Theory Manual

for the

MYSTRAN General Purpose Finite Element Structural Analysis Computer Program

(Open Source Version)

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MYSTRAN

www.mystran.com

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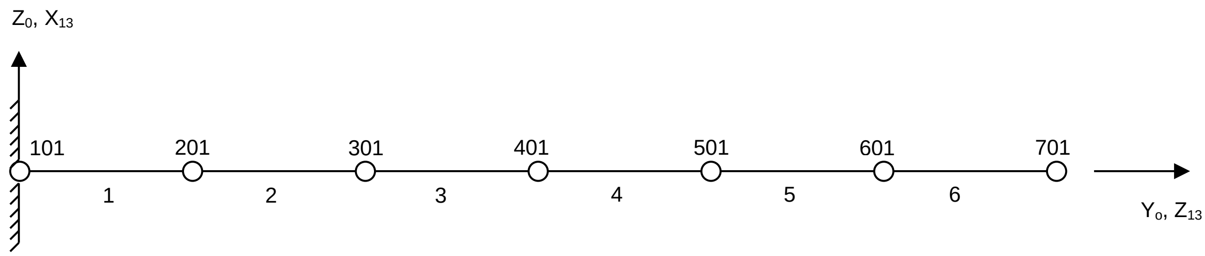
1 MYSTRAN Sample Problem

This example problem shows the input and output for a simple rod with 7 grids and 6 elements. The rod is subjected to loads in two subcases as described below:



The basic coordinate system is the X_0, Y_0, Z_0 system shown (with X_0 in the direction of Y_o cross Z_0). In addition, rectangular coordinate system X_{13}, Y_{13}, Z_{13} (with X_{13} in the same direction as Z_0) is also shown and will be used in the input data in order to help explain the use of coordinate systems. The basic system does not have to be defined explicitly. It is implied through the model grid coordinates and any other coordinate systems (other than basic) which might be referenced in field 3 of the Bulk Data GRID entry. Coordinate system 13 must be defined via a CORD2R Bulk data entry.

The grid point IDs are 101-701 and the rod element IDs are 1-6. The total length is 60 inches consisting of 6 elements of 10 inches each. All of the rods have the same cross-sectional area of 0.6 inch^2. The material is aluminum with a Young's modulus of 1x10^7. The model is constrained at the left end. Several loads are applied in two subcases.

<u>Subcase 35</u> consists of a 120 lb load at grid 701

$$P = \begin{Bmatrix} P_{101} \\ P_{201} \\ P_{301} \\ P_{401} \\ P_{501} \\ P_{601} \\ P_{701} \end{Bmatrix} = \begin{Bmatrix} 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 0. \\ 120. \end{Bmatrix}$$

<u>Subcase 8</u> consists of a 240 lb load at grid 201, a 150. Lb load at grid 301 and a 200 lb load at grid 401

$$P = \begin{Bmatrix} P_{101} \\ P_{201} \\ P_{301} \\ P_{401} \\ P_{501} \\ P_{601} \\ P_{701} \end{Bmatrix} = \begin{Bmatrix} 0. \\ 240. \\ 150. \\ 200. \\ 0. \\ 0. \\ 0. \end{Bmatrix}$$

The output, which includes an echo of the input data deck, is shown on the following pages. Note the following about the OUTPUT:

- The input data consists of everything from the ID entry through the ENDDATA entry, and consists of the Executive Control, Case Control and Bulk Data Decks. Entries that begin with a $ sign (and have anything after $ in the entry) are commentary and are ignored.

  - The Executive Control Deck begins with the optional ID entry, has the mandatory SOL entry (1 for statics) and ends with the mandatory CEND entry. All Executive Control entries are free field in that they may be anywhere within the 80 columns of an entry.

  - The Case Control Deck begins with the entry following CEND (which in this case is a TITLE Case Control entry) and ends with the mandatory BEGIN BULK entry. The entries in between can be in any order that makes sense. That is, if there are no subcases, the data can be in any order. When there are subcases, as is the case for this example, the entries between one SUBCASE entry and another apply only to that subcase. Anything "above" the subcase level pertains to all subcases, unless overridden in a subcase. All Case Control entries are free field.

    - The SPC = 19 entry requests that a Bulk Data SPC (or SPC1, SPCADD) with set ID = 19 be used in defining the single point constraints for the model.

    - The following three entries request various outputs (displacements, etc) with ALL meaning that displacements for all grids (DISP = ALL), applied loads for all grids (OLOAD = ALL) and forces of single point constraint (SPCF = ALL). As these are "above" the subcase ;evel, they apply to all subcases (unless a subcase requests output of the same type for a different set of grids or elements)

    - Subcase 35 (the first subcase in Case Control) is defined with its own subtitle and with LOAD = 191 requesting that a Bulk Data entry with set ID of 191 define the loads for this subcase (which requires that the load be defined on a LOAD, FORCE, MOMENT, GRAV od PLOAD2 Bulk Data entry). In this case, Bulk Data entry FORCE with a set ID od 191 contains the load definition for this subcase. Element engineering force and stress output is requested for this subcase (in addition to the requests above the subcase level). .

    - Subcase 8 (the second subcase in Case Control) is defined with its own subtitle (notice the order doesn't matter) and requesting load set 26 in Bulk Data to define the load. There is also another output request (for nodal element forces) for set 98. Set 98 is defined as 2,5. Since set 98 output is requested as element forces, the 2,5 is interpreted as the element numbers for which nodal element forces will be output in this subcase only. If the request had been above the subcase level (as DISP = ALL, etc) the request would have been honored for both subcases.

  - The Bulk Data Deck begins with the entry immediately following BEGIN BULK and ends with the mandatory ENDDATA entry. The logical entries in between can be in any order with the exception that any one logical entry must be in order. Thus the MAT1 logical entry, which has one parent entry and one continuation entry must be entered together and in the order shown.

- Coordinate system 13 is defined on the CORD2R Bulk Data entry with 13 as the coordinate system ID in field 2. The reference system in field 3 is, in this case, the basic system. It does not have to be. Coordinate system 13 could use some other coordinate system as its reference, and so on. However, the last system in the chain would have to have the basic system as its reference. The nine real numbers on the remainder of the CORD2R logical entry describe three points in coordinates of the reference (basic) system. The first three numbers are the coordinates of the origin of coordinate system 13, which is at the origin of the basic system. The next three numbers are the coordinates of a point on the Z_{13} axis, which is in the direction of the Y_0. The next three numbers (on the continuation entry) are the coordinates of a point in the $X_{13} – Z_{13}$ plane. Thus it is seen that this CORD2R entry describes coordinate system 13 as seen on the figure above.

- The seven grid points of the model are defined on the GRID entries. Note that field 3 (coordinate systems for grid coordinates) is blank indicating the basic coordinate system for grid locations for all seven grids. Field 7, the global coordinate system for each grid is also the basic system for grids 101 through 601. Grid 701, however uses coordinate system 13 as its global system. Field 8 of the GRID entries is for “permanent” single point constraints. Note that 13456 are the permanent single point constraints for grids 101 - 601. Since the rod can only take axial load and torque, only global degrees of freedom that are for displacement along the rod, or rotation about its axis can possibly have stiffness. Since grids 101 - 601, have the basic system as global, degrees of freedom 1346 will be singular and must therefore be removed via single point constraints at these grids. In addition, since the PROD entry has zero torsional constant (field 4 of PROD is blank), there will be no stiffness for global degree of freedom 5 at grids 101 - 601. Thus, field 7 of the grid 101 - 601 entries have 13456 constrained. These constraints do not have to appear on the GRID entry, they can be on SPC (or SPC1) entries as well. Because they appear on the GRID entry these constraints will be used regardless of whether an SPC = SID entry appears in Case Control. Grid 701, on the other hand, uses coordinate system 13 as its global coordinate system. Thus, by the same reasoning as above, global degrees of freedom 12456 are taken as permanent single point constraints.

- The connection entries for the rod elements are the six CROD’s whose element IDs are indicated in field 2. Field 3 (with 16 in it) is the property ID and points to the PROD, ID = 16) for the rod elements properties, which are all the same in this example. Fields 4 and 5 give the grids to which the elements are attached.

- The PROD 16 entry points to a material entry (ID = 20) in field 3 and gives the rod cross-sectional area in field 4.

- The material properties are defined on the MAT1 with ID = 20. Only Young’s modulus is needed for this example but a material density of 0.1 is also entered in field 6.

- Case Control had a request for single point constraint set. The SPC entry, with set ID 19, specifies the remaining constraint of zero displacement in global degree of freedom 2 at grid 101. This could have been included with the constraints specified in field 7 of the GRID 101 entry, in which case the SPC = 19 would not have been needed in Case Control.

        - Case Control had a request for load set 191 for subcase 35. The FORCE Bulk Data entry with ID = 191 is the ID requested for this subcase and defines a 120 lb load at grid 701. The coordinate system for this load definition is coordinate system 13 (indicated by the 13 in field 4). Since the components of the load vector are 0., 0., 1. (fields6-8) this indicates a force in the Z_{13} direction which is along the axis of the rod.

        - Case Control also had a request for load set 26 for subcase 8. As shown above, this loading condition has axial loads on three grid points. As such, these could have been defined using three FORCE Bulk Data entries, all with set ID = 26. However, the LOAD (load combining) Bulk Data entry will be used for illustrative purposes. The LOAD entry has set ID = 26 which is the ID requested for this subcase. It defines a load that is a linear combination of load sets 39, 5 and 178, where the loads for sets 39, 5 and 178 are specified on the FORCE Bulk Data entries below the LOAD 26 entry. The linear combination on LOAD 26 is:

$$P_{set\,26} = 2(4P_{set\,39} + 3P_{set\,5} + P_{set\,178}) = \begin{Bmatrix} 0. \\ 240. \\ 150. \\ 200. \\ 0. \\ 0. \\ 0. \end{Bmatrix}$$

        - The PARAM GRDPNT 101 requests that the Grid Point Weight Generator calculate the total model mass properties relative to grid point 101.

        - The PARAM PRTDOF 1 requests printing of the degree of freedom table.

        - The ENDDATA signifies the end of the Bulk Data Deck.

- The remainder of the output for the sample problem is shown on the pages following the ENDDATA

    - The next of page lists some informational messages printed out as MYSTRAN executes.

    - The degree of freedom table is printed as requested via the Bulk Data PARAM PRTDOF entry. It shows the degree of freedom numbers for each of the displacement sets and is in internal degree of freedom order. Note on this listing that the A-set (analysis set) has six degrees of freedom and these are the axial degrees of freedom of the rod at the “free” grids, namely 201 – 701. Note that for grids 201 – 601, the A-set degree of freedom is in the “2” direction. This is the global “2” direction for these grids, which is the basic Y_0 system. Note also that grid 701 has its A-set degree of freedom as “3” which, since the global system for this grid is coordinate system 13, is in the Z_{13} direction

    - The Grid Point Weight Generator (GPWG) calculates the model total mass properties and prints them. In this example problem, 0.1 was the “mass” density on the MAT1 Bulk data entry. This happens to be the weight density of the aluminum material of which the rod is made. Thus, the units for the GPWG output are lb.

- The following couple of pages list some informational messages printed out as MYSTRAN executes.

- The remainder of the output shows the items requested in Case Control for each subcase. The output shows the subcase number at the beginning of each subcases' output. The output values are easily verified as being correct with some simple hand calculations. Note the following:

  - Displacement, applied load and constraint force output are for grids and all have headings "T1", etc, where

    T1 is translation in the global X direction of that grid

    T2 is translation in the global Y direction of that grid

    T3 is translation in the global Z direction of that grid

    R1 is rotation about the global X axis

    R2 is rotation about the global Y axis

    R3 is rotation about the global Z axis

  - Grids 201 – 601 have T2 displacements since they use the basic system as global and T2 is in the Y_0 direction. Grid 701, however, uses coordinate system 13 as global and has T3 displacement since T3 is in the Z_{13} direction

  - Element engineering forces and stresses are output in the local element coordinate system for each element. See Figure 3-2 for the rod element local axes.

  - Element node forces are output in the same format as grid point displacements, that is, forces at the grids in global coordinate directions

```
   119150503

 MYSTRAN Version  2.06  Jan 19 2006 by Dr Bill Case

 >> MYSTRAN BEGIN:  1/19/2006 at 15: 5: 3. 15 The input file is
EXAMPLE1.DAT

 >> LINK  1 BEGIN

ID ROD SAMPLE PROBLEM FOR USERS MANUAL
SOL 1
CEND
TITLE = ROD WITH AXIAL LOADS IN 2 SUBCASES
ECHO  = UNSORT
SPC   = 19
DISP     = ALL
OLOAD    = ALL
SPCF     = ALL
SUBCASE 35
  SUBTITLE = 120 LB LOAD ON GRID 701
  ELFORCE  = ALL
  STRESS   = ALL
  LOAD = 191
SUBCASE 8
  SET 98 = 2,5
  LOAD = 26
  ELFORCE(NODE) = 98
  SUBTITLE = 240 LB ON GRID 201 + 150 LB ON GRID 301 + 200 LB ON GRID 401
BEGIN BULK
$
CORD2R  13      0       0.      0.      0.      0.      1.      0.      +CORD13
+CORD13 0.      0.      1.
$
GRID    701             0.      60.     0.      13      12456
GRID    601             0.      50.     0.              13456
GRID    501             0.      40.     0.              13456
GRID    401             0.      30.     0.              13456
GRID    301             0.      20.     0.              13456
GRID    201             0.      10.     0.              13456
GRID    101             0.      0.      0.              13456
$
CROD    1       16      101     201
CROD    2       16      201     301
CROD    3       16      301     401
CROD    4       16      401     501
CROD    5       16      501     601
CROD    6       16      601     701
$
PROD    16      20      .6
$
MAT1    20      1.+7            .33     .1      1.                      +MAT1
```

```
 *INFORMATION: MAT1 ENTRY       20 HAD FIELD FOR G  BLANK. MYSTRAN CALCULATED G  =  3.759398E+06
+MAT1   10000.  10000.  10000.
$
SPC1    19      2       101
$
FORCE   191     701     13      120.    0.      0.      1.
$
LOAD    26      2.0     4.0     39      3.0     5       1.0     178
FORCE   39      201     0       30.     0.      1.      0.
FORCE   5       301     13      25.     0.      0.      1.
FORCE   178     401     0       100.    0.      1.      0.
$
PARAM   GRDPNT  101
PARAM   PRTDOF  1
DEBUG   200     1
$
ENDDATA
```

```
 *INFORMATION: SPARSE MATRICES ARE STORED IN SYM    FORMAT

 *INFORMATION: BANDIT WAS CALLED TO RESEQUENCE THE GRIDS AND HAS RETURNED WITH ERROR  =        0

 *INFORMATION: FILE
EXAMPLE1.SEQ
               CONTAINING THE BULK DATA SEQGP CARD IMAGES (NEEDED FOR AUTO GRID POINT SEQUENCING REQUESTED BY
               THE USER VIA PARAM GRIDSEQ BANDIT  ), DOES NOT EXIST

               IT MAY BE THAT BANDIT FOUND THAT NO RESEQUENCING WAS NEEDED OR DUE TO ERROR IN RUNNING BANDIT.

               MAKE SURE BANDIT HAS RUN SUCCESSFULLY (CHECK FILE BANDIT.OUT IN THE DIRECTORY WHERE MYSTRAN.EXE RESIDES).

 *INFORMATION: SUBR AUTO_SEQ_PROC DID NOT SEQUENCE ALL OF THE        7 GRIDS. ONLY        0 GRIDS WERE SEQUENCED.
               MYSTRAN WILL DEFAULT TO A SEQUENCE THAT IS IN GRID NUMERICAL ORDER
```

DEGREE OF FREEDOM TABLE SORTED ON GRID POINT (TDOF)
(Before any AUTOSPC)

EXTERNAL GRD-COMP NUMBER	INTERNAL GRD-COMP NUMBER	G	M	N	SA	SB	SG	SZ	SE	S	F	O	A	R	L
101-1	1-1	1	0	1	0	0	1	1	0	1	0	0	0	0	0
-2	-2	2	0	2	0	1	0	2	0	2	0	0	0	0	0
-3	-3	3	0	3	0	0	2	3	0	3	0	0	0	0	0
-4	-4	4	0	4	0	0	3	4	0	4	0	0	0	0	0
-5	-5	5	0	5	0	0	4	5	0	5	0	0	0	0	0
-6	-6	6	0	6	0	0	5	6	0	6	0	0	0	0	0
201-1	2-1	7	0	7	0	0	6	7	0	7	0	0	0	0	0
-2	-2	8	0	8	0	0	0	0	0	0	1	0	1	0	1
-3	-3	9	0	9	0	0	7	8	0	8	0	0	0	0	0
-4	-4	10	0	10	0	0	8	9	0	9	0	0	0	0	0
-5	-5	11	0	11	0	0	9	10	0	10	0	0	0	0	0
-6	-6	12	0	12	0	0	10	11	0	11	0	0	0	0	0
301-1	3-1	13	0	13	0	0	11	12	0	12	0	0	0	0	0
-2	-2	14	0	14	0	0	0	0	0	0	2	0	2	0	2
-3	-3	15	0	15	0	0	12	13	0	13	0	0	0	0	0
-4	-4	16	0	16	0	0	13	14	0	14	0	0	0	0	0
-5	-5	17	0	17	0	0	14	15	0	15	0	0	0	0	0
-6	-6	18	0	18	0	0	15	16	0	16	0	0	0	0	0
401-1	4-1	19	0	19	0	0	16	17	0	17	0	0	0	0	0
-2	-2	20	0	20	0	0	0	0	0	0	3	0	3	0	3
-3	-3	21	0	21	0	0	17	18	0	18	0	0	0	0	0
-4	-4	22	0	22	0	0	18	19	0	19	0	0	0	0	0
-5	-5	23	0	23	0	0	19	20	0	20	0	0	0	0	0
-6	-6	24	0	24	0	0	20	21	0	21	0	0	0	0	0
501-1	5-1	25	0	25	0	0	21	22	0	22	0	0	0	0	0
-2	-2	26	0	26	0	0	0	0	0	0	4	0	4	0	4
-3	-3	27	0	27	0	0	22	23	0	23	0	0	0	0	0
-4	-4	28	0	28	0	0	23	24	0	24	0	0	0	0	0
-5	-5	29	0	29	0	0	24	25	0	25	0	0	0	0	0
-6	-6	30	0	30	0	0	25	26	0	26	0	0	0	0	0
601-1	6-1	31	0	31	0	0	26	27	0	27	0	0	0	0	0
-2	-2	32	0	32	0	0	0	0	0	0	5	0	5	0	5
-3	-3	33	0	33	0	0	27	28	0	28	0	0	0	0	0
-4	-4	34	0	34	0	0	28	29	0	29	0	0	0	0	0
-5	-5	35	0	35	0	0	29	30	0	30	0	0	0	0	0
-6	-6	36	0	36	0	0	30	31	0	31	0	0	0	0	0
701-1	7-1	37	0	37	0	0	31	32	0	32	0	0	0	0	0
-2	-2	38	0	38	0	0	32	33	0	33	0	0	0	0	0
-3	-3	39	0	39	0	0	0	0	0	0	6	0	6	0	6
-4	-4	40	0	40	0	0	33	34	0	34	0	0	0	0	0
-5	-5	41	0	41	0	0	34	35	0	35	0	0	0	0	0
-6	-6	42	0	42	0	0	35	36	0	36	0	0	0	0	0
TOTAL NUMBER OF DOF:		42	0	42	0	1	35	36	0	36	6	0	6		

```
                    O U T P U T   F R O M   G R I D   P O I N T   W E I G H T   G E N E R A T O R
                                    REFERENCE POINT IS GRID POINT      101

                                          TOTAL MASS =  3.600000E+00

                                                X             Y             Z
                          C.G. LOCATION :  0.000000E+00  3.000000E+01  0.000000E+00
                           (RELATIVE TO REFERENCE POINT IN BASIC COORDINATE SYSTEM)

                       M.O.I. MATRIX - ABOUT REFERENCE POINT IN BASIC COORDINATE SYSTEM
                                ***                                        ***
                                *  4.380000E+03  0.000000E+00  0.000000E+00 *
                                *  0.000000E+00  0.000000E+00  0.000000E+00 *
                                *  0.000000E+00  0.000000E+00  4.380000E+03 *
                                ***                                        ***

                            M.O.I. MATRIX - ABOUT C.G. IN BASIC COORDINATE SYSTEM
                                ***                                        ***
                                *  1.140000E+03  0.000000E+00  0.000000E+00 *
                                *  0.000000E+00  0.000000E+00  0.000000E+00 *
                                *  0.000000E+00  0.000000E+00  1.140000E+03 *
                                ***                                        ***

                              M.O.I. MATRIX - ABOUT C.G. IN PRINCIPAL DIRECTIONS
                                ***                                        ***
                                *  0.000000E+00  0.000000E+00  0.000000E+00 *
                                *  0.000000E+00  1.140000E+03  0.000000E+00 *
                                *  0.000000E+00  0.000000E+00  1.140000E+03 *
                                ***                                        ***

                        TRANSFORMATION FROM BASIC COORDINATES TO PRINCIPAL DIRECTIONS
                                ***                                        ***
                                *  0.000000E+00  1.000000E+00  0.000000E+00 *
                                *  1.000000E+00  0.000000E+00  0.000000E+00 *
                                *  0.000000E+00  0.000000E+00  1.000000E+00 *
                                ***                                        ***
```

```
*INFORMATION: LTERM_MGGE ESTIMATE OF THE NUMBER OF NONZEROS IN MASS MATRIX MGGE IS   =          468

*INFORMATION: NUMBER OF NONZERO TERMS IN THE MGG MASS MATRIX IS                      =            7

*INFORMATION: NUMBER OF NONZERO TERMS IN THE MGG MASS MATRIX IS                      =            7

*INFORMATION: MAX NUMBER OF NONZERO TERMS IN A ROW OF THE G-SET MASS MATRIX          =            1

*INFORMATION: LTERM_KGG ESTIMATE OF THE NUMBER OF NONZEROS IN STIFF MATRIX KGG IS    =          468

*INFORMATION: NUMBER OF NONZERO TERMS IN THE KGG STIFFNESS MATRIX IS                 =           13


*INFORMATION: MAX NUMBER OF NONZERO TERMS IN A ROW OF THE G-SET STIFFNESS MATRIX     =            2

*INFORMATION: NUMBER OF GRID POINTS                                                  =            7
*INFORMATION: NUMBER OF G SET DEGREES OF FREEDOM (NDOFG)                             =           42


>> LINK  1 END

>> LINK  2 BEGIN

*INFORMATION: BASED ON PARAMETER AUTOSPC_NSET =  1 MYSTRAN IS CHECKING KNN TO SEE IF THERE ARE NULL ROWS THAT SHOULD BE AUTOSPC'd


*INFORMATION: MYSTRAN FOUND NO N-SET DOF's THAT WERE SINGULAR AND THAT WERE NOT ALREADY MEMBERS OF THE S-SET


*INFORMATION: AUTOSPC Summary, Overall: after identification of all AUTOSPC's

                                    AUTOSPC_RAT = 1.000000E-06

                      Number of DOF's identified for AUTOSPC in component  1         =            0
                      Number of DOF's identified for AUTOSPC in component  2         =            0
                      Number of DOF's identified for AUTOSPC in component  3         =            0
                      Number of DOF's identified for AUTOSPC in component  4         =            0
                      Number of DOF's identified for AUTOSPC in component  5         =            0
                      Number of DOF's identified for AUTOSPC in component  6         =            0
                                                                                       ------------
                      Total number of DOF's identified overall                       =            0

*INFORMATION: NUMBER OF  M SET DEGREES OF FREEDOM (NDOFM)                            =            0
*INFORMATION: NUMBER OF  N SET DEGREES OF FREEDOM (NDOFN)                            =           42
*INFORMATION: NUMBER OF  S SET DEGREES OF FREEDOM (NDOFS)                            =           36
*INFORMATION: NUMBER OF SA SET DEGREES OF FREEDOM (NDOFSA)                           =            0
*INFORMATION: NUMBER OF  F SET DEGREES OF FREEDOM (NDOFF)                            =            6
*INFORMATION: NUMBER OF  O SET DEGREES OF FREEDOM (NDOFO)                            =            0
*INFORMATION: NUMBER OF  A SET DEGREES OF FREEDOM (NDOFA)                            =            6
*INFORMATION: NUMBER OF  R SET DEGREES OF FREEDOM (NDOFR)                            =            0
*INFORMATION: NUMBER OF  L SET DEGREES OF FREEDOM (NDOFL)                            =            6
```

```
>> LINK  2 END


>> LINK  3 BEGIN

*INFORMATION: NUMBER OF SUPERDIAGONALS IN THE UPPER TRIANGLE OF MATRIX        KLL   =           1

*INFORMATION: MAXIMUM DIAGONAL TERM IN MATRIX         KLL                           =  1.200000E+06 Occurs in row/col no.        1
*INFORMATION: MINIMUM DIAGONAL TERM IN MATRIX         KLL                           =  6.000000E+05 Occurs in row/col no.        6
*INFORMATION: RATIO OF MAX TO MIN DIAGONALS IN MATRIX         KLL                   =  2.000000E+00

*INFORMATION: MAX RATIO OF MATRIX DIAGONAL TO FACTOR DIAGONAL FOR MATRIX         KLL =  1.897367E+03 Occurs in row/col no.        6


*INFORMATION: FOR INTERNAL SUBCASE NUMBER        1 EPSILON ERROR ESTIMATE            =  1.421085E-15 Based on U'*(K*U - P)/(U'*P)

*INFORMATION: FOR INTERNAL SUBCASE NUMBER        2 EPSILON ERROR ESTIMATE            =  1.104361E-15 Based on U'*(K*U - P)/(U'*P)


>> LINK  3 END


>> LINK  5 BEGIN


>> LINK  5 END


>> LINK  9 BEGIN
```

```
SUBCASE       35
ROD WITH AXIAL LOADS IN 2 SUBCASES
120 LB LOAD ON GRID 701

                                                D I S P L A C E M E N T S
                                         (in global coordinate system at each grid)
          GRID     COORD      T1            T2            T3            R1            R2            R3
                    SYS
           101        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           201        0  0.000000E+00  2.000000E-04  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           301        0  0.000000E+00  4.000000E-04  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           401        0  0.000000E+00  6.000000E-04  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           501        0  0.000000E+00  8.000000E-04  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           601        0  0.000000E+00  1.000000E-03  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           701       13  0.000000E+00  0.000000E+00  1.200000E-03  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
MAX (for output set):    0.000000E+00  1.000000E-03  1.200000E-03  0.000000E+00  0.000000E+00  0.000000E+00
MIN (for output set):    0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

ABS (for output set):    0.000000E+00  1.000000E-03  1.200000E-03  0.000000E+00  0.000000E+00  0.000000E+00


SUBCASE       35
ROD WITH AXIAL LOADS IN 2 SUBCASES
120 LB LOAD ON GRID 701

                                              A P P L I E D    F O R C E S
                                         (in global coordinate system at each grid)
          GRID     COORD      T1            T2            T3            R1            R2            R3
                    SYS
           101        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           201        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           301        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           401        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           501        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           601        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           701       13  0.000000E+00  0.000000E+00  1.200000E+02  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
MAX (for output set):    0.000000E+00  0.000000E+00  1.200000E+02  0.000000E+00  0.000000E+00  0.000000E+00
MIN (for output set):    0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

ABS (for output set):    0.000000E+00  0.000000E+00  1.200000E+02  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
APPLIED FORCE TOTALS: not printed since all grids do not have the same global coordinate system
```

```
SUBCASE       35
ROD WITH AXIAL LOADS IN 2 SUBCASES
120 LB LOAD ON GRID 701

                                                                S P C   F O R C E S
                                                    (in global coordinate system at each grid)
          GRID     COORD      T1            T2            T3            R1            R2            R3
                    SYS
           101        0  0.000000E+00 -1.200000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           201        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           301        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           401        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           501        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           601        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           701       13  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
MAX (for output set):    0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
MIN (for output set):    0.000000E+00 -1.200000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

ABS (for output set):    0.000000E+00  1.200000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
    SPC FORCE TOTALS: not printed since all grids do not have the same global coordinate system


SUBCASE       35
ROD WITH AXIAL LOADS IN 2 SUBCASES
120 LB LOAD ON GRID 701

                                             E L E M E N T   E N G I N E E R I N G   F O R C E S
                                                  F O R   E L E M E N T   T Y P E   R O D
  Element     Axial         Torque     Element     Axial         Torque     Element     Axial         Torque
     ID       Force                       ID       Force                       ID       Force
        1  1.200000E+02  0.000000E+00        2  1.200000E+02  0.000000E+00        3  1.200000E+02  0.000000E+00
        4  1.200000E+02  0.000000E+00        5  1.200000E+02  0.000000E+00        6  1.200000E+02  0.000000E+00


SUBCASE       35
ROD WITH AXIAL LOADS IN 2 SUBCASES
120 LB LOAD ON GRID 701

             E L E M E N T   S T R E S S E S   I N   L O C A L   E L E M E N T   C O O R D I N A T E   S Y S T E M
                                             F O R   E L E M E N T   T Y P E   R O D
 Element     Axial          Safety    Torsional         Safety    Element     Axial       Safety    Torsional     Safety
    ID       Stress         Margin      Stress          Margin       ID       Stress      Margin      Stress      Margin
        1  2.000000E+02  4.90E+01   0.000000E+00                      2  2.000000E+02  4.90E+01   0.000000E+00
        3  2.000000E+02  4.90E+01   0.000000E+00                      4  2.000000E+02  4.90E+01   0.000000E+00
        5  2.000000E+02  4.90E+01   0.000000E+00                      6  2.000000E+02  4.90E+01   0.000000E+00
```

