10 Appendix D: Craig-Bampton Model Generation

10.1 Craig-Bampton Equations of Motion for Substructures

MYSTRAN has the capability to generate Craig-Bampton (CB) models via SOL 31 (or SOL GEN CB MODEL). This solution sequence calculates the fixed-base modes of a substructure and generates all of the matrices needed to couple the substructure to other CB models. This appendix describes the Craig-Bampton method and its implementation in MYSTRAN and includes an example problem to explain the input and output for SOL 31.

Craig and Bampton[1] are credited with the first unified approach to modal synthesis, or substructuring for dynamic analysis, using fixed interface flexible modes augmented by boundary constraint modes to describe each substructure. Their work was a simplification of earlier work by Hurty[2] who first introduced the concept for substructures with redundant boundary degrees of freedom (DOF's).

In order to explain the Craig-Bampton (CB) method, consider a structure represented by the picture below that is comprised of several (in this case 5) substructures connected at an arbitrary number of points:

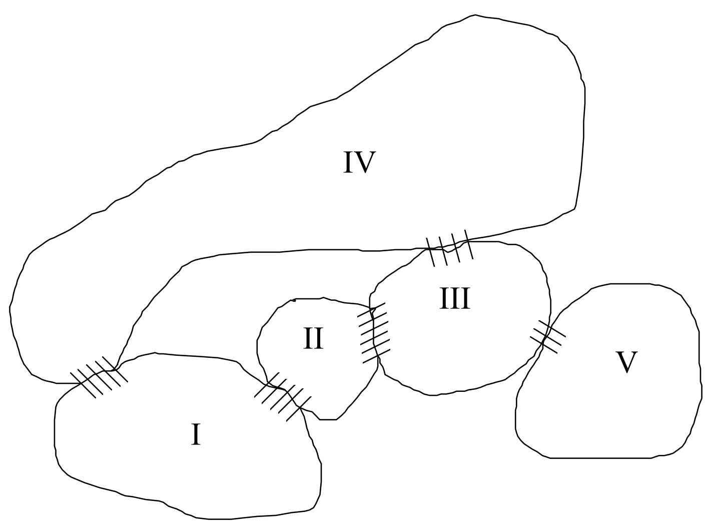

Figure 10.1 - Overall Structure Composed of Several Substructures

Each substructure is joined to one or more other substructures at some number of interface, or boundary, DOF's (indicated by the hatched areas in the above picture. The complete structure, consisting of the connected substructures, may or may not be restrained from free body motion. For any one of the substructures (j = I, II, III, etc.) the G-set equations of motion (ignoring damping for the moment) are:

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[1] Craig, R.R. and Bampton, M.C.C. "*Coupling of Substructures for Dynamic Analysis*", AIAA Journal, Vol. 6, No. 7, July 1968, pp 1313-1319

[2] Hurty, W.C. "*Dynamic Analysis of Structural Systems Using Component Modes*", AIAA Journal, Vol. 3, No. 4, April 1965, pp 678-685

$$M^j_{GG}\ddot{u}^j_G + K^j_{GG}u^j_G = P^j_G + Q^j_G$$

where

$$Q^j_G = Q^{m^j}_G + Q^{s^j}_G + Q^{r^j}_G$$

$$u^j_G = \begin{Bmatrix} u^j_A \\ u^j_O \\ u^j_S \\ u^j_M \end{Bmatrix} = \begin{Bmatrix} \text{analysis DOF's} \\ \text{omitted DOF's} \\ \text{SPC'd DOF's} \\ \text{MPC'd DOF's} \end{Bmatrix}$$

and 10-1

P^j_G = applied loads on the G-set

$Q^{m^j}_G$ = constraint forces due to multi-point constraints (MPC's)

$Q^{s^j}_G$ = constraint forces due to single point constraints (SPC's)

$Q^{r^j}_G$ = interface forces at boundaries between substructures

In MYSTRAN nomenclature, the G-set is reduced to the A-set by the elimination of the M-set multi-point constraints, the S-set single point constraints and the O-set omitted DOF's (using OMIT's or ASET's). The A-set DOF's for this substructure must contain all DOF's that will be connected to other substructures The resulting A-set equations of motion (dropping the j superscript notation for each substructure) are:

$$M_{AA}\ddot{u}_A + K_{AA}u_A = P_A + Q^r_A \qquad 10\text{-}2$$

where the A set matrices are mathematical reductions from the G-set (see Appendix B for details)

Partition 2 into the R-set and L-set, where, the R-set represents the boundary DOF's in which this substructure connects with other substructures and the L-set are all free interior DOF's in this substructure

$$\begin{bmatrix} M_{RR} & M^T_{LR} \\ M_{LR} & M_{LL} \end{bmatrix}\begin{Bmatrix} \ddot{u}_R \\ \ddot{u}_L \end{Bmatrix} + \begin{bmatrix} K_{RR} & K^T_{LR} \\ K_{LR} & K_{LL} \end{bmatrix}\begin{Bmatrix} u_R \\ u_L \end{Bmatrix} = \begin{Bmatrix} P_R \\ P_L \end{Bmatrix} + \begin{Bmatrix} Q^r_R \\ o \end{Bmatrix} \qquad 10\text{-}3$$

Notice at this point that there remain forces of constraint only at the substructure attach points as the L-set represents all free DOF's for this substructure.

At this point we can introduce the transformation from the physical displacements in equation (3) to what are known as the CB DOF's; namely the flexible mode DOF's and the boundary (R-set) DOF's. In order to show that this is not any further approximation to equation 3, consider the following argument:

1) the $u_A = \begin{Bmatrix} u_R \\ u_L \end{Bmatrix}$ DOF's are clearly a complete set of DOF's for the substructure in that, once they are known, the complete g-set DOF's for this substructure can be determined.

2) similarly, a new set of DOF's for the substructure,

$$u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad \text{10-4}$$

are a complete set of DOF's if ξ_N are the generalized DOF's for flexible modes when $u_R = 0$

3) Thus we can take u_L to be a linear combination of u_R and ξ_N or:

$$u_L = D_{LR}u_R + \Phi_{LN}\xi_N \qquad \text{10-5}$$

if we insist that:

a) Φ_{LN} are shapes when $u_R = 0$ and ξ_N are modal DOF's. That is, the columns of Φ_{LN} are the flexible modes, φ_L^i, when the boundary is fixed. The i-th column of the modal matrix Φ_{LN} is φ_L^i.

b) D_{LR} are shapes when $\xi_N = 0$. That is, the columns of D_{LR} are the L-set shapes for unit motions of the R-set when the flexible mode DOF's are zero.

The φ_L^i are easy to understand. They are the eigenvectors resulting from solving an eigenvalue problem from equations 3 with $u_R = 0$. This eigenvalue problem would be:

$$(K_{LL} - \omega^2 M_{LL})\varphi_L = 0 \qquad \text{10-6}$$

This requires that the determinant of the coefficient matrix on the left side of equation 6 be zero:

$$\left| K_{LL} - \omega^2 M_{LL} \right| = 0 \quad \text{which yields N eigenvalues } \omega_1^2, \omega_2^2 \ldots, \omega_N^2 > 0 \qquad \text{10-7}$$

The i-th eigenvector, φ_L^i, is then determined by solving the equation:

$$(K_{LL} - \omega_i^2 M_{LL})\varphi_L^i = 0 \quad \text{for } i = 1, 2, \ldots, N \qquad \text{10-8}$$

Solution of equation 8 requires that one element of φ_L^i be arbitrarily set (the φ_L^i are shapes and their amplitude does not matter). Once equation 8 is solved, the modal matrix is:

$$\Phi_{LN} = \begin{bmatrix} \varphi_l^1 & \varphi_l^2 & \cdots & \varphi_L^N \end{bmatrix} \qquad \text{10-9}$$

The D_{LR} can also be explained easily. As stated above, the D_{LR} are shapes when the flexible mode response is zero. We can see from equation 5 that a column of D_{LR} represents the displacements at the L-set DOF's due to motion at one of the R-set DOF's while all other R-set DOF's are zero (as well as all $\xi_N = 0$). We can therefore solve for D_{LR} from equation 3 by taking all applied forces and accelerations equal to zero and solving the statics problem:

$$\begin{bmatrix} K_{RR} & K_{LR}^T \\ K_{LR} & K_{LL} \end{bmatrix} \begin{Bmatrix} u_R \\ u_L^s \end{Bmatrix} = \begin{Bmatrix} Q_R^r \\ o \end{Bmatrix} \qquad \text{10-10}$$

where u_L^s are static displacements of the L-set. From the second row of equation 10, solve for u_L^s in terms of u_R :

$$u_L^s = -K_{LL}^{-1} K_{LR} u_R = D_{LR} u_R$$
or
$$D_{LR} = -K_{LL}^{-1} K_{LR} \qquad \text{10-11}$$

Thus, the CB DOF's are contained in u_X (equation 4) and the transformation between u_X and u_A is:

$$\begin{Bmatrix} u_R \\ u_L \end{Bmatrix} = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad \text{10-12}$$

where I is an R x R identity matrix. Equation 12 can be written as:

$$u_A = \Psi_{AX} u_X$$
where
$$\Psi_{AX} = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix}, \quad u_A = \begin{Bmatrix} u_R \\ u_L \end{Bmatrix}, \quad u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad \text{10-13}$$

Ψ_{AX} is the CB transformation matrix and is of A-set size. In MYSTRAN this is called matrix PHIXA. When expanded to G-set size, PHIXA becomes matrix PHIXG:

$$\boxed{\begin{aligned} & u_G = \Psi_{GX} u_X \\ & \Psi_{GX} = \text{matrix data block PHIXG} \\ & \text{PHIXG} = \text{PHIXA expanded to G-set} \end{aligned}} \qquad \text{10-14}$$

