### Results of plate element runs Dec 2005

In order to check the various plate elements (QUAD4 and TRIA3 Mindlin elements, QUAD4K and TRIA3K Kirchoff elements) several test runs were made. Appendices A-1 through A-4 and B-1 through B-4 show the results. All of the runs are for a 2"x2" square plate with only ¼ of the plate modeled due to symmetry. Two plate thicknesses were analyzed (0.02" thin plate and 0.20" thick plate). The plates were subjected to two sets of loadings in each of two sets of boundary conditions:

- Simply supported plate subjected to:
  - uniform pressure load and
- Stress-free plate (supported in rigid body mode only) subjected to:
  - constant bulk temperature change and
  - temperature gradient through the plate thickness

The plate physical and material properties were:

- Length and width = 2.0"
- $E = 10.9 \times 10^6$
- $G = 4.19 \times 10^6$
- Poissons ratio = 0.3
- Pressure load =  $0.05 \text{ lb/in}^2$

The following is a key to the tables in the appendices:

- Appendices A-1 through A-4 are results when the QUAD4 element is the original element (MIN4) with:
  - App A-1: Simply supported 0.02" thick plate with pressure load
  - App A-2: Simply supported 0.20"thick plate with pressure load
  - App A-3: Stress-free 0.02" thick plate with bulk and gradient thermal load
  - App A-4: Stress-free 0.20" thick plate with bulk and gradient thermal load
- Appendices B-1 through B-4 are results when the QUAD4 element is the new element (MIN4T) with:
  - App B-1: Simply supported 0.02" thick plate with pressure load
  - App B-2: Simply supported 0.20"thick plate with pressure load
  - App B-3: Stress-free 0.02" thick plate with bulk and gradient thermal load
  - App B-4: Stress-free 0.20" thick plate with bulk and gradient thermal load

Each appendix has results for a convergence study in which the plate mesh was increased from a 2x2 mesh (for the  $\frac{1}{4}$  plate) to 5x5, 10x10, 20x20, 50x50, 100x100 and finally a 200x200 mesh. In these, the elements are all square. In addition to this convergence study, the appendices also have the results of runs wherein the model with a 10x10 mesh had elements distorted (no longer square). This was done by applying a random perturbation to the grids which are interior to the plate with a successfully more severe

random coefficient. Figures 1-9 show the meshes for the 10x10 plate with the various distortions analyzed. As the random coefficient increased there came a time where the mesh distortion was so great that some elements had fatal errors (e.g. an interior angle of a quad element exceeded  $180^{\circ}$ ).

One important finding was that the original QUAD4 Mindlin element (known internally to MYSTRAN as MIN4) has a flaw when the mesh is not rectangular. Appendices A-3 and A-4, table 4, show that with only a small distortion of the mesh from square elements, the stresses are not as close to zero as one would like. The similar results for the newer QUAD4 element (MIN4T) are shown in Appendices B-3 and B-4, table 4. As seen the stresses for the MIN4T are quite small as are those for the other plate elements (Mindlin triangle and Kirchoff triangle and quad).

Table 1 below gives the theoretical results for the displacements and forces to compare with the tables in the appendices for the pressure load results. Equations for the theoretical results were taken from reference 1 for thin plates and from references 2 and 3 for moderately thick plates. The equations were solved in fortran programs written by the author. For the thin plate a comparison could be made for the max values of displacements and forces based on tables in reference 1 that give nondimensional coefficients for simply supported plates under a uniform pressure load. For the thick plate there were no comparison values available other than to use the thick plate equations for the thin plate. The table shows these values as well.

