MYSTRAN Test Problem
Hand Calculations

Foreward

This document contains hand calculations that were made to check on the results of MYSTRAN test problem runs back in the early days when MYSTRAN code was being developed. The specific test problems contained herein were devised to check out specific details of MYSTRAN features in order to verify the adequacy of the code. The 27 test problem calculations in this document are only part of the test problem suite used check the veracity of MYSTRAN. The complete suite of test problems currently numbers over 50. The input Bulk Data Files (BDF's) and outputs for all of the test problems, including the ones for the which hand calculations in this document were made, can be seen in the folder named Test Problems on the GitHub MYSTRAN repository

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BAR-AUTOSPC-CHECK

BAR-AUTOSPC-CHECK: 1-CBAR with end load. Skew coord. system and test of AUTOSPC (9/23/03)

$I1 = I2 = 0.1$

$A = 0$, $J = 0$

$\ell = 10$

$E = 10^7$

Ans. for Global coord sys is basic at G.P. 102

The displ and rotation at the end are

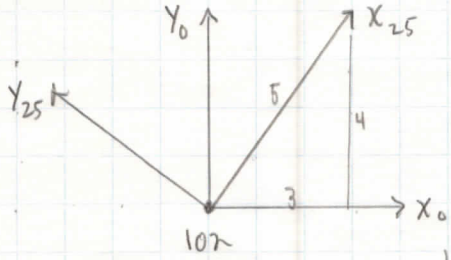
ans. in basic coord system

$$v_0 = \frac{P\ell^3}{3EI_1} = \frac{(100)(10^3)}{3(10^7)(0.1)} = 3.333333\times10^{-2} \text{ in } Y_0 \text{ direction}$$

$$\theta_{z_0} = \frac{P\ell^2}{EI} = \frac{(100)(10^2)}{2(10^7)(0.1)} = 5.000000\times10^{-3} \text{ about } z_0 \text{ axis}$$

MYSTRAN gets these answers and AUTOSPC's components. Table at G.P. 102 in basic coord sys

BAR-AUTOSPC-CHECK: global coord sys is #25 - skewed at 53.1° angle to X_0 axis (3-4-5 triangle)



Since $A, J = 0$, MYSTRAN AUTOSPC's components 2 and 5 at G.P. 102 in coord sys 25 (since Y_{25} is closer to X_0 axis than X_{25} is). Based on this and the v_0, θ_{z_0} above:

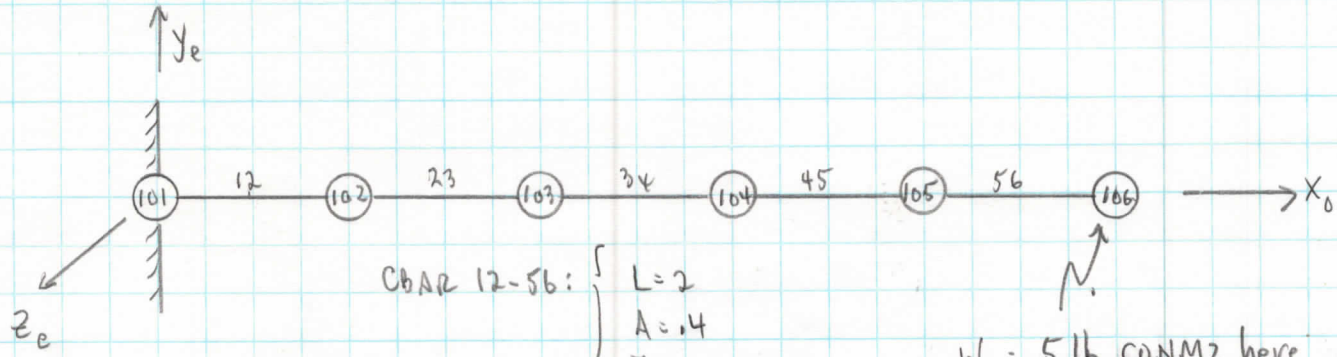
$$u_{25} = \left(\frac{5}{4}\right)v_0 = \frac{5}{4}\left(3.333333\times10^{-2}\right) = 4.166667\times10^{-2}$$

$$\theta_{z_{25}} = \theta_{z_0} = 5.000000\times10^{-3} \quad (\text{since } z_{25} \parallel z_0)$$

MYSTRAN gets these answers.

BAR-GRAV

<u>BAR-GRAV</u> : BAR Model with gravity loading (check on applied load calc)



CBAR 12-56: $\begin{cases} L = 2 \\ A = .4 \\ I_1 = 2. \\ I_2 = 3 \\ E = 1\times10^7,\ \gamma = 0.1 \end{cases}$

W_6 = 5 lb CONM2 here

offset $\left.\begin{array}{l} d_x = .2 \\ d_y = .3 \\ d_z = .4 \end{array}\right\}$ basic

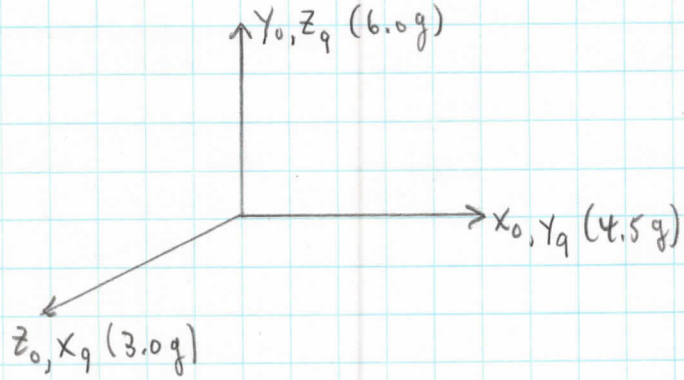
Each CBAR weigh $AL\gamma = (.4)(2)(.1) = .08$ with ½ going to each CBAR grid.

The 5 lb CONM2 has a grid mass matrix (also incl CBAR 56 weight) of:

$$W_{66} = \begin{bmatrix} W_6 + \frac{AL\gamma}{2} & 0 & 0 & 0 & W_6 d_z & -W_6 d_y \\ 0 & W_6 + \frac{AL\gamma}{2} & 0 & -W_6 d_z & 0 & W d_x \\ 0 & 0 & W_6 + \frac{AL\gamma}{2} & W_6 d_y & -W d_x & 0 \\ & & & W_6(d_y^2 + d_z^2) & -W_6 d_x d_y & -W_6 d_x d_z \\ & \text{Sym} & & & W_6(d_x^2 + d_z^2) & -W_6 d_y d_z \\ & & & \text{Sym} & & W_6(d_x^2 + d_y^2) \end{bmatrix}, \quad \begin{array}{l} W_6 = 5 \\ d_x = .2 \\ d_y = .3 \\ d_z = .4 \end{array} \qquad (1)$$

$$W_6 + \frac{AL\gamma}{2} = 5.04 \qquad (2)$$

The grav forces at grid 106 are W_{66} times the grav accel. vector. Since global is basic and the grav load is specified in coord sys 9:



Accel at all grids is;

$$a_g = \begin{Bmatrix} 4.5 \\ 6.0 \\ 3.0 \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}_{\text{basic/global}} \qquad (3)$$

The grav forces at 106 are W_{66} times a_g which is P_{106}

$$P_{106} = \left[\begin{array}{c|c|c|c|c|c} 5.04 & 0 & 0 & 0 & 2.0 & -1.5 \\ 0 & 5.04 & 0 & -2.0 & 0 & 1.0 \\ 0 & 0 & 5.04 & 1.5 & -1.0 & 0 \\ \hline 0 & -2.0 & 1.5 & 1.25 & -0.30 & -0.40 \\ 2.0 & 0 & -1.0 & -0.30 & 1.00 & -0.60 \\ -1.5 & 1.0 & 0 & -0.40 & -0.60 & 0.65 \end{array}\right] \left\{\begin{array}{c} 4.5 \\ 6.0 \\ 3.0 \\ \hline 0 \\ 0 \\ 0 \end{array}\right\} = \left\{\begin{array}{c} 22.68 \\ 30.24 \\ 15.12 \\ \hline -7.50 \\ 6.00 \\ -0.75 \end{array}\right\} \qquad (4)$$

The remainder of the grids have only the bar weight at the grid times a_g

Grid 101 has .04 lb weight and 102-105 have .08 lb so

$$P_{101} = .04 \begin{bmatrix} 4.5 \\ 6.0 \\ 3.0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.18 \\ 0.24 \\ 0.12 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad P_{102} = P_{103} = P_{104} = P_{105} = .08 \begin{bmatrix} 4.5 \\ 6.0 \\ 3.0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.36 \\ 0.48 \\ 0.24 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (5)$$

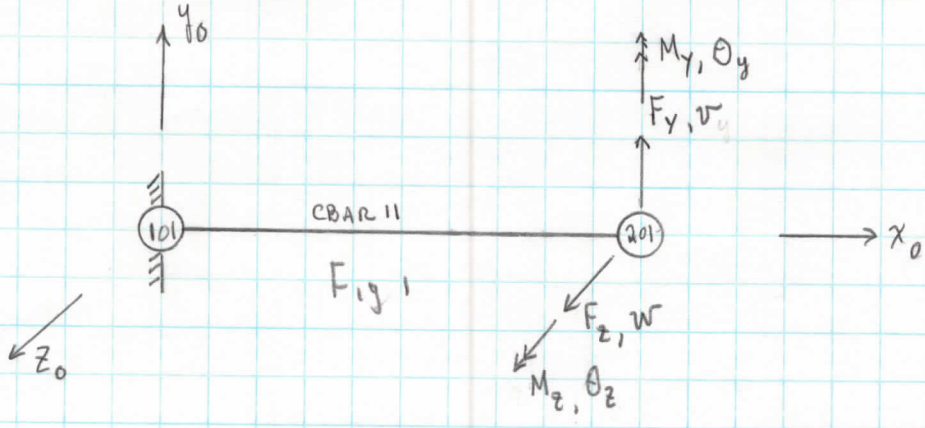
This results in the applied load matrix for all grids (eqn (4), (5)):

Applied Loads

Grid	CID	T1	T2	T3	R1	R2	R3
101	0	0.18	0.24	0.12	0	0	0
102	0	0.36	0.48	0.24	0	0	0
103	0	"	"	"	0	0	0
104	0	"	"	"	0	0	0
105	0	"	"	"	0	0	0
106	0	22.68	30.24	15.12	-7.50	6.00	-0.75

BAR-I12

BAR-I12: 1 CBAR, 2 Grid model. Bar has nonzero I12



$$\text{CBAR 11}: \begin{cases} L = 10 \\ I_1 = 5. \\ I_2 = 4 \\ I_{12} = 2 \\ E = 10^7 \end{cases} \qquad (1)$$

$$Ku = P \quad ,$$

where

$$u = \begin{bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{bmatrix}, \quad P = \begin{bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 6 \\ 9 \end{bmatrix} \qquad (2)$$

$$K = \begin{bmatrix} R_1 & \beta & \frac{l}{2}\beta & -\frac{l}{2}R_1 \\ \beta & R_2 & \frac{l}{2}R_2 & -\frac{l}{2}\beta \\ \frac{l}{2}\beta & \frac{l}{2}R_2 & \hat{k}_2 & -\frac{l^2}{3}\beta \\ -\frac{l}{2}R_1 & -\frac{l}{2}\beta & -\frac{l^2}{2}\beta & \hat{k}_1 \end{bmatrix} \qquad (3)$$

$$\left.\begin{aligned} R_1 &= \frac{12EI_1}{L^3} = \frac{(12)(10^7)}{1000}\cdot E_1 = 1.2\times10^5 I_1 = 6\times10^5 \\ R_2 &= \frac{12EI_2}{L^3} = 1.2\times10^5 I_2 = 4.8\times10^5 \\ \beta &= \frac{12EI_{12}}{L^3} = 1.2\times10^5 I_{12} = 2.4\times10^5 \end{aligned}\right\}$$

$$\left.\begin{aligned} \hat{k}_1 &= \frac{L^2}{4}R_1 + \frac{EI_1}{L} = \frac{100}{4}(6\times10^5) + \frac{(10^7)(5)}{10} = 1.5\times10^7 + .5\times10^7 = 2.0\times10^7 \\ \hat{k}_2 &= \frac{L^2}{4}R_2 + \frac{EI_2}{L} = \frac{100}{4}(4.8\times10^5) + \frac{(10^7)(4)}{10} = 1.2\times10^7 + .4\times10^7 = 1.6\times10^7 \end{aligned}\right\} \qquad (4)$$

Subst (4) into (3)

$$10^4 \begin{bmatrix} 60 & 24 & 120 & -300 \\ 24 & 48 & 240 & -120 \\ 120 & 240 & 1600 & -800 \\ -300 & -120 & -800 & 2000 \end{bmatrix} \begin{bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 6 \\ 9 \end{bmatrix} \begin{matrix} R_{11} \\ R_{21} \\ R_{31} \\ R_{41} \end{matrix} \qquad (5)$$

Solve by Gaussian elimination.

$$\begin{matrix} \\ 0.4\,R_{11} - R_{21} \longrightarrow \\ 2\,R_{11} - R_{31} \longrightarrow \\ 5\,R_{11} + R_{41} \longrightarrow \end{matrix} \quad 10^4 \begin{bmatrix} 60 & 24 & 120 & -300 \\ 0 & -38.4 & -192 & 0 \\ 0 & -192 & -1360 & 200 \\ 0 & 0 & -200 & 500 \end{bmatrix} \begin{bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} 6 \\ 14.4 \\ 6 \\ 39 \end{bmatrix} \begin{matrix} R_{12} \\ R_{22} \\ R_{32} \\ R_{42} \end{matrix}$$

$$\begin{matrix} \\ \\ 5\,R_{22} - R_{23} \\ \\ \end{matrix} \quad 10^4 \begin{bmatrix} 60 & 24 & 120 & -300 \\ 0 & -38.4 & -192 & 0 \\ 0 & 0 & 400 & -200 \\ 0 & 0 & -200 & 500 \end{bmatrix} \begin{bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} 6 \\ 14.4 \\ 66 \\ 39 \end{bmatrix} \begin{matrix} R_{13} \\ R_{23} \\ R_{33} \\ R_{43} \end{matrix}$$

$$\begin{matrix} \\ \\ \\ R_{33} + 2R_{34} \end{matrix} \quad 10^4 \begin{bmatrix} 60 & 24 & 120 & -300 \\ 0 & -38.4 & -192 & 0 \\ 0 & 0 & 400 & -200 \\ 0 & 0 & 0 & 800 \end{bmatrix} \begin{bmatrix} v \\ w \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} 6 \\ 14.4 \\ 66 \\ 144 \end{bmatrix} \qquad (6)$$

Solve (1) by back substitution:

from 4th row

$$800\,\theta_z = 144\times10^{-4}$$

$$\therefore\ \theta_z = 1.8\times10^{-5} \qquad (7)$$

Subst (7) into 3rd row of (1)

$$400\,\theta_y - 200\left(1.8\times10^{-5}\right) = 66\times10^{-4}$$

$$\theta_y = \frac{1}{400}\left[66+36\right]\times10^{-4} = 2.55\times10^{-5} \qquad (8)$$

Subst (7), (8) into 2nd row of (6)

$$-38.4\,w - 192\left(2.55\times10^{-5}\right) = 14.4\times10^{-4}$$

$$w = \frac{-1}{38.4}\left[14.4+192(.255)\right]10^{-4} = -1.65\times10^{-4} \qquad (9)$$

Subst (7)-(9) into 1st row of (6):

$$60v + 24\left(1.65\times10^{-4}\right) + 120\left(.255\times10^{-4}\right) - 300\left(1.8\times10^{-4}\right) = 6\times10^{-4}$$

$$v = \frac{1}{6}\left[6 - 24(-1.65) - 120(.255) + 300(.18)\right]10^{-4}$$

$$v = \frac{1}{6}\left(6+39.6-30.6+54\right)10^{-4} = \frac{69}{6}\times10^{-4} = 1.15\times10^{-4}$$

Displacements

Grid	CID	T1	T2	T3	R1	R2	R3
101	0	0	0	0	0	0	0
201	0	0	1.15×10^{-4}	-1.65×10^{-4}	0	2.55×10^{-5}	1.8×10^{-5}

Elem Engineering Forces

from Fig 1 and loads in eqn (1):

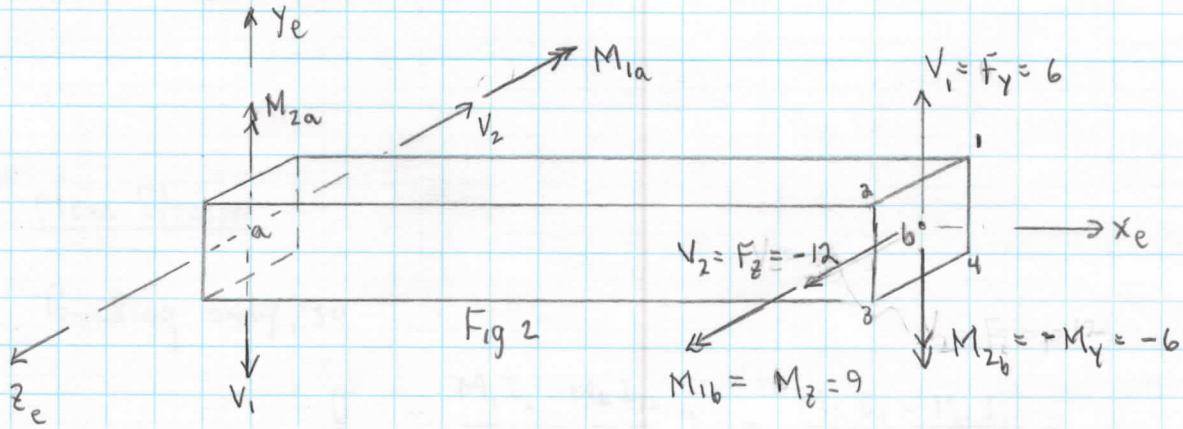


Fig 2

From equil:

$$M_{1a} = M_{1b} + V_1 L = 9 + (6)(10) = 69$$

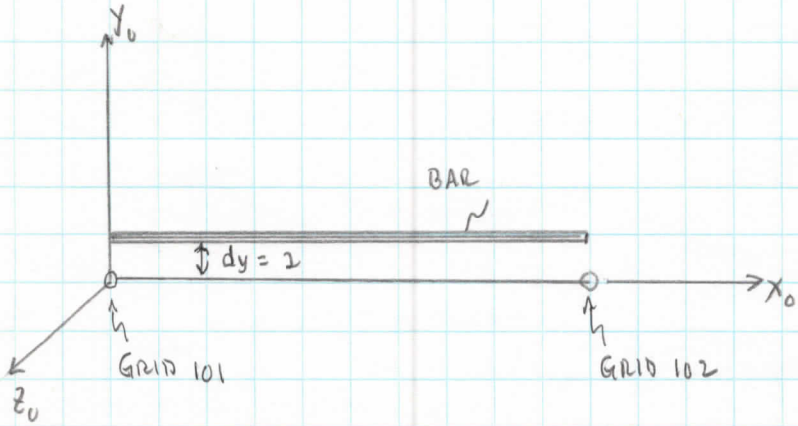
$$M_{2a} = M_{2b} + V_2 L = -6 - (12)(10) = -126$$

Elem	Bend Mom-End a		Bend Mom-End b		Shear		Axial Force	Torque
	plane 1	plane 2	plane 1	plane 2	plane 1	plane 2		
11	69	-126	9	-6	6	-12	0	0

BAR-OFFSET

BAR-OFFSET

1-BAR element offset from both grids an amount $d_y = 2''$:



The relationship between grid displ's and BAR end node displ's is :

$$\begin{bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}_{BAR} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -d_y \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_y & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{D_i} \begin{bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}_{GRID} \quad \text{at BAR end 1 and end 2} \qquad (1)$$

D_i $\begin{cases} i=1: \text{BAR end 1} \\ i=2: \text{BAR end 2} \end{cases}$

$P=1,\ M=1$

lat defl/P $\quad \dfrac{Pl^3}{3EI} = \dfrac{(1)(10^3)}{(3)(.1)(10^7)} = 3.3333\times10^{-4}$ in

long defl/P $\quad \dfrac{Pl}{AE} = \dfrac{(1)(10)}{(.1)(10^7)} = 1\times10^{-5}$ in

lat rot / P $\quad \dfrac{Pl^2}{2EI} = \dfrac{(1)(10^2)}{(2)(10^7)(.1)} = 5\times10^{-5}$ rad

lat defl/M $\quad \dfrac{Ml^2}{2EI} = \dfrac{(1)(10^2)}{(2)(10^7)(.1)} = 5\times10^{-5}$ in

lat rot/M $\quad \dfrac{Ml}{EI} = \dfrac{(1)(10)}{(10^7)(.1)} = 1\times10^{-5}$

$G = \dfrac{E}{2(1+\nu)} = \dfrac{10^7}{2(1+.25)} = 4\times10^6$

$\dfrac{l}{GJ} = \dfrac{10}{(4\times10^6)(1)} = 2.5\times10^{-5}$

$$\begin{bmatrix} v \\ \theta_z \end{bmatrix} = \begin{bmatrix} l^3/3EI & l^2/2EI \\ l^2/2EI & l/EI \end{bmatrix} \begin{Bmatrix} P \\ M \end{Bmatrix} = 10^{-5}\begin{bmatrix} 33.333 & 5 \\ 5 & 1 \end{bmatrix} \begin{Bmatrix} P \\ M \end{Bmatrix}$$

$$\begin{Bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix}_{bar} = \underbrace{\begin{bmatrix} l/AE & & & & & \\ & l^3/3EI_{zz} & & & & l^2/2EI_{zz} \\ & & l^3/3EI_{yy} & & -l^2/2EI_{yy} & \\ & & & l/GJ_{xx} & & \\ & & -l^2/2EI_{yy} & & l/EI_{yy} & \\ & l^2/2EI_{zz} & & & & l/EI_{zz} \end{bmatrix}}_{\text{compliance (pp 101 constrained)}} \begin{Bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}_{bar}$$

(2)

$$K_{\text{NO OFFSET}} = 10^4 \begin{array}{c} \begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\ \left[\begin{array}{cccccc|cccccc} 10 & & & & & & -10 & & & & & \\ & 1.2 & & & & 6 & & -1.2 & & & & 6 \\ & & 1.2 & & -6 & & & & -1.2 & & -6 & \\ & & & 4. & & & & & & -4 & & \\ & & -6 & & 40 & & & & 6. & & 20 & \\ & 6. & & & & 40 & & -6. & & & & 20 \\ \hline -10. & & & & & & 10. & & & & & \\ & -1.2 & & & & -6, & & 1.2 & & & & -6 \\ & & -1.2 & & 6 & & & & 1.2 & & 6. & \\ & & & -4 & & & & & & 4 & & \\ & & -6. & & 20 & & & & 6 & & 40 & \\ & 6 & & & & 20 & & -6. & & & & 40 \end{array}\right] \end{array} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline x \\ 120 \\ 200 \\ -30 \\ -30 \\ 19 \end{array}$$

$\times 10^{-3}$ displs w/ offset

-20	-20	-200
-144 + 114	-30	-300
-240 + 180	-60	-600
120	120	1200
1200 - 600	600	6000
-720 + 380	-340	-3400
20	20	200
144 - 114	30	300
240 - 180	60	600
-120	-120	-1200
1200 - 1200	0	0
-720 + 760	40	400

(3)

MYSTERY: calc'd these PEG which are wrong because PEG was calc'd using KE without offset

$$D_1^T K_{11} D_2 = 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & & & & & \\ & 1.2 & & & & 6 \\ & & 1.2 & & -6 & \\ & & & 4 & & \\ & & -6 & & 40 & \\ & 6 & & & & 40 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ 6 \\ \\ \\ \\ 40 \end{matrix}$$

$$= 10^4 \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & 2 & 1 & & \\ & & & & 1 & \\ -2 & & & & & 1 \end{bmatrix} \begin{bmatrix} 10 & & & & & -20 \\ & 1.2 & & & & 6 \\ & & 1.2 & 2.4 & -6 & \\ & & & 4 & & \\ & & -6 & -12 & 40 & \\ & 6 & & & & -40 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 10 & & & & & -20 \\ & 1.2 & & & & 6 \\ & & 1.2 & 2.4 & -6 & \\ & & 2.4 & 8.8 & -12 & \\ & & -6 & -12 & 40 & \\ -20 & 6 & & & & 80 \end{bmatrix} \checkmark$$

(4)

$$D_1^T K_{12} D_2 = 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10 & & & & & \\ & -1.2 & & & & 6 \\ & & -1.2 & & -6 & \\ & & & -4 & & \\ & & 6 & & 20 & \\ & -6 & & & & 20 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \\ -6 \\ \\ \\ \\ 20 \end{matrix}$$

$$= 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10 & & & & & 20 \\ & -1.2 & & & & 6 \\ & & -1.2 & -2.4 & -6 & \\ & & & -4 & & \\ & & 6 & 12 & 20 & \\ & -6 & & & & 20 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} -10 & & & & & 20 \\ & -1.2 & & & & 6 \\ & & -1.2 & -2.4 & -1 & \\ & & -2.4 & -8.8 & -12 & \\ & & 6 & 12 & 20 & \\ 20 & -6 & & & & -20 \end{bmatrix} \checkmark \qquad (5)$$

$$D_2 K_{22} D_2^T = 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & & & & & \\ & 1.2 & & & & -6 \\ & & 1.2 & & 6 & \\ & & & 4 & & \\ & & 6 & & 40 & \\ & -6 & & & & 40 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & & & & & -20 \\ & 1.2 & & & & -6 \\ & & 1.2 & 2.4 & 6 & \\ & & & 4 & & \\ & & 6 & 12 & 40 & \\ & -6 & & & & 40 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 10 & & & & & -20 \\ & 1.2 & & & & -6 \\ & & 1.2 & 2.4 & 6 & \\ & & 2.4 & 8.8 & 12 & \\ & & 6 & 12 & 40 & \\ -20 & -6 & & & & 80 \end{bmatrix} \checkmark \qquad (c)$$

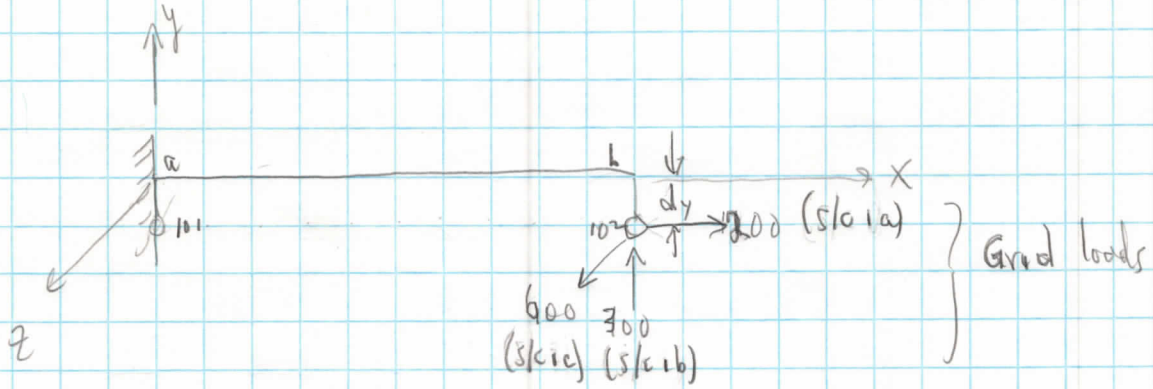
$K_{OFFSET} = 10^4$

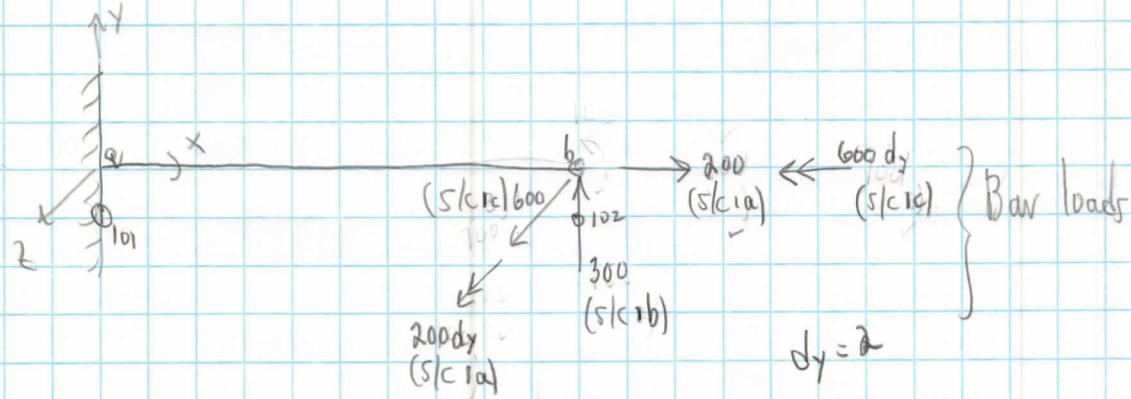
(from MYSTRAN)

(will be checked later)

$$
\begin{array}{c}
\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array} \\
\left[\begin{array}{cccccc|cccccc}
10 & & & & & -20 & -10 & & & & & 20 \\
 & 1.2 & & & & 6 & & -1.2 & & & & 6 \\
 & & 1.2 & 2.4 & -6 & & & & -1.2 & -2.4 & -6. & \\
 & & 2.4 & 8.8 & -12 & & & & -2.4 & -8.8 & -12. & \\
 & & -6 & -12 & 40 & & & & 6. & 12 & 20 & \\
-20 & 6. & & & & 80 & 20 & -6 & & & & -20 \\
\hline
-10 & & & & & 20 & 10 & & & & & -20 \\
 & -1.2 & & & & -6. & & 1.2 & & & & -6. \\
 & & -1.2 & -2.4 & 6. & & & & 1.2 & 2.4 & 6. & \\
 & & -2.4 & -8.8 & 12. & & & & 2.4 & 8.8 & 12 & \\
 & & -6 & -12 & 20 & & & & 6. & 12 & 40 & \\
20 & 6. & & & & -20 & -20 & -6 & & & & 80
\end{array}\right]
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array}
$$

(7)





$$
\begin{Bmatrix} P_{x_b} \\ P_{y_b} \\ P_{z_b} \\ M_{x_b} \\ M_{y_b} \\ M_{z_b} \end{Bmatrix}_{BAR}
=
\underbrace{\begin{Bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ 200dy \end{Bmatrix}}_{\substack{s/c\ 1a \\ \text{from} \\ 200\ lb \\ \text{axial}}}
+
\underbrace{\begin{Bmatrix} 0 \\ 300 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}}_{\substack{s/c\ 1b \\ \text{from} \\ 300\ lb \\ V_y}}
+
\underbrace{\begin{Bmatrix} 0 \\ 0 \\ 600 \\ -600dy \\ 0 \\ 0 \end{Bmatrix}}_{\substack{s/c\ 1c \\ \text{From} \\ 600\ lb \\ V_z}}
=
\underbrace{\begin{Bmatrix} 200 \\ 300 \\ 600 \\ -600dy \\ 0 \\ 200dy \end{Bmatrix}}_{\substack{s/c\ 1 \\ \text{total}}}
\qquad (8)
$$

Using (7) and (8)

$$\begin{bmatrix} u_b \\ v_b \\ w_b \\ \theta_{x_b} \\ \theta_{y_b} \\ \theta_{z_b} \end{bmatrix}_{BAR} = 10^{-5} \begin{bmatrix} 1. & & & & & \\ & 33.3333 & & & & 5. \\ & & 33.3333 & & -5. & \\ & & & 2.5 & & \\ & & -5. & & 1 & \\ & 5. & & & & 1 \end{bmatrix} \begin{bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{bmatrix}_{BAR} \qquad (9)$$

$$\begin{bmatrix} u_b \\ v_b \\ w_b \\ \theta_{x_b} \\ \theta_{y_b} \\ \theta_{z_b} \end{bmatrix}_{BAR} = 10^{-5} \begin{bmatrix} 1. & & & & & \\ & 33.3333 & & & & 5. \\ & & 33.3333 & & -5. & \\ & & & 2.5 & & \\ & & -5. & & 1. & \\ & 5. & & & & 1. \end{bmatrix} \begin{bmatrix} 200 + 0 + 0 \\ 0 + 300 + 0 \\ 0 + 0 + 600 \\ 0 + 0 - 600 d_y \\ 0 + 0 + 0 \\ 200 d_y + 0 + 0 \end{bmatrix}$$

$$\begin{bmatrix} u. \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}_{BAR} = 10^{-3} \underbrace{\begin{bmatrix} 2 \\ 10 d_y \\ 0 \\ 0 \\ 0 \\ 2 d_y \end{bmatrix}}_{1a} + \underbrace{\begin{bmatrix} 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 15. \end{bmatrix}}_{1b} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 200 \\ -15 d_y \\ -30 \\ 0 \end{bmatrix}}_{1c}{}_{BAR} = 10^{-3} \begin{bmatrix} 2 \\ 10 d_y + 100 \\ 200 \\ -15 d_y \\ -30 \\ 2 d_y + 15 \end{bmatrix}_{BAR} = \begin{bmatrix} .002 \\ .120 \\ .200 \\ -.030 \\ -.030 \\ .019 \end{bmatrix}_{BAR} \qquad (10)$$

From Figure on pg 8

$$\begin{bmatrix} u_{102} \\ v_{102} \\ w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \theta_{z_{102}} \end{bmatrix}_{GRID} = \begin{bmatrix} 1 & & & & & d_y \\ & 1 & & & & \\ & & 1 & -d_y & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} u_b \\ v_b \\ w_b \\ \theta_{x_b} \\ \theta_{y_b} \\ \theta_{z_b} \end{bmatrix}_{BAR} \qquad (9)$$

$d_y = 2$

$$\begin{bmatrix} u_{102} \\ v_{102} \\ w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \theta_{z_{102}} \end{bmatrix}_{GRID} = 10^{-3} \begin{bmatrix} 1 & & & & & 2 \\ & 1 & & & & \\ & & 1 & -2 & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix} \left(\underbrace{\begin{bmatrix} 2 \\ 20 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}}_{1a} + \underbrace{\begin{bmatrix} 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 15 \end{bmatrix}}_{1b} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 200 \\ -30 \\ -30 \\ 0 \end{bmatrix}}_{1c} \right)$$

$$\begin{bmatrix} u_{102} \\ v_{102} \\ w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \theta_{z_{102}} \end{bmatrix}_{GRID} = 10^{-3} \begin{bmatrix} 10 \\ 20 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 30 \\ 100 \\ 0 \\ 0 \\ 0 \\ 15 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 260 \\ -30 \\ -30 \\ 0 \end{bmatrix} = 10^{-3} \begin{bmatrix} 40 \\ 120 \\ 260 \\ -30 \\ -30 \\ 19 \end{bmatrix} = \begin{bmatrix} .04 \\ .12 \\ .26 \\ -.03 \\ -.03 \\ .019 \end{bmatrix} \qquad (10)$$

BAR Forces

From (3), (10) and $u_{101} = 0$, calc BAR forces in local sys at BAR ends

$$
\begin{bmatrix} P_{xa} \\ P_{ya} \\ P_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \end{bmatrix}_{BAR}
\overset{\#10}{=}
\left[\begin{array}{cccccc|cccccc}
10 & & & & & & -10 & & & & & \\
 & 1.2 & & & & 6 & & -1.2 & & & & 6 \\
 & & 1.2 & & -6 & & & & -1.2 & & -6 & \\
 & & & 4 & & & & & & -4 & & \\
 & & -6 & & 40 & & & & 6 & & 20 & \\
 & 6 & & & & 40 & & -6 & & & & 20 \\
\hline
 & & & & & & 10 & & & & & \\
 & & & & & & & 1.2 & & & & -6 \\
 & & & & & & & & 1.2 & & 6 & \\
 & & & & & & & & & 4 & & \\
 & & & & & & & & 6 & & 40 & \\
 & & & & & & & -6 & & & & 40
\end{array}\right]
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ .002 \\ .1200 \\ .200 \\ -.030 \\ -.030 \\ .019 \end{bmatrix}_{BAR}
$$

(11)

VEL FROM

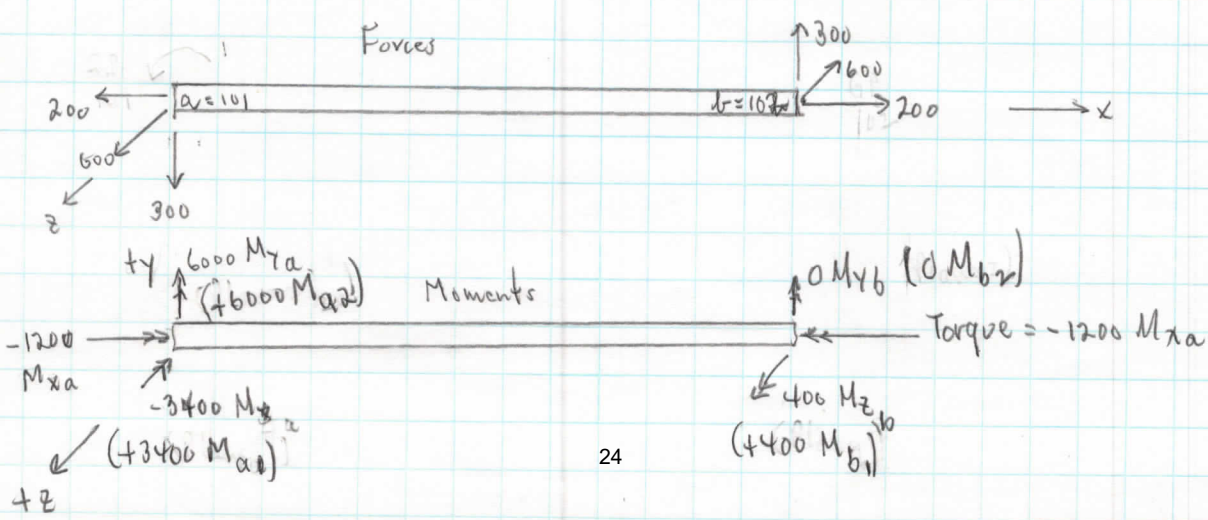
BAR loads in local coords (also global)

$$\begin{bmatrix} P_{x_a} \\ P_{y_a} \\ P_{z_a} \\ M_{x_a} \\ M_{y_a} \\ M_{z_a} \end{bmatrix} = 10^4 \begin{bmatrix} -.002 \\ -.144 + .114 \\ -.240 + .180 \\ -.120 \\ 1.200 - .600 \\ -.720 + .380 \end{bmatrix} = \begin{bmatrix} -200 \\ -300 \\ -600 \\ 1200 \\ 6000 \\ -3400 \end{bmatrix}$$

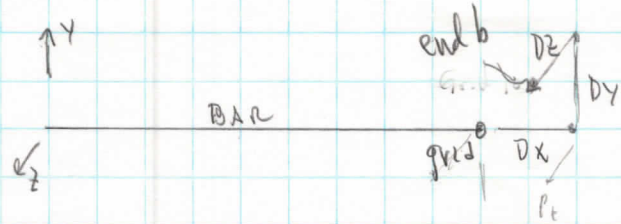
$$\begin{bmatrix} P_{x_b} \\ P_{y_b} \\ P_{z_b} \\ M_{x_b} \\ M_{y_b} \\ M_{z_b} \end{bmatrix}_{BAR} \begin{bmatrix} .002 \\ .144 - .114 \\ .240 - .180 \\ -.120 \\ 1.200 - 1.200 \\ -.720 + .760 \end{bmatrix}_{BAR} \begin{bmatrix} 200 \\ 300 \\ 600 \\ -1200 \\ 0 \\ 400 \end{bmatrix}_{BAR}$$

PEL from MYSTRAN (See BAR-OFFSET.BUG)

(12)



Transform to grid



$$\begin{Bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}_{grid} = \left[\begin{array}{ccc|ccc} 1 & & & 0 & 0 & 0 \\ & 1 & & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ \hline 0 & -D_z & D_y & 1 & 0 & 0 \\ D_z & 0 & -D_x & 0 & 1 & 0 \\ -D_y & D_x & 0 & 0 & 0 & 1 \end{array}\right] \begin{Bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}_{BAR}$$

$DY = 2$ others 0

At G.P. 101: using (12)

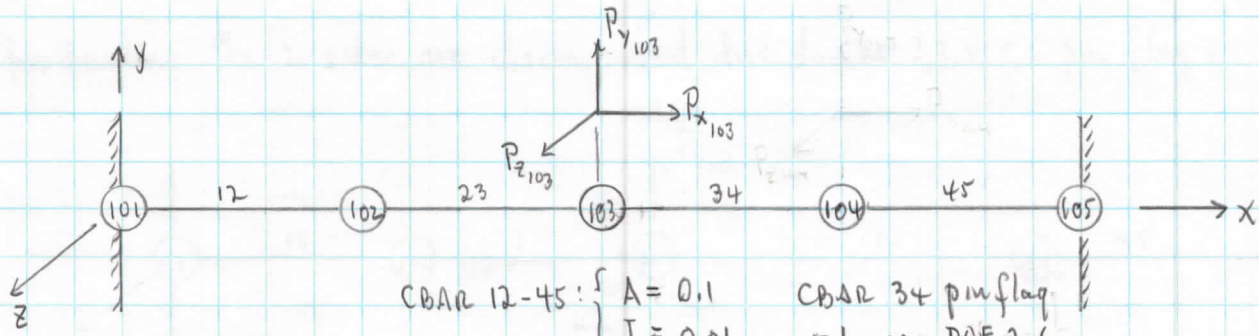
$$\begin{Bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}_{Grid\ 101} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & -2 & 1 & & \\ & & & & 1 & \\ -2 & & & & & 1 \end{bmatrix} \begin{Bmatrix} -200 \\ -300 \\ -600 \\ 1200 \\ 6000 \\ -3400 \end{Bmatrix} = \begin{Bmatrix} -200 \\ -300 \\ -600 \\ 0 \\ 6000 \\ -3000 \end{Bmatrix}_{Grid\ 101}$$

At G.P. 102

$$\begin{Bmatrix} P_x \\ P_y \\ P_z \\ M_x \\ M_y \\ M_z \end{Bmatrix}_{Grid\ 102} = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & -2 & 1 & & \\ & & & & 1 & \\ -2 & & & & & 1 \end{bmatrix} \begin{Bmatrix} 200 \\ 300 \\ 600 \\ -1200 \\ 0 \\ 400 \end{Bmatrix} = \begin{Bmatrix} 200 \\ 300 \\ 600 \\ 0 \\ 0 \\ 0 \end{Bmatrix}_{Grid\ 102}$$

BAR-PINFLAG

BAR-PINFLAG



CBAR 12-45: $\begin{cases} A = 0.1 \\ I_1 = 0.01 \\ I_2 = 0.02 \\ J = 0.03 \\ L = 1'' \end{cases}$

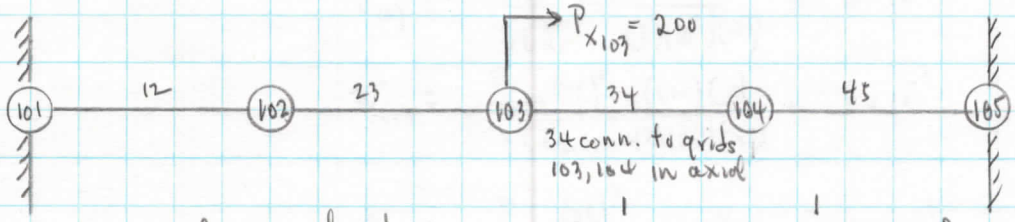
CBAR 34 pinflag releases DOF 2-6 from grid 103

Applied loads

$$P_{X_{103}} = 200$$
$$P_{Y_{103}} = 300$$
$$P_{Z_{103}} = 600$$

Since CBAR 34 is only connected to grid 103 in DOF 1 then this can be viewed as one problem for axial and another for bending

for axial:



$$k_a = \text{axial stiffness} = \frac{1}{\frac{1}{k_{12}} + \frac{1}{k_{23}}} + \frac{1}{\frac{1}{k_{34}} + \frac{1}{k_{35}}} \quad \text{for grid 103}$$

where $k_{ij} = \frac{AE}{L}$

$$= \frac{(0.1)(10^7)}{1} = 10^6$$

so $$k_a = \frac{1}{2/10^6} + \frac{1}{2/10^6} = 10^6$$

$$u_{103} = \frac{P_{X_{103}}}{k_a} = \frac{200}{10^6} = 2\times10^{-4} \quad \text{and} \quad \therefore u_{102} = u_{104} = \frac{1}{2}u_{103} = 1\times10^{-4}$$

Axial loads in elems: CBAR 12,23 : 100# tension $\left\{\right.$ ½ of appl load goes to 12,23 other

CBAR 34,45 : 100# compr. $\left\{\right.$ ½ goes to 34,45 due to parallel springs

BAR-THERM-CONSTR

BAR-THERM-CONSTR: 2 CBAR, 3 GRID model constr at both ends w/ thermal loads
Not stress-free expansion.

