

## **10 Appendix D: Craig-Bampton Model Generation**

## 10.1 Craig-Bampton Equations of Motion for Substructures

MYSTRAN has the capability to generate Craig-Bampton (CB) models via SOL 31 (or SOL GEN CB MODEL). This solution sequence calculates the fixed-base modes of a substructure and generates all of the matrices needed to couple the substructure to other CB models. This appendix describes the Craig-Bampton method and its implementation in MYSTRAN and includes an example problem to explain the input and output for SOL 31.

Craig and Bampton<sup>1</sup> are credited with the first unified approach to modal synthesis, or substructuring for dynamic analysis, using fixed interface flexible modes augmented by boundary constraint modes to describe each substructure. Their work was a simplification of earlier work by Hurty<sup>2</sup> who first introduced the concept for substructures with redundant boundary degrees of freedom (DOF's).

In order to explain the Craig-Bampton (CB) method, consider a structure represented by the picture below that is comprised of several (in this case 5) substructures connected at an arbitrary number of points:

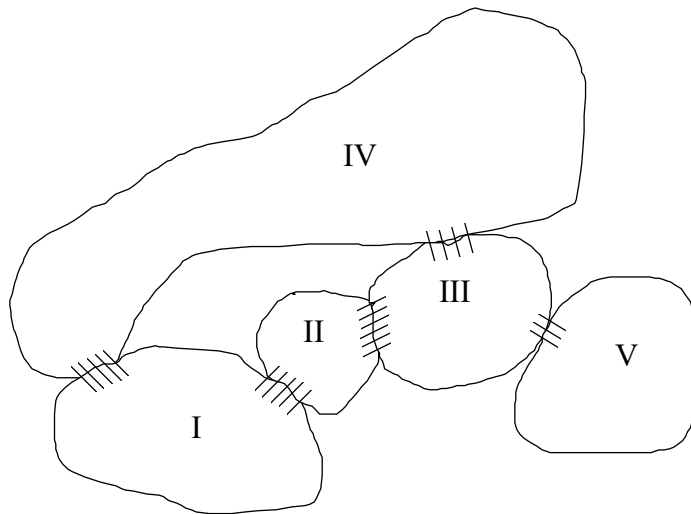


Figure 10.1 - Overall Structure Composed of Several Substructures

Each substructure is joined to one or more other substructures at some number of interface, or boundary, DOF's (indicated by the hatched areas in the above picture. The complete structure, consisting of the connected substructures, may or may not be restrained from free body motion. For any one of the substructures ( $j = I, II, III, \text{ etc.}$ ) the G-set equations of motion (ignoring damping for the moment) are:

<sup>1</sup> Craig, R.R. and Bampton, M.C.C. "Coupling of Substructures for Dynamic Analysis", AIAA Journal, Vol. 6, No. 7, July 1968, pp 1313-1319

<sup>2</sup> Hurty, W.C. "Dynamic Analysis of Structural Systems Using Component Modes", AIAA Journal, Vol. 3, No. 4, April 1965, pp 678-685

$$M_{GG}^j \ddot{u}_G^j + K_{GG}^j u_G^j = P_G^j + Q_G^j$$

where

$$Q_G^j = Q_G^{mj} + Q_G^{sj} + Q_G^{rj}$$

$$u_G^j = \begin{Bmatrix} u_A^j \\ u_O^j \\ u_S^j \\ u_M^j \end{Bmatrix} = \begin{Bmatrix} \text{analysis DOF's} \\ \text{omitted DOF's} \\ \text{SPC'd DOF's} \\ \text{MPC'd DOF's} \end{Bmatrix}$$

and

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$P_G^j$  = applied loads on the G-set

$Q_G^{mj}$  = constraint forces due to multi-point constraints (MPC's)

$Q_G^{sj}$  = constraint forces due to single point constraints (SPC's)

$Q_G^{rj}$  = interface forces at boundaries between substructures

In MYSTRAN nomenclature, the G-set is reduced to the A-set by the elimination of the M-set multi-point constraints, the S-set single point constraints and the O-set omitted DOF's (using OMIT's or ASET's). The A-set DOF's for this substructure must contain all DOF's that will be connected to other substructures. The resulting A-set equations of motion (dropping the j superscript notation for each substructure) are:

$$M_{AA} \ddot{u}_A + K_{AA} u_A = P_A + Q_A^r \quad 10-2$$

where the A set matrices are mathematical reductions from the G-set (see Appendix B for details)

Partition 2 into the R-set and L-set, where, the R-set represents the boundary DOF's in which this substructure connects with other substructures and the L-set are all free interior DOF's in this substructure

$$\begin{bmatrix} M_{RR} & M_{LR}^T \\ M_{LR} & M_{LL} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{u}_L \end{Bmatrix} + \begin{bmatrix} K_{RR} & K_{LR}^T \\ K_{LR} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_R \\ u_L \end{Bmatrix} = \begin{Bmatrix} P_R \\ P_L \end{Bmatrix} + \begin{Bmatrix} Q_R^r \\ o \end{Bmatrix} \quad 10-3$$

Notice at this point that there remain forces of constraint only at the substructure attach points as the L-set represents all free DOF's for this substructure.

At this point we can introduce the transformation from the physical displacements in equation (3) to what are known as the CB DOF's; namely the flexible mode DOF's and the boundary (R-set) DOF's. In order to show that this is not any further approximation to equation 3, consider the following argument:

- 1) the  $u_A = \begin{Bmatrix} u_R \\ u_L \end{Bmatrix}$  DOF's are clearly a complete set of DOF's for the substructure in that, once they are known, the complete g-set DOF's for this substructure can be determined.

2) similarly, a new set of DOF's for the substructure,

$$\mathbf{u}_X = \begin{Bmatrix} \mathbf{u}_R \\ \xi_N \end{Bmatrix} \quad 10-4$$

are a complete set of DOF's if  $\xi_N$  are the generalized DOF's for flexible modes when  $\mathbf{u}_R = 0$

3) Thus we can take  $\mathbf{u}_L$  to be a linear combination of  $\mathbf{u}_R$  and  $\xi_N$  or:

$$\mathbf{u}_L = \mathbf{D}_{LR}\mathbf{u}_R + \Phi_{LN}\xi_N \quad 10-5$$

if we insist that:

a)  $\Phi_{LN}$  are shapes when  $\mathbf{u}_R = 0$  and  $\xi_N$  are modal DOF's. That is, the columns of  $\Phi_{LN}$  are the flexible modes,  $\phi_L^i$ , when the boundary is fixed. The i-th column of the modal matrix  $\Phi_{LN}$  is  $\phi_L^i$ .

b)  $\mathbf{D}_{LR}$  are shapes when  $\xi_N = 0$ . That is, the columns of  $\mathbf{D}_{LR}$  are the L-set shapes for unit motions of the R-set when the flexible mode DOF's are zero.

The  $\phi_L^i$  are easy to understand. They are the eigenvectors resulting from solving an eigenvalue problem from equations 3 with  $\mathbf{u}_R = 0$ . This eigenvalue problem would be:

$$(\mathbf{K}_{LL} - \omega^2 \mathbf{M}_{LL})\phi_L = 0 \quad 10-6$$

This requires that the determinant of the coefficient matrix on the left side of equation 6 be zero:

$$|\mathbf{K}_{LL} - \omega^2 \mathbf{M}_{LL}| = 0 \quad \text{which yields } N \text{ eigenvalues } \omega_1^2, \omega_2^2, \dots, \omega_N^2 > 0 \quad 10-7$$

The i-th eigenvector,  $\phi_L^i$ , is then determined by solving the equation:

$$(\mathbf{K}_{LL} - \omega_i^2 \mathbf{M}_{LL})\phi_L^i = 0 \quad \text{for } i = 1, 2, \dots, N \quad 10-8$$

Solution of equation 8 requires that one element of  $\phi_L^i$  be arbitrarily set (the  $\phi_L^i$  are shapes and their amplitude does not matter). Once equation 8 is solved, the modal matrix is:

$$\Phi_{LN} = \begin{bmatrix} \phi_L^1 & \phi_L^2 & \dots & \phi_L^N \end{bmatrix} \quad 10-9$$

The  $\mathbf{D}_{LR}$  can also be explained easily. As stated above, the  $\mathbf{D}_{LR}$  are shapes when the flexible mode response is zero. We can see from equation 5 that a column of  $\mathbf{D}_{LR}$  represents the displacements at the L-set DOF's due to motion at one of the R-set DOF's while all other R-set DOF's are zero (as well

as all  $\xi_N = 0$  ). We can therefore solve for  $D_{LR}$  from equation 3 by taking all applied forces and accelerations equal to zero and solving the statics problem:

$$\begin{bmatrix} K_{RR} & K_{LR}^T \\ K_{LR} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_R \\ u_L^s \end{Bmatrix} = \begin{Bmatrix} Q_R^r \\ 0 \end{Bmatrix} \quad 10-10$$

where  $u_L^s$  are static displacements of the L-set. From the second row of equation 10, solve for  $u_L^s$  in terms of  $u_R$  :

$$\begin{aligned} u_L^s &= -K_{LL}^{-1} K_{LR} u_R = D_{LR} u_R \\ \text{or} \\ D_{LR} &= -K_{LL}^{-1} K_{LR} \end{aligned} \quad 10-11$$

Thus, the CB DOF's are contained in  $u_X$  (equation 4) and the transformation between  $u_X$  and  $u_A$  is:

$$\begin{Bmatrix} u_R \\ u_L \end{Bmatrix} = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \quad 10-12$$

where  $I$  is an  $R \times R$  identity matrix. Equation 12 can be written as:

$$\begin{aligned} u_A &= \Psi_{AX} u_X \\ \text{where} \\ \Psi_{AX} &= \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix}, \quad u_A = \begin{Bmatrix} u_R \\ u_L \end{Bmatrix}, \quad u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \end{aligned} \quad 10-13$$