Note that when all flexible modes of the substructure are used in u_X equation 13 is exact. In practice, all modes are never used since this would defeat the purpose of making the transformation (which is to find a smaller set of DOF's which are nonetheless an accurate representation of the A-set). Substituting equation 13 into equation 2 and premultiplying the result by the transpose of Ψ_{AX} yields:

$$M_{XX} \ddot{u}_X + K_{XX} u_X = P_X + Q_X^r \qquad \text{10-15}$$

where:

$$M_{XX} = \Psi_{AX}^{T} M_{AA} \Psi_{AX} = \begin{bmatrix} I & D_{LR}^{T} \\ 0 & \Phi_{LN}^{T} \end{bmatrix} \begin{bmatrix} M_{RR} & M_{LR}^{T} \\ M_{LR} & M_{LL} \end{bmatrix} \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} = \begin{bmatrix} m_{RR} & m_{NR}^{T} \\ m_{NR} & m_{NN} \end{bmatrix}$$

$$K_{XX} = \Psi_{AX}^{T} K_{AA} \Psi_{AX} = \begin{bmatrix} I & D_{LR}^{T} \\ 0 & \Phi_{LN}^{T} \end{bmatrix} \begin{bmatrix} K_{RR} & K_{LR}^{T} \\ K_{LR} & K_{LL} \end{bmatrix} \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix} = \begin{bmatrix} k_{RR} & 0 \\ 0 & k_{NN} \end{bmatrix}$$

10-16

$$P_{X} = \Psi_{AX}^{T} P_{A} = \begin{bmatrix} I & D_{LR}^{T} \\ 0 & \Phi_{LN}^{T} \end{bmatrix} \begin{Bmatrix} P_{R} \\ P_{L} \end{Bmatrix} = \begin{Bmatrix} P'_{R} \\ \Xi_{N} \end{Bmatrix}, \quad P'_{R} = P_{R} + D_{LR}^{T} P_{L}, \quad \Xi_{N} = \Phi_{LN}^{T} P_{L}$$

$$Q_{X}^{r} = \Psi_{AX}^{T} Q_{A}^{r} = \begin{bmatrix} I & D_{LR}^{T} \\ 0 & \Phi_{LN}^{T} \end{bmatrix} \begin{Bmatrix} Q_{R}^{r} \\ o \end{Bmatrix} = \begin{Bmatrix} Q_{R}^{r} \\ 0 \end{Bmatrix}$$

and:

$$\begin{aligned} m_{RR} &= M_{RR} + M_{LR}^{T} D_{LR} + (M_{LR}^{T} D_{LR})^{T} + D_{LR}^{T} M_{LL} D_{LR} \\ m_{NR} &= \Phi_{LN}^{T} (M_{LR} + M_{LL} D_{LR}) \\ m_{NN} &= \Phi_{LN}^{T} M_{LL} \Phi_{LN} \\ k_{RR} &= K_{RR} + K_{LR}^{T} D_{LR} \\ k_{NN} &= \Phi_{LN}^{T} K_{LL} \Phi_{LN} \end{aligned} \qquad \text{10-17}$$

m_{NN}, k_{NN} are diagonal matrices of generalized maesses and stiffnesses, respectively.

Equations 15 for the i-th substructure can be written as:

$$\boxed{\begin{bmatrix} m_{RR} & m_{NR}^{T} \\ m_{NR} & m_{NN} \end{bmatrix} \begin{Bmatrix} \ddot{u}_{R} \\ \ddot{\xi}_{N} \end{Bmatrix} + \begin{bmatrix} k_{RR} & 0 \\ 0 & k_{NN} \end{bmatrix} \begin{Bmatrix} u_{R} \\ \xi_{N} \end{Bmatrix} = \begin{Bmatrix} P'_{R} \\ \Xi_{N} \end{Bmatrix} + \begin{Bmatrix} Q_{R}^{r} \\ 0 \end{Bmatrix}} \qquad \text{10-18}$$

The off-diagonal terms in the above stiffness matrix are zero due to the definition of D_{LR} in equation 11. In addition, matrix k_{RR} in equation 18 is null if the boundary is a determinant interface. Equations 14 and 18 are the Craig-Bampton equations of motion for the i-th substructure. The P'_{R} are due to applied loads on the R and L-set DOF's (see equation 16) and the Q_{R}^{r} are the interface forces where substructures connect. Once the equations are developed for all substructures, the individual substructures can be connected and the resulting equations solved for the combined R-set and N-set DOF's u_{R} and ξ_{N} for all substructures. Once this is done, the forces of inter-connection, or substructure interface forces, (that is, the Q_{R}^{r}) can be solved from the individual substructure equations in the top row of equation 18. Equation 14 is used to obtain displacements for all G-set DOF's.

Each organization that is developing a substructure in CB format would deliver the above coefficient matrices in equations 14 and 18 to the organization that is doing the combined structure analysis. In addition, Displacement and Load Transformation Matrices (DTM's and LTM's) collectively known as Output Transformation Matrices, (OTM's), described below, are also delivered as part of the CB model.

10.2 Development of Displ Output Transformation Matrices (Displ OTM's)

Typically, a set of displacement output transformation matrices (displ OTM's, or DTM's for short), is delivered with a Craig-Bampton model to the organization that will couple all substructures and solve for the primary unknowns (u_R and ξ_N and Q_R^r) in order that desired displacements at some of the substructure G-set DOF's may be obtained along with the coupled solution.

Once the combined structure has been solved for the primary variables, the original u_L physical DOF's could be determined from equation 5 and then element forces and stresses could be determined from the u_R and u_L displacements . This is called recovery of the u_L DOF's and element forces and stresses using the Modal Displacement Method (MDM). However, as is often the case, equations 18 are solved using a severely truncated set of modes for each substructure. While this may not compromise the accuracy of the solutions for u_R and ξ_N, it could compromise the accuracy of element forces and stresses calculated using displacements determined from equation 5 with the truncated set of modes. In order to avoid this problem, the u_L DOF's can be found using the Modal Acceleration Method (MAM), described below. It should be noted that the MAM described below *ignores* damping forces so that it is only useful when the damping is small (e.g. less than 10% or so).

From the bottom row of equation 3, solve for u_L in terms of the other variables in the equation:

$$\begin{aligned} u_L &= -K_{LL}^{-1}(M_{LR}\ddot{u}_R + M_{LL}\ddot{u}_L) - K_{LL}^{-1}K_{LR}u_R + K_{LL}^{-1}P_L \\ &= -K_{LL}^{-1}(M_{LR}\ddot{u}_R + M_{LL}\ddot{u}_L) + D_{LR}u_R + K_{LL}^{-1}P_L \end{aligned} \qquad \text{10-19}$$

Differentiate equation 5 twice and use the result for $\ddot{u}_L$ in equation 19, to get:

$$u_L = \left[-K_{LL}^{-1}(M_{LR} + M_{LL}D_{LR}) \;\middle|\; -K_{LL}^{-1}M_{LL}\Phi_{LN} \;\middle|\; D_{LR} \right] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + K_{LL}^{-1}P \qquad \text{10-20}$$

The term $K_{LL}^{-1}M_{LL}\Phi_{LN}$ in equation 20. can be written in a form more convenient for calculation. From equation 8 it can be seen that:

$$K_{LL}^{-1}M_{LL}\phi_L^i = \frac{1}{\omega_i^2}\varphi_L^i$$

so that

$$K_{LL}^{-1}M_{LL}\begin{bmatrix}\varphi_l^1 & \varphi_l^2 & \cdots & \varphi_L^N\end{bmatrix}=\begin{bmatrix}\varphi_l^1 & \varphi_l^2 & \cdots & \varphi_L^N\end{bmatrix}\begin{bmatrix}\omega_1^{-2} & & & \\ & \omega_2^{-2} & & \\ & & \ddots & \\ & & & \omega_N^{-2}\end{bmatrix}$$

or

$$K_{LL}^{-1}M_{LL}\Phi_{LN}=\Phi_{LN}\Omega_{NN}^{-2} \qquad \text{10-21}$$

where

$$\Omega_{NN}^{-2}=\begin{bmatrix}\omega_1^{-2} & & & \\ & \omega_2^{-2} & & \\ & & \ddots & \\ & & & \omega_N^{-2}\end{bmatrix} \qquad \text{10-22}$$

substitute equation 21 into equation 20 to get:

$$u_L=\left[-K_{LL}^{-1}(M_{LR}+M_{LL}D_{LR})\;\middle|\;-\Phi_{LN}\Omega_{NN}^{-2}\;\middle|\;D_{LR}\right]\begin{Bmatrix}\ddot{u}_R\\ \ddot{\xi}_N\\ u_R\end{Bmatrix}+K_{LL}^{-1}P_L \qquad \text{10-23}$$

The various terms in the coefficient matrices in equation 23 are known as Displacement Transformation Matrices (DTM's). Equation 23 can be written as:

$$u_L=\left[DTM1_{LR}\;\middle|\;DTM2_{LN}\;\middle|\;DTM3_{LR}\right]\begin{Bmatrix}\ddot{u}_R\\ \ddot{\xi}_N\\ u_R\end{Bmatrix}+DTM4_{LL}P_l \qquad \text{10-24}$$

where

$$\begin{aligned}DTM1_{LR}&=-K_{LL}^{-1}(M_{LR}+M_{LL}D_{LT})\\ DTM2_{LN}&=-\Phi_{LN}\Omega_{NN}^{-2}\\ DTM3_{LR}&=D_{LR}\\ DTM4_{LL}&=K_{LL}^{-1}\end{aligned} \qquad \text{10-25}$$

Equations 24 and 25 represent the MAM for recovering displacements for the L-set, for the i-th substructure, once the assembled substructure equations have been solved for the u_R and q_N DOF's. Once the L-set displacements have been found, recovery of the remaining displacements in the G-set is accomplished through the transformation matrices used in their elimination from equation 1 (for details see Appendix B). At the G-set level, equation 24 is:

$$u_G = \left[DTM1_{GR} \;|\; DTM2_{GN} \;|\; DTM3_{GR}\right]\begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + DTM4_{GL}P_L$$

or

$$u_G = \Gamma_{GZ}u_Z + DTM4_{GL}P_L \qquad \text{10-26}$$

where

$$\Gamma_{GZ} = \left[DTM1_{GR} \;|\; DTM2_{GN} \;|\; DTM3_{GR}\right] = DTM_{GZ}$$

and

$$u_Z = \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix}$$, where u_Z are the Craig-Bampton Degrees of freedom (CB_DOF's)

.

where each of the G-set DTM's in equation 26 is obtained from the L-set DTM's in equation 25 through the normal recovery operations to build back up to the G-set from the L-set. The coefficient matrix in equation 26 that has DTM's 1 - 3 in it is called matrix PHIZG. The table below explains the meaning of each of the DTM's in equation 26:

Table 10.1

i-th col of:	Represents:
$DTM1_{GR}$	displ's of G-set due to a unit accel of the i-th interface DOF (all other R, N set DOF's zero)
$DTM2_{GN}$	displ's of G-set due to a unit accel of the i-th flex mode DOF (all other R, N set DOF's zero)
$DTM3_{GR}$	displ's of G-set due to a unit displ of the i-th interface DOF (all other R, N set DOF's zero)
$DTM4_{GL}$	displ's of G-set due to a unit force on the i-th L-set DOF (all other L-set forces zero)

10.3 Development of Load Output Transformation Matrices (Load OTM's)

Once the G-set displacements have been found, substructure element forces and stresses, as well as grid point forces, can be recovered and assembled into a Loads Output Transformation Matrix, or Load OTM (more commonly referred to as LTM). There are several types of quantities one may desire in an LTM. Equations are developed, below, for several types of LTM quantities typically used in CB analyses.