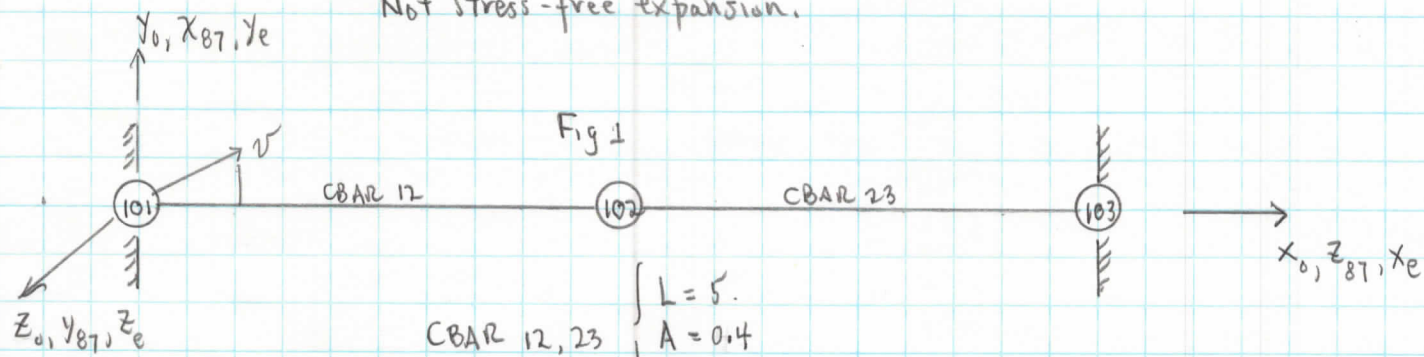


Fig 1

CBAR 12, 23
$L = 5.$
$A = 0.4$
$I_1 = 4.$
$I_2 = 5.$
$E = 1\times10^7$, $\alpha = 20\times10^{-6}$, $T_{ref} = 21.$

Subcase 21 is a thermal load consisting of:

a) Linear temp dist along length ($\Delta T = 0$ at G.P. 101, to $\Delta T = 4$ at G.P. 103

b) $+2°$ Gradient through Y_e depth of CBARS 12, 23 } Y_e, Z_e are local
c) $-4°$ " " Z_e " " " " " } elem axes where $Y_e = Y_0$, $Z_e = Z_0$

We need to determine equivalent thermal loads for the temp. distribution.

For an element

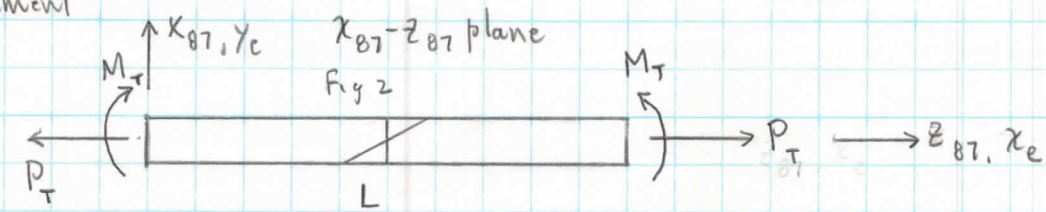


Fig 2

For a + gradient on a cant. beam (see BAR-THERM-FREE) the rotation at end is

$$\theta = \pm \alpha T' L \qquad (1)$$

For a moment on a cant. beam

$$\theta = \pm \frac{M_T L}{EI} \qquad (2)$$

Equating the 2 we get

$$M_T = \pm EI\alpha T' \qquad EI_1 \alpha L T' = (1\times10^7)(4)(20\times10^{-6})T' = 800\,T'$$
$$EI_2 \alpha L T' = (1\times10^7)(5)(20\times10^{-6})T' = 1000\,T' \qquad (3)$$

also, for a bulk temp change ΔT the axial displ of one end rel. to the other is

$$\Delta = \alpha L \Delta T \qquad (4)$$

and the displ for a load P_T is

$$\Delta = \frac{P_T L}{AE} \qquad (5)$$

equating the 2

$$P_T = AE\alpha\Delta T \qquad AE\alpha = \Delta T = (.4)(1\times10^7)(20\times10^{-6})\Delta T = 80.\ \Delta T \qquad (6)$$

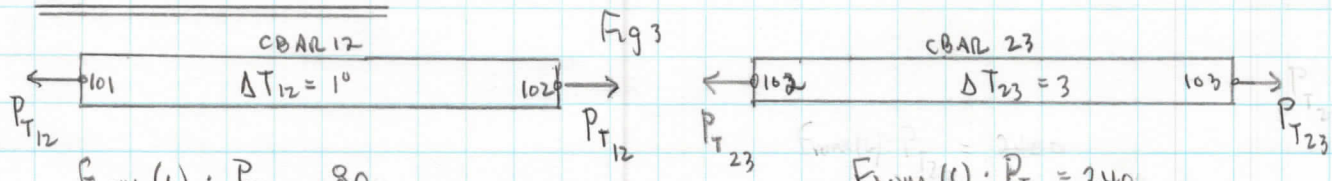
from (6), calc equiv thermal loads due to $\Delta T = 1°$ for elem 12 (avg of G.P.'s 101, 102 = 0, 2°)

and $\Delta T = 3°$ for elem 23 (avg of G.P.'s 102, 103 = 2° and 4°)

Applied Equiv. Thermal Loads

<u>Axial Loads Due To ΔT</u>

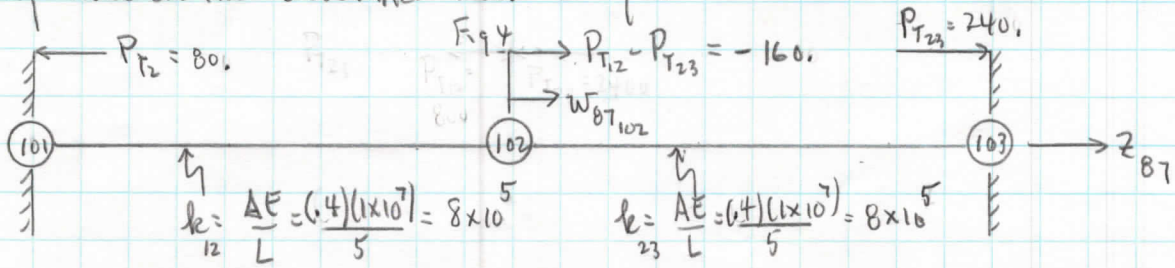
Fig 3



From (6): $P_{T_{12}} = 80.$ From (6): $P_{T_{23}} = 240.$

Combining these on the assembled beam we get

Fig 4



$$\therefore w_{87_{102}} = \frac{P_{T_{12}} - P_{T_{23}}}{k_{12} + k_{23}} = -\frac{160.}{16\times10^5} = -1\times10^{-4} \qquad (7)$$

and applied load (therm equivalent)

$$P_g = \begin{Bmatrix} P_{101} \\ P_{102} \\ P_{103} \end{Bmatrix} = \begin{Bmatrix} -80. \\ -160. \\ 240. \end{Bmatrix} \begin{matrix} \text{in } z_{87} \\ '' \\ '' \end{matrix} \qquad (8)$$

Applied Equiv. Thermal loads, con't

Moments About Y_{87} Due To $T'_{Y_e} = 2°/in$

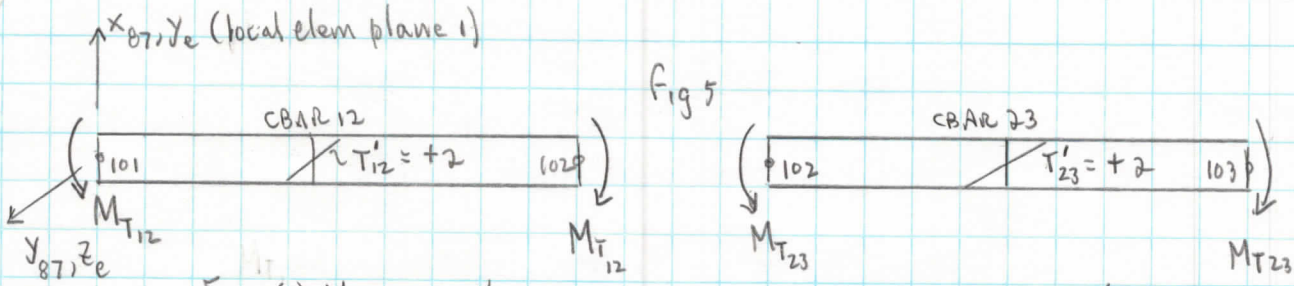


Fig 5

From (3) $M_{T_{12}} = 800 \cdot T' = 1600$ From (3) $M_{T_{23}} = 800\, T' = 1600$ (8)

Combining these on the assembled beam we get

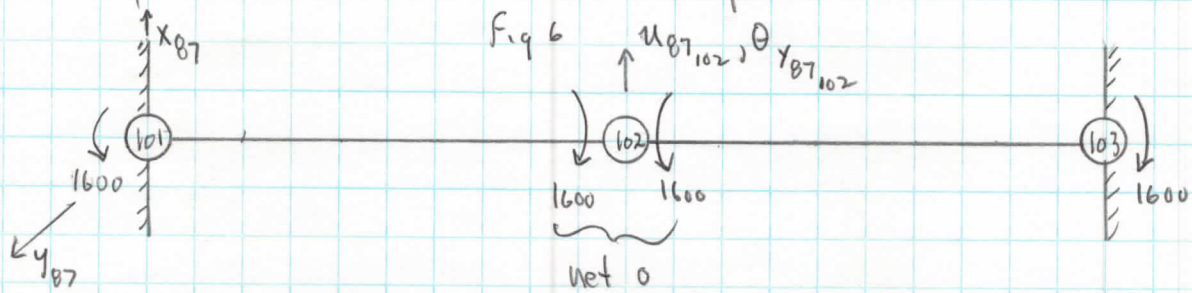


Fig 6

As a result of this there is no net load on G.P. 102 due to the 2° grad T'_{Y_e} so:

$$u_{87_{102}} = \theta_{Y_{87_{102}}} = 0 \;, \quad P_g = \begin{cases} +1600 & \text{G.P. 101 } \theta_{Y_{87}} \\ 0 & \text{G.P. 102 } \theta_{Y_{87}} \\ -1600 & \text{G.P. 103 } \theta_{Y_{87}} \end{cases} \qquad (9)$$

Moments About X_{87} Due To $T'_{Z_e} = -4°/in$

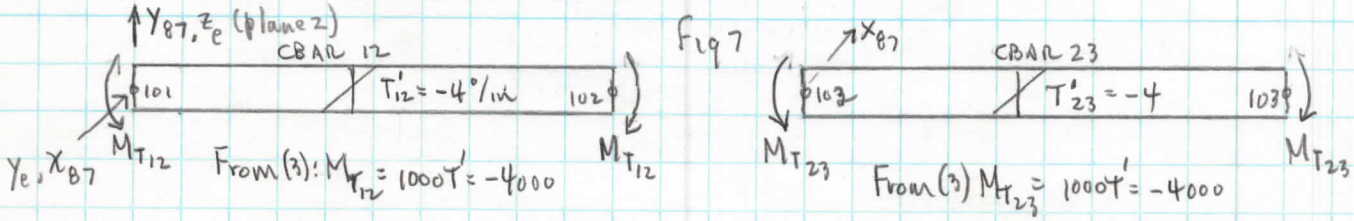


Fig 7

From (3): $M_{T_{12}} = 1000T' = -4000$ From (3) $M_{T_{23}} = 1000T' = -4000$

Combining on assembled model

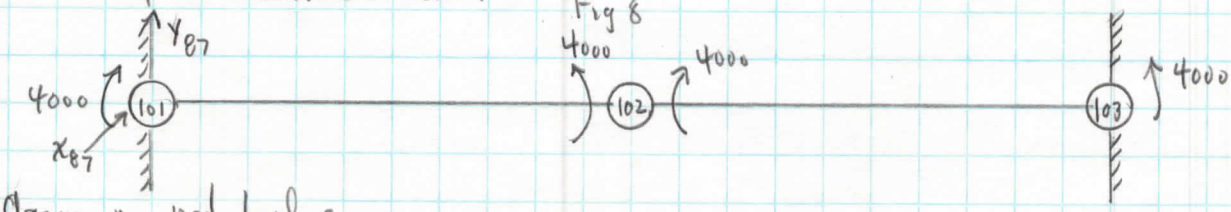


Fig 8

Again, no net load so

$$v_{87_{102}} = \theta_{X_{87_{102}}} = 0 \qquad \text{and applied load (equiv thermal): } P_g = \begin{cases} 4000 & \text{G.P. 101 about } \theta_{X_{87}} \\ 0 & \text{G.P. 102 " "} \\ -4000 & \text{G.P. 103 " "} \end{cases} \qquad (10)$$

from (7), (9), (10)

Displacements

Grid	CID	T1	T2	T3	R1	R2	R3
101	87	0	0	0	0	0	0
102	87	0	0	-1×10^{-4}	0	0	0
103	87	0	0	0	0	0	0

Applied Loads (Equiv. Thermal)

Grid	CID	T1	T2	T3	R1	R2	R3
101	87	0	0	-80.	4000	1600	0
102	87	0	0	-160.	0	0	0
103	87	0	0	240.	-4000	-1600	0

There are SPC forces. SPC Moments at 101, 103 are opposite sign to the equiv. thermal moments on prev. page (to force G.P. equil of the appl + SPC moments since that is all that is applied there). There is an axial SPC force which is most easily calc'd by letting the beam expand freely in z_{87} and then calc load to put ends at 0 displ. The free expansion of whole beam is $\Delta = \alpha(2L)\Delta T_{avg} = (20\times10^{-6})(10)(2^\circ) = 4\times10^{-4}$. To put this back to zero, need compr. force of $P = k\Delta = \left(\frac{8\times10^6}{2}\right)(4\times10^{-4}) = 1600$ lb (NOTE: overall k of beam is the 2 CBARS in series $= 8\times10^6/2$ and $\Delta T_{avg} = \frac{\Delta T_{103} - \Delta T_{101}}{2} = \frac{4-0}{2} = 2$

SPC forces

Grid	CID	T1	T2	T3	R1	R2	R3
101	87	0	0	160.	-4000	-1600	0
102	87	0	0	0	0	0	0
103	87	0	0	-160.	4000	1600	0

Element Engineering Forces

Element engineering forces are easily deduced logically and with aid of the applied equiv. thermal load, SPC forces

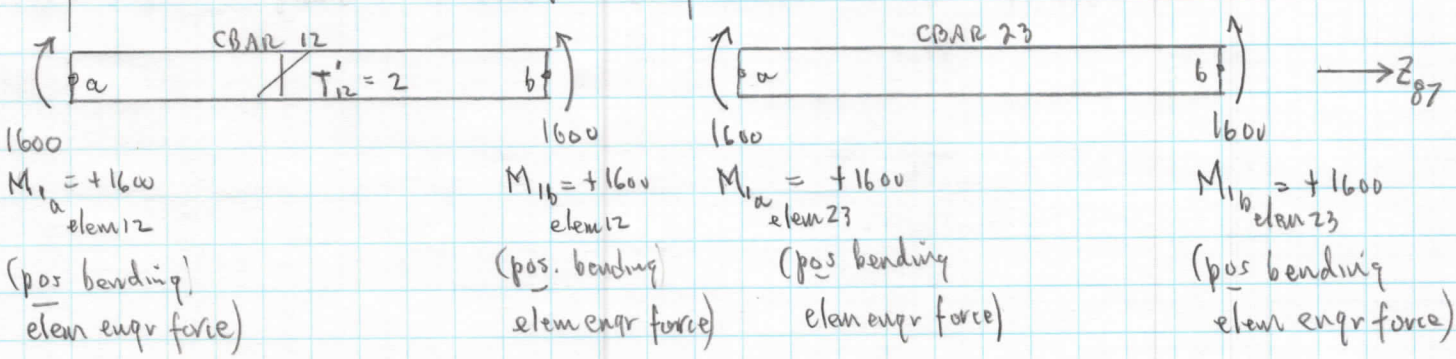
From the T3 SPC force it is seen that the whole beam is in 1600 lb compression

$$\text{Engr. Axial Force CBAR 12, 23} = -160$$

$$\therefore \sigma_{axial} = -160./A = -400\,psi \qquad (11)$$

From fig 5 it is seen that the 1600 in-lb moments (appl. equiv thermal) were put on CBARS 12, 23 as an equiv load to produce free thermal expansion due to the $T' = 2$ in y_e (x_{87}) direction. Since the overall displ's and rotations due to the $T' = 2$ were zero ($u_{87_{1,2}}, \theta_{y_{87_{1,2}}} = 0$) then a 1600 in-lb moment is needed to deflect each BAR back to zero displ & rotation. Therefore the elem engr moments due to the $T' = 2$ in x_{87} are

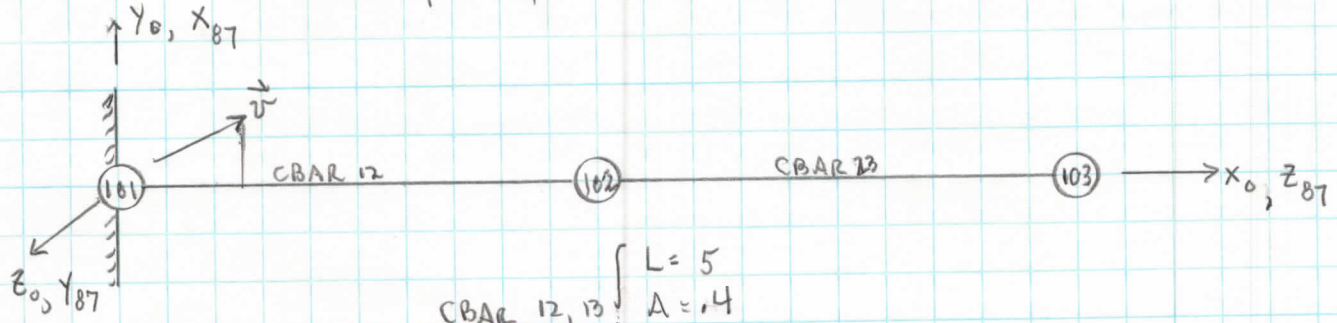
Elem Engr Bending Moments - Plane 1



The fact that the $T' = +2$ means that top is warm (and will expand) and bottom is cool (will contract) and in order to counteract this we need a positive moment, as shown.

BAR-THERM-FREE

BAR-THERM-FREE : 2 CBAR, 3 GRID model constr on one end w/ thermal loads
Stress-free expansion. Uses coord 87 for global



CBAR 12, 13 $\begin{cases} L = 5 \\ A = .4 \\ I_1 = 4. \\ I_2 = 5. \\ E = 1\times10^7,\ \alpha = 20\times10^{-6},\ T_{ref} = 21 \end{cases}$

Subcase 21 is a thermal load consisting of

a) Linear temp distr. along length ($\Delta T = 0$ at G.P. 101 to $\Delta T = 4$ at G.P. 103)

b) $+2^\circ$ Gradient through Y_e depth of CBARS 12, 23 } Y_e, Z_e are local

c) -4° " " Z_e depth " " " " } elem axes where $Y_e = Y_0$, $Z_e = Z_0$

a) Linear temp gradient

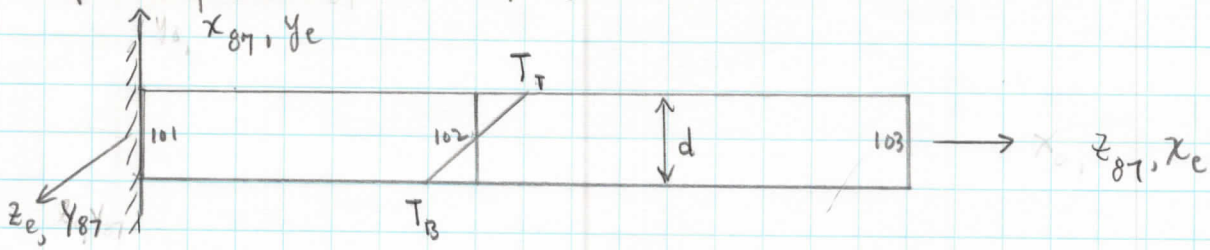
$\Delta T_{101} = 0$
$\Delta T_{102} = 2^\circ$
$\Delta T_{103} = 4^\circ$

CBAR 12 has avg ΔT_{12} of 1°
CBAR 23 " " ΔT_{23} " 3°

$$w_{102_{87}} = \alpha L \Delta T_{12} = (20\times10^{-6})(5)(1) = 1\times10^{-4}$$

$$w_{103_{87}} = w_{102_{87}} + \alpha L \Delta T_{23} = 1\times10^{-4} + (20\times10^{-6})(5)(3) = 4\times10^{-4}$$

For free expansion due to gradient:



The gradient is

$$T' = \frac{T_T - T_B}{d}$$

The displ along the top and bottom are

$$u_T = \alpha T_T z_{87}, \quad u_B = \alpha T_B z_{87}$$

The bending rotation due to the differential displ bet top/bottom is:

$$\theta = \pm \frac{u_T - u_B}{d} = \pm \alpha \left(\frac{T_T - T_B}{d}\right) z_{87} = \pm \alpha T' z_{87}$$

The lateral displ is

$$\Delta = \pm \int \theta \, dz_{87} = \pm \alpha T' \frac{z_{87}^2}{2}$$

The $\pm$ depends on the convention for + rotation for + T'. For example, a pos T' through y_e causes a neg rotation about y_{87} while a pos gradient through z_e causes a pos rotation about x_{87} so that

<u>2° Grad in y_e</u>:

$$u_{87_{101}} = 0, \quad \theta_{y_{87_{101}}} = 0$$

$$u_{87_{102}} = -\alpha T' \left(\frac{5^2}{2}\right) = -(20\times10^{-6})(2)(12.5) = -5\times10^{-4} \text{ in}$$

$$\theta_{y_{87_{102}}} = -\alpha T'(5) = -(20\times10^{-6})(2)(5) = -2\times10^{-4} \text{ rad}$$

$$u_{87_{103}} = -\alpha T'\left(\frac{10^2}{2}\right) = -2\times10^{-3} \text{ in}$$

$$\theta_{y_{87_{103}}} = -\alpha T'(10) = -4\times10^{-4} \text{ rad}$$

$-4°$ Grad in z_e

$$v_{87_{101}} = 0 \quad , \quad \theta_{x_{87_{101}}} = 0$$

$$v_{87_{102}} = -\alpha T' \frac{z_{87}^2}{2} = -(20\times10^{-6})(-4)\left(\frac{5^2}{2}\right) = 1\times10^{-3} \text{ in}$$

$$\theta_{x_{87_{102}}} = \alpha T' z_{87} = (20\times10^{-6})(-4)(5) = -4\times10^{-4} \text{ rad}$$

$$v_{87_{103}} = -\alpha T' \frac{z_{87}^2}{2} = -(20\times10^{-6})(-4)\left(\frac{10^2}{2}\right) = 4\times10^{-3} \text{ in}$$

$$\theta_{x_{87_{103}}} = \alpha T' z_{87} = (20\times10^{-6})(-4)(10) = -8\times10^{-2} \text{ rad}$$

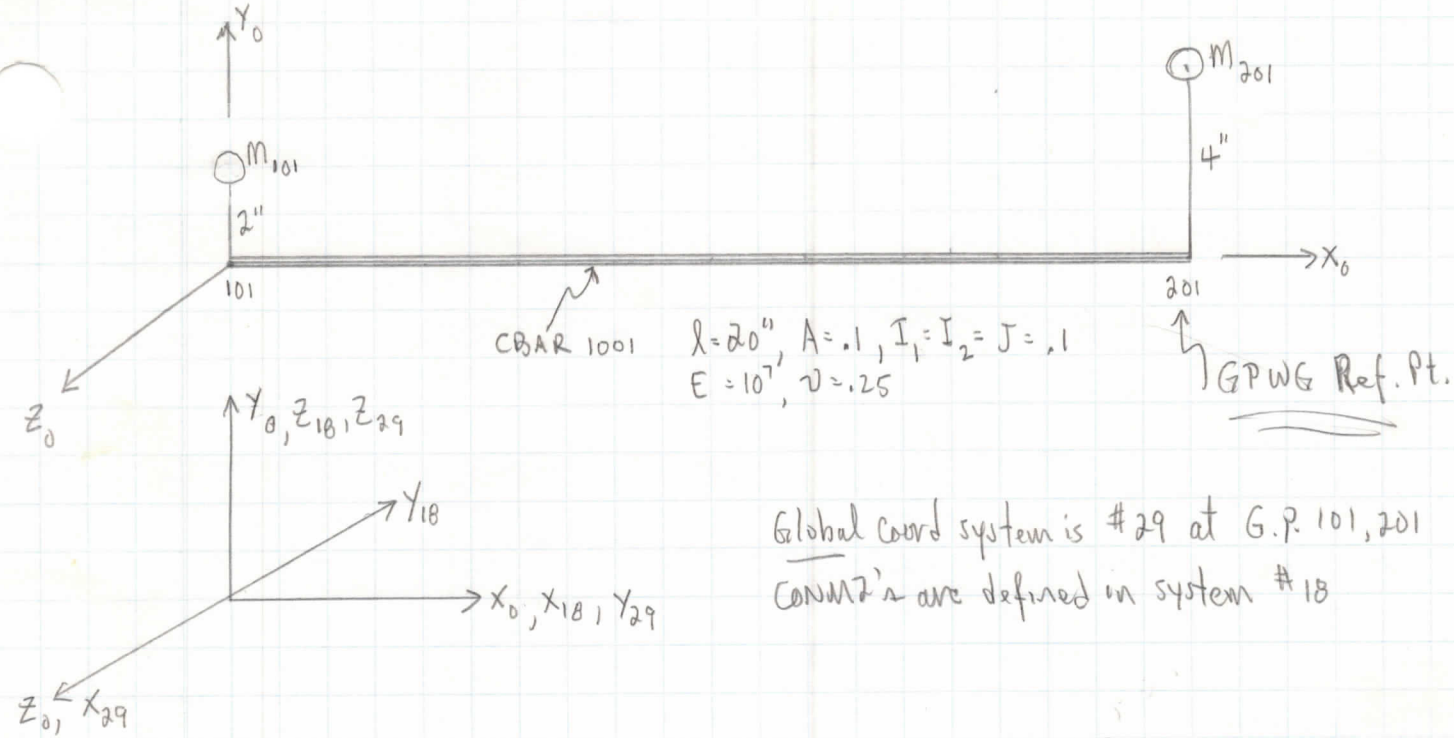
Summary

Grid	u_{87}	v_{87}	w_{87}	$\theta_{x_{87}}$	$\theta_{y_{87}}$	$\theta_{z_{87}}$
101	0	0	0	0	0	0
102	-5×10^{-4}	1×10^{-3}	1×10^{-4}	-4×10^{-4}	-2×10^{-4}	0
103	-2×10^{-3}	4×10^{-3}	4×10^{-4}	-8×10^{-4}	-4×10^{-4}	0

All elem forces and stresses = 0, All SPC forces = 0

CONM2-A

CONM2A.DAT



Global coord system is #29 at G.P. 101, 201

CONM2's are defined in system #18

At G.P. 101 CONM2 11 has $m_{101} = 100^{\#}$, $d_{18}^{101} = \begin{Bmatrix} 0 \\ 0 \\ 2 \end{Bmatrix}$, $I_{18,18}^{101} = \begin{bmatrix} 105. & & \text{sym} \\ 110. & 111. & \\ 115. & 116. & 116. \end{bmatrix}$

At G.P. 201 CONM2 21 has $m_{201} = 300^{\#}$, $d_{18}^{201} = \begin{Bmatrix} 0 \\ 0 \\ 4 \end{Bmatrix}$, $I_{18,18}^{201} = \begin{bmatrix} 206. & & \text{sym} \\ 211. & 211. & \\ 216. & 216. & 217. \end{bmatrix}$

Transformation Matrices

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_0 = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}}_{T_{0,18}} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{18}$$

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_0 = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}}_{T_{0,29}} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{29}$$

Transform CONM2 Values From Coord #18 To Basic (At the Mass)

$$d_0^{101} = T_{0,18}\, d_{18}^{101} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2 \\ 0 \end{Bmatrix}$$

$$d_0^{201} = T_{0,18}\, d_{18}^{201} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 4 \\ 0 \end{Bmatrix}$$

$$I_{0,0}^{101} = T_{0,18}\, I_{18,18}^{101}\, T_{0,18}^{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 105. & 110. & 115. \\ 110. & 111. & 116. \\ 115. & 116. & 116. \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 105 & 115 & -110 \\ 110 & 116. & -111 \\ 115 & 116. & -116 \end{bmatrix} = \begin{bmatrix} 105 & 115 & -110 \\ 115. & 116. & -116. \\ -110 & -116. & 111 \end{bmatrix}$$

$$I_{0,0}^{201} = T_{0,18}\, I_{18,18}^{201}\, T_{0,18}^{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 206 & 211. & 216. \\ 211 & 211. & 216 \\ 216 & 216. & 217 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 206 & 216. & -211 \\ 211 & 216. & -211 \\ 216 & 217 & -216 \end{bmatrix} = \begin{bmatrix} 206 & 216 & -211 \\ 216. & 217 & -216 \\ -211 & -216 & 211 \end{bmatrix}$$

The GPWG Reference Pt. is G.P. 201. Translate I_{00}^{101} to G.P. 201:

1) Mass at GP 101 ($d_x = 20$, $d_y = -2$, $d_z = 0$, $m = 100$)

$$(I_{xx})_{201} = (I'_{xx})_{101} + m(d_y^2 + d_z^2) = 105 + 100(2^2 + 0^2) = 505$$

$$(I_{yy})_{201} = (I'_{yy})_{101} + m(d_x^2 + d_z^2) = 116 + 100(20^2 + 0^2) = 40{,}116$$

$$(I_{zz})_{201} = (I'_{zz})_{101} + m(d_x^2 + d_y^2) = 111 + 100(20^2 + 2^2) = 40{,}511$$

$$(I_{yx})_{201} = (I'_{yx})_{101} - m d_y d_x = 115 - 100(-2)(20) = 4115$$

$$(I_{zx})_{201} = (I'_{zx})_{101} - m d_z d_x = -110 - 100(0)(20) = -110$$

$$\underbrace{(I_{zy})_{201}}_{\text{MoI's of G.P 101 mass translated to Ref. Pt. (GP 201)}} = \underbrace{(I'_{zy})_{201}}_{\text{MoI's of GP 101 Mass from above } I_{0,0}^{101} \text{ matrix}} - m d_z d_y = -116 - m(0)(-2) = -116$$

2) <u>Mass at GP. 201</u> ($d_x = 0,\ d_y = -4,\ d_z = 0$)

$$(I_{xx})_{201} = (I_{x'x'})_{201} + m(d_y^2 + d_z^2) = 206 + 300(-4^2 + 0^2) = 5006$$

$$(I_{yy})_{201} = (I_{y'y'})_{201} + m(d_x^2 + d_z^2) = 217 + 300(0^2 + 0^2) = 217$$

$$(I_{zz})_{201} = (I_{z'z'})_{201} + m(d_x^2 + d_y^2) = 211 + 300(0^2 + (-4)^2) = 5011$$

$$(I_{yx})_{201} = (I_{y'x'})_{201} - m d_y d_x = 216 - 300(-4)(0) = 216$$

$$(I_{zx})_{201} = (I_{z'x'})_{201} - m d_z d_x = -211 - 300(0)(0) = -211$$

$$(I_{zy})_{201} = (I_{z'y'})_{201} - m d_z d_y = -216 - 300(0)(-4) = -216$$

3) Sum 101+201 to get MOI' about Ref. Pt:

$$(I_{xx})_{201} = 505 + 5006 = 5{,}511$$

$$(I_{yy})_{201} = 40{,}116 + 217 = 40{,}333.$$

$$(I_{zz})_{201} = 40{,}511 + 5{,}011 = 45{,}522$$

$$(I_{yx})_{201} = 4{,}115 + 216 = 4{,}331$$

$$(I_{zx})_{201} = -110 - 211 = -321$$

$$(I_{zy})_{201} = -116 - 216 = -332$$

$$I_{REF\,PT} = \begin{bmatrix} 5511. & 4331. & -321. \\ 4331. & 40333. & -332. \\ -321. & -332. & 45522. \end{bmatrix}$$

also agrees w/ MYSTRAN

Calculate Total Mass & C.G.!

$$M_{Tot} = 100 + 300 = 400 \text{ lb total}$$

$$\bar{x}_{cg} = \frac{(-20)(100)}{100+300} = -5'' \text{ from G.P. 201 in } X_0 \text{ direction} \therefore X_{cg} = -5$$

$$\bar{y}_{cg} = \frac{(2)(100)+(-7)(300)}{100+300} = 3.5'' \text{ from GP 201 in } Y_0 \text{ direction} \therefore Y_{CG} = 3.5$$

$$\bar{z}_{cg} = 0 \qquad \underbrace{\therefore Z_{cg} = 0}_{\text{rel. to G.P. 201}}$$

Calculate MoI's about C.G. In Basic Coords

$$\left.\begin{aligned}
(I_{xx})_{REF} &= (I_{xx})_{CG} + M_{Tot}(d_y^2 + d_z^2) \\
(I_{yy})_{REF} &= (I_{yy})_{CG} + M_{Tot}(d_x^2 + d_z^2) \\
(I_{zz})_{REF} &= (I_{zz})_{CG} + M_{Tot}(d_x^2 + d_y^2) \\
(I_{yx})_{REF} &= (I_{yx})_{CG} - M_{Tot}\, d_y d_x \\
(I_{zx})_{REF} &= (I_{zx})_{CG} - M_{Tot}\, d_z d_x \\
(I_{zy})_{REF} &= (I_{zy})_{CG} - M_{Tot}\, d_z d_y
\end{aligned}\right\}$$

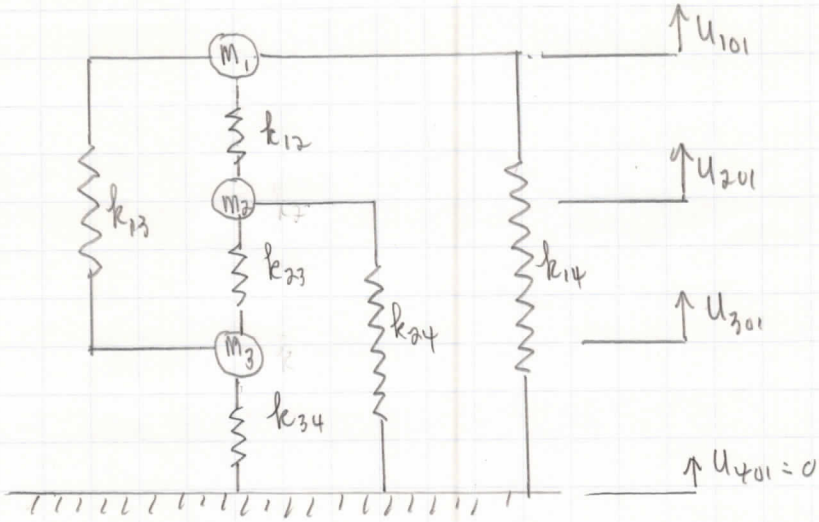
$d_x = X_{REF} - X_{CG} = 5$

$d_y = Y_{REF} - Y_{CG} = 0 - 3.5 = -3.5$

$d_z = Z_{REF} - Z_{CG} = 0$

$$\left.\begin{aligned}
\therefore (I_{xx})_{CG} &= 5511 - 400(-3.5^2 + 0^2) = 611 \\
(I_{yy})_{CG} &= 40{,}333 - 400(5^2 + 0^2) = 30{,}333 \\
(I_{zz})_{CG} &= 45{,}522 - 400(5^2 + 3.5^2) = 30{,}622 \\
(I_{yx})_{CG} &= 4{,}331 + 400(-3.5)(5) = -2{,}669 \\
(I_{zx})_{CG} &= -321 + 400(0)(5) = -321 \\
(I_{zy})_{CG} &= -332 + 400(0)(-3.5) = -332
\end{aligned}\right\} \text{agrees w/ MYSTRAN}$$

To get principal MoI's use eigenvalue soln with a model shown below



Then

$$K = \begin{bmatrix} k_{12}+k_{13}+k_{14} & -k_{12} & -k_{31} \\ -k_{12} & k_{12}+k_{23}+k_{24} & -k_{23} \\ -k_{31} & -k_{23} & k_{13}+k_{23}+k_{34} \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Use

$$k_{12} = 2669, \quad k_{23} = 332, \quad k_{34} = 29969$$

$$k_{13} = 321, \quad k_{14} = -2379, \quad k_{24} = 27332$$

Then

$$I_{CG} = K = \begin{bmatrix} 621 & -2669 & -321 \\ -2669 & 30333 & -332 \\ -321 & -332 & 30622 \end{bmatrix}$$

(same as I_{CG} on previous page)

The eigenvalues and eigenvectors are shown in the following SOL 3 run (CONM2A-PMOI)
The eigenvalues are the PMOI's and the eigenvectors are <u>rows</u> of the transformation from basic to principal directions

23162444

```
MYSTRAN Version 1.00   Sept, 2001
(C) Copyright 2001 William Case

>>>>> LINK  1 BEGIN                 16:24:44.500      4/23/2002   The input file is \SW01\D\Runs\NewRuns\CONM2A-PMOI.DAT

*INFORMATION: PARAMETER EPSIL        1 CHANGED FROM  1.000000E-15 TO  2.220446E-16 BASED ON LAPACK MACHINE PARAMETERS

ID PRINC,MOI
TIME 10
SOL 3
APP DISP
DEBUG   13       1
 *WARNING    : PRIOR CARD NOT PROCESSED BY MYSTRAN
DEBUG   21       1
 *WARNING    : PRIOR CARD NOT PROCESSED BY MYSTRAN
CEND
TITLE = EIGENVALUE RUN WITH CELAS'S TO GET PMOI'S OF CONM2A.DAT
SUBTITLE = K MATRIX FROM CELAS'S GIVE 6x6 R.B. MASS MATRIX FROM CONM32A.DAT
LABEL = EIGENVALUES OF THIS RUN WILL BE PMOI'S OF PROBLEM IN CONM2A
METHOD = 1
DISP=ALL
BEGIN BULK
GRDSET                                                      23456
GRID    101             0.      0.      0.
GRID    201             0.      0.      0.
GRID    301             0.      0.      0.
GRID    401             0.      0.      0.                  123456
CELAS1  12      12      101     1       201     1
CELAS1  23      23      201     1       301     1
CELAS1  31      31      301     1       101     1
CELAS1  10      10      101     1       401     1
CELAS1  20      20      201     1       401     1
CELAS1  30      30      301     1       401     1
PELAS   12      2669.
PELAS   23      332.
PELAS   31      321.
PELAS   10      -2379.
PELAS   20      27332.
PELAS   30      29969.
CONM2   1       101     0       1.0
CONM2   2       201     0       1.0
CONM2   3       301     0       1.0
EIGR    1       GIV     0.      1.E8    3       3                       +E1
+E1     MASS
PARAM   SEQUENCE2
ENDDATA
```

E I G E N V A L U E A N A L Y S S U M M A R Y (GIV)

NUMBER OF EIGENVALUES EXTRACTED 3

LARGEST OFF-DIAGONAL GENERALIZED MASS TERM
(WITH DIAGONAL TERMS = 1.0) 5.0E-16

MODE PAIR . . . . . 2 . . . 3

NUMBER OF OFF DIAGONAL GENERALIZED MASS
TERMS FAILING CRITERION OF 1.0E-06. 0

*INFORMATION: EIGENVECTORS WILL BE RENORMALIZED BASED ON GENERALIZED MASS

R E A L E I G E N V A L U E S

MODE NUMBER	EXTRACTION ORDER	EIGENVALUE	RADIANS	CYCLES	GENERALIZED MASS	GENERALIZED STIFFNESS
1	1	3.691981E+02	1.921453E+01	3.058087E+00	1.000000E+00	3.691981E+02
2	2	3.029495E+04	1.740545E+02	2.770163E+01	1.000000E+00	3.029495E+04
3	3	3.090185E+04	1.757892E+02	2.797772E+01	1.000000E+00	3.090185E+04

princ MoI's

EIGENVECTOR 1
EIGENVALUE RUN WITH CELAS'S TO GET PMOI'S OF CONM2A.DA
K MATRIX FROM CELAS'S GIVE 6x6 R.B. MASS MATRIX FROM CONM32A.DAT
EIGENVALUES OF THIS RUN WILL BE PMOI'S OF PROBLEM IN CONM2A

E I G E N V E C T O R

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
101	0	-9.959787E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
201	0	-8.884384E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
301	0	-1.154291E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
401	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

EIGENVECTOR 2
EIGENVALUE RUN WITH CELAS'S TO GET PMOI'S OF CONM2A.DAT
K MATRIX FROM CELAS'S GIVE 6x6 R.B. MASS MATRIX FROM CONM32A.DAT
EIGENVALUES OF THIS RUN WILL BE PMOI'S OF PROBLEM IN CONM2A

E I G E N V E C T O R

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
101	0	7.337512E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
201	0	-7.349859E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
301	0	-6.741007E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
401	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

EIGENVECTOR 3
EIGENVALUE RUN WITH CELAS'S TO GET PMOI'S OF CONM2A.DAT
K MATRIX FROM CELAS'S GIVE 6x6 R.B. MASS MATRIX FROM CONM32A.DAT
EIGENVALUES OF THIS RUN WILL BE PMOI'S OF PROBLEM IN CONM2A

E I G E N V E C T O R

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
101	0	-5.140582E-02	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
201	0	6.722369E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
301	0	-7.385492E-01	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00
401	0	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00