SUBCASE 8
ROD WITH AXIAL LOADS IN 2 SUBCASES
240 LB ON GRID 201 + 150 LB ON GRID 301 + 200 LB ON GRID 401

D I S P L A C E M E N T S
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
101	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
201	0	0.000000E+00	9.833333E-04	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
301	0	0.000000E+00	1.566667E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
401	0	0.000000E+00	1.900000E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
501	0	0.000000E+00	1.900000E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
601	0	0.000000E+00	1.900000E-03	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
701	13	0.000000E+00	0.000000E+00	1.900000E-03	0.000000E+00	0.000000E+00	0.000000E+00
MAX (for output set):		0.000000E+00	1.900000E-03	1.900000E-03	0.000000E+00	0.000000E+00	0.000000E+00
MIN (for output set):		0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
ABS (for output set):		0.000000E+00	1.900000E-03	1.900000E-03	0.000000E+00	0.000000E+00	0.000000E+00

SUBCASE 8
ROD WITH AXIAL LOADS IN 2 SUBCASES
240 LB ON GRID 201 + 150 LB ON GRID 301 + 200 LB ON GRID 401

A P P L I E D F O R C E S
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
101	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
201	0	0.000000E+00	2.400000E+02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
301	0	0.000000E+00	1.500000E+02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
401	0	0.000000E+00	2.000000E+02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
501	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
601	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
701	13	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
MAX (for output set):		0.000000E+00	2.400000E+02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
MIN (for output set):		0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
ABS (for output set):		0.000000E+00	2.400000E+02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

APPLIED FORCE TOTALS: not printed since all grids do not have the same global coordinate system

```
SUBCASE        8
ROD WITH AXIAL LOADS IN 2 SUBCASES
240 LB ON GRID 201 + 150 LB ON GRID 301 + 200 LB ON GRID 401

                                                        S P C   F O R C E S
                                             (in global coordinate system at each grid)
          GRID     COORD      T1            T2            T3            R1            R2            R3
                    SYS
           101        0  0.000000E+00 -5.900000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           201        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           301        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           401        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           501        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           601        0  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
           701       13  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
MAX (for output set):    0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
MIN (for output set):    0.000000E+00 -5.900000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

ABS (for output set):    0.000000E+00  5.900000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
    SPC FORCE TOTALS: not printed since all grids do not have the same global coordinate system


SUBCASE        8
ROD WITH AXIAL LOADS IN 2 SUBCASES
240 LB ON GRID 201 + 150 LB ON GRID 301 + 200 LB ON GRID 401

                                E L E M   N O D A L   F O R C E S   I N   G L O B A L   C O O R D S
                                             F O R   E L E M E N T   T Y P E   R O D
     Element     Grid         T1            T2            T3            R1            R2            R3
        ID      Point
             2      201  0.000000E+00 -3.500000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                    301  0.000000E+00  3.500000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
             5      501  0.000000E+00 -2.273737E-13  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                    601  0.000000E+00  2.273737E-13  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
                        ------------- ------------- ------------- ------------- ------------- -------------
MAX (for output set):    0.000000E+00  3.500000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
MIN (for output set):    0.000000E+00 -3.500000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

ABS (for output set):    0.000000E+00  3.500000E+02  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00

>> LINK  9 END

>> MYSTRAN END  :  1/19/2006 at 15: 5: 3.8. The output file is:
   EXAMPLE1.F06
MYSTRAN terminated normally. Total CPU time =  1.56E-01 seconds
```

2 Equations for the reduction of the G-set to the A-set and solution for displacements and constraint forces

2.1 Introduction

As discussed in Section 3.6, MYSTRAN builds the original stiffness and mass matrices based on the G-set, which has 6 degrees of freedom per grid specified in the Bulk Data deck. The stiffness matrix is by definition singular as, at this point, there have been no constraints imposed. There are two type of constraints MYSTRAN allows; single point constraints and multi-point constraints as discussed earlier in this manual. In order to apply boundary conditions that restrain the model from rigid body motion, single point constraints must be used. Multi-point constraints (using rigid elements or Bulk Data MPC entries) are used to express some degrees of freedom (DOF's) of the model as being rigidly restrained to some other DOF's. Thus, MYSTRAN must reduce the G-set stiffness, mass, and loads to the independent A-set DOF's

The discussion below shows the process that MYSTRAN uses to solve for the displacements and constraint forces by going through a systematic reduction of the G-set to the N-set then to the F-set and finally to the L-set which represent the independent DOF's. These equations can then be solved for the L-set DOF's. The other DOF displacements, as well as constraint forces, can then be recovered. Element forces and stresses are obtained from the displacements as discussed in Appendix C. The process in this appendix uses the displacement set notation developed in Section 3.6 which should be reviewed prior to this section. In general, the matrix notation used in this development is such that the matrix subscripts describe the matrix size. Thus, K_{GG} is a matrix which has G rows and columns, R_{CG} is a matrix that has C rows and G columns and R^T_{CG} is the transpose of R_{CG} and has G rows and C columns. If a matrix has only one column, it would exhibit only one subscript, as in Y_S which is an S x 1 matrix of single point constraint values

2.2 Reduction of the G-set to the N-set

In terms of this G-set, the equations of motion for the structure can be written as:

$$\boxed{\begin{gathered} M_{GG}\ddot{U}_G + K_{GG}U_G = P_G + R^T{}_{CG}q_C \\ \\ R_{CG}U_G = Y_C \end{gathered}} \tag{8-1}$$

In the first of equations 8.1 M_{GG} is the G-set mass matrix, K_{GG} is the G-set stiffness matrix, U_G are the G-set displacements, P_G are the applied loads on the G-set DOF's and q_C are the independent, generalized, constraint forces (due to single and multi-point constraints). The second of 8.1 expresses the constraints (both single and multi-point constraints) wherein C is the number of constraint equations, R_{CG} is a constraint coefficient matrix and Y_C is a vector of constraint values. For example, if all of the constraints were single point constraints, then all of the coefficients in any one row of R_{CG} would be zero except for one unity value. In addition, if all of these single point constraints were for DOF's that are grounded, then all of the Y_C values would be zero and these single point constraints would all have the form of $u_i = 0$.

The unknowns in 8.1 are the U_G displacements and the q_C generalized constraint forces and there are G+C equations to solve for these unknowns. As will be explained later, direct solution of the q_C constraint forces will not be made.

The q_C generalized forces of constraint do not necessarily have any physical meaning. Rather, the G-set nodal forces of constraint are of interest and are expressed in terms of the q_C as:

$$Q_G = R^T{}_{CG}q_C \tag{8-2}$$

In order to reduce 8.1 the G-set is partitioned into the N and M-sets, where the M DOF's are to be eliminated using the multi-point constraints (from rigid elements as well as MPC Bulk Data entries defined by the user in the input data deck). The U_N are the remainder of the DOF's in the G-set. Thus, write U_G as:

$$U_G = \begin{Bmatrix} U_N \\ U_M \end{Bmatrix} \tag{8-3}$$

The number of constraints is C which is equal to M+S (where S is the number of DOF's in the S set). Thus, partition q_C and Y_C as:

$$q_C = \begin{Bmatrix} q_S \\ q_M \end{Bmatrix}$$
$$Y_C = \begin{Bmatrix} Y_S \\ 0_M \end{Bmatrix} \tag{8-4}$$

0_M is a column vector of M zeros. That is, only the S-set can have nonzero constraint values.

With the second of 8.4 in mind, partition the second of equations 8.1 using 8.3 as:

$$\begin{bmatrix} R_{SN} & 0_{SM} \\ R_{MN} & R_{MM} \end{bmatrix} \begin{Bmatrix} U_N \\ U_M \end{Bmatrix} = \begin{Bmatrix} Y_S \\ 0_M \end{Bmatrix} \tag{8-5}$$

The 0_{SM} partition is an S x M matrix of zero's. This is required by the form of the single point constraint equations which are all of the form $u_i = Y_i$ where Y_i is a constant (zero or some enforced displacement value).

Using 8.3, partition the first of equations 8.1 as:

$$\begin{bmatrix} \bar{M}_{NN} & M_{NM} \\ M^T{}_{NM} & M_{MM} \end{bmatrix} \begin{Bmatrix} \ddot{U}_N \\ \ddot{U}_M \end{Bmatrix} + \begin{bmatrix} \bar{K}_{NN} & K_{NM} \\ K^T{}_{NM} & K_{MM} \end{bmatrix} \begin{Bmatrix} U_N \\ U_M \end{Bmatrix} = \begin{Bmatrix} \bar{P}_N \\ P_M \end{Bmatrix} + \begin{bmatrix} R^T_{SN} & R^T_{MN} \\ 0^T_{SM} & R^T_{MM} \end{bmatrix} \begin{Bmatrix} q_S \\ q_M \end{Bmatrix} \tag{8-6}$$

The bars over the N-set mass, stiffness and loads matrices are used for convenience to distinguish these terms from those that will result from the reduction of the G-set to the N-set. From the second of the constraint equations in 8.5 solve for U_M in terms of U_N:

$$U_M = G_{MN} U_N \tag{8-7}$$

where

$$G_{MN} = -(R^{-1}_{MM} R_{MN}) \tag{8-8}$$

Using 8.7, equation 8.3 can be written as:

$$U_G \equiv \begin{Bmatrix} U_N \\ U_M \end{Bmatrix} = \begin{bmatrix} I_{NN} \\ G_{MN} \end{bmatrix} U_N \tag{8-9}$$

where I_{NN} is an identity matrix of size N.

Substitute 8.9 into 8.6 and premultiply the result by the transpose of the coefficient matrix in 8.9. The result can be written as:

$$M_{NN}\ddot{U}_N + K_{NN}U_N = P_N + \begin{bmatrix} R^T_{SN} & (R^T_{MN} + G^T_{MN}R^T_{MM}) \end{bmatrix} \begin{Bmatrix} q_S \\ q_M \end{Bmatrix} \tag{8-10}$$

where:

$$K_{NN} = \bar{K}_{NN} + K_{NM}G_{MN} + (K_{NM}G_{MN})^T + G_{MN}^T K_{MM} G_{MN}$$

$$M_{NN} = \bar{M}_{NN} + M_{NM}G_{MN} + (M_{NM}G_{MN})^T + G_{MN}^T M_{MM} G_{MN} \tag{8-11}$$

$$P_N = \bar{P}_N + G_{MN}^T P_M$$

M_{NN} , K_{NN} and P_N are the reduced N-set mass stiffness and loads. Note that P_N is not the set of applied loads on the N-set if there are applied loads on the M-set as expressed by the second of equations 8.11 ($\bar{P}_N$ are the applied loads on the N set).

In addition, the second term in the square brackets in 8.10 is zero by the definition of G_{MN} in 8.8 so that 8.10 and 8.5 can be written as:

$$\boxed{M_{NN}\ddot{U}_N + K_{NN}U_N = P_N + R_{SN}^T q_S} \tag{8-12}$$

2.3 Reduction of the N-set to the F-set

The N-set can now be partitioned into the F and S-sets where the S DOF's are to be eliminated using the single point constraints identified by the user in the input data deck. The F-set are the remainder of the DOF's in the N-set and are known as the "free" DOF's (i.e. those that have no constraints imposed on them). Thus, partition U_N into U_F and U_S:

$$U_N = \begin{Bmatrix} U_F \\ U_S \end{Bmatrix} \tag{8-13}$$

Rewrite equation 8.5 in terms of the F, S and M-sets with the restriction that the single point constraints are of the form $u_i = Y_i$ where Y_i is a constant (zero or some enforced displacement value), using:

$$\begin{aligned} R_{SN} &= \begin{bmatrix} 0_{SF} & I_{SS} \end{bmatrix} \\ R_{MN} &= \begin{bmatrix} R_{MF} & R_{MS} \end{bmatrix} \end{aligned} \tag{8-14}$$

where O_{SF} is an S x F matrix of zeros and I_{SS} is an S size identity matrix. Equation 8.5 can be written as:

$$\begin{bmatrix} 0_{SF} & I_{SS} & O_{SM} \\ R_{MF} & R_{MS} & R_{MM} \end{bmatrix} \begin{Bmatrix} U_F \\ U_S \\ U_M \end{Bmatrix} = \begin{Bmatrix} Y_S \\ 0_M \end{Bmatrix} \tag{8-15}$$

Substitute 8.13 and the first of 8.14 into 8.12 and partition the mass, stiffness and load matrices into the F and S-sets to get:

$$\begin{bmatrix} M_{FF} & M_{FS} \\ M_{FS}^T & M_{SS} \end{bmatrix} \begin{Bmatrix} \ddot{U}_F \\ \ddot{U}_S \end{Bmatrix} + \begin{bmatrix} K_{FF} & K_{FS} \\ K_{FS}^T & K_{SS} \end{bmatrix} \begin{Bmatrix} U_F \\ U_S \end{Bmatrix} = \begin{Bmatrix} \bar{P}_F \\ P_S \end{Bmatrix} + \begin{bmatrix} O_{FS} \\ I_{SS} \end{bmatrix} q_S \tag{8-16}$$

Note that 0_{SF} is the transpose of 0_{FS} and is an S x F matrix of zero's. From the first of 8.15 it is seen that the single point constraints are of the form:

$$U_S = Y_S = \text{ constants} \tag{8-17}$$

where Y_S is a column matrix of known constant displacement values (either zero or some enforced displacement). This agrees with the single point constraint form discussed above; that is, single point constraints express one DOF as being equal to a constant.

Substituting 8.17 into the first of 8.16 results in the equations for the F-set displacements:

$$\boxed{M_{FF}\ddot{U}_F + K_{FF}U_F = P_F} \tag{8-18}$$

where

$$P_F = \bar{P}_F - K_{FS}Y_S \tag{8-19}$$

At this point the F-set equations in 8.18 can be solved for since there are F unknowns and F equations with which to solve for them. However, MYSTRAN also allows for a Guyan reduction which, although not generally used in static analysis, may be relevant for eigenvalue analysis. In eigenvalue analyses by the GIV method (see EIGR Bulk Data entry), the mass matrix must be nonsingular. In a situation where the model has no mass for the rotational DOF's, the mass matrix would be singular. Guyan reduction to statically condense massless DOF's will result in a nonsingular mass matrix. Thus, if the user identifies an O set, there is a further reduction; that from the F-set to the A-set

2.4 Reduction of the F-set to the A-set

The F-set is partitioned into the A and O-sets where the O DOF's are to be eliminated using Guyan reduction identified by the user either through the use of ASET/ASET1 or OMIT/OMIT1 entries in the input data deck. The A-set are the remainder of the DOF's in the F-set and are known as the "analysis" DOF's. Thus, partition U_F into U_A and U_O:

$$U_F = \begin{Bmatrix} U_A \\ U_O \end{Bmatrix} \tag{8-20}$$

Substitute 8.20 into 8.18 and partition the stiffness and load matrices into the A and O-sets to get:

$$\begin{bmatrix} \bar{M}_{AA} & M_{AO} \\ M_{AO}^T & M_{OO} \end{bmatrix} \begin{Bmatrix} \ddot{U}_A \\ \ddot{U}_O \end{Bmatrix} + \begin{bmatrix} \bar{K}_{AA} & K_{AO} \\ K_{AO}^T & K_{OO} \end{bmatrix} \begin{Bmatrix} U_A \\ U_O \end{Bmatrix} = \begin{Bmatrix} \bar{P}_A \\ P_O \end{Bmatrix} \tag{8-21}$$

Guyan reduction is only exact, in general, for a statics problem. In a dynamic problem it is only exact if there is no mass on the O-set. In order to explain the Guyan reduction, consider equation 8.21 for a statics problem:

In a static analysis ($\ddot{U}$=0) the second of 8.21 can be used to get:

$$\begin{bmatrix} \bar{K}_{AA} & K_{AO} \\ K_{AO}^T & K_{OO} \end{bmatrix} \begin{Bmatrix} U_A \\ U_O \end{Bmatrix} = \begin{Bmatrix} \bar{P}_A \\ P_O \end{Bmatrix} \tag{8-22}$$

From the 2nd of 8.22 we can solve for U_O in terms of U_A. We can then write:

$$
\begin{Bmatrix} U_A \\ U_O \end{Bmatrix} = \begin{bmatrix} I_{AA} \\ G_{OA} \end{bmatrix} U_A + \begin{Bmatrix} 0 \\ U_O^0 \end{Bmatrix}
$$

where

$$
G_{OA} = -K_{OO}^{-1} K_{AO}^T \tag{8-23}
$$

and

$$
U_O^0 = K_{OO}^{-1} P_O
$$

The first part of the first equation in 8.23 suggests the possibility of using:

$$
\begin{Bmatrix} U_A \\ U_O \end{Bmatrix} = \begin{bmatrix} I_{AA} \\ G_{OA} \end{bmatrix} U_A \tag{8-24}
$$

Using 8.24 in 8.22 and premutiplying by the transpose of the coefficient matrix in 8.24 yields:

$$
\begin{bmatrix} I_{AA} & G_{OA}^T \end{bmatrix} \begin{bmatrix} \bar{K}_{AA} & K_{AO} \\ K_{AO}^T & K_{OO} \end{bmatrix} \begin{bmatrix} I_{AA} \\ G_{OA} \end{bmatrix} \begin{Bmatrix} U_A \\ U_O \end{Bmatrix} = \begin{bmatrix} I_{AA} & G_{OA}^T \end{bmatrix} \begin{Bmatrix} \bar{P}_A \\ P_O \end{Bmatrix}
$$

or

$$
K_{AA} U_A = P_A
$$

where

$$
K_{AA} = \bar{K}_{AA} + K_{AO} G_{OA} + (K_{AO} G_{OA})^T + G_{MN}^T K_{OO} G_{OA} = \bar{K}_{AA} + K_{AO} G_{OA} \text{ (by virtue of definition of } G_{OA}\text{)}
$$

and

$$
P_A = \bar{P}_A + G_{OA}^T P_O
$$

(8-25)

Which is exactly what would have been found if 8.23 had been substituted into 8.22 for U_O.

Equation 8.24 to can be used as a way to eliminate the O-set degrees of freedom for the dynamic system of equations in 8.21. This would be an approximation unless there was no mass associated with the O-set degrees of freedom and is the classic Guyan reduction approximation made in dynamic analyses in which the O-set is eliminated by static condensation (i.e. using the G_{OA} in equation 8.23). Using 8.24 in 8.21 yields

$$
\begin{bmatrix} I_{AA} & G_{OA}^T \end{bmatrix} \begin{bmatrix} \bar{M}_{AA} & M_{AO} \\ M_{AO}^T & M_{OO} \end{bmatrix} \begin{bmatrix} I_{AA} \\ G_{OA} \end{bmatrix} \begin{Bmatrix} \ddot{U}_A \\ \ddot{U}_O \end{Bmatrix} + \begin{bmatrix} I_{AA} & G_{OA}^T \end{bmatrix} \begin{bmatrix} \bar{K}_{AA} & K_{AO} \\ K_{AO}^T & K_{OO} \end{bmatrix} \begin{bmatrix} I_{AA} \\ G_{OA} \end{bmatrix} \begin{Bmatrix} U_A \\ U_O \end{Bmatrix} = \begin{bmatrix} I_{AA} & G_{OA}^T \end{bmatrix} \begin{Bmatrix} \bar{P}_A \\ P_O \end{Bmatrix}
$$

(8-26)

where:

$$M_{AA}\ddot{U}_A + K_{AA}U_A = P_A$$

where

$$M_{AA} = \bar{M}_{AA} + M_{AO}G_{OA} + (M_{AO}G_{OA})^T + G_{OA}^T M_{OO} G_{OA} \tag{8-27}$$

$$K_{AA} = \bar{K}_{AA} + K_{AO}G_{OA}$$

$$P_A = \bar{P}_A + G_{OA}^T P_O$$

Now, equation 8.27 can be solved for the A-set DOF displacements. The process of recovering the displacements of the O, S and M-set displacements is accomplished by reversing the process we just went through in the reduction. First, the O set displacements are recovered using 8.23. The combination of the A and O-sets yields the F-set. The S-set is given by 8.17. The combination of the F and S-sets yields the N-set. The M-set is recovered from the N-set by 8.7 and the combination of the N and M-sets yield the complete model displacements in the G-set.

