KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Electrical Engineering Department

EE562 – DIGITAL SIGNAL PROSSING I

Term Paper

Interpolation of Seismic Data Using Compressive Sensing

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Submitted to

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I. Introduction:

Seismic exploration is an important step in oil& gas discovery. The aim of seismic surveying is to get images of multiple sub-surfaces to estimate the possible locations of hydrocarbon reservoirs. However, this task encounters many challenges. One of the challenges is the loss of some traces.

Missing traces and irregularities are typical in seismic exploration due to ocean currents, drilling platforms, mountains, rivers, and so on. Even if the data is occasionally regular in one domain, it becomes irregular when it is sorted into another. Missing or irregular traces will cause issues with seismic processing such as wide azimuth data processing, migration, and surface related multiple removal (SRME). They result in low picture gather quality, significant aliasing, and other issues. Interpolation is required to overcome these challenges.

In past few years, Compressive Sensing (CS) has been used into seismic exploration to improve the efficiency of seismic data collecting. The data generated by the compressive sensing approach are exceedingly irregular, making conventional processing problematic unless the data are preprocessed by interpolation [1].

There are several interpolation techniques which can be classified into two categories [1]. The first kind is model-driven and based on wave equations. The approach works effectively when the subsurface model is correct. However, because an exact model is sometimes difficult to obtain, the approach is rarely employed. The second kind is data-driven and is based on signal processing. Methods based on prediction filtering and math transformations are mostly included in this category. The older approaches are more efficient and perform well for data with consistently missing traces, but they struggle with data with random missed traces. For different math transformations, the efficiency and impact of the latter procedures change. To summarize, each approach has pros and disadvantages.

The goal of this project is to interpolate the data such that the lost traces are recovered and subsurface is estimated accurately.

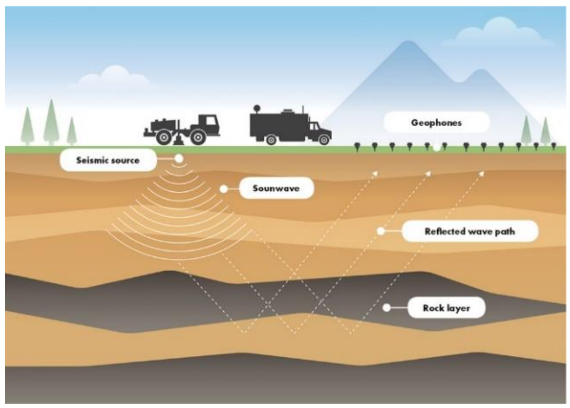


Fig. 1. An example of seismic exploration.

II. Literature Review:

Herrmann, et al. [2] follow the principle of compressive sensing, and they argued that randomized subsampling may be used to rebuild seismic wavefields with a manageable inaccuracy. they demonstrated that CS may be effectively utilized to seismic data collection using precisely constructed numerical experiments on synthetic and real data, resulting in a new age group of randomized acquisition and processing approaches.

They stated that curvelets perform the best in terms of recovery, comparable to wave atoms, and wavelets, which is dependable with the directional nature of seismic wavefronts, using cautiously calculated experiments and the introduction of performance events for nonlinear approximation and recovery errors. This finding is noteworthy for two reasons: (i) it emphasizes the importance of sparsity promotion, which offsets the "costs" of redundancy; and (ii) it demonstrates that the relative sparsity ratio, rather than the absolute number of significant coefficients, effectively determines recovery performance. Their discovery of much better recovery for simultaneous-source acquisition verifies compression sensing expectations.

Cao, Jingjie, et al. [3] apply CS theory to the problem of seismic interpolation and data restoration. They employed The curvelet transform as their sparse transform, and piecewise sampling is used to increase restoration quality. The piecewise sampling approach allows them to manage the sample gap while maintaining unpredictability. The numerical findings suggest that this sampling strategy outperforms random sampling. They describe two rapid approaches for solving the L_0 and L_1 minimization models in computing. Their approaches were shown to be substantially quicker than the IST and SPGL1 methods. They contend that the curvelet transform may be utilized for de-noising, multiple removal, and migration, and that the sampling technique and sparse transform have a significant impact on interpolation.