10.3.1 LTM Terms for Substructure Interface Forces

From the top row of equation 18, the interface forces can be determined once the substructures have been coupled and the u_R and ξ_N solved. The interface forces are:

$$Q_R^r = m_{RR}\ddot{u}_R + m_{NR}^T\ddot{\xi}_N + k_{RR}u_R - P_R'$$

or

$$Q_R^r = \begin{bmatrix} m_{RR} & m_{NR}^T & k_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - I_{RR}P_R' \qquad \text{10-27}$$

where I_{RR} is an RxR identity matrix. Equation 27 can be written as:

$$Q_R^r = \begin{bmatrix} LTM21_{RR} & LTM22_{RN} & LTM23_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - LTM24_{RR}P_R^r$$

or

$$Q_R^r = J_{RZ}U_Z - I_{RR}P_R$$

where

$$J_{RZ} = \begin{bmatrix} LTM21_{RR} & LTM22_{RN} & LTM23_{RR} \end{bmatrix} = LTM2_{RZ} \qquad \text{10-28}$$

$$LTM21_{RR} = m_{RR}$$

$$LTM22_{RN} = m_{NR}^T$$

$$LTM23_{RR} = k_{RR}$$

$$LTM24_{RR} = I_{RR}$$

10.3.2 LTM Terms for Net cg Loads

Terms can also be included in the overall LTM that will recover what are known as "net" accelerations at the center of gravity (cg) of the CB model. These are termed Net Load factors (NLF's) and represent rigid body accelerations of the cg due to the reaction (or interface) forces, Q_R^r. The development below demonstrates how these are determined.

Define:

$$
\begin{aligned}
u_{cg} &= \text{6 x 1 matrix of rigid body displacements of the cg of the substructure} \\
u_{R_{rb}} &= \text{r x 1 vector of rigid body displacements at the r DOF} \\
T_{R6} &= \text{r x 6 matrix where each column represents rigid body displacements of} \\
&\quad \text{the r DOF due to a unit motion in one DOF at the cg} \\
Q_{cg} &= \text{6 x 1 vector of forces at the cg that are static equivalents of } Q_r^r
\end{aligned} \qquad \text{10-29}
$$

Then:

$$
\begin{aligned}
u_{R_{rb}} &= T_{R6} u_{cg} \\
&\text{and} \\
Q_{cg} &= T_{R6}^T Q_R^r
\end{aligned} \qquad \text{10-30}
$$

Substitute equation 27 into 30 for Q_R^r :

$$
Q_{cg} = T_{R6}^T (m_{RR} \ddot{u}_R + m_{NR}^T \ddot{\xi}_N + k_{RR} u_R - P_R') \qquad \text{10-31}
$$

For rigid body motion:

$$
Q_{cg} = m_{cg} \ddot{u}_{cg} \qquad \text{10-32}
$$

where m_{cg} is the 6 x 6 rigid body mass matrix relative to the cg and is equal to:

$$
m_{cg} = T_{R6}^T m_{RR} T_{R6} \qquad \text{10-33}
$$

and m_{RR} is given in equation 17. From equations 31 through 33 we can write the cg acceleration net load factors (NLF's) as:

$$
\ddot{u}_{cg} = m_{cg}^{-1} Q_{cg} = m_{cg}^{-1} T_{R6}^T \begin{bmatrix} m_{RR} & m_{NR}^T & k_{RR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - m_{cg}^{-1} T_{R6}^T P_R' \qquad \text{10-34}
$$

However, $T_{R6}^T k_{RR} = 0$ since the columns of T_{R6} are rigid body modes. Therefore:

$$
\ddot{u}_{cg} = m_{cg}^{-1} Q_{cg} = m_{cg}^{-1} T_{R6}^T \begin{bmatrix} m_{RR} & m_{NR}^T & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - m_{cg}^{-1} T_{R6}^T P_R' \qquad \text{10-35}
$$

which can be written as:

$$
\begin{aligned}
&\ddot{u}_{cg} = \begin{bmatrix} LTM11_{6R} & LTM12_{6N} & 0_{6R} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} - \begin{bmatrix} LTM14_{6R} \end{bmatrix} P'_R \\
&\text{where} \\
&LTM11_{6R} = m_{cg}^{-1} T_{R6}^T m_{RR} \\
&LTM12_{6N} = m_{cg}^{-1} T_{R6}^T m_{NR}^T \\
&LTM14_{6R} = m_{cg}^{-1} T_{R6}^T \\
&LTM1_{6Z} = \begin{bmatrix} LTM11_{6R} & LTM12_{6N} & 0 \end{bmatrix}
\end{aligned}
\qquad (10\text{-}36)
$$

10.3.3 LTM Terms for Element Forces and Stresses

In MYSTRAN, element forces and stresses are obtained from the G-set displacement vector and the individual element stiffness matrices. Equation 26 is the G-set displacement vector:

$$
u_G = \begin{bmatrix} DTM1_{GR} \mid DTM2_{GN} \mid DTM3_{GR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + DTM4_{GL} P_L = \Gamma_{GZ} u_Z + DTM4_{GL} P_L
$$

Thus the columns of each of the DTM's represents G-set displacements per unit value of one of the variables $\ddot{u}_R, \ddot{\xi}_N, u_R, P_L$ as described in Table 10.1. Therefore, each of the DTM's can be used as if they were a matrix of displacements in calculating element forces and stresses to give:

$$
\begin{aligned}
&f_e = \begin{bmatrix} LTM31_{eR} \mid LTM32_{eN} \mid LTM33_{eR} \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} + LTM34_{eL} P_L \\
&\text{where} \\
&f_e = \text{vector of element forces and stresses (e = number of finite elements)} \\
&LTM31_{eR} = \text{matrix of element forces and stresses due to G-set displ's } DTM1_{GR} \\
&LTM32_{eN} = \text{matrix of element forces and stresses due to G-set displ's } DTM2_{GN} \\
&LTM33_{eR} = \text{matrix of element forces and stresses due to G-set displ's } DTM3_{GR} \\
&LTM34_{eL} = \text{matrix of element forces and stresses due to G-set displ's } DTM4_{GL} \\
&LTM3_{eZ} = \begin{bmatrix} LTM31_{eR} \mid LTM32_{eN} \mid LTM33_{eR} \end{bmatrix}
\end{aligned}
\qquad (10\text{-}37)
$$

10.3.4 LTM Terms for Grid Point Forces due to multi-point constraints (MPC's)

There are cases in CB analyses in which the forces due to MPC's are of interest. As an example, if a user wishes to determine a load in a bolt at an interface between components, it is common to model the bolt as an MPC where two coincident grids are constrained to have the same displacements. This section develops the equations for determining an LTM for grid point MPC forces.

Equation 1 for the i-th substructure (dropping the superscript-j notation):

$$M_{GG}\ddot{u}_G + K_{GG}u_G = P_G + Q_G^s + Q_G^m + Q_G^r \qquad 10\text{-}38$$

As described in section 10.1 the Q constraint forces on the right side of equation 38 are the constraint forces on the S-set SPC DOF's, the M-set MPC DOF's and on the R-set boundary DOF's respectively. Since all of the boundary DOF's are contained in the R-set there should be no constraint forces on the S-set. That is, all S-set DOF's should be the result of removing singularities and not the result of grounding the model[3]. With this assumption, as well as the assumption that there are no applied loads on the M-st degrees of freedom the following equation is valid for the MPC forces on the M-set grids:

$$Q_G^m = M_{GG}\ddot{u}_G + K_{GG}u_G - Q_G^r \qquad 10\text{-}39$$

We want to get 39 in a form like the other LTM'; that is, in terms of u_Z .

From equation 26 with applied loads zero:

$$u_G = \Gamma_{GZ}u_Z, \quad u_Z = \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} \qquad 10\text{-}40$$

The g-set DOF vector can also be written using equation 14:

$$u_G = \Psi_{GX}u_X, \quad u_X = \begin{Bmatrix} u_R \\ \xi_N \end{Bmatrix} \qquad 10\text{-}41$$

Differentiating twice:

$$\ddot{u}_G = \Psi_{GX}\ddot{u}_X$$

This can also be written as:

$$\ddot{u}_G = \begin{bmatrix} \Psi_{GX} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_X \\ u_R \end{Bmatrix} \qquad 10\text{-}42$$

Partition the x DOF's into R and N as in equation 13. This will require partitioning Ψ_{GX} into sub-matrices for the R and N also, so that equation 42 can be written as:

[3] This should be verified by the user by inspection of the forces of single point constraint in the output from the analysis

$$\ddot{u}_G = \begin{bmatrix} \Psi_{GR} & \Psi_{GN} & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \\ u_R \end{Bmatrix} = \Psi'_{GZ} u_Z$$

where 10-43

$$\Psi'_{GZ} = \begin{bmatrix} \Psi_{GR} & \Psi_{GN} & 0 \end{bmatrix} = \begin{bmatrix} \Psi_{GX} & 0 \end{bmatrix}$$

.