>>>>> LINK 9 END 16:24:45.141

Mass Matrix

G.P. mass matrices in local coords at the mass (based on mass, MOI's input on CONM2 cards):

CONM2 ID

$$M^{11}_{\substack{\text{mass},21\\ \text{local}=18}} = \left[\begin{array}{ccc|ccc} 100 & & & & & \\ & 100 & & & & \\ & & 100 & & & \\ \hline & & & 105 & 110 & 115 \\ & & & 110 & 111 & 116 \\ & & & 115 & 116 & 116 \end{array}\right], \quad M^{21}_{\substack{\text{mass},21\\ \text{local}=18}} = \left[\begin{array}{ccc|ccc} 300 & & & & & \\ & 300 & & & & \\ & & 300 & & & \\ \hline & & & 206 & 211 & 216 \\ & & & 211 & 211 & 216 \\ & & & 216 & 216 & 217 \end{array}\right]$$

(where "local" means CONM2 coord system in field 4 of CONM2 B.D. card)

note: only lower triangle input on CONM2 card

In subr CONM2_PROC_1 these are transformed to basic coords at the mass. The local system for the CONM2's is coord system 18. The transformation to basic from 18 was given earlier:

$$T_{0,18} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

We want

$$T_{18,0} \equiv T^T_{0,18} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Then

$$I^i_{\substack{\text{mass},i\\ \text{basic}}} = T^T_{18,0}\, I^i_{\substack{\text{mass},i,\\ 18}}\, T_{18,0} \qquad \left(I^i = \text{MOI portion of } M^i_{\substack{\text{mass,}\\ \text{local}}} \right)$$

$$I^{11}_{\substack{\text{mass},11\\ \text{basic}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 105 & 110 & 115 \\ 110 & 111 & 116 \\ 115 & 116 & 116 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 105 & 115 & -110 \\ 115 & 116 & -116 \\ -110 & -116 & 111 \end{bmatrix}$$

$$I^{21}_{\substack{\text{mass},21\\ \text{basic}}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 206 & 211 & 216 \\ 211 & 211 & 216 \\ 216 & 216 & 217 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 206 & 216 & -211 \\ 216 & 217 & -216 \\ -211 & -216 & 211 \end{bmatrix}$$

So

$$M^{11}_{\substack{\text{mass,11,}\\ \text{basic}}} = \left[\begin{array}{ccc|ccc} 100 & & & & & \\ & 100 & & & & \\ & & 100 & & & \\ \hline & & & 105 & 115 & -110 \\ & & & 115 & 116 & -116 \\ & & & -110 & -116 & 111 \end{array}\right], \quad M^{21}_{\substack{\text{mass,21,}\\ \text{basic}}} = \left[\begin{array}{ccc|ccc} 300 & & & & & \\ & 300 & & & & \\ & & 300 & & & \\ \hline & & & 206 & 216 & -211 \\ & & & 216 & 217 & -216 \\ & & & -211 & -216 & 211 \end{array}\right]$$

these agree w/ outputs from subr CONM2_PROC_1 for mass, MoI terms in "RCONM2 in basic coords at the mass". Keep in mind that RCONM2 has only lower portion of 3x3 MoI partition and that they are in the order $I_{xx}, I_{yx}, I_{yy}, I_{zx}, I_{zy}, I_{zz}$

In subr CONM2_PROC_2 the above matrices are transformed to the grid in basic and then to the grid in global. The transformations between the mass and grid for the 2 CONM2's are (in basic coords):

$$\begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_{x_0} \\ \theta_{y_0} \\ \theta_{z_0} \end{bmatrix}_{\text{at mass 11}} = \underbrace{\left[\begin{array}{ccc|ccc} 1 & & & & & -2 \\ & 1 & & & & \\ & & 1 & 2 & & \\ \hline & & & 1 & & \\ & 0 & & & 1 & \\ & & & & & 1 \end{array}\right]}_{T_{11,101}} \begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_{x_0} \\ \theta_{y_0} \\ \theta_{z_0} \end{bmatrix}_{\text{at GP 101}}, \quad \begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_{x_0} \\ \theta_{y_0} \\ \theta_{z_0} \end{bmatrix}_{\text{at mass 21}} = \underbrace{\left[\begin{array}{ccc|ccc} 1 & & & & & -4 \\ & 1 & & & & \\ & & 1 & 4 & & \\ \hline & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{array}\right]}_{T_{21,201}} \begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta_{x_0} \\ \theta_{y_0} \\ \theta_{z_0} \end{bmatrix}$$

so

$$M^{11}_{\substack{\text{grid 101}\\ \text{basic}}} = T^T_{11,101} \, M^{11}_{\substack{\text{mass 11,}\\ \text{basic}}} \, T_{11,101}$$

and

$$M^{21}_{\substack{\text{grid 201}\\ \text{basic}}} = T^T_{21,201} \, M^{21}_{\substack{\text{mass 21,}\\ \text{basic}}} \, T_{21,201}$$

$$M^{11}_{\substack{\text{grid 101,}\\ \text{basic}}} = \left[\begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ \hline & & 2 & 1 & & \\ & & & & 1 & \\ -2 & & & & & 1 \end{array}\right] \left[\begin{array}{ccc|ccc} 100 & & & & & \\ & 100 & & & 0 & \\ & & 100 & & & \\ \hline & & & 105 & 115 & -110 \\ & 0 & & 115 & 116 & -116 \\ & & & -110 & -116 & 111 \end{array}\right] \left[\begin{array}{ccc|ccc} 1 & & & & & -2 \\ & 1 & & & & \\ & & 1 & 2 & & \\ \hline & & & 1 & & \\ & 0 & & & 1 & \\ & & & & & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 100 & & & & & -200 \\ & 100 & & & & \\ & & 100 & 200 & & \\ \hline & & 200 & 505 & 115 & -110 \\ & & & 115 & 116 & -116 \\ -200 & & & -110 & -116 & 511 \end{array}\right]$$

$$M^{21}_{\substack{\text{grid 201,}\\ \text{basic}}} = \left[\begin{array}{ccc|ccc} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ \hline & & 4 & 1 & & \\ & & & & 1 & \\ -4 & & & & & 1 \end{array}\right] \left[\begin{array}{ccc|ccc} 300 & & & & & \\ & 300 & & & 0 & \\ & & 300 & & & \\ \hline & & & 706 & 216 & -211 \\ & 0 & & 216 & 217 & -216 \\ & & & -211 & -216 & 211 \end{array}\right] \left[\begin{array}{ccc|ccc} 1 & & & & & -4 \\ & 1 & & & & \\ & & 1 & 4 & & \\ \hline & & & 1 & & \\ & 0 & & & 1 & \\ & & & & & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 300 & & & & & -1200 \\ & 300 & & & & \\ & & 300 & 1200 & & \\ \hline & & 1200 & 5006 & 216 & -211 \\ & & & 216 & 217 & -216 \\ -1200 & & & -211 & -216 & 5011 \end{array}\right]$$

The MOI terms agree w/ outputs from subr CONM2-PROC-2 for Mass, MOI terms in "RCONM2 in basic coords at the grid ..." . The mass coupling terms are also OK since they are Mass*d values and the d values in RCONM2 are correct

We now need to transform from basic to global at the grid. Global is coord sys 29 so (from earlier)

$$T_{0,29} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad \bar{T}_{0,29} = \begin{bmatrix} T_{0,29} & 0 \\ 0 & T_{0,29} \end{bmatrix} = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & 0 & \\ 1 & 0 & 0 & & & \\ \hline & & & 0 & 1 & 0 \\ & 0 & & 0 & 0 & 1 \\ & & & 1 & 0 & 0 \end{array}\right]$$

so

$$M^{i}_{\substack{\text{grid } j\\ \text{global}=29}} = \bar{T}^{T}_{0,29} \, M^{i}_{\substack{\text{grid } j\\ \text{basic}}} \, \bar{T}_{0,29}$$

$$M''_{\substack{grid\,101,\\ global=29}} = \left[\begin{array}{ccc|ccc|ccc|ccc}
0 & 0 & 1 & & & & 100 & & & & & -200 \\
1 & 0 & 0 & & 0 & & & 100 & & & & \\
0 & 1 & 0 & & & & & & 100 & 200 & & \\
\hline
 & & & 0 & 0 & 1 & & & 200 & 505 & 115 & -110 \\
 & 0 & & 1 & 0 & 0 & & & & 115 & 116 & -116 \\
 & & & 0 & 1 & 0 & -200 & & & -110 & -116 & 511
\end{array}\right]
\left[\begin{array}{ccc|ccc}
0 & 1 & 0 & & & \\
0 & 0 & 1 & & 0 & \\
1 & 0 & 0 & & & \\
\hline
 & & & 0 & 1 & 0 \\
 & 0 & & 0 & 0 & 1 \\
 & & & 1 & 0 & 0
\end{array}\right]
= \left[\begin{array}{ccc|ccc}
100 & & & 0 & 200 & 0 \\
 & 100 & & -200 & 0 & 0 \\
 & & 100 & 0 & 0 & 0 \\
\hline
0 & -200 & 0 & 511 & -110 & -116 \\
200 & 0 & 0 & -110 & 505 & 115 \\
0 & 0 & 0 & -116 & 115 & 116
\end{array}\right]$$

$$M^{201}_{\substack{grid\,201\\ global=29}} = \left[\begin{array}{ccc|ccc|ccc|ccc}
0 & 0 & 1 & & & & 300 & & & & & -1200 \\
1 & 0 & 0 & & 0 & & & 300 & & & & \\
0 & 1 & 0 & & & & & & 300 & 1200 & & \\
\hline
 & & & 0 & 0 & 1 & & & 1200 & 5006 & 216 & -211 \\
 & 0 & & 1 & 0 & 0 & & & & 216 & 217 & -216 \\
 & & & 0 & 1 & 0 & -1200 & & & 211 & -216 & 5011
\end{array}\right]
\left[\begin{array}{ccc|ccc}
0 & 1 & 0 & & & \\
0 & 0 & 1 & & 0 & \\
1 & 0 & 0 & & & \\
\hline
 & & & 0 & 1 & 0 \\
 & 0 & & 0 & 0 & 1 \\
 & & & 1 & 0 & 0
\end{array}\right]
= \left[\begin{array}{ccc|ccc}
300 & & & 0 & 1200 & 0 \\
 & 300 & & -1200 & 0 & 0 \\
 & & 300 & 0 & 0 & 0 \\
\hline
0 & -1200 & 0 & 5011 & -211 & -216 \\
1200 & 0 & 0 & -211 & 5006 & 216 \\
0 & 0 & 0 & -216 & 216 & 217
\end{array}\right]$$

The MGGL & MO3 terms agree w/ outputs from CONM2_PROC_2 for "RCONM2 in global coords at the grid..."

All terms agree with outputs from subr MGG_MASS_MATRIX for "In MGG_MASS_MATRIX: MGG after processing CONM2's"

Hand Calc of Principal MoI's

Solve
$$\left(I_{CG} - \lambda I\right) Q = 0$$

for PMoI's (λ's) and principal directions, Q.

First get eigenvalues:

$$\begin{vmatrix} I_{11}-\lambda & I_{21} & I_{31} \\ I_{21} & I_{22}-\lambda & I_{32} \\ I_{31} & I_{32} & I_{33}-\lambda \end{vmatrix} = 0$$

$$(I_{11}-\lambda)(I_{22}-\lambda)(I_{33}-\lambda) + I_{21}I_{32}I_{31} + I_{31}I_{32}I_{21} - \left\{(I_{22}-\lambda)I_{31}^2 + (I_{11}-\lambda)I_{32}^2 + (I_{33}-\lambda)I_{21}^2\right\} = 0$$

$$-\lambda^3 + (I_{11}+I_{22}+I_{33})\lambda^2 - (I_{11}I_{22}+I_{11}I_{33}+I_{22}I_{33})\lambda + \Big[I_{11}I_{22}I_{33} - (I_{11}-\lambda)I_{32}^2 - (I_{22}-\lambda)I_{31}^2 - (I_{33}-\lambda)I_{21}^2 + 2I_{21}I_{31}I_{32}\Big] = 0$$

$$\lambda^3 - (I_{11}+I_{22}+I_{33})\lambda^2 + (I_{11}I_{22}+I_{11}I_{33}+I_{22}I_{33} - I_{32}^2 - I_{31}^2 - I_{21}^2)\lambda - \Big(I_{11}I_{22}I_{33} - I_{11}I_{32}^2 - I_{22}I_{31}^2 - I_{33}I_{21}^2 + 2I_{21}I_{31}I_{32}\Big) = 0$$

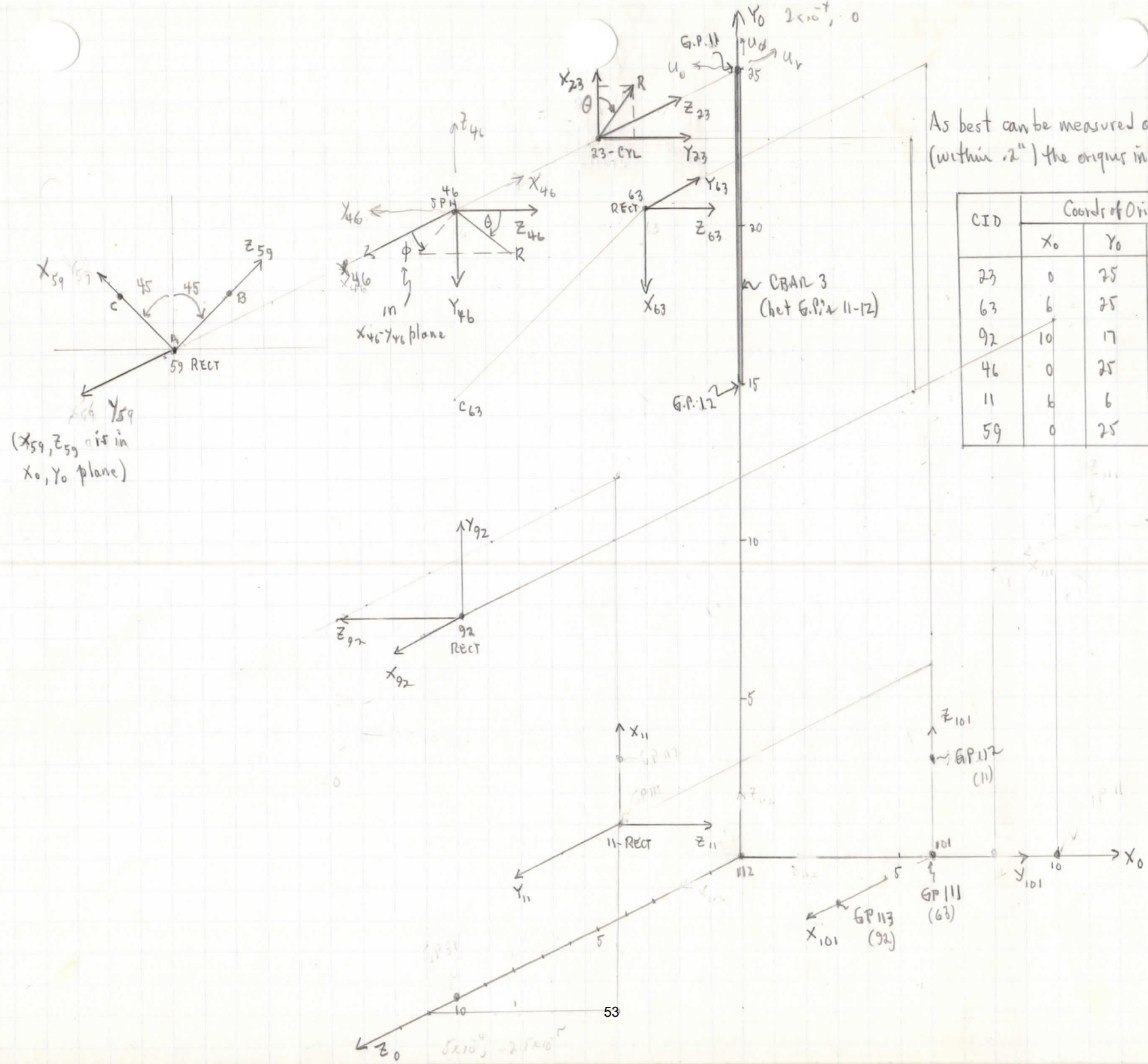
Or
$$\lambda^3 + p\lambda^2 + q\lambda + r = 0$$

where
$$p = -(I_{11}+I_{22}+I_{33})$$
$$q = (I_{11}I_{22}+I_{11}I_{33}+I_{22}I_{33} - I_{21}^2 - I_{31}^2 - I_{32}^2)$$
$$r = -(I_{11}I_{22}I_{33} - I_{11}I_{32}^2 - I_{22}I_{31}^2 - I_{33}I_{21}^2 + 2I_{21}I_{31}I_{32})$$

?

(unfinished - use output from COINMZA-PMOI given earlier)

CORD3-0



As best can be measured on this dwg (within .2") the origins in basic are

CID	Coords of Origin		
	X_0	Y_0	Z_0
23	0	25	5
63	6	25	10
92	10	17	21
46	0	25	10
11	6	6	11
59	0	25	20

CORD3.DAT

```
ID CASE,CORDTST
SOL 1
TIME 7
CEND
TITLE =  TEST COORDINATE SYSTEM ROUTINES
LOAD = 1
SPC = 1
DISP = ALL
OLOAD = ALL
SPCFORCE = ALL
ELFORCE1 = ALL
STRESS = ALL
ELDATA(1,PRINT) = ALL
ELDATA(2,PRINT) = ALL
ELDATA(3,PRINT) = ALL
ELDATA(4,PRINT) = ALL
ELDATA(5,PRINT) = ALL
BEGIN BULK
SPC1    1       123456  12
FORCE   1       11              10.     1.      2.      3.
$
GRID    11      0       0.      25.     0.      46
GRID    12      0       0.      15.     0.      46
CBAR    3       10      11      12      0.      0.      1.
PBAR    10      20      1.      2.      2.      4.
$
CORD2C  23      92      -16.    8.      10.     -18.    8.      10.     +CD3
+CD3    -16.    10.     10.
$
CORD2R  63      46      6.      0.      0.      8.      0.      0.      +CD6
+CD6    6.      90.     90.
$
CORD2R  92      11      11.     10.     4.      11.     10.     2.      +CD2
+CD2    11.     12.     4.
$
CORD2S  46      23      0.      0.      -5.     5.      90.     -5.     +CD4
+CD4    0.      0.      -10.
$
CORD2R  11      0       6.      6.      11.     7.      6.      11.     +CD1
+CD1    6.      8.      11.
$
CORD2R  59      23      0.      0.      -15.    2.      45.     -15.    +CD5
+CD5    2.      -45.    -15.
$
MAT1    20      10.+06  4.0+06          .1                              +MAT1
+MAT1   10000.
PARAM   SEQUENCE 2
PARAM   PRTBASIC 1
PARAM   PRTDOF   1
PARAM   PRTSCP   1
PARAM   PRTCORD  2
PARAM   PRTSTIFF 1      1       1       1
PARAM   PRTMASS  1      1       1       1
PARAM   PRTFOR   1      1       1       1
PARAM   PRTDISP  1      1       1       1
ENDDATA
```

<u>CORD3.DAT</u> (continued) Coordinate Transformations (by inspection)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{T_{0,11}} \checkmark \begin{Bmatrix} u_{11} \\ v_{11} \\ w_{11} \end{Bmatrix}$$

$$\begin{Bmatrix} u_{11} \\ v_{11} \\ w_{11} \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{T_{11,92}} \checkmark \begin{Bmatrix} u_{92} \\ v_{92} \\ w_{92} \end{Bmatrix}$$

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{T_{0,92}} \checkmark \begin{Bmatrix} u_{92} \\ v_{92} \\ w_{92} \end{Bmatrix}$$

$$\begin{Bmatrix} u_{92} \\ v_{92} \\ w_{92} \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}}_{T_{92,23}} \checkmark \begin{Bmatrix} u_{23} \\ v_{23} \\ w_{23} \end{Bmatrix}$$

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{T_{0,23}} \checkmark \begin{Bmatrix} u_{23} \\ v_{23} \\ w_{23} \end{Bmatrix}$$

$$\begin{Bmatrix} u_{23} \\ v_{23} \\ w_{23} \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}}_{T_{23,46}} \checkmark \begin{Bmatrix} u_{46} \\ v_{46} \\ w_{46} \end{Bmatrix}$$

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{T_{0,46}} \checkmark \begin{Bmatrix} u_{46} \\ v_{46} \\ w_{46} \end{Bmatrix}$$

$$\begin{Bmatrix} u_{23} \\ v_{23} \\ w_{23} \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & -1 & 0 \end{bmatrix}}_{T_{23,59}} \checkmark \begin{Bmatrix} u_{59} \\ v_{59} \\ w_{59} \end{Bmatrix}$$

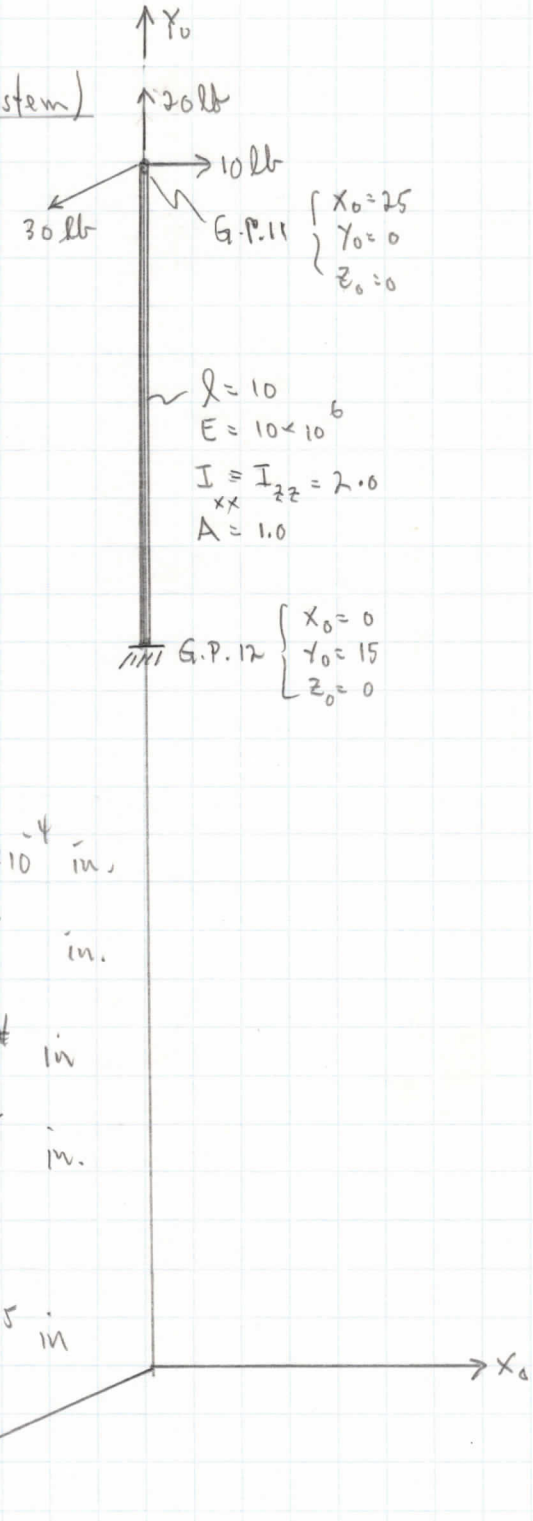
$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{bmatrix}}_{T_{0,59}} \checkmark \begin{Bmatrix} u_{59} \\ v_{59} \\ w_{59} \end{Bmatrix}$$

$$\begin{Bmatrix} u_{46} \\ v_{46} \\ w_{46} \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_{46,63}} \checkmark \begin{Bmatrix} u_{63} \\ v_{63} \\ w_{63} \end{Bmatrix}$$

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \end{Bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}}_{T_{0,63}} \checkmark \begin{Bmatrix} u_{63} \\ v_{63} \\ w_{63} \end{Bmatrix}$$

✓ : Agrees w/ MYSTRAN

Deflections of Beam (Using basic as global coord. system)



The lateral deflection and slope are:

$$w = \frac{Pl^3}{3EI} = \frac{P}{3}\frac{(10^3)}{(10\times10^6)(2)} = \frac{P}{6}\times10^{-4}$$

$$\theta = \pm\frac{Pl^2}{2EI} = \pm\frac{P}{2}\frac{(10^2)}{(10\times10^6)(2)} = \pm\frac{P}{4}\times10^{-5}$$

The longitudinal displacement u

$$u = \frac{Pl}{AE} = P\frac{(10)}{(1)(10\times10^6)} = 10^{-6}P$$

$\therefore$ $T1_0 = \frac{P}{6}\times10^{-4}$ with $P = 10^{\#}$; $T1_0 = 1.66667\times10^{-4}$ in.

$T2_0 = 10^{-6}P$ with $P = 20^{\#}$; $T2_0 = 2.0\times10^{-5}$ in.

$T3_0 = \frac{P}{6}\times10^{-4}$ with $P = 30^{\#}$; $T3_0 = 5.0\times10^{-4}$ in

$R1_0 = \frac{P}{4}\times10^{-4}$ with $P = 30^{\#}$; $R1_0 = 7.5\times10^{-5}$ in.

$R2_0 = 0$ (no torsional moment)

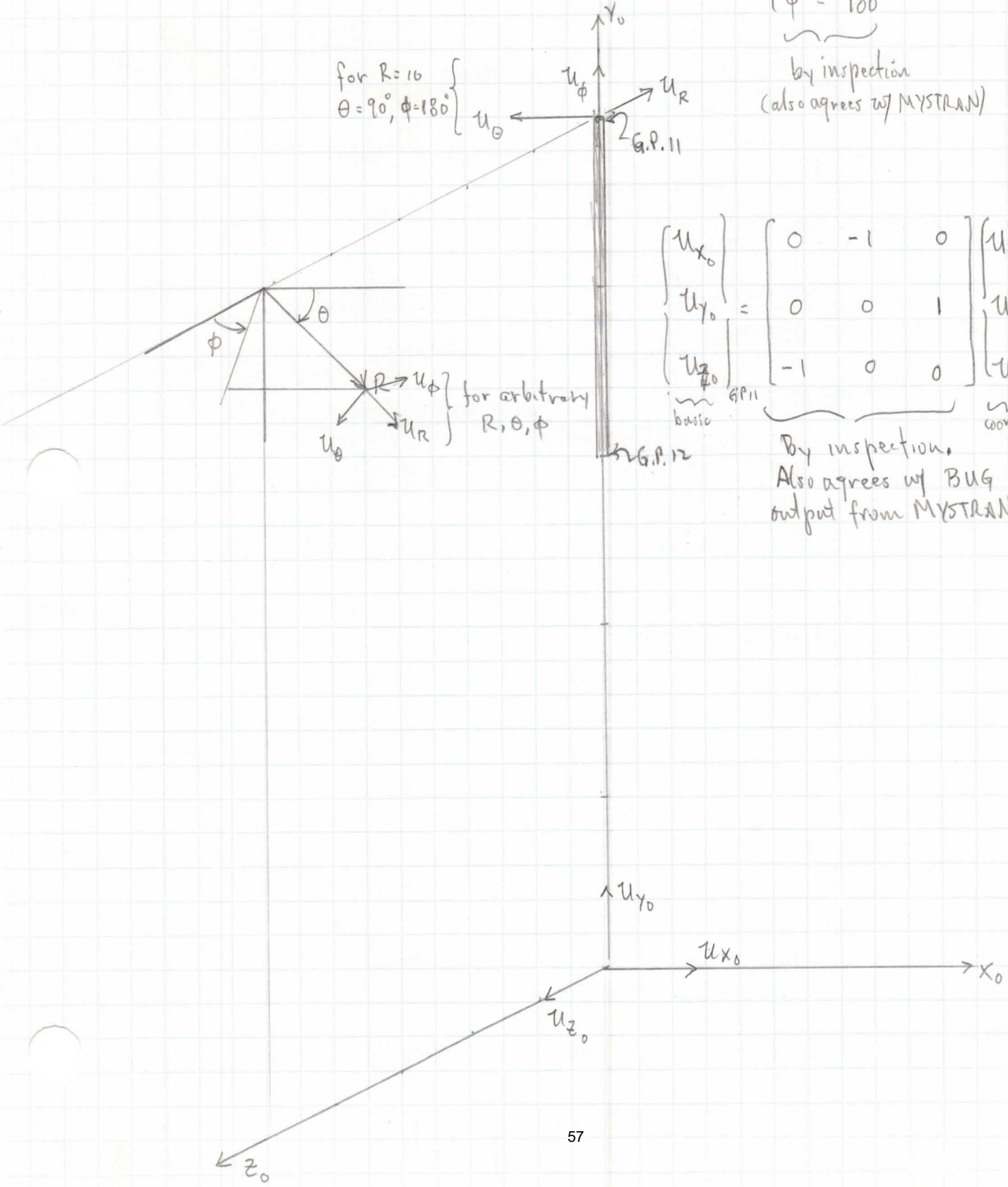
$R3_0 = -\frac{P}{4}\times10^{-4}$ with $P = 10^{\#}$; $R3_0 = -2.5\times10^{-5}$ in

Check Final Global-To-Basic Transformation

Grid 11: Global is spherical system 46

Coords of G.P. 11 Relative to Origin of Coord 46: $\begin{cases} R = 10 \\ \theta = 90^\circ \\ \phi = 180^\circ \end{cases}$

by inspection (also agrees w/ MYSTRAN)



$$\underbrace{\begin{bmatrix} u_{X_0} \\ u_{Y_0} \\ u_{Z_0} \end{bmatrix}_{GP11}}_{\text{basic}} = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}}_{} \underbrace{\begin{bmatrix} u_R \\ u_\theta \\ u_\phi \end{bmatrix}_{GP11}}_{\text{coord sys 46}}$$

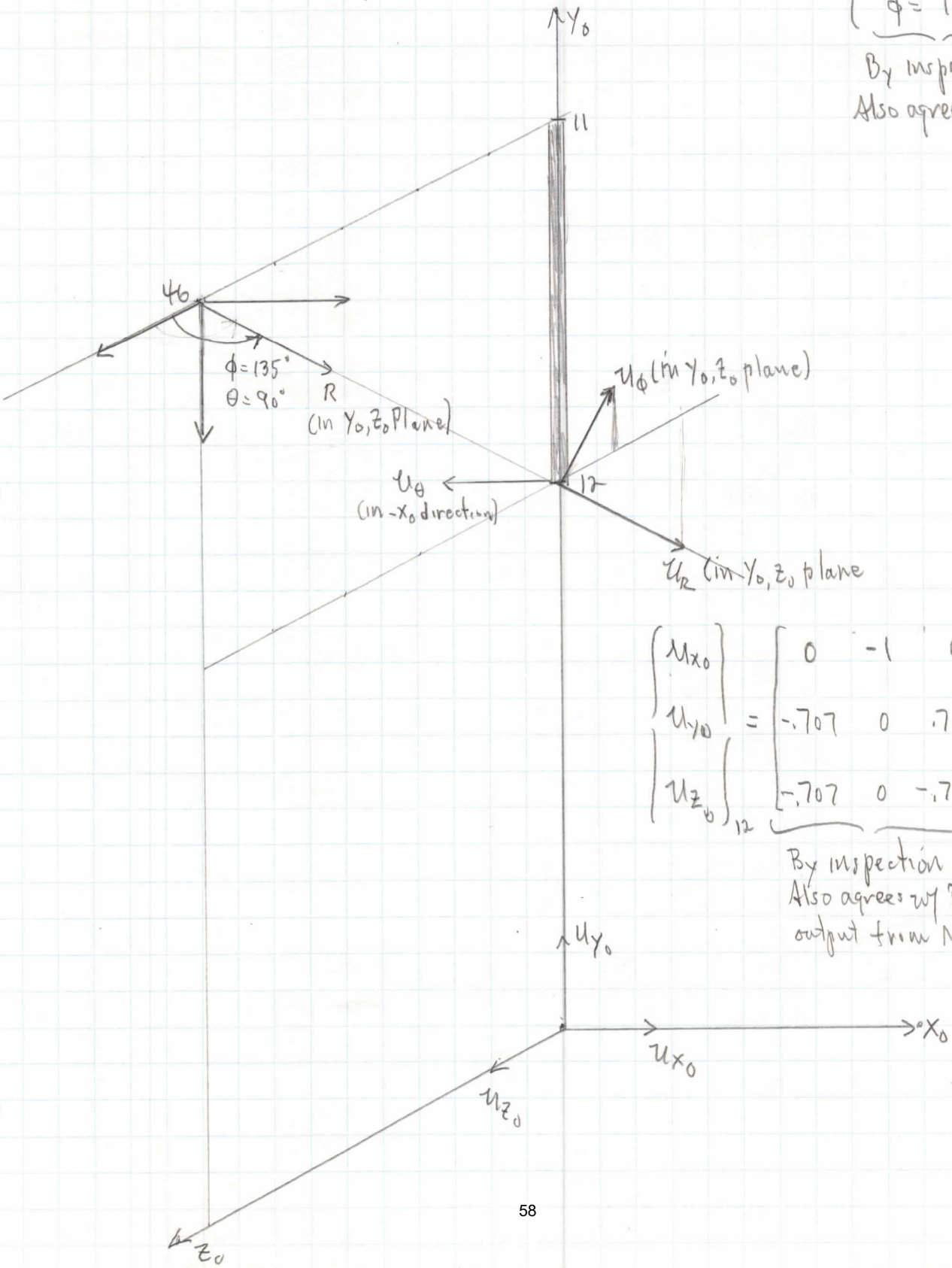
By inspection. Also agrees w/ BUG output from MYSTRAN

2) Grid 12: Global is spherical system 46

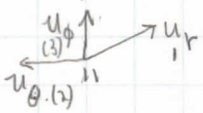
Coords of GP 12 Relative to Origin of Coord 46 is:

$$\begin{cases} R = 10\sqrt{2} \\ \theta = 90^\circ \\ \phi = 135^\circ \end{cases}$$

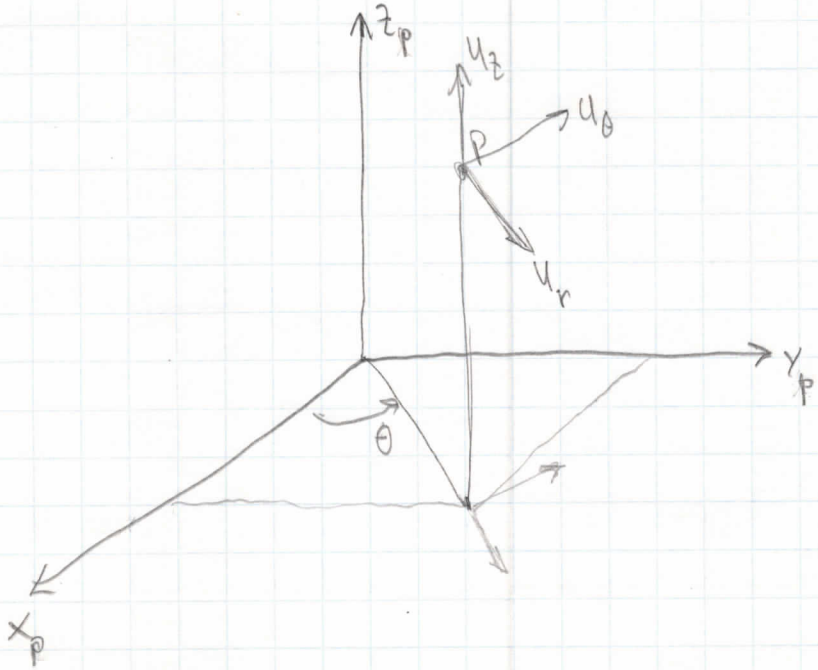
By inspection.
Also agrees w/ MYSTRAN



$$\begin{Bmatrix} u_{x_0} \\ u_{y_0} \\ u_{z_0} \end{Bmatrix}_{12} = \underbrace{\begin{bmatrix} 0 & -1 & 0 \\ -.707 & 0 & .707 \\ -.707 & 0 & -.707 \end{bmatrix}}_{\text{By inspection. Also agrees w/ BUG output from MYSTRAN}} \begin{Bmatrix} u_R \\ u_\theta \\ u_\phi \end{Bmatrix}_{12}$$

G.P. 11 DOF	G.P. 11 Displacements For Various Choices of Global Coord. System						
	0 (Basic)	23 (Cyl.)	63 (Rect.)	92 (Rect)	46 (Sph.)	11 (Rect)	59 (Rect)
T1	1.666667×10^{-4}	2.0×10^{-5}	-2.0×10^{-5}	5×10^{-4}	-5.0×10^{-4}	2.0×10^{-5}	-1.037090×10^{-4}
T2	2.0×10^{-5}	1.6667×10^{-4}	-5.0×10^{-4}	2×10^{-5}	-1.666667×10^{-4}	5.0×10^{-4}	5.0×10^{-4}
T3	5.0×10^{-4}	-5.0×10^{-4}	1.66667×10^{-4}	-1.666667×10^{-4}	2.0×10^{-5}	1.666667×10^{-4}	1.319933×10^{-4}
R1	7.5×10^{-5}	0	0	-2.5×10^{-5}	2.5×10^{-5}	0	-5.303301×10^{-5}
R2	0	7.5×10^{-5}	2.5×10^{-5}	0	-7.5×10^{-5}	-2.5×10^{-5}	-2.5×10^{-5}
R3	-2.5×10^{-5}	2.5×10^{-5}	7.5×10^{-5}	-7.5×10^{-5}	0	7.5×10^{-5}	5.303301×10^{-5}
	See analysis of beam on previous page ✓ agrees w/ MYSTRAN	G.P. 11 is at $R=0, \theta=0$ so use prime. direction of coord sys. #23 $T1_{23}=T2_0$ $T2_{23}=T1_0$ $T3_{23}=-T3_0$ $R1_{23}=R2_0$ $R2_{23}=R1_0$ $R3_{23}=-R3_0$ ✓ agrees w/ MYSTRAN	$T1_{63}=-T2_0$ $T2_{63}=-T3_0$ $T3_{63}=T1_0$ $R1_{63}=-R2_0$ $R2_{63}=-R3_0$ $R3_{63}=R1_0$ ✓ agree w/ MYSTRAN	$T1_{92}=T3_0$ $T2_{92}=T2_0$ $T3_{92}=-T1_0$ $R1_{92}=R3_0$ $R2_{92}=R2_0$ $R3_{92}=-R1_0$ ✓ agree w/ MYSTRAN	G.P. 11 is at $R=10, \theta=90, \phi=180$  so $T1_{46}=-T3_0$ $T2_{46}=-T1_0$ $T3_{46}=T2_0$ $R1_{46}=-R3_0$ $R2_{46}=-R1_0$ $R3_{46}=R2_0$ ✓ agree w/ MYSTRAN	$T1_{11}=T2_0$ $T2_{11}=T3_0$ $T3_{11}=T1_0$ $R1_{11}=R2_0$ $R2_{11}=R3_0$ $R3_{11}=R1_0$ ✓ agree w/ MYSTRAN	$T1_{59}=\frac{\sqrt{2}}{2}(-T1_0+T2_0)$ $T2_{59}=T3_0$ $T3_{59}=\frac{\sqrt{2}}{2}(T1_0+T2_0)$ $R1_{59}=\frac{\sqrt{2}}{2}(-R1_0+R2_0)$ $R2_{59}=R3_0$ $R3_{59}=\frac{\sqrt{2}}{2}(R1_0+R2_0)$ ✓ agree w/ MYSTRAN
	CORD3A-0.F06	CORD3A-23.F06	CORD3A-63.F06	CORD3A-92.F06	CORD3A-46.F06	CORD3A-11.F06	CORD3A-59.F06

Cylindrical Coord. System:

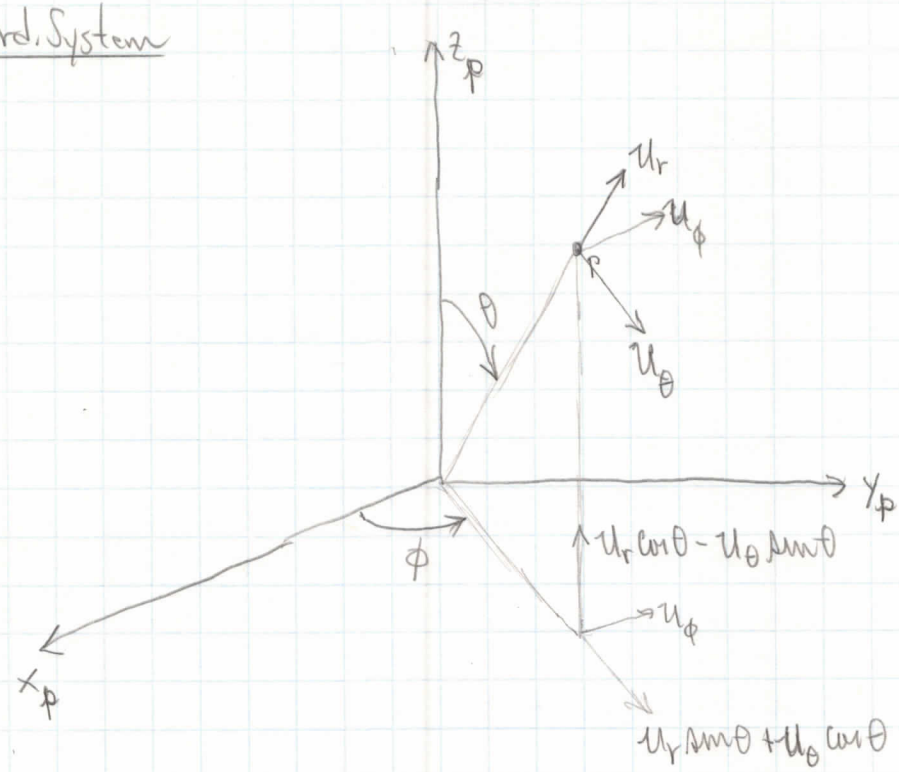


x_p, y_p, z_p: Principal Directions of the Cylindrical System

$$\begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_{x_p} \\ u_{y_p} \\ u_{z_p} \end{Bmatrix}$$

$$\begin{Bmatrix} u_{x_p} \\ u_{y_p} \\ u_{z_p} \end{Bmatrix} = \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Matrix TPL in Subroutine GEN_TOL}} \begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix}$$

