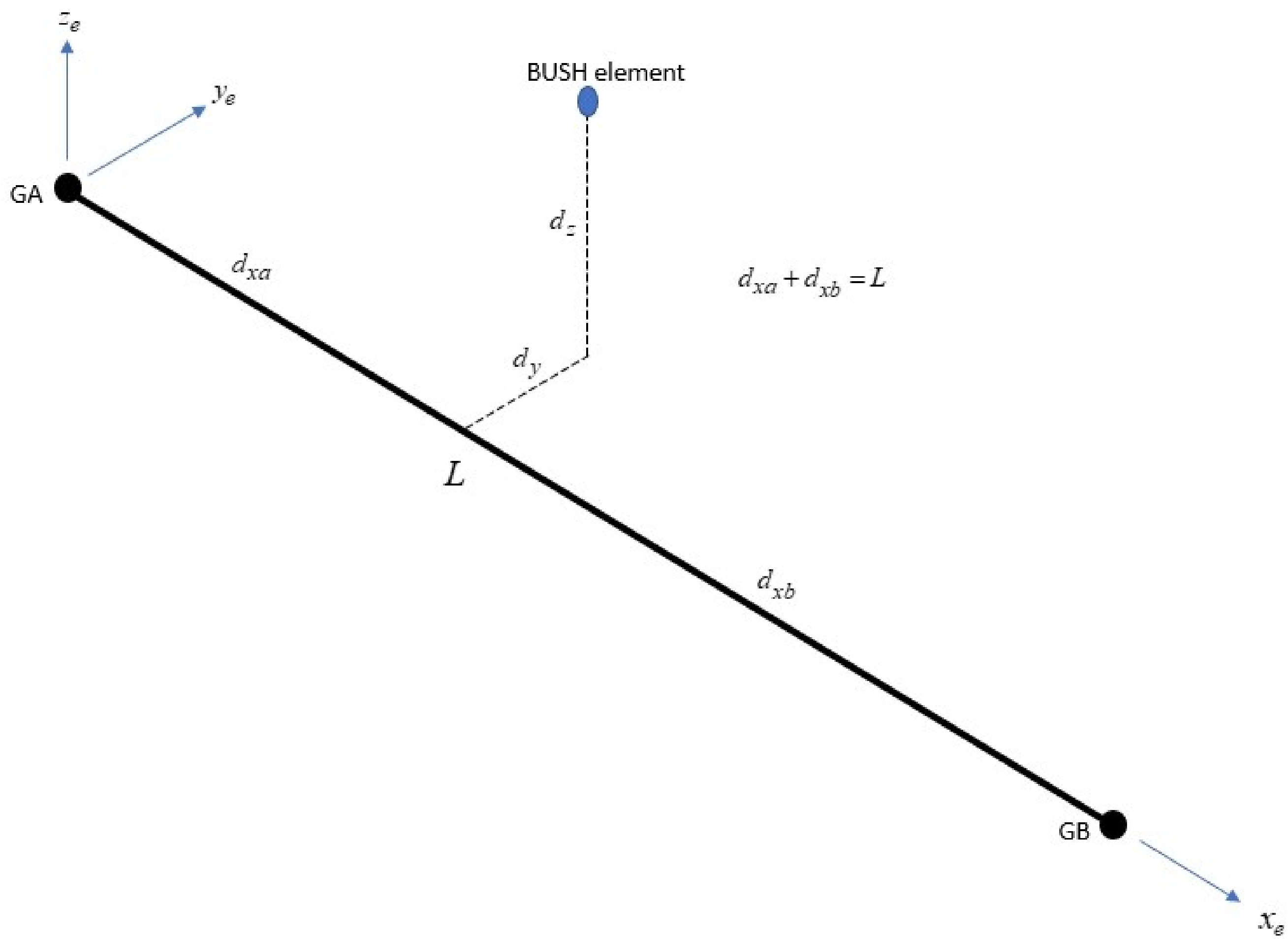
12 Appendix F: Equations for the BUSH element

BUSH Element Geometry

(in local element coordinates)



The stiffness equations for the BUSH element can be expressed as:

$$Ku = F$$

where K is a 12x12 matrix and u and F are the 12 degree of freedom (6 at each of the 2 grids) displacements and node forces. For the sake of clarity, rather than showing the whole 12x12 stiffness matrix, express the above equation in grid partitioned form as:

$$\begin{bmatrix} K_{aa} & K_{ab} \\ K_{ab}^T & K_{bb} \end{bmatrix} \begin{Bmatrix} u_a \\ u_b \end{Bmatrix} = \begin{Bmatrix} F_a \\ F_b \end{Bmatrix}$$