2.5 Reduction of the A-set to the L-set

The A-set is partitioned into the L and R-sets where the R DOF's are boundary DOF's where one substructure attaches to another in Craig-Bampton (CB) analyses. The modal properties of the substructure in CB analysis are fixed boundary modes so that, for the modal portion of CB, the R-set are constrained to zero. The development of the subsequent CB equations of motion in terms of the modal and boundary DOF's will not be presented here. See Appendix D and reference 11 for a complete discussion of CB analyses. For other analyses there is no R-set so that the L set is the same as the A set for solution of the independent degrees of freedom

$$U_A = \begin{Bmatrix} U_L \\ U_R \end{Bmatrix}$$

2.6 Solution for constraint forces

The constraint forces are recovered as follows. Rewrite 8.2 by partitioning Q_G into Q_F, Q_N and Q_M and partitioning q_C into q_S and q_M. Using the coefficient matrix in 8.15 for R_{CG} we get, for Q_G:

$$Q_G = \begin{Bmatrix} Q_F \\ Q_S \\ Q_M \end{Bmatrix} = \begin{bmatrix} 0_{FS} & R_{MF}^T \\ I_{SS} & R_{MS}^T \\ 0_{MS} & R_{MM}^T \end{bmatrix} \begin{Bmatrix} q_S \\ q_M \end{Bmatrix} \tag{8-28}$$

As discussed earlier, the distinction between the q and Q is that the former are generalized forces of constraint and the later are physical constraint forces on the DOF's of the model. It is the Q constraint forces that are of interest.

Rewrite 8.28 as:

$$Q_G = \begin{Bmatrix} 0_F \\ q_S \\ 0_M \end{Bmatrix} + \begin{bmatrix} R_{MF}^T \\ R_{MS}^T \\ R_{MM}^T \end{bmatrix} q_M \tag{8-29}$$

where 0_F and 0_M are null column matrices of size F and M.

Equation 8.29 can be written as:

$$Q_G = Q_{G_{SPC}} + Q_{G_{MPC}} \tag{8-30}$$

The first term in 8.30 represents the forces of single point constraint and the second the forces of multi-point constraint. Comparing 8.29 and 8.30:

$$\begin{aligned} Q_{G_{SPC}} &= \begin{Bmatrix} 0_F \\ q_S \\ 0_M \end{Bmatrix} \\ Q_{G_{MPC}} &= \begin{bmatrix} R_{MF}^T \\ R_{MS}^T \\ R_{MM}^T \end{bmatrix} q_M \end{aligned} \tag{8-31}$$

From the first of 8.31 it is seen that the grid point SPC constraint forces are equal to the generalized q_S forces. Using 8.17 and the second of 8.16 (keeping in mind that the derivatives of the S-set degrees of freedom are zero due to 8.17) the q_S, or Q_S is:

$$\boxed{Q_{G_{SPC}} \equiv \begin{Bmatrix} 0_F \\ Q_{S_{SPC}} \\ 0_M \end{Bmatrix} = \begin{Bmatrix} 0_F \\ M_{FF}\ddot{U}_{FF} + K_{FS}^T U_F + K_{SS} Y_S - P_S \\ 0_M \end{Bmatrix}} \tag{8-32}$$

Thus, there are SPC forces only on the S-set DOF's

From the second of 8.31 and using 8.14 it is seen that the MPC forces can be written as:

$$Q_{G_{MPC}} = \begin{bmatrix} R_{MN}^T \\ R_{MM}^T \end{bmatrix} q_M \tag{8-33}$$

From 8.7 and the second of 8.6, solve for q_M :

$$q_M = R_{MM}^{-T}[(M_{NM}^T + M_{MM}G_{MN})\ddot{U}_N + (K_{NM}^T + K_{MM}G_{MN})U_N - P_M] \tag{8-34}$$

Substituting 8.34 into 8.33 yields:

$$Q_{G_{MPC}} = \begin{bmatrix} R_{MN}^T R_{MM}^{-T} \\ I_{MM} \end{bmatrix} [(M_{NM}^T + M_{MM}G_{MN})\ddot{U}_N + (K_{NM}^T + K_{MM}G_{MN})U_N - P_M] \tag{8-35}$$

Using 8.8 this becomes:

$$\boxed{Q_{G_{MPC}} \equiv \begin{Bmatrix} Q_{N_{MPC}} \\ Q_{M_{MPC}} \end{Bmatrix} = \begin{bmatrix} -G_{MN}^T \\ I_{MM} \end{bmatrix} [(M_{NM}^T + M_{MM}G_{MN})\ddot{U}_N + (K_{NM}^T + K_{MM}G_{MN})U_N - P_M]} \qquad (8\text{-}36)$$

This can also be written as:

$$\boxed{\begin{aligned} & Q_{G_{MPC}} \equiv \begin{Bmatrix} Q_{N_{MPC}} \\ Q_{M_{MPC}} \end{Bmatrix} \\ & \text{with} \\ & Q_{M_{MPC}} = L_{MN}\ddot{U}_N + H_{mn}U_n - P_m \\ & Q_{N_{MPC}} = -G_{mn}^T Q_{M_{MPC}} \\ & \text{where} \\ & H_{mn} = (K_{NM}^T + K_{MM}G_{MN}) \\ & L_{MN} = (M_{NM}^T + M_{MM}G_{MN}) \end{aligned}} \qquad (8\text{-}37)$$

There are MPC forces on the N-set (which includes the F and S-sets) as well as on the M-set. Equations 8.32 and 8.36 (or 8.37) are used to determine the constraint forces once the U_G are found.

This completes the derivation of the solution for the G-set displacements and the constraint forces. However, it is of interest to demonstrate that the constraint forces satisfy the principal of virtual work (that is, constraint forces do no virtual work).

Let W_C be the work done by the constraint forces and δW_C the virtual work done by the constraint forces. Write δW_C as:

$$\delta W_C = \delta W_{SPC} + \delta W_{MPC} = 0$$

where

$$\delta W_{SPC} = \text{virtaul work of the SPC single point constraint forces} \qquad (8\text{-}38)$$

and

$$\delta W_{MPC} = \text{virtaul work of the MPC multi-point constraint forces}$$

The virtual work of the constraint forces is equal to the constraint forces moving through a virtual displacement, δU. Thus:

$$\delta W_{SPC} = Q_{S_{SPC}}^T \delta U_S \qquad (8\text{-}39)$$

By virtue of 8.17:

$$\delta U_S = \delta Y_S = 0_S \tag{8-40}$$

That is, the virtual displacements of the S-set are zero since Y_S contains specified values (zero or some enforced displacement). Therefore:

$$\delta W_{spc} = 0 \tag{8-41}$$

Thus δW_{MPC} must also be zero by virtue of the first of 8.38. This virtual work of the MPC forces can be written as a combination of the virtual work of the MPC forces on the N and M-sets as follows:

$$\delta W_{MPC} = Q^T_{N_{MPC}} \delta U_N + Q^T_{M_{MPC}} \delta U_M \tag{8-42}$$

Using 8.7 this can be written as:

$$\delta W_{MPC} = (Q^T_{N_{MPC}} + Q^T_{M_{MPC}} G_{MN}) \delta U_N \tag{8-43}$$

using 8-41:

$$\delta W_{MPC} = (Q_{N_{MPC}} + G^T_{MN} Q_{M_{MPC}})^T \delta U_N = 0 \tag{8-44}$$

Since the virtual displacements of the N-set are not necessarily zero this requires that:

$$Q_{N_{MPC}} = -G^T_{MN} Q_{M_{MPC}} \tag{8-45}$$

This agrees with 8.36. Thus, the constraint forces developed above are consistent with the principal of virtual work.

3 Equations for Element Stress/Strain Recovery

3.1 General discussion

For the 2D plate elements and 3D solid elements arrays called STRAIN and STRESS are calculated for each element. For 1D elements.. like the rod and beam. only the STRESS array is calculated. Both arrays STRAIN and STRESS can contain up to 9 rows and there is one of each these calculated for every subcase. The STRAIN and STRESS arrays are further subdivided as shown below:

$$\text{STRAIN} = \begin{Bmatrix} \text{STRAIN}_1 \\ \text{STRAIN}_2 \\ \text{STRAIN}_3 \end{Bmatrix} \quad , \quad \text{STRESS} = \begin{Bmatrix} \text{STRESS}_1 \\ \text{STRESS}_2 \\ \text{STRESS}_3 \end{Bmatrix} \qquad (9\text{-}1)$$

where STRAIN_i and STRESS_i each have 3 rows

for 2D and 3D elements:

$\text{STRAIN}_I = (\text{BEi}) * \text{U}_e$

and

$$\text{STRESS}_i = (\text{DE}_i) * \text{STRAIN}_I - (\text{STEi}) \qquad (9\text{-}2)$$

for 1D elements stresses are calculated directly from displacements:

$\text{STRESS}_i = (\text{SEi}) * \text{U}_e - (\text{STEi})$

U_e are the displacements of the nodes of the element in the local element coordinate system (see Figures 3-2 through 3-6 in the main body of this manual) and are obtained from the G-set displacements, the solution for which is discussed in Appendix B. These G-set displacements for the nodes of an element are transformed to the local element coordinate system to obtain U_e which has a number of rows equal to 6n where n is the number of nodes for the element (e.g. n=4 for a quadrilateral plate element). There is one U_e for each subcase in the solution. The BE_i arrays each have 3 rows and 6n columns and are based on the strain-displacement relationships for individual elements. The SE_i are equal to material matrices times the BE_i. The STEi arrays contain the thermal stress effects, if there are any, and have 3 rows and as many columns as there are thermal subcases.. That is, if the input data deck has 5 subcases and two of these have thermal loads, then STE_i will have only 2 columns while U_e will have 5 columns. If a user outputs the SE_i and STE_i arrays, it is their responsibility to keep track of which subcases the columns of STE_i belong. MYSTRAN does this internally for its stress output calculations.

The following sections show what is contained in arrays STRESS_i for each of the element types. In that manner, it will be obvious how MYSTRAN uses the SEi and STEi arrays, generated internally in MYSTRAN, to obtain stresses. If desired, they are available to be output to a text or unformatted binary file through use of the Case Control entry ELDATA. They need not be output for the user to obtain element stresses, however, which are available in the normal text output file through use of the Case Control entry STRESS.

3.2 Rod element

The rod geometry and loading is shown in Figure 3-2 in the main body of this manual. It is a very simple element and has only two stresses that can be output: the axial stress and the torsional stress. It only uses the first 2 rows of array STRESS_1 with row 1 being the axial stress and row 2 the torsional stress. Array STRESS_1 is:

$$\text{STRESS}_1 = \begin{Bmatrix} \sigma_{axial} \\ \tau \\ 0 \end{Bmatrix} \qquad (9\text{-}3)$$

As an example of what is in arrays SE1 and STE1 for a simple element, the arrays are shown below for this rod element. More complicated elements won't have a simple closed form for these matrices and will not be shown.

Array SE1 for the rod element is:

$$SE1 = \frac{1}{L}\begin{bmatrix} -E & 0 & 0 & 0 & 0 & 0 & E & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -C*G & 0 & 0 & 0 & 0 & 0 & C*G & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9-4}$$

E and G are Young's modulus and shear modulus from the Bulk Data material entry for the element, L is the element length and C is the torsional stress recovery coefficient from a PROD entry.

Array STE1 would have the following column for each subcase that has a thermal load:

$$STE1 = E\alpha(\bar{T} - T_{ref})\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \tag{9-5}$$

α and T_{ref} are the coefficient of thermal expansion and reference temperature from the material Bulk Data entry for the element and $\bar{T}$ is the average element temperature for the thermal subcase.

3.3 Bar element

The bar element geometry and loading is shown in Figures 3-3 and 3-4 in the main body of this manual. For the bar element, array STRESS uses all 3 rows of $STRESS_1$ and $STRESS_2$. The first row of $STRESS_1$ contains the actual axial stress in the bar and the third row of $STRESS_2$ contains the actual torsional stress. The second and third rows of $STRESS_1$ and the first two rows of $STRESS_2$ are not actual stress values. Rather, they are the four independent parameters needed to determine the bending stresses at points in the bar cross-section. Thus:

$$STRESS_1 = \begin{Bmatrix} \sigma_{axial} \\ \kappa'_{1a} \\ \kappa'_{1b} \end{Bmatrix} \quad , \quad STRESS_2 = \begin{Bmatrix} \kappa'_{2a} \\ \kappa'_{2b} \\ \tau \end{Bmatrix}$$

where

$$\kappa'_{1a} = \frac{M_{1a}I_2 - M_{2a}I_{12}}{I_1I_2 - I_{12}^2} \quad , \quad \kappa'_{1b} = \frac{M_{1b}I_2 - M_{2b}I_{12}}{I_1I_2 - I_{12}^2}$$

$$\kappa'_{2a} = \frac{M_{2a}I_1 - M_{1a}I_{12}}{I_1I_2 - I_{12}^2} \quad , \quad \kappa'_{2b} = \frac{M_{2b}I_1 - M_{1b}I_{12}}{I_1I_2 - I_{12}^2} \tag{9-6}$$

and

$\sigma_{axial} =$ Axial stress at the neutral axis

$\tau =$ Torsional stress

$I_1, I_2, I_{12} =$ the moments of inertia of the bar on the PBAR entry for this bar element

$M_{1a}, M_{2a}, M_{1b}, M_{2b} =$ the moments in planes 1 and 2 at ends a and b of the bar

(9-7)

This can be put into the form of equation 9.2 as:

$$
\begin{aligned}
&STRESS_1 = SE1 * U_e - STE1 \\
&STRESS_2 = SE2 * U_e - STE2 \\
&\text{where} \\
&SE1 = \begin{bmatrix} B_1K_{aa} & B_1K_{ab} \end{bmatrix} \quad , \quad STE1 = B_1K_{aa}\bar{A}T' \\
&SE2 = \begin{bmatrix} B_2K_{aa} & B_2K_{ab} \end{bmatrix} \quad , \quad STE1 = B_2K_{aa}\bar{A}T'
\end{aligned}
$$

K_{aa} and K_{ab} are 6x6 partitions from the 1st 6 rows of the bar element stiffness matrix and B_1, B_2 and $\bar{A}$
are matrices of element properties as shown below:

$$
B_1 = \begin{bmatrix} -1/A & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\Delta_{12} & -\Delta_1 \\ 0 & 0 & 0 & 0 & \Delta_2 & \Delta_{12} \end{bmatrix}
$$

$$
B_2 = \begin{bmatrix} 0 & \Delta_1 L & -\Delta_{12}L & 0 & -\Delta_{12} & -\Delta_1 \\ 0 & -\Delta_{12}L & \Delta_2 L & 0 & \Delta_2 & \Delta_{12} \\ 0 & 0 & 0 & -C/J & 0 & 0 \end{bmatrix}
$$

$$
\bar{A} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & L\Delta_1 I_1/6 & L\Delta_1 I_1/3 & -L\Delta_{12} I_2/6 & L\Delta_{12} I_2/3 \\
0 & -L\Delta_{12} I_1/6 & -L\Delta_{12} I_1/3 & L\Delta_2 I_2/6 & L\Delta_2 I_2/3 \\
0 & 0 & 0 & 0 & 0 \\
0 & -\Delta_{12} I_1/2 & -\Delta_{12} I_1/2 & \Delta_2 I_2/2 & \Delta_2 I_2/2 \\
0 & -\Delta_1 I_1/2 & -\Delta_1 I_1/2 & \Delta_{12} I_2/2 & \Delta_{12} I_2/2
\end{bmatrix}
$$

and

$$
T' = \begin{Bmatrix} \bar{T} - T_{ref} \\ T'_{1a} \\ T'_{1b} \\ T'_{2a} \\ T'_{2b} \end{Bmatrix} = \begin{Bmatrix} \text{avg bulk temp above material ref temp} \\ \text{gradient through bar in plane 1 at end a} \\ \text{gradient through bar in plane 1 at end b} \\ \text{gradient through bar in plane 2 at end a} \\ \text{gradient through bar in plane 2 at end b} \end{Bmatrix}
$$

with the following bar properties:

$$L = \text{bar length}$$
$$A = \text{cross-sectional area}$$
$$I_1 = \text{area moment of inertia in plane 1}$$
$$I_2 = \text{area moment of inertia in plane 1}$$
$$I_{12} = \text{product of inertia}$$
$$\Delta_1 = \frac{I_2}{I_1 I_2 - I_{12}^2}$$
$$\Delta_2 = \frac{I_1}{I_1 I_2 - I_{12}^2}$$
$$\Delta_{12} = \frac{I_{12}}{I_1 I_2 - I_{12}^2}$$

Stresses due to bending (i.e. not including axial stress at the neutral axis) at ends a and b of the bar element are obtained from:

$$\sigma_a = -(\kappa'_{1a}\bar{y}_e + \kappa'_{2a}\bar{z}_e) \quad , \quad \sigma_b = -(\kappa'_{1b}\bar{y}_e + \kappa'_{2b}\bar{z}_e) \tag{9-8}$$

where σ_a , σ_b are the bending stresses at ends a and b of the bar and $\bar{y}_e$, $\bar{z}_e$ are the coordinates of a point on the bar cross section as measured in the local element coordinate system (see Figure 3-3 in the main body of this manual). It should be noted that temperature distributions through the depth of the bar that are higher order than linear are ignored

3.4 Plate elements

Triangular and quadrilateral plate element geometry, loading and stress conventions are shown in Figures 3-5 and 3-6 in the main body of this manual. They can use all three of the $STRESS_i$ arrays.

3.4.1 Membrane stresses

$STRESS_1$ contains the membrane stresses (at the plate mid-plane)

$$STRESS_1 = \left\{ \begin{matrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{matrix} \right\}_{z=0} \tag{9-9}$$

This can be put into the form of equation 9.2 as:

$$\begin{aligned} &STRESS_1 = (SE1) * U_e - (STE1) \\ &\text{where} \\ &SE1 = E_m B_m \quad \text{and} \qquad STE1 = E_m \alpha (T - T_{ref}) \end{aligned} \tag{9-10}$$

E_m is the 3x3 membrane material matrix, B_m is the element membrane strain-displacement matrix (developed internally in MYSTRAN), α is the 3x1 vector of coefficients of thermal expansion for the material, T is the element average bulk temperature and T_{ref} is the reference temperature for the element material.

3.4.2 Bending stresses

$STRESS_2$, times a fiber distance, contains the stresses due to bending, where:

$$STRESS_2 = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{9-11}$$

This can be put into the form of equation 9.2 as:

$$\begin{aligned} & STRESS_2 = (SE2) * U_e - (STE2) \\ & \text{where} \\ & SE2 = E_b B_b \quad \text{and} \qquad STE2 = E_b \alpha T' \end{aligned} \tag{9-12}$$

E_b is the 3x3 bending material matrix, B_b is the element bending strain-displacement matrix (developed internally in MYSTRAN), α is the 3x1 vector of coefficients of thermal expansion for the material and T' is the temperature gradient through the thickness of the plate element.

3.4.3 Combined membrane and bending stresses

The total bending and in-plane shear stresses at a fiber distance z are obtained from $STRESS_1$ and $STRESS_2$ as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = STRESS_1 + z(STRESS_2) \tag{9-13}$$

3.4.4 Transverse shear stresses

The average transverse shear stresses through the thickness of the plate (for TRIA3 and QUAD4 elements only) are obtained from $STRESS_3$:

$$STRESS_3 = \begin{Bmatrix} \tau_{zx} \\ \tau_{zy} \\ 0 \end{Bmatrix} \tag{9-14}$$

This can be put into the form of equation 9.2 as

$$STRESS_3 = SE3$$

where

$$SE3 = E_s B_s$$

E_s is the 3x3 transverse shear material matrix and B_s is the element transverse shear strain-displacement matrix (developed internally in MYSTRAN).

The transverse shear stresses are not output in the normal output file even if stress output is requested in Case Control. However, the transverse shear stress resultants (integrals of shear stress through thickness) are output if there is a request in Case Control for element engineering forces

3.5 Solid elements

For the 3D solid elements HEXA, PENTA and TETRA arrays STRAIN and STRESS contain only the 6 actual strains and stresses for a 3D solid:

$$\text{STRAIN} = (\text{BE}) * U_e = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

The BE are strain-displacement matrices that are based on the shape functions chosen for the particular 3D solid element. Once the strains have been calculated the stresses are determined from:

$$\text{STRESS} = (\text{ES}) * (\text{STRAIN} - \text{ALPT}) = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

ES is the 6x6 material matrix for a solid and ALPT is the thermal distortion portion of the strains. For a homogeneous isotropic material these are:

$$\text{ES} = \begin{bmatrix} (1-\upsilon)E_0 & \upsilon E_0 & \upsilon E_0 & 0 & 0 & 0 \\ \upsilon E_0 & (1-\upsilon)E_0 & \upsilon E_0 & 0 & 0 & 0 \\ \upsilon E_0 & \upsilon E_0 & (1-\upsilon)E_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} , \quad E_0 = \frac{E}{(1+\upsilon)(1-2\upsilon)} \quad \text{and} \quad G = \frac{E}{2(1+\upsilon)}$$

$$\text{ALPT} = \begin{Bmatrix} \alpha \\ \alpha \\ \alpha \\ 0 \\ 0 \\ 0 \end{Bmatrix} (T - T_{ref})$$

MYSTRAN does allow anisotropic element properties for solids and, in that case, ES and ALPT are different

4 Craig-Bampton Model Generation

4.1 Craig-Bampton Equations of Motion for Substructures

MYSTRAN has the capability to generate Craig-Bampton (CB) models via SOL 31 (or SOL GEN CB MODEL). This solution sequence calculates the fixed-base modes of a substructure and generates all of the matrices needed to couple the substructure to other CB models. This appendix describes the Craig-Bampton method and its implementation in MYSTRAN and includes an example problem to explain the input and output for SOL 31.

Craig and Bampton[1] are credited with the first unified approach to modal synthesis, or substructuring for dynamic analysis, using fixed interface flexible modes augmented by boundary constraint modes to describe each substructure. Their work was a simplification of earlier work by Hurty[2] who first introduced the concept for substructures with redundant boundary degrees of freedom (DOF's).

In order to explain the Craig-Bampton (CB) method, consider a structure represented by the picture below that is comprised of several (in this case 5) substructures connected at an arbitrary number of points:

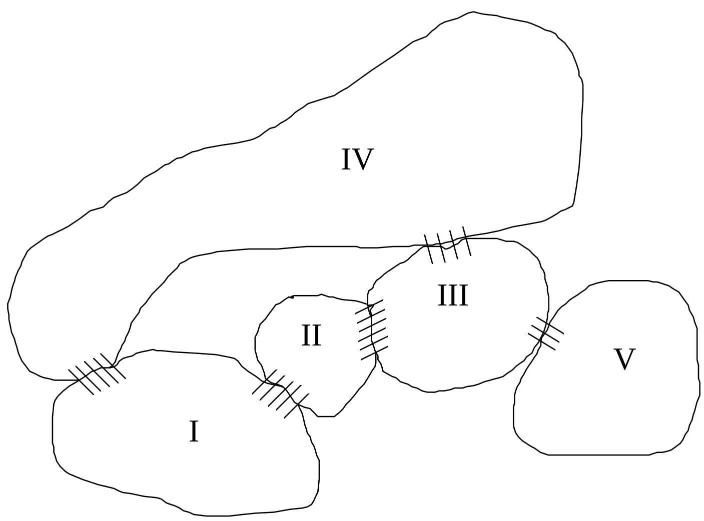


Figure 10.1 - Overall Structure Composed of Several Substructures

Each substructure is joined to one or more other substructures at some number of interface, or boundary, DOF's (indicated by the hatched areas in the above picture. The complete structure, consisting of the connected substructures, may or may not be restrained from free body motion. For any one of the substructures (j = I, II, III, etc.) the G-set equations of motion (ignoring damping for the moment) are:

[1] Craig, R.R. and Bampton, M.C.C. "*Coupling of Substructures for Dynamic Analysis*", AIAA Journal, Vol. 6, No. 7, July 1968, pp 1313-1319
[2] Hurty, W.C. "*Dynamic Analysis of Structural Systems Using Component Modes*", AIAA Journal, Vol. 3, No. 4, April 1965, pp 678-685

$$M^j_{GG}\ddot{u}^j_G + K^j_{GG}u^j_G = P^j_G + Q^j_G$$

where

$$Q^j_G = Q^{m^j}_G + Q^{s^j}_G + Q^{r^j}_G$$

$$u^j_G = \begin{Bmatrix} u^j_A \\ u^j_O \\ u^j_S \\ u^j_M \end{Bmatrix} = \begin{Bmatrix} \text{analysis DOF's} \\ \text{omitted DOF's} \\ \text{SPC'd DOF's} \\ \text{MPC'd DOF's} \end{Bmatrix}$$

and 10-1

P^j_G = applied loads on the G-set

$Q^{m^j}_G$ = constraint forces due to multi-point constraints (MPC's)

$Q^{s^j}_G$ = constraint forces due to single point constraints (SPC's)

$Q^{r^j}_G$ = interface forces at boundaries between substructures

In MYSTRAN nomenclature, the G-set is reduced to the A-set by the elimination of the M-set multi-point constraints, the S-set single point constraints and the O-set omitted DOF's (using OMIT's or ASET's). The A-set DOF's for this substructure must contain all DOF's that will be connected to other substructures The resulting A-set equations of motion (dropping the j superscript notation for each substructure) are:

$$M_{AA}\ddot{u}_A + K_{AA}u_A = P_A + Q^r_A \qquad 10\text{-}2$$

where the A set matrices are mathematical reductions from the G-set (see Appendix B for details)

Partition 2 into the R-set and L-set, where, the R-set represents the boundary DOF's in which this substructure connects with other substructures and the L-set are all free interior DOF's in this substructure

$$\begin{bmatrix} M_{RR} & M^T_{LR} \\ M_{LR} & M_{LL} \end{bmatrix}\begin{Bmatrix} \ddot{u}_R \\ \ddot{u}_L \end{Bmatrix} + \begin{bmatrix} K_{RR} & K^T_{LR} \\ K_{LR} & K_{LL} \end{bmatrix}\begin{Bmatrix} u_R \\ u_L \end{Bmatrix} = \begin{Bmatrix} P_R \\ P_L \end{Bmatrix} + \begin{Bmatrix} Q^r_R \\ o \end{Bmatrix} \qquad 10\text{-}3$$

Notice at this point that there remain forces of constraint only at the substructure attach points as the L-set represents all free DOF's for this substructure.