Substitute equations 40 and 43 into 39 for u_G and $\ddot{u}_G$ respectively to get:

$$Q_G^m = M_{GG}\Psi'_{GZ}u_Z + K_{GG}\Gamma_{GZ}u_Z - Q_G^r \tag{10-44}$$

We need to express the boundary constraint forces in equation 44 in terms of the u_Z vector as we did for the inertia and stiffness terms. From 28:

$$Q_R^r = J_{RZ}u_Z - I_{RR}P_R \tag{10-45}$$

The Q_R^r boundary forces on the R-set can be expanded from the R-set to the G-set Q_G^r by adding zero rows to 45 for the M, S, O-sets (all of the G-set but the R degrees of freedom) to give

$$Q_G^r = J_{GZ}u_Z - I_{GR}P_R \tag{10-46}$$

where J_{GZ} is J_{RZ} expanded to G-set size by addition of zero rows for M, S, O-sets and I_{GR} is expanded from I_{RR} in the same fashion (recall I_{RR} is an R size identity matrix). Substituting 46 into 44 we get::

$$Q_G^m = (M_{GG}\Psi'_{GZ} + K_{GG}\Gamma_{GZ} - J_{GZ})u_Z$$

or

$$Q_G^m = LTM4_{GZ}\, u_Z \tag{10-47}$$

where

$$LTM4_{GZ} = (M_{GG}\Psi'_{GZ} + K_{GG}\Gamma_{GZ} - J_{GZ})$$

$LTM4_{GZ}$ is the LTM for MPC forces at grids that have no applied load

10.4 Development of Acceleration Output Transfer Matrices (Accel OTM)

In addition to the displacement and load output transformation matrices (DTM's and LTM's) it is common to supply acceleration output transformation matrices (accel OTM's or ATM's for short). From equation 10-12 and differentiating twice we obtain:

$$\begin{Bmatrix} \ddot{u}_R \\ \ddot{u}_L \end{Bmatrix} = [ATM] \begin{Bmatrix} \ddot{u}_R \\ \ddot{\xi}_N \end{Bmatrix}$$

where 10-48

$$ATM = \begin{bmatrix} I & 0 \\ D_{LR} & \Phi_{LN} \end{bmatrix}$$

ATM is the acceleration transfer matrix. Notice that the “degrees of freedom” for the ATM are the accelerations of the boundary and modal degrees of freedom whereas all of the other OTM's have as degrees of freedom: boundary accelerations, modal accelerations and boundary displacements. This is due to the use of the modal acceleration method for recovery of displacements and element forces.

10.5 Correspondence between matrix names and CB Equation Variables

The table below shows the correspondence between variables introduced in the above equations and matrix data block names in the DMAP program in Section 10.5. Any of these may be output in a MYSTRAN CB model generation analysis using the Executive Control entry OUTPUT4.

Table 10-2
Matrices that can be written to OUTPUT4 files

	MYSTRAN Matrix Name (OUTPUT4 matrices)	NASTRAN DMAP Name	CB equation variable in Appendix D (where applicable)	Matrix size[1]	Partition rows and/or cols
1	CG_LTM		$\left[LTM11_{6r} \quad LTM12_{6N} \quad 0 \right]$	6x(2R+N)	
2	DLR	DM	D_{LR}	LxR	rows and cols
3	EIGEN_VAL	LAMA	Ω^2_{NN}	NxN	
4	EIGEN_VEC	PHIG	Φ_{GN} , (Φ_{LN} with rows expanded to G-set)	GxN	rows
5	GEN_MASS	MI	m_{NN}	Nx1 vector of diag. terms	
6	IF_LTM		$\left[LTM21_{RR} \quad LTM22_{RN} \quad LTM23_{RR} \right]$	Rx(2R+N)	rows
7	KAA	KAA	K_{AA}	AxA	rows and cols
8	KGG	KGG	K_{GG}	GxG	rows and cols
9	KLL	KLL	K_{LL}	LxL	rows and cols
10	KRL	KLR(t)	K_{LR}	LxR	rows and cols
11	KRR	KRR	K_{RR}	RxR	rows and cols
12	KRRcb	KBB	$k_{RR} = K_{RR} + K^T_{LR} D_{LR}$	RxR	rows and cols
13	KXX	KRRGN	K_{XX}	(R+N)x(R+N)	
14	LTM	LTM	CG_LTM and IF_LTM merged	(6+R)x(2R+N)	
15	MCG	RBMCG	m_{cg}	6x6	
16	MEFFMASS		Modal effective mass	Nx6	
17	MPFACTOR		Modal participation factors	Nx6 or NxR	
18	MAA		M_{AA}	AxA	rows and cols
19	MGG		M_{GG}	GxG	rows and cols
20	MLL	MLL	M_{LL}	LxL	rows and cols
21	MRL	MRL	M_{RL}	RxL	rows and cols
22	MRN		$m_{RN} = m^T_{NR}$	RxN	rows
23	MRR	MRR	M_{RR}	RxR	rows and cols

Table 10-2 (con't)

	MYSTRAN Matrix Name (OUTPUT4 matrices)	NASTRAN DMAP Name	CB equation variable in Appendix D (where applicable)	Matrix size[4]	Partition rows and/or cols
24	MRRcb	MBB	$m_{RR} = M_{RR} + M_{LR}^T D_{LR} + (M_{LR}^T D_{LR})^T + D_{LR}^T M_{LL} D_{LR}$	RxR	rows and cols
25	MXX	MRRGN	$M_{XX} = \begin{bmatrix} m_{RR} & m_{NR}^T \\ m_{NR} & m_{NN} \end{bmatrix}$	(R+N)x(R+N)	
26	PA		(A-set static reduced loads - only used in statics)		Rows
27	PG		(G-set static loads - only used in statics)		Rows
28	PL		(L-set static reduced loads - only used in statics)		rows
29	PHIXG	PHIXG	Ψ_{AX} , (Ψ_{AX} with rows expanded to G-set)	Gx(R+N)	rows
30	PHIZG		The G-set displacement transformation matrix is written out in the F06 file under "C B D I S P L A C E M E N T O T M"	Gx(2R+N)	rows
31	RBM0		Rigid body mass matrix relative to the basic origin	6x6	
32	TR6_0	RBR	T_{R6} : rigid body displacement matrix for R-set relative to the model basic coordinate system	Rx6	rows
33	TR6_CG	RBRCG	T_{R6} : rigid body displacement matrix for R-set relative to the model CG	Rx6	rows

Notes:

a. (t) indicates matrix transposition

b. Matrix m_{RR} will be singular if there are rotational DOF's but no rotational inertia in the R-set, in which case small rotational inertias may have to be added at these DOF's.

c. Matrix k_{RR} is null if the boundary is a determinant set of DOF's.

d. Matrix m_{RR} is the rigid body mass matrix if the boundary is a determinant set of DOF's

[4] Matrix size given in rows x columns where R means the size of the R-set, L is the size of the L-set, A is the size of the A-set, G is the size of the G-set and N is the number of eigenvectors. See section 3.6 for definition of the complete displacement set notation

10.6 Craig-Bampton model generation example problem

The figure below shows a small example problem that is a frame made of CBAR's that is a substructure assumed to be attached to some other structure in DOF's 1,2,3 at grids 11 and 13 and in DOF's 2,3 at grid 12. The example problem F06 file (with the input echo'd) is shown on the following pages. This section will discuss the input and output in an effort to explain the Craig-Bampton model generation process.

Equation 10.26 defines the Craig-Bampton degrees of freedom (CB-DOF's) as U_z which, for this example, consists of the 18 DOF's:

- 8 boundary acceleration DOF's, $\ddot{u}_R$
- 2 modal acceleration DOF's, $\ddot{\xi}_N$ (see EIGRL request for 2 modes to be extracted)
- 8 boundary displacement DOF's, u_R

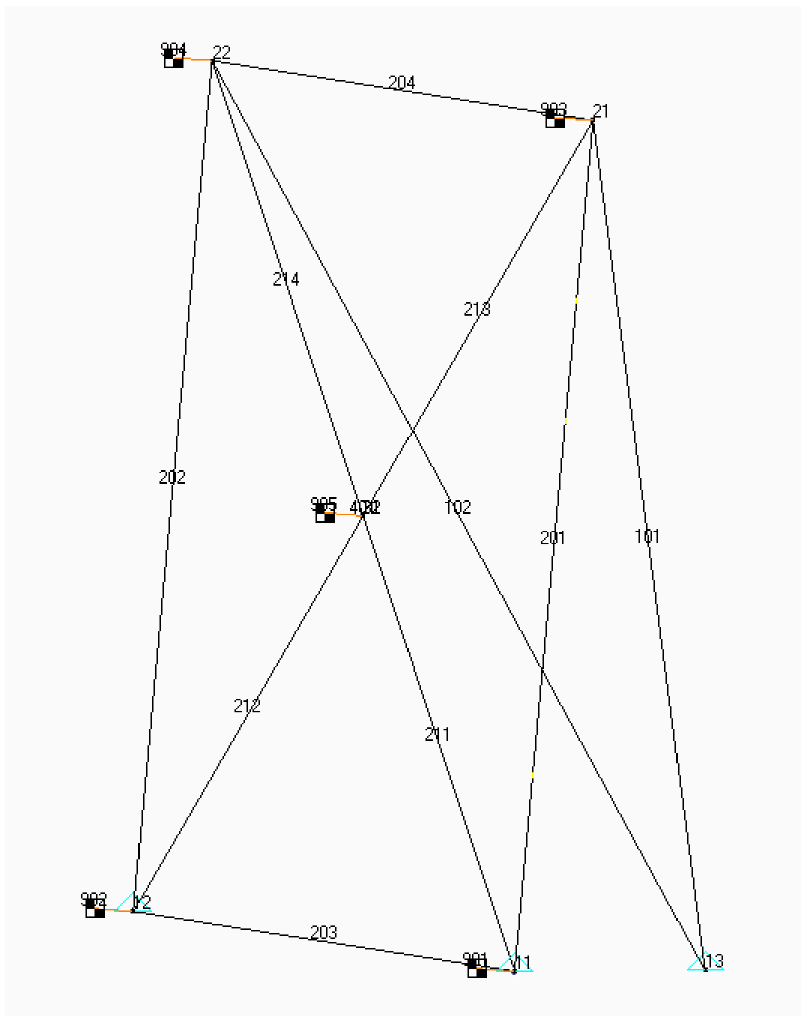

Figure 10.2 – Example CB model: CB-EXAMPLE-12b.DAT

Notes on section 10.6.1: CB-EXAMPLE-12b.F06

The echo of the input shows the following salient points for a CB model generation (much like a SOL 3 eigenvalue analysis in terms of input data):

- Executive Control:

    - SOL 31 indicates CB model generation

    - The OUTPUT4 commands show the matrices that will be written in a format the same as NASTRAN OUTPUT4 files. These matrix data blocks are ones that are listed on Table 10.2 as allowable OUTPUT4 matrices. Notice that several are written to unit 21 while others are written to unit 22. As explained in section 5.1 of the MYSTRAN Users Reference Manual, unit numbers 21 through 27 are valid for writing OUTPUT4 matrices.