Chengbo, et al. [1] created interpolated compressive sensing, a unique data reconstruction approach that permits for a discrepancy between the notional grid to which the data are reconstructed and the actual grid to which the data are gathered. Any dictionary applied in the CS data reconstruction model may be applied to the regular nominal grid using their technique. The interpolated restriction operator specifies the connection between the observed and nominal grids. The interpolated restriction operator, in turn, accommodates for the observed grid's reduced size as well as when a point on the observed grid does not match to a nominal grid point.

The previously mentioned point is achieved using Lagrange interpolation in the restriction operator.

Huang, et al. [4] research improved the compressive sensing approach to address the issue of processing data with often missing traces and then used to interpolate the OBN data to enhance the reliability of the OBN time-lapse data and the migration performance.

III. Proposed Solution:

The proposed solution is to use compressive sensing to interpolate the missing traces. Compressive sensing is a signal processing technique introduced in [5] that promises to reconstruct a signal f from a highly incomplete set of measurements b. There are two conditions for the successful reconstruction of the signal. The first is the sparsity, meaning that the signal needs to be sparse in some domain. The second is the irregularity of the sampling, which is applied through the isometry property [6]. Compressive sensing is robust in reconstructing signals with irregularly missed points, which is the case of the seismic interpolation problem as we cannot control which traces are lost in the acquisition.

Starting from the measurement vector $b \in \mathbb{R}^m$ we want to recover the signal $f \in \mathbb{R}^n, n > m$.

$$b = \phi f$$

Where $\phi \in \mathbb{R}^{m \times n}$ is the measurement matrix. To do this, we need to find a transformation matrix ψ such that

$$f = \psi x$$

Where x is a sparse vector representing f in the transform domain described by ψ. Now let

$$A = \phi\psi$$

and

$$A\hat{x} = b$$

Using the definition of A and b we can write $\hat{x}$ as

$$\hat{x} = \psi^{-1}\phi^{+}\phi f$$

$$\hat{x} = \psi^{-1} f$$

Where the superscript (+) denotes the pseudo-inverse. However, this is an underdetermined system with an infinite number of solutions and the above solution does not guarantee the sparsity of $\hat{x}$. Therefore, we solve the following optimization problem instead

$$\min\|\hat{x}\|_1 \quad s.t. \quad A\hat{x} = b$$

Where $\|.\|_1$ denotes the $L1$ norm of a vector defined as the sum of the absolute value of all its elements

$$\|\hat{x}\|_1 = \sum_{r=1}^{n} |\hat{x}_r|$$

Using this sparse vector $\hat{x}$ that has the minimum $L1$ norm we can recover the original signal f (with a high probability) using

$$f = \psi\hat{x}$$

The workflow of compressive sensing is summarized in Fig. 2

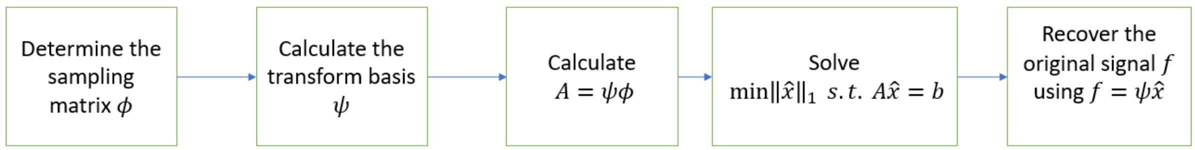


Fig. 2 Compressive sensing workflow

IV. Results:

1. Compressive Sensing on a Sinusoidal Signal

To prove the concept of compressive sensing, we started by a simple example of a sinusoidal signal. Let

$$f_a(t) = \sin(1394\pi t) + \sin(3266\pi t)$$

If we sample f_a at 40 kHz for 1/8 seconds, we get a vector f that has 5000 samples. Taking 500 random samples of f, we get b as illustrated below. Note the sparsity of x in the DCT domain.