At this point we can introduce the transformation from the physical displacements in equation (3) to what are known as the CB DOF's; namely the flexible mode DOF's and the boundary (R-set) DOF's. In order to show that this is not any further approximation to equation 3, consider the following argument:

1) the $u_A = \begin{Bmatrix} u_R \\ u_L \end{Bmatrix}$ DOF's are clearly a complete set of DOF's for the substructure in that, once they are known, the complete g-set DOF's for this substructure can be determined.

2) similarly, a new set of DOF's for the substructure,

$$u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad 10\text{-}4$$

are a complete set of DOF's if ξ_N are the generalized DOF's for flexible modes when $u_R = 0$

3) Thus we can take u_L to be a linear combination of u_R and ξ_N or:

$$u_L = D_{LR}u_R + \Phi_{LN}\xi_N \qquad 10\text{-}5$$

if we insist that:

a) Φ_{LN} are shapes when $u_R = 0$ and ξ_N are modal DOF's. That is, the columns of Φ_{LN} are the flexible modes, φ_L^i, when the boundary is fixed. The i-th column of the modal matrix Φ_{LN} is φ_L^i.

b) D_{LR} are shapes when $\xi_N = 0$. That is, the columns of D_{LR} are the L-set shapes for unit motions of the R-set when the flexible mode DOF's are zero.

The φ_L^i are easy to understand. They are the eigenvectors resulting from solving an eigenvalue problem from equations 3 with $u_R = 0$. This eigenvalue problem would be:

$$(K_{LL} - \omega^2 M_{LL})\varphi_L = 0 \qquad 10\text{-}6$$

This requires that the determinant of the coefficient matrix on the left side of equation 6 be zero:

$$\left|K_{LL} - \omega^2 M_{LL}\right| = 0 \quad \text{which yields N eigenvalues } \omega_1^2, \omega_2^2 \ldots, \omega_N^2 > 0 \qquad 10\text{-}7$$

The i-th eigenvector, φ_L^i, is then determined by solving the equation:

$$(K_{LL} - \omega_i^2 M_{LL})\varphi_L^i = 0 \quad \text{for } i = 1, 2, \ldots, N \qquad 10\text{-}8$$

Solution of equation 8 requires that one element of φ_L^i be arbitrarily set (the φ_L^i are shapes and their amplitude does not matter). Once equation 8 is solved, the modal matrix is:

$$\Phi_{LN} = \begin{bmatrix} \varphi_l^1 & \varphi_l^2 & \cdots & \varphi_L^N \end{bmatrix} \qquad 10\text{-}9$$

The D_{LR} can also be explained easily. As stated above, the D_{LR} are shapes when the flexible mode response is zero. We can see from equation 5 that a column of D_{LR} represents the displacements at the L-set DOF's due to motion at one of the R-set DOF's while all other R-set DOF's are zero (as well as all $\xi_N = 0$). We can therefore solve for D_{LR} from equation 3 by taking all applied forces and accelerations equal to zero and solving the statics problem:

$$\begin{bmatrix} K_{RR} & K_{LR}^T \\ K_{LR} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_R \\ u_L^s \end{Bmatrix} = \begin{Bmatrix} Q_R^r \\ o \end{Bmatrix} \qquad 10\text{-}10$$

where u_L^s are static displacements of the L-set. From the second row of equation 10, solve for u_L^s in terms of u_R :

$$\begin{gathered} u_L^s = -K_{LL}^{-1} K_{LR} u_R = D_{LR} u_R \\ \text{or} \\ D_{LR} = -K_{LL}^{-1} K_{LR} \end{gathered} \qquad 10\text{-}11$$

Thus, the CB DOF's are contained in u_X (equation 4) and the transformation between u_X and u_A is:

$$\begin{Bmatrix} u_R \\ u_L \end{Bmatrix} = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad 10\text{-}12$$

where I is an R x R identity matrix. Equation 12 can be written as:

$$\begin{gathered} u_A = \Psi_{AX} u_X \\ \text{where} \\ \Psi_{AX} = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix}, \quad u_A = \begin{Bmatrix} u_R \\ u_L \end{Bmatrix}, \quad u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \end{gathered} \qquad 10\text{-}13$$

Ψ_{AX} is the CB transformation matrix and is of A-set size. In MYSTRAN this is called matrix PHIXA. When expanded to G-set size, PHIXA becomes matrix PHIXG:

$$\boxed{\begin{aligned} & u_G = \Psi_{GX} u_X \\ & \Psi_{GX} = \text{matrix data block PHIXG} \\ & \text{PHIXG} = \text{PHIXA expanded to G-set} \end{aligned}} \qquad 10\text{-}14$$

Note that when all flexible modes of the substructure are used in u_X equation 13 is exact. In practice, all modes are never used since this would defeat the purpose of making the transformation (which is to find a smaller set of DOF's which are nonetheless an accurate representation of the A-set). Substituting equation 13 into equation 2 and premultiplying the result by the transpose of Ψ_{AX} yields:

$$M_{XX} \ddot{u}_X + K_{XX} u_X = P_X + Q_X^r \qquad 10\text{-}15$$

where:

$$M_{XX} = \Psi^T_{AX} M_{AA} \Psi_{AX} = \begin{bmatrix} I & D^T_{LR} \\ 0 & \Phi^T_{LN} \end{bmatrix} \begin{bmatrix} M_{RR} & M^T_{LR} \\ M_{LR} & M_{LL} \end{bmatrix} \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} = \begin{bmatrix} m_{RR} & m^T_{NR} \\ m_{NR} & m_{NN} \end{bmatrix}$$

$$K_{XX} = \Psi^T_{AX} K_{AA} \Psi_{AX} = \begin{bmatrix} I & D^T_{LR} \\ 0 & \Phi^T_{LN} \end{bmatrix} \begin{bmatrix} K_{RR} & K^T_{LR} \\ K_{LR} & K_{LL} \end{bmatrix} \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} = \begin{bmatrix} k_{RR} & 0 \\ 0 & k_{NN} \end{bmatrix}$$

10-16

$$P_X = \Psi^T_{AX} P_A = \begin{bmatrix} I & D^T_{LR} \\ 0 & \Phi^T_{LN} \end{bmatrix} \begin{Bmatrix} P_R \\ P_L \end{Bmatrix} = \begin{Bmatrix} P'_R \\ \Xi_N \end{Bmatrix}, \quad P'_R = P_R + D^T_{LR} P_L, \quad \Xi_N = \Phi^T_{LN} P_L$$

$$Q^r_X = \Psi^T_{AX} Q^r_A = \begin{bmatrix} I & D^T_{LR} \\ 0 & \Phi^T_{LN} \end{bmatrix} \begin{Bmatrix} Q^r_R \\ o \end{Bmatrix} = \begin{Bmatrix} Q^r_R \\ 0 \end{Bmatrix}$$

and:

$$\begin{aligned} m_{RR} &= M_{RR} + M^T_{LR} D_{LR} + (M^T_{LR} D_{LR})^T + D^T_{LR} M_{LL} D_{LR} \\ m_{NR} &= \Phi^T_{LN} (M_{LR} + M_{LL} D_{LR}) \\ m_{NN} &= \Phi^T_{LN} M_{LL} \Phi_{LN} \\ k_{RR} &= K_{RR} + K^T_{LR} D_{LR} \\ k_{NN} &= \Phi^T_{LN} K_{LL} \Phi_{LN} \end{aligned}$$

10-17

m_{NN}, k_{NN} are diagonal matrices of generalized maesses and stiffnesses, respectively.

Equations 15 for the i-th substructure can be written as:

$$\boxed{\begin{bmatrix} m_{RR} & m^T_{NR} \\ m_{NR} & m_{NN} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \end{Bmatrix} + \begin{bmatrix} k_{RR} & 0 \\ 0 & k_{NN} \end{bmatrix} \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} = \begin{Bmatrix} P'_R \\ \Xi_N \end{Bmatrix} + \begin{Bmatrix} Q^r_R \\ 0 \end{Bmatrix}}$$

10-18

The off-diagonal terms in the above stiffness matrix are zero due to the definition of D_{LR} in equation 11. In addition, matrix k_{RR} in equation 18 is null if the boundary is a determinant interface. Equations 14 and 18 are the Craig-Bampton equations of motion for the i-th substructure. The P'_R are due to applied loads on the R and L-set DOF's (see equation 16) and the Q^r_R are the interface forces where substructures connect. Once the equations are developed for all substructures, the individual substructures can be connected and the resulting equations solved for the combined R-set and N-set DOF's u_R and ξ_N for all substructures. Once this is done, the forces of inter-connection, or substructure interface forces, (that is, the Q_R^r) can be solved from the individual substructure equations in the top row of equation 18. Equation 14 is used to obtain displacements for all G-set DOF's.

Each organization that is developing a substructure in CB format would deliver the above coefficient matrices in equations 14 and 18 to the organization that is doing the combined structure analysis. In addition, Displacement and Load Transformation Matrices (DTM's and LTM's) collectively known as Output Transformation Matrices, (OTM's), described below, are also delivered as part of the CB model.

4.2 Development of Displ Output Transformation Matrices (Displ OTM's)

Typically, a set of displacement output transformation matrices (displ OTM's, or DTM's for short), is delivered with a Craig-Bampton model to the organization that will couple all substructures and solve for the primary unknowns (u_R and ξ_N and Q_R^r) in order that desired displacements at some of the substructure G-set DOF's may be obtained along with the coupled solution.

Once the combined structure has been solved for the primary variables, the original u_L physical DOF's could be determined from equation 5 and then element forces and stresses could be determined from the u_R and u_L displacements . This is called recovery of the u_L DOF's and element forces and stresses using the Modal Displacement Method (MDM). However, as is often the case, equations 18 are solved using a severely truncated set of modes for each substructure. While this may not compromise the accuracy of the solutions for u_R and ξ_N, it could compromise the accuracy of element forces and stresses calculated using displacements determined from equation 5 with the truncated set of modes. In order to avoid this problem, the u_L DOF's can be found using the Modal Acceleration Method (MAM), described below. It should be noted that the MAM described below *ignores* damping forces so that it is only useful when the damping is small (e.g. less than 10% or so).

From the bottom row of equation 3, solve for u_L in terms of the other variables in the equation:

$$\begin{aligned} u_L &= -K_{LL}^{-1}(M_{LR}\ddot{u}_R + M_{LL}\ddot{u}_L) - K_{LL}^{-1}K_{LR}u_R + K_{LL}^{-1}P_L \\ &= -K_{LL}^{-1}(M_{LR}\ddot{u}_R + M_{LL}\ddot{u}_L) + D_{LR}u_R + K_{LL}^{-1}P_L \end{aligned} \qquad \text{10-19}$$

Differentiate equation 5 twice and use the result for $\ddot{u}_L$ in equation 19, to get:

$$u_L = \left[-K_{LL}^{-1}(M_{LR} + M_{LL}D_{LR}) \mid -K_{LL}^{-1}M_{LL}\Phi_{LN} \mid D_{LR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + K_{LL}^{-1}P \qquad \text{10-20}$$

The term $K_{LL}^{-1}M_{LL}\Phi_{LN}$ in equation 20. can be written in a form more convenient for calculation. From equation 8 it can be seen that:

$$K_{LL}^{-1}M_{LL}\phi_L^i = \frac{1}{\omega_i^2}\varphi_L^i$$

so that

$$K_{LL}^{-1}M_{LL}\begin{bmatrix}\varphi_l^1 & \varphi_l^2 & \cdots & \varphi_L^N\end{bmatrix}=\begin{bmatrix}\varphi_l^1 & \varphi_l^2 & \cdots & \varphi_L^N\end{bmatrix}\begin{bmatrix}\omega_1^{-2} & & & \\ & \omega_2^{-2} & & \\ & & \ddots & \\ & & & \omega_N^{-2}\end{bmatrix}$$

or

$$K_{LL}^{-1}M_{LL}\Phi_{LN}=\Phi_{LN}\Omega_{NN}^{-2} \qquad \text{10-21}$$

where

$$\Omega_{NN}^{-2}=\begin{bmatrix}\omega_1^{-2} & & & \\ & \omega_2^{-2} & & \\ & & \ddots & \\ & & & \omega_N^{-2}\end{bmatrix} \qquad \text{10-22}$$

substitute equation 21 into equation 20 to get:

$$u_L=\left[-K_{LL}^{-1}(M_{LR}+M_{LL}D_{LR}) \;\middle|\; -\Phi_{LN}\Omega_{NN}^{-2} \;\middle|\; D_{LR}\right]\begin{Bmatrix}\ddot{u}_R\\ \ddot{\xi}_N\\ u_R\end{Bmatrix}+K_{LL}^{-1}P_L \qquad \text{10-23}$$

The various terms in the coefficient matrices in equation 23 are known as Displacement Transformation Matrices (DTM's). Equation 23 can be written as:

$$u_L=\left[DTM1_{LR} \;\middle|\; DTM2_{LN} \;\middle|\; DTM3_{LR}\right]\begin{Bmatrix}\ddot{u}_R\\ \ddot{\xi}_N\\ u_R\end{Bmatrix}+DTM4_{LL}P_l \qquad \text{10-24}$$

where

$$\begin{aligned}DTM1_{LR}&=-K_{LL}^{-1}(M_{LR}+M_{LL}D_{LT})\\ DTM2_{LN}&=-\Phi_{LN}\Omega_{NN}^{-2}\\ DTM3_{LR}&=D_{LR}\\ DTM4_{LL}&=K_{LL}^{-1}\end{aligned} \qquad \text{10-25}$$

Equations 24 and 25 represent the MAM for recovering displacements for the L-set, for the i-th substructure, once the assembled substructure equations have been solved for the u_R and q_N DOF's. Once the L-set displacements have been found, recovery of the remaining displacements in the G-set is accomplished through the transformation matrices used in their elimination from equation 1 (for details see Appendix B). At the G-set level, equation 24 is:

$$u_G = \left[DTM1_{GR} \mid DTM2_{GN} \mid DTM3_{GR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + DTM4_{GL} P_L$$

or

$$u_G = \Gamma_{GZ} u_Z + DTM4_{GL} P_L \qquad \text{10-26}$$

where

$$\Gamma_{GZ} = \left[DTM1_{GR} \mid DTM2_{GN} \mid DTM3_{GR} \right] = DTM_{GZ}$$

and

$$u_Z = \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix}$$, where u_Z are the Craig-Bampton Degrees of freedom (CB_DOF's)

.

where each of the G-set DTM's in equation 26 is obtained from the L-set DTM's in equation 25 through the normal recovery operations to build back up to the G-set from the L-set. The coefficient matrix in equation 26 that has DTM's 1 - 3 in it is called matrix PHIZG. The table below explains the meaning of each of the DTM's in equation 26:

Table 10.1

i-th col of:	Represents:
$DTM1_{GR}$	displ's of G-set due to a unit accel of the i-th interface DOF (all other R, N set DOF's zero)
$DTM2_{GN}$	displ's of G-set due to a unit accel of the i-th flex mode DOF (all other R, N set DOF's zero)
$DTM3_{GR}$	displ's of G-set due to a unit displ of the i-th interface DOF (all other R, N set DOF's zero)
$DTM4_{GL}$	displ's of G-set due to a unit force on the i-th L-set DOF (all other L-set forces zero)

4.3 Development of Load Output Transformation Matrices (Load OTM's)

Once the G-set displacements have been found, substructure element forces and stresses, as well as grid point forces, can be recovered and assembled into a Loads Output Transformation Matrix, or Load OTM (more commonly referred to as LTM). There are several types of quantities one may desire in an LTM. Equations are developed, below, for several types of LTM quantities typically used in CB analyses.

4.3.1 LTM Terms for Substructure Interface Forces

From the top row of equation 18, the interface forces can be determined once the substructures have been coupled and the u_R and ξ_N solved. The interface forces are:

$$Q_R^r = m_{RR}\ddot{u}_R + m_{NR}^T\ddot{\xi}_N + k_{RR}u_R - P_R'$$

or

$$Q_R^r = \begin{bmatrix} m_{RR} & m_{NR}^T & k_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - I_{RR}P_R' \qquad \text{10-27}$$

where I_{RR} is an RxR identity matrix. Equation 27 can be written as:

$$Q_R^r = \begin{bmatrix} LTM21_{RR} & LTM22_{RN} & LTM23_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - LTM24_{RR}P_R^r$$

or

$$Q_R^r = J_{RZ}U_Z - I_{RR}P_R$$

where

$$J_{RZ} = \begin{bmatrix} LTM21_{RR} & LTM22_{RN} & LTM23_{RR} \end{bmatrix} = LTM2_{RZ} \qquad \text{10-28}$$

$$LTM21_{RR} = m_{RR}$$

$$LTM22_{RN} = m_{NR}^T$$

$$LTM23_{RR} = k_{RR}$$

$$LTM24_{RR} = I_{RR}$$

4.3.2 LTM Terms for Net cg Loads

Terms can also be included in the overall LTM that will recover what are known as "net" accelerations at the center of gravity (cg) of the CB model. These are termed Net Load factors (NLF's) and represent rigid body accelerations of the cg due to the reaction (or interface) forces, Q_R^r. The development below demonstrates how these are determined.

Define:

$$u_{cg} = \text{6 x 1 matrix of rigid body displacements of the cg of the substructure}$$
$$u_{R_{rb}} = \text{r x 1 vector of rigid body displacements at the r DOF}$$
$$T_{R6} = \text{r x 6 matrix where each column represents rigid body displacements of the r DOF due to a unit motion in one DOF at the cg} \qquad \text{10-29}$$
$$Q_{cg} = \text{6 x 1 vector of forces at the cg that are static equivalents of } Q_r^r$$

Then:

$$u_{R_{rb}} = T_{R6} u_{cg}$$
$$\text{and} \qquad \text{10-30}$$
$$Q_{cg} = T_{R6}^T Q_R^r$$

Substitute equation 27 into 30 for Q_R^r:

$$Q_{cg} = T_{R6}^T (m_{RR}\ddot{u}_R + m_{NR}^T \ddot{\xi}_N + k_{RR}u_R - P_R') \qquad \text{10-31}$$

For rigid body motion:

$$Q_{cg} = m_{cg}\ddot{u}_{cg} \qquad \text{10-32}$$

where m_{cg} is the 6 x 6 rigid body mass matrix relative to the cg and is equal to:

$$m_{cg} = T_{R6}^T m_{RR} T_{R6} \qquad \text{10-33}$$

and m_{RR} is given in equation 17. From equations 31 through 33 we can write the cg acceleration net load factors (NLF's) as:

$$\ddot{u}_{cg} = m_{cg}^{-1} Q_{cg} = m_{cg}^{-1} T_{R6}^T \begin{bmatrix} m_{RR} & m_{NR}^T & k_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - m_{cg}^{-1} T_{R6}^T P_R' \qquad \text{10-34}$$

However, $T_{R6}^T k_{RR} = 0$ since the columns of T_{R6} are rigid body modes. Therefore:

$$\ddot{u}_{cg} = m_{cg}^{-1} Q_{cg} = m_{cg}^{-1} T_{R6}^T \begin{bmatrix} m_{RR} & m_{NR}^T & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - m_{cg}^{-1} T_{R6}^T P_R' \qquad \text{10-35}$$

which can be written as:

$$
\begin{aligned}
&\ddot{u}_{cg} = \begin{bmatrix} LTM11_{6R} & LTM12_{6N} & 0_{6R} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - \begin{bmatrix} LTM14_{6R} \end{bmatrix} P'_R \\
&\text{where} \\
&LTM11_{6R} = m_{cg}^{-1} T_{R6}^T m_{RR} \\
&LTM12_{6N} = m_{cg}^{-1} T_{R6}^T m_{NR}^T \\
&LTM14_{6R} = m_{cg}^{-1} T_{R6}^T \\
&LTM1_{6Z} = \begin{bmatrix} LTM11_{6R} & LTM12_{6N} & 0 \end{bmatrix}
\end{aligned}
\qquad \text{10-36}
$$

4.3.3 LTM Terms for Element Forces and Stresses

In MYSTRAN, element forces and stresses are obtained from the G-set displacement vector and the individual element stiffness matrices. Equation 26 is the G-set displacement vector:

$$
u_G = \begin{bmatrix} DTM1_{GR} & | & DTM2_{GN} & | & DTM3_{GR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + DTM4_{GL} P_L = \Gamma_{GZ} u_Z + DTM4_{GL} P_L
$$

Thus the columns of each of the DTM's represents G-set displacements per unit value of one of the variables $\ddot{u}_R, \ddot{\xi}_N, u_R, P_L$ as described in Table 10.1. Therefore, each of the DTM's can be used as if they were a matrix of displacements in calculating element forces and stresses to give:

$$
\begin{aligned}
&f_e = \begin{bmatrix} LTM31_{eR} & | & LTM32_{eN} & | & LTM33_{eR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + LTM34_{eL} P_L \\
&\text{where} \\
&f_e = \text{vector of element forces and stresses (e = number of finite elements)} \\
&LTM31_{eR} = \text{matrix of element forces and stresses due to G-set displ's } DTM1_{GR} \\
&LTM32_{eN} = \text{matrix of element forces and stresses due to G-set displ's } DTM2_{GN} \\
&LTM33_{eR} = \text{matrix of element forces and stresses due to G-set displ's } DTM3_{GR} \\
&LTM34_{eL} = \text{matrix of element forces and stresses due to G-set displ's } DTM4_{GL} \\
&LTM3_{eZ} = \begin{bmatrix} LTM31_{eR} & | & LTM32_{eN} & | & LTM33_{eR} \end{bmatrix}
\end{aligned}
\qquad \text{10-37}
$$

4.3.4 LTM Terms for Grid Point Forces due to multi-point constraints (MPC's)

There are cases in CB analyses in which the forces due to MPC's are of interest. As an example, if a user wishes to determine a load in a bolt at an interface between components, it is common to model the bolt as an MPC where two coincident grids are constrained to have the same displacements. This section develops the equations for determining an LTM for grid point MPC forces.

Equation 1 for the i-th substructure (dropping the superscript-j notation):

$$M_{GG}\ddot{u}_G + K_{GG}u_G = P_G + Q_G^s + Q_G^m + Q_G^r \qquad 10\text{-}38$$

As described in section 10.1 the Q constraint forces on the right side of equation 38 are the constraint forces on the S-set SPC DOF's, the M-set MPC DOF's and on the R-set boundary DOF's respectively. Since all of the boundary DOF's are contained in the R-set there should be no constraint forces on the S-set. That is, all S-set DOF's should be the result of removing singularities and not the result of grounding the model[3]. With this assumption, as well as the assumption that there are no applied loads on the M-st degrees of freedom the following equation is valid for the MPC forces on the M-set grids:

$$Q_G^m = M_{GG}\ddot{u}_G + K_{GG}u_G - Q_G^r \qquad 10\text{-}39$$

We want to get 39 in a form like the other LTM'; that is, in terms of u_Z .