If we denote κ_i (i=1,…6) as the 6 stiffness values from the PBUSH Bulk Data entry then the above partitions are:

$$K_{aa} = \begin{bmatrix} \kappa_1 & 0 & 0 & 0 & d_z\kappa_1 & -d_y\kappa_1 \\ 0 & \kappa_2 & 0 & -d_z\kappa_2 & 0 & d_{xa}\kappa_2 \\ 0 & 0 & \kappa_3 & d_y\kappa_3 & -d_{xa}\kappa_3 & 0 \\ 0 & -d_z\kappa_2 & d_y\kappa_3 & \kappa_4 + d_y^2\kappa_3 + d_z^2\kappa_2 & -d_{xa}d_y\kappa_3 & -d_{xa}d_z\kappa_2 \\ d_z\kappa_1 & 0 & -d_{xa}\kappa_3 & -d_{xa}d_y\kappa_3 & \kappa_5 + d_{xa}^2\kappa_3 + d_z^2\kappa_1 & -d_yd_z\kappa_1 \\ -d_y\kappa_1 & d_{xa}\kappa_2 & 0 & -d_{xa}d_z\kappa_2 & -d_yd_z\kappa_1 & \kappa_6 + d_{xa}^2\kappa_2 + d_y^2\kappa_1 \end{bmatrix}$$

$$K_{ab} = \begin{bmatrix} -\kappa_1 & 0 & 0 & 0 & -d_z\kappa_1 & d_y\kappa_1 \\ 0 & -\kappa_2 & 0 & d_z\kappa_2 & 0 & d_{xb}\kappa_2 \\ 0 & 0 & -\kappa_3 & -d_y\kappa_3 & -d_{xb}\kappa_3 & 0 \\ 0 & d_z\kappa_2 & -d_y\kappa_3 & -(\kappa_4 + d_y^2\kappa_3 + d_z^2\kappa_2) & -d_{xb}d_y\kappa_3 & -d_{xb}d_z\kappa_2 \\ -d_z\kappa_1 & 0 & d_{xa}\kappa_3 & d_{xa}d_y\kappa_3 & -\kappa_5 + d_{xa}d_{xb}\kappa_3 - d_z^2\kappa_1 & d_yd_z\kappa_1 \\ d_y\kappa_1 & -d_{xa}\kappa_2 & 0 & d_{xa}d_z\kappa_2 & d_yd_z\kappa_1 & -\kappa_6 + d_{xa}d_{xb}\kappa_2 - d_y^2\kappa_1 \end{bmatrix}$$