- Case Control:

    - METHOD = 1 is to be used for a normal eigenvalue analysis (same as if SOL were 3)

    - Outputs (ACCE, DISP, ELFORCE, STRESS) are for Output Transformation Matrices (OTM's) for the specified sets. These will be written to the text F06 file. In addition they will be written to binary files (same name, CB-EXAMPLE-12b) with extension OP8 for the element related OTM;s (ELFORCE, STRESS in this case and OP9 for the grid related OTM's (ACCE, DISP in this case)

- Bulk Data:

    - Shows the model for this example (notice it has mostly CBAR's but there is also a RBE2)

    - Degrees of freedom at the boundary where this substructure attaches to other substructures are defined with the SUPORT Bulk Data entry. This is the same procedure that is used in CB analyses by the NASTRAN DMAP (Direct Matrix Abstraction Program) method familiar to NASTRAN CB analysts.

    - Eigenvalue extraction, EIGRL requesting 2 modes to be extracted

The delineated F06 output begins on the page following the input model echo and shows the following:

- Eigenvalues extracted

- Messages on the matrices requested to be written to OUTPUT4 files

- For the first 3 of the 18 CB_DOF's in this example the following output (requested in Case Control) is shown (other 15 were left out for clarity):

    - Displacement OTM for the requested grids (see Case Control command DISP = 102)

    - Element engineering force OTM (see Case Control command ELFORCE = 201)

    - Element stress OTM (see Case Control command STRESS = 202)

- Acceleration OTM. As shown in equation 10.48 the acceleration OTM has columns for $\ddot{u}_R$ and $\ddot{\xi}_N$ but not u_R . For this example, there are 10 columns in the acceleration OTM (8 boundary acceleration DOF's and 2 modal acceleration DOF's)

Notes on section 10.6.2: OUTPUT4 matrices written to CB-EXAMPLE-12b.OP1 and OP2

As shown in the Executive Control section of the F06 file in section 10.6.1, there were 3 matrices requested to be written to unit 21 and 4 to unit 22. These binary files, translated to text, are shown in section 10.6.2. The number of actual columns for each matrix is indicated in Table 10.2 but only the first 5 of the columns are shown here for the sake of brevity. These are several of the important CB matrices needed to couple this CB substructure to other substructures in a combined analysis. The binary OUTPUT4 files are written in the same format as the NASTRAN OUTPUT4 binary files.

Notes on section 10.6.3: Displ and elem force/stress OTM's written to CB-EXAMPLE-12b.OP1, OP2

Any output requests in Case Control for grid related outputs (e.g. DISPL, ACCEL) and element force/stress outputs (e.g. ELFORCE, STRESS) are written to the text F06 file and also written to OUTPUT4 binary files (automatically; that is, no formal OUTPUT4 request is needed). The element related OTM's are always written to a file with the same filename as the F06 file but with extension OP8. The grid related OTM's are written to a file with extension OP9.

The first page of section 10.6.3 is a text translation of the element related OTM's written to file CB-EXAMPLE-12b.OP8. The values are the same as was written to the F06 file for element forces and stresses but are also written to binary files in OUTPUT4 format to be used in analyses that couple the CB substructures. In order to explain the contents of the binary OP8 file, a text file with extension OT8 is also automatically written (provided any Case Control requests are included for element forces/stresses) describing the contents of the OP8 binary file. This OT8 text file gives an overview of the OP8 binary file and then goes on to describe each row written to the OP8 file.

The next several pages show the same type of information on the grid related OTM's written to binary file with extension OP9 (with text description in OT9). Again, this is the grid related outputs requested in Case Control and also written to the F06 text file.
*

10.6.1 CB-EXAMPLE-12-b.F06

(delineated – some output not included here for the sake of clarity)

```
1030180330

 MYSTRAN Version 3.00   Oct 20 2006 by Dr Bill Case (this TRIAL edition is SP protected)

 >> MYSTRAN BEGIN  : 10/30/2006 at 18: 3:30.640 The input file is CB-EXAMPLE-12-b.DAT

 >> LINK  1 BEGIN


SOL 31
$
OUTPUT4   CG_LTM   , IF_LTM   ,         ,        ,        //-1/21 $
OUTPUT4   KRRGN    , RBMCG    , MRRGN   ,        , RBRCG //-1/22 $
OUTPUT4   MR       ,          ,         ,        ,        //-1/21 $
CEND
TITLE  = TEST OF CRAIG-BAMPTON SOLUTION
SUBTI  = FRAME USING CBAR's
SPC    = 1
METHOD = 1
ECHO     = UNSORT
$
SET 101  = 32
SET 102  = 22, 32
SET 201  = 211, 212
SET 202  = 201
$
ACCE     = 101
DISP     = 102
ELFORCE  = 201
STRESS   = 202
MEFFMASS = ALL
MPFACTOR = ALL
$
BEGIN BULK
$
EIGRL   1                       2       2       DPB     -1.     MASS
$
EIGR    2        MGIV                   1       24                      +E1
+E1     MASS
GRID    11              0.      0.      0.
GRID    12              100.    0.      0.
GRID    13              50.     0.      50.
GRID    21              0.      100.    0.
GRID    22              100.    100.    0.
GRID    31              50.     50.     0.
GRID    32              50.     50.     0.
$
RBE2    401     31      123456  32
```

```
$
$ Frame support bars
$
CBAR    101     1       13      21      0.0     0.5     1.0             +C1
+C1     56      456
CBAR    102     1       13      22      0.0     0.5     1.0             +C2
+C2     56      456
$
$ Edge bars
$
CBAR    201     2       11      21      0.0     0.0     1.0
CBAR    202     2       12      22      0.0     0.0     1.0
CBAR    203     2       11      12      0.0     0.0     1.0
CBAR    204     2       21      22      0.0     0.0     1.0
$
$ Diag bars
$
CBAR    211     3       11      31      0.0     0.0     1.0
CBAR    212     3       12      31      0.0     0.0     1.0
CBAR    213     3       21      31      0.0     0.0     1.0
CBAR    214     3       22      31      0.0     0.0     1.0
$
PBAR    1       1       0.36    0.09    0.09    0.18
PBAR    2       1       0.10    10.0    10.0    20.0
PBAR    3       1       6.0     6.0     6.0     12.0
$
MAT1    1       10.+6           0.3     0.1
 *INFORMATION: MAT1 ENTRY        1 HAD FIELD FOR G  BLANK. MYSTRAN CALCULATED G  =  3.846154E+06
$
CONM2   901     11              150.0   0.0     0.0     -5.0
CONM2   902     12              150.0   0.0     0.0     -5.0
CONM2   903     21              150.0   0.0     0.0     -5.0
CONM2   904     22              150.0   0.0     0.0     -5.0
CONM2   905     32              150.0   0.0     0.0     -5.0
$
SPC1    1       456     13
$
$ BOUNDARY DOF'S
$
SUPORT  11      123     12      23      13      123
$
PARAM   WTMASS   .002591
$
ENDDATA
```

```
E I G E N V A L U E   A N A L Y S I S   S U M M A R Y   (LANCZOS  Mode 2 DPB     Shift eigen = -1.00E+00)

         NUMBER OF EIGENVALUES EXTRACTED  . . . . . .         2

         LARGEST OFF-DIAGONAL GENERALIZED MASS TERM    -2.7E-13 (Vecs renormed to 1.0 for gen masses)

                                                   . . .      2
                   MODE PAIR . . . . . . . . . .
                                                   . . .      1

         NUMBER OF OFF DIAGONAL GENERALIZED MASS
         TERMS FAILING CRITERION OF  1.0E-04. . . . .         0


                                      R E A L   E I G E N V A L U E S
  MODE  EXTRACTION      EIGENVALUE           RADIANS              CYCLES            GENERALIZED          GENERALIZED
 NUMBER   ORDER                                                                        MASS               STIFFNESS

     1       1        3.895211E+03        6.241163E+01        9.933119E+00        1.000000E+00        3.895211E+03
     2       2        7.011163E+03        8.373269E+01        1.332647E+01        1.000000E+00        7.011163E+03


>> LINK  4 END


>> LINK  6 BEGIN
```

```
*INFORMATION: THE FOLLOWING   7 MATRICES WILL BE WRITTEN TO  2 OUTPUT4 FILES IN THE ORDER LISTED BELOW:

               OUTPUT4 file on unit  21 has been created as: CB-EXAMPLE-12-b.OP1 and will contain the matrices:
                       ( 1) CG_LTM         :       6 rows and      18 cols   This is MYSTRAN matrix CG_LTM
                       ( 2) IF_LTM         :       8 rows and      18 cols   This is MYSTRAN matrix IF_LTM
                       ( 3) MR             :       8 rows and       8 cols   This is MYSTRAN matrix MRRcb

               OUTPUT4 file on unit  22 has been created as: CB-EXAMPLE-12-b.OP2 and will contain the matrices:
                       ( 1) KRRGN          :      10 rows and      10 cols   This is MYSTRAN matrix KXX
                       ( 2) RBMCG          :       6 rows and       6 cols   This is MYSTRAN matrix MCG
                       ( 3) MRRGN          :      10 rows and      10 cols   This is MYSTRAN matrix MXX
                       ( 4) RBRCG          :       8 rows and       6 cols   This is MYSTRAN matrix TR6


 >> LINK  6 END


 >> LINK  5 BEGIN


 >> LINK  5 END


 >> LINK  9 BEGIN
```