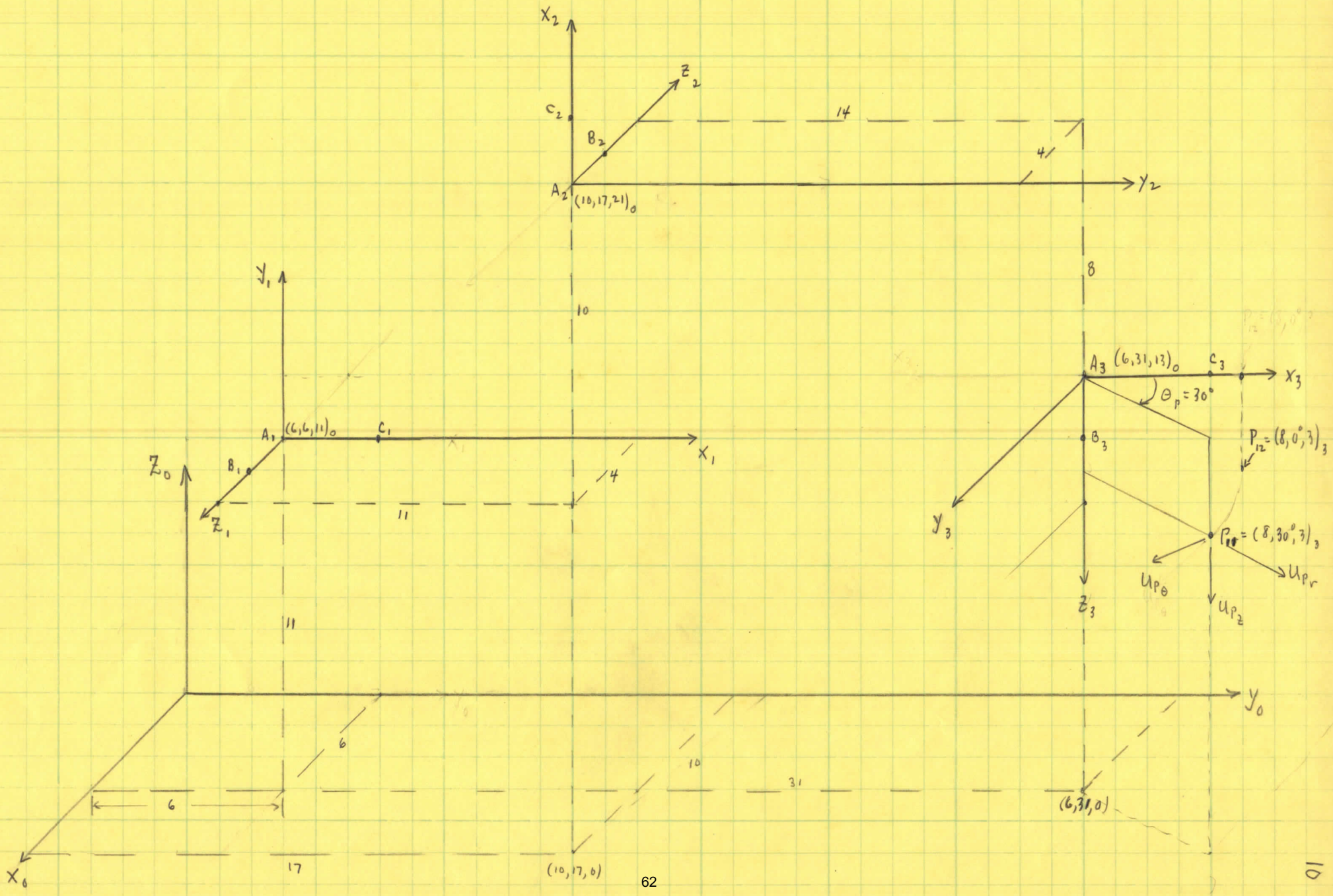
Spherical Coord. System



x_p, y_p, z_p: Principal Directions of the Spherical Coord. Sys.

$$\begin{Bmatrix} u_{x_p} \\ u_{y_p} \\ u_{z_p} \end{Bmatrix} = \underbrace{\begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}}_{\text{Matrix TPL in subroutine GEN_TOL}} \begin{Bmatrix} u_r \\ u_\theta \\ u_\phi \end{Bmatrix}$$

CORDI.DAT

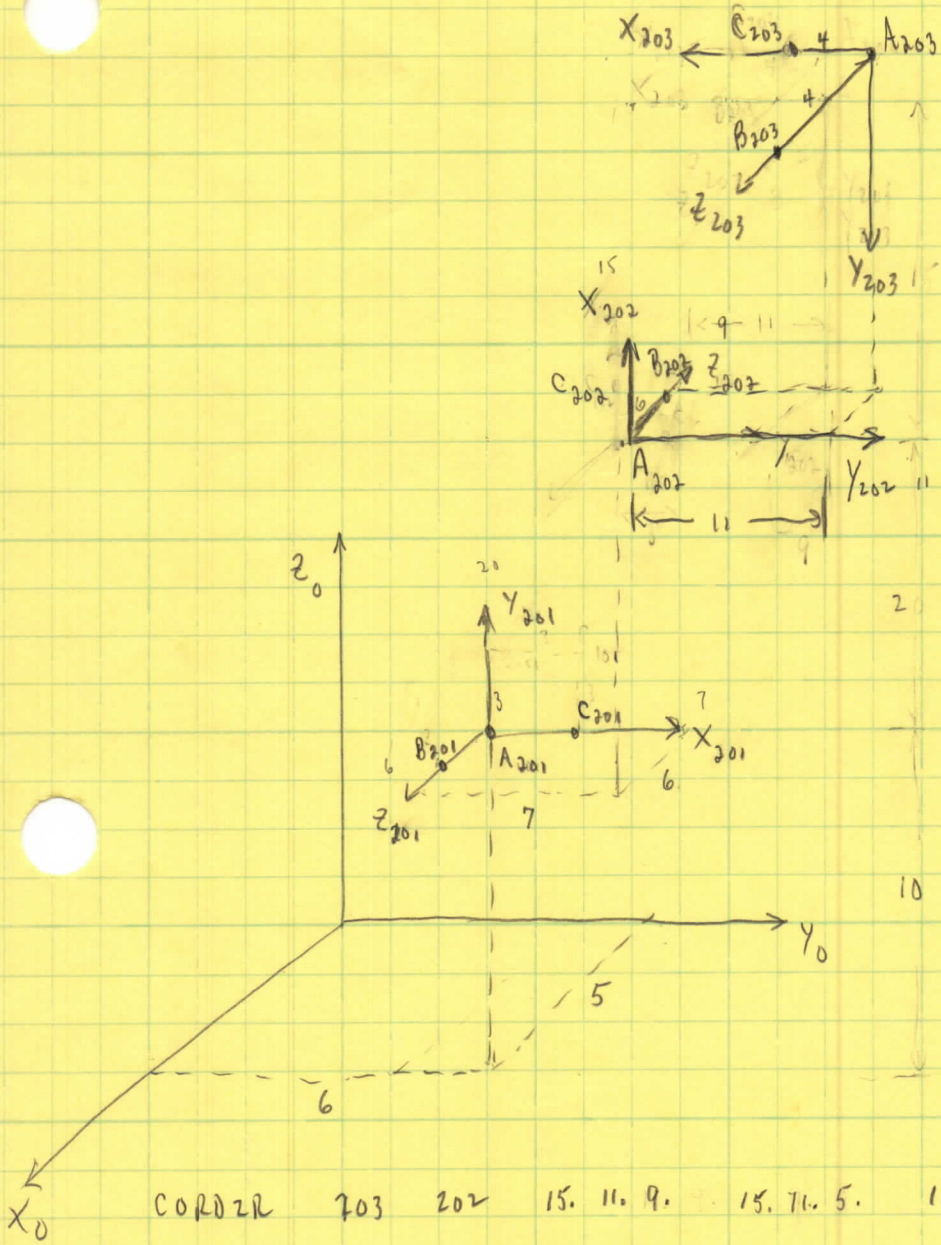

X_2
Z_2
C_2
B_2
A_2
$(10,17,21)_0$
14
4
Y_2
8
10
Y_1
A_1
$(6,6,11)_0$
C_1
X_1
B_1
Z_1
11
4
Z_0
11
Y_0
6
10
31
6
$(6,31,0)$
X_0
17
$(10,17,0)$
A_3
$(6,31,13)_0$
C_3
X_3
$\theta_p = 30°$
B_3
$P_{12} = (8,0°,3)_3$
Y_3
$P_{12} = (8,30°,3)_3$
u_{P_θ}
u_{P_r}
Z_3
u_{P_z}

CORDZR	1	0	6. 6. 11	7. 6. 11.	6. 8. 11.
CORDZR	2	1	11. 10. 4.	11. 10. 2.	13. 10. 4
CORDZC	3	2	-8 14. 4.	-10. 14. 4	-8. 16. 4.

The coords of all origins (in basic) are shown on previous figure.

The coords of P_{11}.... (in basic) are

$$(\underset{\sim}{P_{11}})_0 = (\underset{\sim}{A_3})_0 + \begin{bmatrix} 8\sin 30^\circ & (\Delta X_0) \\ 8\cos 30 & (\Delta Y_0) \\ -3 & (\Delta Z_0) \end{bmatrix} = \begin{bmatrix} 6 \\ 31 \\ 13 \end{bmatrix} + \begin{bmatrix} 4 \\ 6.9282032 \\ -3 \end{bmatrix} = \begin{bmatrix} 10 \\ 37.9282032 \\ 10 \end{bmatrix}$$



	X_0	Y_0	Z_0
A_{201}	5.	6.	10.
	+6	+7	+20
A_{202}	11	13	30
	-9	+11	+15
A_{203}	2	24	45

CORDZR 203 202 15. 11. 9. 15. 11. 5. 15. 7. 9.

CORDZR 202 201 7. 20 6. 7. 20. 0 7 23. 6.

CORDZR 201 0 5. 6. 10. 8 6 10 5. 9. 10.

$$\underset{\sim}{u}_2 = \underset{\sim}{T}_{2-3}\,\underset{\sim}{u}_3$$

$$\therefore \underset{\sim}{T}_{2-3} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

transfer from 203 to 202

$$\underset{\sim}{u}_1 = \underset{\sim}{T}_{1-2}\,\underset{\sim}{u}_2$$

$$\therefore T_{1-2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\underset{\sim}{u}_0 = \underset{\sim}{T}_{0-1}\,\underset{\sim}{u}_1$$

$$\therefore \underset{\sim}{T}_{0-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Then

$$\underset{\sim}{T}_{0,2} = T_{0-1}T_{1-2} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \checkmark$$

$$T_{0-3} = \underbrace{T_{0-1}\,T_{1-2}}_{T_{0-2}}\,T_{2-3} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$\checkmark$

The coords of pt A_3 in the 0 system are calculated in steps:

First: Coords of A_3 in the #2 system $\equiv \underset{\sim}{A}_{3,2} = \begin{Bmatrix} 15 \\ 11 \\ 9 \end{Bmatrix}$

Second: Coords of A_3 in the #1 system are $A_{3,1}$

$$A_{3,1} = A_{2,1} + T_{1-2} A_{3,2}$$

$$= \begin{Bmatrix} 7 \\ 20 \\ 6 \end{Bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} 15 \\ 11 \\ 9 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 20 \\ 6 \end{Bmatrix} + \begin{Bmatrix} 11 \\ 15 \\ -9 \end{Bmatrix} = \begin{Bmatrix} 18 \\ 35 \\ -3 \end{Bmatrix}$$

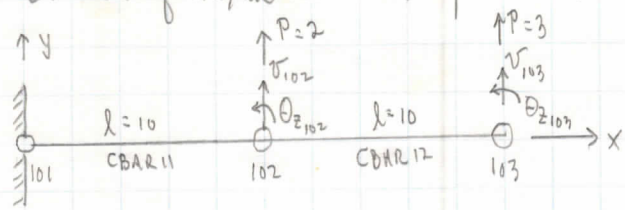
Third: Coords of A_3 in the 0 system are $A_{3,0}$

$$A_{3,0} = A_{1,0} + T_{0-1} A_{3,1}$$

$$= \begin{Bmatrix} 5 \\ 6 \\ 10 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} 18 \\ 35 \\ -3 \end{Bmatrix} = \begin{Bmatrix} 5 \\ 6 \\ 10 \end{Bmatrix} + \begin{Bmatrix} -3 \\ 18 \\ 35 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 24 \\ 45 \end{Bmatrix}$$

GUYAN-01

GUYAN-01 2 Cell beam checkout of Guyan reduction algorithm (2 ASET DOF's)

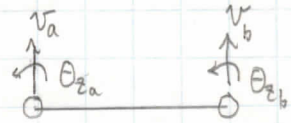


(Grids ordered 103, 101, 102
DOF's 103-T2 and 103 R3 are A-set

$E = 10\times10^6$ (both CBAR's)
$I = 0.1$ (" ")

The free-free stiffness matrix for each beam is: (bending in 1 plane)

$$K_{ee} = \begin{array}{c} \begin{array}{cccc} v_a & \theta_{z_a} & v_b & \theta_{z_b} \end{array} \\ \begin{bmatrix} 12EI/l^3 & 6EI/l^2 & -12EI/l^3 & 6EI/l^2 \\ 6EI/l^2 & 4EI/l & -6EI/l^2 & 2EI/l \\ -12EI/l^3 & -6EI/l^2 & 12EI/l^3 & -6EI/l^2 \\ 6EI/l^2 & 2EI/l & -6EI/l^2 & 4EI/l \end{bmatrix} \end{array} \begin{array}{c} v_a \\ \theta_{z_a} \\ v_b \\ \theta_{z_b} \end{array}$$



Using E, I, l above:

$$K_{ee} = \begin{bmatrix} 12000 & 60000 & -12000 & 60000 \\ 60000 & 400000 & -60000 & 200000 \\ -12000 & -60000 & 12000 & -60000 \\ 60000 & 200000 & -60000 & 400000 \end{bmatrix}$$

for CBAR 11 & 12

The K_{ff} stiffness matrix is

$$K_{ff} = \begin{array}{c} \begin{array}{cccc} 102\text{-}T2 & 102\text{-}R3 & 103\text{-}T2 & 103\ R3 \end{array} \\ \begin{bmatrix} 24000 & 0 & -12000 & 60000 \\ 0 & 800000 & -60000 & 200000 \\ -12000 & -60000 & 12000 & -60000 \\ 60000 & 200000 & -60000 & 400000. \end{bmatrix} \end{array} \begin{array}{l} 102\text{-}T2 \\ 102\text{-}R3 \\ 103\text{-}T2 \\ 103\text{-}R3 \end{array} \quad P_f = \begin{array}{c} \begin{Bmatrix} 2. \\ 0 \\ 3. \\ 0 \end{Bmatrix} \end{array} \begin{array}{l} 102\text{-}T2 \\ 102\text{-}R3 \\ 103\text{-}T2 \\ 103\text{-}R3 \end{array}$$

Rearrange according to the grid order run in GUYAN-01.F06:

$$K_{ff} = \begin{array}{c} \begin{array}{cccc} 103\text{-}T2 & 103\text{-}R3 & 102\text{-}T2 & 102\text{-}R3 \end{array} \\ \begin{bmatrix} 12000 & -60000 & -12000 & -60000 \\ -60000 & 400000 & 60000 & 200000 \\ -12000 & 60000 & 24000 & 0 \\ -60000 & 200000 & 0 & 800000 \end{bmatrix} \end{array} \begin{array}{l} 103\text{-}T2 \\ 103\text{-}R2 \\ 102\text{-}T2 \\ 102\text{-}R3 \end{array} , \quad P_f = \begin{Bmatrix} 3. \\ 0. \\ 2. \\ 0. \end{Bmatrix} \begin{array}{l} 103\text{-}T2 \\ 103\text{-}R3 \\ 102\text{-}T2 \\ 102\text{-}R3 \end{array}$$

$$K_{ff} = \begin{bmatrix} \bar{K}_{aa} & K_{ao} \\ K_{ao}^T & K_{oo} \end{bmatrix}, \quad P_f = \begin{Bmatrix} \bar{P}_a \\ P_o \end{Bmatrix}, \quad u_f = \begin{Bmatrix} u_a \\ u_o \end{Bmatrix}$$

Solve for u_o then u_a

$$u_o = K_{oo}^{-1} P_o - K_{oo}^{-1} K_{ao}^T u_a \equiv u_o^0 + G_{oa} u_a$$

where $\quad u_o^0 = K_{oo}^{-1} P_o$

and $\quad G_{oa} = -K_{oo}^{-1} K_{ao}^T$

$$K_{aa} u_a = P_a \quad \begin{cases} K_{aa} = \bar{K}_{aa} + K_{ao} G_{oa} \\ P_a = \bar{P}_a + G_{oa}^T P_o \end{cases}$$

By comparison:

$$\bar{K}_{aa} = \begin{bmatrix} 12000 & -60000 \\ -60000 & 400000 \end{bmatrix}, \quad \bar{P}_a = \begin{Bmatrix} 3 \\ 0 \end{Bmatrix}$$

$$K_{ao} = \begin{bmatrix} -12000 & -60000 \\ 60000 & 200000 \end{bmatrix}$$

$$K_{oo} = \begin{bmatrix} 24000 & 0 \\ 0 & 800000 \end{bmatrix}, \quad P_o = \begin{Bmatrix} 2 \\ 0 \end{Bmatrix}$$

Invert K_{oo}:

$$K_{oo}^{-1} = 10^{-6} \begin{bmatrix} 41.666667 & 0 \\ 0 & 1.25 \end{bmatrix}, \quad u_{o_o} = K_{oo}^{-1} P_o = 10^{-5} \begin{Bmatrix} 8.333333 \\ 0 \end{Bmatrix}$$

Then

$$G_{oa} = -K_{oo}^{-1} K_{ao}^T = -10^{-6} \begin{bmatrix} 41.666667 & 0 \\ 0 & 1.25 \end{bmatrix} \begin{bmatrix} -12000 & 60000 \\ -60000 & 200000 \end{bmatrix}$$

$$G_{oa} = \begin{bmatrix} .5 & -2.5 \\ .075 & -.25 \end{bmatrix}$$ QED (from GUYAN-01.506)

$$K_{ao} G_{oa} = \begin{bmatrix} -12000 & -60000 \\ 60000 & 200000 \end{bmatrix} \begin{bmatrix} .5 & -2.5 \\ .075 & -.25 \end{bmatrix} = \begin{bmatrix} -10500 & 45000 \\ 45000 & -200000 \end{bmatrix}$$

$$G_{oa}^T P_o = \begin{bmatrix} .5 & .075 \\ -2.5 & -.25 \end{bmatrix} \begin{Bmatrix} 2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -5 \end{Bmatrix}$$

Then

$$K_{aa} = \bar{K}_{aa} + K_{ao}G_{oa} = \begin{bmatrix} 12000 & -60000 \\ -60000 & 400000 \end{bmatrix} + \begin{bmatrix} -10500 & 45000 \\ 45000 & -200000 \end{bmatrix}$$

$$\therefore K_{aa} = \begin{bmatrix} 1500 & -15000 \\ -15000 & 200000 \end{bmatrix}$$ QED from GUYAN-01.F06

and

$$P_a = \bar{P}_a + G_{oa}^T P_o = \begin{Bmatrix} 3 \\ 0 \end{Bmatrix} + \begin{bmatrix} .5 & .075 \\ -2.5 & -.25 \end{bmatrix} \begin{Bmatrix} 2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 1 \\ -5 \end{Bmatrix}$$

$$\therefore P_a = \begin{Bmatrix} 4 \\ -5 \end{Bmatrix}$$ QED from GUYAN-01.F06

$$u_a = K_{aa}^{-1} P_a$$

$$K_{aa}^{-1} = \frac{1}{\underbrace{(1500)(200000)}_{3\times10^8} - \underbrace{(15000)^2}_{2.25\times10^8}} \begin{bmatrix} 200000 & 15000 \\ 15000 & 1500 \end{bmatrix} = 10^{-3} \begin{bmatrix} 2.666667 & .2000000 \\ .2000000 & .0200000 \end{bmatrix}$$

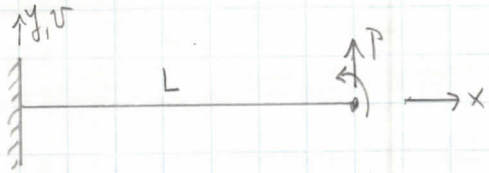
$$u_a = 10^{-3} \begin{bmatrix} 2.666667 & .2000000 \\ .2000000 & .0200000 \end{bmatrix} \begin{Bmatrix} 4 \\ -5 \end{Bmatrix} = \begin{Bmatrix} 9.666667 \times 10^{-3} \\ 7.000000 \times 10^{-4} \end{Bmatrix} \begin{matrix} 103\text{-}T2 \\ 103\text{-}R3 \end{matrix}$$ agree w/ MYSTRAN

$$u_o = u_o^o + G_{oa} u_a = \begin{Bmatrix} 8.333333 \times 10^{-5} \\ 0 \end{Bmatrix} + \begin{bmatrix} .5 & -2.5 \\ .075 & -.25 \end{bmatrix} \begin{Bmatrix} 9.666667 \times 10^{-3} \\ 7.000000 \times 10^{-4} \end{Bmatrix}$$

$$\therefore u_o = \begin{Bmatrix} 8.333333 \times 10^{-5} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 3.083333 \times 10^{-3} \\ 5.500000 \times 10^{-3} \end{Bmatrix} = \begin{Bmatrix} 3.166667 \times 10^{-3} \\ 5.500000 \times 10^{-4} \end{Bmatrix} \begin{matrix} 102\text{-}T2 \\ 102\text{-}R3 \end{matrix}$$ agree w/ MYSTRAN

Independent Solution

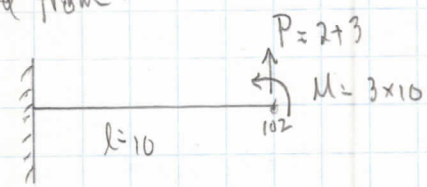
Calculate theoretical displacements of beam from beam theory:



at tip $v = \dfrac{Pl^3}{3EI} + \dfrac{Ml^2}{2EI}$

$$\theta_z = \frac{Pl^2}{2EI} + \frac{Ml}{EI}$$

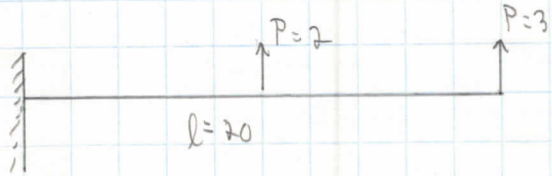
Dspl at G.P. 102 is found from:



$$\therefore v_{102} = \frac{(5)(10^3)}{(3)(10^7)(.1)} + \frac{(30)(10^2)}{(2)(10^7)(.1)} = 10^{-3}\left(\frac{5}{3} + 1.5\right) = 3.1666667\times10^{-3} \text{ QED}$$

$$\theta_{z_{102}} = \frac{(5)(10^2)}{(2)(10^7)(.1)} + \frac{(30)(10)}{(10^7)(.1)} = 10^{-4}(2.5 + 3) = 5.5\times10^{-4} \text{ QED}$$

Dspl at G.P. 103 is found from



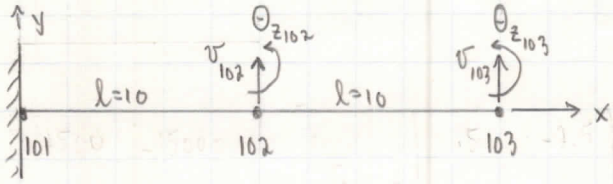
$$v_{103} = v_{102} + 10\theta_{z_{102}} + \left(\frac{Pl^3}{3EI}\right)_{\substack{P=3\\ l=10}} = 3.1666667\times10^{-3} + 5.5\times10^{-3} + \frac{(3)(1000)}{3(10^7)(.1)} = 9.6666667\times10^{-3} \text{ QED}$$

$$\theta_{z_{103}} = \theta_{z_{102}} + \left(\frac{Pl^2}{2EI}\right)_{\substack{P=3\\ l=10}} = 5.5\times10^{-4} + \frac{(3)(100)}{2(10^7)(.1)} = 7.0\times10^{-4} \text{ QED}$$

GUYAN-03

GUYAN-03: Eigenvalues of 2-DOF A-Set Beam from GUYAN-01

(GIV, MGIV, INV)



$$\text{A-set} = \begin{bmatrix} v_{103} \\ \theta_{z103} \end{bmatrix}, \quad \text{F-set} = \begin{bmatrix} v_{102} \\ \theta_{z102} \\ v_{103} \\ \theta_{z103} \end{bmatrix}$$

$E = 10 \times 10^6$, $\gamma = 0.1$ (wgt density on MAT1 card) } both Bars

$A = 0.01$, $I = 0.1$

There are no CONM2's so mass matrix is due to elem. mass only, Each BAR mass

$$m_e = A \times \gamma \times l = (.01)(.1)(10) = .01 \text{ lb}$$

and is split evenly between the 2 connecting grids. Therefore,

$$M_{ff} = \frac{1}{386} \begin{array}{c} \begin{array}{cccc} 103\text{-}T2 & 103\text{-}R3 & 102\text{-}T2 & 102\text{-}R3 \end{array} \\ \begin{bmatrix} 0.005 & & & \\ & 0 & & \\ & & 0.01 & \\ & & & 0 \end{bmatrix} \end{array} \begin{array}{l} 103\text{-}T2 \\ 103\text{-}R3 \\ 102\text{-}T2 \\ 102\text{-}R3 \end{array} = \begin{bmatrix} \bar{M}_{aa} & M_{ao} \\ M_{ao}^T & M_{oo} \end{bmatrix}$$

where

$$\bar{M}_{aa} = \frac{1}{386}\begin{bmatrix} 0.005 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_{oo} = \frac{1}{386}\begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_{ao} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and the reduced A-set mass matrix is:

$$M_{aa} = \bar{M}_{aa} + M_{ao}G_{oa} + (M_{ao}G_{oa})^T + G_{oa}^T M_{oo} G_{oa}$$

where

$$G_{oa} = \begin{bmatrix} .5 & -2.5 \\ .075 & -.25 \end{bmatrix} \quad \text{(see GUYAN-01 problem)}$$

$$G_{oa}^T M_{oo} G_{oa} = \frac{1}{386}\begin{bmatrix} .5 & .075 \\ -2.5 & -.25 \end{bmatrix}\begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} .5 & -2.5 \\ .075 & -.25 \end{bmatrix} = \frac{1}{386}\begin{bmatrix} .0025 & -.0125 \\ -.0125 & .0625 \end{bmatrix}$$

$$\therefore M_{aa} = \frac{1}{g}\begin{bmatrix} .0075 & -.0125 \\ -.0125 & .0625 \end{bmatrix}, \quad \frac{1}{g} = .002591$$

$$K_{aa} = \begin{bmatrix} 1500 & -15000 \\ -15000 & 200000 \end{bmatrix}$$ See GUYAN-01 problem)

So the eigenvalue problem is

$$(K_{aa} - \lambda M_{aa}) u_a = 0$$

which requires

$$|K_{aa} - \lambda M_{aa}| = 0$$

or

$$\begin{vmatrix} 1500g - .0075\lambda & -15000g + .0125\lambda \\ -15000g + .0125\lambda & 200000g - .0625\lambda \end{vmatrix} = 0$$

Expand

$$[1500g - .0075\lambda][200000g - .0625\lambda] - [-15000g + .0125\lambda]^2 = 0$$

Or

$$3\times10^8 g^2 - (1500 + 93.75)g\lambda + .00046875\lambda^2 - 2.25\times10^8 g^2 + 375\lambda - .00015625\lambda^2 = 0$$

$$.0003125\lambda^2 - 1218.75g\lambda + .75\times10^8 g^2 = 0$$

Divide by .000625

$$\lambda^2 - 3.9\times10^6 g\lambda + 0.24\times10^{12} g^2 = 0$$

or

$$\lambda^2 + b\lambda + c = 0 \quad \begin{cases} b = -3.9\times10^6 g \\ c = 0.24\times10^{12} g^2 \end{cases}$$

$$\lambda_{1,2} = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c} \qquad f_{1,2} = \frac{1}{2\pi}\sqrt{-\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}}$$

$$\lambda_{1,2} = 1.95\times10^{6}g \pm \sqrt{(1.95\times10^{6}g)^2 - .24\times10^{12}g^2}$$

$$= \left[1.955\times10^{6} \pm \sqrt{3.8025\times10^{12} - .24\times10^{12}}\right]g$$

$$= \left[1.95\times10^{6} \pm \sqrt{3.5625\times10^{12}}\right]g$$

$$= (1.95 \pm 1.887459860881769$$

$$\lambda_{1,2} = \begin{Bmatrix} 6.254139 & \times10^{4}g \\ 3.837459 & \times10^{6}g \end{Bmatrix} \quad f_{1,2} = \frac{1}{2\pi}\sqrt{\lambda_{1,2}} = \begin{Bmatrix} 39.80191 \\ 311.7758 \end{Bmatrix}$$ exact in MYSTRAN

The eigenvectors are found from

$$(K - \lambda_i M)\phi^i = 0$$

$$\begin{bmatrix} 1500 - .0075\lambda_i & -15000 + .0125\lambda_i \\ -15000 + .0125\lambda_i & 200000 - .0625\lambda_i \end{bmatrix} \begin{Bmatrix} \phi_{1i} \\ \phi_{2i} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Let $\phi_{1i} = 1.0$

$$(1500 - .0075\lambda_i) + (-15000 + .0125\lambda_i)\phi_{2i} = 0$$

$$\phi_{2i} = \frac{1500 - .0075\lambda_i}{15000 - .0125\lambda_i}$$

$$\phi_{2i} = \begin{cases} \dfrac{1030.939}{14218.23} = .0725082 4 & i=1 \\[2ex] \dfrac{-27280.94}{-32968.24} = .8274916 & i=2 \end{cases}$$ Same (diff in 7th digit) as MYSTRAN

For the A-set

$$\phi^1 = \begin{Bmatrix} 1 \\ .0725082 4 \end{Bmatrix} \begin{matrix} 103\text{-}T2 \\ 103\text{-}R3 \end{matrix}$$

$$\phi^2 = \begin{Bmatrix} 1 \\ .8274916 \end{Bmatrix} \begin{matrix} 103\text{-}T2 \\ 103\text{-}R3 \end{matrix}$$

The gen. mass matrix is

$$\phi_a^\dagger M_{aa} \phi_a = \frac{1}{g}\begin{bmatrix} 1 & .07250824 \\ 1 & .8274916 \end{bmatrix}\begin{bmatrix} .0075 & -.0125 \\ -.0125 & .0625 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ .07250824 & .8274916 \end{bmatrix}$$

$$= \frac{1}{g}\begin{bmatrix} & \text{"} & \end{bmatrix}\begin{bmatrix} .006593647 & -.002843675 \\ -.007968235 & .03921823 \end{bmatrix}$$

$$= \frac{1}{g}\begin{bmatrix} .006015884 & -3\times10^{-8} \\ -5\times10^{-10} & .02960911 \end{bmatrix}$$

(MAPLE, w/ double prec., gets the diag. the same (to 7 sig. figs) and the off-diags to 1.1×10^{-18})

GUYAN-03 : Cholesky Decomp of M

$$M = LDL^T = (LD^{1/2})(LD^{1/2})^T = M^{1/2}M^{T/2}$$

$$M = \frac{1}{g}\begin{bmatrix} .0075 & -.0125 \\ -.0125 & .0625 \end{bmatrix} = (LD^{1/2})(LD^{1/2})^T = M^{1/2}M^{T/2}$$

$M^{-T/2}M^{1/2}M^{-1/2}M^{-T/2}$

$$M = \frac{1}{g}\begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}\underbrace{\begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}\begin{bmatrix} 1 & l_{21} \\ 0 & 1 \end{bmatrix}}_{\begin{bmatrix} d_{11} & d_{11}l_{21} \\ 0 & d_{22} \end{bmatrix}}$$

$$M\ddot{x} + Kx = 0$$

$$M^{1/2}M^{T/2}\ddot{x} + Kx = 0, \quad M^{T/2}x = w, \quad x = M^{-T/2}w$$

$$M^{1/2}\ddot{w} + KM^{-T/2}w = 0$$

$$\ddot{w} + M^{-1/2}KM^{-T/2}w = 0$$

$$\ddot{w} + Aw = 0, \quad A = M^{-1/2}KM^{-T/2}$$

$$K = \begin{bmatrix} 1500 & -15000 \\ -15000 & 200000 \end{bmatrix}$$

$$M = \frac{1}{g}\begin{bmatrix} d_{11} & d_{11}l_{21} \\ d_{11}l_{21} & d_{22} + d_{11}l_{21}^2 \end{bmatrix}$$

$d_{11} = .0075$

$d_{11}l_{21} = -.0125 \quad \therefore l_{21} = -.0125/.0075 = -1.66666$

$d_{22} + d_{11}l_{21}^2 = .0625 \quad \therefore d_{22} = .0625 - d_{11}l_{21}^2 = .0625 - .0075(-1.66666)^2 = .0625 - \frac{.0125^2}{.0075}$

$d_{22} = .0625 - .02083333 = .04166666$

$$L = \begin{bmatrix} 1 & 0 \\ -1.66666 & 1 \end{bmatrix} \quad D = \frac{1}{g}\begin{bmatrix} .0075 & 0 \\ 0 & .04166666 \end{bmatrix}, \quad D^{1/2} = \frac{1}{\sqrt{g}}\begin{bmatrix} .08660254 & 0 \\ 0 & .2041241 \end{bmatrix}$$

$$(LD^{1/2}) = \frac{1}{\sqrt{g}}\begin{bmatrix} 1 & 0 \\ -1.66666 & 1 \end{bmatrix}\begin{bmatrix} .08660254 & 0 \\ 0 & .2041241 \end{bmatrix} = \frac{1}{\sqrt{g}}\begin{bmatrix} .08660254 & 0 \\ -.1443757 & .2041241 \end{bmatrix} \equiv M^{1/2}$$

$$(LD^{1/2})^T = \frac{1}{\sqrt{g}}\begin{bmatrix} .08660254 & -.1443757 \\ 0 & .2041241 \end{bmatrix} \equiv M^{T/2}$$

$$(LD^{1/2})(LD^{1/2})^T = \frac{1}{g}\begin{bmatrix} .08660254 & 0 \\ -.1443757 & .2041241 \end{bmatrix}\begin{bmatrix} .08660254 & -.1443757 \\ 0 & .2041241 \end{bmatrix} = \frac{1}{g}\begin{bmatrix} .0075 & -.01250330 2 \\ -.01250330 2 & .0625\ 10981 \end{bmatrix}$$

$$M^{-1/2} \equiv (M^{1/2})^{-1} = (LD^{1/2})^{-1} = \frac{\sqrt{g}}{(.08660254)(.2041241)}\begin{bmatrix} .2041241 & 0 \\ .1443757 & .08660254 \end{bmatrix} = \sqrt{g}\begin{bmatrix} 11.547005 & 0 \\ 8.167125 & 4.898981 \end{bmatrix}$$

Then

$$A = M^{-1/2}KM^{-T/2} = g\begin{bmatrix} 11.547005 & 0 \\ 8.167125 & 4.898981 \end{bmatrix}\underbrace{\begin{bmatrix} 1500 & -15000 \\ -15000 & 200000 \end{bmatrix}\begin{bmatrix} 11.547005 & 8.167125 \\ 0 & 4.898981 \end{bmatrix}}_{\begin{bmatrix} 17320.51 & -61234.03 \\ -173205.1 & 857289.3 \end{bmatrix}}$$

$$A = M^{-1/2}KM^{-T/2} = g\begin{bmatrix} 200000. & -707069.7 \\ -707069.7 & 3699738. \end{bmatrix}$$

this should be

$$g\begin{bmatrix} 200000 & -707106.8 \\ -707106.8 & 3700000 \end{bmatrix}$$

must need higher precision in calcs

GUYAN-03-GIV

Use inverse power method to check eigenvalue

$$K_{aa} = \begin{bmatrix} 1500 & -15000 \\ -15000 & 200{,}000 \end{bmatrix}$$

$$M_{aa} = \begin{bmatrix} .0075 & -.0125 \\ -.0125 & .0625 \end{bmatrix}$$

$$K_{aa}^{-1} = \frac{1}{(3-2.25)10^8} \begin{bmatrix} 200{,}000 & 15000 \\ 15000 & 1500 \end{bmatrix}$$

$$= \frac{4}{3} \times 10^{-8} \begin{bmatrix} 200000 & 15000 \\ 15000 & 1500 \end{bmatrix}$$

$$K_{aa}^{-1} = \begin{bmatrix} 2.666667 \times 10^{-3} & 2 \times 10^{-4} \\ 2 \times 10^{-4} & 2 \times 10^{-5} \end{bmatrix} \begin{matrix} 1.5 \times 10^{-3} & -1.5 \times 10^{-4} \\ -1.5 \times 10^{-4} & 2 \times 10^{-5} \end{matrix}$$

$$K_{aa}^{-1} M = \begin{bmatrix} 2.666667 \times 10^{-3} & 2 \times 10^{-4} \\ 2 \times 10^{-4} & 2 \times 10^{-5} \end{bmatrix} \begin{bmatrix} 7.5 \times 10^{-3} & -1.25 \times 10^{-2} \\ -1.25 \times 10^{-2} & 6.25 \times 10^{-2} \end{bmatrix}$$

$$= \begin{bmatrix} 17.5 \times 10^{-6} & -20.83333 \times 10^{-6} \\ 1.25 \times 10^{-6} & -1.25 \times 10^{-6} \end{bmatrix}$$

$$u_2 = 10^{-6}\alpha \begin{bmatrix} 17.5 & -20.8333333 \\ 1.25 & -1.25 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 10^{-6}\alpha \begin{Bmatrix} 3.333333 \\ 0 \end{Bmatrix} = 3.333333\alpha \times 10^{-6} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$$

$$u_3 = 10^{-6}\alpha \begin{bmatrix} 17.5 & -20.833333 \\ 1.25 & -1.25 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = 10^{-6}\alpha \begin{Bmatrix} 17.5 \\ 1.25 \end{Bmatrix} = 17.5 \cdot 10^{-6}\alpha \begin{Bmatrix} 1 \\ .07142857 \end{Bmatrix}$$

$$u_4 = 10^{-6}\alpha \begin{bmatrix} 17.5 & -20.83333 \\ 1.25 & -1.25 \end{bmatrix} \begin{Bmatrix} 1 \\ .07142857 \end{Bmatrix} = 10^{-6}\alpha \begin{Bmatrix} 16.011905 \\ 1.160717643 \end{Bmatrix} = 16.011905\alpha \times 10^{-6} \begin{Bmatrix} 1 \\ .072490914 \end{Bmatrix}$$

$$u_5 = 10^{-6}\alpha \begin{bmatrix} 17.5 & -20.833333 \\ 1.25 & -1.25 \end{bmatrix} \begin{Bmatrix} 1 \\ .072490914 \end{Bmatrix} = 10^{-6}\alpha \begin{Bmatrix} 15.98977258 \\ 1.159386355 \end{Bmatrix} = 15.98977 \times 10^{-6}\alpha \begin{Bmatrix} 1 \\ .07250799 \end{Bmatrix}$$

$$u_6 = 10^{-6}\alpha \begin{bmatrix} 17.5 & -20.83333 \\ 1.25 & -1.25 \end{bmatrix} \begin{Bmatrix} 1 \\ .07250799 \end{Bmatrix} = 10^{-6}\alpha \begin{Bmatrix} 15.98941677 \\ 1.159365006 \end{Bmatrix} = 15.98942 \times 10^{-6}\alpha \begin{Bmatrix} 1 \\ .0725083 \end{Bmatrix}$$

$$15.98942 \times 10^{-6}\alpha = 1$$

$$\alpha = 6.254136 \times 10^{4} = \lambda \quad (\text{since } \lambda_0 = 0)$$

(MYSTRAN gets $\lambda = 6.254139 \times 10^{4}$)

RADIAL-BARS-CYL-GLOBAL-END-LOADS

RADIAL-BARS-CYL-GLOBAL-END-LOADS 6 radial bars attached to a central point which is grounded. Each bar has end loads. Global is cyl.