From equation 26 with applied loads zero:

$$u_G = \Gamma_{GZ}u_Z, \quad u_Z = \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} \qquad 10\text{-}40$$

The g-set DOF vector can also be written using equation 14:

$$u_G = \Psi_{GX}u_X, \quad u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad 10\text{-}41$$

Differentiating twice:

$$\ddot{u}_G = \Psi_{GX}\ddot{u}_X$$

This can also be written as:

$$\ddot{u}_G = \begin{bmatrix} \Psi_{GX} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_X \\ u_R \end{Bmatrix} \qquad 10\text{-}42$$

Partition the x DOF's into R and N as in equation 13. This will require partitioning Ψ_{GX} into sub-matrices for the R and N also, so that equation 42 can be written as:

[3] This should be verified by the user by inspection of the forces of single point constraint in the output from the analysis

$$\ddot{u}_G = \begin{bmatrix} \Psi_{GR} & \Psi_{GN} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} = \Psi'_{GZ} u_Z$$

where 10-43

$$\Psi'_{GZ} = \begin{bmatrix} \Psi_{GR} & \Psi_{GN} & 0 \end{bmatrix} = \begin{bmatrix} \Psi_{GX} & 0 \end{bmatrix}$$

.

Substitute equations 40 and 43 into 39 for u_G and $\ddot{u}_G$ respectively to get:

$$Q^m_G = M_{GG}\Psi'_{GZ}u_Z + K_{GG}\Gamma_{GZ}u_Z - Q^r_G \qquad \text{10-44}$$

We need to express the boundary constraint forces in equation 44 in terms of the u_Z vector as we did for the inertia and stiffness terms. From 28:

$$Q^r_R = J_{RZ}u_Z - I_{RR}P_R \qquad \text{10-45}$$

The Q^r_R boundary forces on the R-set can be expanded from the R-set to the G-set Q^r_G by adding zero rows to 45 for the M, S, O-sets (all of the G-set but the R degrees of freedom) to give

$$Q^r_G = J_{GZ}u_Z - I_{GR}P_R \qquad \text{10-46}$$

where J_{GZ} is J_{RZ} expanded to G-set size by addition of zero rows for M, S, O-sets and I_{GR} is expanded from I_{RR} in the same fashion (recall I_{RR} is an R size identity matrix). Substituting 46 into 44 we get::

$$Q^m_G = (M_{GG}\Psi'_{GZ} + K_{GG}\Gamma_{GZ} - J_{GZ})u_Z$$

or

$$Q^m_G = LTM4_{GZ}\,u_Z \qquad \text{10-47}$$

where

$$LTM4_{GZ} = (M_{GG}\Psi'_{GZ} + K_{GG}\Gamma_{GZ} - J_{GZ})$$

$LTM4_{GZ}$ is the LTM for MPC forces at grids that have no applied load

4.4 Development of Acceleration Output Transfer Matrices (Accel OTM)

In addition to the displacement and load output transformation matrices (DTM's and LTM's) it is common to supply acceleration output transformation matrices (accel OTM's or ATM's for short). From equation 10-12 and differentiating twice we obtain:

$$\begin{Bmatrix} \ddot{u}_R \\ \ddot{u}_L \end{Bmatrix} = [ATM] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \end{Bmatrix}$$

where 10-48

$$ATM = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix}$$

ATM is the acceleration transfer matrix. Notice that the "degrees of freedom" for the ATM are the accelerations of the boundary and modal degrees of freedom whereas all of the other OTM's have as degrees of freedom: boundary accelerations, modal accelerations and boundary displacements. This is due to the use of the modal acceleration method for recovery of displacements and element forces.

4.5 Correspondence between matrix names and CB Equation Variables

The table below shows the correspondence between variables introduced in the above equations and matrix data block names in the DMAP program in Section 10.5. Any of these may be output in a MYSTRAN CB model generation analysis using the Executive Control entry OUTPUT4.

Table 10-2
Matrices that can be written to OUTPUT4 files

	MYSTRAN Matrix Name (OUTPUT4 matrices)	NASTRAN DMAP Name	CB equation variable in Appendix D (where applicable)	Matrix size[1]	Partition rows and/or cols
1	CG_LTM		$\left[LTM11_{6r} \quad LTM12_{6N} \quad 0\right]$	6x(2R+N)	
2	DLR	DM	D_{LR}	LxR	rows and cols
3	EIGEN_VAL	LAMA	Ω_{NN}^{2}	NxN	
4	EIGEN_VEC	PHIG	Φ_{GN} , (Φ_{LN} with rows expanded to G-set)	GxN	rows
5	GEN_MASS	MI	m_{NN}	Nx1 vector of diag. terms	
6	IF_LTM		$\left[LTM21_{RR} \quad LTM22_{RN} \quad LTM23_{RR}\right]$	Rx(2R+N)	rows
7	KAA	KAA	K_{AA}	AxA	rows and cols
8	KGG	KGG	K_{GG}	GxG	rows and cols
9	KLL	KLL	K_{LL}	LxL	rows and cols
10	KRL	KLR(t)	K_{LR}	LxR	rows and cols
11	KRR	KRR	K_{RR}	RxR	rows and cols
12	KRRcb	KBB	$k_{RR} = K_{RR} + K_{LR}^{T} D_{LR}$	RxR	rows and cols
13	KXX	KRRGN	K_{XX}	(R+N)x(R+N)	
14	LTM	LTM	CG_LTM and IF_LTM merged	(6+R)x(2R+N)	
15	MCG	RBMCG	m_{cg}	6x6	
16	MEFFMASS		Modal effective mass	Nx6	
17	MPFACTOR		Modal participation factors	Nx6 or NxR	
18	MAA		M_{AA}	AxA	rows and cols
19	MGG		M_{GG}	GxG	rows and cols
20	MLL	MLL	M_{LL}	LxL	rows and cols
21	MRL	MRL	M_{RL}	RxL	rows and cols
22	MRN		$m_{RN} = m_{NR}^{T}$	RxN	rows
23	MRR	MRR	M_{RR}	RxR	rows and cols

Table 10-2 (con't)

	MYSTRAN Matrix Name (OUTPUT4 matrices)	NASTRAN DMAP Name	CB equation variable in Appendix D (where applicable)	Matrix size[4]	Partition rows and/or cols
24	MRRcb	MBB	$m_{RR} = M_{RR} + M_{LR}^T D_{LR} + (M_{LR}^T D_{LR})^T + D_{LR}^T M_{LL} D_{LR}$	RxR	rows and cols
25	MXX	MRRGN	$M_{XX} = \begin{bmatrix} m_{RR} & m_{NR}^T \\ m_{NR} & m_{NN} \end{bmatrix}$	(R+N)x(R+N)	
26	PA		(A-set static reduced loads - only used in statics)		Rows
27	PG		(G-set static loads - only used in statics)		Rows
28	PL		(L-set static reduced loads - only used in statics)		rows
29	PHIXG	PHIXG	Ψ_{AX} , (Ψ_{AX} with rows expanded to G-set)	Gx(R+N)	rows
30	PHIZG		The G-set displacement transformation matrix is written out in the F06 file under "C B D I S P L A C E M E N T O T M"	Gx(2R+N)	rows
31	RBM0		Rigid body mass matrix relative to the basic origin	6x6	
32	TR6_0	RBR	T_{R6} : rigid body displacement matrix for R-set relative to the model basic coordinate system	Rx6	rows
33	TR6_CG	RBRCG	T_{R6} : rigid body displacement matrix for R-set relative to the model CG	Rx6	rows

Notes:

a. (t) indicates matrix transposition

b. Matrix m_{RR} will be singular if there are rotational DOF's but no rotational inertia in the R-set, in which case small rotational inertias may have to be added at these DOF's.

c. Matrix k_{RR} is null if the boundary is a determinant set of DOF's.

d. Matrix m_{RR} is the rigid body mass matrix if the boundary is a determinant set of DOF's

[4] Matrix size given in rows x columns where R means the size of the R-set, L is the size of the L-set, A is the size of the A-set, G is the size of the G-set and N is the number of eigenvectors. See section 3.6 for definition of the complete displacement set notation

4.6 Craig-Bampton model generation example problem

The figure below shows a small example problem that is a frame made of CBAR's that is a substructure assumed to be attached to some other structure in DOF's 1,2,3 at grids 11 and 13 and in DOF's 2,3 at grid 12. The example problem F06 file (with the input echo'd) is shown on the following pages. This section will discuss the input and output in an effort to explain the Craig-Bampton model generation process.

Equation 10.26 defines the Craig-Bampton degrees of freedom (CB-DOF's) as U_z which, for this example, consists of the 18 DOF's:

- 8 boundary acceleration DOF's, $\ddot{u}_R$
- 2 modal acceleration DOF's, $\ddot{\xi}_N$ (see EIGRL request for 2 modes to be extracted)
- 8 boundary displacement DOF's, u_R

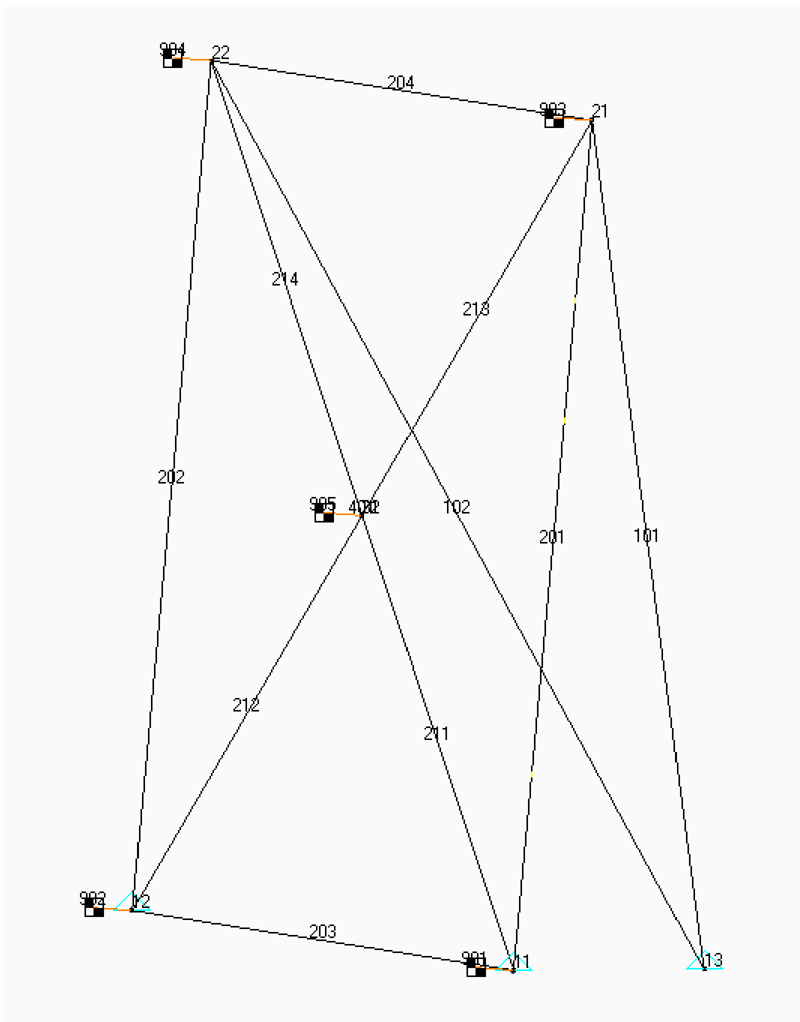


Figure 10.2 – Example CB model: CB-EXAMPLE-12b.DAT

Notes on section 10.6.1: CB-EXAMPLE-12b.F06

The echo of the input shows the following salient points for a CB model generation (much like a SOL 3 eigenvalue analysis in terms of input data):

- Executive Control:

    - SOL 31 indicates CB model generation

    - The OUTPUT4 commands show the matrices that will be written in a format the same as NASTRAN OUTPUT4 files. These matrix data blocks are ones that are listed on Table 10.2 as allowable OUTPUT4 matrices. Notice that several are written to unit 21 while others are written to unit 22. As explained in section 5.1 of the MYSTRAN Users Reference Manual, unit numbers 21 through 27 are valid for writing OUTPUT4 matrices.

- Case Control:

    - METHOD = 1 is to be used for a normal eigenvalue analysis (same as if SOL were 3)

    - Outputs (ACCE, DISP, ELFORCE, STRESS) are for Output Transformation Matrices (OTM's) for the specified sets. These will be written to the text F06 file. In addition they will be written to binary files (same name, CB-EXAMPLE-12b) with extension OP8 for the element related OTM;s (ELFORCE, STRESS in this case and OP9 for the grid related OTM's (ACCE, DISP in this case)

- Bulk Data:

    - Shows the model for this example (notice it has mostly CBAR's but there is also a RBE2)

    - Degrees of freedom at the boundary where this substructure attaches to other substructures are defined with the SUPORT Bulk Data entry. This is the same procedure that is used in CB analyses by the NASTRAN DMAP (Direct Matrix Abstraction Program) method familiar to NASTRAN CB analysts.

    - Eigenvalue extraction, EIGRL requesting 2 modes to be extracted

The delineated F06 output begins on the page following the input model echo and shows the following:

- Eigenvalues extracted

- Messages on the matrices requested to be written to OUTPUT4 files

- For the first 3 of the 18 CB_DOF's in this example the following output (requested in Case Control) is shown (other 15 were left out for clarity):

    - Displacement OTM for the requested grids (see Case Control command DISP = 102)

    - Element engineering force OTM (see Case Control command ELFORCE = 201)

    - Element stress OTM (see Case Control command STRESS = 202)

- Acceleration OTM. As shown in equation 10.48 the acceleration OTM has columns for $\ddot{u}_R$ and $\ddot{\xi}_N$ but not u_R. For this example, there are 10 columns in the acceleration OTM (8 boundary acceleration DOF's and 2 modal acceleration DOF's)

Notes on section 10.6.2: OUTPUT4 matrices written to CB-EXAMPLE-12b.OP1 and OP2

As shown in the Executive Control section of the F06 file in section 10.6.1, there were 3 matrices requested to be written to unit 21 and 4 to unit 22. These binary files, translated to text, are shown in section 10.6.2. The number of actual columns for each matrix is indicated in Table 10.2 but only the first 5 of the columns are shown here for the sake of brevity. These are several of the important CB matrices needed to couple this CB substructure to other substructures in a combined analysis. The binary OUTPUT4 files are written in the same format as the NASTRAN OUTPUT4 binary files.

Notes on section 10.6.3: Displ and elem force/stress OTM's written to CB-EXAMPLE-12b.OP1, OP2

Any output requests in Case Control for grid related outputs (e.g. DISPL, ACCEL) and element force/stress outputs (e.g. ELFORCE, STRESS) are written to the text F06 file and also written to OUTPUT4 binary files (automatically; that is, no formal OUTPUT4 request is needed). The element related OTM's are always written to a file with the same filename as the F06 file but with extension OP8. The grid related OTM's are written to a file with extension OP9.

The first page of section 10.6.3 is a text translation of the element related OTM's written to file CB-EXAMPLE-12b.OP8. The values are the same as was written to the F06 file for element forces and stresses but are also written to binary files in OUTPUT4 format to be used in analyses that couple the CB substructures. In order to explain the contents of the binary OP8 file, a text file with extension OT8 is also automatically written (provided any Case Control requests are included for element forces/stresses) describing the contents of the OP8 binary file. This OT8 text file gives an overview of the OP8 binary file and then goes on to describe each row written to the OP8 file.

The next several pages show the same type of information on the grid related OTM's written to binary file with extension OP9 (with text description in OT9). Again, this is the grid related outputs requested in Case Control and also written to the F06 text file.
*

4.6.1 CB-EXAMPLE-12-b.F06

(delineated – some output not included here for the sake of clarity)

```
1030180330

 MYSTRAN Version 3.00   Oct 20 2006 by Dr Bill Case (this TRIAL edition is SP protected)

 >> MYSTRAN BEGIN  : 10/30/2006 at 18: 3:30.640 The input file is CB-EXAMPLE-12-
b.DAT

 >> LINK  1 BEGIN


SOL 31
$
OUTPUT4   CG_LTM   , IF_LTM   ,         ,        ,        //-1/21 $
OUTPUT4   KRRGN    , RBMCG    , MRRGN   ,        , RBRCG //-1/22 $
OUTPUT4   MR       ,          ,         ,        ,        //-1/21 $
CEND
TITLE  = TEST OF CRAIG-BAMPTON SOLUTION
SUBTI  = FRAME USING CBAR's
SPC    = 1
METHOD = 1
ECHO     = UNSORT
$
SET 101  = 32
SET 102  = 22, 32
SET 201  = 211, 212
SET 202  = 201
$
ACCE     = 101
DISP     = 102
ELFORCE  = 201
STRESS   = 202
MEFFMASS = ALL
MPFACTOR = ALL
$
BEGIN BULK
$
EIGRL   1                       2       2       DPB     -1.     MASS
$
EIGR    2        MGIV                   1       24                      +E1
+E1     MASS
GRID    11              0.      0.      0.
GRID    12              100.    0.      0.
GRID    13              50.     0.      50.
GRID    21              0.      100.    0.
GRID    22              100.    100.    0.
GRID    31              50.     50.     0.
GRID    32              50.     50.     0.
$
RBE2    401     31      123456  32
```

```
$
$ Frame support bars
$
CBAR    101     1       13      21      0.0     0.5     1.0             +C1
+C1     56      456
CBAR    102     1       13      22      0.0     0.5     1.0             +C2
+C2     56      456
$
$ Edge bars
$
CBAR    201     2       11      21      0.0     0.0     1.0
CBAR    202     2       12      22      0.0     0.0     1.0
CBAR    203     2       11      12      0.0     0.0     1.0
CBAR    204     2       21      22      0.0     0.0     1.0
$
$ Diag bars
$
CBAR    211     3       11      31      0.0     0.0     1.0
CBAR    212     3       12      31      0.0     0.0     1.0
CBAR    213     3       21      31      0.0     0.0     1.0
CBAR    214     3       22      31      0.0     0.0     1.0
$
PBAR    1       1       0.36    0.09    0.09    0.18
PBAR    2       1       0.10    10.0    10.0    20.0
PBAR    3       1       6.0     6.0     6.0     12.0
$
MAT1    1       10.+6           0.3     0.1
 *INFORMATION: MAT1 ENTRY        1 HAD FIELD FOR G  BLANK. MYSTRAN CALCULATED G  =  3.846154E+06
$
CONM2   901     11              150.0   0.0     0.0     -5.0
CONM2   902     12              150.0   0.0     0.0     -5.0
CONM2   903     21              150.0   0.0     0.0     -5.0
CONM2   904     22              150.0   0.0     0.0     -5.0
CONM2   905     32              150.0   0.0     0.0     -5.0
$
SPC1    1       456     13
$
$ BOUNDARY DOF'S
$
SUPORT  11      123     12      23      13      123
$
PARAM   WTMASS   .002591
$
ENDDATA
```

```
             E I G E N V A L U E   A N A L Y S I S   S U M M A R Y   (LANCZOS  Mode 2 DPB     Shift eigen = -1.00E+00)

                      NUMBER OF EIGENVALUES EXTRACTED  . . . . . .         2

                      LARGEST OFF-DIAGONAL GENERALIZED MASS TERM    -2.7E-13 (Vecs renormed to 1.0 for gen masses)

                                                            . . .          2
                                MODE PAIR . . . . . . . . . .
                                                            . . .          1

                      NUMBER OF OFF DIAGONAL GENERALIZED MASS
                      TERMS FAILING CRITERION OF  1.0E-04. . . . .         0


                                              R E A L   E I G E N V A L U E S
   MODE  EXTRACTION      EIGENVALUE            RADIANS             CYCLES            GENERALIZED         GENERALIZED
  NUMBER   ORDER                                                                         MASS              STIFFNESS

      1       1        3.895211E+03        6.241163E+01        9.933119E+00        1.000000E+00        3.895211E+03
      2       2        7.011163E+03        8.373269E+01        1.332647E+01        1.000000E+00        7.011163E+03


>> LINK  4 END


>> LINK  6 BEGIN
```

```
*INFORMATION: THE FOLLOWING   7 MATRICES WILL BE WRITTEN TO  2 OUTPUT4 FILES IN THE ORDER LISTED BELOW:

               OUTPUT4 file on unit  21 has been created as: CB-EXAMPLE-12-b.OP1 and will contain the matrices:
                       ( 1) CG_LTM          :        6 rows and       18 cols   This is MYSTRAN matrix CG_LTM
                       ( 2) IF_LTM          :        8 rows and       18 cols   This is MYSTRAN matrix IF_LTM
                       ( 3) MR              :        8 rows and        8 cols   This is MYSTRAN matrix MRRcb

               OUTPUT4 file on unit  22 has been created as: CB-EXAMPLE-12-b.OP2 and will contain the matrices:
                       ( 1) KRRGN           :       10 rows and       10 cols   This is MYSTRAN matrix KXX
                       ( 2) RBMCG           :        6 rows and        6 cols   This is MYSTRAN matrix MCG
                       ( 3) MRRGN           :       10 rows and       10 cols   This is MYSTRAN matrix MXX
                       ( 4) RBRCG           :        8 rows and        6 cols   This is MYSTRAN matrix TR6


 >> LINK  6 END


 >> LINK  5 BEGIN


 >> LINK  5 END


 >> LINK  9 BEGIN
```

OUTPUT FOR CRAIG-BAMPTON DOF 1 OF 18

C B D I S P L A C E M E N T O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	-1.412939E-05	1.622140E-05	8.242222E-05	5.883709E-07	-1.667433E-06	5.125151E-07
32	0	1.051041E-05	-9.465944E-06	-3.182887E-06	-1.086181E-07	-9.450720E-07	2.106009E-07

C B E L E M E N T E N G I N E E R I N G F O R C E O T M
F O R E L E M E N T T Y P E B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	2.091876E-01	7.894539E-01	1.515607E+00	-1.439344E+00	-1.847556E-02	3.151997E-02	6.266800E-01	9.672846E-03
212	-1.133151E-01	-1.008960E-02	-1.725401E+00	-6.166148E-02	2.279833E-02	7.293366E-04	-2.953611E-01	-4.720428E-03

C B E L E M E N T S T R E S S O T M I N L O C A L E L E M E N T C O O R D I N A T E S Y S T E M
F O R E L E M E N T T Y P E B A R

Element ID	SA1 / SB1	SA2 / SB2	SA3 / SB3	SA4 / SB4	Axial Stress	SA-Max / SB-Max	SA-Min / SB-Min	M.S.-T / M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-2.748670E+00	-2.748670E+00	-2.748670E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		-2.748670E+00	-2.748670E+00	

OUTPUT FOR CRAIG-BAMPTON DOF 2 OF 18

C B D I S P L A C E M E N T O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	-7.600290E-05	8.243595E-05	3.128787E-04	1.925291E-06	2.220055E-06	1.292053E-07
32	0	-5.990878E-05	6.308617E-05	3.224179E-04	3.643362E-06	4.904270E-07	3.218612E-08

C B E L E M E N T E N G I N E E R I N G F O R C E O T M
F O R E L E M E N T T Y P E B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	3.640634E+00	-2.875040E+00	-7.752079E+00	4.486528E+00	1.611173E-01	-1.041083E-01	1.906435E+00	-5.333935E-03
212	3.789705E+00	2.992877E+00	-6.061077E+00	-4.713484E+00	1.393111E-01	1.089844E-01	1.808077E+00	5.333935E-03

C B E L E M E N T S T R E S S O T M I N L O C A L E L E M E N T C O O R D I N A T E S Y S T E M
F O R E L E M E N T T Y P E B A R

Element ID	SA1 / SB1	SA2 / SB2	SA3 / SB3	SA4 / SB4	Axial Stress	SA-Max / SB-Max	SA-Min / SB-Min	M.S.-T / M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	7.582667E+00	7.582667E+00	7.582667E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		7.582667E+00	7.582667E+00	

OUTPUT FOR CRAIG-BAMPTON DOF 3 OF 18

C B D I S P L A C E M E N T O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	3.800145E-05	-4.121798E-05	-1.564393E-04	-9.626456E-07	-1.110028E-06	-6.460267E-08
32	0	2.995439E-05	-3.154308E-05	-1.612090E-04	-1.821681E-06	-2.452135E-07	-1.609306E-08

C B E L E M E N T E N G I N E E R I N G F O R C E O T M
F O R E L E M E N T T Y P E B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	-1.820317E+00	1.437520E+00	3.876039E+00	-2.243264E+00	-8.055864E-02	5.205414E-02	-9.532175E-01	2.666968E-03
212	-1.894852E+00	-1.496438E+00	3.030538E+00	2.356742E+00	-6.965554E-02	-5.449220E-02	-9.040385E-01	-2.666968E-03