$\Psi_{AX}$  is the CB transformation matrix and is of A-set size. In MYSTRAN this is called matrix PHIXA. When expanded to G-set size, PHIXA becomes matrix PHIXG:

$$\begin{aligned} u_G &= \Psi_{GX} u_X \\ \Psi_{GX} &= \text{matrix data block PHIXG} \\ \text{PHIXG} &= \text{PHIXA expanded to G-set} \end{aligned}$$

$$\quad 10-14$$

Note that when all flexible modes of the substructure are used in  $u_X$  equation 13 is exact. In practice, all modes are never used since this would defeat the purpose of making the transformation (which is to find a smaller set of DOF's which are nonetheless an accurate representation of the A-set). Substituting equation 13 into equation 2 and premultiplying the result by the transpose of  $\Psi_{AX}$  yields:

$$M_{xx} \ddot{u}_X + K_{xx} u_X = P_X + Q_X^r \quad 10-15$$

where:

$$\mathbf{M}_{XX} = \Psi_{AX}^T \mathbf{M}_{AA} \Psi_{AX} = \begin{bmatrix} \mathbf{I} & \mathbf{D}_{LR}^T \\ 0 & \Phi_{LN}^T \end{bmatrix} \begin{bmatrix} \mathbf{M}_{RR} & \mathbf{M}_{LR}^T \\ \mathbf{M}_{LR} & \mathbf{M}_{LL} \end{bmatrix} \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{D}_{LR} & \Phi_{LN} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_{RR} & \mathbf{m}_{NR}^T \\ \mathbf{m}_{NR} & \mathbf{m}_{NN} \end{bmatrix}$$

$$\mathbf{K}_{XX} = \Psi_{AX}^T \mathbf{K}_{AA} \Psi_{AX} = \begin{bmatrix} \mathbf{I} & \mathbf{D}_{LR}^T \\ 0 & \Phi_{LN}^T \end{bmatrix} \begin{bmatrix} \mathbf{K}_{RR} & \mathbf{K}_{LR}^T \\ \mathbf{K}_{LR} & \mathbf{K}_{LL} \end{bmatrix} \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{D}_{LR} & \Phi_{LN} \end{bmatrix} = \begin{bmatrix} \mathbf{k}_{RR} & 0 \\ 0 & \mathbf{k}_{NN} \end{bmatrix}$$

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$$\mathbf{P}_X = \Psi_{AX}^T \mathbf{P}_A = \begin{bmatrix} \mathbf{I} & \mathbf{D}_{LR}^T \\ 0 & \Phi_{LN}^T \end{bmatrix} \begin{Bmatrix} \mathbf{P}_R \\ \mathbf{P}_L \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}'_R \\ \Xi_N \end{Bmatrix}, \quad \mathbf{P}'_R = \mathbf{P}_R + \mathbf{D}_{LR}^T \mathbf{P}_L, \quad \Xi_N = \Phi_{LN}^T \mathbf{P}_L$$

$$\mathbf{Q}_X^r = \Psi_{AX}^T \mathbf{Q}_A^r = \begin{bmatrix} \mathbf{I} & \mathbf{D}_{LR}^T \\ 0 & \Phi_{LN}^T \end{bmatrix} \begin{Bmatrix} \mathbf{Q}_R^r \\ \mathbf{o} \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q}_R^r \\ 0 \end{Bmatrix}$$

and:

$$\begin{aligned} \mathbf{m}_{RR} &= \mathbf{M}_{RR} + \mathbf{M}_{LR}^T \mathbf{D}_{LR} + (\mathbf{M}_{LR}^T \mathbf{D}_{LR})^T + \mathbf{D}_{LR}^T \mathbf{M}_{LL} \mathbf{D}_{LR} \\ \mathbf{m}_{NR} &= \Phi_{LN}^T (\mathbf{M}_{LR} + \mathbf{M}_{LL} \mathbf{D}_{LR}) \\ \mathbf{m}_{NN} &= \Phi_{LN}^T \mathbf{M}_{LL} \Phi_{LN} \\ \mathbf{k}_{RR} &= \mathbf{K}_{RR} + \mathbf{K}_{LR}^T \mathbf{D}_{LR} \\ \mathbf{k}_{NN} &= \Phi_{LN}^T \mathbf{K}_{LL} \Phi_{LN} \end{aligned}$$

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$\mathbf{m}_{NN}$ ,  $\mathbf{k}_{NN}$  are diagonal matrices of generalized masses and stiffnesses, respectively.

Equations 15 for the i-th substructure can be written as:

$$\begin{bmatrix} \mathbf{m}_{RR} & \mathbf{m}_{NR}^T \\ \mathbf{m}_{NR} & \mathbf{m}_{NN} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_R \\ \ddot{\xi}_N \end{Bmatrix} + \begin{bmatrix} \mathbf{k}_{RR} & 0 \\ 0 & \mathbf{k}_{NN} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_R \\ \xi_N \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}'_R \\ \Xi_N \end{Bmatrix} + \begin{Bmatrix} \mathbf{Q}_R^r \\ 0 \end{Bmatrix}$$

10-18

The off-diagonal terms in the above stiffness matrix are zero due to the definition of  $\mathbf{D}_{LR}$  in equation 11. In addition, matrix  $\mathbf{k}_{RR}$  in equation 18 is null if the boundary is a determinant interface. Equations 14 and 18 are the Craig-Bampton equations of motion for the i-th substructure. The  $\mathbf{P}'_R$  are due to applied loads on the R and L-set DOF's (see equation 16) and the  $\mathbf{Q}_R^r$  are the interface forces where substructures connect. Once the equations are developed for all substructures, the individual substructures can be connected and the resulting equations solved for the combined R-set and N-set DOF's  $\mathbf{u}_R$  and  $\xi_N$  for all substructures. Once this is done, the forces of inter-connection, or substructure interface forces, (that is, the  $\mathbf{Q}_R^r$ ) can be solved from the individual substructure

equations in the top row of equation 18. Equation 14 is used to obtain displacements for all G-set DOF's.

Each organization that is developing a substructure in CB format would deliver the above coefficient matrices in equations 14 and 18 to the organization that is doing the combined structure analysis. In addition, Displacement and Load Transformation Matrices (DTM's and LTM's) collectively known as Output Transformation Matrices, (OTM's), described below, are also delivered as part of the CB model.

## 10.2 Development of Displ Output Transformation Matrices (Displ OTM's)

Typically, a set of displacement output transformation matrices (displ OTM's, or DTM's for short), is delivered with a Craig-Bampton model to the organization that will couple all substructures and solve for the primary unknowns ( $u_R$  and  $\xi_N$  and  $Q_R^r$ ) in order that desired displacements at some of the substructure G-set DOF's may be obtained along with the coupled solution.

Once the combined structure has been solved for the primary variables, the original  $u_L$  physical DOF's could be determined from equation 5 and then element forces and stresses could be determined from the  $u_R$  and  $u_L$  displacements. This is called recovery of the  $u_L$  DOF's and element forces and stresses using the Modal Displacement Method (MDM). However, as is often the case, equations 18 are solved using a severely truncated set of modes for each substructure. While this may not compromise the accuracy of the solutions for  $u_R$  and  $\xi_N$ , it could compromise the accuracy of element forces and stresses calculated using displacements determined from equation 5 with the truncated set of modes. In order to avoid this problem, the  $u_L$  DOF's can be found using the Modal Acceleration Method (MAM), described below. It should be noted that the MAM described below *ignores* damping forces so that it is only useful when the damping is small (e.g. less than 10% or so).

From the bottom row of equation 3, solve for  $u_L$  in terms of the other variables in the equation:

$$\begin{aligned} u_L &= -K_{LL}^{-1}(M_{LR}\ddot{u}_R + M_{LL}\ddot{u}_L) - K_{LL}^{-1}K_{LR}u_R + K_{LL}^{-1}P_L \\ &= -K_{LL}^{-1}(M_{LR}\ddot{u}_R + M_{LL}\ddot{u}_L) + D_{LR}u_R + K_{LL}^{-1}P_L \end{aligned} \quad 10-19$$

Differentiate equation 5 twice and use the result for  $\ddot{u}_L$  in equation 19, to get:

$$u_L = \left[ -K_{LL}^{-1}(M_{LR} + M_{LL}D_{LR}) \mid -K_{LL}^{-1}M_{LL}\Phi_{LN} \mid D_{LR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + K_{LL}^{-1}P \quad 10-20$$

The term  $K_{LL}^{-1}M_{LL}\Phi_{LN}$  in equation 20. can be written in a form more convenient for calculation. From equation 8 it can be seen that:

$$K_{LL}^{-1}M_{LL}\phi_L^i = \frac{1}{\omega_i^2} \phi_L^i$$

so that

$$K_{LL}^{-1} M_{LL} \begin{bmatrix} \varphi_1^1 & \varphi_1^2 & \cdots & \varphi_L^N \end{bmatrix} = \begin{bmatrix} \varphi_1^1 & \varphi_1^2 & \cdots & \varphi_L^N \end{bmatrix} \begin{bmatrix} \omega_1^{-2} & & & \\ & \omega_2^{-2} & & \\ & & \ddots & \\ & & & \omega_N^{-2} \end{bmatrix}$$

or

$$K_{LL}^{-1} M_{LL} \Phi_{LN} = \Phi_{LN} \Omega_{NN}^{-2} \quad 10-21$$

where

$$\Omega_{NN}^{-2} = \begin{bmatrix} \omega_1^{-2} & & & \\ & \omega_2^{-2} & & \\ & & \ddots & \\ & & & \omega_N^{-2} \end{bmatrix} \quad 10-22$$

substitute equation 21 into equation 20 to get:

$$u_L = \left[ -K_{LL}^{-1} (M_{LR} + M_{LL} D_{LR}) \mid -\Phi_{LN} \Omega_{NN}^{-2} \mid D_{LR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + K_{LL}^{-1} P_L \quad 10-23$$

The various terms in the coefficient matrices in equation 23 are known as Displacement Transformation Matrices (DTM's). Equation 23 can be written as:

$$u_L = \left[ DTM1_{LR} \mid DTM2_{LN} \mid DTM3_{LR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + DTM4_{LL} P_L \quad 10-24$$

where

$$\begin{aligned} DTM1_{LR} &= -K_{LL}^{-1} (M_{LR} + M_{LL} D_{LT}) \\ DTM2_{LN} &= -\Phi_{LN} \Omega_{NN}^{-2} \\ DTM3_{LR} &= D_{LR} \\ DTM4_{LL} &= K_{LL}^{-1} \end{aligned} \quad 10-25$$