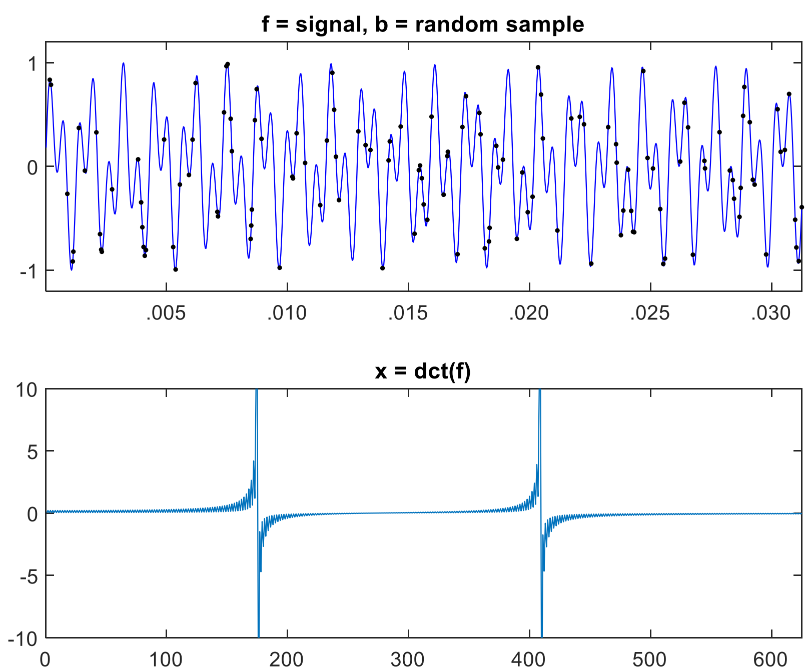


Fig. 3 Top: Random samples of the original signal generated by the "A" key on a touch-tone phone. Bottom: The discrete cosine transform of the signal.

Applying the procedure illustrated earlier, we got:

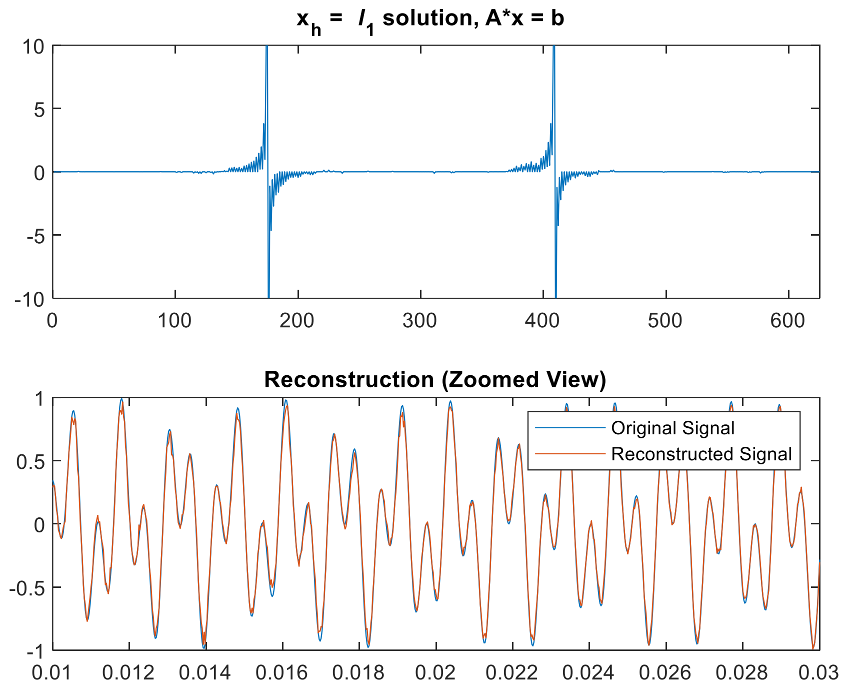
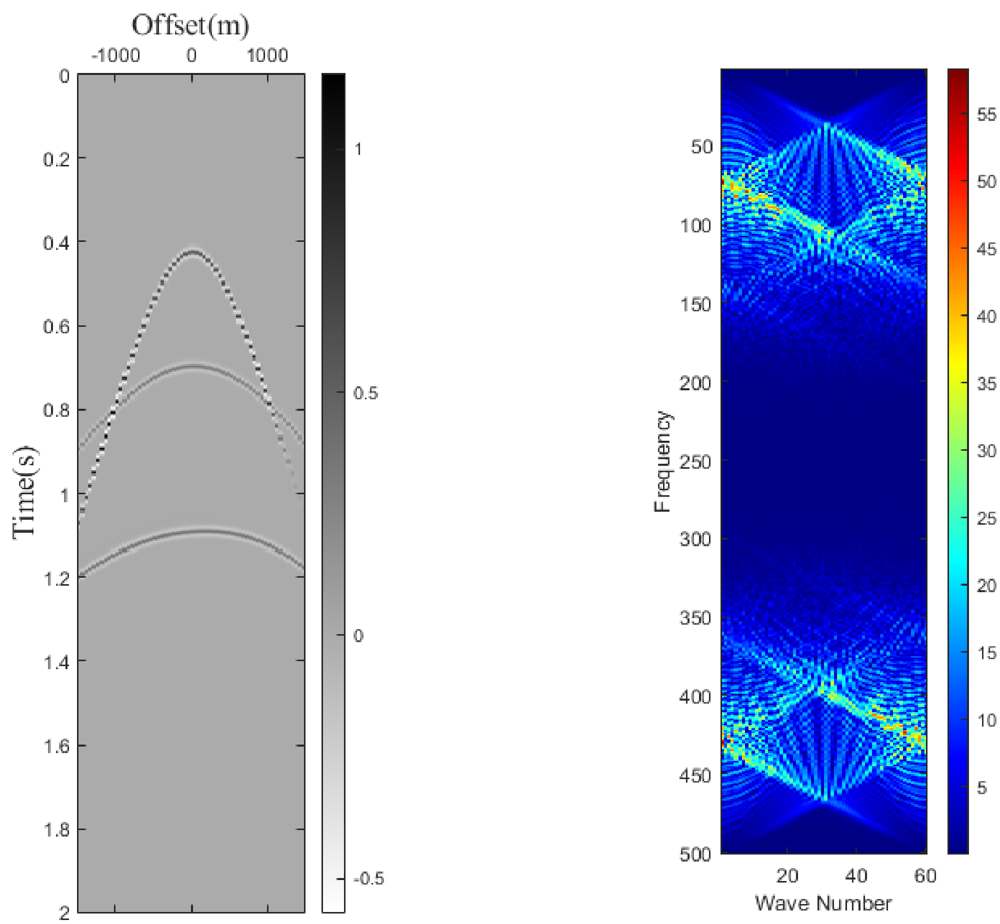


Fig. 4. The l1 solution to Ax = b leads to idct(x), a signal that is nearly identical to the original.

2. Compressive Sensing on Synthetic Seismic Data

We applied compressive sensing on the synthetic data image shown in Fig. 5.



(a) (b)

Fig. 5 (a) The synthetic seismic image (b) its 2D Fourier transform

Fig. 6 shows the result of applying compressive sensing to the synthetic seismic image with 52% missing traces compared to linear interpolation. Fig. 7 presents the results of running the algorithm for different percentages of traces. Unfortunately, the algorithm did not produce good results. We expect the reason for that to be not satisfying the sparsity condition, because seismic images are not sparse in the DCT domain. Fig. 8 shows the DCT of the synthetic seismic data, we see that the DCT coefficients are scattered along the spectrum. Fig. 9 shows the SNR curve of applying compressive sensing on the synthetic seismic data.