OUTPUT FOR CRAIG-BAMPTON DOF 1 OF 18

C B   D I S P L A C E M E N T   O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	-1.412939E-05	1.622140E-05	8.242222E-05	5.883709E-07	-1.667433E-06	5.125151E-07
32	0	1.051041E-05	-9.465944E-06	-3.182887E-06	-1.086181E-07	-9.450720E-07	2.106009E-07

C B   E L E M E N T   E N G I N E E R I N G   F O R C E   O T M
F O R   E L E M E N T   T Y P E   B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	2.091876E-01	7.894539E-01	1.515607E+00	-1.439344E+00	-1.847556E-02	3.151997E-02	6.266800E-01	9.672846E-03
212	-1.133151E-01	-1.008960E-02	-1.725401E+00	-6.166148E-02	2.279833E-02	7.293366E-04	-2.953611E-01	-4.720428E-03

C B   E L E M E N T   S T R E S S   O T M   I N   L O C A L   E L E M E N T   C O O R D I N A T E   S Y S T E M
F O R   E L E M E N T   T Y P E   B A R

Element	SA1	SA2	SA3	SA4	Axial	SA-Max	SA-Min	M.S.-T
ID	SB1	SB2	SB3	SB4	Stress	SB-Max	SB-Min	M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-2.748670E+00	-2.748670E+00	-2.748670E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		-2.748670E+00	-2.748670E+00	

OUTPUT FOR CRAIG-BAMPTON DOF 2 OF 18

C B   D I S P L A C E M E N T   O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	-7.600290E-05	8.243595E-05	3.128787E-04	1.925291E-06	2.220055E-06	1.292053E-07
32	0	-5.990878E-05	6.308617E-05	3.224179E-04	3.643362E-06	4.904270E-07	3.218612E-08

C B   E L E M E N T   E N G I N E E R I N G   F O R C E   O T M
F O R   E L E M E N T   T Y P E   B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	3.640634E+00	-2.875040E+00	-7.752079E+00	4.486528E+00	1.611173E-01	-1.041083E-01	1.906435E+00	-5.333935E-03
212	3.789705E+00	2.992877E+00	-6.061077E+00	-4.713484E+00	1.393111E-01	1.089844E-01	1.808077E+00	5.333935E-03

C B   E L E M E N T   S T R E S S   O T M   I N   L O C A L   E L E M E N T   C O O R D I N A T E   S Y S T E M
F O R   E L E M E N T   T Y P E   B A R

Element	SA1	SA2	SA3	SA4	Axial	SA-Max	SA-Min	M.S.-T
ID	SB1	SB2	SB3	SB4	Stress	SB-Max	SB-Min	M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	7.582667E+00	7.582667E+00	7.582667E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		7.582667E+00	7.582667E+00	

OUTPUT FOR CRAIG-BAMPTON DOF 3 OF 18

C B D I S P L A C E M E N T O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
22	0	3.800145E-05	-4.121798E-05	-1.564393E-04	-9.626456E-07	-1.110028E-06	-6.460267E-08
32	0	2.995439E-05	-3.154308E-05	-1.612090E-04	-1.821681E-06	-2.452135E-07	-1.609306E-08

C B E L E M E N T E N G I N E E R I N G F O R C E O T M
F O R E L E M E N T T Y P E B A R

Element	Bend-Moment End A		Bend-Moment End B		- Shear -		Axial	Torque
ID	Plane 1	Plane 2	Plane 1	Plane 2	Plane 1	Plane 2	Force	
211	-1.820317E+00	1.437520E+00	3.876039E+00	-2.243264E+00	-8.055864E-02	5.205414E-02	-9.532175E-01	2.666968E-03
212	-1.894852E+00	-1.496438E+00	3.030538E+00	2.356742E+00	-6.965554E-02	-5.449220E-02	-9.040385E-01	-2.666968E-03

C B E L E M E N T S T R E S S O T M I N L O C A L E L E M E N T C O O R D I N A T E S Y S T E M
F O R E L E M E N T T Y P E B A R

Element	SA1	SA2	SA3	SA4	Axial	SA-Max	SA-Min	M.S.-T
ID	SB1	SB2	SB3	SB4	Stress	SB-Max	SB-Min	M.S.-C
201	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00	-3.791334E+00	-3.791334E+00	-3.791334E+00	
	0.000000E+00	0.000000E+00	0.000000E+00	0.000000E+00		-3.791334E+00	-3.791334E+00	

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(output for the 4th – 18th CB DOF deleted)

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL 1 OF 10

C B A C C E L E R A T I O N O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
32	0	2.199853E-02	-2.028331E-02	-1.681579E-02	-3.363157E-04	8.006145E-03	5.254334E-04

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL 2 OF 10

C B A C C E L E R A T I O N O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
32	0	0.000000E+00	0.000000E+00	-1.000000E+00	-2.000000E-02	0.000000E+00	0.000000E+00

OUTPUT FOR CRAIG-BAMPTON ACCEL OTM COL 3 OF 10

C B A C C E L E R A T I O N O T M
(in global coordinate system at each grid)

GRID	COORD SYS	T1	T2	T3	R1	R2	R3
32	0	0.000000E+00	0.000000E+00	5.000000E-01	1.000000E-02	0.000000E+00	0.000000E+00

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(output for the 4^{th} – 10^{th} Accel OTM columns deleted)

```
                    M O D A L   P A R T I C I P A T I O N   F A C T O R S
                         (dimensionless, in coordinate sys       0)
  MODE       T1            T2            T3            R1            R2            R3
   NUM
     1  1.227574E-01 -1.758352E+00  8.791759E-01  1.259087E+00  6.535370E-02 -5.341716E-01
     2  6.061630E-01  1.829524E-01 -9.147622E-02 -4.910542E-01 -1.366914E-01 -4.626569E-01

------------------------------------------------------------------------------------------------------------------------------------

                E F F E C T I V E   M O D A L   M A S S E S   O R   W E I G H T S
                               (in coordinate system       0)
                   Units are same as units for mass input in the Bulk Data Deck
  MODE       T1            T2            T3            R1            R2            R3
   NUM
     1  6.532677E+01  4.179096E+01  4.694259E+02  3.836785E+05  3.287406E+04  3.611917E+02
     2  7.948285E+00  9.016521E-01  1.363070E+01  1.674257E+00  6.082279E+05  4.781873E+05
      ------------- ------------- ------------- ------------- ------------- -------------
        Sum all modes:  7.327506E+01  4.269261E+01  4.830566E+02  3.836801E+05  6.411019E+05  4.785485E+05
     Total model mass:  9.325238E+02  9.325238E+02  9.325238E+02  4.105260E+06  4.094237E+06  8.139951E+06
 Modes % of total mass*:          7.86          4.58         51.80          9.35         15.66          5.88

 *If all modes are calculated the % of total mass should be 100% of the free mass (i.e. not counting mass at constrained DOF's).
  Percentages are only printed for components that have finite model mass.
                                                                      -----

 >> LINK  9 END

 >> MYSTRAN END    : 10/30/2006 at 18: 3:31.562
```

10.6.2 OUTPUT4 matrices written to CB-EXAMPLE-12-b.OP1 and OP2

(OUTPUT4 matrices requested in Exec Control)

OUTPUT4 matrices requested in Exec Control to be written to file CB-EXAMPLE-12-b.OP1 (on unit 21)

(note: only 1st 5 columns written here for the sake of clarity)

```
    CG_LTM                 NCOLS =        18        NROWS =        6        FORM   =        2        PREC   =        2
              1                       2                       3                       4                       5
1   -6.65821789802521E-05    1.29562159612018E-17   -6.47810798060089E-18   -1.29549999999999E-03    6.47766872193621E-05 .......
2   -2.99785601343913E-05   -1.96135553418977E-04    1.04193052213477E-04    1.39356777670951E-03   -6.70858061739371E-05 .......
3   -4.35697030582909E-05   -2.59100000000000E-03    1.30775055100798E-03    1.29550000000001E-03    6.19839872966866E-04 .......
4   -3.33844454038618E-04   -2.00000000000000E-02    9.80743672854175E-03    1.00000000000000E-02   -5.07064059129018E-03 .......
5    8.13687816036514E-03    1.47885176327023E-16   -7.39425881635114E-17   -7.78457159844592E-17   -5.93156091981744E-03 .......
6    5.63393757592496E-04    8.55130582230051E-17   -4.27565291115026E-17    9.99999999999996E-03    2.81696878796245E-04 .......

    IF_LTM                 NCOLS =        18        NROWS =        8        FORM   =        2        PREC   =        2
              1                       2                       3                       4                       5
1    6.02957424769077E-01    7.32039059471622E-02   -3.66019529735811E-02    3.35492666170908E-02   -7.19015457719424E-02 .......
2    7.32039059471623E-02    4.25469107253153E+00   -2.12163357113457E+00   -2.21879607113459E+00   -1.10665832128050E-01 .......
3   -3.66019529735811E-02   -2.12163357113457E+00    1.07224071582968E+00    1.10939803556729E+00    5.53329160640251E-02 .......
4    3.35492666170908E-02   -2.21879607113459E+00    1.10939803556729E+00    3.26418464157067E+00    1.75366508593570E-02 .......
5   -7.19015457719424E-02   -1.10665832128050E-01    5.53329160640251E-02    1.75366508593570E-02    4.96481812094837E-01 .......
6   -6.65046890695409E-01   -7.32039059471504E-02    3.66019529735752E-02   -1.24163383728600E+00    1.32307347677584E-01 .......
7   -1.34708893096271E-01   -2.21879607113459E+00    1.10939803556729E+00    2.54146535101691E-01    3.05700710026811E-02 .......
8    6.78737140960850E-02   -1.83869738075211E-01    9.19348690376054E-02    8.11498842422746E-02    2.62006997196796E-02 .......

    MR                     NCOLS =         8        NROWS =        8        FORM   =        1        PREC   =        2
              1                       2                       3                       4                       5
1    6.02957424769077E-01    7.32039059471622E-02   -3.66019529735811E-02    3.35492666170908E-02   -7.19015457719424E-02 .......
2    7.32039059471623E-02    4.25469107253153E+00   -2.12163357113457E+00   -2.21879607113459E+00   -1.10665832128050E-01 .......
3   -3.66019529735811E-02   -2.12163357113457E+00    1.07224071582968E+00    1.10939803556729E+00    5.53329160640251E-02 .......
4    3.35492666170908E-02   -2.21879607113459E+00    1.10939803556729E+00    3.26418464157067E+00    1.75366508593570E-02 .......
5   -7.19015457719424E-02   -1.10665832128050E-01    5.53329160640251E-02    1.75366508593570E-02    4.96481812094837E-01 .......
6   -6.65046890695409E-01   -7.32039059471504E-02    3.66019529735752E-02   -1.24163383728600E+00    1.32307347677584E-01 .......
7   -1.34708893096271E-01   -2.21879607113459E+00    1.10939803556729E+00    2.54146535101691E-01    3.05700710026811E-02 .......
8    6.78737140960850E-02   -1.83869738075211E-01    9.19348690376054E-02    8.11498842422746E-02    2.62006997196796E-02 .......
```