Equations 24 and 25 represent the MAM for recovering displacements for the L-set, for the i-th substructure, once the assembled substructure equations have been solved for the  $u_R$  and  $q_N$  DOF's. Once the L-set displacements have been found, recovery of the remaining displacements in



the G-set is accomplished through the transformation matrices used in their elimination from equation 1 (for details see Appendix B). At the G-set level, equation 24 is:

$$u_G = [DTM1_{GR} \mid DTM2_{GN} \mid DTM3_{GR}] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + DTM4_{GL} P_L$$

or

$$u_G = \Gamma_{GZ} u_Z + DTM4_{GL} P_L$$

where

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$$\Gamma_{GZ} = [DTM1_{GR} \mid DTM2_{GN} \mid DTM3_{GR}] = DTM_{GZ}$$

and

$$u_Z = \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix}, \quad \text{where } u_Z \text{ are the Craig-Bampton Degrees of freedom (CB\_DOF's)}$$

where each of the G-set DTM's in equation 26 is obtained from the L-set DTM's in equation 25 through the normal recovery operations to build back up to the G-set from the L-set. The coefficient matrix in equation 26 that has DTM's 1 - 3 in it is called matrix PHIZG. The table below explains the meaning of each of the DTM's in equation 26:

Table 10.1

i-th col of:	Represents:
$DTM1_{GR}$	displ's of G-set due to a unit accel of the i-th interface DOF (all other R, N set DOF's zero)
$DTM2_{GN}$	displ's of G-set due to a unit accel of the i-th flex mode DOF (all other R, N set DOF's zero)
$DTM3_{GR}$	displ's of G-set due to a unit displ of the i-th interface DOF (all other R, N set DOF's zero)
$DTM4_{GL}$	displ's of G-set due to a unit force on the i-th L-set DOF (all other L-set forces zero)

### 10.3 Development of Load Output Transformation Matrices (Load OTM's)

Once the G-set displacements have been found, substructure element forces and stresses, as well as grid point forces, can be recovered and assembled into a Loads Output Transformation Matrix, or Load OTM (more commonly referred to as LTM). There are several types of quantities one may desire in an LTM. Equations are developed, below, for several types of LTM quantities typically used in CB analyses.

#### 10.3.1 LTM Terms for Substructure Interface Forces

From the top row of equation 18, the interface forces can be determined once the substructures have been coupled and the  $u_R$  and  $\xi_N$  solved. The interface forces are:

$$Q_R^r = m_{RR} \ddot{u}_R + m_{NR}^T \ddot{\xi}_N + k_{RR} u_R - P_R'$$

or

$$Q_R^r = \begin{bmatrix} m_{RR} & m_{NR}^T & k_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - I_{RR} P_R' \quad 10-27$$

where  $I_{RR}$  is an  $R \times R$  identity matrix. Equation 27 can be written as:

$$Q_R^r = \begin{bmatrix} LTM21_{RR} & LTM22_{RN} & LTM23_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - LTM24_{RR} P_R^r$$

or

$$Q_R^r = J_{RZ} U_Z - I_{RR} P_R^r$$

where

$$J_{RZ} = \begin{bmatrix} LTM21_{RR} & LTM22_{RN} & LTM23_{RR} \end{bmatrix} = LTM2_{RZ} \quad 10-28$$

$$LTM21_{RR} = m_{RR}$$

$$LTM22_{RN} = m_{NR}^T$$

$$LTM23_{RR} = k_{RR}$$

$$LTM24_{RR} = I_{RR}$$

#### 10.3.2 LTM Terms for Net cg Loads

Terms can also be included in the overall LTM that will recover what are known as “net” accelerations at the center of gravity (cg) of the CB model. These are termed Net Load factors (NLF's) and represent rigid body accelerations of the cg due to the reaction (or interface) forces,  $Q_R^r$ . The development below demonstrates how these are determined.

Define:

$u_{cg} = 6 \times 1$  matrix of rigid body displacements of the cg of the substructure

$u_{R_{rb}} = r \times 1$  vector of rigid body displacements at the  $r$  DOF

$T_{R6} = r \times 6$  matrix where each column represents rigid body displacements of the  $r$  DOF due to a unit motion in one DOF at the cg 10-29

$Q_{cg} = 6 \times 1$  vector of forces at the cg that are static equivalents of  $Q_r^r$

Then:

$$\begin{aligned} u_{R_{rb}} &= T_{R6} u_{cg} \\ \text{and} \\ Q_{cg} &= T_{R6}^T Q_r^r \end{aligned} \quad 10-30$$

Substitute equation 27 into 30 for  $Q_r^r$ :

$$Q_{cg} = T_{R6}^T (m_{RR} \ddot{u}_R + m_{NR}^T \ddot{\xi}_N + k_{RR} u_R - P_R') \quad 10-31$$

For rigid body motion:

$$Q_{cg} = m_{cg} \ddot{u}_{cg} \quad 10-32$$

where  $m_{cg}$  is the  $6 \times 6$  rigid body mass matrix relative to the cg and is equal to:

$$m_{cg} = T_{R6}^T m_{RR} T_{R6} \quad 10-33$$

and  $m_{RR}$  is given in equation 17. From equations 31 through 33 we can write the cg acceleration net load factors (NLF's) as:

$$\ddot{u}_{cg} = m_{cg}^{-1} Q_{cg} = m_{cg}^{-1} T_{R6}^T \left[ m_{RR} \quad m_{NR}^T \quad k_{RR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - m_{cg}^{-1} T_{R6}^T P_R' \quad 10-34$$

However,  $T_{R6}^T k_{RR} = 0$  since the columns of  $T_{R6}$  are rigid body modes. Therefore:

$$\ddot{u}_{cg} = m_{cg}^{-1} Q_{cg} = m_{cg}^{-1} T_{R6}^T \left[ m_{RR} \quad m_{NR}^T \quad 0 \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - m_{cg}^{-1} T_{R6}^T P_R' \quad 10-35$$

which can be written as:

$$\ddot{u}_{cg} = \begin{bmatrix} \text{LTM11}_{6R} & \text{LTM12}_{6N} & 0_{6R} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\zeta}_N \\ u_R \end{Bmatrix} - [\text{LTM14}_{6R}] P'_R$$

where

$$\begin{aligned} \text{LTM11}_{6R} &= m_{cg}^{-1} T_{R6}^T m_{RR} \\ \text{LTM12}_{6N} &= m_{cg}^{-1} T_{R6}^T m_{NR}^T \\ \text{LTM14}_{6R} &= m_{cg}^{-1} T_{R6}^T \\ \text{LTM1}_{6Z} &= [\text{LTM11}_{6R} \quad \text{LTM12}_{6N} \quad 0] \end{aligned}$$

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### 10.3.3 LTM Terms for Element Forces and Stresses

In MYSTRAN, element forces and stresses are obtained from the G-set displacement vector and the individual element stiffness matrices. Equation 26 is the G-set displacement vector:

$$u_G = [\text{DTM1}_{GR} \mid \text{DTM2}_{GN} \mid \text{DTM3}_{GR}] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\zeta}_N \\ u_R \end{Bmatrix} + \text{DTM4}_{GL} P_L = \Gamma_{GZ} u_Z + \text{DTM4}_{GL} P_L$$

Thus the columns of each of the DTM's represents G-set displacements per unit value of one of the variables  $\ddot{u}_R, \ddot{\zeta}_N, u_R, P_L$  as described in Table 10.1. Therefore, each of the DTM's can be used as if they were a matrix of displacements in calculating element forces and stresses to give:

$$f_e = [\text{LTM31}_{eR} \mid \text{LTM32}_{eN} \mid \text{LTM33}_{eR}] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\zeta}_N \\ u_R \end{Bmatrix} + \text{LTM34}_{eL} P_L$$

where

$f_e$  = vector of element forces and stresses (e = number of finite elements )

$\text{LTM31}_{eR}$  = matrix of element forces and stresses due to G-set displ's  $\text{DTM1}_{GR}$

$\text{LTM32}_{eN}$  = matrix of element forces and stresses due to G-set displ's  $\text{DTM2}_{GN}$

$\text{LTM33}_{eR}$  = matrix of element forces and stresses due to G-set displ's  $\text{DTM3}_{GR}$

$\text{LTM34}_{eL}$  = matrix of element forces and stresses due to G-set displ's  $\text{DTM4}_{GL}$

$\text{LTM3}_{eZ} = [\text{LTM31}_{eR} \mid \text{LTM32}_{eN} \mid \text{LTM33}_{eR}]$

10-37

### 10.3.4 LTM Terms for Grid Point Forces due to multi-point constraints (MPC's)

There are cases in CB analyses in which the forces due to MPC's are of interest. As an example, if a user wishes to determine a load in a bolt at an interface between components, it is common to model the bolt as an MPC where two coincident grids are constrained to have the same displacements. This section develops the equations for determining an LTM for grid point MPC forces.

Equation 1 for the i-th substructure (dropping the superscript-j notation):

$$\mathbf{M}_{GG}\ddot{\mathbf{u}}_G + \mathbf{K}_{GG}\mathbf{u}_G = \mathbf{P}_G + \mathbf{Q}_G^s + \mathbf{Q}_G^m + \mathbf{Q}_G^r \quad 10-38$$

As described in section 10.1 the Q constraint forces on the right side of equation 38 are the constraint forces on the S-set SPC DOF's, the M-set MPC DOF's and on the R-set boundary DOF's respectively. Since all of the boundary DOF's are contained in the R-set there should be no constraint forces on the S-set. That is, all S-set DOF's should be the result of removing singularities and not the result of grounding the model<sup>3</sup>. With this assumption, as well as the assumption that there are no applied loads on the M-set degrees of freedom the following equation is valid for the MPC forces on the M-set grids:

$$\mathbf{Q}_G^m = \mathbf{M}_{GG}\ddot{\mathbf{u}}_G + \mathbf{K}_{GG}\mathbf{u}_G - \mathbf{Q}_G^r \quad 10-39$$

We want to get 39 in a form like the other LTM'; that is, in terms of  $\mathbf{u}_Z$ .