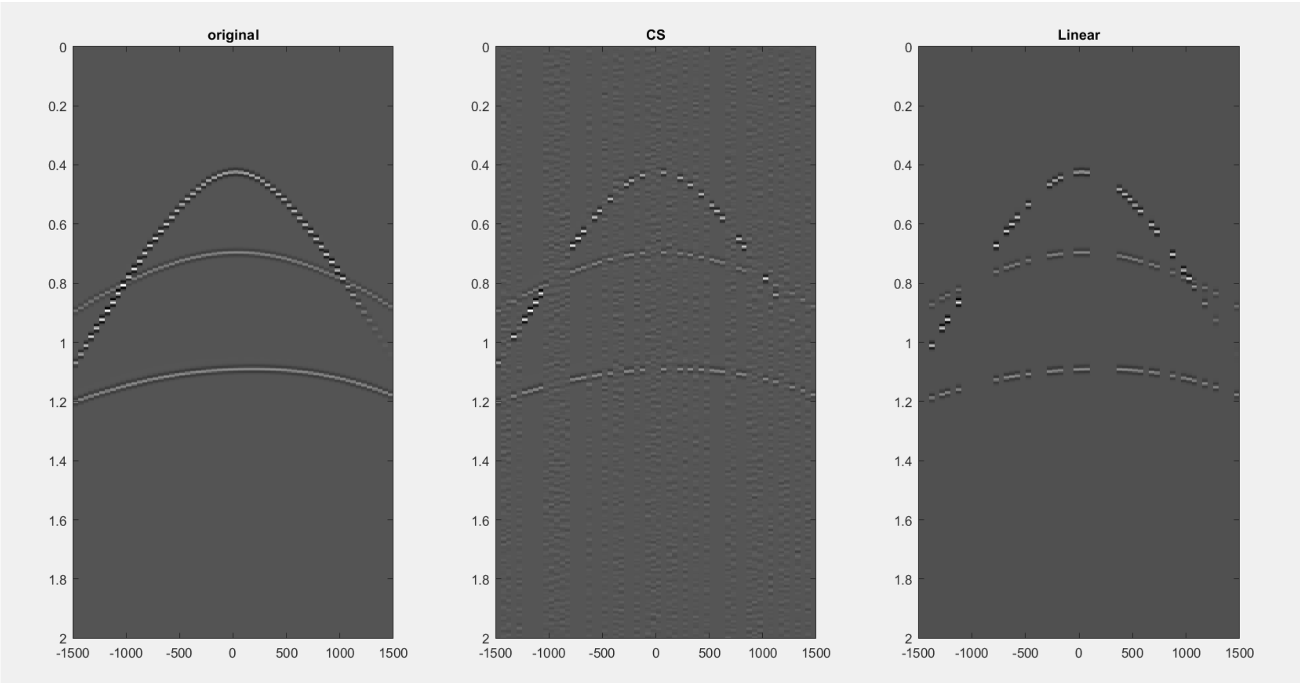
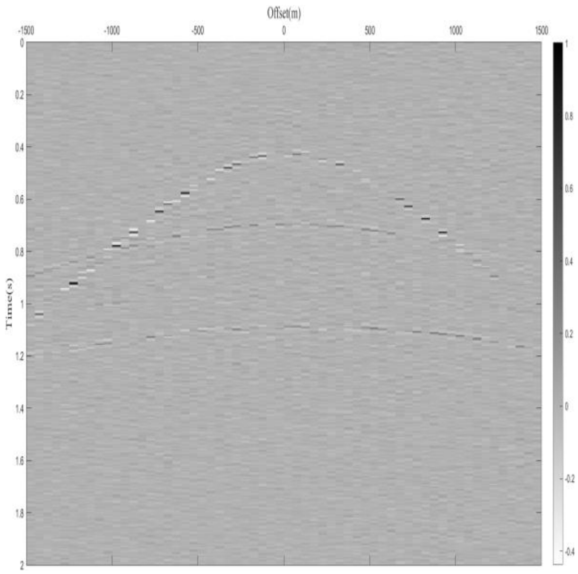
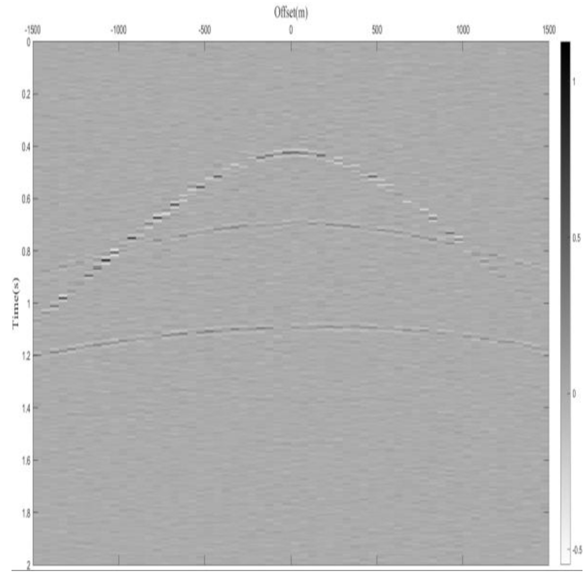