C B E L E M E N T S T R E S S O T M I N L O C A L E L E M E N T C O O R D I N A T E S Y S T E M
F O R E L E M E N T T Y P E B A R

Element	SA1	SA2	SA3	SA4	Axial	SA-Max	SA-Min	M.S.-T
ID	SB1	SB2	SB3	SB4	Stress	SB-Max	SB-Min	M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.791334E+00	-3.791334E+00	-3.791334E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		-3.791334E+00	-3.791334E+00	

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(output for the 4th – 18th CB DOF deleted)

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL 1 OF 10

```
                                                C B   A C C E L E R A T I O N   O T M
                                             (in global coordinate system at each grid)
           GRID     COORD      T1             T2             T3             R1             R2             R3
                     SYS
             32        0  2.199853E-02 -2.028331E-02 -1.681579E-02 -3.363157E-04  8.006145E-03  5.254334E-04
```

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL 2 OF 10

```
                                                C B   A C C E L E R A T I O N   O T M
                                             (in global coordinate system at each grid)
           GRID     COORD      T1             T2             T3             R1             R2             R3
                     SYS
             32        0  0.000000E+00  0.000000E+00 -1.000000E+00 -2.000000E-02  0.000000E+00  0.000000E+00
```

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL 3 OF 10

```
                                                C B   A C C E L E R A T I O N   O T M
                                             (in global coordinate system at each grid)
           GRID     COORD      T1             T2             T3             R1             R2             R3
                     SYS
             32        0  0.000000E+00  0.000000E+00  5.000000E-01  1.000000E-02  0.000000E+00  0.000000E+00
```

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(output for the 4th – 10th Accel OTM columns deleted)

```
                         M O D A L   P A R T I C I P A T I O N   F A C T O R S
                              (dimensionless, in coordinate sys       0)
           MODE       T1            T2            T3            R1            R2            R3
            NUM
              1  1.227574E-01 -1.758352E+00  8.791759E-01  1.259087E+00  6.535370E-02 -5.341716E-01
              2  6.061630E-01  1.829524E-01 -9.147622E-02 -4.910542E-01 -1.366914E-01 -4.626569E-01

------------------------------------------------------------------------------------------------------------------------------------

                   E F F E C T I V E   M O D A L   M A S S E S   O R   W E I G H T S
                                    (in coordinate system       0)
                         Units are same as units for mass input in the Bulk Data Deck
           MODE       T1            T2            T3            R1            R2            R3
            NUM
              1  6.532677E+01  4.179096E+01  4.694259E+02  3.836785E+05  3.287406E+04  3.611917E+02
              2  7.948285E+00  9.016521E-01  1.363070E+01  1.674257E+00  6.082279E+05  4.781873E+05
                 ------------- ------------- ------------- ------------- ------------- -------------
      Sum all modes:  7.327506E+01  4.269261E+01  4.830566E+02  3.836801E+05  6.411019E+05  4.785485E+05
   Total model mass:  9.325238E+02  9.325238E+02  9.325238E+02  4.105260E+06  4.094237E+06  8.139951E+06
Modes % of total mass*:         7.86          4.58         51.80          9.35         15.66          5.88

*If all modes are calculated the % of total mass should be 100% of the free mass (i.e. not counting mass at constrained DOF's).
 Percentages are only printed for components that have finite model mass.
                                                                 -----

>> LINK  9 END

>> MYSTRAN END    : 10/30/2006 at 18: 3:31.562
```

4.6.2 OUTPUT4 matrices written to CB-EXAMPLE-12-b.OP1 and OP2

(OUTPUT4 matrices requested in Exec Control)

OUTPUT4 matrices requested in Exec Control to be written to file CB-EXAMPLE-12-b.OP1 (on unit 21)

(note: only 1st 5 columns written here for the sake of clarity)

```
     CG_LTM                  NCOLS =       18         NROWS =       6          FORM  =       2          PREC  =       2
               1                       2                       3                       4                       5
1   -6.65821789802521E-05    1.29562159612018E-17   -6.47810798060089E-18   -1.29549999999999E-03    6.47766872193621E-05 .......
2   -2.99785601343913E-05   -1.96135553418977E-04    1.04193052213477E-04    1.39356777670951E-03   -6.70858061739371E-05 .......
3   -4.35697030582909E-05   -2.59100000000000E-03    1.30775055100798E-03    1.29550000000001E-03    6.19839872966866E-04 .......
4   -3.33844454038618E-04   -2.00000000000000E-02    9.80743672854175E-03    1.00000000000000E-02   -5.07064059129018E-03 .......
5    8.13687816036514E-03    1.47885176327023E-16   -7.39425881635114E-17   -7.78457159844592E-17   -5.93156091981744E-03 .......
6    5.63393757592496E-04    8.55130582230051E-17   -4.27565291115026E-17    9.99999999999996E-03    2.81696878796245E-04 .......

     IF_LTM                  NCOLS =       18         NROWS =       8          FORM  =       2          PREC  =       2
               1                       2                       3                       4                       5
1    6.02957424769077E-01    7.32039059471622E-02   -3.66019529735811E-02    3.35492666170908E-02   -7.19015457719424E-02 .......
2    7.32039059471623E-02    4.25469107253153E+00   -2.12163357113457E+00   -2.21879607113459E+00   -1.10665832128050E-01 .......
3   -3.66019529735811E-02   -2.12163357113457E+00    1.07224071582968E+00    1.10939803556729E+00    5.53329160640251E-02 .......
4    3.35492666170908E-02   -2.21879607113459E+00    1.10939803556729E+00    3.26418464157067E+00    1.75366508593570E-02 .......
5   -7.19015457719424E-02   -1.10665832128050E-01    5.53329160640251E-02    1.75366508593570E-02    4.96481812094837E-01 .......
6   -6.65046890695409E-01   -7.32039059471504E-02    3.66019529735752E-02   -1.24163383728600E+00    1.32307347677584E-01 .......
7   -1.34708893096271E-01   -2.21879607113459E+00    1.10939803556729E+00    2.54146535101691E-01    3.05700710026811E-02 .......
8    6.78737140960850E-02   -1.83869738075211E-01    9.19348690376054E-02    8.11498842422746E-02    2.62006997196796E-02 .......

     MR                      NCOLS =        8         NROWS =       8          FORM  =       1          PREC  =       2
               1                       2                       3                       4                       5
1    6.02957424769077E-01    7.32039059471622E-02   -3.66019529735811E-02    3.35492666170908E-02   -7.19015457719424E-02 .......
2    7.32039059471623E-02    4.25469107253153E+00   -2.12163357113457E+00   -2.21879607113459E+00   -1.10665832128050E-01 .......
3   -3.66019529735811E-02   -2.12163357113457E+00    1.07224071582968E+00    1.10939803556729E+00    5.53329160640251E-02 .......
4    3.35492666170908E-02   -2.21879607113459E+00    1.10939803556729E+00    3.26418464157067E+00    1.75366508593570E-02 .......
5   -7.19015457719424E-02   -1.10665832128050E-01    5.53329160640251E-02    1.75366508593570E-02    4.96481812094837E-01 .......
6   -6.65046890695409E-01   -7.32039059471504E-02    3.66019529735752E-02   -1.24163383728600E+00    1.32307347677584E-01 .......
7   -1.34708893096271E-01   -2.21879607113459E+00    1.10939803556729E+00    2.54146535101691E-01    3.05700710026811E-02 .......
8    6.78737140960850E-02   -1.83869738075211E-01    9.19348690376054E-02    8.11498842422746E-02    2.62006997196796E-02 .......
```

OUTPUT4 matrices requested in Exec Control to be written to file CB-EXAMPLE-12-b.OP2 (on unit 22)

(note: only 1^{st} 5 columns written here the sake of clarity)

```
        KRRGN                  NCOLS =         10         NROWS =         10         FORM   =          1         PREC   =          2
                 1                      2                      3                      4                      5
  1     1.19504240447136E+03  -3.63797880709171E-12   1.81898940354586E-12   1.54614099301398E-11   5.97521202235677E+02 .......
  2    -5.45696821063757E-12   0.00000000000000E+00   0.00000000000000E+00   1.81898940354586E-12   0.00000000000000E+00 .......
  3     2.72848410531878E-12   0.00000000000000E+00   0.00000000000000E+00  -9.09494701772928E-13   0.00000000000000E+00 .......
  4     2.08011385893769E-11   0.00000000000000E+00   0.00000000000000E+00  -1.16415321826935E-10   9.43778388773353E-12 .......
  5     5.97521202235677E+02  -1.13686837721616E-13   5.68434188608080E-14  -1.59161572810262E-12   2.98760601117838E+02 .......
  6    -1.19504240447137E+03   0.00000000000000E+00   0.00000000000000E+00  -1.79397829924710E-10  -5.97521202235685E+02 .......
  7    -2.98427949019242E-13   0.00000000000000E+00   0.00000000000000E+00  -4.31782609666698E-10  -2.76401124210679E-12 .......
  8    -5.97521202235677E+02  -1.81898940354586E-12   9.09494701772928E-13   1.36424205265939E-12  -2.98760601117839E+02 .......
  9     0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00 .......
 10     0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00 .......

        RBMCG                  NCOLS =          6         NROWS =          6         FORM   =          2         PREC   =          2
                 1                      2                      3                      4                      5
  1     2.41616914133782E+00  -3.35287353436797E-14  -6.52256026967279E-15  -1.34114941374719E-13  -3.97903932025656E-13 .......
  2    -3.30846461338297E-14   2.41616914133786E+00   2.33146835171283E-14   7.74491581978509E-13   2.89102075612391E-13 .......
  3    -6.52256026967279E-15   2.27734497926235E-14   2.41616914133783E+00  -9.59232693276135E-14  -7.10542735760100E-14 .......
  4    -1.35891298214119E-13   7.81374964731185E-13  -1.24344978758018E-13   4.56169135583651E+03  -3.86535248253495E-12 .......
  5    -3.92130772297605E-13   2.88435941797616E-13  -6.75015598972095E-14  -4.09272615797818E-12   4.53313153018053E+03 .......
  6     1.99662508748588E-12   4.26325641456060E-14  -3.62376795237651E-13  -1.36424205265939E-11   2.85598256559946E+01 .......

        MRRGN                  NCOLS =         10         NROWS =         10         FORM   =          1         PREC   =          2
                 1                      2                      3                      4                      5
  1     6.02957424769077E-01   7.32039059471622E-02  -3.66019529735811E-02   3.35492666170908E-02  -7.19015457719424E-02 .......
  2     7.32039059471623E-02   4.25469107253153E+00  -2.12163357113457E+00  -2.21879607113459E+00  -1.10665832128050E-01 .......
  3    -3.66019529735811E-02  -2.12163357113457E+00   1.07224071582968E+00   1.10939803556729E+00   5.53329160640251E-02 .......
  4     3.35492666170908E-02  -2.21879607113459E+00   1.10939803556729E+00   3.26418464157067E+00   1.75366508593570E-02 .......
  5    -7.19015457719424E-02  -1.10665832128050E-01   5.53329160640251E-02   1.75366508593570E-02   4.96481812094837E-01 .......
  6    -6.65046890695409E-01  -7.32039059471504E-02   3.66019529735752E-02  -1.24163383728600E+00   1.32307347677584E-01 .......
  7    -1.34708893096271E-01  -2.21879607113459E+00   1.10939803556729E+00   2.54146535101691E-01   3.05700710026811E-02 .......
  8     6.78737140960850E-02  -1.83869738075211E-01   9.19348690376054E-02   8.11498842422746E-02   2.62006997196796E-02 .......
  9     1.22757372107055E-01  -1.75835189695839E+00   8.79175948479194E-01   1.25908689725916E+00   6.53537005701318E-02 .......
 10     6.06162990294928E-01   1.82952442095713E-01  -9.14762210478567E-02  -4.91054200271590E-01  -1.36691428775775E-01 .......

        RBRCG                  NCOLS =          6         NROWS =          8         FORM   =          2         PREC   =          2
                 1                      2                      3                      4                      5
  1     1.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   5.37849392786371E+01 .......
  2     0.00000000000000E+00   1.00000000000000E+00   0.00000000000000E+00  -5.37849392786371E+01   0.00000000000000E+00 .......
  3     0.00000000000000E+00   0.00000000000000E+00   1.00000000000000E+00  -5.00000000000000E+01   0.00000000000000E+00 .......
  4     0.00000000000000E+00   1.00000000000000E+00   0.00000000000000E+00  -3.78493927863709E+00   0.00000000000000E+00 .......
  5     0.00000000000000E+00   0.00000000000000E+00   1.00000000000000E+00  -5.00000000000000E+01  -5.00000000000000E+01 .......
  6     1.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   0.00000000000000E+00   3.78493927863709E+00 .......
  7     0.00000000000000E+00   1.00000000000000E+00   0.00000000000000E+00  -3.78493927863709E+00   0.00000000000000E+00 .......
  8     0.00000000000000E+00   0.00000000000000E+00   1.00000000000000E+00  -5.00000000000000E+01   5.00000000000000E+01 .......
```

4.6.3 Displ and Element force/stress OTM's written to CB-EXAMPLE-12-b.OP8 and OP9

(OTM's requested in Case Control)

CB-EXAMPLE-12-b.OP8 binary file of element force/stress OTM's requested in Case Control

(note: only 1st 5 columns written here the sake of clarity)

```
       OTM_ELFE                NCOLS =       18        NROWS =       16        FORM  =        2        PREC  =        2
                  1                       2                       3                       4                       5
  1    2.09187572390564E-01    3.64063384390388E+00   -1.82031692195194E+00   -1.84227921264778E+00   -9.14925412689932E-01 .......
  2    7.89453912890167E-01   -2.87503976462738E+00    1.43751988231369E+00    1.92080844772306E+00   -1.26234542491864E-01 .......
  3    1.51560714339846E+00   -7.75207867487571E+00    3.87603933743785E+00    3.62690741509324E+00    1.45527637571713E+00 .......
  4   -1.43934432738336E+00    4.48652751792572E+00   -2.24326375896286E+00   -2.73874759882899E+00    2.35906653084923E-01 .......
  5   -1.84755627546901E-02    1.61117285562758E-01   -8.05586427813792E-02   -7.73459790410093E-02   -3.35197151472623E-02 .......
  6    3.15199669918811E-02   -1.04108282913086E-01    5.20541414565432E-02    6.58960735567147E-02   -5.12144990278700E-03 .......
  7    6.26679968599842E-01    1.90643492900070E+00   -9.53217464500349E-01   -1.19040949990613E-01   -1.14791218537626E-01 .......
  8    9.67284596743351E-03   -5.33393540270422E-03    2.66696770135211E-03   -5.34876839175438E-02    8.35971431688627E-04 .......
  9   -1.13315069892136E-01    3.78970456518829E+00   -1.89485228259414E+00   -1.26147862482940E+00   -9.55864075040792E-01 .......
 10   -1.00896004659258E-02    2.99287680850590E+00   -1.49643840425295E+00   -4.03697533588189E+00   -1.41398274167766E-02 .......
 11   -1.72540058669802E+00   -6.06107677196644E+00    3.03053838598322E+00    2.53928832803047E+00    1.96715396237338E+00 .......
 12   -6.16614847670031E-02   -4.71348398353008E+00    2.35674199176504E+00    6.82365970711492E+00    3.39064169416761E-02 .......
 13    2.27983320157212E-02    1.39311085669760E-01   -6.96555428348799E-02   -5.37509617215390E-02   -4.13377175157231E-02 .......
 14    7.29336582157196E-04    1.08984399486375E-01   -5.44921997431877E-02   -1.53592573737906E-01   -6.79476503928156E-04 .......
 15   -2.95361107284698E-01    1.80807707871691E+00   -9.04038539358453E-01   -1.95832712226347E+00    3.00896480121837E-03 .......
 16   -4.72042770150405E-03    5.33393540270377E-03   -2.66696770135189E-03   -1.12160973347287E-01   -3.69369770142806E-03 .......

       OTM_STRE                NCOLS =       18        NROWS =       18        FORM  =        2        PREC  =        2
                  1                       2                       3                       4                       5
  1    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
  2    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
  3    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
  4    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
  5   -2.74867035744303E+00    7.58266712433821E+00   -3.79133356216910E+00   -1.07520478850513E+00    4.30045958649968E-01 .......
  6   -2.74867035744303E+00    7.58266712433821E+00   -3.79133356216910E+00   -1.07520478850513E+00    4.30045958649968E-01 .......
  7   -2.74867035744303E+00    7.58266712433821E+00   -3.79133356216910E+00   -1.07520478850513E+00    4.30045958649968E-01 .......
  8   -1.00000000000000E+00   -1.00000000000000E+00   -1.00000000000000E+00   -1.00000000000000E+00   -1.00000000000000E+00 .......
  9    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
 10    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
 11    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
 12    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
 13    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
 14   -2.74867035744303E+00    7.58266712433821E+00   -3.79133356216910E+00   -1.07520478850513E+00    4.30045958649968E-01 .......
 15   -2.74867035744303E+00    7.58266712433821E+00   -3.79133356216910E+00   -1.07520478850513E+00    4.30045958649968E-01 .......
 16   -2.74867035744303E+00    7.58266712433821E+00   -3.79133356216910E+00   -1.07520478850513E+00    4.30045958649968E-01 .......
 17   -1.00000000000000E+00   -1.00000000000000E+00   -1.00000000000000E+00   -1.00000000000000E+00   -1.00000000000000E+00 .......
 18    1.00000000000000E+10    1.00000000000000E+10    1.00000000000000E+10    1.00000000000000E+10    1.00000000000000E+10 .......
```

CB-EXAMPLE-12-b.OT8 text file descriptor of rows in above binary file for element related OTM's

```
This text file describes the rows of the elem related OTM matrices written to unformatted file: CB-EXAMPLE-12-b.OP8
-----------------------------------------------------------------------------------------------

The description for each of the matrices has the headers:
       ROW        : row number in the individual OTM described
       DESCRIPTION: what OTM is this
       TYPE       : element type
       EID        : element ID
Then, for the element nodal force OTM:
       GRID       : grid number of the element that the OTM is for
       COMP       : displacement component number (1,2,3 translations and 4,5,6 rotations)
and for element engineering force and element stress OTMs:
       ITEM       : element force or stress item (axial force, torque, etc)

The number of rows for each OTM depends on the output requests, by the user, in Case Control

The number of cols for each OTM depends on the number of support DOFs (NDOFR) and the number of eigenvecors (NVEC)where:
       NDOFR =        8
       NVEC  =        2

This text file has descriptions for the following element related OTMs from CB-EXAMPLE-12-
b.OP8
       Element engr  force OTM (matrix OTM_ELFE) with 2*NDOFR + NVEC =       18 cols
       Element stress      OTM (matrix OTM_STRE) with 2*NDOFR + NVEC =       18 cols

---------------------------------------------------------------------------------
       Explanation of rows of       16 row by       18 col matrix OTM_ELFE

   ROW           DESCRIPTION             TYPE     EID             ITEM
 ------- ------------------------------ -------- -------    --------------------
       1 Element engineering force      BAR          211    M1a: Mom Plane1 EndA
       2 Element engineering force      BAR          211    M1b: Mom Plane2 EndA
       3 Element engineering force      BAR          211    M2a: Mom Plane1 EndB
       4 Element engineering force      BAR          211    M2b: Mom Plane2 EndB
       5 Element engineering force      BAR          211    V1 : Shear Plane1
       6 Element engineering force      BAR          211    V2 : Shear Plane2
       7 Element engineering force      BAR          211    FX : Axial force
       8 Element engineering force      BAR          211    T  : Torque

       9 Element engineering force      BAR          212    M1a: Mom Plane1 EndA
      10 Element engineering force      BAR          212    M1b: Mom Plane2 EndA
      11 Element engineering force      BAR          212    M2a: Mom Plane1 EndB
      12 Element engineering force      BAR          212    M2b: Mom Plane2 EndB
      13 Element engineering force      BAR          212    V1 : Shear Plane1
      14 Element engineering force      BAR          212    V2 : Shear Plane2
      15 Element engineering force      BAR          212    FX : Axial force
      16 Element engineering force      BAR          212    T  : Torque
```

Explanation of rows of 18 row by 18 col matrix OTM_STRE

ROW	DESCRIPTION	TYPE	EID	ITEM
1	Element stress	BAR	201	SA1: Stress Pt1 EndA
2	Element stress	BAR	201	SA2: Stress Pt2 EndA
3	Element stress	BAR	201	SA3: Stress Pt3 EndA
4	Element stress	BAR	201	SA4: Stress Pt4 EndA
5	Element stress	BAR	201	Axial Stress
6	Element stress	BAR	201	SA-Max
7	Element stress	BAR	201	SA-Min
8	Element stress	BAR	201	MS-Tension
9	Element stress	BAR	201	Torsional Stress
10	Element stress	BAR	201	SB1: Stress Pt1 EndB
11	Element stress	BAR	201	SB2: Stress Pt2 EndB
12	Element stress	BAR	201	SB3: Stress Pt3 EndB
13	Element stress	BAR	201	SB4: Stress Pt4 EndB
14	Element stress	BAR	201	Axial stress
15	Element stress	BAR	201	SB-Max
16	Element stress	BAR	201	SB-Min
17	Element stress	BAR	201	MS-Compression
18	Element stress	BAR	201	MS-Torsion

CB-EXAMPLE-12-b.OP9 binary file of displacement OTM's requested in Case Control

(note: only 1st 5 columns written here the sake of clarity)

OTM_ACCE   NCOLS = 10   NROWS = 6   FORM = 2   PREC = 2

	1	2	3	4	5	
1	2.19985250269592E-02	0.00000000000000E+00	0.00000000000000E+00	-5.00000000000004E-01	1.09992625134795E-02	....
2	-2.02833087802606E-02	0.00000000000000E+00	0.00000000000000E+00	5.00000000000004E-01	-1.01416543901302E-02	....
3	-1.68157865913898E-02	-1.00000000000000E+00	5.00000000000000E-01	5.00000000000005E-01	2.41592106704306E-01	....
4	-3.36315731827796E-04	-2.00000000000000E-02	1.00000000000000E-02	1.00000000000001E-02	-5.16815786591390E-03	....
5	8.00614495648658E-03	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	-5.99692752175671E-03	....
6	5.25433423070610E-04	0.00000000000000E+00	0.00000000000000E+00	9.99999999999992E-03	2.62716711535305E-04	....

OTM_DISP   NCOLS = 18   NROWS = 12   FORM = 2   PREC = 2

	1	2	3	4	5	
1	-1.41293911043985E-05	-7.60029025912968E-05	3.80014512956484E-05	1.29492635368416E-04	3.14571590643487E-06	....
2	1.62214021120513E-05	8.24359519633505E-05	-4.12179759816752E-05	-1.30161832591346E-04	-3.52963231517632E-06	....
3	8.24222187730972E-05	3.12878663301563E-04	-1.56439331650781E-04	-2.40634384994669E-04	-1.68993616070736E-05	....
4	5.88370868696758E-07	1.92529119983460E-06	-9.62645599917302E-07	-2.07019101770705E-06	1.88916538580397E-07	....
5	-1.66743323917105E-06	2.22005501168008E-06	-1.11002750584004E-06	-1.14971054599053E-06	-8.88454144573320E-08	....
6	5.12515138397389E-07	1.29205343624621E-07	-6.46026718123106E-08	-1.07589130445167E-06	-9.61720937623318E-08	....
7	1.05104109813473E-05	-5.99087762260462E-05	2.99543881130231E-05	6.53233961326989E-05	-1.57813540011406E-06	....
8	-9.46594436701425E-06	6.30861677743807E-05	-3.15430838871904E-05	-6.55217977160166E-05	1.38681670255135E-06	....
9	-3.18288681491121E-06	3.22417925611894E-04	-1.61208962805947E-04	-1.96081126486432E-04	-3.61627931263323E-05	....
10	-1.08618067423320E-07	3.64336233382231E-06	-1.82168116691115E-06	-2.63986785628832E-06	-3.24126419085498E-08	....
11	-9.45071958677177E-07	4.90427017653186E-07	-2.45213508826593E-07	-2.21449664764883E-07	1.36502293189118E-07	....
12	2.10600905814006E-07	3.21861205426993E-08	-1.60930602713497E-08	-6.09852683088454E-07	-3.82285587596693E-08	....