From equation 26 with applied loads zero:

$$\mathbf{u}_G = \Gamma_{GZ}\mathbf{u}_Z, \quad \mathbf{u}_Z = \begin{Bmatrix} \ddot{\mathbf{u}}_R \\ \ddot{\xi}_N \\ \mathbf{u}_R \end{Bmatrix} \quad 10-40$$

The g-set DOF vector can also be written using equation 14:

$$\mathbf{u}_G = \Psi_{GX}\mathbf{u}_X, \quad \mathbf{u}_X = \begin{Bmatrix} \mathbf{u}_R \\ \xi_N \end{Bmatrix} \quad 10-41$$

Differentiating twice:

$$\ddot{\mathbf{u}}_G = \Psi_{GX}\ddot{\mathbf{u}}_X$$

This can also be written as:

$$\ddot{\mathbf{u}}_G = \begin{bmatrix} \Psi_{GX} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_X \\ \mathbf{u}_R \end{Bmatrix} \quad 10-42$$

Partition the x DOF's into R and N as in equation 13. This will require partitioning  $\Psi_{GX}$  into sub-matrices for the R and N also, so that equation 42 can be written as:

---

<sup>3</sup> This should be verified by the user by inspection of the forces of single point constraint in the output from the analysis

$$\ddot{\mathbf{u}}_G = \begin{bmatrix} \Psi_{GR} & \Psi_{GN} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_R \\ \ddot{\boldsymbol{\zeta}}_N \\ \mathbf{u}_R \end{Bmatrix} = \Psi'_{GZ} \mathbf{u}_Z$$

where

$$\Psi'_{GZ} = \begin{bmatrix} \Psi_{GR} & \Psi_{GN} & 0 \end{bmatrix} = \begin{bmatrix} \Psi_{GX} & 0 \end{bmatrix}$$

10-43

Substitute equations 40 and 43 into 39 for  $\mathbf{u}_G$  and  $\ddot{\mathbf{u}}_G$  respectively to get:

$$\mathbf{Q}_G^m = \mathbf{M}_{GG} \Psi'_{GZ} \mathbf{u}_Z + \mathbf{K}_{GG} \Gamma_{GZ} \mathbf{u}_Z - \mathbf{Q}_G^r \quad 10-44$$

We need to express the boundary constraint forces in equation 44 in terms of the  $\mathbf{u}_Z$  vector as we did for the inertia and stiffness terms. From 28:

$$\mathbf{Q}_R^r = \mathbf{J}_{RZ} \mathbf{u}_Z - \mathbf{I}_{RR} \mathbf{P}_R \quad 10-45$$

The  $\mathbf{Q}_R^r$  boundary forces on the R-set can be expanded from the R-set to the G-set  $\mathbf{Q}_G^r$  by adding zero rows to 45 for the M, S, O-sets (all of the G-set but the R degrees of freedom) to give

$$\mathbf{Q}_G^r = \mathbf{J}_{GZ} \mathbf{u}_Z - \mathbf{I}_{GR} \mathbf{P}_R \quad 10-46$$

where  $\mathbf{J}_{GZ}$  is  $\mathbf{J}_{RZ}$  expanded to G-set size by addition of zero rows for M, S, O-sets and  $\mathbf{I}_{GR}$  is expanded from  $\mathbf{I}_{RR}$  in the same fashion (recall  $\mathbf{I}_{RR}$  is an R size identity matrix). Substituting 46 into 44 we get::

$$\mathbf{Q}_G^m = (\mathbf{M}_{GG} \Psi'_{GZ} + \mathbf{K}_{GG} \Gamma_{GZ} - \mathbf{J}_{GZ}) \mathbf{u}_Z$$

or

$$\mathbf{Q}_G^m = \mathbf{LTM4}_{GZ} \mathbf{u}_Z \quad 10-47$$

where

$$\mathbf{LTM4}_{GZ} = (\mathbf{M}_{GG} \Psi'_{GZ} + \mathbf{K}_{GG} \Gamma_{GZ} - \mathbf{J}_{GZ})$$

$\mathbf{LTM4}_{GZ}$  is the LTM for MPC forces at grids that have no applied load

#### 10.4 Development of Acceleration Output Transfer Matrices (Accel OTM)

In addition to the displacement and load output transformation matrices (DTM's and LTM's) it is common to supply acceleration output transformation matrices (accel OTM's or ATM's for short). From equation 10-12 and differentiating twice we obtain:

$$\begin{Bmatrix} \ddot{u}_R \\ \ddot{u}_L \end{Bmatrix} = [ATM] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \end{Bmatrix}$$

where

10-48

$$ATM = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix}$$

ATM is the acceleration transfer matrix. Notice that the “degrees of freedom” for the ATM are the accelerations of the boundary and modal degrees of freedom whereas all of the other OTM's have as degrees of freedom: boundary accelerations, modal accelerations and boundary displacements. This is due to the use of the modal acceleration method for recovery of displacements and element forces.

## 10.5 Correspondence between matrix names and CB Equation Variables

The table below shows the correspondence between variables introduced in the above equations and matrix data block names in the DMAP program in Section 10.5. Any of these may be output in a MYSTRAN CB model generation analysis using the Executive Control entry OUTPUT4.

**Table 10-2**  
**Matrices that can be written to OUTPUT4 files**

	MYSTRAN Matrix Name (OUTPUT4 matrices)	NASTRAN DMAP Name	CB equation variable in Appendix D (where applicable)	Matrix size <sup>1</sup>	Partition rows and/or cols
1	CG_LTM		$[LTM11_{6r} \quad LTM12_{6N} \quad 0]$	$6 \times (2R+N)$	
2	DLR	DM	$D_{LR}$	$L \times R$	rows and cols
3	EIGEN_VAL	LAMA	$\Omega_{NN}^2$	$N \times N$	
4	EIGEN_VEC	PHIG	$\Phi_{GN}$ , ( $\Phi_{LN}$ with rows expanded to G-set)	$G \times N$	rows
5	GEN_MASS	MI	$m_{NN}$	$N \times 1$ vector of diag. terms	
6	IF_LTM		$[LTM21_{RR} \quad LTM22_{RN} \quad LTM23_{RR}]$	$R \times (2R+N)$	rows
7	KAA	KAA	$K_{AA}$	$A \times A$	rows and cols
8	KGG	KGG	$K_{GG}$	$G \times G$	rows and cols
9	KLL	KLL	$K_{LL}$	$L \times L$	rows and cols
10	KRL	KLR(t)	$K_{LR}$	$L \times R$	rows and cols
11	KRR	KRR	$K_{RR}$	$R \times R$	rows and cols
12	KRRcb	KBB	$k_{RR} = K_{RR} + K_{LR}^T D_{LR}$	$R \times R$	rows and cols
13	KXX	KRRGN	$K_{XX}$	$(R+N) \times (R+N)$	
14	LTM	LTM	CG_LTM and IF_LTM merged	$(6+R) \times (2R+N)$	
15	MCG	RBMCG	$m_{cg}$	$6 \times 6$	
16	MEFFMASS		Modal effective mass	$N \times 6$	
17	MPFACTOR		Modal participation factors	$N \times 6$ or $N \times R$	
18	MAA		$M_{AA}$	$A \times A$	rows and cols
19	MGG		$M_{GG}$	$G \times G$	rows and cols
20	MLL	MLL	$M_{LL}$	$L \times L$	rows and cols
21	MRL	MRL	$M_{RL}$	$R \times L$	rows and cols
22	MRN		$m_{RN} = m_{NR}^T$	$R \times N$	rows
23	MRR	MRR	$M_{RR}$	$R \times R$	rows and cols



**Table 10-2 (con't)**

	MYSTRAN Matrix Name (OUTPUT4 matrices)	NASTRAN DMAP Name	CB equation variable in Appendix D (where applicable)	Matrix size <sup>4</sup>	Partition rows and/or cols
24	MRRcb	MBB	$m_{RR} = M_{RR} + M_{LR}^T D_{LR} + (M_{LR}^T D_{LR})^T + D_{LR}^T M_{LL} D_{LR}$	RxR	rows and cols
25	MXX	MRRGN	$M_{XX} = \begin{bmatrix} m_{RR} & m_{NR}^T \\ m_{NR} & m_{NN} \end{bmatrix}$	(R+N)x(R+N)	
26	PA		(A-set static reduced loads - only used in statics)		Rows
27	PG		(G-set static loads - only used in statics)		Rows
28	PL		(L-set static reduced loads - only used in statics)		rows
29	PHIXG	PHIXG	$\Psi_{AX}$ , ( $\Psi_{AX}$ with rows expanded to G-set)	Gx(R+N)	rows
30	PHIZG		The G-set displacement transformation matrix is written out in the F06 file under "C B D I S P L A C E M E N T O T M"	Gx(2R+N)	rows
31	RBM0		Rigid body mass matrix relative to the basic origin	6x6	
32	TR6_0	RBR	$T_{R6}$ : rigid body displacement matrix for R-set relative to the model basic coordinate system	Rx6	rows
33	TR6_CG	RBR CG	$T_{R6}$ : rigid body displacement matrix for R-set relative to the model CG	Rx6	rows

Notes:

- (t) indicates matrix transposition
- Matrix  $m_{RR}$  will be singular if there are rotational DOF's but no rotational inertia in the R-set, in which case small rotational inertias may have to be added at these DOF's.
- Matrix  $k_{RR}$  is null if the boundary is a determinant set of DOF's.
- Matrix  $m_{RR}$  is the rigid body mass matrix if the boundary is a determinant set of DOF's

<sup>4</sup> Matrix size given in rows x columns where R means the size of the R-set, L is the size of the L-set, A is the size of the A-set, G is the size of the G-set and N is the number of eigenvectors. See section 3.6 for definition of the complete displacement set notation

## 10.6 Craig-Bampton model generation example problem

The figure below shows a small example problem that is a frame made of CBAR's that is a substructure assumed to be attached to some other structure in DOF's 1,2,3 at grids 11 and 13 and in DOF's 2,3 at grid 12. The example problem F06 file (with the input echo'd) is shown on the following pages. This section will discuss the input and output in an effort to explain the Craig-Bampton model generation process.