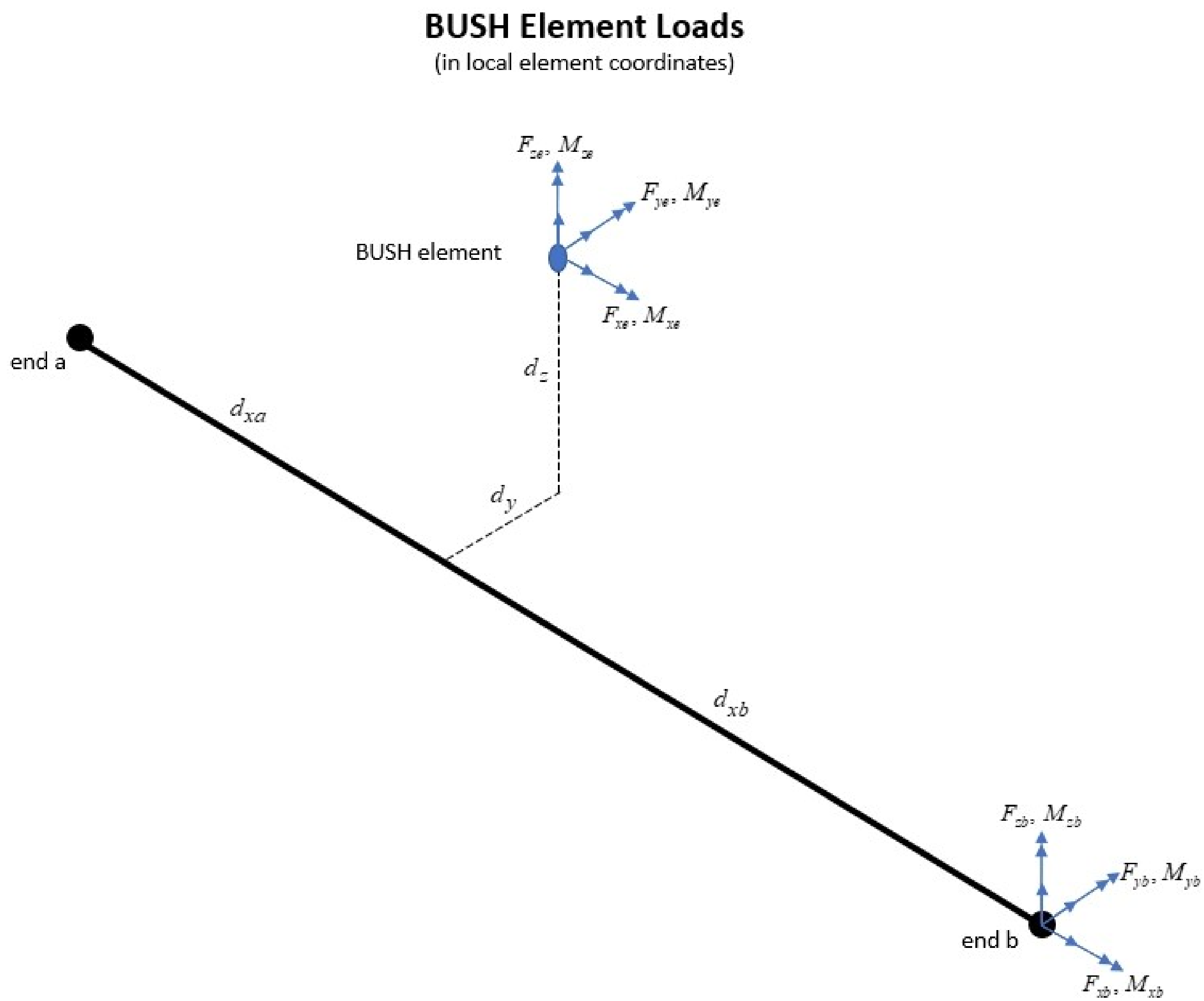
$$K_{bb} = \begin{bmatrix} \kappa_1 & 0 & 0 & 0 & d_z\kappa_1 & -d_y\kappa_1 \\ 0 & \kappa_2 & 0 & -d_z\kappa_2 & 0 & -d_{xb}\kappa_2 \\ 0 & 0 & \kappa_3 & d_y\kappa_3 & d_{xb}\kappa_3 & 0 \\ 0 & -d_z\kappa_2 & d_y\kappa_3 & \kappa_4 + d_y^2\kappa_3 + d_z^2\kappa_2 & d_{xa}d_y\kappa_3 & d_{xb}d_z\kappa_2 \\ d_z\kappa_1 & 0 & -d_{xb}\kappa_3 & d_{xb}d_y\kappa_3 & \kappa_5 + d_{xb}^2\kappa_3 + d_z^2\kappa_1 & -d_y d_z\kappa_1 \\ -d_y\kappa_1 & d_{xb}\kappa_2 & 0 & d_{xb}d_z\kappa_2 & -d_y d_z\kappa_1 & \kappa_6 + d_{xb}^2\kappa_2 + d_y^2\kappa_1 \end{bmatrix}$$

An image of the full 12x12 matrix with the above partitions is shown below:

$$K = \left[\begin{array}{cccccc|cccccc} K_1 & 0 & 0 & 0 & d_zK_1 & -d_yK_1 & -K_1 & 0 & 0 & 0 & -d_zK_1 & d_yK_1 \\ 0 & K_2 & 0 & -d_zK_2 & 0 & d_{xa}K_2 & 0 & -K_2 & 0 & d_zK_2 & 0 & d_{xb}K_2 \\ 0 & 0 & K_3 & d_yK_3 & -d_{xa}K_3 & 0 & 0 & 0 & -K_3 & -d_yK_3 & -d_{xb}K_3 & 0 \\ 0 & -d_zK_2 & d_yK_3 & K_4 + d_y^2K_3 + d_z^2K_2 & -d_{xa}d_yK_3 & -d_{xa}d_zK_2 & 0 & d_zK_2 & -d_yK_3 & -(K_4 + d_y^2K_3 + d_z^2K_2) & -d_{xb}d_yK_3 & -d_{xb}d_zK_2 \\ d_zK_1 & 0 & -d_{xa}K_3 & -d_{xa}d_yK_3 & K_5 + d_{xa}^2K_3 + d_z^2K_1 & -d_yd_zK_1 & -d_zK_1 & 0 & d_{xa}K_3 & d_{xa}d_yK_3 & -K_5 + d_{xa}d_{xb}K_3 - d_z^2K_1 & d_yd_zK_1 \\ -d_yK_1 & d_{xa}K_2 & 0 & -d_{xa}d_zK_2 & -d_yd_zK_1 & K_6 + d_{xa}^2K_2 + d_y^2K_1 & d_yK_1 & -d_{xa}K_2 & 0 & d_{xa}d_zK_2 & d_yd_zK_1 & -K_6 + d_{xa}d_{xb}K_2 - d_y^2K_1 \\ \hline -K_1 & 0 & 0 & 0 & -d_zK_1 & d_yK_1 & K_1 & 0 & 0 & 0 & d_zK_1 & -d_yK_1 \\ 0 & -K_2 & 0 & d_zK_2 & 0 & -d_{xa}K_2 & 0 & K_2 & 0 & -d_zK_2 & 0 & -d_{xb}K_2 \\ 0 & 0 & -K_3 & -d_yK_3 & d_{xa}K_3 & 0 & 0 & 0 & K_3 & d_yK_3 & d_{xb}K_3 & 0 \\ 0 & d_zK_2 & -d_yK_3 & -(K_4 + d_y^2K_3 + d_z^2K_2) & d_{xa}d_yK_3 & d_{xa}d_zK_2 & 0 & -d_zK_2 & d_yK_3 & K_4 + d_y^2K_3 + d_z^2K_2 & d_{xb}d_yK_3 & d_{xb}d_zK_2 \\ -d_zK_1 & 0 & -d_{xb}K_3 & -d_{xb}d_yK_3 & -K_5 + d_{xa}d_{xb}K_3 - d_z^2K_1 & d_yd_zK_1 & d_zK_1 & 0 & d_{xb}K_3 & d_{xb}d_yK_3 & K_5 + d_{xb}^2K_3 + d_z^2K_1 & -d_yd_zK_1 \\ d_yK_1 & d_{xb}K_2 & 0 & -d_{xb}d_zK_2 & d_yd_zK_1 & -K_6 + d_{xa}d_{xb}K_2 - d_y^2K_1 & -d_yK_1 & -d_{xb}K_2 & 0 & d_{xb}d_zK_2 & -d_yd_zK_1 & K_6 + d_{xb}^2K_2 + d_y^2K_1 \end{array}\right]$$

Note that the partitions K_{aa} and K_{bb} are symmetric

The element engineering forces can be derived using the figure below:



The engineering forces in the BUSH element are:

$$
\begin{aligned}
F_{xe} &= F_{xb} \\
F_{ye} &= F_{yb} \\
F_{ze} &= F_{zb} \\
M_{xe} &= F_{yb} d_z - F_{zb} d_y + M_{xb} \\
M_{ye} &= -F_{xb} d_z - F_{zb} d_{xb} + M_{yb} \\
M_{ze} &= F_{xb} d_y + F_{yb} d_{xb} + M_{zb}
\end{aligned}
$$

This can be put into a form which includes all nodal forces as:

$$
\begin{Bmatrix} F_{xe} \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze} \end{Bmatrix}
=
\left[\begin{array}{cccccc|cccccc}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & d_z & -d_y & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -d_z & 0 & -d_{xb} & 0 & . & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & d_y & d_{xb} & 0 & 0 & 0 & .
\end{array}\right]
\left\{\begin{array}{c} F_{xa} \\ F_{ya} \\ F_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \\ \hline F_{xb} \\ F_{yb} \\ F_{zb} \\ M_{xb} \\ M_{yb} \\ M_{zb} \end{array}\right\}
$$

The 6x12 transformation matrix in the above equation is used in the MYSTRAN code to transform the element nodal forces to element engineering forces

The engineering forces in the BUSH element are:

$$
\begin{aligned}
F_{xe} &= F_{xa} \\
F_{ye} &= F_{ya} \\
F_{ze} &= F_{za} \\
M_{xe} &= F_{ya} d_z - F_{za} d_y + M_{xa} \\
M_{ye} &= -F_{xa} d_z + F_{za} d_{xa} + M_{ya} \\
M_{ze} &= F_{xa} d_y - F_{yb} d_{xa} + M_{za}
\end{aligned}
$$

This can be put into a form which includes all nodal forces as:

$$
\begin{Bmatrix} F_{xe} \\ F_{ye} \\ F_{ze} \\ M_{xe} \\ M_{ye} \\ M_{ze} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & d_z & -d_y & 1 & 0 & 0 \\ -d_z & 0 & d_{xa} & 0 & 1 & 0 \\ d_y & -d_{xa} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} F_{xa} \\ F_{ya} \\ F_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \end{Bmatrix}
$$

The 6x transformation matrix in the above equation is used in the MYSTRAN code to transform the element nodal forces to element engineering forces