OUTPUT4 matrices requested in Exec Control to be written to file CB-EXAMPLE-12-b.OP2 (on unit 22)

(note: only 1st 5 columns written here the sake of clarity)

```
       KRRGN                  NCOLS =       10        NROWS =       10        FORM   =        1         PREC   =        2
                 1                      2                       3                       4                       5
  1    1.19504240447136E+03   -3.63797880709171E-12    1.81898940354586E-12    1.54614099301398E-11    5.97521202235677E+02 .......
  2   -5.45696821063757E-12    0.00000000000000E+00    0.00000000000000E+00    1.81898940354586E-12    0.00000000000000E+00 .......
  3    2.72848410531878E-12    0.00000000000000E+00    0.00000000000000E+00   -9.09494701772928E-13    0.00000000000000E+00 .......
  4    2.08011385893769E-11    0.00000000000000E+00    0.00000000000000E+00   -1.16415321826935E-10    9.43778388773353E-12 .......
  5    5.97521202235677E+02   -1.13686837721616E-13    5.68434188608080E-14   -1.59161572810262E-12    2.98760601117838E+02 .......
  6   -1.19504240447137E+03    0.00000000000000E+00    0.00000000000000E+00   -1.79397829924710E-10   -5.97521202235685E+02 .......
  7   -2.98427949019242E-13    0.00000000000000E+00    0.00000000000000E+00   -4.31782609666698E-10   -2.76401124210679E-12 .......
  8   -5.97521202235677E+02   -1.81898940354586E-12    9.09494701772928E-13    1.36424205265939E-12   -2.98760601117839E+02 .......
  9    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......
 10    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00 .......

       RBMCG                  NCOLS =        6        NROWS =        6        FORM   =        2         PREC   =        2
                 1                      2                       3                       4                       5
  1    2.41616914133782E+00   -3.35287353436797E-14   -6.52256026967279E-15   -1.34114941374719E-13   -3.97903932025656E-13 .......
  2   -3.30846461338297E-14    2.41616914133786E+00    2.33146835171283E-14    7.74491581978509E-13    2.89102075612391E-13 .......
  3   -6.52256026967279E-15    2.27734497926235E-14    2.41616914133783E+00   -9.59232693276135E-14   -7.10542735760100E-14 .......
  4   -1.35891298214119E-13    7.81374964731185E-13   -1.24344978758018E-13    4.56169135583651E+03   -3.86535248253495E-12 .......
  5   -3.92130772297605E-13    2.88435941797616E-13   -6.75015598972095E-14   -4.09272615797818E-12    4.53313153018053E+03 .......
  6    1.99662508748588E-12    4.26325641456060E-14   -3.62376795237651E-13   -1.36424205265939E-11    2.85598256559946E+01 .......

       MRRGN                  NCOLS =       10        NROWS =       10        FORM   =        1         PREC   =        2
                 1                      2                       3                       4                       5
  1    6.02957424769077E-01    7.32039059471622E-02   -3.66019529735811E-02    3.35492666170908E-02   -7.19015457719424E-02 .......
  2    7.32039059471623E-02    4.25469107253153E+00   -2.12163357113457E+00   -2.21879607113459E+00   -1.10665832128050E-01 .......
  3   -3.66019529735811E-02   -2.12163357113457E+00    1.07224071582968E+00    1.10939803556729E+00    5.53329160640251E-02 .......
  4    3.35492666170908E-02   -2.21879607113459E+00    1.10939803556729E+00    3.26418464157067E+00    1.75366508593570E-02 .......
  5   -7.19015457719424E-02   -1.10665832128050E-01    5.53329160640251E-02    1.75366508593570E-02    4.96481812094837E-01 .......
  6   -6.65046890695409E-01   -7.32039059471504E-02    3.66019529735752E-02   -1.24163383728600E+00    1.32307347677584E-01 .......
  7   -1.34708893096271E-01   -2.21879607113459E+00    1.10939803556729E+00    2.54146535101691E-01    3.05700710026811E-02 .......
  8    6.78737140960850E-02   -1.83869738075211E-01    9.19348690376054E-02    8.11498842422746E-02    2.62006997196796E-02 .......
  9    1.22757372107055E-01   -1.75835189695839E+00    8.79175948479194E-01    1.25908689725916E+00    6.53537005701318E-02 .......
 10    6.06162990294928E-01    1.82952442095713E-01   -9.14762210478567E-02   -4.91054200271590E-01   -1.36691428775775E-01 .......

       RBRCG                  NCOLS =        6        NROWS =        8        FORM   =        2         PREC   =        2
                 1                      2                       3                       4                       5
  1    1.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    5.37849392786371E+01 .......
  2    0.00000000000000E+00    1.00000000000000E+00    0.00000000000000E+00   -5.37849392786371E+01    0.00000000000000E+00 .......
  3    0.00000000000000E+00    0.00000000000000E+00    1.00000000000000E+00   -5.00000000000000E+01    0.00000000000000E+00 .......
  4    0.00000000000000E+00    1.00000000000000E+00    0.00000000000000E+00   -3.78493927863709E+00    0.00000000000000E+00 .......
  5    0.00000000000000E+00    0.00000000000000E+00    1.00000000000000E+00   -5.00000000000000E+01   -5.00000000000000E+01 .......
  6    1.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    0.00000000000000E+00    3.78493927863709E+00 .......
  7    0.00000000000000E+00    1.00000000000000E+00    0.00000000000000E+00   -3.78493927863709E+00    0.00000000000000E+00 .......
  8    0.00000000000000E+00    0.00000000000000E+00    1.00000000000000E+00   -5.00000000000000E+01    5.00000000000000E+01 .......
```

10.6.3 Displ and Element force/stress OTM’s written to CB-EXAMPLE-12-b.OP8 and OP9

(OTM’s requested in Case Control)

CB-EXAMPLE-12-b.OP8 binary file of element force/stress OTM's requested in Case Control

(note: only 1st 5 columns written here the sake of clarity)

OTM_ELFE NCOLS = 18 NROWS = 16 FORM = 2 PREC = 2

	1	2	3	4	5	
1	2.09187572390564E-01	3.64063384390388E+00	-1.82031692195194E+00	-1.84227921264778E+00	-9.14925412689932E-01	....
2	7.89453912890167E-01	-2.87503976462738E+00	1.43751988231369E+00	1.92080844772306E+00	-1.26234542491864E-01	....
3	1.51560714339846E+00	-7.75207867487571E+00	3.87603933743785E+00	3.62690741509324E+00	1.45527637571713E+00	....
4	-1.43934432738336E+00	4.48652751792572E+00	-2.24326375896286E+00	-2.73874759882899E+00	2.35906653084923E-01	....
5	-1.84755627546901E-02	1.61117285562758E-01	-8.05586427813792E-02	-7.73459790410093E-02	-3.35197151472623E-02	....
6	3.15199669918811E-02	-1.04108282913086E-01	5.20541414565432E-02	6.58960735567147E-02	-5.12144990278700E-03	....
7	6.26679968599842E-01	1.90643492900070E+00	-9.53217464500349E-01	-1.19040949990613E-01	-1.14791218537626E-01	....
8	9.67284596743351E-03	-5.33393540270422E-03	2.66696770135211E-03	-5.34876839175438E-02	8.35971431688627E-04	....
9	-1.13315069892136E-01	3.78970456518829E+00	-1.89485228259414E+00	-1.26147862482940E+00	-9.55864075040792E-01	....
10	-1.00896004659258E-02	2.99287680850590E+00	-1.49643840425295E+00	-4.03697533588189E+00	-1.41398274167766E-02	....
11	-1.72540058669802E+00	-6.06107677196644E+00	3.03053838598322E+00	2.53928832803047E+00	1.96715396237338E+00	....
12	-6.16614847670031E-02	-4.71348398353008E+00	2.35674199176504E+00	6.82365970711492E+00	3.39064169416761E-02	....
13	2.27983320157212E-02	1.39311085669760E-01	-6.96555428348799E-02	-5.37509617215390E-02	-4.13377175157231E-02	....
14	7.29336582157196E-04	1.08984399486375E-01	-5.44921997431877E-02	-1.53592573737906E-01	-6.79476503928156E-04	....
15	-2.95361107284698E-01	1.80807707871691E+00	-9.04038539358453E-01	-1.95832712226347E+00	3.00896480121837E-03	....
16	-4.72042770150405E-03	5.33393540270377E-03	-2.66696770135189E-03	-1.12160973347287E-01	-3.69369770142806E-03	....