Equation 10.26 defines the Craig-Bampton degrees of freedom (CB-DOF's) as  $U_z$  which, for this example, consists of the 18 DOF's:

- 8 boundary acceleration DOF's,  $\ddot{U}_R$
- 2 modal acceleration DOF's,  $\ddot{\xi}_N$  (see EIGRL request for 2 modes to be extracted)
- 8 boundary displacement DOF's,  $U_R$

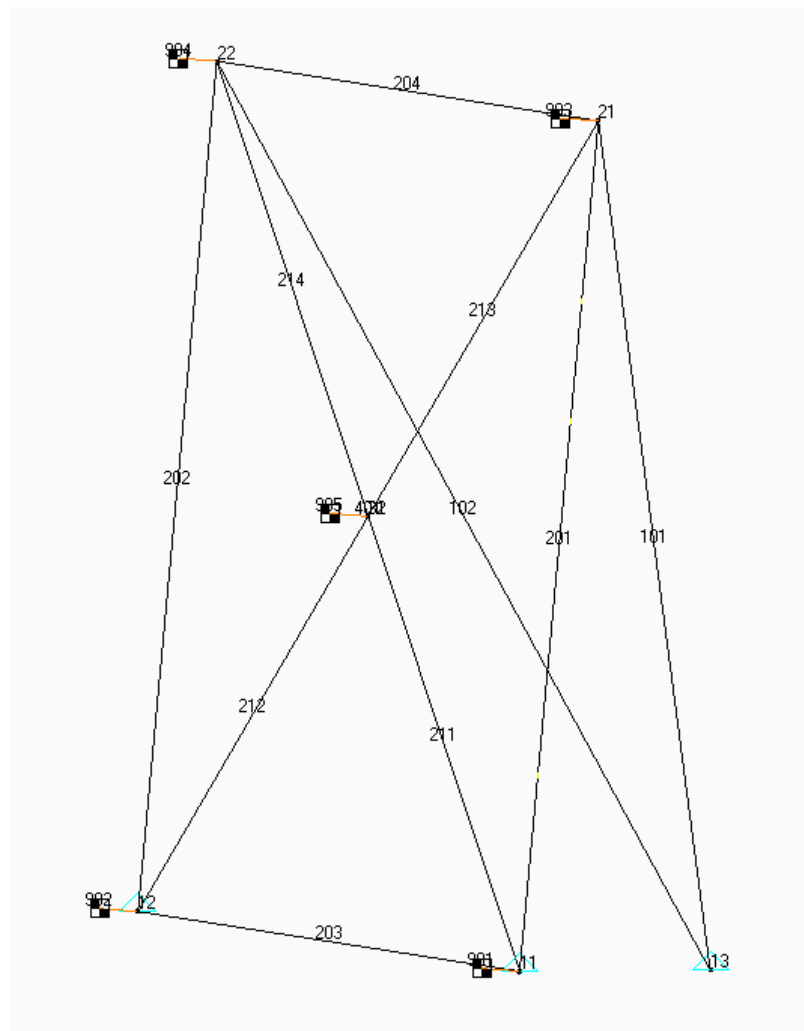


Figure 10.2 – Example CB model: CB-EXAMPLE-12b.DAT

### Notes on section 10.6.1: CB-EXAMPLE-12b.F06

The echo of the input shows the following salient points for a CB model generation (much like a SOL 3 eigenvalue analysis in terms of input data):

- Executive Control:
  - SOL 31 indicates CB model generation
  - The OUTPUT4 commands show the matrices that will be written in a format the same as NASTRAN OUTPUT4 files. These matrix data blocks are ones that are listed on Table 10.2 as allowable OUTPUT4 matrices. Notice that several are written to unit 21 while others are written to unit 22. As explained in section 5.1 of the MYSTRAN Users Reference Manual, unit numbers 21 through 27 are valid for writing OUTPUT4 matrices.
- Case Control:
  - METHOD = 1 is to be used for a normal eigenvalue analysis (same as if SOL were 3)
  - Outputs (ACCE, DISP, ELFORCE, STRESS) are for Output Transformation Matrices (OTM's) for the specified sets. These will be written to the text F06 file. In addition they will be written to binary files (same name, CB-EXAMPLE-12b) with extension OP8 for the element related OTM's (ELFORCE, STRESS in this case and OP9 for the grid related OTM's (ACCE, DISP in this case)
- Bulk Data:
  - Shows the model for this example (notice it has mostly CBAR's but there is also a RBE2)
  - Degrees of freedom at the boundary where this substructure attaches to other substructures are defined with the SUPORT Bulk Data entry. This is the same procedure that is used in CB analyses by the NASTRAN DMAP (Direct Matrix Abstraction Program) method familiar to NASTRAN CB analysts.
  - Eigenvalue extraction, EIGRL requesting 2 modes to be extracted

The delineated F06 output begins on the page following the input model echo and shows the following:

- Eigenvalues extracted
- Messages on the matrices requested to be written to OUTPUT4 files
- For the first 3 of the 18 CB\_DOF's in this example the following output (requested in Case Control) is shown (other 15 were left out for clarity):
  - Displacement OTM for the requested grids (see Case Control command DISP = 102)
  - Element engineering force OTM (see Case Control command ELFORCE = 201)
  - Element stress OTM (see Case Control command STRESS = 202)
- Acceleration OTM. As shown in equation 10.48 the acceleration OTM has columns for  $\ddot{u}_R$  and  $\ddot{\xi}_N$  but not  $u_R$ . For this example, there are 10 columns in the acceleration OTM (8 boundary acceleration DOF's and 2 modal acceleration DOF's)

#### Notes on section 10.6.2: OUTPUT4 matrices written to CB-EXAMPLE-12b.OP1 and OP2

As shown in the Executive Control section of the F06 file in section 10.6.1, there were 3 matrices requested to be written to unit 21 and 4 to unit 22. These binary files, translated to text, are shown in section 10.6.2. The number of actual columns for each matrix is indicated in Table 10.2 but only the first 5 of the columns are shown here for the sake of brevity. These are several of the important CB matrices needed to couple this CB substructure to other substructures in a combined analysis. The binary OUTPUT4 files are written in the same format as the NASTRAN OUTPUT4 binary files.

#### Notes on section 10.6.3: Displ and elem force/stress OTM's written to CB-EXAMPLE-12b.OP1, OP2

Any output requests in Case Control for grid related outputs (e.g. DISPL, ACCEL) and element force/stress outputs (e.g. ELFORCE, STRESS) are written to the text F06 file and also written to OUTPUT4 binary files (automatically; that is, no formal OUTPUT4 request is needed). The element related OTM's are always written to a file with the same filename as the F06 file but with extension OP8. The grid related OTM's are written to a file with extension OP9.

The first page of section 10.6.3 is a text translation of the element related OTM's written to file CB-EXAMPLE-12b.OP8. The values are the same as was written to the F06 file for element forces and stresses but are also written to binary files in OUTPUT4 format to be used in analyses that couple the CB substructures. In order to explain the contents of the binary OP8 file, a text file with extension OT8 is also automatically written (provided any Case Control requests are included for element forces/stresses) describing the contents of the OP8 binary file. This OT8 text file gives an overview of the OP8 binary file and then goes on to describe each row written to the OP8 file.

The next several pages show the same type of information on the grid related OTM's written to binary file with extension OP9 (with text description in OT9). Again, this is the grid related outputs requested in Case Control and also written to the F06 text file.

\*

### **10.6.1 CB-EXAMPLE-12-b.F06**

(delineated – some output not included here for the sake of clarity)

1030180330

MYSTRAN Version 3.00 Oct 20 2006 by Dr Bill Case (this TRIAL edition is SP protected)

>> MYSTRAN BEGIN : 10/30/2006 at 18: 3:30.640 The input file is CB-EXAMPLE-12-b.DAT

>> LINK 1 BEGIN

SOL 31

\$  
OUTPUT4 CG\_LTM , IF\_LTM , , , //-1/21 \$  
OUTPUT4 KRRGN , RBMCG , MRRGN , , RBRCG //-1/22 \$  
OUTPUT4 MR , , , , , //-1/21 \$  
CEND

TITLE = TEST OF CRAIG-BAMPTON SOLUTION

SUBTI = FRAME USING CBAR's

SPC = 1

METHOD = 1

ECHO = UNSORT

\$

SET 101 = 32

SET 102 = 22, 32

SET 201 = 211, 212

SET 202 = 201

\$

ACCE = 101

DISP = 102

ELFORCE = 201

STRESS = 202

MEFFMASS = ALL

MPFACTOR = ALL

\$

BEGIN BULK

\$

EIGRL 1 2 2 DPB -1. MASS

\$

EIGR 2 MGIV 1 24 +E1

+E1 MASS

GRID 11 0. 0. 0.

GRID 12 100. 0. 0.

GRID 13 50. 0. 50.

GRID 21 0. 100. 0.

GRID 22 100. 100. 0.

GRID 31 50. 50. 0.

GRID 32 50. 50. 0.

\$

RBE2 401 31 123456 32

```

$
$ Frame support bars
$
CBAR      101      1      13      21      0.0      0.5      1.0      +C1
+C1       56      456
CBAR      102      1      13      22      0.0      0.5      1.0      +C2
+C2       56      456
$
$ Edge bars
$
CBAR      201      2      11      21      0.0      0.0      1.0
CBAR      202      2      12      22      0.0      0.0      1.0
CBAR      203      2      11      12      0.0      0.0      1.0
CBAR      204      2      21      22      0.0      0.0      1.0
$
$ Diag bars
$
CBAR      211      3      11      31      0.0      0.0      1.0
CBAR      212      3      12      31      0.0      0.0      1.0
CBAR      213      3      21      31      0.0      0.0      1.0
CBAR      214      3      22      31      0.0      0.0      1.0
$
PBAR      1        1      0.36    0.09    0.09    0.18
PBAR      2        1      0.10    10.0    10.0    20.0
PBAR      3        1      6.0     6.0     6.0     12.0
$
MAT1      1        10.+6      0.3     0.1
*INFORMATION: MAT1 ENTRY      1 HAD FIELD FOR G BLANK. MYSTRAN CALCULATED G = 3.846154E+06
$
CONM2     901      11      150.0    0.0     0.0     -5.0
CONM2     902      12      150.0    0.0     0.0     -5.0
CONM2     903      21      150.0    0.0     0.0     -5.0
CONM2     904      22      150.0    0.0     0.0     -5.0
CONM2     905      32      150.0    0.0     0.0     -5.0
$
SPC1      1        456      13
$
$ BOUNDARY DOF'S
$
SUPT      11      123      12      23      13      123
$
PARAM     WTMASS    .002591
$
ENDDATA