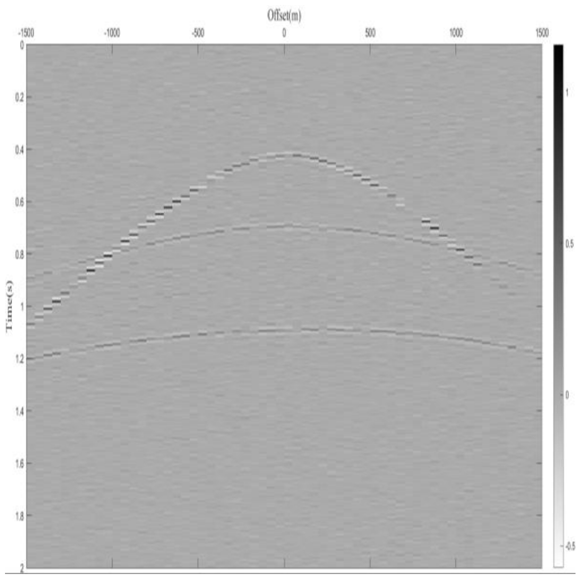
Fig.6 comparison between original seismic image and interpolated one using CS and linear interpolation for 48% available traces



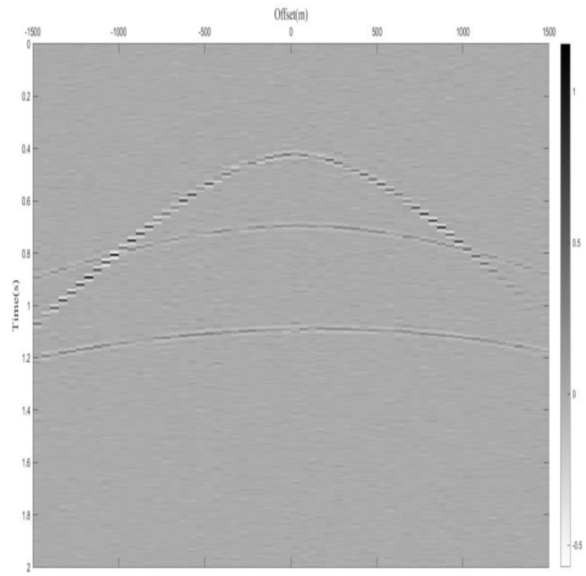
(a)