CB-EXAMPLE-12-b.OT9 text file descriptor of rows in above binary file for grid related OTM's

```
This text file describes the rows of the grid related OTM matrices written to unformatted file: CB-EXAMPLE-12-
b.OP9
-----------------------------------------------------------------------------------------------

The description for each of the matrices has the headers:
       ROW        : row number in the individual OTM described
       DESCRIPTION: what OTM is this
       GRID       : grid number for this row of the OTM
       COMP       : displacement component number (1,2,3 translations and 4,5,6 rotations)

The number of rows for each OTM depends on the output requests, by the user, in Case Control

The number of cols for each OTM depends on the number of support DOFs (NDOFR) and the number of eigenvecors (NVEC)where:
       NDOFR =        8
       NVEC  =        2

This text file has descriptions for the following grid relatad OTMs from CB-EXAMPLE-
12b.OP9
       Acceleration OTM (matrix OTM_ACCE) with   NDOFR + NVEC =       10 cols
       Displacement OTM (matrix OTM_DISP) with 2*NDOFR + NVEC =       18 cols

---------------------------------------------------------------------------------
Explanation of rows of        6 row by       10 col matrix OTM_ACCE

   ROW            DESCRIPTION             GRID     COMP
 ------- ------------------------------  -------    ----
       1 Acceleration                        32       1
       2 Acceleration                        32       2
       3 Acceleration                        32       3
       4 Acceleration                        32       4
       5 Acceleration                        32       5
       6 Acceleration                        32       6

---------------------------------------------------------------------------------
Explanation of rows of       12 row by       18 col matrix OTM_DISP

   ROW            DESCRIPTION             GRID     COMP
 ------- ------------------------------  -------    ----
       1 Displacement                        22       1
       2 Displacement                        22       2
       3 Displacement                        22       3
       4 Displacement                        22       4
       5 Displacement                        22       5
       6 Displacement                        22       6

       7 Displacement                        32       1
       8 Displacement                        32       2
       9 Displacement                        32       3
      10 Displacement                        32       4
      11 Displacement                        32       5
```

12	Displacement	32	6

5 Derivation of RBE3 Element Constraint Equations

5.1 Introduction

The RBE3 element is used for distributing applied loads and mass from a reference point to other points in the finite element model. The geometry and loads for a RBE3 are shown in Figure 1. Point d in the figure is the RBE3 reference (or dependent) point and is the grid where loads will be applied by the user. The RBE3 element will distribute these loads to other, independent, points i = 1,…,N, in the model, where N is the total number of independent grid points defined on the RBE3 Bulk Data entry. The RBE3 is not intended to add stiffness to the model as does a RBE2 element. As such, the RBE3 reference point should not be a grid that is attached to other elements in the model – it should be a stand alone grid only connected to other grids through the REB3 element definition. The following describes the nomenclature used in this appendix in deriving the "constraint" equations used in MYSTRAN for the RBE3 element.

Superscripts denote the location of a quantity:
"d" refers to the reference (or dependent) grid on the RBE3
"i" refers to the independent grids, the locations where the loads on point d will be distributed

$$X, Y, Z = \text{coordinate system axes}$$

$$u_x, u_y, u_z = \text{ displacements in the x, y, z directions}$$

$$\theta_x, \theta_y, \theta_z = \text{ rotations about the x, y, z axes}$$

$$F_x, F_y, F_z = \text{ forces in the x, y, z directions}$$

$$M_x, M_y, M_z = \text{ moments about the x, y, z axes}$$

$$d_x^i, d_y^i, d_z^i = \text{ position of point i relative to the RBE3 reference point, d}$$

For the sake of simplicity and clarity, the following derivation of the RBE3 equations is done for conditions where the global coordinate systems of all grid points involved in the RBE3 are the same and are rectangular. The code in the MYSTRAN program is written for general conditions where the global system of all points may be different and non-rectangular.

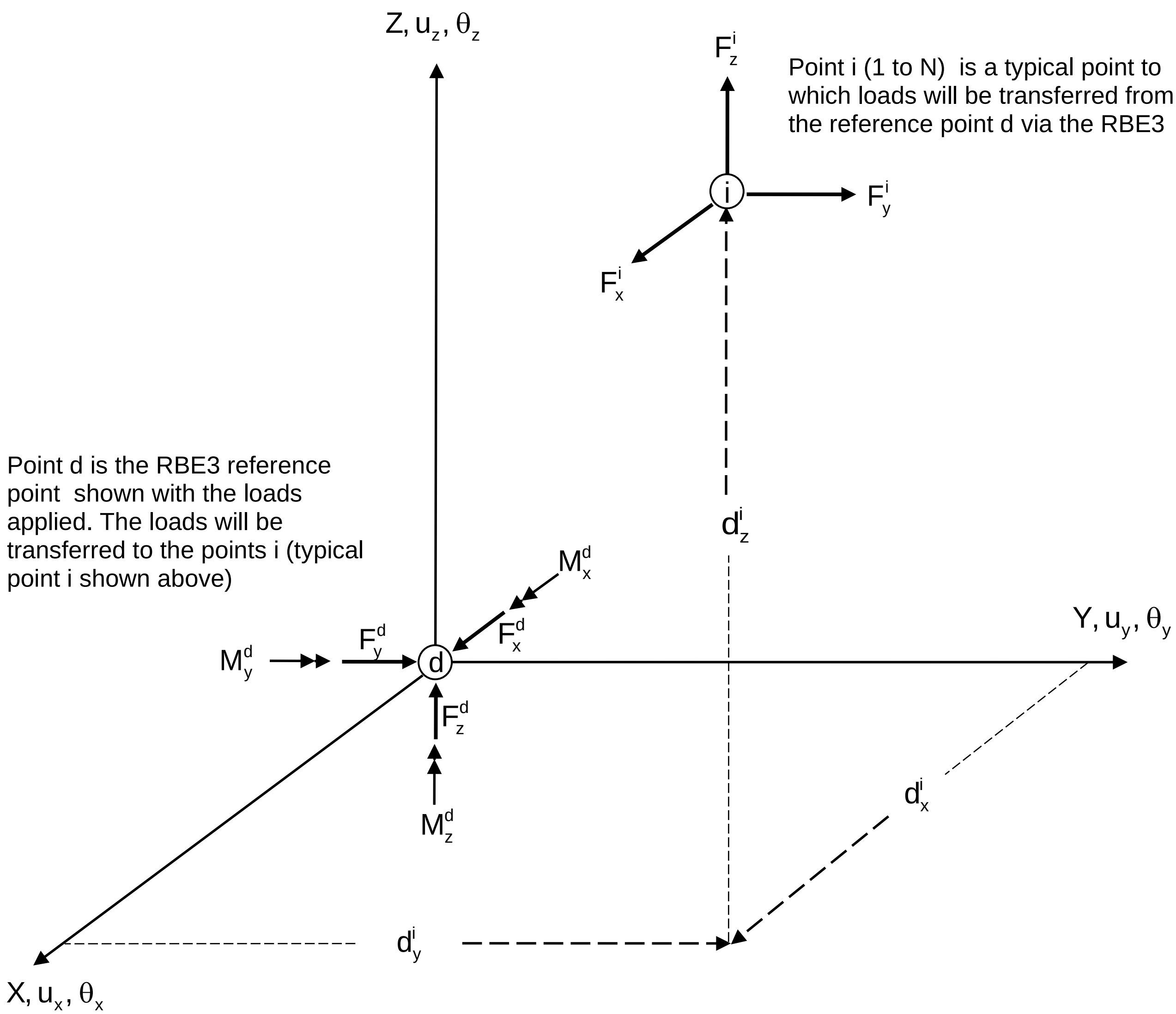


Fig 1: RBE3 geometry and loads

5.2 *Equations for translational force components*

In this section 3 equations will be developed that relate the forces applied at the RBE3 reference point to those where the loads will be distributed (points i = 1,…,N).

The sum of the forces on the points i = 1,…,N must equal the forces on the reference point d. Thus:

$$\sum_{i=1}^{N} F_x^i = F_x^d \quad , \quad \sum_{i=1}^{N} F_y^i = F_y^d \quad , \quad \sum_{i=1}^{N} F_z^i = F_z^d \qquad 11\text{-}1$$

The moments at reference point due to the forces at the points i are:

$$\sum_{i=1}^{N} (F_z^i d_y^i - F_y^i d_z^i) = M_x^d \quad , \quad \sum_{i=1}^{N} (F_x^i d_z^i - F_z^i d_x^i) = M_y^d \quad , \quad \sum_{i=1}^{N} (F_y^i d_x^i - F_x^i d_y^i) = M_z^d \qquad 11\text{-}2$$

Write the F_x^i, etc, as:

$$F_x^i = \frac{\omega_i}{W_T} F_x^d \quad , \quad F_y^i = \frac{\omega_i}{W_T} F_y^d \quad , \quad F_z^i = \frac{\omega_i}{W_T} F_z^d \qquad 11\text{-}3$$

where ω_i is the weighting factor (the WTi on the RBE3 Bulk Data entry) for the ith force and:

$$W_T = \sum_{i=1}^{N} \omega_i \qquad 11\text{-}4$$

Equations 3 and 4 are sufficient for equations 1. Substitute equations 3 and 4 into 2 to get the following 3 equations:

$$\frac{F_z^d}{W_T} \sum_{i=1}^{N} \omega_i d_y^i - \frac{F_y^d}{W_T} \sum_{i=1}^{N} \omega_i d_z^i = M_x^d \qquad 11\text{-}5$$

$$\frac{F_x^d}{W_T} \sum_{i=1}^{N} \omega_i d_z^i - \frac{F_z^d}{W_T} \sum_{i=1}^{N} \omega_i d_x^i = M_y^d \qquad 11\text{-}6$$

$$\frac{F_y^d}{W_T} \sum_{i=1}^{N} \omega_i d_x^i - \frac{F_x^d}{W_T} \sum_{i=1}^{N} \omega_i d_y^i = M_z^d \qquad 11\text{-}7$$

Define:

$$\bar{d}_x = \frac{1}{W_T} \sum_{i=1}^{N} \omega_i d_x^i \quad , \quad \bar{d}_y = \frac{1}{W_T} \sum_{i=1}^{N} \omega_i d_y^i \quad , \quad \bar{d}_z = \frac{1}{W_T} \sum_{i=1}^{N} \omega_i d_z^i \qquad 11\text{-}8$$

Using equation 8, equations 5-7 become:

$$F_z^d \bar{d}_y - F_y^d \bar{d}_z = M_x^d \qquad 11\text{-}9$$

$$F_x^d \bar{d}_z - F_z^d \bar{d}_z = M_y^d \qquad 11\text{-}10$$

$$F_y^d \bar{d}_x - F_x^d \bar{d}_y = M_z^d \qquad 11\text{-}11$$

The work done by the forces and moments at the reference point, d, is Ω_d:

$$\Omega_d = F_x^d u_x^d + F_y^d u_y^d + F_z^d u_z^d + M_x^d \theta_x^d + M_y^d \theta_y^d + M_z^d \theta_z^d \qquad 11\text{-}12$$

where u, θ are the displacements and rotations of the reference point in the x, y, z directions. Similarly, the work done by the forces on the points I = 1,…,N is:

$$\Omega_N = \sum_{i=1}^{N} (F_x^i u_x^i + F_y^i u_y^i + F_z^i u_z^i) \qquad 11\text{-}13$$

The u_x^i, ec, are the displacements in the x, y and z directions at point I. Substitute equation 3 into 12 and 9, 10 and 11 into 12 and equate the work done by the two systems of forces:

$$F_x^d u_x^d + F_y^d u_y^d + F_z^d u_z^d + (F_z^d \bar{d}_y - F_y^d \bar{d}_z)\theta_x^d + (F_x^d \bar{d}_z - F_z^d \bar{d}_z)\theta_y^d + (F_y^d \bar{d}_x - F_x^d \bar{d}_y)\theta_z^d =$$
$$\sum_{i=1}^{N} \left(\frac{\omega_i}{W_T} F_x^d u_x^i + \frac{\omega_i}{W_T} F_y^d u_y^i + \frac{\omega_i}{W_T} F_z^d u_z^i\right)$$

Rearrange:

$$\begin{aligned}
&(u_x^d + \bar{d}_z \theta_y^d - \bar{d}_y \theta_z^d - \sum_{i=1}^{N} \frac{\omega_i}{W_T} u_x^i) F_x^d + \\
&(u_y^d + \bar{d}_z \theta_x^d - \bar{d}_x \theta_z^d - \sum_{i=1}^{N} \frac{\omega_i}{W_T} u_y^i) F_y^d + \\
&(u_z^d + \bar{d}_y \theta_x^d - \bar{d}_x \theta_y^d - \sum_{i=1}^{N} \frac{\omega_i}{W_T} u_z^i) F_z^d = 0
\end{aligned} \qquad 11\text{-}14$$

Since the F_x^d, F_y^d and F_z^d are independent and, in general, not zero, equation 14 requires that:

$$\boxed{\begin{aligned}
&(u_x^d + \bar{d}_z \theta_y^d - \bar{d}_y \theta_z^d - \sum_{i=1}^{N} \frac{\omega_i}{W_T} u_x^i) = 0 \\
&(u_y^d - \bar{d}_z \theta_x^d + \bar{d}_x \theta_z^d - \sum_{i=1}^{N} \frac{\omega_i}{W_T} u_y^i) = 0 \\
&(u_z^d + \bar{d}_y \theta_x^d - \bar{d}_x \theta_y^d - \sum_{i=1}^{N} \frac{\omega_i}{W_T} u_z^i) = 0
\end{aligned}} \qquad 11\text{-}15$$

Equation 15 represents 3 constraint equations for the RBE3. However, there are only 3 equations and 6 unknowns. This will be resolved in the next section where we develop 3 more equations based on the moments at the reference point.

5.3 Equations for rotational moment components

In addition to the 3 equations developed in the last section there are also 3 equations that relate the moments applied at the RBE3 reference point to those where the loads will be distributed (points i = 1,…,N).

Figure 2 shows how the forces in the y-z plane relate to the RBE3 reference point moment about the x axis:

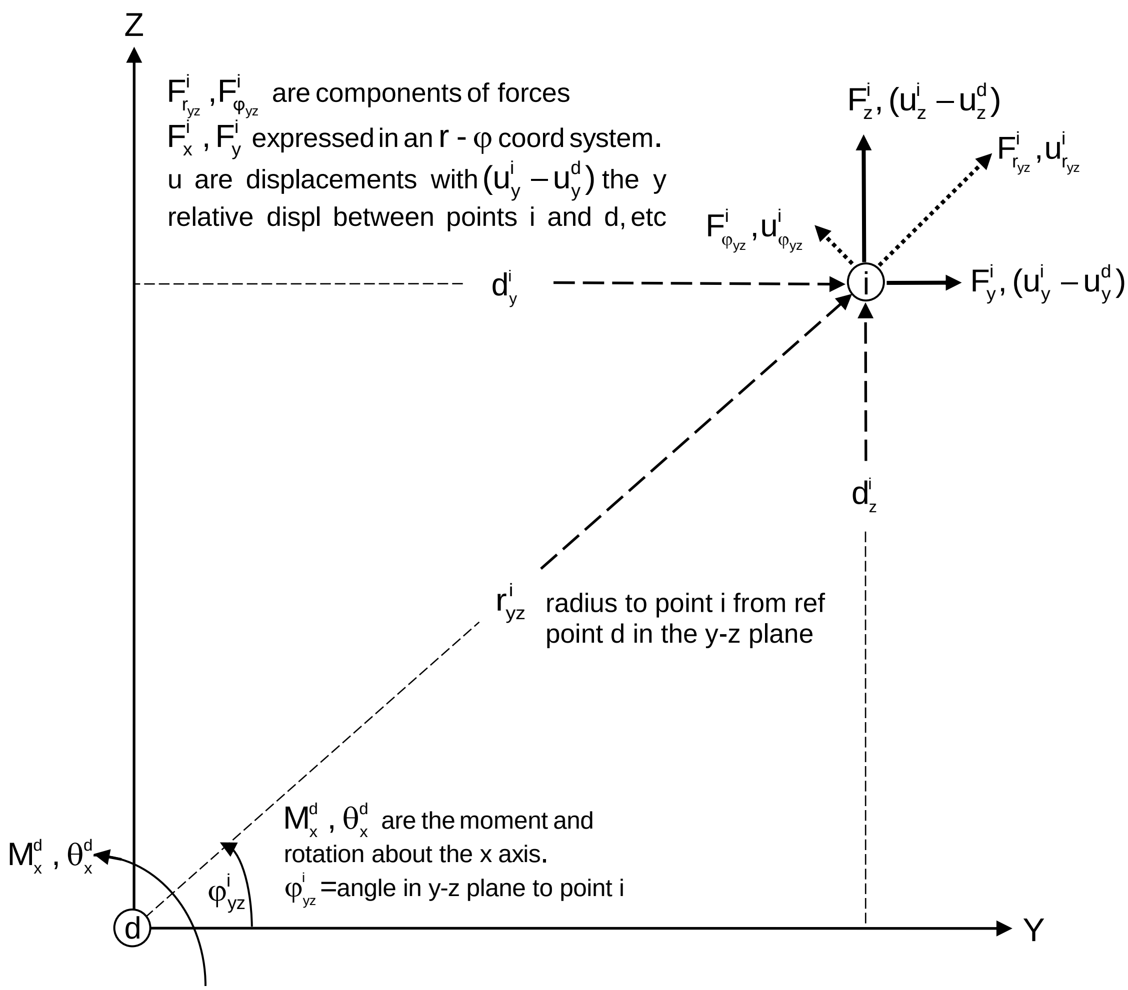


Figure 2: Relationship of moments and forces in the y-z plane

Using the r-φ components of the forces, the moments about the x axis of the forces at the i = 1,..,N points is:

$$\sum_{i=1}^{N} F^i_{\varphi_{yz}} r^i_{yz} = M^d_x \qquad 11\text{-}16$$

As before, express the forces at the i points using the weighting factors, ω_i:

$$F^i_{\varphi_{yz}} = \frac{\omega^i r^i_{yz}}{\sum_{i=1}^{N} \omega^i r^{i^2}_{yz}} M^d_x \qquad 11\text{-}17$$

Note that if equation 17 were substituted into 16 it would be seen that 17 is a valid representation of the tangential force components.

The work done by M^d_x must equal that due to all of the $F^i_{\varphi_{yz}}$, or:

$$\sum F^i_{\varphi_{yz}} u^i_{\varphi_{yz}} = M^d_x \theta^d_x \qquad 11\text{-}18$$

where $u^i_{\varphi_{yz}}$ is the tangential component of displacement at independent grid i in the y-z plane. Substitute equation 17 into 18:

$$\sum_{i=1}^{N} \frac{\omega^i r^i_{yz}}{\sum_{i=1}^{N} \omega^i r^{i^2}_{yz}} M^d_x u^i_{\varphi_{yz}} = M^d_x \theta^d_x$$

or:

$$\theta^d_x = \frac{\sum_{i=1}^{n} \omega^i r^i_{yz} u^i_{\varphi_{yz}}}{\sum_{i=1}^{N} \omega^i r^{i^2}_{yz}} \qquad 11\text{-}19$$

From Figure 2 it can be seen that:

$$\begin{aligned} u^i_{\varphi_{yz}} &= (u^i_z - u^d_z)\cos\varphi^i_{yz} - (u^i_y - u^d_y)\sin\varphi^i_{yz} \\ &= (u^i_z - u^d_z)\frac{d^i_y}{r^i_{yz}} - (u^i_y - u^d_y)\frac{d^i_z}{r^i_{yz}} \end{aligned}$$

Therefore:

$$r^i_{yz} u^i_{\varphi_{yz}} = (u^i_z - u^d_z)d^i_y - (u^i_y - u^d_y)d^i_z \qquad 11\text{-}20$$

Define:

$$\bar{e}^i_{yz} = \frac{1}{W_T}\sum_{i=1}^{N} \omega^i r^{i^2}_{\phi_{yz}} \equiv \frac{1}{W_T}\sum_{i=1}^{N} \omega^i (d^{i^2}_y + d^{i^2}_z) \qquad 11\text{-}21$$

Substitute equations 20 and 21 into 19

$$
\begin{aligned}
\theta_x^d &= \frac{1}{W_T \bar{e}_{yz}^i}\left[\sum_{i=1}^{N}\omega^i(u_z^i - u_z^d)d_y^i - \sum_{i=1}^{N}\omega^i(u_y^i - u_y^d)d_z^i\right] \\
&= \frac{1}{W_T \bar{e}_{yz}^i}\left[-(\sum_{i=1}^{N}\omega^i d_y^i)u_z^d + (\sum_{i=1}^{N}\omega^i d_z^i)u_y^d + \sum_{i=1}^{N}\omega^i d_y^i u_z^i - \sum_{i=1}^{N}\omega^i d_z^i u_y^i\right] \qquad \text{11-22} \\
&= \frac{1}{\bar{e}_{yz}^i}\left[-\bar{d}_y u_z^d + \bar{d}_z u_y^d + \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_y^i u_z^i - \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_z^i u_y^i\right]
\end{aligned}
$$

In reference to Figures 3 and 4, define:

$$
\begin{aligned}
&\bar{e}_{zx}^i = \frac{1}{W_T}\sum_{i=1}^{N}\omega^i r_{\phi_{zx}}^{i^2} \equiv \frac{1}{W_T}\sum_{i=1}^{N}\omega^i (d_z^{i^2} + d_x^{i^2}) \\
&\text{and} \qquad \text{11-23} \\
&\bar{e}_{xy}^i = \frac{1}{W_T}\sum_{i=1}^{N}\omega^i r_{\phi_{xy}}^{i^2} \equiv \frac{1}{W_T}\sum_{i=1}^{N}\omega^i (d_x^{i^2} + d_y^{i^2})
\end{aligned}
$$

Then, θ_y^d and θ_z^d , by similar reasoning for θ_x^a in equation 22 are:

$$
\begin{aligned}
\theta_y^d &= \frac{1}{W_T \bar{e}_{zx}^i}\left[\sum_{i=1}^{N}\omega^i(u_x^i - u_x^d)d_z^i - \sum_{i=1}^{N}\omega^i(u_z^i - u_z^d)d_x^i\right] \qquad \text{11-24} \\
&= \frac{1}{\bar{e}_{zx}^i}\left[-\bar{d}_z u_x^d + \bar{d}_x u_z^d + \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_z^i u_x^i - \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_x^i u_z^i\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\theta_z^d &= \frac{1}{W_T \bar{e}_{xy}^i}\left[\sum_{i=1}^{N}\omega^i(u_y^i - u_y^d)d_x^i - \sum_{i=1}^{N}\omega^i(u_x^i - u_x^a)d_y^i\right] \qquad \text{11-25} \\
&= \frac{1}{\bar{e}_{xy}^i}\left[-\bar{d}_x u_y^d + \bar{d}_y u_x^d + \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_x^i u_y^i - \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_y^i u_x^i\right]
\end{aligned}
$$

Thus, for the rotations:

$$
\boxed{
\begin{aligned}
&\bar{e}_{yz}\theta_x^d - \bar{d}_z u_y^d + \bar{d}_y u_z^d + \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_z^i u_y^i - \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_y^i u_z^i = 0 \\
&\bar{e}_{zx}\theta_y^d + \bar{d}_z u_x^d - \bar{d}_x u_z^d - \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_z^i u_x^i + \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_x^i u_z^i = 0 \\
&\bar{e}_{xy}\theta_z^d - \bar{d}_y u_x^d + \bar{d}_x u_y^d + \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_y^i u_x^i - \frac{1}{W_T}\sum_{i=1}^{N}\omega^i d_x^i u_y^i = 0
\end{aligned}
} \qquad \text{11-26}
$$

Equations 15 and 26 constitute 6 equations in the 6 unknown displacements and rotations at point a. They are summarized in matrix notation below at the end of this appendix.