```

E I G E N V A L U E   A N A L Y S I S   S U M M A R Y   (LANCZOS   Mode 2 DPB   Shift eigen = -1.00E+00)

NUMBER OF EIGENVALUES EXTRACTED . . . . . 2

LARGEST OFF-DIAGONAL GENERALIZED MASS TERM -2.7E-13 (Vecs renormed to 1.0 for gen masses)

MODE PAIR . . . . . 2

. . . . . 1

NUMBER OF OFF DIAGONAL GENERALIZED MASS  
TERMS FAILING CRITERION OF 1.0E-04. . . . . 0

MODE NUMBER	EXTRACTION ORDER	EIGENVALUE	R E A L RADIANS	E I G E N V A L U E S CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	1	3.895211E+03	6.241163E+01	9.933119E+00	1.000000E+00	3.895211E+03
2	2	7.011163E+03	8.373269E+01	1.332647E+01	1.000000E+00	7.011163E+03

>> LINK 4 END

>> LINK 6 BEGIN



\*INFORMATION: THE FOLLOWING 7 MATRICES WILL BE WRITTEN TO 2 OUTPUT4 FILES IN THE ORDER LISTED BELOW:

OUTPUT4 file on unit 21 has been created as: CB-EXAMPLE-12-b.OP1 and will contain the matrices:

( 1) CG_LTM	:	6 rows and	18 cols	This is MYSTRAN matrix CG_LTM
( 2) IF_LTM	:	8 rows and	18 cols	This is MYSTRAN matrix IF_LTM
( 3) MR	:	8 rows and	8 cols	This is MYSTRAN matrix MRRcb

OUTPUT4 file on unit 22 has been created as: CB-EXAMPLE-12-b.OP2 and will contain the matrices:

( 1) KRRGN	:	10 rows and	10 cols	This is MYSTRAN matrix KXX
( 2) RBMCG	:	6 rows and	6 cols	This is MYSTRAN matrix MCG
( 3) MRRGN	:	10 rows and	10 cols	This is MYSTRAN matrix MXX
( 4) RBRCG	:	8 rows and	6 cols	This is MYSTRAN matrix TR6

>> LINK 6 END

>> LINK 5 BEGIN

>> LINK 5 END

>> LINK 9 BEGIN

C B D I S P L A C E M E N T O T M  
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	-1.412939E-05	1.622140E-05	8.242222E-05	5.883709E-07	-1.667433E-06	5.125151E-07
32	0	1.051041E-05	-9.465944E-06	-3.182887E-06	-1.086181E-07	-9.450720E-07	2.106009E-07

C B E L E M E N T E N G I N E E R I N G F O R C E O T M  
F O R E L E M E N T T Y P E B A R

Element ID	Bend-Moment Plane 1	End A Plane 2	Bend-Moment Plane 1	End B Plane 2	- Shear - Plane 1	Axial Plane 2	Torque Force
211	2.091876E-01	7.894539E-01	1.515607E+00	-1.439344E+00	-1.847556E-02	3.151997E-02	6.266800E-01
212	-1.133151E-01	-1.008960E-02	-1.725401E+00	-6.166148E-02	2.279833E-02	7.293366E-04	-2.953611E-01

C B E L E M E N T S T R E S S O T M I N L O C A L E L E M E N T C O O R D I N A T E S Y S T E M  
F O R E L E M E N T T Y P E B A R

Element ID	SA1 SB1	SA2 SB2	SA3 SB3	SA4 SB4	Axial Stress	SA-Max SB-Max	SA-Min SB-Min	M.S.-T M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-2.748670E+00	-2.748670E+00	-2.748670E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		-2.748670E+00	-2.748670E+00	

C B D I S P L A C E M E N T O T M  
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	-7.600290E-05	8.243595E-05	3.128787E-04	1.925291E-06	2.220055E-06	1.292053E-07
32	0	-5.990878E-05	6.308617E-05	3.224179E-04	3.643362E-06	4.904270E-07	3.218612E-08

C B E L E M E N T E N G I N E E R I N G F O R C E O T M  
F O R E L E M E N T T Y P E B A R

Element ID	Bend-Moment Plane 1	End A Plane 2	Bend-Moment Plane 1	End B Plane 2	- Shear - Plane 1	Axial Plane 2	Torque Force
211	3.640634E+00	-2.875040E+00	-7.752079E+00	4.486528E+00	1.611173E-01	-1.041083E-01	1.906435E+00
212	3.789705E+00	2.992877E+00	-6.061077E+00	-4.713484E+00	1.393111E-01	1.089844E-01	1.808077E+00

C B E L E M E N T S T R E S S O T M I N L O C A L E L E M E N T C O O R D I N A T E S Y S T E M  
F O R E L E M E N T T Y P E B A R

Element ID	SA1 SB1	SA2 SB2	SA3 SB3	SA4 SB4	Axial Stress	SA-Max SB-Max	SA-Min SB-Min	M.S.-T M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	7.582667E+00	7.582667E+00	7.582667E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		7.582667E+00	7.582667E+00	

C B D I S P L A C E M E N T O T M  
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	3.800145E-05	-4.121798E-05	-1.564393E-04	-9.626456E-07	-1.110028E-06	-6.460267E-08
32	0	2.995439E-05	-3.154308E-05	-1.612090E-04	-1.821681E-06	-2.452135E-07	-1.609306E-08

C B      E L E M E N T      E N G I N E E R I N G      F O R C E      O T M  
         F O R      E L E M E N T      T Y P E      B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	-1.820317E+00	1.437520E+00	3.876039E+00	-2.243264E+00	-8.055864E-02	5.205414E-02	-9.532175E-01	2.666968E-03
212	-1.894852E+00	-1.496438E+00	3.030538E+00	2.356742E+00	-6.965554E-02	-5.449220E-02	-9.040385E-01	-2.666968E-03

C B		E L E M E N T		S T R E S S		O T M		I N		L O C A L		E L E M E N T		C O O R D I N A T E		S Y S T E M	
				F O R		E L E M E N T		T Y P E		B A R							
Element	SA1	SA2	SA3	SA4	Axial	SA-Max	SA-Min	M.S.-T									
ID	SB1	SB2	SB3	SB4	Stress	SB-Max	SB-Min	M.S.-C									
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.791334E+00	-3.791334E+00	-3.791334E+00										
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		-3.791334E+00	-3.791334E+00										

[illegible]

(output for the 4<sup>th</sup> – 18<sup>th</sup> CB DOF deleted)

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL      1 OF      10

		C B   A C C E L E R A T I O N   O T M						
		(in global coordinate system at each grid)						
GRID	COORD	T1	T2	T3	R1	R2	R3	
	SYS							
32	0	2.199853E-02	-2.028331E-02	-1.681579E-02	-3.363157E-04	8.006145E-03	5.254334E-04	

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL      2 OF      10

		C B   A C C E L E R A T I O N   O T M						
		(in global coordinate system at each grid)						
GRID	COORD	T1	T2	T3	R1	R2	R3	
	SYS							
32	0	0.000000E+00	0.000000E+00	-1.000000E+00	-2.000000E-02	0.000000E+00	0.000000E+00	

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL      3 OF      10

		C B   A C C E L E R A T I O N   O T M						
		(in global coordinate system at each grid)						
GRID	COORD	T1	T2	T3	R1	R2	R3	
	SYS							
32	0	0.000000E+00	0.000000E+00	5.000000E-01	1.000000E-02	0.000000E+00	0.000000E+00	

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(output for the 4<sup>th</sup> – 10<sup>th</sup> Accel OTM columns deleted)

M O D A L P A R T I C I P A T I O N F A C T O R S						
(dimensionless, in coordinate sys 0)						
MODE	T1	T2	T3	R1	R2	R3
NUM						
1	1.227574E-01	-1.758352E+00	8.791759E-01	1.259087E+00	6.535370E-02	-5.341716E-01
2	6.061630E-01	1.829524E-01	-9.147622E-02	-4.910542E-01	-1.366914E-01	-4.626569E-01

-----

E F F E C T I V E M O D A L M A S S E S O R W E I G H T S						
(in coordinate system 0)						
Units are same as units for mass input in the Bulk Data Deck						
MODE	T1	T2	T3	R1	R2	R3
NUM						
1	6.532677E+01	4.179096E+01	4.694259E+02	3.836785E+05	3.287406E+04	3.611917E+02
2	7.948285E+00	9.016521E-01	1.363070E+01	1.674257E+00	6.082279E+05	4.781873E+05
-----						
Sum all modes:	7.327506E+01	4.269261E+01	4.830566E+02	3.836801E+05	6.411019E+05	4.785485E+05
Total model mass:	9.325238E+02	9.325238E+02	9.325238E+02	4.105260E+06	4.094237E+06	8.139951E+06
Modes % of total mass*:	7.86	4.58	51.80	9.35	15.66	5.88

\*If all modes are calculated the % of total mass should be 100% of the free mass (i.e. not counting mass at constrained DOF's).  
Percentages are only printed for components that have finite model mass.

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>> LINK 9 END

>> MYSTRAN END : 10/30/2006 at 18: 3:31.562

### **10.6.2 OUTPUT4 matrices written to CB-EXAMPLE-12-b.OP1 and OP2**

(OUTPUT4 matrices requested in Exec Control)

**OUTPUT4 matrices requested in Exec Control to be written to file CB-EXAMPLE-12-b.OP1 (on unit 21)**

(note: only 1<sup>st</sup> 5 columns written here for the sake of clarity)

	CG_LTM	NCOLS =	18	NROWS =	6	FORM =	2	PREC =	2
	1	2	3	4	5				
1	-6.65821789802521E-05	1.29562159612018E-17	-6.47810798060089E-18	-1.29549999999999E-03	6.47766872193621E-05	.....			
2	-2.99785601343913E-05	-1.96135553418977E-04	1.04193052213477E-04	1.39356777670951E-03	-6.70858061739371E-05	.....			
3	-4.35697030582909E-05	-2.59100000000000E-03	1.30775055100798E-03	1.29550000000001E-03	6.19839872966866E-04	.....			
4	-3.33844454038618E-04	-2.00000000000000E-02	9.80743672854175E-03	1.00000000000000E-02	-5.07064059129018E-03	.....			
5	8.13687816036514E-03	1.47885176327023E-16	-7.39425881635114E-17	-7.78457159844592E-17	-5.93156091981744E-03	.....			
6	5.63393757592496E-04	8.55130582230051E-17	-4.27565291115026E-17	9.9999999999996E-03	2.81696878796245E-04	.....			