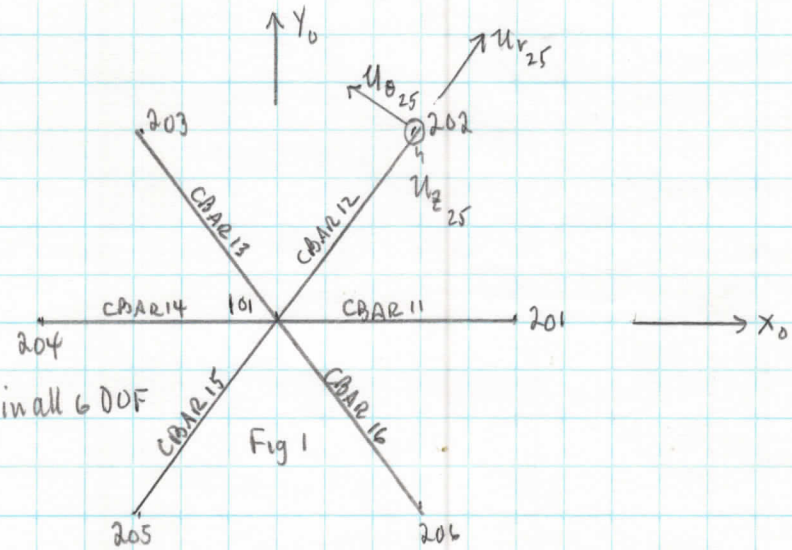


Fig 1

Grid 101 SPC'd to in all 6 DOF

CBARS 11-16 : J=
L=10
A=.4
I_1=4
I_2=5
E=1x10^7

Plane 1 of BARS is r-z
" 2 " " is r-θ

The global coord sys is CORD2C 25 :

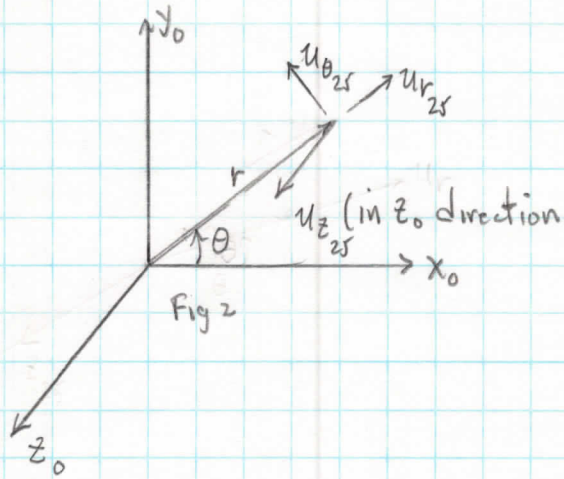
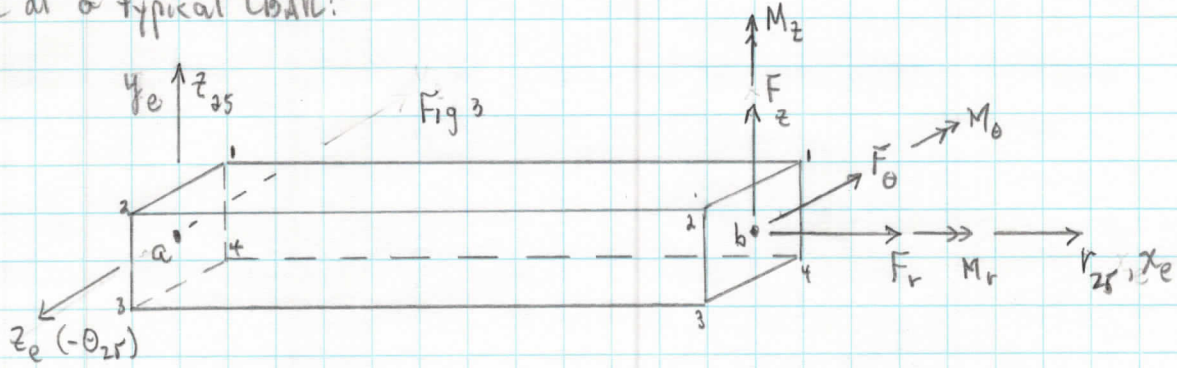


Fig 2

The CBARS have end loads that are all the same and are specified in coord 25

Look at a typical CBAR:



End a for all bars is G.P. 101 and is constrained in all 6 DOF. The applied loads (at the 201-206 G.P.'s) are all the same

$$F_r = 4.$$

$$F_\theta = 6.$$

$$F_z = 12$$

$$M_r = 3.$$

$$M_\theta = 6.$$

$$M_z = 9.$$

The displ's of each CBAR at end b are:

$$u_{r_{25}} = \frac{F_r L}{AE} = \frac{(4)(10)}{(.4)(10^7)} = 1\times10^{-5}$$

$$u_{\theta_{25}} = \frac{F_\theta L^3}{3EI_2} + \frac{M_z L^2}{2EI_2} = \frac{(6)(1000)}{(3)(10^7)(5)} + \frac{(9)(100)}{(2)(10^7)(5)} = 4\times10^{-5} + .9\times10^{-5} = 4.9\times10^{-5}$$

$$u_{z_{25}} = \frac{F_z L^3}{3EI_1} - \frac{M_\theta L^2}{2EI_1} = \frac{(12)(1000)}{(3)(10^7)(4)} - \frac{(6)(100)}{(2)(10^7)(4)} = 10\times10^{-5} - .75\times10^{-5} = 9.25\times10^{-5}$$

$$\theta_{r_{25}} = \frac{M_r L}{GJ} = \frac{(3)(10)}{(4\times10^6)(1.5)} = 5\times10^{-6}$$

$$\theta_{\theta_{25}} = -\frac{F_z L^2}{2EI_1} + \frac{M_\theta L}{EI_1} = -\frac{(12)(100)}{(2)(10^7)(4)} + \frac{(6)(10)}{(10^7)(4)} = -1.5\times10^{-5} + .15\times10^{-5} = -1.35\times10^{-5}$$

$$\theta_{z_{25}} = \frac{F_\theta L^2}{2EI_2} + \frac{M_z L}{EI_2} = \frac{(6)(100)}{(2)(10^7)(5)} + \frac{(9)(10)}{(10^7)(5)} = 6\times10^{-6} + 1.8\times10^{-6} = 7.8\times10^{-6}$$

The constr. forces can be deduced from the following

1) effect of F_r forces is 0 since they all pass thru G.P. 101 and sum to 0

2) " " F_θ forces is only to produce a M_z mom. about G.P. 101

3) " " F_z forces is to produce a F_z const force at G.P. 101 $= \sum_{elem} F_z$

4) " " M_r mom's cancell

5) " " M_θ " "

6) " " M_z " adds to $M_{z_{101}}$ caused by elem F_θ in (2) above

$$\therefore F_{r_{101}} = 0$$

$$F_{\theta_{101}} = 0$$

$$F_{z_{101}} = -6 F_{z_{elem}} = (6)(12) = -72$$

$$M_{r_{101}} = 0$$

$$M_{\theta_{101}} = 0$$

$$M_{z_{101}} = -6 F_{\theta_{elem}} L - 6 M_z = -6\left[(6)(10) + 9\right] = -414.$$

SPC forces at G.P. 101

(-) sign due to these reacting applied forces

	Grid	CID	T1	T2	T3	R1	R2	R3
Displ.	201-206	25	1×10^{-5}	4.9×10^{-5}	9.25×10^{-5}	5×10^{-6}	-1.35×10^{-5}	7.8×10^{-6}
Applied Load	201-206	25	4.	6.	12.	3.	6.	9
SPC Force	201-206	25	0	0	-72	0	0	-414

Elem Engineering Forces

Elem engr forces can be deduced from Fig 3 considering

plane 1 is x_e-y_e (or r_{2r}, z_{2r})

plane 2 is x_e-z_e

+ bending puts compr on top (at $+y_e$, or $+z_e$)

Thus, at end b

M_{1_b} = mom in plane 1 at end b = $-M_\theta = -6.$

M_{2_b} = " " " 2 " " b = $-M_z = -9.$

V_1 = shear in plane 1 = $F_z = 12.$

V_2 = " " " 2 = $-F_\theta = -6.$

At end a

$$M_{1_a} = M_{1_b} + V_1 L = -6 + (12)(10) = 114.$$

$$M_{2_a} = M_{2_b} + V_2 L = -9 - (6)(10) = -69$$

and torque, axial force:

$$T = M_r = 3. \quad , \quad P = F_r = 4.$$

Elem Engr Forces

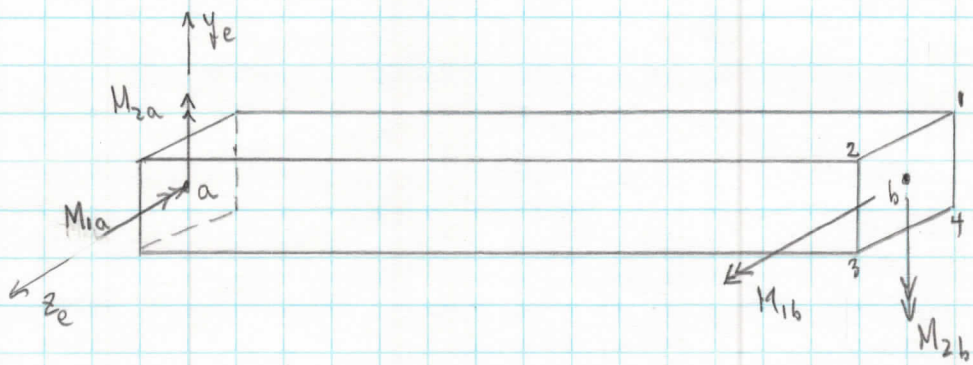
Elem	Bend. Mom - End a		Bend. Mom - End b		Shear		Axial Force	Torque
	plane 1	plane 2	plane 1	plane 2	plane 1	plane 2		
12-16	114.	-69.	-6.	-9.	12.	-6.	4.	3.

Elem Stresses

The locations of the 4 stress pts:

pt 1 : $(.2, -.3)_{y_e, z_e}$
pt 2 : $(.2, .3)_{y_e, z_e}$
pt 3 : $(-.2, .3)_{y_e, z_e}$
pt 4 : $(-.2, -.3)_{y_e, z_e}$

The moments are



$I_1 = 4$
$I_2 = 5$

For a point $p: (y_e, z_e)$ on the cross-section the stresses at end a and end b are

$$\sigma_{a_p} = -\left(\frac{M_{1a} y_e}{I_1} + \frac{M_{2a} z_e}{I_2}\right) \quad (I_{12} = 0) \qquad M_{1a} = 114,\ M_{2a} = -69.$$

$$\sigma_{b_p} = -\left(\frac{M_{1b} y_e}{I_1} + \frac{M_{2b} z_e}{I_2}\right) \quad " \qquad M_{1b} = -6,\ M_{2b} = -9$$

So;

$$\sigma_{a_1} = -\frac{(114)(.2)}{4} - \frac{(-69)(-.3)}{5} = -5.70 - 4.14 = -9.84, \quad \sigma_{b_1} = -\frac{(-6)(.2)}{4} - \frac{(-9)(-.3)}{5} = 0.30 - 0.54 = -0.24$$

$$\sigma_{a_2} = -\frac{(114)(.2)}{4} - \frac{(-69)(.3)}{5} = -5.70 + 4.14 = -1.56, \quad \sigma_{b_2} = -\frac{(-6)(.2)}{4} - \frac{(-9)(.3)}{5} = 0.30 + 0.54 = 0.84$$

$$\sigma_{a_3} = -\frac{(114)(-.2)}{4} - \frac{(-69)(.3)}{5} = 5.70 + 4.14 = 9.84, \quad \sigma_{b_3} = -\frac{(-6)(-.2)}{4} - \frac{(-9)(.3)}{5} = -0.30 + 0.54 = 0.24$$

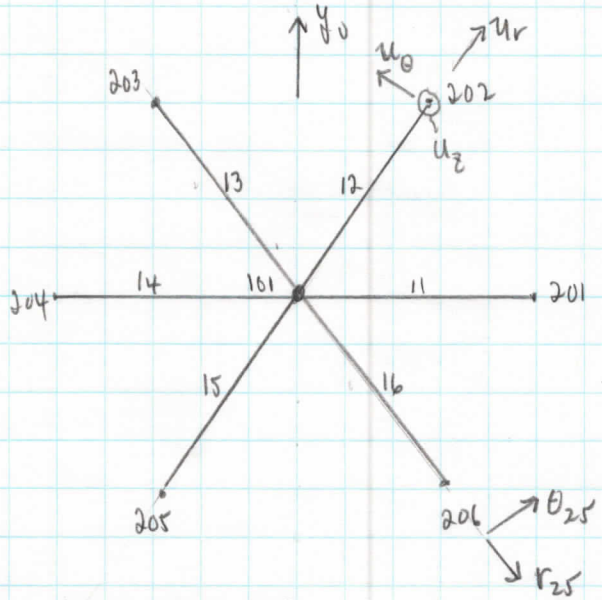
$$\sigma_{a_4} = -\frac{(114)(-.2)}{4} - \frac{(-69)(-.3)}{5} = 5.70 - 4.14 = 1.56, \quad \sigma_{b_4} = -\frac{(-6)(-.2)}{4} - \frac{(-9)(-.3)}{5} = -0.30 - 0.54 = -0.84$$

and

$$\sigma_{axial} = \frac{P_{axial}}{A} = \frac{4}{.4} = 10$$

RADIAL-BARS-CYL-GLOBAL-RB-DISP

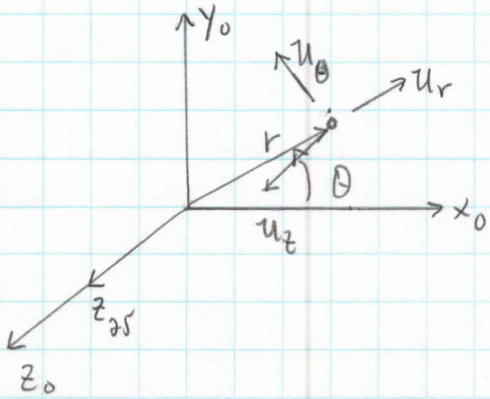
RADIAL-BARS-CYL-GLOBAL-RB-DISP : 6 radial bars attached to a central point which is given RB displ. in all 6 DOF's. Global Coord Sys is cylindrical



CBAR's 11-16: $L = 10$, $A = .4$, $I_1 = 4.$, $I_2 = 5.$, $E = 1\times10^7$

Plane 1 of BARS is r-z
" 2 " " " r-θ

The global system is COORDSE 25:

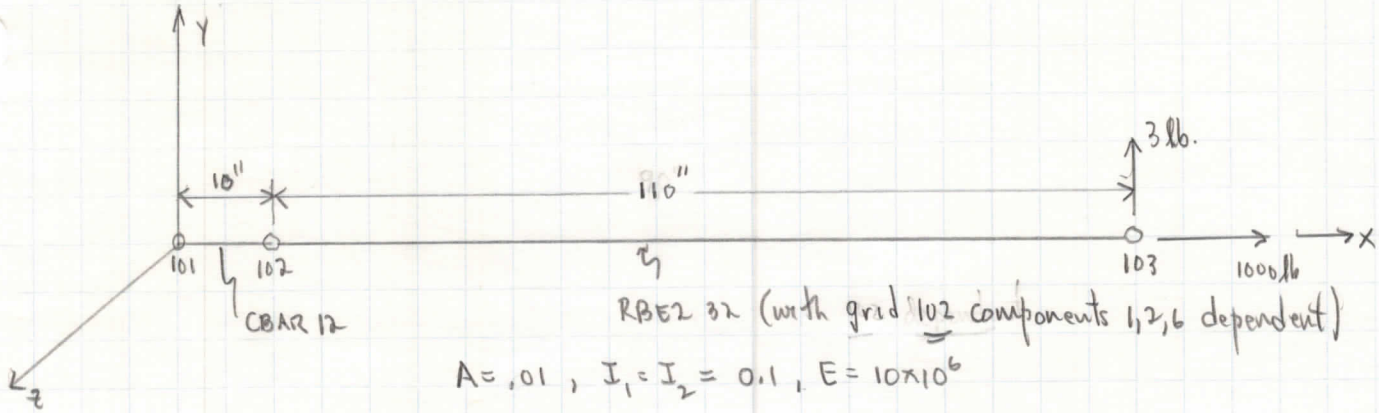


Grid 101 is given 1.0 displ in all 6 cyl. DOF's. Output displ's are in sys. 25 and when F06 file is scanned they don't look RB since all of a given rotation are not equal (except for R3). However, this is OK since R1 = rot. about r and R2 = rot about θ and these are diff. for each grid.

Look at BUG file UEB - which shows elem displ's in basic. Then they do show all R1 = 1.0, all R2 = 1.0, all R3 = 1.0 which is. RB.

RBE2-01-CBAR-01

RBE2-01-CBAR-01- (With G.P. 102, comp 1,2,6 dependent, shows singular K_{NN} after LINK1 AUTOSPC)



$A = .01$, $I_1 = I_2 = 0.1$, $E = 10 \times 10^6$

$$K_{GG} u_G = 10^4 \begin{array}{c} \begin{array}{cccccc|ccc|cccccc|ccc} \multicolumn{6}{c}{101} & \multicolumn{3}{c}{102} & \multicolumn{6}{c}{103} & \multicolumn{3}{c}{102} \\ 1 & 2 & 3 & 4 & 5 & 6 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 & 6 \end{array} \\ \left[\begin{array}{cccccc|ccc|cccccc|ccc} 1 & & & & & & & & & & & & & & & -1 & & \\ & 1.2 & & & & 6 & & & & & & & & & & & -1.2 & 6 \\ & & 1.2 & & -6 & & -1.2 & & -6 & & & & & & & & & \\ & & & 3.759 & & & & -3.759 & & & & & 0 & & & & & \\ & & -6 & & 40 & & 6 & & 20 & & & & & & & & & \\ & 6 & & & & 40 & & & & & & & & & & & -6 & 20 \\ \hline & & -1.2 & & 6 & & 1.2 & & 6 & & & & & & & & & \\ & & & & & & & 3.759 & & & & & 0 & & & & 0 & \\ & & -6 & & 20 & & 6 & & 40 & & & & & & & & & \\ \hline & & & & & & & & & & & & & & & & & \\ & & & 0 & & & & 0 & & & & 0 & & & & & 0 & \\ & & & & & & & & & & & & & & & & & \\ \hline -1 & & & & & -6 & & & & & & & & & & 1 & & \\ & -1.2 & & & & & & 0 & & & & 0 & & & & & 1.2 & -6 \\ & 6 & & & & 20 & & & & & & & & & & & -6 & 40 \end{array}\right] \end{array} \begin{array}{c} \begin{bmatrix} u_{101} \\ v_{101} \\ w_{101} \\ \theta_{x101} \\ \theta_{y101} \\ \theta_{z101} \\ \hline w_{102} \\ \theta_{x102} \\ \theta_{y102} \\ \hline u_{103} \\ v_{103} \\ w_{103} \\ \theta_{x103} \\ \theta_{y103} \\ \theta_{z103} \\ \hline u_{102} \\ v_{102} \\ \theta_{z102} \end{bmatrix} \end{array}, \quad P_g = \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 1000 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \quad (1)$$

K_{NM}^T $\qquad$ K_{MM} $\qquad$ $\begin{Bmatrix} \bar{P}_n \\ P_m \end{Bmatrix}$

Constraints

Grid 101 all 6 components = 0

Grids 101-103 components 3,4,5 = 0 (x-y plane problem

$$K_{GG} = \begin{bmatrix} \bar{K}_{NN} & K_{NM} \\ K_{NM}^T & K_{MM} \end{bmatrix}, \quad u_G = \begin{bmatrix} I \\ G_{MN} \end{bmatrix} u_N \tag{2}$$

$$\bar{K}_{NN} = 10^4 \left[\begin{array}{ccc|ccc|ccc}
1 & & & & & & & & \\
 & 1.2 & & & & 6 & & 0 & \\
 & & 1.2 & & -6 & & -1.2 & & -6 \\
 & & & 3.759 & & & & -3.759 & \\
 & & -6 & & 40 & & 6 & & 20 \\
 & 6 & & & & 40 & & & \\
\hline
 & & -1.2 & & 6 & & 1.2 & & 6 \\
 & & & & & & & 3.759 & \\
 & & -6 & & 20 & & 6 & & 40 \\
\hline
 & & & 0 & & & & 0 & \\
 & & & & & & & & \\
\end{array}\right], \quad K_{NM} = 10^4 \left[\begin{array}{ccc}
-1 & & \\
 & -1.2 & 6 \\
 & & \\
 & -6 & 20 \\
\hline
 & 0 & \\
\hline
 & 0 & \\
\end{array}\right], \quad K_{MM} = 10^4 \begin{bmatrix} 1 & & \\ & 1.2 & -6 \\ & -6 & 40 \end{bmatrix} \tag{3}$$

The RBE2 has components 1,2,6 constrained at G.P. 102:

$$\begin{aligned} u_{102} &= u_{103} \\ v_{102} &= v_{103} - 110\,\theta_{z_{103}} \\ \underbrace{\theta_{z_{102}}}_{u_M} &= \theta_{z_{102}} \end{aligned} \tag{4}$$

$$\begin{Bmatrix} u_{102} \\ v_{102} \\ \theta_{z_{102}} \end{Bmatrix} = \underbrace{\left[\begin{array}{c|c|ccc} 0 & 0 & 1 & & \\ & & & 1 & -110 \\ & & & & 1 \end{array}\right]}_{G_{MN}} \begin{Bmatrix} u_{101} \\ v_{101} \\ w_{101} \\ \theta_{x_{101}} \\ \theta_{y_{101}} \\ \theta_{z_{101}} \\ \hline w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \hline u_{103} \\ v_{103} \\ w_{103} \\ \theta_{x_{103}} \\ \theta_{y_{103}} \\ \theta_{z_{103}} \end{Bmatrix} \tag{5}$$

$$K_{NN} = \bar{K}_{NN} + K_{NM}G_{MN} + (K_{NM}G_{MN})^T + G_{MN}^T K_{MM} G_{MN}$$

$$K_{NM}G_{MN} = 10^4 \left[\begin{array}{c|c|c} \begin{matrix} -1 & & \\ & -1.2 & 6 \\ & & \\ & -6 & 20 \end{matrix} & \begin{matrix} & & \\ & & \\ & & \\ & & \end{matrix} & \begin{matrix} 1 & & \\ & 1 & \\ & & -110 \\ & & 1 \end{matrix} \\ \hline 0 & 0 & 0 \\ \hline 0 & & \end{array}\right] \tag{6}$$

$$= 10^4 \left[\begin{array}{c|c|c} \begin{matrix} & & \\ & & \\ & & \\ & & \end{matrix} & \begin{matrix} & \\ & \\ & \\ & \end{matrix} & \begin{matrix} -1 & & \\ & -1.2 & 138 \\ & & \\ & -6 & 680 \end{matrix} \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right] \tag{7}$$

$$G_{MN}^{T} K_{MM} G_{MN} = 10^{4} \left[\begin{array}{c|c|c|c} 0 & & & \\ \hline 0 & \begin{bmatrix} 1 & \\ 1.2 & -6 \\ -6 & 40 \end{bmatrix} & 0 & 0 \\ \hline \begin{matrix} 1 \\ 1 \\ \\ -110 \end{matrix} & & \begin{bmatrix} 1 & & \\ & 1 & -110 \\ & & 1 \end{bmatrix} & \begin{bmatrix} 1 & & \\ & 1.2 & -138 \\ & -6 & 700 \end{bmatrix} \end{array}\right] \tag{8}$$

$$= 10^{4} \left[\begin{array}{c|c|c|c} 0 & 0 & 0 & \\ \hline 0 & 0 & 0 & \\ \hline 0 & 0 & \begin{matrix} 1 & & \\ & 1.2 & -138 \\ & -138 & 15880 \end{matrix} & \end{array}\right] \tag{9}$$

$$\therefore K_{NN} = 10^4 \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array}
\begin{array}{c}
\begin{array}{cccccc|ccc|cccccc} \multicolumn{6}{c}{101} & \multicolumn{3}{c}{102} & \multicolumn{6}{c}{103} \\ 1 & 2 & 3 & 4 & 5 & 6 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\
\left[\begin{array}{cccccc|ccc|cccccc}
1 & & & & & & & & & -1 & & & & & \\
 & 1.2 & & & & 6 & & & & & -1.2 & & & & 114 \\
 & & 1.2 & & -6 & & -1.2 & & -6 & & & & & & \\
 & & & 3.759 & & & & -3.759 & & & & & & & \\
 & & -6 & & 40 & & 6 & & 20 & & & & & & \\
 & 6 & & & & 40 & & & & & -6 & & & & 560 \\
\hline
 & & -1.2 & & 6 & & 1.2 & & 6 & & & & & & \\
 & & & & & & & 3.759 & & & & 0 & & & \\
 & & -6 & & 20 & & 6 & & 40 & & & & & & \\
\hline
-1 & & & & & & & & & 1 & & & & & \\
 & -1.2 & & & & -6 & & & & & 1.2 & & & & -138 \\
 & & & & & & & 0 & & & & 0 & & & \\
 & & & & & & & & & & & & 0 & & \\
 & & & & & & & & & & & & & 0 & \\
 & 114 & & & & 560 & & & & & -138 & & & & 15880
\end{array}\right]
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 3 \\ 4 \\ 5 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\begin{array}{l} \\ \\ \\ 101 \\ \\ \\ \\ 102 \\ \\ \\ 103 \\ \\ \\ \\ \end{array}
\quad 15 \times 15$$

$$, \; P_n = \bar{P}_n \quad (P_m = 0) \qquad
\left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 1000 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right] \quad (10)$$

Note the 3 singular DOF's in K_{NN} (Grid 103 comps 3,4,5). In addition, prior to the reduction of K_{GG} to K_{NN}, Grid 103 had <u>no</u> stiffness but comps 1,2,6 now have stiffness due to the m set reduction.

The AUTOSPC_NSET processor will AUTOSPC comps 3,4,5 at 103. The SPC at 101 eliminates all 6 comps at grid 101 to give the f-set stiffness:

$$K_{FF}u_F = P_F \Rightarrow 10^4 \begin{array}{c} \\ \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{cccccc} & 102 & & & 103 & \\ 3 & 4 & 5 & 1 & 2 & 6 \end{array} \\ \left[\begin{array}{ccc|ccc} 1.2 & & 6 & & & \\ & 3759 & & & 0 & \\ 6 & & 40 & & & \\ \hline & & & 1 & & \\ & 0 & & & 1.2 & -138 \\ & & & & -138 & 15880 \end{array}\right] \end{array} \begin{bmatrix} w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ u_{103} \\ v_{103} \\ \theta_{z_{103}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1000 \\ 3 \\ 0 \end{bmatrix} \begin{array}{c} P_{z_{102}} \\ M_{x_{102}} \\ M_{y_{102}} \\ P_{x_{103}} \\ P_{y_{103}} \\ M_{z_{103}} \end{array} \tag{11}$$

Grid 102 comps 3-4-5 have no load and are not coupled to 103 comps 1-2-6 so

$$\begin{bmatrix} w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

Solve for 103 1-2-6:

$$\begin{bmatrix} 1 & & \\ & 1.2 & -138 \\ & -138 & 15880 \end{bmatrix} \begin{bmatrix} u_{103} \\ v_{103} \\ \theta_{z_{103}} \end{bmatrix} = 10^{-4} \begin{bmatrix} 1000 \\ 3 \\ 0 \end{bmatrix} \tag{13}$$

$$\therefore u_{103} = 0.1 \tag{14}$$

Mult row 1 by $\frac{138}{1.2}$ and add to row 3 to get

$$\begin{bmatrix} 1.2 & -138 \\ 0 & \left(15880 - \frac{138^2}{1.2}\right) \end{bmatrix} \begin{bmatrix} v_{103} \\ \theta_{z_{103}} \end{bmatrix} = 10^{-4} \begin{bmatrix} 3 \\ \left(3 - \frac{(138)}{1.2}\right) \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} 1.2 & -138 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} v_{103} \\ \theta_{z_{103}} \end{Bmatrix} = \begin{Bmatrix} 3 \\ 345 \end{Bmatrix} \times 10^{-4} \tag{16}$$

$$\therefore \theta_{z_{103}} = 3.45 \times 10^{-3} \tag{17}$$

Subst this into 1st row:

$$1.2\, v_{103} - 138(3.45 \times 10^{-3}) = 3 \times 10^{-4}$$

$$v_{103} = \frac{1}{1.2}\left[4761 + 3\right]10^{-4} = 0.397 \tag{18}$$

Now we can get comps 1-2-6 at 102 from the constr. eqns on page 2:

$$\left.\begin{aligned} u_{102} &= u_{103} = 0.1 \\ v_{102} &= v_{103} - 110\,\theta_{z_{103}} = 0.397 - (110)(3.45 \times 10^{-3}) = .0175 \\ \theta_{z_{102}} &= \theta_{z_{103}} = 3.45 \times 10^{-3} \end{aligned}\right\} \tag{19}$$

Displacements

Grid	CID	T1	T2	T3	R1	R2	R3
101	0	0	0	0	0	0	0
102	0	0.1	.0175	0	0	0	.00345
103	0	0.1	.397	0	0	0	.00345

The MPC forces are

$$Q_{M_{MPC}} = H_{MN} U_N - \cancel{P_M}^{\,0} \tag{20}$$

$$Q_{N_{MPC}} = -G^T_{MN} Q_{M_{MPC}} \tag{21}$$

where

$$H_{MN} = K^T_{NM} + K_{MM} G_{MN}$$

from (1) and (8)

$$H_{MN} = 10^4 \left[\begin{array}{cc|c|cc} -1 & -6 & & 1 & \\ -1.2 & & 0 & 1.2 & -138 \\ 6 & 20 & & -6 & 700 \end{array}\right] \tag{22}$$

$$U_N = \left[\begin{array}{c} u_{101} \\ v_{101} \\ w_{101} \\ \theta_{x_{101}} \\ \theta_{y_{101}} \\ \theta_{z_{101}} \\ \hline w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \hline u_{103} \\ v_{103} \\ w_{103} \\ \theta_{x_{103}} \\ \theta_{y_{103}} \\ \theta_{z_{103}} \end{array}\right] = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0.1 \\ .397 \\ 0 \\ 0 \\ 0 \\ .00345 \end{array}\right] \tag{23}$$

$$\therefore\ Q_{M_{MPC}} = \left\{\begin{array}{c} 0.1 \\ (1.2)(.397)-(138)(.00345) \\ (-6)(.397)+(700)(.00345) \end{array}\right\} 10^4 = \left\{\begin{array}{c} 1000 \\ 3 \\ 330 \end{array}\right\} \begin{array}{l} \text{on } 102\text{-}T1 \\ \text{on } 102\text{-}T2 \\ \text{on } 102\text{-}R3 \end{array} \tag{24}$$

and from G_{mm} in (1) with (21) and (24)

$$Q_{N_{MPC}} = -\begin{bmatrix} & & \\ & 0 & \\ & & \\ \hline & 0 & \\ \hline 1 & & \\ & 1 & \\ & & \\ & & \\ & & \\ -110 & & 1 \end{bmatrix} \begin{Bmatrix} 1000 \\ 3 \\ 330 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ \hline -1000 \\ -3 \\ 0 \\ 0 \\ 0 \\ 330-330=0 \end{Bmatrix} \begin{array}{l} \text{on 101 -T1} \\ \text{-T2} \\ \text{-T3} \\ \text{-T4} \\ \text{-T5} \\ \text{-T6} \\ \text{on 102 -T3} \\ \text{-T4} \\ \text{-T5} \\ \text{on 103 -T1} \\ \text{-T2} \\ \text{-T3} \\ \text{-R1} \\ \text{-R2} \\ \text{-R3} \end{array}$$

Thus

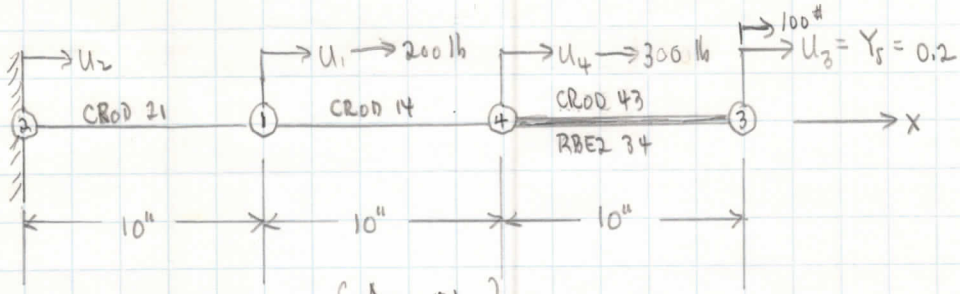
MPC Forces

Grid	CID	T1	T2	T3	R1	R2	R3
101	0	0	0	0	0	0	0
102	0	1000.	3	0	0	0	330
103	0	-1000	3	0	0	0	0

RBE2-01-CROD-03

RBE2-01-CROD-03

Example: Rod With Rigid Element And Enforced Displ. (Rigid Elem indep. DOF is an spc'd DOF)



All CROD's have $\left\{ \begin{array}{l} A = .01 \\ E = 10\times10^6 \\ \ell = 10 \end{array} \right\}$ so $\frac{AE}{\ell} = 10^4$

The eqns of motion are:

$$K_{gg} u_g = P_g + Q_g \quad \text{where } Q_g = R_{cg}^T q_c \tag{1}$$

$$R_{cg} u_g = Y_c \tag{2}$$

where q_c are generalized constraint forces, Y_c are constrained displ's for the c constraints

Since only motion (and stiffness) exist for the x direction, take

$$u_g = \left\{ \begin{array}{c} \left.\begin{array}{c} \left.\begin{array}{c} u_1 \\ u_2 \end{array}\right\} = u_f \\ \\ \end{array}\right\} u_n \\ \left.\begin{array}{c} u_3 \\ u_4 \end{array}\right\} = u_s \end{array} \right\} \tag{3}$$

$= u_m$

The constraints are

$$\left. \begin{array}{l} u_2 = 0 \\ u_3 = 0.2 \\ u_4 = u_3 \end{array} \right\} \tag{4}$$

Put (4) in matrix form of (2)

$$\left[\begin{array}{c|cc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & -1 \end{array}\right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.2 \\ 0 \end{Bmatrix} \quad \text{i.e. } R_{cg} u_g = Y_c \tag{5}$$

Eqn (5) is of the form required for R_{cg}, namely:

$$\begin{bmatrix} \theta_{sf} & I_{ss} & \theta_{sm} \\ \underbrace{R_{mf} \quad R_{ms}}_{R_{mn}} & & R_{mm} \end{bmatrix} \begin{Bmatrix} u_f \\ u_s \\ u_m \end{Bmatrix} = \begin{Bmatrix} Y_s \\ \theta_m \end{Bmatrix} \tag{6}$$

where

$$R_{mn} = [0 \quad 0 \quad 1] \tag{7}$$

$$R_{mm} = [-1] \tag{8}$$

So

$$G_{mn} = -R_{mm}^{-1} R_{mn} = \begin{array}{c} \begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \end{array} \tag{9}$$

The g-set stiffness, loads are:

$$K_{gg} = 10^4 \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \left[\begin{array}{ccc|c} 2 & -1 & & -1 \\ -1 & 1 & & \\ & & 1 & -1 \\ \hline -1 & & -1 & 2 \end{array}\right] \end{array} \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \qquad P_g = \begin{Bmatrix} 200 \\ 0 \\ 100 \\ \hline 300 \end{Bmatrix} \tag{10}$$

So that

$$\bar{K}_{nn} = 10^4 \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K_{nm} = 10^4 \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}, \quad K_{mm} = 2 \times 10^4, \quad \bar{P}_n = \begin{Bmatrix} 200 \\ 0 \\ 100 \end{Bmatrix}, \quad P_m = [300] \tag{11}$$

Then
$$K_{nn} = \bar{K}_{nn} + (K_{nm}G_{mn}) + (K_{nm}G_{mn})^T + G_{mn}^T K_{mm} G_{mn}$$

where

$$K_{nm}G_{mn} = 10^4 \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = 10^4 \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$G_{mn}^T K_{mm} G_{mn} = 10^4 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = 10^4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Thus

$$10^4 K_{nn} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore \quad K_{nn} = 10^4 \begin{array}{c} \begin{array}{ccc} u_1 & u_2 & u_3 \end{array} \\ \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \qquad (12)$$

Also

$$P_n = \bar{P}_n + G_{mn}^T P_m = \begin{Bmatrix} 200 \\ 0 \\ 100 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (200) = \begin{Bmatrix} 200 \\ 0 \\ 400 \end{Bmatrix} \qquad (13)$$

Partition n set into f, s sets

$$K_{nn} = 10^4 \left[\begin{array}{c|cc} 2 & -1 & -1 \\ \hline -1 & 1 & 0 \\ -1 & 0 & 1 \end{array}\right] = \begin{bmatrix} K_{ff} & K_{fs} \\ K_{fs}^T & K_{ss} \end{bmatrix} \tag{14}$$

So

$$K_{ff} = [2\times10^4], \quad K_{fs} = 10^4 \begin{bmatrix} -1 & -1 \end{bmatrix}, \quad K_{ss} = 10^4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

Also

$$P_n = \left\{\begin{array}{c} 200 \\ \hline 0 \\ 400 \end{array}\right\} = \begin{Bmatrix} \bar{P}_f \\ P_s \end{Bmatrix} \tag{16}$$

So

$$\bar{P}_f = \{200\}, \quad P_s = \begin{Bmatrix} 0 \\ 400 \end{Bmatrix} \tag{17}$$

Then

$$P_f = \bar{P}_f - K_{fs} Y_s = \{200\} - 10^4 \begin{bmatrix} -1 & -1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2 \end{Bmatrix}$$

$$\therefore P_f = \{2{,}200\} \tag{18}$$

There is no o set so the f set eqns are the a set:

$$K_{ff} u_f = P_f \quad \text{or} \quad K_{aa} u_a = P_a \tag{19}$$

$$[2\times10^4] u_1 = 2{,}200$$

$$\therefore u_1 = 0.11 \quad (u_f) \tag{20}$$

Then

$$u_n = \begin{Bmatrix} u_f \\ u_s \end{Bmatrix} = \begin{Bmatrix} u_1 \\ \hline u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0.11 \\ 0 \\ 0.2 \end{Bmatrix} \tag{21}$$

and

$$u_g = \begin{Bmatrix} u_n \\ u_m \end{Bmatrix}$$

where u_m is DOF 4 and

$$u_4 = u_3$$

So

$$u_g = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0.11 \\ 0 \\ 0.2 \\ 0.2 \end{Bmatrix} \tag{22}$$

Now recover q_s:

$$q_s = K_{fs}^T u_f + K_{ss} Y_s - P_s \tag{23}$$

$$q_s = 10^4 \begin{bmatrix} -1 \\ -1 \end{bmatrix} \{0.11\} + 10^4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 400 \end{Bmatrix}$$

$$Q_{s_{SPC}} = q_s = \begin{Bmatrix} -1100 + 0 - 0 \\ -1100 + 2000 - 400 \end{Bmatrix} = \begin{Bmatrix} -1100 \\ 500 \end{Bmatrix} \begin{matrix} \text{on DOF 2} \\ \text{on DOF 3} \end{matrix} \tag{24}$$

(nodal forces due to SPC's on S-set DOF's)

and

$$Q_{g_{SPC}} = \begin{Bmatrix} \quad \\ -1100 \\ 500 \\ 0 \end{Bmatrix} \begin{matrix} \text{on DOF 1} \\ \text{" " 2} \\ \text{" " 3} \\ \text{" " 4} \end{matrix}$$

Now recover MPC forces:

$$Q_{m_{MPC}} = (K_{nm}^T + K_{mn}G_{mn})u_n - P_m$$

$$= \left(10^4\begin{bmatrix}-1 & 0 & -1\end{bmatrix} + 10^4\begin{bmatrix}2\end{bmatrix}\begin{bmatrix}0 & 0 & 1\end{bmatrix}\right)\begin{Bmatrix}0.11\\0\\0.2\end{Bmatrix} - 300$$

$$= 10^4\begin{bmatrix}-1 & 0 & 1\end{bmatrix}\begin{Bmatrix}0.11\\0\\0.2\end{Bmatrix} - 300$$

$$= 900 - 300$$

$$= 600 \quad \text{on m set DOF } u_4$$

and

$$Q_{n_{MPC}} = -G_{mn}^T Q_{m_{MPC}}$$

Use (9)

$$Q_{n_{MPC}} = -\begin{bmatrix}0\\0\\1\end{bmatrix}600 = \begin{Bmatrix}0\\0\\-600\end{Bmatrix}\begin{matrix}\text{DOF 1}\\ \text{" 2}\\ \text{" 3}\end{matrix} \quad \text{and} \quad Q_{g_{MPC}} = \begin{Bmatrix}0\\0\\-600\\600\end{Bmatrix}\begin{matrix}\text{DOF 1}\\ \text{" 2}\\ \text{" 3}\\ \text{" 4}\end{matrix}$$

So the sum of constraint forces are:

$$Q_g = Q_{g_{SPC}} + Q_{g_{MPC}} = \underbrace{\begin{Bmatrix}0\\-1100\\500\\0\end{Bmatrix}}_{\text{g-set SPC constraint forces}} + \underbrace{\begin{Bmatrix}0\\0\\-600\\600\end{Bmatrix}}_{\text{g-set MPC constraint forces}} = \begin{Bmatrix}0\\-1100\\-100\\600\end{Bmatrix}\begin{matrix}\text{DOF 1}\\ \text{" 2}\\ \text{" 3}\\ \text{" 4}\end{matrix}$$

Note also that Q_g could have been found from eqn (1) as

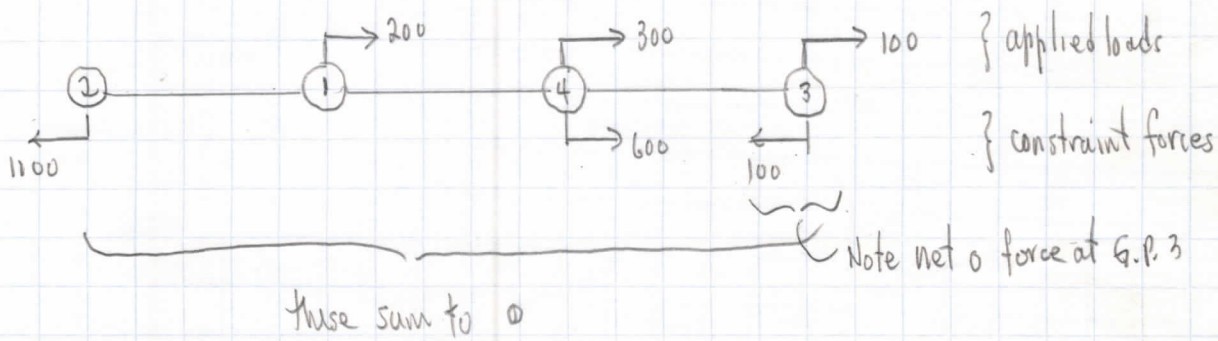
$$Q_g = K_{gg} u_g - P_g \tag{29}$$

where K_{gg} and P_g are in eqn 10 and u_g in eqn (22);

$$\therefore Q_g = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = 10^4 \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix} \begin{Bmatrix} 0.11 \\ 0 \\ 0.2 \\ 0.2 \end{Bmatrix} - \begin{Bmatrix} 200 \\ 0 \\ 100 \\ 300 \end{Bmatrix}$$

$$= \begin{Bmatrix} 200 \\ -1100 \\ 0 \\ 900 \end{Bmatrix} - \begin{Bmatrix} 200 \\ 0 \\ 100 \\ 300 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1100 \\ -100 \\ 600 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \tag{30}$$

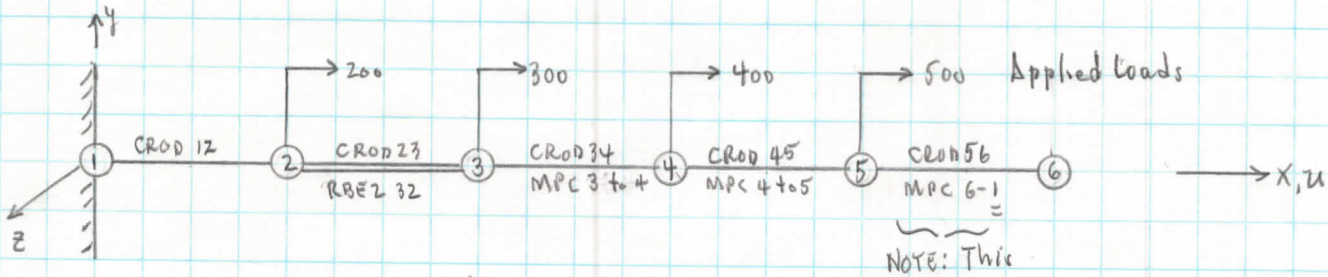
which is the same as (28). Equilibrium of the applied and constraint forces is shown below:



Note also that this set of forces would result in $u_3 = u_4$ since there is no net force at G.P. 3 to cause strain in the element between G.P. 4 and G.P. 3 (which was stated to be rigid)

RBE2-01-CROD-05-MPC-03

RBE2-01-CROD-05-MPC-03

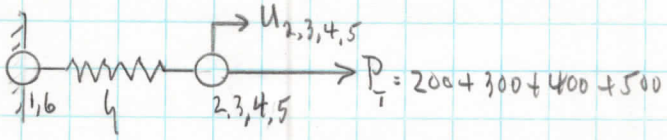


NOTE: This makes $u_6 = 0$ since u_1 is SPC'd

CRODS 12 → 56 have $\begin{cases} A = .01 \\ L = 10 \\ E = 10^7 \end{cases}$

Due to the RBE2 and MPC's, elems 23, 34, 45 are essentially rigid (i.e. the constr's will make displ's at both ends equal. Thus grids 2,3,4,5 will have same disp and grids 1,6 will have $u = 0$ ($u_6 = 0$ since it is MPC'd to u_1 which is SPC'd)

To calc the displ's of 2-5 consider the problem to be as if



$$k_T = \frac{2AE}{L} \text{ (CROD's 12, 56)} = \frac{(2)(.01)(10^7)}{10} = 2\times10^4$$

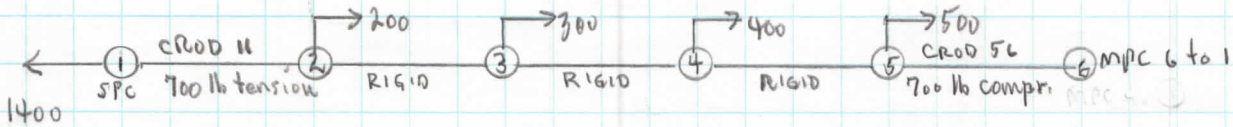
$$u_{2,3,4,5} = \frac{P_T}{k_T} = \frac{1400}{2\times10^4} = .07$$

The 2 CROD's, which have the same stiffness (i.e. $k_T/2$) will share the 1400 lb P_T equally so that engr force in CROD 12 and 56 will be 700 lb. The others will be 0 since they are rigid (due to RBE2 or MPC)