OTM_STRE NCOLS = 18 NROWS = 18 FORM = 2 PREC = 2

	1	2	3	4	5	
1	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
2	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
3	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
4	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
5	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	....
6	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	....
7	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	....
8	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	....
9	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
10	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
11	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
12	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
13	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	....
14	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	....
15	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	....
16	-2.74867035744303E+00	7.58266712433821E+00	-3.79133356216910E+00	-1.07520478850513E+00	4.30045958649968E-01	....
17	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	-1.00000000000000E+00	....
18	1.00000000000000E+10	1.00000000000000E+10	1.00000000000000E+10	1.00000000000000E+10	1.00000000000000E+10	....

CB-EXAMPLE-12-b.OT8 text file descriptor of rows in above binary file for element related OTM's

```
This text file describes the rows of the elem related OTM matrices written to unformatted file: CB-EXAMPLE-12-b.OP8
-----------------------------------------------------------------------------------------------

The description for each of the matrices has the headers:
       ROW        : row number in the individual OTM described
       DESCRIPTION: what OTM is this
       TYPE       : element type
       EID        : element ID
Then, for the element nodal force OTM:
       GRID       : grid number of the element that the OTM is for
       COMP       : displacement component number (1,2,3 translations and 4,5,6 rotations)
and for element engineering force and element stress OTMs:
       ITEM       : element force or stress item (axial force, torque, etc)

The number of rows for each OTM depends on the output requests, by the user, in Case Control

The number of cols for each OTM depends on the number of support DOFs (NDOFR) and the number of eigenvecors (NVEC)where:
       NDOFR =        8
       NVEC  =        2

This text file has descriptions for the following element related OTMs from CB-EXAMPLE-12-b.OP8
       Element engr  force OTM (matrix OTM_ELFE) with 2*NDOFR + NVEC =       18 cols
       Element stress      OTM (matrix OTM_STRE) with 2*NDOFR + NVEC =       18 cols

---------------------------------------------------------------------------------
       Explanation of rows of       16 row by       18 col matrix OTM_ELFE

   ROW            DESCRIPTION             TYPE     EID             ITEM
 ------- ------------------------------ -------- -------    --------------------
       1 Element engineering force      BAR          211    M1a: Mom Plane1 EndA
       2 Element engineering force      BAR          211    M1b: Mom Plane2 EndA
       3 Element engineering force      BAR          211    M2a: Mom Plane1 EndB
       4 Element engineering force      BAR          211    M2b: Mom Plane2 EndB
       5 Element engineering force      BAR          211    V1 : Shear Plane1
       6 Element engineering force      BAR          211    V2 : Shear Plane2
       7 Element engineering force      BAR          211    FX : Axial force
       8 Element engineering force      BAR          211    T  : Torque

       9 Element engineering force      BAR          212    M1a: Mom Plane1 EndA
      10 Element engineering force      BAR          212    M1b: Mom Plane2 EndA
      11 Element engineering force      BAR          212    M2a: Mom Plane1 EndB
      12 Element engineering force      BAR          212    M2b: Mom Plane2 EndB
      13 Element engineering force      BAR          212    V1 : Shear Plane1
      14 Element engineering force      BAR          212    V2 : Shear Plane2
      15 Element engineering force      BAR          212    FX : Axial force
      16 Element engineering force      BAR          212    T  : Torque
```

```
--------------------------------------------------------------------------------
      Explanation of rows of       18 row by       18 col matrix OTM_STRE

   ROW            DESCRIPTION             TYPE     EID            ITEM
 ------- ------------------------------ -------- -------    --------------------
       1 Element stress                 BAR          201    SA1: Stress Pt1 EndA
       2 Element stress                 BAR          201    SA2: Stress Pt2 EndA
       3 Element stress                 BAR          201    SA3: Stress Pt3 EndA
       4 Element stress                 BAR          201    SA4: Stress Pt4 EndA
       5 Element stress                 BAR          201    Axial Stress
       6 Element stress                 BAR          201    SA-Max
       7 Element stress                 BAR          201    SA-Min
       8 Element stress                 BAR          201    MS-Tension
       9 Element stress                 BAR          201    Torsional Stress
      10 Element stress                 BAR          201    SB1: Stress Pt1 EndB
      11 Element stress                 BAR          201    SB2: Stress Pt2 EndB
      12 Element stress                 BAR          201    SB3: Stress Pt3 EndB
      13 Element stress                 BAR          201    SB4: Stress Pt4 EndB
      14 Element stress                 BAR          201    Axial stress
      15 Element stress                 BAR          201    SB-Max
      16 Element stress                 BAR          201    SB-Min
      17 Element stress                 BAR          201    MS-Compression
      18 Element stress                 BAR          201    MS-Torsion
```

CB-EXAMPLE-12-b.OP9 binary file of displacement OTM's requested in Case Control

(note: only 1st 5 columns written here the sake of clarity)

	OTM_ACCE	NCOLS = 10	NROWS = 6	FORM = 2	PREC = 2	
	1	2	3	4	5	
1	2.19985250269592E-02	0.00000000000000E+00	0.00000000000000E+00	-5.00000000000004E-01	1.09992625134795E-02	....
2	-2.02833087802606E-02	0.00000000000000E+00	0.00000000000000E+00	5.00000000000004E-01	-1.01416543901302E-02	....
3	-1.68157865913898E-02	-1.00000000000000E+00	5.00000000000000E-01	5.00000000000005E-01	2.41592106704306E-01	....
4	-3.36315731827796E-04	-2.00000000000000E-02	1.00000000000000E-02	1.00000000000001E-02	-5.16815786591390E-03	....
5	8.00614495648658E-03	0.00000000000000E+00	0.00000000000000E+00	0.00000000000000E+00	-5.99692752175671E-03	....
6	5.25433423070610E-04	0.00000000000000E+00	0.00000000000000E+00	9.99999999999992E-03	2.62716711535305E-04	....

	OTM_DISP	NCOLS = 18	NROWS = 12	FORM = 2	PREC = 2	
	1	2	3	4	5	
1	-1.41293911043985E-05	-7.60029025912968E-05	3.80014512956484E-05	1.29492635368416E-04	3.14571590643487E-06	....
2	1.62214021120513E-05	8.24359519633505E-05	-4.12179759816752E-05	-1.30161832591346E-04	-3.52963231517632E-06	....
3	8.24222187730972E-05	3.12878663301563E-04	-1.56439331650781E-04	-2.40634384994669E-04	-1.68993616070736E-05	....
4	5.88370868696758E-07	1.92529119983460E-06	-9.62645599917302E-07	-2.07019101770705E-06	1.88916538580397E-07	....
5	-1.66743323917105E-06	2.22005501168008E-06	-1.11002750584004E-06	-1.14971054599053E-06	-8.88454144573320E-08	....
6	5.12515138397389E-07	1.29205343624621E-07	-6.46026718123106E-08	-1.07589130445167E-06	-9.61720937623318E-08	....
7	1.05104109813473E-05	-5.99087762260462E-05	2.99543881130231E-05	6.53233961326989E-05	-1.57813540011406E-06	....
8	-9.46594436701425E-06	6.30861677743807E-05	-3.15430838871904E-05	-6.55217977160166E-05	1.38681670255135E-06	....
9	-3.18288681491121E-06	3.22417925611894E-04	-1.61208962805947E-04	-1.96081126486432E-04	-3.61627931263323E-05	....
10	-1.08618067423320E-07	3.64336233382231E-06	-1.82168116691115E-06	-2.63986785628832E-06	-3.24126419085498E-08	....
11	-9.45071958677177E-07	4.90427017653186E-07	-2.45213508826593E-07	-2.21449664764883E-07	1.36502293189118E-07	....
12	2.10600905814006E-07	3.21861205426993E-08	-1.60930602713497E-08	-6.09852683088454E-07	-3.82285587596693E-08	....

CB-EXAMPLE-12-b.OT9 text file descriptor of rows in above binary file for grid related OTM's

```
This text file describes the rows of the grid related OTM matrices written to unformatted file: CB-EXAMPLE-12-b.OP9
-----------------------------------------------------------------------------------------------

The description for each of the matrices has the headers:
       ROW        : row number in the individual OTM described
       DESCRIPTION: what OTM is this
       GRID       : grid number for this row of the OTM
       COMP       : displacement component number (1,2,3 translations and 4,5,6 rotations)

The number of rows for each OTM depends on the output requests, by the user, in Case Control

The number of cols for each OTM depends on the number of support DOFs (NDOFR) and the number of eigenvecors (NVEC)where:
       NDOFR =        8
       NVEC  =        2

This text file has descriptions for the following grid relatad OTMs from CB-EXAMPLE-12-b.OP9
       Acceleration OTM (matrix OTM_ACCE) with   NDOFR + NVEC =       10 cols
       Displacement OTM (matrix OTM_DISP) with 2*NDOFR + NVEC =       18 cols

--------------------------------------------------------------------------------
Explanation of rows of        6 row by       10 col matrix OTM_ACCE

   ROW           DESCRIPTION              GRID     COMP
 ------- ------------------------------  -------    ----
       1 Acceleration                         32       1
       2 Acceleration                         32       2
       3 Acceleration                         32       3
       4 Acceleration                         32       4
       5 Acceleration                         32       5
       6 Acceleration                         32       6

--------------------------------------------------------------------------------
Explanation of rows of       12 row by       18 col matrix OTM_DISP

   ROW           DESCRIPTION              GRID     COMP
 ------- ------------------------------  -------    ----
       1 Displacement                         22       1
       2 Displacement                         22       2
       3 Displacement                         22       3
       4 Displacement                         22       4
       5 Displacement                         22       5
       6 Displacement                         22       6

       7 Displacement                         32       1
       8 Displacement                         32       2
       9 Displacement                         32       3
      10 Displacement                         32       4
      11 Displacement                         32       5
      12 Displacement                         32       6
```