	IF_LTM	NCOLS =	18	NROWS =	8	FORM =	2	PREC =	2
	1	2	3	4	5				
1	6.02957424769077E-01	7.32039059471622E-02	-3.66019529735811E-02	3.35492666170908E-02	-7.19015457719424E-02	.....			
2	7.32039059471623E-02	4.25469107253153E+00	-2.12163357113457E+00	-2.21879607113459E+00	-1.10665832128050E-01	.....			
3	-3.66019529735811E-02	-2.12163357113457E+00	1.07224071582968E+00	1.10939803556729E+00	5.53329160640251E-02	.....			
4	3.35492666170908E-02	-2.21879607113459E+00	1.10939803556729E+00	3.26418464157067E+00	1.75366508593570E-02	.....			
5	-7.19015457719424E-02	-1.10665832128050E-01	5.53329160640251E-02	1.75366508593570E-02	4.96481812094837E-01	.....			
6	-6.65046890695409E-01	-7.32039059471504E-02	3.66019529735752E-02	-1.24163383728600E+00	1.32307347677584E-01	.....			
7	-1.34708893096271E-01	-2.21879607113459E+00	1.10939803556729E+00	2.54146535101691E-01	3.05700710026811E-02	.....			
8	6.78737140960850E-02	-1.83869738075211E-01	9.19348690376054E-02	8.11498842422746E-02	2.62006997196796E-02	.....			

	MR	NCOLS =	8	NROWS =	8	FORM =	1	PREC =	2
	1	2	3	4	5				
1	6.02957424769077E-01	7.32039059471622E-02	-3.66019529735811E-02	3.35492666170908E-02	-7.19015457719424E-02	.....			
2	7.32039059471623E-02	4.25469107253153E+00	-2.12163357113457E+00	-2.21879607113459E+00	-1.10665832128050E-01	.....			
3	-3.66019529735811E-02	-2.12163357113457E+00	1.07224071582968E+00	1.10939803556729E+00	5.53329160640251E-02	.....			
4	3.35492666170908E-02	-2.21879607113459E+00	1.10939803556729E+00	3.26418464157067E+00	1.75366508593570E-02	.....			
5	-7.19015457719424E-02	-1.10665832128050E-01	5.53329160640251E-02	1.75366508593570E-02	4.96481812094837E-01	.....			
6	-6.65046890695409E-01	-7.32039059471504E-02	3.66019529735752E-02	-1.24163383728600E+00	1.32307347677584E-01	.....			
7	-1.34708893096271E-01	-2.21879607113459E+00	1.10939803556729E+00	2.54146535101691E-01	3.05700710026811E-02	.....			
8	6.78737140960850E-02	-1.83869738075211E-01	9.19348690376054E-02	8.11498842422746E-02	2.62006997196796E-02	.....			

**OUTPUT4 matrices requested in Exec Control to be written to file CB-EXAMPLE-12-b.OP2 (on unit 22)**

(note: only 1<sup>st</sup> 5 columns written here the sake of clarity)

	KRRGN	NCOLS =	10	NROWS =	10	FORM =	1	PREC =	2	
	1	2	3	4	5					
1	1.19504240447136E+03	-3.63797880709171E-12	1.81898940354586E-12	1.54614099301398E-11	5.97521202235677E+02	.....				
2	-5.45696821063757E-12	0.00000000000000E+00	0.00000000000000E+00	1.81898940354586E-12	0.00000000000000E+00	.....				
3	2.72848410531878E-12	0.00000000000000E+00	0.00000000000000E+00	-9.09494701772928E-13	0.00000000000000E+00	.....				
4	2.08011385893769E-11	0.00000000000000E+00	0.00000000000000E+00	-1.16415321826935E-10	9.43778388773353E-12	.....				
5	5.97521202235677E+02	-1.13686837721616E-13	5.68434188608080E-14	-1.59161572810262E-12	2.98760601117838E+02	.....				
6	-1.19504240447137E+03	0.00000000000000E+00	0.00000000000000E+00	-1.79397829924710E-10	-5.97521202235685E+02	.....				
7	-2.98427949019242E-13	0.00000000000000E+00	0.00000000000000E+00	-4.31782609666698E-10	-2.76401124210679E-12	.....				
8	-5.97521202235677E+02	-1.81898940354586E-12	9.09494701772928E-13	1.36424205265939E-12	-2.98760601117839E+02	.....				
9	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....				
10	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....				

	RBMCG	NCOLS =	6	NROWS =	6	FORM =	2	PREC =	2	
	1	2	3	4	5					
1	2.41616914133782E+00	-3.35287353436797E-14	-6.52256026967279E-15	-1.34114941374719E-13	-3.97903932025656E-13	.....				
2	-3.30846461338297E-14	2.41616914133786E+00	2.33146835171283E-14	7.74491581978509E-13	2.89102075612391E-13	.....				
3	-6.52256026967279E-15	2.27734497926235E-14	2.41616914133783E+00	-9.59232693276135E-14	-7.10542735760100E-14	.....				
4	-1.35891298214119E-13	7.81374964731185E-13	-1.24344978758018E-13	4.56169135583651E+03	-3.86535248253495E-12	.....				
5	-3.92130772297605E-13	2.88435941797616E-13	-6.75015598972095E-14	-4.09272615797818E-12	4.53313153018053E+03	.....				
6	1.99662508748588E-12	4.26325641456060E-14	-3.62376795237651E-13	-1.36424205265939E-11	2.85598256559946E+01	.....				

	MRRGN	NCOLS =	10	NROWS =	10	FORM =	1	PREC =	2	
	1	2	3	4	5					
1	6.02957424769077E-01	7.32039059471622E-02	-3.66019529735811E-02	3.35492666170908E-02	-7.19015457719424E-02	.....				
2	7.32039059471623E-02	4.25469107253153E+00	-2.12163357113457E+00	-2.21879607113459E+00	-1.10665832128050E-01	.....				
3	-3.66019529735811E-02	-2.12163357113457E+00	1.07224071582968E+00	1.10939803556729E+00	5.53329160640251E-02	.....				
4	3.35492666170908E-02	-2.21879607113459E+00	1.10939803556729E+00	3.26418464157067E+00	1.75366508593570E-02	.....				
5	-7.19015457719424E-02	-1.10665832128050E-01	5.53329160640251E-02	1.75366508593570E-02	4.96481812094837E-01	.....				
6	-6.65046890695409E-01	-7.32039059471504E-02	3.66019529735752E-02	-1.24163383728600E+00	1.32307347677584E-01	.....				
7	-1.34708893096271E-01	-2.21879607113459E+00	1.10939803556729E+00	2.54146535101691E-01	3.05700710026811E-02	.....				
8	6.78737140960850E-02	-1.83869738075211E-01	9.19348690376054E-02	8.11498842422746E-02	2.62006997196796E-02	.....				
9	1.22757372107055E-01	-1.75835189695839E+00	8.79175948479194E-01	1.25908689725916E+00	6.53537005701318E-02	.....				
10	6.06162990294928E-01	1.82952442095713E-01	-9.14762210478567E-02	-4.91054200271590E-01	-1.36691428775775E-01	.....				

	RBRCG	NCOLS =	6	NROWS =	8	FORM =	2	PREC =	2	
	1	2	3	4	5					
1	1.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	5.37849392786371E+01	.....				
2	0.00000000000000E+00	1.00000000000000E+00	0.00000000000000E+00	-5.37849392786371E+01	0.00000000000000E+00	.....				
3	0.00000000000000E+00	0.00000000000000E+00	1.00000000000000E+00	-5.00000000000000E+01	0.00000000000000E+00	.....				
4	0.00000000000000E+00	1.00000000000000E+00	0.00000000000000E+00	-3.78493927863709E+00	0.00000000000000E+00	.....				
5	0.00000000000000E+00	0.00000000000000E+00	1.00000000000000E+00	-5.00000000000000E+01	-5.00000000000000E+01	.....				
6	1.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	3.78493927863709E+00	.....				
7	0.00000000000000E+00	1.00000000000000E+00	0.00000000000000E+00	-3.78493927863709E+00	0.00000000000000E+00	.....				
8	0.00000000000000E+00	0.00000000000000E+00	1.00000000000000E+00	-5.00000000000000E+01	5.00000000000000E+01	.....				



### **10.6.3 Displ and Element force/stress OTM's written to CB-EXAMPLE-12-b.OP8 and OP9**

(OTM's requested in Case Control)

**CB-EXAMPLE-12-b.OP8 binary file of element force/stress OTM's requested in Case Control**

(note: only 1<sup>st</sup> 5 columns written here the sake of clarity)

	OTM_ELFE	NCOLS =	18	NROWS =	16	FORM =	2	PREC =	2
	1	2	3	4	5				
1	2.09187572390564E-01	3.64063384390388E+00	-1.82031692195194E+00	-1.84227921264778E+00	-9.14925412689932E-01	.....			
2	7.89453912890167E-01	-2.87503976462738E+00	1.43751988231369E+00	1.92080844772306E+00	-1.26234542491864E-01	.....			
3	1.51560714339846E+00	-7.75207867487571E+00	3.87603933743785E+00	3.62690741509324E+00	1.45527637571713E+00	.....			
4	-1.43934432738336E+00	4.48652751792572E+00	-2.24326375896286E+00	-2.73874759882899E+00	2.35906653084923E-01	.....			
5	-1.84755627546901E-02	1.61117285562758E-01	-8.05586427813792E-02	-7.73459790410093E-02	-3.35197151472623E-02	.....			
6	3.15199669918811E-02	-1.04108282913086E-01	5.20541414565432E-02	6.58960735567147E-02	-5.12144990278700E-03	.....			
7	6.26679968599842E-01	1.90643492900070E+00	-9.53217464500349E-01	-1.19040949990613E-01	-1.14791218537626E-01	.....			
8	9.67284596743351E-03	-5.33393540270422E-03	2.66696770135211E-03	-5.34876839175438E-02	8.35971431688627E-04	.....			
9	-1.13315069892136E-01	3.78970456518829E+00	-1.89485228259414E+00	-1.26147862482940E+00	-9.55864075040792E-01	.....			
10	-1.00896004659258E-02	2.99287680850590E+00	-1.49643840425295E+00	-4.03697533588189E+00	-1.41398274167766E-02	.....			
11	-1.72540058669802E+00	-6.06107677196644E+00	3.03053838598322E+00	2.53928832803047E+00	1.96715396237338E+00	.....			
12	-6.16614847670031E-02	-4.71348398353008E+00	2.35674199176504E+00	6.82365970711492E+00	3.39064169416761E-02	.....			
13	2.27983320157212E-02	1.39311085669760E-01	-6.96555428348799E-02	-5.37509617215390E-02	-4.13377175157231E-02	.....			
14	7.29336582157196E-04	1.08984399486375E-01	-5.44921997431877E-02	-1.53592573737906E-01	-6.79476503928156E-04	.....			
15	-2.95361107284698E-01	1.80807707871691E+00	-9.04038539358453E-01	-1.95832712226347E+00	3.00896480121837E-03	.....			
16	-4.72042770150405E-03	5.33393540270377E-03	-2.66696770135189E-03	-1.12160973347287E-01	-3.69369770142806E-03	.....			