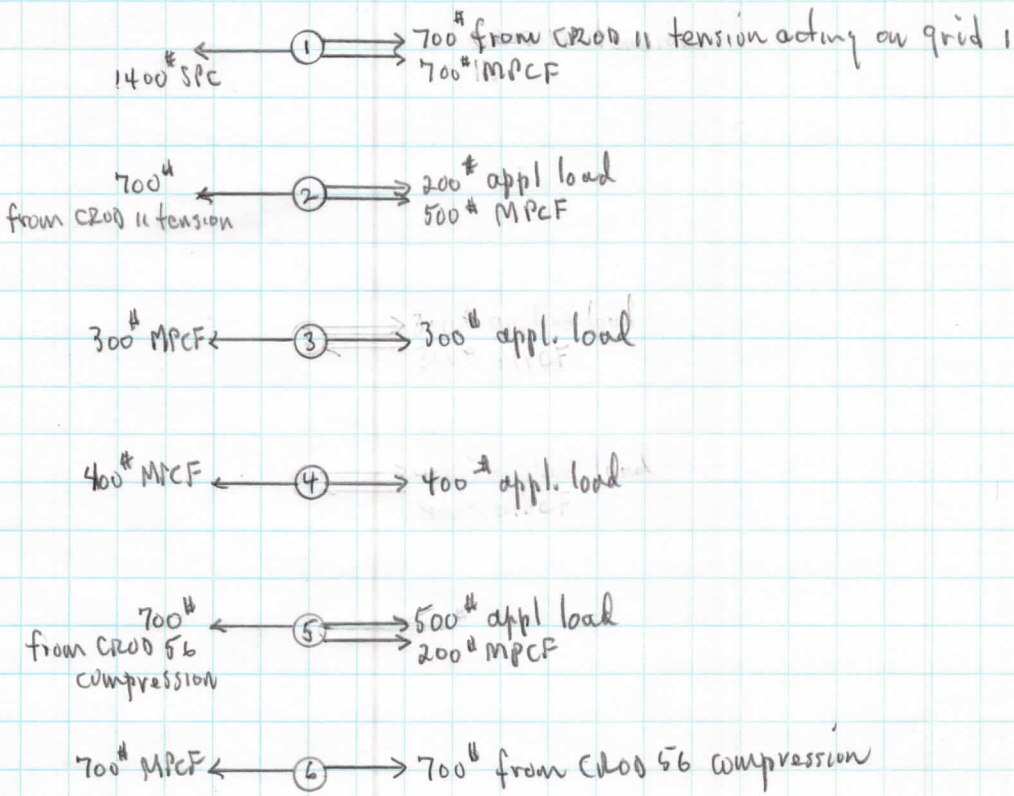
The SPC and MPC forces can be determined logically as shown on next page

Since the only SPC is $u_1 = 0$ then $Q_{s_1} = -(P_2 + P_3 + P_4 + P_5) = -1400$ (now redundant)

This leaves us with the following free body diagram



The remainder of the forces needed <u>to put each grid in force balance</u> are <u>MPC forces</u> so that, looking at each grid:

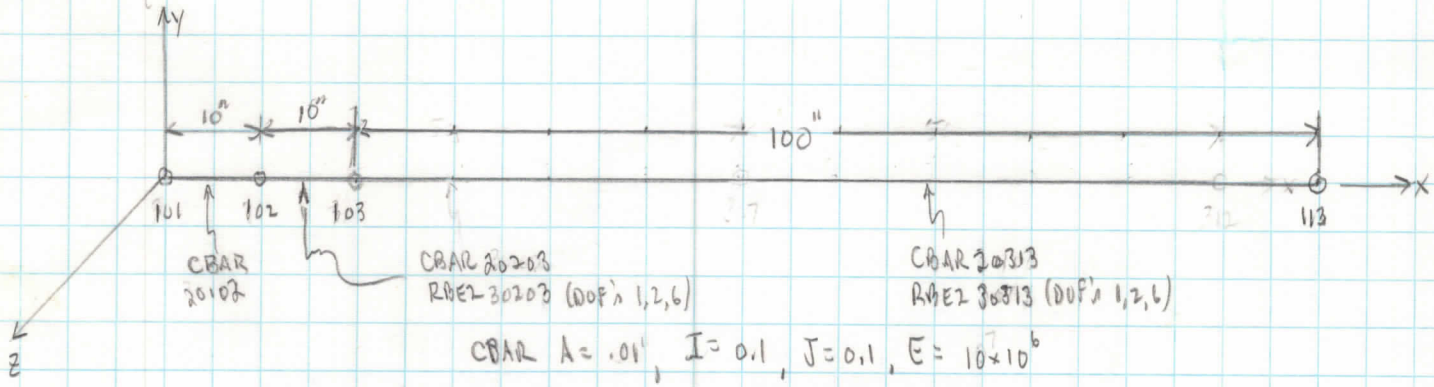


So the MPC forces are

$$Q_{MPC_1} = 700$$
$$Q_{MPC_2} = 500$$
$$Q_{MPC_3} = -300$$
$$Q_{MPC_4} = -400$$
$$Q_{MPC_5} = 200$$
$$Q_{MPC_6} = -700$$

RBE2-02-CBAR-03

RBE2-02-CBAR-03 — 3 CBAR's and 2 RBE2's



CBAR $A = .01$, $I = 0.1$, $J = 0.1$, $E = 10 \times 10^6$

Note: 103-1, 103-2, 103-6, 113-1, 113-2, 113-6 are M-set
102-1, 102-2, 102-6 are F, A sets
All others are S-set
Loads are on 113-1, 113-2 (M-set DOF's)

$$\begin{array}{c} \begin{array}{cc} 101 & 102 \end{array} \\ \left[\begin{array}{cccccc|cccccc} 1 & & & & & & -1 & & & & & \\ & 1.2 & & & & 6 & & -1.2 & & & & 6 \\ & & 1.2 & & -6 & & & & -1.2 & & -6 & \\ & & & 3.759 & & & & & & -3.759 & & \\ & & -6 & & 40 & & & & 6 & & 20 & \\ & 6 & & & & 40 & & -6 & & & & 20 \\ \hline -1 & & & & & & 1 & & & & & \\ & -1.2 & & & & -6 & & 1.2 & & & & -6 \\ & & -1.2 & & 6 & & & & 1.2 & & 6 & \\ & & & -3.759 & & & & & & 3.759 & & \\ & & -6 & & 20 & & & & 6 & & 40 & \\ & 6 & & & & 20 & & -6 & & & & 40 \end{array}\right] \end{array} \begin{array}{c} \\ 101 \\ \\ \\ \\ \\ 102 \\ \\ \\ \\ \\ \end{array} \times 10^4 = K^e_{20102}$$

$$\begin{array}{c} \begin{array}{cc} 102 & 103 \end{array} \\ \left[\begin{array}{cccccc|cccccc} 1 & & & & & & -1 & & & & & \\ & 1.2 & & & & 6 & & -1.2 & & & & 6 \\ & & 1.2 & & -6 & & & & -1.2 & & -6 & \\ & & & 3.759 & & & & & & -3.759 & & \\ & & -6 & & 40 & & & & 6 & & 20 & \\ & 6 & & & & 40 & & -6 & & & & 20 \\ \hline -1 & & & & & & 1 & & & & & \\ & -1.2 & & & & -6 & & 1.2 & & & & -6 \\ & & -1.2 & & 6 & & & & 1.2 & & 6 & \\ & & & -3.759 & & & & & & 3.759 & & \\ & & -6 & & 20 & & & & 6 & & 40 & \\ & 6 & & & & 20 & & -6 & & & & 40 \end{array}\right] \end{array} \begin{array}{c} \\ 102 \\ \\ \\ \\ \\ 103 \\ \\ \\ \\ \\ \end{array} 10^4 = K^e_{20303}$$

$$\begin{array}{c} \begin{array}{cc} 103 & 113 \end{array} \\ \left[\begin{array}{cccccc|cccccc} .1 & & & & & & -.1 & & & & & \\ & .0012 & & & & .06 & & -.0012 & & & & .06 \\ & & .0012 & & -.06 & & & & -.0012 & & -.06 & \\ & & & .376 & & & & & & -.376 & & \\ & & -.06 & & 4 & & & & .06 & & 2 & \\ & .06 & & & & 4 & & -.06 & & & & 2 \\ \hline -.1 & & & & & & .1 & & & & & \\ & -.0012 & & & & -.06 & & .0012 & & & & -.06 \\ & & -.0012 & & .06 & & & & .0012 & & .06 & \\ & & & -.376 & & & & & & .376 & & \\ & & .06 & & 2 & & & & .06 & & 4 & \\ & .06 & & & & 2 & & -.06 & & & & 4 \end{array}\right] \end{array} \begin{array}{c} \\ 103 \\ \\ \\ \\ \\ 113 \\ \\ \\ \\ \\ \end{array} 10^4 = K^e_{20313}$$

$K_{GG} = 10^4$

101						102						103						113						
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	
1						-1																		1
	1.2				6		-1.2				6													2
		1.2		-6				-1.2		-6														3
			3.759						-3.759															4
		-6		40				6		20														5
	6				40		-6				20													6
-1						1						-1												1
	-1.2				-6		2.496						-1.2				.6							2
		-1.2		6				2.496						-1.2		-6								3
			-3.759						7.518						-3.759									4
		-6		20						80				6		20								5
	6				20						80		-6				20							6
						-1						1.1						-.1						1
							-1.2				-6		1.2017				-5.94		-.0012				.06	2
								-1.2		.6				1.2017		5.94				-.0012		-.06		3
									-3.759						4.135						-.376			4
								-6		20				5.94		44				.06		2		5
							6				20		-5.94				44		-.06				2	6
												-.1						.1						1
													-.0012				-.06		.0012				-.06	2
														-.0012		.06				.0012		.06		3
															-.376						.376			4
														-.06		2				.06		4		5
													.06				2		-.06				4	6

101 102 103 113

Arrange into n, m order

$K_{GG} = 10^4$

101						102						103			113			103			113			
1	2	3	4	5	6	1	2	3	4	5	6	3	4	5	3	4	5	1	2	6	1	2	6	
1						-1																		1
	1.2				6		-1.2				6													2
		1.2		-6				-1.2		-6														3
			3.759						-3.759															4
		-6		40				6		20														5
	6				40		-6				20													6
-1						2												-1						1
	-1.2				-6		2.4												-1.2	6				2
		-1.2		6				2.4				-1.2		-6										3
			-3.759						7.518				-3.759											4
		-6		20						80		6		20										5
	6				20						80								-6	20				6
								-1.2		6		1.2017		5.94	-.0012		-.06							3
									-3.759				4.135			-.376								4
								-6		20		5.94		44	.06		2							5
												-.0012		.06	.0012		.06							3
													-.376			.376								4
												-.06		2	.06		4							5
						-1												1.1			-.1			1
							-1.2				-6								1.2017	-5.94		-.0012	.06	2
							6				20								-5.94	44		-.06	2	6
																		-.1			.1			1
																			-.0012	-.06		.0012	-.06	2
																			.06	2		-.06	4	6

101, 102, 103, 113

Agrees w/ MYSTRAN (as KGG and as partitions)

$$K_{GG} = \left[\begin{array}{c||c} \bar{K}_{NN} & K_{NM} \\ \hline\hline K_{NM}^T & K_{MM} \end{array}\right]$$

$\bar{K}_{NN}$ has 42 nonzeros on, and above, diagonal
K_{NM} has 5 nonzeros total
K_{MM} has 13 nonzeros on, and above, diagonal

The constraint equations due to the rigid elements are

RBE2 30203

$$u_{103} = u_{102} \quad \text{or} \quad u_{103} - u_{102} = 0 \qquad (a)$$

$$v_{103} = v_{102} + 10\,\theta_{z_{102}} \quad \text{or} \quad v_{103} - v_{102} - 10\,\theta_{x_{102}} = 0 \qquad (b)$$

$$\theta_{z_{103}} = \theta_{z_{102}} \quad \text{or} \quad \theta_{z_{103}} - \theta_{z_{102}} = 0 \qquad (c)$$

RBE2 30313

$$u_{113} = u_{103} \quad \text{or} \quad u_{113} - u_{103} = 0 \qquad (d)$$

$$v_{113} = v_{103} + 100\,\theta_{z_{103}} \quad \text{or} \quad v_{113} - v_{103} - 100\,\theta_{z_{103}} = 0 \qquad (e)$$

$$\theta_{z_{113}} = \theta_{z_{103}} \quad \text{or} \quad \theta_{z_{113}} - \theta_{z_{103}} = 0 \qquad (f)$$

$$\begin{array}{c}
 \\ (a) \\ (b) \\ (c) \\ (d) \\ (e) \\ (f)
\end{array}
\underbrace{\left[\begin{array}{cccccc|cccccc|ccc|ccc|ccc}
 & & \overset{101}{w} & & & & & & \overset{102}{w} & & & & & \overset{103}{\theta_x} & & & \overset{113}{\theta_x} & & & & & \\
u & v & & \theta_x & \theta_y & \theta_z & u & v & & \theta_x & \theta_y & \theta_z & w & & \theta_y & w & & \theta_y & u & v & \theta_z & u & v & \theta_z \\
 & & & & & & -1 & & & & & & & & & & & & 1 & & & & & \\
 & & & & & & & -1 & & & & -10 & & & & & & & & 1 & & & & \\
 & & & & & & & & & & & -1 & & & & & & & & & 1 & & & \\
 & & & & & & & & & & & & & & & & & & -1 & & & 1 & & \\
 & & & & & & & & & & & & & & & & & & & -1 & -100 & & 1 & \\
 & & & & & & & & & & & & & & & & & & & & -1 & & & 1
\end{array}\right]}_{\substack{R_{mn} \\ \text{(agrees w/ MYSTRAN)}}\ \ \substack{R_{mM} \\ \text{(agrees w/ MYSTRAN)}}}
\begin{bmatrix}
u_{101} \\ v_{101} \\ w_{101} \\ \theta_{x_{101}} \\ \theta_{y_{101}} \\ \theta_{z_{101}} \\ \hline
u_{102} \\ v_{102} \\ w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \theta_{z_{102}} \\ \hline
u_{103} \\ v_{103} \\ w_{103} \\ \theta_{x_{103}} \\ \theta_{y_{103}} \\ \theta_{z_{103}} \\ \hline
u_{113} \\ v_{113} \\ w_{113} \\ \theta_{x_{113}} \\ \theta_{y_{113}} \\ \theta_{z_{113}}
\end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We can generate G_{MN} by inspection of eqns (a)-(f) on previous page rather than inverting R_{MN}:

$$u_M = G_{MN}\, u_N$$

$$\begin{Bmatrix} u_{103} \\ v_{103} \\ \theta_{z_{103}} \\ \hline u_{113} \\ v_{113} \\ \theta_{z_{113}} \end{Bmatrix} \approx \underbrace{\left[\begin{array}{c|c|c|c} 0 & \begin{matrix} 1 & & \\ & 1 & \\ & & 1 \end{matrix} & \begin{matrix} 10 \\ \\ 1 \end{matrix} & 0 & 0 \\ \hline 0 & \begin{matrix} 1 & & \\ & 1 & \\ & & 1 \end{matrix} & \begin{matrix} 110 \\ \\ 1 \end{matrix} & 0 & 0 \end{array}\right]}_{G_{MN}} \begin{Bmatrix} u_{101} \\ v_{101} \\ w_{101} \\ \theta_{x_{101}} \\ \theta_{y_{101}} \\ \theta_{z_{101}} \\ \hline u_{102} \\ v_{102} \\ w_{102} \\ \theta_{x_{102}} \\ \theta_{y_{102}} \\ \theta_{z_{102}} \\ \hline w_{103} \\ \theta_{x_{103}} \\ \theta_{y_{103}} \\ \hline w_{113} \\ \theta_{x_{113}} \\ \theta_{y_{113}} \end{Bmatrix}$$

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$$K_{NM} G_{MN} = 10^4 \left[\begin{array}{c|c} 0 & 0 \\ \hline \begin{matrix} -1 & & \\ & -1.2 & 6 \\ & & \\ & -6 & 20 \end{matrix} & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \end{array}\right] \left[\begin{array}{c|c|c|c} 0 & \begin{matrix} 1 & & \\ & 1 & \\ & & 10 \\ & & 1 \end{matrix} & 0 & 0 \\ \hline 0 & \begin{matrix} 1 & 1 & \\ & & 110 \\ & & 1 \end{matrix} & 0 & 0 \end{array}\right]$$

$$K_{NM} G_{MN} = 10^4 \left[\begin{array}{c|c|c|c} 0 & 0 & 0 & 0 \\ \hline 0 & \begin{matrix} -1 & & \\ & -1.2 & -6 \\ & -6 & -40 \end{matrix} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

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$$K_{NM}G_{MN} + (K_{NM}G_{MN})^T = 10^4 \begin{bmatrix} 0 & 0 & 0 & 0 \\ \begin{matrix} & & & \\ & & & \\ -2 & & & \\ & -2.4 & & -12 \\ & & & \\ & -12 & & -80 \end{matrix} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Agrees w/ MYSTRAN

$$\bar{K}_{NN} + K_{NM}G_{MN} + (K_{NM}G_{MN})^T = 10^4 \begin{bmatrix} \begin{matrix} 1 & & & & & \\ & 1.2 & & & & 6 \\ & & 1.2 & & -6 & \\ & & & 3.759 & & \\ & & -6 & & 40 & \\ & 6 & & & & 40 \end{matrix} & \begin{matrix} -1 & & & & & \\ & -1.2 & & & & 6 \\ & & -1.2 & & -6 & \\ & & & -3.759 & & \\ & & 6 & & 20 & \\ & -6 & & & & 20 \end{matrix} & \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix} & \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix} \\ \begin{matrix} -1 & & & & & \\ & -1.2 & & & & -6 \\ & & -1.2 & & 6 & \\ & & & -3.759 & & \\ & & -6 & & 20 & \\ & 6 & & & & 20 \end{matrix} & \begin{matrix} 0 & & & & & \\ & 0 & & & & -12 \\ & & 2.4 & & & \\ & & & 7.518 & & \\ & & & & 80 & \\ & -12 & & & & 0 \end{matrix} & \begin{matrix} & & & & & \\ & & & & & \\ -1.2 & & -6 & & & \\ & -3.759 & & & & \\ 6 & & 20 & & & \\ & & & & & \end{matrix} & \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix} \\ \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix} & \begin{matrix} & & & & & \\ & & & & & \\ -1.2 & & 6 & & & \\ & -3.759 & & & & \\ -6 & & 20 & & & \\ & & & & & \end{matrix} & \begin{matrix} 1.2017 & & 5.94 & & & \\ & 4.135 & & & & \\ 5.94 & & 44 & & & \\ & & & & & \end{matrix} & \begin{matrix} -.0017 & & -.06 & & & \\ & -.376 & & & & \\ .06 & & 2 & & & \end{matrix} \\ \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix} & \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix} & \begin{matrix} -.0017 & & .06 & & & \\ & -.376 & & & & \\ -.06 & & 2 & & & \end{matrix} & \begin{matrix} .0017 & & .06 & & & \\ & .376 & & & & \\ .06 & & 4 & & & \end{matrix} \end{bmatrix}$$

Agrees w/ MYSTRAN

$$K_{MM} G_{MN} = 10^4 \left[\begin{array}{cc|cc} 1.1 & & -.1 & \\ 1.2012 & -5.94 & -.0012 & .06 \\ -5.94 & 44 & -.06 & 2 \\ \hline -.1 & & .1 & \\ -.0012 & -.06 & .0012 & -.06 \\ .06 & 2 & -.06 & 4 \end{array}\right] \left[\begin{array}{c|c|c|c} 0 & \begin{matrix} 1 & & \\ & 1 & \\ & & 10 \\ & & 1 \end{matrix} & 0 & 0 \\ \hline 0 & \begin{matrix} 1 & & \\ & 1 & \\ & & 110 \\ & & 1 \end{matrix} & 0 & 0 \end{array}\right]$$

$$= 10^4 \left[\begin{array}{c|c|c|c} 0 & \begin{matrix} 1 & & \\ & 1.2 & 6 \\ & -6 & -20 \end{matrix} & 0 & 0 \\ \hline 0 & & 0 & 0 \end{array}\right]$$

$$G_{MN}^T K_{MM} G_{MN} = 10^4 \left[\begin{array}{cc|cc} 0 & & 0 & \\ \hline 1 & & 1 & \\ & 1 & & 1 \\ 10 & 1 & 110 & 1 \\ \hline 0 & & 0 & \\ \hline 0 & & 0 & \end{array}\right] \left[\begin{array}{c|c|c|c} 0 & \begin{matrix} 1 & & \\ & 1.2 & 6 \\ & -6 & -20 \end{matrix} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

$$G_{MN}^T K_{MM} G_{MN} = 10^4 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \begin{matrix} 1 & & & \\ & 1.2 & & 6 \\ & & & \\ & 6 & & 40 \end{matrix} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Agrees w/ MYSTRAN

$$\therefore K_{NN} = 10^4 \begin{array}{c|cccccc|cccccc|ccc|ccc|c}
 & & & 101 & & & & & & 102 & & & & & 103 & & & 104 & & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 & 3 & 4 & 5 & 6 & 3 & 4 & 5 & 3 & 4 & 5 & \\
 & S & S & S & S & S & S & F & F & S & S & S & F & S & S & S & S & S & S & \\
\hline
 & 1 & & & & & & -1 & & & & & & & & & & & & 1 \\
 & & 1.2 & & & & 6 & & -1.2 & & & & 6 & & & & & & & 2 \\
 & & & 1.2 & & -6 & & & & -1.2 & & & -6 & & & & & & & 3 \\
 & & & & 3.759 & & & & & & -3.759 & & & & & & & & & 4 \\
 & & & -6 & & 40 & & & & 6 & & 20 & & & & & & & & 5 \\
 & & 6 & & & & 40 & & -6 & & & & 20 & & & & & & & 6 \\
\hline
 & 1 & & & & & & 1 & & & & & & & & & & & & 1 \\
 & & -1.2 & & & & -6 & & 1.2 & & & & -6 & & & & & & & 2 \\
 & & & -1.2 & & 6 & & & & 2.4 & & & & -1.2 & & -6 & & & & 3 \\
 & & & & -3.759 & & & & & & 7.518 & & & & -3.759 & & & & & 4 \\
 & & & -6 & & 20 & & & & & & 80 & & 6 & & 20 & & & & 5 \\
 & & 6 & & & & 20 & & -6 & & & & 40 & & & & & & & 6 \\
\hline
 & & & & & & & & & -1.2 & & 6 & & 1.2012 & & 5.94 & -.0012 & & -.06 & 3 \\
 & & & & & & & & & & -3.759 & & & & 4.135 & & & -.376 & & 4 \\
 & & & & & & & & & -6 & & 20 & & 5.94 & & 44 & .06 & & 2 & 5 \\
\hline
 & & & & & & & & & & & & & -.0012 & & .06 & .0012 & & .06 & 3 \\
 & & & & & & & & & & & & & & -.376 & & & .376 & & 4 \\
 & & & & & & & & & & & & & -.06 & & 2 & .06 & & 4 & 5
\end{array}$$

Agrees w/ MYSTRAN

43 nonzeros on, and above, the diagonal

The loads are on M-set DOF's 113-1, 113-2 only

$$\therefore \bar{P}_N = 0 \;, \quad P_M = \begin{array}{c} \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right\} \begin{array}{l} 103\text{-}1 \\ 103\text{-}2 \\ 103\text{-}6 \end{array} \\ \left\{ \begin{array}{c} 1000 \\ 3 \\ 0 \end{array} \right\} \begin{array}{l} 113\text{-}1 \\ 113\text{-}2 \\ 113\text{-}6 \end{array} \end{array}$$

$$\therefore P_N = \bar{P}_N + G_{MN}^T P_M \Rightarrow G_{MN}^T P_M$$

$$P_N = \left[\begin{array}{cc|cc} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \hline 1 & & 1 & \\ & 1 & & 1 \\ & & & \\ & & & \\ & & & \\ & 10 & 1 & 110 \quad 1 \\ \hline & & & \\ & & & \\ & & & \\ \hline & & & \\ & & & \\ & & & \end{array}\right] \begin{array}{c} \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right\} \\ \left\{ \begin{array}{c} 1000 \\ 3 \\ 0 \end{array} \right\} \end{array} = \begin{array}{c} \left\{ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline 1000 \\ 3 \\ 0 \\ 0 \\ 0 \\ 330 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{array} \right\} \end{array} \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \hline 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \hline 3 \\ 4 \\ 5 \\ \hline 3 \\ 4 \\ 5 \end{array} \begin{array}{l} 101 \\ \\ \\ \\ \\ \\ \\ \\ \\ 102 \\ \\ \\ \\ \\ \\ 103 \\ \\ \\ 104 \end{array} \begin{array}{l} \\ \\ \\ \\ \\ \\ \\ \text{F-set DOF} \\ \text{F-set DOF} \\ \\ \\ \\ \text{F-set DOF} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array}$$

DOF's 102-1, 102-2, 102-6 are F (and A) sets - all others are S-set

$$\therefore K_{AA} = K_{FF} = 10^4 \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 102\ 2 & 6 \end{array} \\ \left[\begin{array}{c|c|c} 1 & 0 & 0 \\ \hline 0 & 1.2 & -6 \\ \hline 0 & -6 & 40 \end{array}\right] \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 6 \end{array}$$

and

$$P_A = P_F = \begin{array}{c} \left[\begin{array}{c} 1000 \\ 3 \\ 330 \end{array}\right] \end{array}\begin{array}{c} 1 \\ 2 \\ 6 \end{array}$$

Solve

$$K_{AA}\, u_A = P_A$$

$$10^4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.2 & -6 \\ 0 & -6 & 40 \end{bmatrix} \begin{Bmatrix} u_{102} \\ v_{102} \\ \theta_{z_{102}} \end{Bmatrix} = \begin{Bmatrix} 1000 \\ 3 \\ 330 \end{Bmatrix}$$

$$\therefore u_{102} = \frac{1000}{10^4} = 0.1$$

and v, θ_z are found from:

$$10^4 \begin{bmatrix} 1.2 & -6 \\ -6 & 40 \end{bmatrix} \begin{Bmatrix} v_{102} \\ \theta_{z_{102}} \end{Bmatrix} = \begin{Bmatrix} 3 \\ 330 \end{Bmatrix}$$

$$\begin{Bmatrix} v_{102} \\ \theta_{z_{102}} \end{Bmatrix} = 10^{-4} \frac{1}{(1.2)(40) - 6^2} \begin{bmatrix} 40 & 6 \\ 6 & 1.2 \end{bmatrix} \begin{Bmatrix} 3 \\ 330 \end{Bmatrix}$$

$$= 10^{-4} \begin{bmatrix} 3.33333 & .5 \\ .5 & .1 \end{bmatrix} \begin{Bmatrix} 3 \\ 330 \end{Bmatrix}$$

$$u_A = u_F \qquad \begin{Bmatrix} u_{102} \\ v_{102} \\ \theta_{z_{102}} \end{Bmatrix} = \begin{Bmatrix} .1175 \\ .0175 \\ .00345 \end{Bmatrix}$$

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The M-set is recovered from

$$u_M = G_{MN}\, u_N \qquad (u_N \text{ in } u_A \text{ (or } u_F\text{) with all zeros for other DOF's})$$

$$\begin{bmatrix} u_{103} \\ v_{103} \\ \theta_{z,103} \\ u_{113} \\ v_{113} \\ \theta_{z,113} \end{bmatrix} = \left[\begin{array}{cccc|c|c} & & 1 & & & \\ & & & 1 & 10 & \\ & & & & 1 & \\ \hline & & 1 & & & \\ & & & 1 & 110 & \\ & & & & 1 & \end{array}\right] \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline .1 \\ .0175 \\ 0 \\ 0 \\ \hline 0 \\ .00345 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{array}\right] = \begin{array}{c} \begin{bmatrix} .1 \\ .052 \\ .00345 \end{bmatrix} \\ \begin{bmatrix} .1 \\ .397 \\ .00345 \end{bmatrix} \end{array} \begin{array}{l} 103\text{-}1 \\ 103\text{-}2 \\ 103\text{-}6 \\ 113\text{-}1 \\ 113\text{-}2 \\ 113\text{-}6 \end{array}$$

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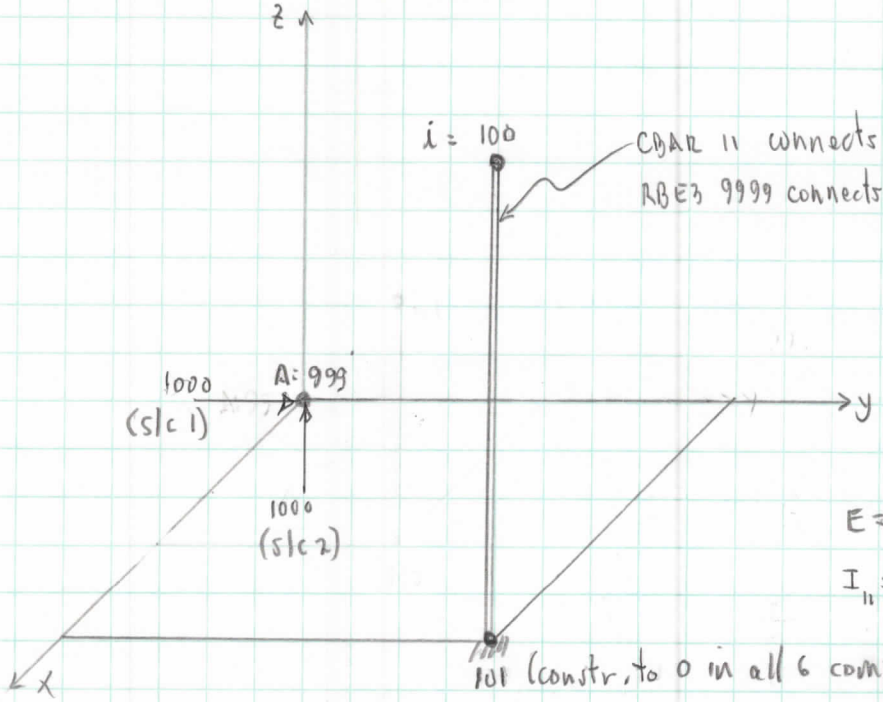
u_a and u_m are the only nonzero disp's

RBE3-01

RBE3-01



	x	y	z
99	0.	0.	0.
100	3.	3.	0.
101	3.	3.	4.

$E = 8\times10^6$, $G = 3.2\times10^6$

$I_{11} = I_{22} = 1.$, $A = 1.$, $J = 1$

$K_{NN}u_N = 10^5$

1 u_{100}	2 v_{100}	3 w_{100}	4 θ_{x100}	5 θ_{y100}	6 θ_{z100}	1 u_{101}	2 v_{101}	3 w_{101}	4 θ_{x101}	5 θ_{y101}	6 θ_{z101}	
15				-30		-15				-30		u_{100}
	15		30				-15		30			v_{100}
		20						-20				w_{100}
	30		80				-30		40			θ_{x100}
-30				80		30				40		θ_{y100}
					8						-8	θ_{z100}
-15				30		15				30		u_{101}
	-15		-30				15		-30			v_{101}
		-20						20				w_{101}
	30		40				-30		80			θ_{x101}
-30				40		30				80		θ_{y101}
					-8						8	θ_{z101}

(1)

(since no elements connect to u_m (grid 999) there are no terms in K_{NN} or K_{mm} and eqn (1) are the only nonzero terms in K_{GG}. The N set consists of grids 100, 101 and the M set is grid 99.

The G set load vector, P_G, only has terms for the M set (no applied loads on the N set):

$$P_G = \begin{Bmatrix} \bar{P}_N \\ P_M \end{Bmatrix} \qquad \bar{P}_N = 0 \text{ no applied loads on the N set} \tag{2}$$

$$P_M = \begin{array}{c} \begin{array}{cc} s/c\ 1 & s/c\ 2 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1000 & 0 \\ 0 & 1000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array} \tag{3}$$

The constraint eqns are

$$R_{MG} u_G = 0 \quad \text{or} \quad \begin{bmatrix} R_{MN} & R_{MM} \end{bmatrix} \begin{Bmatrix} u_N \\ u_M \end{Bmatrix} = 0 \tag{4}$$

where

$$R_{MG} = \begin{array}{c} \begin{array}{cccccc|cccccc|cccccc} \multicolumn{6}{c|}{100} & \multicolumn{6}{c|}{101} & \multicolumn{6}{c}{999} \\ 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 & 3 & 4 & 5 & 6 & 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccccc|cccccc|ccc|ccc} -1 & & & & & & & & & & & & 1 & & & & -4 & 3 \\ & -1 & & & & & & & & & & & & 1 & & 4 & & -3 \\ & & -1 & & & & & & 0 & & & & & & 1 & -3 & 3 & \\ \hline & -4 & 3 & & & & & & & & & & & 4 & -3 & 25 & & \\ 4 & & -3 & & & & & & & & & & -4 & & 3 & & 25 & \\ -3 & 3 & & & & & & & & & & & 3 & -3 & & & & 18 \end{array}\right] \\ \underbrace{\qquad\qquad\qquad\qquad}_{R_{MN}} \qquad \underbrace{\qquad\qquad\qquad\qquad}_{R_{MM}} \end{array} \tag{5}$$

(see attached Appx for details)

Solving (4) for u_M in terms of u_N we get:

$$u_M = \underbrace{-R_{MM}^{-1} R_{MN}}_{G_{MN}} u_N \tag{6}$$

Using G_{MN} from MYSTRAN (see check on G_{MN} in App A):

$$G_{MN} = \begin{array}{c} \begin{array}{cccccc} \overbrace{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}^{100} & \overbrace{1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6}^{101} \end{array} \\ \left[\begin{array}{ccc|ccc|ccc|ccc} 1 & & & & & & & & & & & \\ & 1 & & & 0 & & & 0 & & & 0 & \\ & & 1 & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & 0 & & & 0 & & & 0 & & & 0 & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & 0 & & & 0 & & & 0 & & & 0 & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & 0 & & & 0 & & & 0 & & & 0 & \\ & & & & & & & & & & & \end{array}\right] \end{array} \begin{array}{c} 999-1 \\ -2 \\ -3 \\ -4 \\ -5 \\ -6 \end{array} \qquad (7)$$

The reduced loads on the N set are

$$P_N = \bar{P}_N + G_{MN}^T P_M \qquad (\bar{P}_N = 0 \text{ in this prob})$$

$$P_N = \underbrace{\left[\begin{array}{ccc|ccc|ccc|ccc} 1 & & & & & & & & & & & \\ & 1 & & & 0 & & & & & & & \\ & & 1 & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & 0 & & & 0 & & & & & & & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & 0 & & & 0 & & & & & & & \\ & & & & & & & & & & & \\ \hline & & & & & & & & & & & \\ & 0 & & & 0 & & & & & & & \\ & & & & & & & & & & & \end{array}\right]}_{G_{MN}} \underbrace{\begin{bmatrix} 0 & 0 \\ 1000 & 0 \\ 0 & 1000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{P_M \text{ loads on grid } 999} = \underbrace{\begin{array}{cc} s/c\ 1 & s/c\ 2 \\ \begin{bmatrix} 0 & 0 \\ 1000 & 0 \\ 0 & 1000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}}_{P_N \text{ loads on grid } 100} \qquad (8)$$

From here on the soln for displs is:

$$K_{NN} u_N = P_N \qquad (9)$$

and is not solved here since it is standard MYSTRAN stuff. However, we will go through the calculation of constraint forces.

From MYSTRAN the displs are

$$
u_G = \begin{array}{c}
\begin{array}{cc} \text{s/c 1} & \text{s/c 2} \end{array} \\
\left[\begin{array}{c|c}
0 & 0 \\
2.666667\times10^{-3} & 0 \\
0 & 5.\times10^{-4} \\
-1.0\times10^{-3} & 0 \\
0 & 0 \\
0 & 0 \\
\hline
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\hline
0 & 0 \\
2.666667\times10^{-3} & 0 \\
0 & 5\times10^{-4} \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]
\end{array}
\begin{array}{l}
100\text{-}1 \\ \text{-}2 \\ \text{-}3 \\ \text{-}4 \\ \text{-}5 \\ \text{-}6 \\
101\text{-}1 \\ \text{-}2 \\ \text{-}3 \\ \text{-}4 \\ \text{-}5 \\ \text{-}6 \\
999\text{-}1 \\ \text{-}2 \\ \text{-}3 \\ \text{-}4 \\ \text{-}5 \\ \text{-}6
\end{array}
\begin{array}{l}
\left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\\0\end{array}}\right\} u_F \\
\left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\\0\end{array}}\right\} u_S \\
\left.\vphantom{\begin{array}{c}0\\0\\0\\0\\0\\0\end{array}}\right\} u_M
\end{array}
\qquad (10)
$$

The SPC forces are

$$Q_{s_{spc}} = K_{FS}^T u_F + K_{SS}\cancel{Y_s}^{\,0} - \cancel{P_s}^{\,0} \quad \text{on } u_s \text{ DOF}$$

K_{FS}^T is the 6×6 lower left partition in (1) and u_F is the upper 6×2 partition in (10):

$$
Q_{s_{spc}} = 10^5 \begin{array}{c|cccccc}
 & \multicolumn{6}{c}{\overbrace{\hspace{6em}}^{100}} \\
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
101\text{-}1 & -15 & & & & 30 & \\
\text{-}2 & & -15 & & -30 & & \\
\text{-}3 & & & -20 & & & \\
\text{-}4 & & 30 & & 40 & & \\
\text{-}5 & -30 & & & & 40 & \\
\text{-}6 & & & & & & -8
\end{array}
\left[\begin{array}{c|c}
0 & 0 \\
2.666667\times10^{-3} & 0 \\
0 & 5.\times10^{-4} \\
-1.\times10^{-3} & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]
=
\begin{array}{c}
\begin{array}{cc} \text{s/c 1} & \text{s/c 2} \end{array} \\
\left[\begin{array}{c|c}
0 & 0 \\
-1000 & 0 \\
0 & -1000 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]
\end{array}
\begin{array}{l}
101\text{-}1 \\ \text{-}2 \\ \text{-}3 \\ \text{-}4 \\ \text{-}5 \\ \text{-}6
\end{array}
$$

Same Q_s as MYSTRAN

(11)

The MPC forces are (on N, M sets)

$$Q_{G_{MPC}} = \begin{bmatrix} -G_{MN}^T \\ I_{MM} \end{bmatrix} \left[(\cancel{K_{NM}^T}^{\,0} + \cancel{K_{MM}}^{\,0} G_{MN}) u_N - P_M \right] \quad \text{on G-set} \qquad (12)$$

for this prob.

$$= \begin{bmatrix} G_{NN}^T P_M \\ -P_M \end{bmatrix} \begin{matrix} \text{MPC forces on N set (grids 100, 101} \\ \text{MPC forces on M set (grid 999)} \end{matrix} \qquad (13)$$

from top of (13)

$$Q_{N_{MPC}} = G_{MN}^T P_M = \begin{bmatrix} 1 & & & & & \\ & 1 & & & 0 & \\ & & 1 & & & \\ \hline & 0 & & & 0 & \\ \hline & 0 & & & 0 & \\ \hline & 0 & & & 0 & \end{bmatrix} \begin{array}{c} \begin{array}{cc} \text{s/c 1} & \text{s/c 2} \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1000 & 0 \\ 0 & 1000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} \text{s/c 1} & \text{s/c 2} \end{array} \\ \begin{bmatrix} 0 & 0 \\ 1000 & 0 \\ 0 & 1000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array} \begin{array}{r} 100-1 \\ -2 \\ -3 \\ -4 \\ -5 \\ -6 \\ 101-1 \\ -2 \\ -3 \\ -4 \\ -5 \\ -6 \end{array}$$

MPC forces on grids 100, 101
(same as MYSTRAN

from bottom of (13)

$$Q_{M_{MPC}} = -P_M = \begin{bmatrix} 0 & 0 \\ -1000 & 0 \\ 0 & -1000 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{array}{r} 999-1 \\ -2 \\ -3 \\ -4 \\ -5 \\ -6 \end{array}$$

MPC forces on grid 999
(same as MYSTRAN

Appendix A

(for CBAR_RBE3_3)

The constraint eqns on $u_{x_A}, u_{y_A}, u_{z_A}, \theta_{x_A}, \theta_{y_A}, \theta_{z_A}$ are summarized below:

$$u_{x_A} - \bar{d}_z \theta_{y_A} + \bar{d}_y \theta_{z_A} - \frac{1}{W_T} \sum^N \omega_i u_{x_i} = 0$$

$$u_{y_A} + \bar{d}_z \theta_{x_A} - \bar{d}_x \theta_{z_A} - \frac{1}{W_T} \sum^N \omega_i u_{y_i} = 0$$

$$u_{z_A} - \bar{d}_y \theta_{x_A} + \bar{d}_x \theta_{y_A} - \frac{1}{W_T} \sum^N \omega_i u_{z_i} = 0$$

$$\bar{e}_{yz} \theta_{x_A} + W_T \bar{d}_z u_{y_A} - W_T \bar{d}_y u_{z_A} - \sum^N \omega_i d_{z_i} u_{y_i} + \sum^N \omega_i d_{y_i} u_{z_i} = 0$$

$$\bar{e}_{zx} \theta_{y_A} - W_T \bar{d}_z u_{x_A} + W_T \bar{d}_x u_{z_A} + \sum^N \omega_i d_{z_i} u_{x_i} - \sum^N \omega_i d_{x_i} u_{z_i} = 0$$

$$\bar{e}_{xy} \theta_{z_A} + W_T \bar{d}_y u_{x_A} - W_T \bar{d}_x u_{y_A} - \sum^N \omega_i d_{y_i} u_{x_i} + \sum^N \omega_i d_{x_i} u_{y_i} = 0$$

where

a) $W_T = \sum^N \omega_i$, $\bar{e}_{xy} = \sum^N \omega_i (d_{x_i}^2 + d_{y_i}^2)$, $\bar{e}_{yz} = \sum^N \omega_i (d_{y_i}^2 + d_{z_i}^2)$, $\bar{e}_{zx} = \sum^N \omega_i (d_{z_i}^2 + d_{x_i}^2)$

b) $\bar{d}_x = \sum^N \omega_i d_{x_i}$, $\bar{d}_y = \sum^N \omega_i d_{y_i}$, $\bar{d}_z = \sum^N \omega_i d_{z_i}$

c) $d_{x_i}, d_{y_i}, d_{z_i}$ are the coords of pt i <u>relative</u> to pt A in the global coord. sys at A

d) $u_{x_i}, u_{y_i}, u_{z_i}$ are displ's at i transformed from global coord sys. at i to global coord sys at A

e) $u_{x_A}, u_{y_A}, u_{z_A}, \theta_{x_A}, \theta_{y_A}, \theta_{z_A}$ are displ's at A in the global coord. sys. at A (obviously)

f) The weights ω_i are assigned by the user (and are often 1.0)

The RBE3 connection entry in CBAR_RBE3_3.DAT is

1	2	3	4	5	6	7	8	9
RBE3	9999		999	123456	1.	123	100	
	EID		pt A	Comp's at A		Comp's at i	pt i = 1	

Using the eqns on the prev pg and the data for the prob CBAR_RBE3-3 (with 1 "indep" pt)

$$W_T = \sum \omega_i = 1$$

$$\bar{d}_x = 3, \ \bar{d}_y = 3, \ \bar{d}_z = 4$$

$$d_{x_i} = 3, \ d_{y_i} = 3, \ d_{z_i} = 4$$

$$\bar{e}_{xy} = 3^2 + 3^2 = 18$$

$$\bar{e}_{yz} = 3^2 + 4^2 = 25$$

$$\bar{e}_{zx} = 4^2 + 3^2 = 25$$

So the 6 constraint eqns are (A = 999):

$$\left.\begin{aligned}
u_{x_A} - 4\theta_{y_A} + 3\theta_{z_A} - u_{x_{100}} &= 0 \\
u_{y_A} + 4\theta_{x_A} - 3\theta_{z_A} - u_{y_{100}} &= 0 \\
u_{z_A} - 3\theta_{x_A} + 3\theta_{y_A} - u_{z_{100}} &= 0
\end{aligned}\right\} \text{coeffs for RMG matrix}$$

$$\left.\begin{aligned}
25\theta_{x_A} + 4u_{y_A} - 3u_{z_A} - 4u_{y_{100}} + 3u_{z_{100}} &= 0 \\
25\theta_{y_A} - 4u_{x_A} + 3u_{z_A} + 4u_{x_{100}} - 3u_{z_{100}} &= 0 \\
18\theta_{z_A} + 3u_{x_A} - 3u_{y_A} - 3u_{x_{100}} + 3u_{y_{100}} &= 0
\end{aligned}\right\}$$

which is the same as eqn (5) in the main derivation of sol'n for CBAR_RBE3_3.