(b)



(c)



(d)

Fig.7 The result of applying compressive sensing for different percentages of traces: (a) 20%, (b) 30%, (c) 40%, (d) 50%

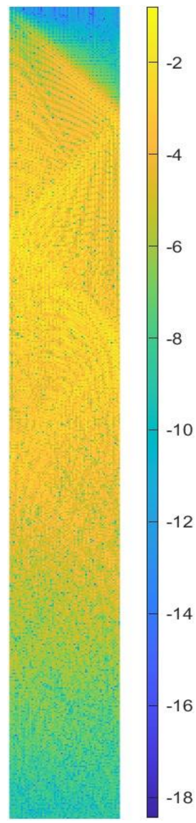


Fig. 8 The DCT of the synthetic seismic image

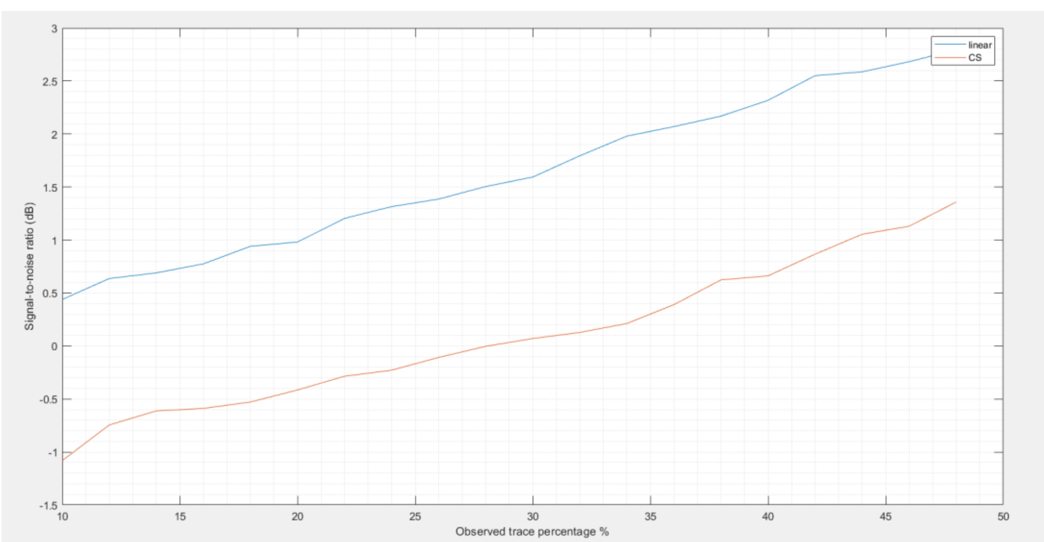


Fig. 9. SNR curves CS vs Linear interpolation for the Seismic data.

V. Attempts to solve the sparsity problem:

Firstly, we tried to use the wavelet transform as our sparsifying transform, but as shown in Fig. 10, the results are disappointing. The SNR experiences a strange behavior, as we increase the number of available traces the signal to noise ratio decreases.