	OTM_STRE	NCOLS =	18	NROWS =	18	FORM =	2	PREC =	2
	1	2	3	4	5				
1	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
2	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
3	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
4	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
5	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	.....			
6	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	.....			
7	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	.....			
8	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	.....			
9	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
10	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
11	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
12	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
13	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	.....			
14	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	.....			
15	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	.....			
16	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	.....			
17	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	.....			
18	1.00000000000000E+10	1.00000000000000E+10	1.00000000000000E+10	1.00000000000000E+10	1.00000000000000E+10	.....			

### CB-EXAMPLE-12-b.OT8 text file descriptor of rows in above binary file for element related OTM's

This text file describes the rows of the elem related OTM matrices written to unformatted file: CB-EXAMPLE-12-b.OP8

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The description for each of the matrices has the headers:

ROW : row number in the individual OTM described  
DESCRIPTION: what OTM is this  
TYPE : element type  
EID : element ID

Then, for the element nodal force OTM:

GRID : grid number of the element that the OTM is for  
COMP : displacement component number (1,2,3 translations and 4,5,6 rotations)

and for element engineering force and element stress OTMs:

ITEM : element force or stress item (axial force, torque, etc)

The number of rows for each OTM depends on the output requests, by the user, in Case Control

The number of cols for each OTM depends on the number of support DOFs (NDOFR) and the number of eigenvectors (NVEC) where:

NDOFR = 8  
NVEC = 2

This text file has descriptions for the following element related OTMs from CB-EXAMPLE-12-b.OP8

Element engr force OTM (matrix OTM\_ELFE) with 2\*NDOFR + NVEC = 18 cols  
Element stress OTM (matrix OTM\_STRE) with 2\*NDOFR + NVEC = 18 cols

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Explanation of rows of		16 row by	18 col matrix OTM_ELFE	
ROW	DESCRIPTION	TYPE	EID	ITEM
-----				
1	Element engineering force	BAR	211	M1a: Mom Plane1 EndA
2	Element engineering force	BAR	211	M1b: Mom Plane2 EndA
3	Element engineering force	BAR	211	M2a: Mom Plane1 EndB
4	Element engineering force	BAR	211	M2b: Mom Plane2 EndB
5	Element engineering force	BAR	211	V1 : Shear Plane1
6	Element engineering force	BAR	211	V2 : Shear Plane2
7	Element engineering force	BAR	211	FX : Axial force
8	Element engineering force	BAR	211	T : Torque
9	Element engineering force	BAR	212	M1a: Mom Plane1 EndA
10	Element engineering force	BAR	212	M1b: Mom Plane2 EndA
11	Element engineering force	BAR	212	M2a: Mom Plane1 EndB
12	Element engineering force	BAR	212	M2b: Mom Plane2 EndB
13	Element engineering force	BAR	212	V1 : Shear Plane1
14	Element engineering force	BAR	212	V2 : Shear Plane2
15	Element engineering force	BAR	212	FX : Axial force
16	Element engineering force	BAR	212	T : Torque

-----				
Explanation of rows of		18 row by	18 col matrix OTM_STRE	
ROW	DESCRIPTION	TYPE	EID	ITEM
-----				
1	Element stress	BAR	201	SA1: Stress Pt1 EndA
2	Element stress	BAR	201	SA2: Stress Pt2 EndA
3	Element stress	BAR	201	SA3: Stress Pt3 EndA
4	Element stress	BAR	201	SA4: Stress Pt4 EndA
5	Element stress	BAR	201	Axial Stress
6	Element stress	BAR	201	SA-Max
7	Element stress	BAR	201	SA-Min
8	Element stress	BAR	201	MS-Tension
9	Element stress	BAR	201	Torsional Stress
10	Element stress	BAR	201	SB1: Stress Pt1 EndB
11	Element stress	BAR	201	SB2: Stress Pt2 EndB
12	Element stress	BAR	201	SB3: Stress Pt3 EndB
13	Element stress	BAR	201	SB4: Stress Pt4 EndB
14	Element stress	BAR	201	Axial stress
15	Element stress	BAR	201	SB-Max
16	Element stress	BAR	201	SB-Min
17	Element stress	BAR	201	MS-Compression
18	Element stress	BAR	201	MS-Torsion

**CB-EXAMPLE-12-b.OP9 binary file of displacement OTM's requested in Case Control**

(note: only 1<sup>st</sup> 5 columns written here the sake of clarity)

	OTM_ACCE	NCOLS =	10	NROWS =	6	FORM =	2	PREC =	2
	1	2	3	4	5				
1	2.19985250269592E-02	0.00000000000000E+00	0.00000000000000E+00	-5.00000000000004E-01	1.09992625134795E-02	.....			
2	-2.02833087802606E-02	0.00000000000000E+00	0.00000000000000E+00	5.00000000000004E-01	-1.01416543901302E-02	.....			
3	-1.68157865913898E-02	-1.00000000000000E+00	5.00000000000000E-01	5.00000000000005E-01	2.41592106704306E-01	.....			
4	-3.36315731827796E-04	-2.00000000000000E-02	1.00000000000000E-02	1.00000000000001E-02	-5.16815786591390E-03	.....			
5	8.00614495648658E-03	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	-5.99692752175671E-03	.....			
6	5.25433423070610E-04	0.00000000000000E+00	0.00000000000000E+00	9.9999999999992E-03	2.62716711535305E-04	.....			

	OTM_DISP	NCOLS =	18	NROWS =	12	FORM =	2	PREC =	2
	1	2	3	4	5				
1	-1.41293911043985E-05	-7.60029025912968E-05	3.80014512956484E-05	1.29492635368416E-04	3.14571590643487E-06	.....			
2	1.62214021120513E-05	8.24359519633505E-05	-4.12179759816752E-05	-1.30161832591346E-04	-3.52963231517632E-06	.....			
3	8.24222187730972E-05	3.12878663301563E-04	-1.56439331650781E-04	-2.40634384994669E-04	-1.68993616070736E-05	.....			
4	5.88370868696758E-07	1.92529119983460E-06	-9.62645599917302E-07	-2.07019101770705E-06	1.88916538580397E-07	.....			
5	-1.66743323917105E-06	2.22005501168008E-06	-1.11002750584004E-06	-1.14971054599053E-06	-8.88454144573320E-08	.....			
6	5.12515138397389E-07	1.29205343624621E-07	-6.46026718123106E-08	-1.07589130445167E-06	-9.61720937623318E-08	.....			
7	1.05104109813473E-05	-5.99087762260462E-05	2.99543881130231E-05	6.53233961326989E-05	-1.57813540011406E-06	.....			
8	-9.46594436701425E-06	6.30861677743807E-05	-3.15430838871904E-05	-6.55217977160166E-05	1.38681670255135E-06	.....			
9	-3.18288681491121E-06	3.22417925611894E-04	-1.61208962805947E-04	-1.96081126486432E-04	-3.61627931263323E-05	.....			
10	-1.08618067423320E-07	3.64336233382231E-06	-1.82168116691115E-06	-2.63986785628832E-06	-3.24126419085498E-08	.....			
11	-9.45071958677177E-07	4.90427017653186E-07	-2.45213508826593E-07	-2.21449664764883E-07	1.36502293189118E-07	.....			
12	2.10600905814006E-07	3.21861205426993E-08	-1.60930602713497E-08	-6.09852683088454E-07	-3.82285587596693E-08	.....			

### CB-EXAMPLE-12-b.OT9 text file descriptor of rows in above binary file for grid related OTM's

This text file describes the rows of the grid related OTM matrices written to unformatted file: CB-EXAMPLE-12-b.OP9

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The description for each of the matrices has the headers:

ROW : row number in the individual OTM described  
DESCRIPTION: what OTM is this  
GRID : grid number for this row of the OTM  
COMP : displacement component number (1,2,3 translations and 4,5,6 rotations)

The number of rows for each OTM depends on the output requests, by the user, in Case Control

The number of cols for each OTM depends on the number of support DOFs (NDOFR) and the number of eigenvectors (NVEC) where:

NDOFR = 8  
NVEC = 2

This text file has descriptions for the following grid related OTMs from CB-EXAMPLE-12-b.OP9

Acceleration OTM (matrix OTM\_ACCE) with NDOFR + NVEC = 10 cols  
Displacement OTM (matrix OTM\_DISP) with 2\*NDOFR + NVEC = 18 cols

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Explanation of rows of 6 row by 10 col matrix OTM\_ACCE

ROW	DESCRIPTION	GRID	COMP
1	Acceleration	32	1
2	Acceleration	32	2
3	Acceleration	32	3
4	Acceleration	32	4
5	Acceleration	32	5
6	Acceleration	32	6

-----

Explanation of rows of 12 row by 18 col matrix OTM\_DISP

ROW	DESCRIPTION	GRID	COMP
1	Displacement	22	1
2	Displacement	22	2
3	Displacement	22	3
4	Displacement	22	4
5	Displacement	22	5
6	Displacement	22	6
7	Displacement	32	1
8	Displacement	32	2
9	Displacement	32	3
10	Displacement	32	4
11	Displacement	32	5
12	Displacement	32	6