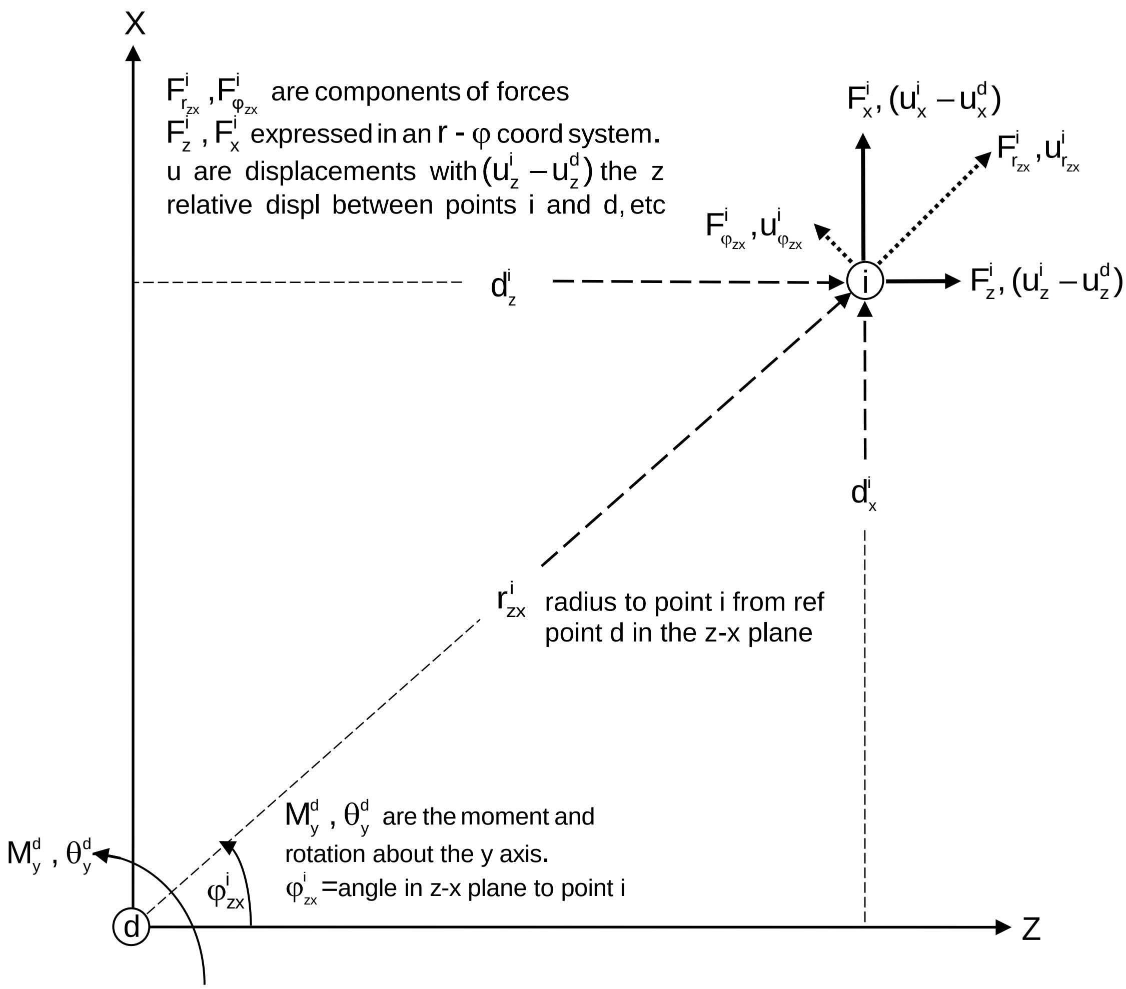


Figure 3: Relationship of moments and forces in the z-x plane

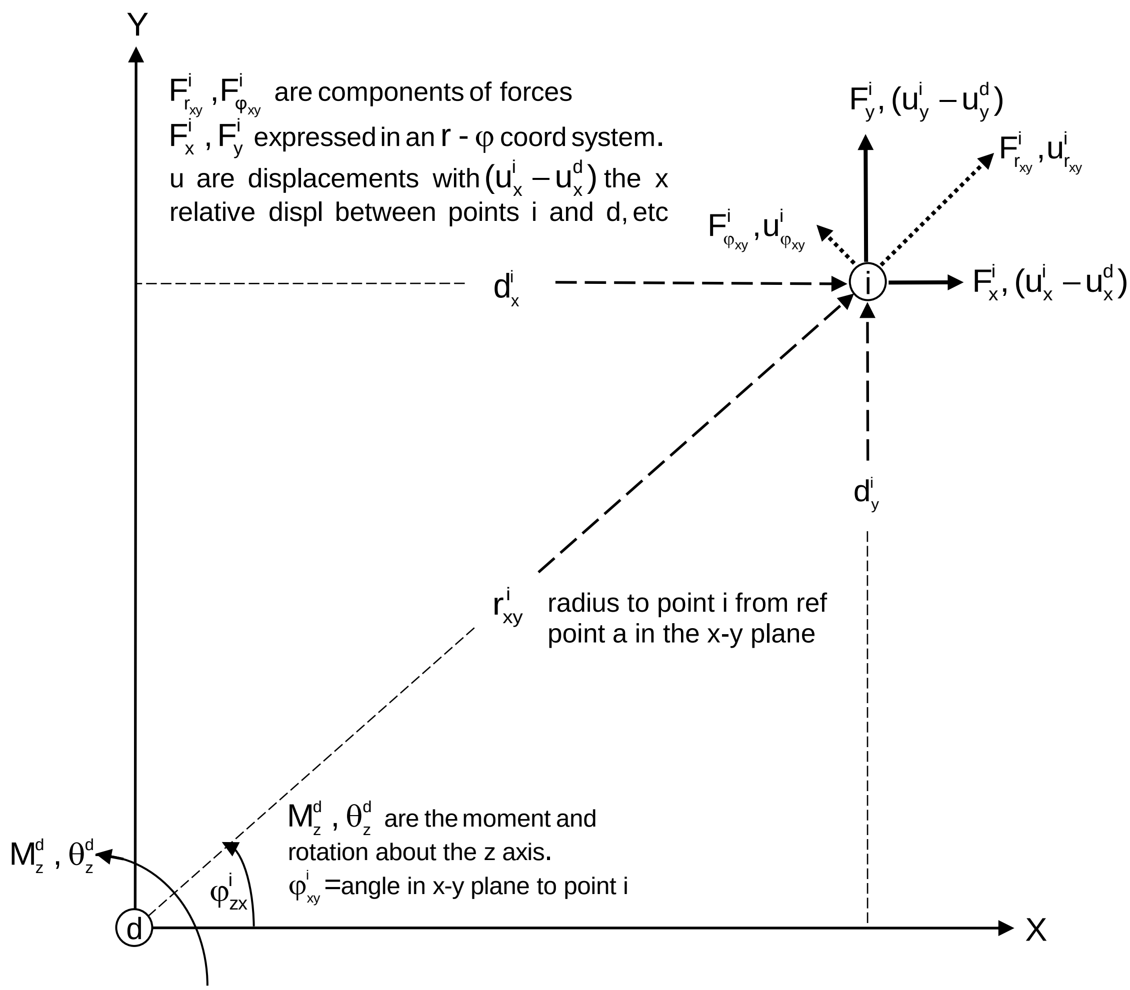


Figure 4: Relationship of moments and forces in the x-y plane

5.4 Summary of equations for the RBE3

In general, the equations for one RBE3 can be represented in matrix notation as:

$$R_{dd}U_d + R_{dN}U_N = 0 \qquad \text{11-27}$$

R_{dd} is the square, d x d, matrix of coefficients for the dependent (or reference) grid denoted as REFGRID in field 4 of the RBE3 Bulk Data entry. It can have up to d = 6 dependent components (REFC in field 5). For all 6 components, R_{dd} and U_d are:

$$R_{dd} = \begin{bmatrix} 1 & 0 & 0 & | & 0 & \bar{d}_z & -\bar{d}_y \\ 0 & 1 & 0 & | & -\bar{d}_z & 0 & \bar{d}_x \\ 0 & 0 & 1 & | & \bar{d}_y & -\bar{d}_x & 0 \\ - & - & - & | & - & - & - \\ 0 & -\bar{d}_z & \bar{d}_y & | & \bar{e}_{yz} & 0 & 0 \\ \bar{d}_z & 0 & -\bar{d}_x & | & 0 & \bar{e}_{zx} & 0 \\ -\bar{d}_y & \bar{d}_x & 0 & | & 0 & 0 & \bar{e}_{xy} \end{bmatrix} \quad , \quad U_d = \begin{Bmatrix} u_x^a \\ u_y^a \\ u_z^a \\ - \\ \theta_x^a \\ \theta_y^a \\ \theta_z^a \end{Bmatrix} \qquad \text{11-28}$$

R_{dN} is a rectangular, d x N, matrix of coefficients for the N independent grids on the RBE3

$$R_{dN} = \frac{1}{W_T}\left[R_{d1} \quad R_{d2} \quad . \quad . \quad . \quad R_{dN}\right] \quad , \quad U_N = \begin{Bmatrix} U_1 \\ U_2 \\ . \\ . \\ . \\ U_N \end{Bmatrix} \qquad \text{11-29}$$

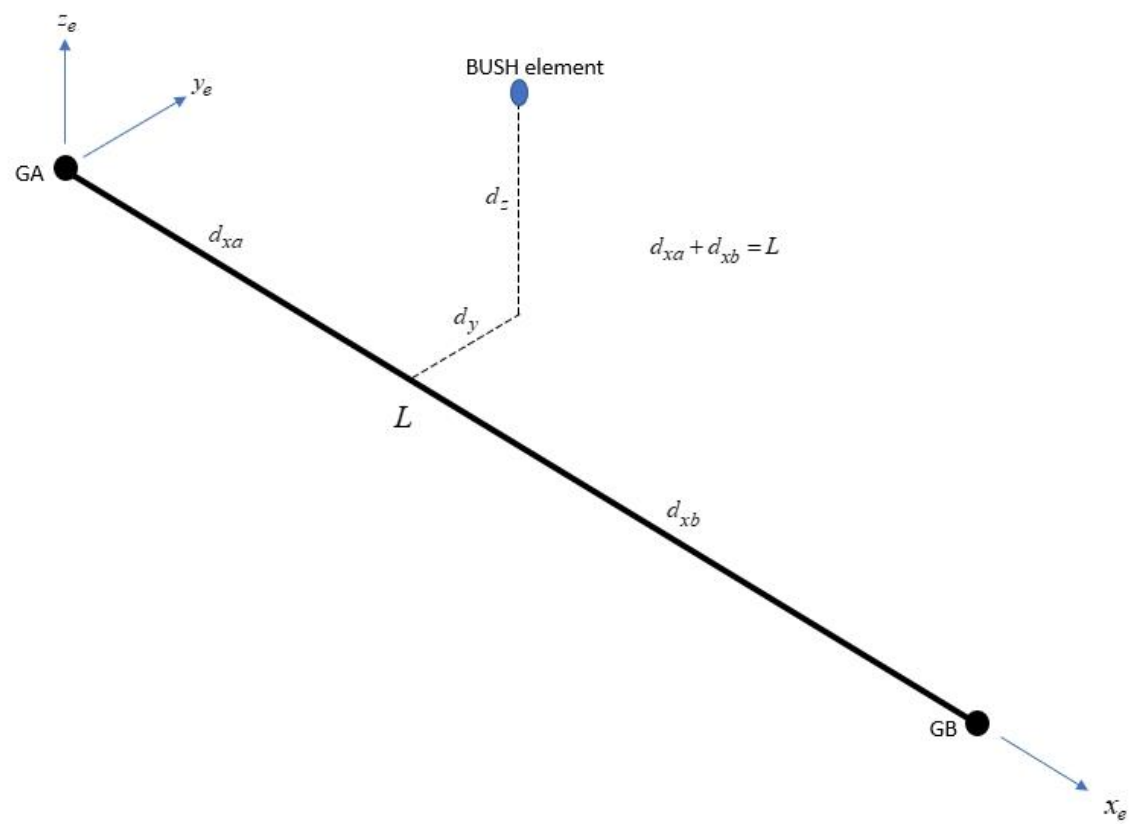
A typical sub-matrix in R_{ai} is of size d by 3 with R_{ai} and U_i. For d = 6:

$$R_{di} = \frac{1}{W_T}\begin{bmatrix} \omega^i & 0 & 0 \\ 0 & \omega^i & 0 \\ 0 & 0 & \omega^i \\ - & - & - \\ 0 & \omega^i d_z^i & -\omega^i d_y^i \\ -\omega^i d_z^i & 0 & \omega^i d_x^i \\ \omega^i d_y^i & -\omega^i d_x^i & 0 \end{bmatrix} \quad , \quad U_i = \begin{Bmatrix} u_x^i \\ u_y^i \\ u_z^i \end{Bmatrix} \qquad \text{11-30}$$

A RBE3 is processed by solving equation 27 for the dependent degrees of freedom, U_d, in terms of the independent degrees of freedom, U_N.

6 Equations for the BUSH Element

BUSH Element Geometry
(in local element coordinates)



The stiffness equations for the BUSH element can be expressed as:

$$Ku = F$$

where K is a 12x12 matrix and u and F are the 12 degree of freedom (6 at each of the 2 grids) displacements and node forces. For the sake of clarity, rather than showing the whole 12x12 stiffness matrix, express the above equation in grid partitioned form as:

$$\begin{bmatrix} K_{aa} & K_{ab} \\ K_{ab}^T & K_{bb} \end{bmatrix} \begin{Bmatrix} u_a \\ u_b \end{Bmatrix} = \begin{Bmatrix} F_a \\ F_b \end{Bmatrix}$$

If we denote κ_i (i=1,…6) as the 6 stiffness values from the PBUSH Bulk Data entry then the above partitions are:

$$K_{aa} = \begin{bmatrix} \kappa_1 & 0 & 0 & 0 & d_z\kappa_1 & -d_y\kappa_1 \\ 0 & \kappa_2 & 0 & -d_z\kappa_2 & 0 & d_{xa}\kappa_2 \\ 0 & 0 & \kappa_3 & d_y\kappa_3 & -d_{xa}\kappa_3 & 0 \\ 0 & -d_z\kappa_2 & d_y\kappa_3 & \kappa_4 + d_y^2\kappa_3 + d_z^2\kappa_2 & -d_{xa}d_y\kappa_3 & -d_{xa}d_z\kappa_2 \\ d_z\kappa_1 & 0 & -d_{xa}\kappa_3 & -d_{xa}d_y\kappa_3 & \kappa_5 + d_{xa}^2\kappa_3 + d_z^2\kappa_1 & -d_yd_z\kappa_1 \\ -d_y\kappa_1 & d_{xa}\kappa_2 & 0 & -d_{xa}d_z\kappa_2 & -d_yd_z\kappa_1 & \kappa_6 + d_{xa}^2\kappa_2 + d_y^2\kappa_1 \end{bmatrix}$$

$$K_{ab} = \begin{bmatrix} -\kappa_1 & 0 & 0 & 0 & -d_z\kappa_1 & d_y\kappa_1 \\ 0 & -\kappa_2 & 0 & d_z\kappa_2 & 0 & d_{xb}\kappa_2 \\ 0 & 0 & -\kappa_3 & -d_y\kappa_3 & -d_{xb}\kappa_3 & 0 \\ 0 & d_z\kappa_2 & -d_y\kappa_3 & -(\kappa_4 + d_y^2\kappa_3 + d_z^2\kappa_2) & -d_{xb}d_y\kappa_3 & -d_{xb}d_z\kappa_2 \\ -d_z\kappa_1 & 0 & d_{xa}\kappa_3 & d_{xa}d_y\kappa_3 & -\kappa_5 + d_{xa}d_{xb}\kappa_3 - d_z^2\kappa_1 & d_yd_z\kappa_1 \\ d_y\kappa_1 & -d_{xa}\kappa_2 & 0 & d_{xa}d_z\kappa_2 & d_yd_z\kappa_1 & -\kappa_6 + d_{xa}d_{xb}\kappa_2 - d_y^2\kappa_1 \end{bmatrix}$$

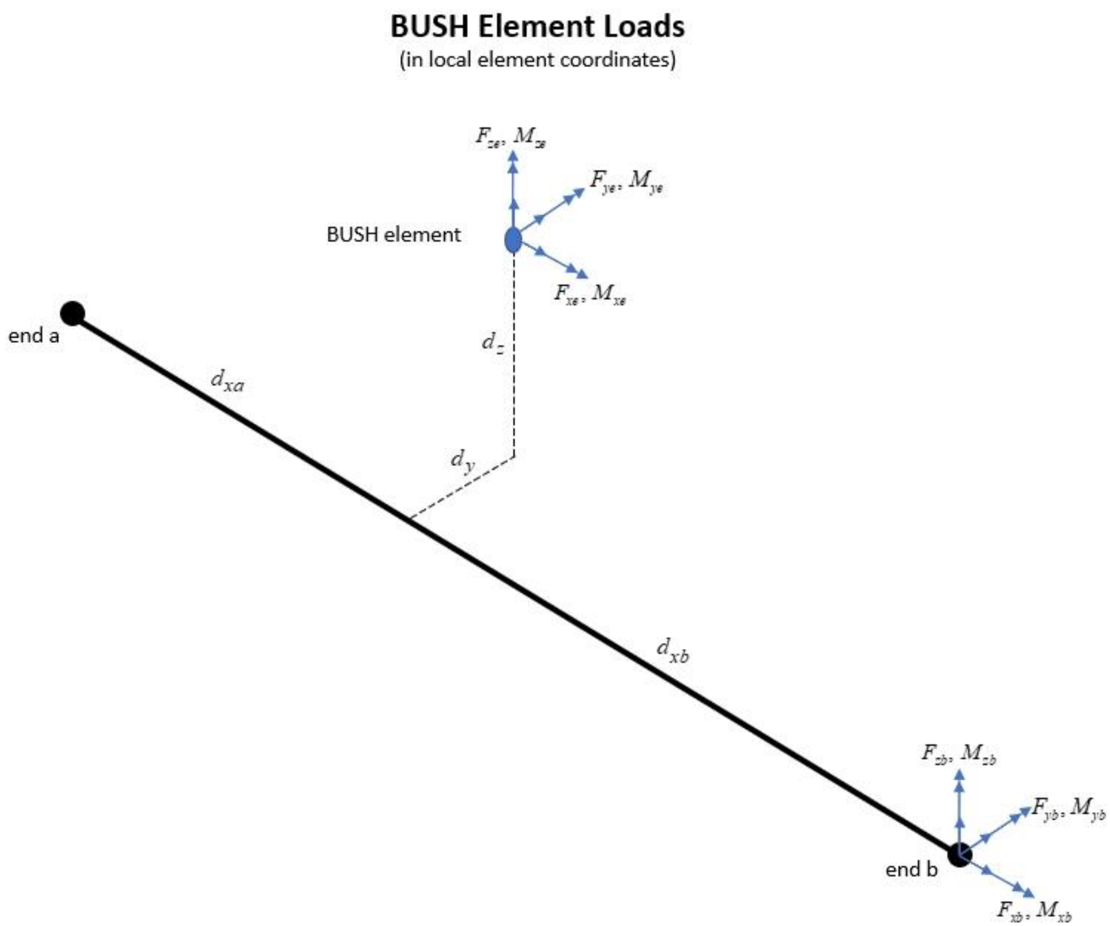
$$K_{bb} = \begin{bmatrix} \kappa_1 & 0 & 0 & 0 & d_z\kappa_1 & -d_y\kappa_1 \\ 0 & \kappa_2 & 0 & -d_z\kappa_2 & 0 & -d_{xb}\kappa_2 \\ 0 & 0 & \kappa_3 & d_y\kappa_3 & d_{xb}\kappa_3 & 0 \\ 0 & -d_z\kappa_2 & d_y\kappa_3 & \kappa_4 + d_y^2\kappa_3 + d_z^2\kappa_2 & d_{xa}d_y\kappa_3 & d_{xb}d_z\kappa_2 \\ d_z\kappa_1 & 0 & -d_{xb}\kappa_3 & d_{xb}d_y\kappa_3 & \kappa_5 + d_{xb}^2\kappa_3 + d_z^2\kappa_1 & -d_yd_z\kappa_1 \\ -d_y\kappa_1 & d_{xb}\kappa_2 & 0 & d_{xb}d_z\kappa_2 & -d_yd_z\kappa_1 & \kappa_6 + d_{xb}^2\kappa_2 + d_y^2\kappa_1 \end{bmatrix}$$

An image of the full 12x12 matrix with the above partitions is shown below:

$$K = \left[\begin{array}{cccccc|cccccc}
K_1 & 0 & 0 & 0 & d_zK_1 & -d_yK_1 & -K_1 & 0 & 0 & 0 & -d_zK_1 & d_yK_1 \\
0 & K_2 & 0 & -d_zK_2 & 0 & d_{xa}K_2 & 0 & -K_2 & 0 & d_zK_2 & 0 & d_{xb}K_2 \\
0 & 0 & K_3 & d_yK_3 & -d_{xa}K_3 & 0 & 0 & 0 & -K_3 & -d_yK_3 & -d_{xb}K_3 & 0 \\
0 & -d_zK_2 & d_yK_3 & K_4 + d_y^2K_3 + d_z^2K_2 & -d_{xa}d_yK_3 & -d_{xa}d_zK_2 & 0 & d_zK_2 & -d_yK_3 & -(K_4 + d_y^2K_3 + d_z^2K_2) & -d_{xb}d_yK_3 & -d_{xb}d_zK_2 \\
d_zK_1 & 0 & -d_{xa}K_3 & -d_{xa}d_yK_3 & K_5 + d_{xa}^2K_3 + d_z^2K_1 & -d_yd_zK_1 & -d_zK_1 & 0 & d_{xa}K_3 & d_{xa}d_yK_3 & -K_5 + d_{xa}d_{xb}K_3 - d_z^2K_1 & d_yd_zK_1 \\
-d_yK_1 & d_{xa}K_2 & 0 & -d_{xa}d_zK_2 & -d_yd_zK_1 & K_6 + d_{xa}^2K_2 + d_y^2K_1 & d_yK_1 & -d_{xa}K_2 & 0 & d_{xa}d_zK_2 & d_yd_zK_1 & -K_6 + d_{xa}d_{xb}K_2 - d_y^2K_1 \\
\hline
-K_1 & 0 & 0 & 0 & -d_zK_1 & d_yK_1 & K_1 & 0 & 0 & 0 & d_zK_1 & -d_yK_1 \\
0 & -K_2 & 0 & d_zK_2 & 0 & -d_{xa}K_2 & 0 & K_2 & 0 & -d_zK_2 & 0 & -d_{xb}K_2 \\
0 & 0 & -K_3 & -d_yK_3 & d_{xa}K_3 & 0 & 0 & 0 & K_3 & d_yK_3 & d_{xb}K_3 & 0 \\
0 & d_zK_2 & -d_yK_3 & -(K_4 + d_y^2K_3 + d_z^2K_2) & d_{xa}d_yK_3 & d_{xa}d_zK_2 & 0 & -d_zK_2 & d_yK_3 & K_4 + d_y^2K_3 + d_z^2K_2 & d_{xb}d_yK_3 & d_{xb}d_zK_2 \\
-d_zK_1 & 0 & -d_{xb}K_3 & -d_{xb}d_yK_3 & -K_5 + d_{xa}d_{xb}K_3 - d_z^2K_1 & d_yd_zK_1 & d_zK_1 & 0 & d_{xb}K_3 & d_{xb}d_yK_3 & K_5 + d_{xb}^2K_3 + d_z^2K_1 & -d_yd_zK_1 \\
d_yK_1 & d_{xb}K_2 & 0 & -d_{xb}d_zK_2 & d_yd_zK_1 & -K_6 + d_{xa}d_{xb}K_2 - d_y^2K_1 & -d_yK_1 & -d_{xb}K_2 & 0 & d_{xb}d_zK_2 & -d_yd_zK_1 & K_6 + d_{xb}^2K_2 + d_y^2K_1
\end{array}\right]$$

Note that the partitions K_{aa} and K_{bb} are symmetric

The element engineering forces can be derived using the figure below:



The engineering forces in the BUSH element are:

$$
\begin{aligned}
F_{xe} &= F_{xb} \\
F_{ye} &= F_{yb} \\
F_{ze} &= F_{zb} \\
M_{xe} &= F_{yb} d_z - F_{zb} d_y + M_{xb} \\
M_{ye} &= -F_{xb} d_z - F_{zb} d_{xb} + M_{yb} \\
M_{ze} &= F_{xb} d_y + F_{yb} d_{xb} + M_{zb}
\end{aligned}
$$

This can be put into a form which includes all nodal forces as:

$$
\begin{Bmatrix} F_{xe} \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze} \end{Bmatrix} = \left[\begin{array}{cccccc|cccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & d_z & -d_y & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -d_z & 0 & -d_{xb} & 0 & . & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & d_y & d_{xb} & 0 & 0 & 0 & . \end{array} \right] \left\{ \begin{array}{c} F_{xa} \\ F_{ya} \\ F_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \\ \hline F_{xb} \\ F_{yb} \\ F_{zb} \\ M_{xb} \\ M_{yb} \\ M_{zb} \end{array} \right\}
$$

The 6x12 transformation matrix in the above equation is used in the MYSTRAN code to transform the element nodal forces to element engineering forces

The engineering forces in the BUSH element are:

$$
\begin{aligned}
F_{xe} &= F_{xa} \\
F_{ye} &= F_{ya} \\
F_{ze} &= F_{za} \\
M_{xe} &= F_{ya}d_z - F_{za}d_y + M_{xa} \\
M_{ye} &= -F_{xa}d_z + F_{za}d_{xa} + M_{ya} \\
M_{ze} &= F_{xa}d_y - F_{yb}d_{xa} + M_{za}
\end{aligned}
$$

This can be put into a form which includes all nodal forces as:

$$
\begin{Bmatrix} F_{xe} \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & d_z & -d_y & 1 & 0 & 0 \\ -d_z & 0 & d_{xa} & 0 & 1 & 0 \\ d_y & -d_{xa} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_{xa} \\ F_{ya} \\ F_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \end{Bmatrix}
$$

The 6x transformation matrix in the above equation is used in the MYSTRAN code to transform the element nodal forces to element engineering forces