Therefore RMG is correct

As a check on G_{MN} in eqns (6), (7) in the main part of this example notice that

$$G_{MN} = -R_{MM}^{-1} R_{MN}$$

so that

$$R_{MM} G_{MN} = -R_{MN}$$

where R_{MM} is in eqn (5) and G_{MN} in eqn (7) so:

$$R_{MM} G_{MN} = \left[\begin{array}{ccc|ccc} 1 & & & & -4 & 3 \\ & 1 & & 4 & & -3 \\ & & 1 & -3 & 3 & \\ \hline & 4 & -3 & 25 & & \\ -4 & & 3 & & 25 & \\ 3 & -3 & & & & 18 \end{array}\right] \left[\begin{array}{ccc|c|c|c} 1 & & & & & \\ & 1 & & 0 & 0 & 0 \\ & & 1 & & & \\ \hline & & & & & \\ & 0 & & 0 & 0 & 0 \\ & & & & & \end{array}\right]$$

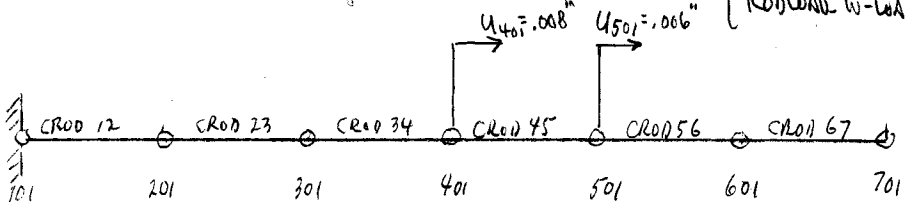
$$= \left[\begin{array}{ccc|c|c|c} 1 & & & & & \\ & 1 & & 0 & 0 & 0 \\ & & 1 & & & \\ \hline & 4 & -3 & & & \\ -4 & & 3 & 0 & 0 & 0 \\ 3 & -3 & & & & \end{array}\right]$$

which is the neg of the R_{MN} partition in eqn (5)

RODDISPL

(Runs RODDISPL-N-LOAD and RODDISPL-W-LOAD)

<u>RODDISPL</u> : Rod with enforced displ's – solved twice — { RODLOAD_N_LOAD with no applied loads; RODLOAD_W_LOAD " applied loads }



CROD	A_e	E_e	l_e	A_eE_e/l_e
12	.6	10×10^6	10	6×10^5
23	.5	↓	↓	5×10^5
34	.4			4×10^5
45	.3			3×10^5
56	.2			2×10^5
67	.1			1×10^5

Grid	Y_s	Applied Load S/C 1	Applied Load S/C 2
101			
201			240 lb
301			150 lb
401			200 lb
501	.008"		
601	.006"	100 lb	
701		120 lb	

Grid	Grid_Seq	Grid-DOF	Loads S/C 1	Loads S/C 2
701	1	701-1	120 lb	
601	2	601-1	100	
201	3	201-1		240
501	4	501-1		
301	5	301-1		150
101	6	101-1		
401	7	401-1		200 lb

$K = 10^5$

	701	601	201	501	301	101	401
F	1 FF	-1 FF					
F	-1 FF	3 FF		-2			
F			11 FF		-5 FF	-6	
		-2		5 SS			-3 SS
F			-5 FF		9 FF		-4
			-6			6 SS	
				-3 SS	-4		7 SS
	F	F	F		F		

	1	2	3	4	5	6	
701	1	2	3	4	5	6	
601	1	2	3	4	5	6	
201	1	2	3	4	5	6	
501	1	2	3	4	5	6	
301	1	2	3	4	5	6	
101	1	2	3	4	5	6	
401	1	2	3	4	5	6	

$P_g =$

	S/C 1	S/C 2						
701	120		1	2	3	4	5	6
601	100		7	8	9	10	11	12
201		240	13	14	15	16	17	18
501			19	20	21	22	23	24
301		150	25	26	27	28	29	30
101			31	32	33	34	35	36
401		280	37	38	39	40	41	42

	701 1	601 1	201 1	301 1	701 2	701 3	701 4	701 5	701 6	601 2	601 3	601 4	601 5	601 6	201 2	201 3	201 4	201 5	201 6	301 2	301 3	301 4	301 5	301 6	501 1	501 2	501 3	501 4	501 5	501 6	101 6	101 1	101 2	101 3	101 4	101 5	101 6	401 1	401 2	401 3	401 4	401 5	401 6
701-1	1	-1																													-6													
601-1	-1	3																									-2																	
701-1			11	-5																																								
301-1			-5	9																																	-4							
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$K_{nn} \approx 10^5$

$= \begin{bmatrix} K_{ff} & K_{fs} \\ K_{fs}^T & K_{ss} \end{bmatrix}$

	S/c 1	S/c 2	
701-1	120		701-1
601-1	100		601-1
701-1		240	701-1
301-1		150	301-1
401-1		200	401-1

$\bar{P}_f$

P_s

$P_n = \begin{Bmatrix} \bar{P}_f \\ P_s \end{Bmatrix} =$

$$K_{ff} U_f = P_f$$

$$P_f = \bar{P}_f - K_{fs} U_s = \bar{P}_f - K_{fs_e} Y_{s_e}$$

where

$$K_{ff} = 10^5 \begin{array}{c} \begin{bmatrix} 1 & -1 & & \\ -1 & 3 & & \\ & & 11 & -5 \\ & & -5 & 9 \end{bmatrix} \end{array} \begin{array}{l} 701\text{-}1 \\ 601\text{-}1 \\ 201\text{-}1 \\ 301\text{-}1 \end{array}$$

K_{fs_e} is the partition from K_{fs} for the col's associated with the enforced displ DOF's (501-TX, 401-TX):

$$K_{fs_e} = 10^5 \begin{array}{c} \begin{array}{cc} 501\text{-}1 & 401\text{-}1 \end{array} \\ \begin{bmatrix} 0 & 0 \\ -2 & 0 \\ 0 & 0 \\ 0 & -4 \end{bmatrix} \end{array}, \quad Y_{s_e} = \begin{Bmatrix} .006 \\ .008 \end{Bmatrix} \begin{array}{l} 501\text{-}1 \\ 401\text{-}1 \end{array}$$

$$\therefore K_{fs_e} Y_{s_e} = 10^5 \begin{bmatrix} 0 & 0 \\ -2 & 0 \\ 0 & 0 \\ 0 & -4 \end{bmatrix} \begin{Bmatrix} .006 \\ .008 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1200 \\ 0 \\ -3200 \end{Bmatrix} \begin{array}{l} 701\text{-}1 \\ 601\text{-}1 \\ 201\text{-}1 \\ 301\text{-}1 \end{array} \quad (= PFYS)$$

$$\bar{P}_f = \underbrace{\begin{array}{c} \begin{array}{cc} S/C\ 1 & S/C\ 2 \end{array} \\ \begin{bmatrix} 120 & 0 \\ 100 & 0 \\ 0 & 240 \\ 0 & 150 \end{bmatrix} \end{array} \begin{array}{l} 701\text{-}1 \\ 601\text{-}1 \\ 201\text{-}1 \\ 301\text{-}1 \end{array}}_{\text{RODLOAD-W-LOAD (2 subcases)}}, \quad \bar{P}_f = \underbrace{\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}}_{\text{RODLOAD-N-LOAD (1 subcase)}}$$

$$\therefore P_f = \underbrace{\begin{bmatrix} 120 & 0 \\ 1300 & 1200 \\ 0 & 240 \\ 3200 & 3350 \end{bmatrix}}_{\text{RODLOAD-W-LOAD (2 subcases)}}, \quad P_F = \underbrace{\begin{Bmatrix} 0 \\ 1200 \\ 0 \\ 3200 \end{Bmatrix}}_{\text{RODLOAD-N-LOAD (1 subcase)}} \begin{array}{l} 701\text{-}1 \\ 601\text{-}1 \\ 201\text{-}1 \\ 301\text{-}1 \end{array}$$

Since there are no OMIT's, the F-set is also the A-set

$$K_{aa} = 10^5 \begin{bmatrix} 1 & -1 & & \\ -1 & 3 & & \\ & & 11 & -5 \\ & & -5 & 9 \end{bmatrix}, \quad P_a = \begin{array}{c} \begin{array}{cc} s/c\,1 & s/c\,2 \end{array} \\ \begin{bmatrix} 120 & 0 \\ 1300 & 1200 \\ 0 & 240 \\ 3200 & 3350 \end{bmatrix} \end{array}$$

Note

$$K_{aa} = \begin{bmatrix} K_{11} & \theta \\ \theta & K_{22} \end{bmatrix}, \quad K_{11} = 10^5 \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}, \quad K_{22} = 10^5 \begin{bmatrix} 11 & -5 \\ -5 & 9 \end{bmatrix}$$

So

$$K_{aa}^{-1} = \begin{bmatrix} K_{11}^{-1} & \theta \\ \theta & K_{22}^{-1} \end{bmatrix}, \quad K_{11}^{-1} = 10^{-5} \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad K_{22}^{-1} = 10^{-5} \begin{bmatrix} 9/74 & 5/74 \\ 5/74 & 11/74 \end{bmatrix}$$

$$\therefore \; u_a = K_{aa}^{-1} P_a = 10^{-5} \begin{bmatrix} 1.5 & 0.5 & & \\ 0.5 & 0.5 & & \\ & & 9/74 & 5/74 \\ & & 5/74 & 11/74 \end{bmatrix} \underbrace{\begin{bmatrix} 120 & 0 \\ 1300 & 1200 \\ 0 & 240 \\ 3200 & 3350 \end{bmatrix}}_{\text{RODLOAD-W-LOAD (2 SUBCASES)}} \text{ and } \underbrace{\begin{bmatrix} 0 \\ 1200 \\ 0 \\ 3200 \end{bmatrix}}_{\text{RODLOAD-N-LOAD (1 SUBCASE)}}$$

$$u_a = u_f = \begin{array}{r} 701\text{-}1 \\ 601\text{-}1 \\ 201\text{-}1 \\ 301\text{-}1 \end{array} \begin{bmatrix} 8.300000 \times 10^{-3} & 6.000000 \times 10^{-3} \\ 7.100000 \times 10^{-3} & 6.000000 \times 10^{-3} \\ 2.162162 \times 10^{-3} & 2.555405 \times 10^{-3} \\ 4.756757 \times 10^{-3} & 5.141892 \times 10^{-3} \end{bmatrix} \text{ and } \begin{bmatrix} 6.000000 \times 10^{-3} \\ 6.000000 \times 10^{-3} \\ 2.162162 \times 10^{-3} \\ 4.756757 \times 10^{-3} \end{bmatrix}$$

Merge in Y_{se} and sort to get u_g:

$$u_g = \begin{array}{r} 101\text{-}1 \\ 201\text{-}1 \\ 301\text{-}1 \\ 401\text{-}1 \\ 501\text{-}1 \\ 601\text{-}1 \\ 701\text{-}1 \end{array} \underbrace{\begin{array}{c} \begin{array}{cc} s/c\,1 & s/c\,2 \end{array} \\ \begin{bmatrix} 0. & 0. \\ 2.162162 \times 10^{-3} & 2.555405 \times 10^{-3} \\ 4.756757 \times 10^{-3} & 5.141892 \times 10^{-3} \\ 8.000000 \times 10^{-3} & 8.000000 \times 10^{-3} \\ 6.000000 \times 10^{-3} & 6.000000 \times 10^{-3} \\ 7.100000 \times 10^{-3} & 6.000000 \times 10^{-3} \\ 8.300000 \times 10^{-3} & 6.000000 \times 10^{-3} \end{bmatrix} \end{array}}_{\text{RODDISPL-W-LOAD}} \text{ and } \underbrace{\begin{bmatrix} 0. \\ 2.162162 \times 10^{-3} \\ 4.756757 \times 10^{-3} \\ 8.000000 \times 10^{-3} \\ 6.000000 \times 10^{-3} \\ 6.000000 \times 10^{-3} \\ 6.000000 \times 10^{-3} \end{bmatrix}}_{\text{RODDISPL-N-LOAD}}$$

Constraint Forces: $Q_s = K_{fs}^T u_f + K_{ss_e} Y_{s_e} - P_s$

$K_{ff} u_f = 10^5$

				2
				3
				4 701
				5
				6
				2
				3
				4 601
				5
				6
				2
				3
				4 201
				5
				6
	-2			1
				2
				3
				4 501
				5
				6
				2
				3
				4 301
				5
				6
		-6		1
				2
				3
				4 101
				5
				6
			-4	1
				2
				3
				4 401
				5
				6

$$\underbrace{\begin{bmatrix} \overbrace{\begin{matrix} 8.300000\times10^{-3} \\ 7.100000\times10^{-3} \end{matrix}}^{s/c1} & \overbrace{\begin{matrix} 6.000000\times10^{-3} \\ 6.000000\times10^{-3} \end{matrix}}^{s/c2} \\ \begin{matrix} 2.162162\times10^{-3} \\ 4.756757\times10^{-3} \end{matrix} & \begin{matrix} 2.555405\times10^{-3} \\ 5.141892\times10^{-2} \end{matrix} \end{bmatrix}}_{\text{RODDISPL-W-LOAD}} \text{ and } \underbrace{\overbrace{\begin{bmatrix} 6.000000\times10^{-3} \\ 6.000000\times10^{-3} \\ 2.162162\times10^{-3} \\ 4.756757\times10^{-3} \end{bmatrix}}^{s/c1}}_{\text{RODDISPL-N-LOAD}} =$$

RODDISPL-W-LOAD s/c1	RODDISPL-W-LOAD s/c2		s/c1 RODDISPL-N-LOAD
-1420	-1200	and	-1200
-1297.297	-1533.243		-1297.297
-1902.703	-2056.757		-1902.703

for either RODDISPL-W-LOAD or RODDISPL-N-LOAD (1 subcase)

| 501-1 | 401-1 | | |

RODDISPL-W-LOAD

s/c1	s/c2	
		2 3 4 5 6 (701)
		2 3 4 5 6 (601)
		2 3 4 5 6 (201)
		1 2 3 4 5 6 (501)
		2 3 4 5 6 (301)
		1 2 3 4 5 6 (101)
	200	1 2 3 4 5 6 (401)

$P_s =$

$P_s = 0$ for RODDISPL-N-LOAD

for RODDISPL-W-LOAD:

	s/c 1	s/c 2		s/c 1	s/c 2		s/c 1	s/c 2		s/c 1	s/c 2	
												2
												3
												4 701
												5
												6
												2
												3
												4 601
												5
												6
												2
												3
												4 201
												5
												6
$Q_s =$	-1420	-1200	+	600	600	$\pm$			=	-820.	-600	1
												2
												3
												4 501
												5
												6
												2
												3
												4 301
												5
												6
	-1297.2872	-1533.2430								-1297.2872	-1533.2430	1
												2
												3
												4 101
												5
												6
	-1902.7028	-2056.7568		3800	3800			-200		1897.2971	1543.2431	1
												2
												3
												4 401
												5
												6
									$\Sigma =$	-220	-590	

for RODDISPL-H-LAND

								2	701
								3	
								4	
								5	
								6	
								2	601
								3	
								4	
								5	
								6	
								2	201
								3	
								4	
								5	
								6	
$Q_s =$	-1200	+	600	+	0	=	-600	1	501
								2	
								3	
								4	
								5	
								6	
								2	301
								3	
								4	
								5	
								6	
	-1297.297				0		-1297.297	1	101
								2	
								3	
								4	
								5	
								6	
	-1902.703		3800		0		1897.297	1	401
								2	
								3	
								4	
								5	
								6	
							$\Sigma = 0$		

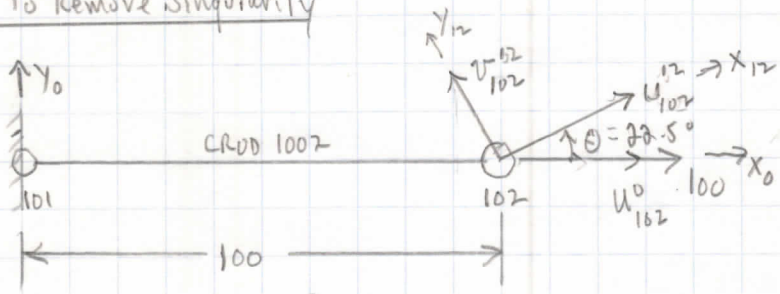
RODLOAD-MPC-FOR-SING

RODLOAD-MPC-FOR-SING

Rod with MPC to Remove Singularity

G.P. 101 Global = 0

G.P. 102 Global = 12



G.P. 102 has Coord Sys 12 as Global

$\cos\theta = .923880$

$\sin\theta = .382683$

$A = 0.1,\ E = 10^7 :\ \frac{AE}{l} = 10^4$

Elem K_e:

$$K_e = \begin{array}{cc} \begin{bmatrix} 10^4 & -10^4 \\ -10^4 & 10^4 \end{bmatrix} & \begin{matrix} u_a \\ u_b \end{matrix} \end{array} \qquad \begin{matrix} u_a = u_{101} \\ u_b = \cos\theta\, u_{102} - \sin\theta\, v_{102} \end{matrix}$$

$$\begin{Bmatrix} u_a \\ u_b \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \end{bmatrix} \begin{Bmatrix} u^0_{101} \\ u^{12}_{102} \\ v^{12}_{102} \end{Bmatrix} \quad \text{or} \quad u_e = T_e u_g$$

The global K for dof's $u_{101}, u_{102}, v_{102}$ is

$$K_{gg} = T_e^T K_e T_e$$

$$= 10^4 \begin{bmatrix} 1 & 0 \\ 0 & c \\ 0 & -s \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \end{bmatrix} \qquad \begin{matrix} c = \cos\theta = .923880 \\ s = \sin\theta = .382683 \end{matrix}$$

(nonzero terms in:) $$K_{gg} = 10^4 \left[\begin{array}{cc|c} 1 & -c & s \\ -c & c^2 & -sc \\ \hline s & -sc & s^2 \end{array}\right] = 10^4 \left[\begin{array}{cc|c} 1 & -.923880 & .382683 \\ -.923880 & .853553 & -.353553 \\ \hline .382683 & -.353553 & .146447 \end{array}\right] \begin{matrix} u^0_{101} \\ u^{12}_{102} \\ v^{12}_{102} \end{matrix}$$

In order to remove the singularity in the Y_0 direction, use MPC:

$$\sin\theta\, u^{12}_{102} + \cos\theta\, v^{12}_{102} = 0$$

So

$$\left[\underbrace{\begin{matrix} 0 & s \end{matrix}}_{R_{mn}} \;\middle|\; \underbrace{c}_{R_{mm}}\right] \begin{Bmatrix} u^0_{101} \\ u^{12}_{102} \\ v^{12}_{102} \end{Bmatrix} = 0$$

$$\left[\begin{matrix} 0 & .382683 \end{matrix} \;\middle|\; .923880\right] \begin{Bmatrix} u^0_{101} \\ u^{12}_{102} \\ v^{12}_{102} \end{Bmatrix} = 0$$

Then

$$G_{mn} = -R_{mm}^{-1} R_{mn} \qquad u_n = \begin{Bmatrix} u^0_{101} \\ u^{12}_{102} \end{Bmatrix}, \; u_m = v^{12}_{102}$$

$$= -\frac{1}{c}\begin{bmatrix} 0 & s \end{bmatrix} = \begin{bmatrix} 0 & -t \end{bmatrix} \qquad t = \frac{s}{c} = \tan\theta = .414213562$$

$$= \begin{bmatrix} 0 & -.414213 \end{bmatrix}$$

Reduce to m set:

$$K_{nn} = \bar{K}_{nn} + K_{nm}G_{mn} + (K_{nm}G_{mn})^T + G_{mn}^T K_{mm} G_{mn}$$

$$\bar{K}_{nn} = 10^4\begin{bmatrix} 1 & -c \\ -c & c^2 \end{bmatrix} = 10^4\begin{bmatrix} 1 & -.923880 \\ -.923880 & .853553 \end{bmatrix}$$

$$K_{nm}G_{mn} = 10^4\begin{bmatrix} s \\ -sc \end{bmatrix}\begin{bmatrix} 0 & -t \end{bmatrix} = 10^4\begin{bmatrix} 0 & -st \\ 0 & sct \end{bmatrix}$$

$$= 10^4\begin{bmatrix} .382683 \\ -.353553 \end{bmatrix}\begin{bmatrix} 0 & -.414214 \end{bmatrix} = 10^4\begin{bmatrix} 0 & -.158513 \\ 0 & .146446 \end{bmatrix} \checkmark$$

$$G_{mn}^T K_{mm} G_{mn} = 10^4\begin{bmatrix} 0 \\ -.414214 \end{bmatrix}\begin{bmatrix} .146447 \end{bmatrix}\begin{bmatrix} 0 & -.414214 \end{bmatrix}$$

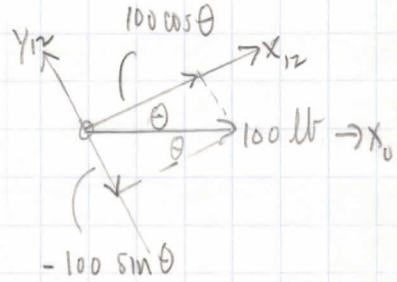
$$= 10^4\begin{bmatrix} 0 & 0 \\ 0 & .0251263 \end{bmatrix}$$

So

$$K_{nn} = 10^4\left\{\begin{bmatrix} 1 & -.923880 \\ -.923880 & .853553 \end{bmatrix} + \begin{bmatrix} 0 & -.158513 \\ 0 & .146446 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -.158513 & .146446 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & .0251263 \end{bmatrix}\right\}$$

$$= 10^4\begin{bmatrix} 1 & -1.08239 \\ -1.08239 & 1.17187 \end{bmatrix} \checkmark \qquad \begin{matrix} u^0_{101} \\ u^{12}_{102} \end{matrix}$$

The loads are



$P = 100$ lb in X_0 direction at G.P. 102

or

$$P_g = \begin{Bmatrix} 0 \\ 100\cos\theta \\ -100\sin\theta \end{Bmatrix} \begin{matrix} u^0_{101} \\ u^{12}_{102} \\ v^{12}_{102} \end{matrix}$$

$$\therefore P_g = \begin{Bmatrix} 0 \\ 92.388 \\ -38.2683 \end{Bmatrix} \checkmark = \begin{Bmatrix} \bar{P}_n \\ P_m \end{Bmatrix}$$

$$P_n = \bar{P}_n + G_{mn}^{\uparrow} P_m = \begin{Bmatrix} 0 \\ 92.388 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -.414214 \end{Bmatrix} \{-38.2683\}$$

$$P_n = \begin{Bmatrix} 0 \\ 108.239 \end{Bmatrix} \checkmark$$

So

$$K_{nn} u_n = P_n + Q_n = 10^4 \left[\begin{array}{c|c} 1 & -1.08239 \\ \hline -1.08239 & 1.17157 \end{array}\right] \left\{\begin{array}{c} u^0_{101} \\ \hline u^{12}_{102} \end{array}\right\} = \left\{\begin{array}{c} 0 \\ \hline 108.239 \end{array}\right\} + \left\{\begin{array}{c} 0 \\ Q_s \end{array}\right\}$$

$$= \begin{bmatrix} K_{ff} & K_{fs} \\ K_{fs} & K_{ss} \end{bmatrix} \begin{Bmatrix} u_f \\ u_s \end{Bmatrix} = \begin{Bmatrix} \bar{P}_n \\ P_m \end{Bmatrix} + \begin{Bmatrix} 0 \\ Q_s \end{Bmatrix}$$

Now $u^0_{101} = 0$ so

$$u^{12}_{102} = \frac{108.239}{1.17157 \times 10^4} = 9.2388 \times 10^{-3} \checkmark$$

and

$$v^{12}_{102} = -\frac{s}{c} u^{12}_{102} = -\frac{.382683}{.923880}\left(9.2388 \times 10^{-3}\right) = -3.82683 \times 10^{-3} \checkmark$$

$$u_f = u^{12}_{102}$$

The SPC forces are

$$Q_s = K_{fs}^T u_f + K_{ss}\cancel{u_s}^{\,0} - \cancel{P_s}^{\,0} = 10^4(-1.08239)(9.23880\times10^{-3}) = 100\ \text{lb at } u_{101}\ \text{dof}$$

The MPC forces are

$$Q_m = K_{nm}^T u_n + K_{mm} u_m - P_m \qquad u_n = \begin{Bmatrix} u_{101} \\ u_{102} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 9.2388\times10^{-3} \end{Bmatrix}$$

$$= 10^4 \begin{bmatrix} .382683 \\ -.353553 \end{bmatrix}^T \begin{Bmatrix} 0 \\ 9.2388\times10^{-3} \end{Bmatrix} + 10^4(.146447)(-3.82683\times10^{-3}) + (38.2683)\Big\}$$

$$= \Big\{ -32.6641 - 5.6043 + 38.2683 \Big\} = \cancel{-.0001}^{\,0}$$

Note also that the displ. in the x_0 direction could have been calculated simply from

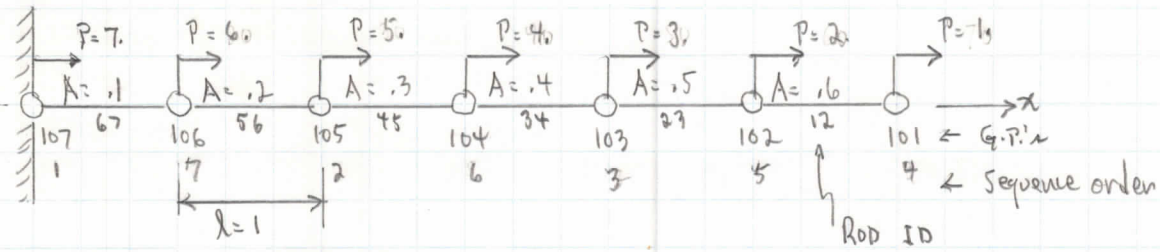
$$u_{102}^0 = \frac{PL}{AE} = \frac{(100)(100)}{(.1)(10^7)} = .0001 = 10\times10^{-3}$$

Then

$$u_{102}^{12} = u_{102}^0 \cos\theta = (10.0)(.923880) = 9.2388\times10^{-3}$$

SOLVE_EQNS_1

SOLVE_EQNS_1.DAT



$$k_e = \frac{A_e E_e}{l_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 10A_e & -10A_e \\ -10A_e & 10A_e \end{bmatrix}$$

$$K = \begin{array}{c} \begin{array}{cccccc} 105 & 103 & 101 & 102 & 104 & 106 \end{array} \\ \begin{bmatrix} 5. & & & & -3. & -2. \\ & 9. & & -5. & -4. & \\ & & 6. & -6 & & \\ & -5 & -6. & 11. & & \\ -3. & -4. & & & 7. & \\ -2. & & & & & 3. \end{bmatrix} \end{array} \begin{array}{c} 105 \\ 103 \\ 101 \\ 102 \\ 104 \\ 106 \end{array}, \quad u = \begin{bmatrix} u_{105} \\ u_{103} \\ u_{101} \\ u_{102} \\ u_{104} \\ u_{106} \end{bmatrix}, \quad P = \begin{bmatrix} 5. \\ 3. \\ 1. \\ 2. \\ 4. \\ 6. \end{bmatrix}$$

Solve

$$Ku = P \tag{1}$$

Factor K into $\mathcal{U}^T D^{-1} \mathcal{U}$

$$\mathcal{U}^T D^{-1} \mathcal{U} u = P \tag{2}$$

Let

$$\mathcal{U} u = y \tag{3}$$

Then, from (2) and (3)

$$\mathcal{U}^T D^{-1} y = P \tag{4}$$

Solve (4) for y in "forward pass", then solve (3) for u in "backward pass"

Method 1: Do Not Equilibrate K:

Triangularize K

Original K

$$\begin{bmatrix} 5. & & & & -3 & -2. \\ & 9. & & -5 & -4 & \\ & & 6 & -6 & & \\ & -5 & -6 & 11. & & \\ -3. & -4 & & & 7. & \\ -2. & & & & & 3. \end{bmatrix} \qquad (a)$$

Add .6 of row 1 (above) to row 5, and Add .4 of row 1 to row 6

$$\begin{bmatrix} 5 & & & & -3 & -2 \\ & 9 & & -5 & -4 & \\ & & 6 & -6 & & \\ & -5 & -6 & 11 & & \\ & -4 & & & 5.2 & -1.2 \\ & & & & -1.2 & 2.2 \end{bmatrix} \qquad (b)$$

Add $\frac{5}{9}$ of row 2 (above) to row 4, and Add 4/9 of row 2 to row 5

$$\begin{bmatrix} 5 & & & & -3 & -2 \\ & 9 & & -5 & -4 & \\ & & 6 & -6 & & \\ & & -6 & \frac{74}{9} & -\frac{20}{9} & \\ & & & -\frac{20}{9} & \frac{154}{45} & -1.2 \\ & & & & -1.2 & 2.2 \end{bmatrix} \qquad (c)$$

Add row 3 (above) to row 4

$$\begin{bmatrix} 5 & & & & -3 & -2 \\ & 9 & & -5 & -4 & \\ & & 6 & -6 & & \\ & & & \frac{20}{9} & -\frac{20}{9} & \\ & & & -\frac{20}{9} & \frac{154}{45} & -1.2 \\ & & & & -1.2 & 2.2 \end{bmatrix} \qquad (d)$$

Add row 4 (above) to row 5

$$\begin{bmatrix} 5 & & & & -3 & -2 \\ & 9 & & -5 & -4 & \\ & & 6 & -6 & & \\ & & & 20/9 & -\frac{20}{9} & \\ & & & & 1.2 & -1.2 \\ & & & & -1.2 & 2.2 \end{bmatrix} \qquad (e)$$

Add row 5 (above) row 6

$$\begin{bmatrix} 5 & & & & -3 & -2 \\ & 9 & & -5 & -4 & \\ & & 6 & -6 & & \\ & & & \frac{20}{9} & -\frac{20}{9} & \\ & & & & 1.2 & -1.2 \\ & & & & & 1 \end{bmatrix} \qquad (f)$$

The matrix in (f) is upper triangular. To make it such that

$$K = U^T D\, U$$

Then divide each row of (f) by the diagonal term in that row.

$$\therefore\ U = \begin{bmatrix} 1 & & & & -3/5 & -2/5 \\ & 1 & & -5/9 & -4/9 & \\ & & 1 & -1 & & \\ & & & 1 & -1 & \\ & & & & 1 & -1 \\ & & & & & 1 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} 5 & & & & & \\ & 9 & & & & \\ & & 6 & & & \\ & & & 20/9 & & \\ & & & & 6/5 & \\ & & & & & 1 \end{bmatrix}$$

Forward Pass

Solve $U^T D x = P$ $\begin{cases} \text{Solve } U^T z = P \text{ for } z \\ \text{then } D x = z \text{ for } x \quad (x = D^{-1} z) \end{cases}$

$$\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & -5/9 & -1 & 1 & & \\ -3/5 & -4/9 & & -1 & 1 & \\ -7/5 & & & & -1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 1 \\ 2 \\ 4 \\ 6 \end{bmatrix}$$

$z_1 = 5$, $x_1 = d_{11}^{-1} z_1 = (1/5)(5) = 1$

$z_2 = 3$, $x_2 = d_{22}^{-1} z_2 = (1/9)(3) = 1/3$

$z_3 = 1$, $x_3 = d_{33}^{-1} z_3 = (1/6)(1) = 1/6$

$z_4 = 2 + z_3 + 5/9\, z_2 = 2 + 1 + 5/3 = 14/3$, $x_4 = d_{44}^{-1} z_4 = (9/20)(14/3) = \frac{42}{20} = 2.1$

$z_5 = 4 + z_4 + \frac{4}{9} z_2 + \frac{3}{5} z_1 = 4 + \frac{14}{3} + \frac{4}{3} + 3 = 13$, $x_5 = d_{55}^{-1} z_5 = (5/6)(13) = 65/6 = 10\frac{5}{6}$

$z_6 = 6 + z_5 + \frac{7}{5} z_1 = 21$, $x_6 = d_{66}^{-1} z_6 = (1)(21) = 21$

Backward Pass

Solve $\quad U u = y \quad$ for displ'ts u

$$\begin{bmatrix} 1 & & & & -3/5 & -2/5 \\ & 1 & & -5/9 & -4/9 & \\ & & 1 & -1 & & \\ & & & 1 & -1 & \\ & & & & 1 & -1 \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/3 \\ 1/6 \\ 21/10 \\ 65/6 \\ 21 \end{bmatrix}$$

$$u_6 = 21$$

$$u_5 = 65/6 + u_6 = 31\tfrac{5}{6} = 31.83333$$

$$u_4 = 21/10 + u_5 = 2.1 + 31.83333 = 33.93333$$

$$u_3 = 1/6 + u_4 = .166667 + 33.93333 = 34.1$$

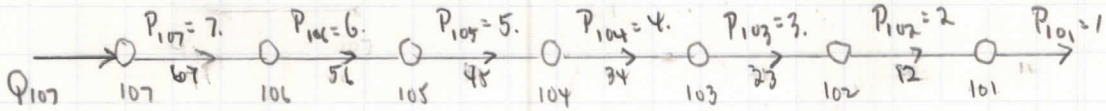
$$u_2 = 1/3 + \tfrac{5}{9} u_4 + \tfrac{4}{9} u_5 = 33.33333$$

$$u_1 = 1 + \tfrac{3}{5} u_5 + \tfrac{2}{5} u_6 = 1 + \tfrac{3}{5}\left(31\tfrac{5}{6}\right) + \tfrac{2}{5}(21) = 28.5$$

$$\begin{bmatrix} u_{101} \\ u_{102} \\ u_{103} \\ u_{104} \\ u_{105} \\ u_{106} \end{bmatrix} = \begin{bmatrix} u_3 \\ u_4 \\ u_2 \\ u_5 \\ u_1 \\ u_6 \end{bmatrix} = \begin{bmatrix} 34.1 \\ 33.933333 \\ 33.333333 \\ 31.833333 \\ 28.5 \\ 21. \end{bmatrix}$$

✓ agrees w/ MYSTRAN

The SPC force at G.P. 107 equilibrates the sum of the applied loads



$$Q_{107} + P_{107} + P_{106} + P_{105} + P_{104} + P_{103} + P_{102} + P_{101} = 0$$

$$Q_{107} = -(7. + 6. + 5. + 4. + 3. + 2. + 1.) = -28.$$ agrees w/ MYSTRAN

The rod loads are

$$\left.\begin{aligned}
P^e_{12} &= P_{101} &&= 1. \\
P^e_{23} &= P_{101} + P_{102} &&= 3. \\
P^e_{34} &= P_{101} + P_{102} + P_{103} &&= 6. \\
P^e_{45} &= P_{101} + P_{102} + P_{103} + P_{104} &&= 10. \\
P^e_{56} &= P_{101} + P_{102} + P_{103} + P_{104} + P_{105} &&= 15. \\
P^e_{67} &= P_{101} + P_{102} + P_{103} + P_{104} + P_{105} + P_{106} &&= 21.
\end{aligned}\right\} \text{tension}$$

Element stresses

$$\sigma_{12} = \frac{P^e_{12}}{A_{12}} = \frac{1.}{.6} = 1.666667 \qquad M.S. = \frac{100}{1.66666} - 1 = 59$$

$$\sigma_{23} = \frac{P^e_{23}}{A_{23}} = \frac{3.}{.5} = 6.0 \qquad M.S. = \frac{100}{6} - 1 = 15.7$$

$$\sigma_{34} = \frac{P^e_{34}}{A_{34}} = \frac{6.}{.4} = 15.0 \qquad M.S. = \frac{100}{15} - 1 = 5.67$$

$$\sigma_{45} = \frac{P^e_{45}}{A_{45}} = \frac{10}{.3} = 33.33333 \qquad M.S. = \frac{100}{33.3333} - 1 = 2$$

$$\sigma_{56} = \frac{P^e_{56}}{A_{56}} = \frac{15.}{.2} = 75.0 \qquad M.S. = \frac{100}{75} - 1 = 0.333$$

$$\sigma_{67} = \frac{P^e_{67}}{A_{67}} = \frac{21}{.1} = 210. \qquad M.S. = \frac{100}{210} - 1 = -.524 \ *$$

Check on Sparse Matrix Add routine

Check on MATADD_SSS subr.

Input is in DRIVERS\MATADD: MATADD.IN1-a, Output is in MATADD.OUT-a

ALPHA = 3, BETA = +2

A

1		113		
	132	133	134	
	142		144	
		153		
161				

B

	212	213		215
	232		234	235
251			254	
				265

=

C

333.	424.	765.		430.
	860.	399.	870.	470.
	426.		432.	
502.		459.	508.	
483.				530

i	I_A(i)
1	1
2	3
3	3
4	6
5	8
6	9
7	9

k	J_A(k)	A(k)
1	1	111.
2	3	113.
3	2	132.
4	3	133.
5	4	134.
6	2	142.
7	4	144.
8	3	153.
9		161.

8-8 6 7

A stored in sparse CRS

i	I_B(i)
1	1
2	4
3	4
4	7
5	7
6	9
7	10

k	J_B(k)	B(k)
1	2	212.
2	3	213.
3	5	215.
4	2	232.
5	4	234.
6	5	235.
7	1	251.
8	4	254.
9	5	265.

B stored in sparse CRS

i	I_C(i)
1	1
2	5
3	5
4	9
5	11
6	14
7	15

k	J_C(k)	C(k)
1	1	3(111)+ 0 = 333.
2	2	0 + 2(212) = 424.
3	3	3(113)+2(213) = 765.
4	5	0 + 2(215) = 430.
5	2	3(132)+2(232) = 860.
6	3	3(133) + 0 = 399.
7	4	3(134)+2(234)= 870.
8	5	0 + 2(235)= 470.
9	2	3(142)+0 = 426.
10	4	3(144)+0 = 432.
11	1	0+2(251) = 502.
12	3	3(153)+0 = 459.
13	4	0 + 2(254) = 508.
14	5	0+2(265) = 530.

C calculated in sparse CRS

Check on MATADD-SSS subr.

Input is in DRIVERS\MATADD : MATADD.IN1-6. Output in MATADD.OUT-6

111.		113.		115.
	122.	123.		
[illegible].	[illegible].	133	134.	
		[illegible].		145.
[illegible].			[illegible]	155.

+

211.	212.			215.
[illegible].	222.	223.	224.	
	[illegible].			235.
	[illegible].		244.	245.
[illegible].			[illegible]	255.

=

322.	212.	113.		330.
	344.	346.	224.	
		133.	134.	235.
			244.	390.
				410.

Input matrices A & B are symmetric and will be stored in CRS sparse format with only the terms on and above the diagonal. Resulting matrix C will also be symmetric and C will only contain terms on and above its diagonal. ~~Red terms not input~~

i	I_A(i)	i	J_A(i)	A(i)
1	1	1	1	111.
2	4	2	3	113.
3	6	3	5	115.
4	8	4	2	122.
	9	5	3	123.
6	10	6	3	133.
		7	4	134.
		8	5	145.
		9	5	155.

A stored sparse CRS

i	I_B(i)	i	J_B(i)	B(i)
1	1	1	1	211.
2	4	2	2	212.
3	7	3	5	215.
4	8	4	2	222.
5	10	5	3	223.
6	11	6	4	224.
		7	5	235.
		8	4	244.
		9	5	245.
		10	5	255.

B stored sparse CRS

i	I_C(i)	i	J_C(i)	Ci
1	1	1	1	111. + 211. = 322.
2	5	2	2	0 + 212. = 212.
3	8	3	3	113. + 0 = 113.
4	11	4	5	115. + 215. = 330.
5	13	5	2	122. + 222. = 344.
6	14	6	3	123. + 223. = 346.
		7	4	0 + 224. = 224.
		8	3	133. + 0 = 133.
		9	4	134. + 0 = 134.
		10	5	0 + 235. = 235.
		11	4	0 + 244. = 244.
		12	5	145. + 245. = 390.
		13	5	155. + 255 = 410.

