(-1)^2(-0.4) + D(-1)^2(-0.3)$$

$$1 = -0.12A + 0.12B - 0.4C - 0.3D$$

$$-0.12A = 1 - 0.12 \times \frac{100}{21} + 0A \times \frac{-1300}{49} + 0.3 \times \frac{350}{9} = \frac{218}{147}$$

$$A = \frac{218}{147} \div -0.12 = -\frac{5450}{441}$$

$$Y(z) = -\frac{5450}{441} \frac{z}{z-1} + \frac{100}{21} \frac{z}{(z-1)^2} - \frac{1300}{49} \frac{z}{z-0.3} + \frac{350}{9} \frac{z}{z-0.4}$$

$$Y(n) = -\frac{5450}{441} u(n) + \frac{100}{21} n u(n) - \frac{1300}{49} (0.3)^n u(n) + \frac{350}{9} (0.4)^n u(n)$$

\therefore poles of $H(z)$ is inside the unit circle \Rightarrow \therefore system is stable