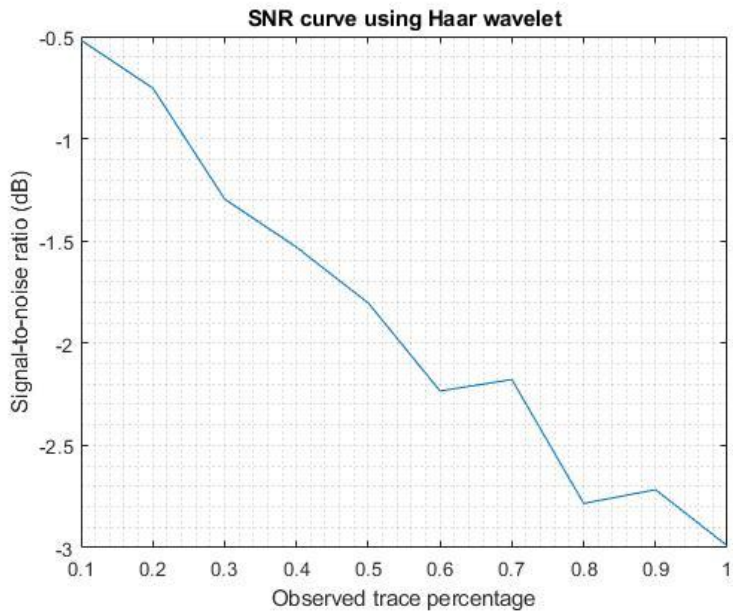


Fig. 10. SNR curves using CS interpolation with Haar wavelet.

Another approach we tried is adopted from Chengbo Li, et al. [1] where they first interpolate the observed data back to nearest points on the nominal grid and then reconstruct using traditional CS. We applied the same idea here but unfortunately the results were still bad.

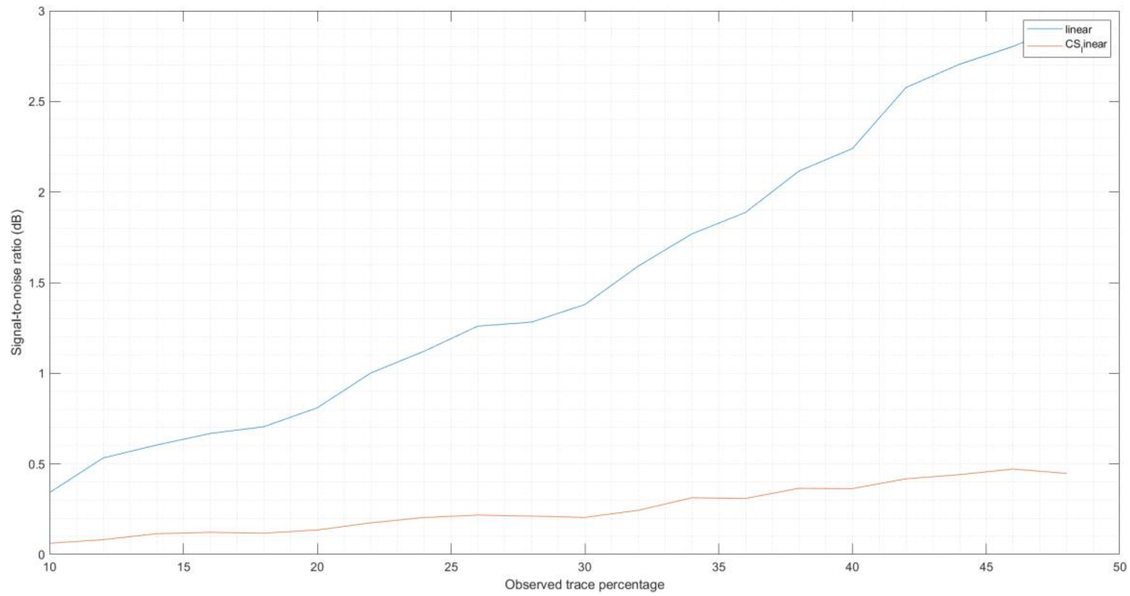


Fig. 11. SNR curves Linear-CS vs Linear interpolation for the Seismic data.

VI. Conclusion

Interpolation of Seismic data traces is a very important topic for the oil and gas industry. One of the interpolation methods is using Compressive Sensing. We managed to apply the theory on a sinusoidal signal perfectly. Then we applied CS to interpolate seismic data, but unfortunately the results were bad compared to basic interpolation methods like linear interpolation. The main obstacle that we faced is finding a sparse transform for seismic data and applying compressive sensing using that transform. We found in the literature that seismic images are sparse in the curvelet transform [3]. However, we could not construct the transform basis ψ and therefore failed in applying compressive sensing on seismic data.

VII. References:

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