C calculated in sparse CRS

Check on MATADD_SSS Subr

Input is in DRIVERS\MATADD_SSS: MATADD.INI-c, Output is MATADD_SSS-c

Input Matrix A is null

Input Matrix B is same as in MATADD_INI-a
B multiplier is 2.0
So result is 2×B from MATADD_INI-a

Input is in DRIVERS\MATADD-SSS: MATADD_INI-d. Output is MATADD-SSS-d

Input matrix A is same as in MATADD_INI-a
A Multiplier is 3.0
B matrix is null
So result is 3×A from MATADD-INI-a

```
PROGRAM DRIVE_MATADD_SSS:
 ------------------------

 Driver for sparse matrix add subr (MATADD_SSS.FOR) checkout: C = alpha*A + beta*B where A, B and result C are in sparse
 Compressed Row Storage (CRS) format

 ****************************************************************************************************************************
 Input matrix A = A        has        5 rows and        5 cols with        9 nonzero terms

 Compressed Row Storage (CRS) format of matrix A:
 ------------------------------------------------

 1) Index, L, and array I_A(L) for matrix A, where I_A(L+1) - I_A(L) is the number of nonzero terms in row L of matrix A.
    (also, I_A(L) is the index, K, in array A(K) where row L begins - up to, but not including, the last entry in I_A(L)).

    L, I_A(L)       =            1           1
    L, I_A(L)       =            2           4
    L, I_A(L)       =            3           6
    L, I_A(L)       =            4           8
    L, I_A(L)       =            5           9
    L, I_A(L)       =            6          10

 2) Index, K, and arrays J_A(K) and A(K). A(K) are the nonzeros in matrix A and J_A(K) is the col number in matrix A for term A(K).

    K, J_A(K), A(K) =            1           1   1.110000E+02
    K, J_A(K), A(K) =            2           3   1.130000E+02
    K, J_A(K), A(K) =            3           5   1.150000E+02
    K, J_A(K), A(K) =            4           2   1.220000E+02
    K, J_A(K), A(K) =            5           3   1.230000E+02
```

```
   K, J_A(K), A(K) =            6           3   1.330000E+02

   K, J_A(K), A(K) =            7           4   1.340000E+02

   K, J_A(K), A(K) =            8           5   1.450000E+02

   K, J_A(K), A(K) =            9           5   1.550000E+02


Full representation of input matrix A:

--------------------------------------


   111.      0.    113.      0.    115.

     0.    122.    123.      0.      0.

     0.      0.    133.    134.      0.

     0.      0.      0.      0.    145.

     0.      0.      0.      0.    155.


*****************************************************************************************************************

Input matrix B = B        has        5 rows and        5 cols with       10 nonzero terms


Compressed Row Storage (CRS) format of matrix B:

-----------------------------------------------


1) Index, L, and array I_B(L) for matrix B, where I_B(L+1) - I_B(L) is the number of nonzero terms in row L of matrix B.

   (also, I_B(L) is the index, K, in array B(K) where row L begins - up to, but not including, the last entry in I_B(L)).


   L, I_B(L)       =            1           1

   L, I_B(L)       =            2           4

   L, I_B(L)       =            3           7

   L, I_B(L)       =            4           8

   L, I_B(L)       =            5          10

   L, I_B(L)       =            6          11
```

2) Index, K, and arrays J_B(K) and B(K). B(K) are the nonzeros in matrix B and J_B(K) is the col number in matrix B for term B(K).

```
K, J_B(K), B(K) =            1           1   2.110000E+02
K, J_B(K), B(K) =            2           2   2.120000E+02
K, J_B(K), B(K) =            3           5   2.150000E+02
K, J_B(K), B(K) =            4           2   2.220000E+02
K, J_B(K), B(K) =            5           3   2.230000E+02
K, J_B(K), B(K) =            6           4   2.240000E+02
K, J_B(K), B(K) =            7           5   2.350000E+02
K, J_B(K), B(K) =            8           4   2.440000E+02
K, J_B(K), B(K) =            9           5   2.450000E+02
K, J_B(K), B(K) =           10           5   2.550000E+02
```

Full representation of input matrix B:

```
211.    212.      0.      0.    215.
  0.    222.    223.    224.      0.
  0.      0.      0.      0.    235.
  0.      0.      0.    244.    245.
  0.      0.      0.      0.    255.
```

```
*****************************************************************************************************************
Full representation of output matrix C after calculating it using MATADD_FFF:
-----------------------------------------------------------------------------

   322.    212.    113.      0.    330.
     0.    344.    346.    224.      0.
     0.      0.    133.    134.    235.
     0.      0.      0.    244.    390.
     0.      0.      0.      0.    410.

Output matrix C = C        has        5 rows and        5 cols with       13 nonzero terms

Compressed Row Storage (CRS) format of matrix C after calculating it using MATADD_FFF:
-------------------------------------------------------------------------------------

1) Index, L, and array I_C(L) for matrix C, where I_C(L+1) - I_C(L) is the number of nonzero terms in row L of matrix C.
   (also, I_C(L) is the index, K, in array C(K) where row L begins - up to, but not including, the last entry in I_C(L)).

   L, I_C(L)       =            1           1
   L, I_C(L)       =            2           5
   L, I_C(L)       =            3           8
   L, I_C(L)       =            4          11
   L, I_C(L)       =            5          13
   L, I_C(L)       =            6          14
```

2) Index, K, and arrays J_C(K) and C(K). C(K) are the nonzeros in matrix C and J_C(K) is the col number in matrix C for term C(K).

K, J_C(K), C(K) =	1	1	3.220000E+02
K, J_C(K), C(K) =	2	2	2.120000E+02
K, J_C(K), C(K) =	3	3	1.130000E+02
K, J_C(K), C(K) =	4	5	3.300000E+02
K, J_C(K), C(K) =	5	2	3.440000E+02
K, J_C(K), C(K) =	6	3	3.460000E+02
K, J_C(K), C(K) =	7	4	2.240000E+02
K, J_C(K), C(K) =	8	3	1.330000E+02
K, J_C(K), C(K) =	9	4	1.340000E+02
K, J_C(K), C(K) =	10	5	2.350000E+02
K, J_C(K), C(K) =	11	4	2.440000E+02
K, J_C(K), C(K) =	12	5	3.900000E+02
K, J_C(K), C(K) =	13	5	4.100000E+02

Check on Sparse Matrix Multiply routine

Example Problem Used To Checkout New Sparse Multiply Routines

MATMULT_SFF
MATMULT_SFS
MATMULT_SSS

Example IN1a (MATMULT_SFF.IN1a, MATMULT_SFS.IN1a, MATMULT_SSS.IN1a)

A × B = C

$$\begin{bmatrix} 11 & 0 & 13 & 0 & 15 \\ 0 & 22 & 0 & 24 & 25 \\ 0 & 0 & 0 & 0 & 0 \\ 41 & 42 & 43 & 44 & 45 \\ 0 & 0 & 0 & 0 & 55 \\ 61 & 62 & 63 & 64 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 13 & 0 \\ 21 & 22 & 23 & 0 \\ 31 & 0 & 33 & 0 \\ 0 & 42 & 43 & 0 \\ 51 & 0 & 53 & 0 \end{bmatrix} = \begin{bmatrix} 1168 & 0 & 1367 & 0 \\ 1737 & 1492 & 2863 & 0 \\ 0 & 0 & 0 & 0 \\ 4510 & 2772 & 7195 & 0 \\ 2805 & 0 & 2915 & 0 \\ 3255 & 4052 & 7050 & 0 \end{bmatrix}$$

~~NOTE: B must be CCS for sparse mult.~~

A (CRS)

L	I_A(L)	K	J_A(K)	A(K)
1	1	1	1	11.
2	4	2	3	13.
3	7	3	5	15.
4	7	4	2	22.
5	12	5	4	24.
6	13	6	5	25.
7	17	7	1	41.
		8	2	42.
		9	3	43.
		10	4	44.
		11	5	45.
		12	5	55.
		13	1	61.
		14	2	62.
		15	3	63.
		16	4	64.

B (CRS)

L	I_B(L)	K	J_B(K)	B(K)
1	1	1	1	13.
2	2	2	1	21.
3	5	3	2	22.
4	7	4	3	23.
5	9	5	1	31.
6	11	6	3	33.
		7	2	42.
		8	3	43.
		9	1	51.
		10	3	53.

C (CRS)

L	I_C(L)	K	J_C(K)	C(K)
1	1	1	1	1168.
2	3	2	3	1367.
3	6	3	1	1737.
4	6	4	2	1492.
5	9	5	3	2863.
6	11	6	1	4510.
7	14	7	2	2772.
		8	3	7195
		9	1	2805
		10	3	2915
		11	1	3255
		12	2	4052.
		13	3	7050

This will be the result for C in
~~MATMULT_SFF.OUTa~~
MATMULT_SFS.OUTa
MATMULT_SSS.OUTa

~~Except that~~ SFF will have it in "full" format as C above in A·B=C

B (CCS)

L	J_B(L)	K	I_B(K)	B(K)
1	1	1	2	21.
2	4	2	3	31.
3	6	3	5	51.
4	11	4	2	22.
5	11	5	4	42.
		6	1	13.
		7	2	23.
		8	3	33.
		9	4	43
		10	5	53

Example IN1-2

```
Sparse matrix multiply: C <= cons*A*B where A and result C are in sparse CRS format and B is in sparse CCS format
 ---------------------------------------------------------------------------------------------------------------


 Input  matrix A represented in sparse compressed row storage format as I_A, J_A, A has  5  rows
 ----------------------------------------------------------------------------------------------------------
 I_A(I) =      1     4     4     7     9    11
 J_A(I) =      1     3     4     1     3     5     1     4     3     5
   A(I) =    111.  113.  114.  113.  133.  135.  114.  144.  135.  155.

 Input  matrix B represented in sparse compressed row (CRS) storage format as I_B, J_B, B has  4  cols
 ---------------------------------------------------------------------------------------------------------------
 J_B(I) =      1     4     6     7     9    11
 I_B(I) =      1     2     4     1     4     3     2     4     1     4
   B(I) =    211.  212.  214.  221.  224.  233.  242.  244.  251.  254.



  Full representation of input  matrix A:
    111.      0.    113.    114.      0.
      0.      0.      0.      0.      0.
    113.      0.    133.      0.    135.
    114.      0.      0.    144.      0.
      0.      0.    135.      0.    155.
```

```
Full representation of input  matrix B:
   211.    212.      0.    214.
   221.      0.      0.    224.
     0.      0.    233.      0.
     0.    242.      0.    244.
   251.      0.      0.    254.

 Full representation of output matrix C:
 23421.  51120.  26329.  51570.
     0.      0.      0.      0.
 57728.  23956.  30989.  58472.
 24054.  59016.      0.  59532.
 38905.      0.  31455.  39370.

I, I_MATOUT(I)            =  1 1
I, I_MATOUT(I)            =  2 5
I, I_MATOUT(I)            =  3 5
I, I_MATOUT(I)            =  4 9
I, I_MATOUT(I)            =  5 12
I, I_MATOUT(I)            =  6 15

K, J_MATOUT(K), MATOUT(K) =   1 1 23421.0
K, J_MATOUT(K), MATOUT(K) =   2 2 51120.0
K, J_MATOUT(K), MATOUT(K) =   3 3 26329.0
K, J_MATOUT(K), MATOUT(K) =   4 4 51570.0
K, J_MATOUT(K), MATOUT(K) =   5 1 57728.0
K, J_MATOUT(K), MATOUT(K) =   6 2 23956.0
K, J_MATOUT(K), MATOUT(K) =   7 3 30989.0
K, J_MATOUT(K), MATOUT(K) =   8 4 58472.0
```

```
K, J_MATOUT(K), MATOUT(K) =   9 1 24054.0

K, J_MATOUT(K), MATOUT(K) =  10 2 59016.0

K, J_MATOUT(K), MATOUT(K) =  11 4 59532.0

K, J_MATOUT(K), MATOUT(K) =  12 1 38905.0

K, J_MATOUT(K), MATOUT(K) =  13 3 31455.0

K, J_MATOUT(K), MATOUT(K) =  14 4 39370.0
```

Example Problem Used To Checkout New Sparse Multiply Routines

MATMULT_SFF
MATMULT_SFS
MATMULT_SSS

Example IN1b (MATMULT_SFF.IN1b, MATMULT_SFS.IN1b, MATMULT_SSS.IN1b : input files) for SYM_A ≠ 'Y'

Example IN1c (MATMULT_SFF.IN1c, MATMULT_SFC.IN1C, MATMULT_SSS.IN1C : input files for SYM_A = 'Y'

A · B = C

A				
111		113	114	
113		133		135
114			144	
		135		155

B			
211	212		214
221			224
		233	
	242		244
251			254

C			
23421	23532+27588= 51120	26329	23754+27816= 51570
0	0	0	0
23843+33885= 57728	23956	30989	24182+34290= 58472
24054	24168+37848= 59016	0	24396+35136= 59532
38905	0	31455	39370

The terms in A in red are not input for A when SYM_A = 'Y' (only when SYM_A ≠ 'Y')

CRS — A stored w/ SYM_A = 'Y'

L	I_A(L)	K	J_A(K)	A(K)
1	1	1	1	111.
2	4	2	3	113.
3	4	3	4	114.
4	6	4	3	133.
5	7	5	5	135.
6	8	6	4	144.
		7	5	155.

CRS — A stored w/ SYM_A ≠ 'Y'

L	I_A(L)	K	J_A(K)	A(K)
1	1	1	1	111.
2	4	2	3	113.
3	4	3	4	114.
4	7	4	1	113.
5	9	5	3	133.
6	11	6	5	135.
		7	1	114.
		8	4	144
		9	3	135.
		10	5	155.

CCS — B stored in CCS

L	J_B(L)	K	I_B(K)	B(K)
1	1	1	1	211.
2	4	2	2	221
3	6	3	5	251.
4	7	4	1	212.
5	11	5	4	242.
		6	3	233.
		7	1	214
		8	2	224.
		9	4	244.
		10	5	254.

CRS — C stored in CRS

L	I_C(L)	K	J_C(K)	C(K)
1	1	1	1	23241.
2	5	2	2	51120.
3	5	3	3	26329
4	9	4	4	51570
5	12	5	1	57728.
6	15.	6	2	23956.
		7	3	30989
		8	4	58472
		9	1	24054
		10	2	59016.
		11	4	59532.
		12	1	38905
		13	3	31455
		14	4	39370

NOTE: B must be stored in CCS sparse format for the matrix mult. routines (sparse ones), i.e., when A and/or C is sparse

This will be the result for C in {MATMULT_SFS.OUTb,c / MATMULT_SSS.OUTb,c and MATMULT_SSS.OUTb,c will have C in full format as shown above in A·B=C

Check on Sparse Matrix Partitioning routine

Example Problems Used To Checkout Matrix Partition Routines: PARTITION_SIZE, PARTITION

Example IN1-1: (All partitions are done with partitioning vectors = 1 (not 2))

$$MATIN = \begin{bmatrix} 11. & 12. & 13. & 14. & 15. \\ 12. & 22. & 23. & 24. & 25. \\ 13. & 23. & 33. & 34. & 35. \\ 14. & 24. & 34. & 44. & 45. \\ 15. & 25. & 35. & 45. & 55. \end{bmatrix}$$

terms in red are not explicitly input since MATIN, in this example, is stored symmetrically (SYM_IN = 'Y')

Run 1-YN (SYM_OUT = 'N')

row part vec = 1 1 1 2 2
col part vec = 1 1 1 2 2

$$MATOUT = \begin{bmatrix} 11. & 12. & 13. \\ 12. & 22. & 23. \\ 13. & 23. & 33. \end{bmatrix}$$

Run 1-YY (SYM_OUT = 'Y')

row_part_vec = 1 1 1 2 2
col_part_vec = 1 1 1 2 2

$$MATOUT = \begin{bmatrix} 11. & 12. & 13. \\ 12. & 22. & 23. \\ 13. & 23. & 33. \end{bmatrix}$$

terms in red not stored (SYM_OUT = 'Y')

Run 2-YN (SYM_OUT = 'N')

row part vec = 2 2 2 1 2
col part vec = 2 2 2 2 1

MATOUT = [45.]

Run 2-YY (SYM_OUT = 'Y')

row part vec = 2 2 2 1 2
col part vec = 2 2 2 2 1

MATOUT = [45.]

Run 3-YN (SYM_OUT = 'N')

row part vec = 2 2 2 2 1
col part vec = 2 2 2 1 2

MATOUT = [45.]

Run 3-YY (SYM_OUT = 'Y')

row part vec = 2 2 2 2 1
col part vec = 2 2 2 1 2

MATOUT is null since we are asking for term 5,4 (which is 45. value) and we want SYM_OUT = 'Y' (no sub diagonals explicit)

Run 4-YN (SYM_OUT = 'N')

row part vec = 2 2 2 1 1
col part vec = 2 2 2 1 1

$$MATOUT = \begin{bmatrix} 44. & 45. \\ 45. & 55. \end{bmatrix}$$

Run 4-YY (SYM_OUT = 'Y')

row part vec = 2 2 2 1 1
col part vec = 2 2 2 1 1

$$MATOUT = \begin{bmatrix} 44. & 45. \\ 45. & 55. \end{bmatrix}$$

Run 5-YN (SYM_OUT = 'N')

row part vec = 2 2 2 1 1
col part vec = 1 1 1 2 2

$$MATOUT = \begin{bmatrix} 14. & 24. & 34. \\ 15. & 25. & 35. \end{bmatrix}$$

Run 5-YY (SYM_OUT = 'Y')

row part vec = 2 2 2 1 1
col part vec = 1 1 1 2 2

ERROR 909 SYM_OUT = 'Y' but MATOUT not square

Also MATOUT is null since we ask for SYM_OUT = 'Y' and the terms that would be partitioned are below the diagonal in MATIN

Run 6-YN (SYM_OUT = 'N')

row part vec = 1 1 2 2 2
col part vec = 2 2 1 1 1

$$MATOUT = \begin{bmatrix} 13. & 14. & 15. \\ 23. & 24. & 25. \end{bmatrix}$$

Run 6-YY (SYM_OUT = 'Y')

row part vec = 1 1 2 2 2
col part vec = 2 2 1 1 1

*Error 909: Asked for SYM_OUT = 'Y' but rows and cols to be partitioned do not form a symmetric matrix

xample IN1-2

$$MATIN = \begin{bmatrix} 11. & 12. & 13. & 14. & 15. & 16. & 17. \\ 21. & 22. & 23. & 24. & 25. & 26. & 27. \\ 31. & 32. & 33. & 34. & 35. & 36. & 37. \\ 41. & 42. & 43. & 44. & 45. & 46. & 47. \end{bmatrix} \qquad SYM_IN = 'N'$$

Run 1-NN (SYM_OUT='N') row part vec = 2 1 2 1 col part vec = 2 2 1 2 1 2 1 MATOUT = $\begin{bmatrix} 23. & 25. & 27. \\ 43. & 45. & 47. \end{bmatrix}$	Run 1-NY (SYM_OUT='Y') row part vec = 2 1 2 1 col part vec = 2 2 1 2 1 2 1 ~~*ERROR 49 Input is not sym. so output can't be~~ (1) ERROR 49 Output cannot be sym for non square matrix
Run 2-NN (SYM_OUT='N') row part vec = 2 1 2 2 col part vec = 1 1 1 1 1 1 1 MATOUT = [21. 22. 23. 24. 25. 26. 27.]	Run 2-NY (SYM_OUT='Y') row part vec = 2 1 2 2 col part vec = 1 1 1 1 1 1 1 Same error as in Run 1-NY above (~~test N~~)
Run 3-NN (SYM_OUT='N') row part vec = 2 2 1 2 col part vec = 2 2 2 2 1 2 2 MATOUT = [35.]	Run 3-NY (SYM_OUT='Y') row part vec = 2 2 1 2 col part vec = 1 1 1 1 1 1 1 ~~Same error 1 as above~~ MATOUT = [35]
Run 4-NN (SYM_OUT='N') row part vec = 2 2 2 1 col part vec = 2 2 2 1 2 2 2 MATOUT = [44.]	Run 4-NY (SYM_OUT='Y') row part vec = 2 2 2 1 col part vec = 2 2 2 1 2 2 2 ~~Same error 1 as above~~ MATOUT = [44]
Run 5-NN (SYM_OUT='N') row part vec = 2 1 2 1 col part vec = 2 2 2 1 2 2 2 MATOUT = $\begin{bmatrix} 24. \\ 44. \end{bmatrix}$	Run 5-NY (SYM_OUT='Y') row part vec = 2 1 2 1 col part vec = 2 2 2 1 2 2 2 Same error 1 as above
Run 6-NN (SYM_OUT='N') row part vec = 2 1 2 2 col part vec = 2 1 1 2 2 2 1 MATOUT = [22. 27. 27.]	Run 6-NY (SYM_OUT='Y') row part vec = 2 1 2 2 col part vec = 2 1 1 2 2 2 1 Same error 1 as above

Example IN1-3

$$\begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 22. & 23. & 0. & 25. & 0. & 0. & 28. & 29. \\ 0. & 23. & 0. & 34. & 35. & 36. & 0. & 38. & 0. \\ 0. & 0. & 34. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 25. & 35. & 0. & 55. & 56. & 0. & 58. & 59. \\ 0. & 0. & 36. & 0. & 56. & 66. & 67. & 68. & 69. \\ 0. & 0. & 0. & 0. & 0. & 67. & 0. & 78. & 0. \\ 0. & 28. & 38. & 0. & 58. & 68. & 78. & 0. & 0. \\ 0. & 29. & 0. & 0. & 59. & 69. & 0. & 0. & 0. \end{bmatrix}$$

SYM_IN = 'Y'

SYM_OUT = 'N'

RUN1-YN

row part vec = 1 1 1 1 1 1 1 1 1
col part vec = 1 1 1 1 1 1 1 1 1

$$\text{MATOUT} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 22. & 23. & 0. & 25. & 0. & 0. & 28. & 29. \\ 0. & 23. & 0. & 34. & 35. & 36. & 0. & 38. & 0. \\ 0. & 0. & 34. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 25. & 35. & 0. & 55. & 56. & 0. & 58. & 59. \\ 0. & 0. & 36. & 0. & 56. & 66. & 67. & 68. & 69. \\ 0. & 0. & 0. & 0. & 0. & 67. & 0. & 78. & 0. \\ 0. & 28. & 38. & 0. & 58. & 68. & 78. & 0. & 0. \\ 0. & 29. & 0. & 0. & 59. & 69. & 0. & 0. & 0. \end{bmatrix}$$

RUN2-YN

row part vec = 1 2 2 1 2 2 1 2 2
col part vec = 1 2 1 2 1 2 2 1 1

$$\text{MATOUT} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0. & 34. & 0. & 0. & 0. \\ 0. & 0. & 0. & 78. & 0. \end{bmatrix}$$

RUN3-YN

row part vec = 2 2 2 2 2 2 1 2 2
col part vec = 1 1 1 1 1 1 2 2 2

MATOUT = [0. 0. 0. 0. 0. 67.]

RUN4-YN

row part vec = 1 2 2 2 2 2 1 2 2
col part vec = 1 1 1 1 1 2 2 2 2

$$\text{MATOUT} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

SYM_OUT = 'Y'

RUN1-YY

row part vec = 1 1 1 1 1 1 1 1 1
col part vec = 1 1 1 1 1 1 1 1 1

$$\text{MATOUT} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 22. & 23. & 0. & 25. & 0. & 0. & 28. & 29. \\ 0. & 23. & 0. & 34. & 35. & 36. & 0. & 38. & 0. \\ 0. & 0. & 34. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 25. & 35. & 0. & 55. & 56. & 0. & 58. & 59. \\ 0. & 0. & 36. & 0. & 56. & 66. & 67. & 68. & 69. \\ 0. & 0. & 0. & 0. & 0. & 67. & 0. & 78. & 0. \\ 0. & 28. & 38. & 0. & 58. & 68. & 78. & 0. & 0. \\ 0. & 29. & 0. & 0. & 59. & 69. & 0. & 0. & 0. \end{bmatrix}$$

RUN2-YY

row part vec = 1 2 2 1 2 2 1 2 2
col part vec = 1 2 1 2 1 2 2 1 1

Error 919 : matrix can't be sym if not square

RUN3-YY

row part vec = 2 2 2 2 2 2 1 2 2
col part vec = 1 1 1 1 1 1 2 2 2

Error 919 (see above)

RUN4-YY

row part vec = 1 2 2 2 2 2 1 2 2
col part vec = 1 1 1 1 1 2 2 2 2

Error 919 (see above)

Example INL-3 (con't)

SYM_OUT = 'N'	SYM_OUT = 'Y'
RUN 5-YN { row part vec = 1 1 1 1 1 1 1 1 2 col part vec = 2 2 2 2 2 2 2 2 1 MATOUT = [0. 0. 28. 29. 38. 0. 0. 0. 58. 59. 68. 69. 78. 0 0. 0.]	RUN 5-YY { row part vec = 1 1 1 1 1 1 1 1 2 col part vec = 2 2 2 2 2 2 2 1 1 Error 919 (see above)
RUN 6-YN { row part vec = 2 2 1 2 1 2 2 2 2 col part vec = 2 1 2 1 2 2 2 2 2 MATOUT = [23. 34. 25. 0.]	RUN 6-YY { row part vec = 2 2 1 2 1 2 2 2 2 col part vec = 2 1 2 1 2 2 2 2 2 Error 916
RUN 7-YN { row part vec = 2 2 2 2 2 2 1 2 1 col part vec = 1 1 1 1 1 1 1 1 1 MATOUT = [0. 0. 0. 0. 0. 67. 0. 78. 0. 0. 29. 0. 0. 59. 69. 0. 0. 0.]	RUN 7-YY { row part vec = 2 2 2 2 2 2 1 2 1 col part vec = 1 1 1 1 1 1 1 1 1 Error 919 (see above)

Example IN1-4

$$\text{MATIN} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 22. & 0. & 24. & 0. & 26. & 0. & 28. & 0. \\ 0. & 32. & 0. & 34. & 0. & 0. & 0. & 0. & 39. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 51. & 52. & 53. & 54. & 0. & 56. & 57. & 58. & 0. \end{bmatrix} \qquad \text{SYM_IN} = \text{'N'}$$

Run 1-NN (SYM_OUT='N')
row part vec = 1 2 1 1 1
col part vec = 1 1 2 2 1 1 2 2 1

$$\text{MATOUT} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. \\ 0. & 32. & 0. & 0. & 39. \\ 0. & 0. & 0. & 0. & 0. \\ 51. & 52. & 0. & 56. & 0. \end{bmatrix}$$

Run 1-NY (SYM_OUT='Y')
row part vec = 1 2 1 1 1
col part vec = 1 1 2 2 1 1 2 2 1

~~Error 9[illegible] Want output sym. but input isn't~~
Error 9[illegible] Want output sym but it isn't square

Run 2-NN (SYM_OUT='N')
row part vec = 1 2 2 1 2
col part vec = 1 1 2 2 1 1 2 2 1

MATOUT is null (consists of cols from MATIN rows that are all 0's)

Run 2-NY (SYM_OUT='Y')
row part vec = 1 2 2 1 2
col part vec = 1 1 2 2 1 1 2 2 1

Error 9[illegible] } Same as above

Example IN1-5

$$\text{MATIN} = \begin{bmatrix} 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \end{bmatrix}$$

SYM-IN = 'N' (MATIN is null)

Run 1-NN (SYM-OUT = 'N') row part vec = 1 2 1 1 1 col part vec = 1 1 2 2 1 1 2 2 1 MATOUT is null	Run 1-NY (SYM-OUT = 'Y') row part vec = 1 2 1 1 1 col part vec = 1 1 2 2 1 1 2 2 1 ~~Error 904 Want sym output but input not sym~~ Error 908 Want output sym but it isn't square
Run 2-NN (SYM-OUT = 'N') row part vec = 1 2 2 1 2 col part vec = 1 1 2 2 1 1 2 2 1 MATOUT is null	Run 2-NY (SYM-OUT = 'Y') row part vec = 1 2 2 1 2 col part vec = 1 1 2 2 1 1 2 2 1 Error 914 ~~920~~ Same as above

Example IN1-2

```
PROGRAM DRIVE_PARTITION:
 ------------------------

 Driver for sparse matrix partition subr (PARTITION_SS.FOR) checkout. Input matrix A and partition B are both in sparse
 Compressed Row Storage (CRS) format.

 ***********************************************************************************************************************
 Input matrix A =    MAT_2 with SYM_A = N has        4 rows and        7 cols with       28 nonzero terms

 Compressed Row Storage (CRS) format of matrix A:
 ------------------------------------------------

 1) Index, L, and array I_A(L) for matrix A, where I_A(L+1) - I_A(L) is the number of nonzero terms in row L of matrix A.
    (also, I_A(L) is the index, K, in array A(K) where row L begins - up to, but not including, the last entry in I_A(L)).

    L, I_A(L)       =            1           1
    L, I_A(L)       =            2           8
    L, I_A(L)       =            3          15
    L, I_A(L)       =            4          22
    L, I_A(L)       =            5          29

 2) Index, K, and arrays J_A(K) and A(K). A(K) are the nonzeros in matrix A and J_A(K) is the col number in matrix A for term A(K).

    K, J_A(K), A(K) =            1           1   1.100000E+01
    K, J_A(K), A(K) =            2           2   1.200000E+01
    K, J_A(K), A(K) =            3           3   1.300000E+01
    K, J_A(K), A(K) =            4           4   1.400000E+01
```

```
K, J_A(K), A(K) =          5          5   1.500000E+01
K, J_A(K), A(K) =          6          6   1.600000E+01
K, J_A(K), A(K) =          7          7   1.700000E+01
K, J_A(K), A(K) =          8          1   2.100000E+01
K, J_A(K), A(K) =          9          2   2.200000E+01
K, J_A(K), A(K) =         10          3   2.300000E+01
K, J_A(K), A(K) =         11          4   2.400000E+01
K, J_A(K), A(K) =         12          5   2.500000E+01
K, J_A(K), A(K) =         13          6   2.600000E+01
K, J_A(K), A(K) =         14          7   2.700000E+01
K, J_A(K), A(K) =         15          1   3.100000E+01
K, J_A(K), A(K) =         16          2   3.200000E+01
K, J_A(K), A(K) =         17          3   3.300000E+01
K, J_A(K), A(K) =         18          4   3.400000E+01
K, J_A(K), A(K) =         19          5   3.500000E+01
K, J_A(K), A(K) =         20          6   3.600000E+01
K, J_A(K), A(K) =         21          7   3.700000E+01
K, J_A(K), A(K) =         22          1   4.100000E+01
K, J_A(K), A(K) =         23          2   4.200000E+01
K, J_A(K), A(K) =         24          3   4.300000E+01
K, J_A(K), A(K) =         25          4   4.400000E+01
K, J_A(K), A(K) =         26          5   4.500000E+01
K, J_A(K), A(K) =         27          6   4.600000E+01
K, J_A(K), A(K) =         28          7   4.700000E+01
```

```
Full representation of input matrix A:
 -------------------------------------


     11.     12.     13.     14.     15.     16.     17.

     21.     22.     23.     24.     25.     26.     27.

     31.     32.     33.     34.     35.     36.     37.

     41.     42.     43.     44.     45.     46.     47.


 !!!!
```

```
*****************************************************************************************************************
 SUMMARY: Compressed Row Storage (CRS) format of matrix B = MAT_21NN:
 -------

 1) Index, L, and array I_B(L) for matrix B, where I_B(L+1) - I_B(L) is the number of nonzero terms in row L of matrix B.
    (also, I_B(L) is the index, K, in array B(K) where row L begins - up to, but not including, the last entry in I_B(L)).

    L, I_B(L)       =            1           1
    L, I_B(L)       =            2           4
    L, I_B(L)       =            3           7


2) Index, K, and arrays J_B(K) and B(K). B(K) are the nonzeros in matrix B and J_B(K) is the col number in matrix B for term B(K).

    K, J_B(K), B(K) =            1           1   2.300000E+01
    K, J_B(K), B(K) =            2           2   2.500000E+01
    K, J_B(K), B(K) =            3           3   2.700000E+01
    K, J_B(K), B(K) =            4           1   4.300000E+01
    K, J_B(K), B(K) =            5           2   4.500000E+01
    K, J_B(K), B(K) =            6           3   4.700000E+01

 Full representation of output matrix B:
 ---------------------------------------

     23.     25.     27.
     43.     45.     47.
```

Check on Sparse Matrix Transpose routine

Example Problem Used To Checkout New Sparse Transpose Routine - MATTRNSP_SS

Example IN1a

$$A = \begin{array}{|c|c|c|c|c|} \hline 11. & 0 & 13. & 0 & 15. \\ \hline 0 & 22. & 0 & 24. & 25. \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hline 41. & 42. & 43. & 44. & 45. \\ \hline 0 & 0 & 0 & 0 & 55. \\ \hline 61. & 62. & 63. & 64. & 0 \\ \hline \end{array}_{6\times 5} \quad , \quad A^T = \begin{array}{|c|c|c|c|c|c|} \hline 11. & 0 & 0 & 41. & 0 & 61. \\ \hline 0 & 22. & 0. & 42. & 0 & 62. \\ \hline 13. & 0. & 0. & 43. & 0 & 63. \\ \hline 0 & 24. & 0 & 44. & 0 & 64. \\ \hline 15. & 25. & 0 & 45. & 55. & 0. \\ \hline \end{array}_{5\times 6}$$

A (CCS)				
L	J_A(L)	K	J_A(K)	A(K)
1	1	1	1	11.
2	4	2	3	13.
3	7	3	5	15.
4	7	4	2	22.
5	12	5	4	24.
6	13	6	5	25.
7	17	7	1	41.
		8	2	42.
		9	3	43.
		10	4	44.
		11	5	45.
		12	5	55.
		13	1	61.
		14	2	62.
		15	3	63.
		16	4	64.

AT (CCS				
L	J_AT(L)	K	J_AT(K)	AT(K)
1	1	1	1	11.
2	4	2	4	41.
3	7	3	6	61.
4	10	4	2	22.
5	13	5	4	42.
6	17	6	6	62.
		7	1	13.
		8	4	43.
		9	6	63.
		10	2	24.
		11	4	44.
		12	6	64.
		13	1	15.
		14	2	25.
		15	4	45.
		16	5	55.

Example Problem Used To Checkout New Sparse Matrix Transpose Routine - MATTRNSP_SS

Example IN1d

$A =$

0.	0.	0.	0.	0.	0.	0.	0.
0.	22.	23.	24.	0.	26.	0.	28.
0.	32.	33.	34.	35	36.	37.	38.
0.	0.	0.	0.	45.	46.	0.	0.
0.	0.	53.	54.	55.	56.	0.	58

$A^T =$

0.	0.	0.	0.	0.
0.	22.	32.	0.	0.
0.	23.	33.	0.	53.
0.	24.	34.	0.	54.
0.	0	35.	45.	55.
0.	26.	36.	46.	56.
0.	0.	37.	0.	0
0.	28.	38.	0.	58.

A (CRS)

L	I_A(L)	K	J_A(K)	A(K)
1	1	1	2	22.
2	1	2	3	23.
3	6	3	4	24.
4	13	4	6	26.
5	15	5	8	28.
6	20	6	2	32.
		7	3	33.
		8	4	34.
		9	5	35.
		10	6	36.
		11	7	37.
		12	8	38.
		13	5	45.
		14	6	46.
		15	3	53.
		16	4	54.
		17	5	55.
		18	6	56.
		19	8	58.

AT = A^T (CRS)

L	I_AT(L)	K	J_AT(K)	AT(K)
1	1	1	2	22.
2	1	2	3	32.
3	3	3	2	23.
4	6	4	3	33.
5	9	5	5	53.
6	12	6	2	24.
7	16	7	3	34.
8	17	8	5	54.
9	20	9	3	35.
		10	4	45.
		11	5	55.
		12	2	26.
		13	3	36.
		14	4	46.
		15	5	56.
		16	3	37.
		17	2	28.
		18	3	38.
		19	5	58.

```
PROGRAM DRIVE_MATTRNSP_SS:
 -------------------------

 Driver for sparse matrix transposition subr (MATTRNSP_SS.FOR) checkout: B = A(transpose) where input A and result B are in sparse
 Compressed Row Storage (CRS) format

 ____________________________________________________________________________________________________________________________________

 ::::::::::::::::::::::::::::::::::::::START DEBUG(85) OUTPUT FROM SUBROUTINE MATTRNSP_SS::::::::::::::::::::::::::::::::::::::::

 SSS SPARSE MATRIX TRANSPOSITION ROUTINE: Transpose matrix A, stored in sparse Compressed Row Storage (CRS) format, to matrix AT,
 ----------------------------------------
 stored in sparse CRS format also

                                        The name of CRS formatted matrix A is: A
                                        The name of CRS formatted matrix B is: AT
                                        The name of CRS formatted matrix C is:
                          Matrix A  has        6 rows and        5 cols and           16 nonzero terms


 ****************************************************************************************************************************
 Input matrix A = A        has        6 rows and        5 cols with       16 nonzero terms

 Compressed Row Storage (CRS) format of matrix A:
 ------------------------------------------------

 1) Index, L, and array I_A(L) for matrix A, where I_A(L+1) - I_A(L) is the number of nonzero terms in row L of matrix A.
    (also, I_A(L) is the index, K, in array A(K) where row L begins - up to, but not including, the last entry in I_A(L)).
```

```
L, I_A(L)       =            1           1
L, I_A(L)       =            2           4
L, I_A(L)       =            3           7
L, I_A(L)       =            4           7
L, I_A(L)       =            5          12
L, I_A(L)       =            6          13
L, I_A(L)       =            7          17
```

2) Index, K, and arrays J_A(K) and A(K). A(K) are the nonzeros in matrix A and J_A(K) is the col number in matrix A for term A(K).

```
K, J_A(K), A(K) =            1           1   1.100000E+01
K, J_A(K), A(K) =            2           3   1.300000E+01
K, J_A(K), A(K) =            3           5   1.500000E+01
K, J_A(K), A(K) =            4           2   2.200000E+01
K, J_A(K), A(K) =            5           4   2.400000E+01
K, J_A(K), A(K) =            6           5   2.500000E+01
K, J_A(K), A(K) =            7           1   4.100000E+01
K, J_A(K), A(K) =            8           2   4.200000E+01
K, J_A(K), A(K) =            9           3   4.300000E+01
K, J_A(K), A(K) =           10           4   4.400000E+01
K, J_A(K), A(K) =           11           5   4.500000E+01
K, J_A(K), A(K) =           12           5   5.500000E+01
K, J_A(K), A(K) =           13           1   6.100000E+01
K, J_A(K), A(K) =           14           2   6.200000E+01
K, J_A(K), A(K) =           15           3   6.300000E+01
K, J_A(K), A(K) =           16           4   6.400000E+01
```

****

Output matrix AT = AT has 5 rows and 6 cols with 16 nonzero terms

Compressed Row Storage (CRS) format of matrix AT after calculating it using MATMULT_FFF:

---

1) Index, L, and array I_AT(L) for matrix AT, where I_AT(L+1) - I_AT(L) is the number of nonzero terms in row L of matrix AT.

(also, I_AT(L) is the index, K, in array AT(K) where row L begins - up to, but not including, the last entry in I_AT(L)).

```
L, I_C(L)       =            1           1
L, I_C(L)       =            2           4
L, I_C(L)       =            3           7
L, I_C(L)       =            4          10
L, I_C(L)       =            5          13
L, I_C(L)       =            6          17
```

2) Index, K, and arrays J_AT(K) and AT(K). AT(K) are the nonzeros in matrix AT and J_AT(K) is the col number in matrix AT for term AT(K).

```
K, J_AT(K), AT(K) =            1           1   1.100000E+01
K, J_AT(K), AT(K) =            2           4   4.100000E+01
K, J_AT(K), AT(K) =            3           6   6.100000E+01
K, J_AT(K), AT(K) =            4           2   2.200000E+01
K, J_AT(K), AT(K) =            5           4   4.200000E+01
K, J_AT(K), AT(K) =            6           6   6.200000E+01
K, J_AT(K), AT(K) =            7           1   1.300000E+01
K, J_AT(K), AT(K) =            8           4   4.300000E+01
K, J_AT(K), AT(K) =            9           6   6.300000E+01
K, J_AT(K), AT(K) =           10           2   2.400000E+01
K, J_AT(K), AT(K) =           11           4   4.400000E+01
K, J_AT(K), AT(K) =           12           6   6.400000E+01
K, J_AT(K), AT(K) =           13           1   1.500000E+01
```

```
K, J_AT(K), AT(K) =          14          2   2.500000E+01
K, J_AT(K), AT(K) =          15          4   4.500000E+01
K, J_AT(K), AT(K) =          16          5   5.500000E+01

::::::::::::::::::::::::::::::::::::::END DEBUG(85) OUTPUT FROM SUBROUTINE MATTRNSP_SS::::::::::::::::::::::::::::::::::::::::::::
```

```
Full representation of input matrix A:
--------------------------------------

    11.      0.     13.      0.     15.
     0.     22.      0.     24.     25.
     0.      0.      0.      0.      0.
    41.     42.     43.     44.     45.
     0.      0.      0.      0.     55.
    61.     62.     63.     64.      0.

Full representation of output matrix AT:
----------------------------------------

    11.      0.      0.     41.      0.     61.
     0.     22.      0.     42.      0.     62.
    13.      0.      0.     43.      0.     63.
     0.     24.      0.     44.      0.     64.
    15.     25.      0.     45.     55.      0.
```

Check on converting sym mat A to non-sym form

Convert Symmetric A to NonSym Form in B

$$
\begin{array}{c}
\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
a_{11} & a_{12} & 0 & 0 & a_{15} & 0 & 0 \\
a_{12} & 0 & a_{23} & 0 & 0 & a_{26} & a_{27} \\
0 & a_{23} & a_{33} & 0 & a_{35} & a_{36} & a_{37} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{15} & 0 & a_{35} & 0 & a_{55} & 0 & a_{57} \\
0 & a_{26} & a_{36} & 0 & 0 & a_{66} & 0 \\
0 & a_{27} & a_{37} & 0 & a_{57} & 0 & a_{77}
\end{bmatrix}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\end{array}
$$

A are terms in black
B are " " " and red
(red are sym. terms)

L	I_A(L)	K	J_A(K)	A(K)
1	1	1	1	a_{11}
2	4 (3)	2	2	a_{12}
3	7 (3)	3	5	a_{15}
4	11 (4)	4	3	a_{23}
5	11 (0)	5	6	a_{26}
6	13 (2)	6	7	a_{27}
7	14 (1)	7	3	a_{33}
8	15 (1)	8	5	a_{35}
		9	6	a_{36}
		10	7	a_{37}
		11	5	a_{55}
		12	7	a_{57}
		13	6	a_{66}
		14	7	a_{77}

L	I_B(L)	K	J_B(K)	B(K)
1	1	1	1	a_{11}
2	4 (3)	2	2	a_{12}
3	8 (4)	3	5	a_{15}
4	13 (5)	4	1	a_{12}
5	13 (0)	5	3	a_{23}
6	17 (4)	6	6	a_{26}
7	20 (3)	7	7	a_{27}
8	24 (4)	8	2	a_{23}
		9	3	a_{33}
		10	5	a_{35}
		11	6	a_{36}
		12	7	a_{37}
		13	1	a_{15}
		14	3	a_{35}
		15	5	a_{55}
		16	7	a_{57}
		17	2	a_{26}
		18	3	a_{36}
		19	6	a_{66}
		20	2	a_{27}
		21	3	a_{37}
		22	5	a_{57}
		23	7	a_{77}

If A has NTERM_A nonzeros and NDIAG_A nonzero diag. terms

$$\text{NTERM_B} = 2[\text{NTERM_A} - \text{NDIAG_A}] + \text{NDIAG_A} = 2\text{NTERM_A} - \text{NDIAG_A}$$

$$= 2(14) - 5 = 23 \checkmark$$

Check on Sparse Matrix Conversion from CRS to CCS

Example Problem Used To Checkout Conversion From Sparse CRS to Sparse CCS Routine: SPARSE_CRS_SPARSE_CCS:

Example IN1d

A =

0.	0.	0.	0.	0.	0.	0.	0.
0.	22.	23.	24.	0.	26.	0.	28.
0.	32.	33	34.	35.	36.	37.	38.
0.	0.	0.	0.	45.	46.	0.	0.
0.	0.	53.	54.	55.	56.	0.	58.

A (CRS)

L	I_A(L)	K	J_A(K)	A(K)
1	1	1	2	22.
2	1	2	3	23.
3	6	3	4	24.
4	13	4	6	26.
5	15	5	8	28.
6	20	6	2	32.
		7	3	33.
		8	4	34.
		9	5	35.
		10	6	36.
		11	7	37.
		12	8	38.
		13	5	45.
		14	6	46.
		15	3	53.
		16	4	54.
		17	5	55.
		18	6	56.
		19	8	58.

A (CCS)

L	J_A(L)	K	I_A(K)	A(K)
1	1	1	2	22.
2	1	2	3	32.
3	3	3	2	23.
4	6	4	3	33.
5	9	5	5	53.
6	12	6	2	24.
7	16	7	3	34.
8	17	8	5	54.
9	20	9	3	35.
		10	4	45
		11	5	55.
		12	2	26.
		13	3	36.
		14	4	46.
		15	5	56.
		16	3	37.
		17	2	28.
		18	3	38.
		19	5	58.