Table 1: Theoretical results for simply supported plate with uniform pressure load

	1	2	3	4		
Max displacement or force	Kirchoff t	thin plate	Mindlin thick plate			
	equation	solution	equation solution			
	t = 0.02"	t = 0.20"	t = 0.02"	t = 0.20"		
displacement (in)	+4.070x10 <sup>-4</sup>	+4.070x10 <sup>-7</sup>	+4.072 x10 <sup>-4</sup>	+4.249x10 <sup>-7</sup>		
bending moment (lb-in/in)	+9.577x10 <sup>-3</sup>	+9.577x10 <sup>-3</sup>	+9.578 x10 <sup>-3</sup>	+9.607x10 <sup>-3</sup>		
twisting moment (lb-in/in)	-6.496x10 <sup>-3</sup>	-6.496x10 <sup>-3</sup>	-6.493 x10 <sup>-3</sup>	-6.225 x10 <sup>-3</sup>		
transverse shear (lb/in)	-3.375x10 <sup>-2</sup>	-3.375x10 <sup>-2</sup>	-3.376 x10 <sup>-2</sup>	-3.374 x10 <sup>-2</sup>		

For a plate whose thickness to length (t/a) is negligible, columns 1 and 3 should be the same. The fact that they are close would seem to validate the thick plate solution programmed. Columns 1 (or 3) and 4 then would be the values to compare with the MYSTRAN QUAD4 and TRIA3 results for the 0.02" and 0.20" plates, respectively. Columns 1 and 2 would be the expected values from MYSTRAN for the QUAD4K and TRIA3K Kirchoff elements for the 0.02" and 0.20" plates, respectively.

Table 2 gives the results for the stress free thermal loadings. Note that the displacements are the same for the thin or thick plate.

Table 2: Theoretical results for simply supported plate with stress free thermal expansion

	1	2
Max displacement or force	1.0 deg bulk temp change	1.0 deg temp gradient
	(all plate thicknesses)	(all plate thicknesses)
displacement (in)	$2.0 \times 10^{-5}$	$-2.0x10^{-5}$
bending moment (lb-in/in)	0	0
twisting moment (lb-in/in)	0	0
transverse shear (lb/in)	0	0

#### References

- 1. "Theory of plates and shells", S. Timoshenko
- 2. "On bending of elastic plates", Eric Reissner, Quarterly of Applied Mathematics, Vol V, 1947, pp 55-68
- 3. "Effect of shear deformations on the bending of elastic plates", V. Salerno and M. Goldberg, J of Applied Mathematics, Transactions of the ASME, March 1960, pp 54-58

Figure 1
AQ3U2S010 – Plate with no element random distortion

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L	2

10001	10002	10003	10004	10005	10006	10007	10008	10009	10010
9001	9002	9003	9004	9005	9006	9007	9008	9009	9010
8001	8002	8003	8004	8005	8006	8007	8008	8009	8010
7001	7002	7003	7004	7005	7006	7007	7008	7009	7010
6001	6002	6003	6004	6005	6006	6007	6008	6009	6010
5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1001	1002	1003	1004	1005	1006	1007	1008	1009	1010

Figure 2
AQ3R2S010a – Plate with random grid disorder coefficient = 0.1

						_	_	_	
10001	10002	10003	10004	10005	10006	10007	10008	10009	10010
9001	9002	9003	9004	9005	9006	9007	9008	9009	9010
8001	8002	8003	8004	8005	8006	8007	8008	8009	8010
7001	7002	7003	7004	7005	7006	7007	7008	7009	7010
6001	6002	6003	6004	6005	6006	6007	6008	6009	6010
5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1001	1002	1003	1004	1005	1006	1007	1008	1009	1010

Figure 3
AQ3R2S010b – Plate with random grid disorder coefficient = 0.2

	10001	10002	10003	10004	10005	10006	10007	10008	10009	10010
	9001	9002	9003	9004	9005	9006	9007	9008	9009	9010
	8001	8002	8003	8004	8005	8006	8007	8008	8009	8010
•	7001	7002	7003	7004	7005	7006	7007	7008	7009	7010
	6001	6002	6003	6004	6005	6006	6007	6008	6009	6010
	5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
	4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
·	3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010

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Figure 4
AQ3R2S010c – Plate with random grid disorder coefficient = 0.3

		_					_		
10001	10002	10003	10004	10005	10006	10007	10008	10009	10010
9001	9002	9003	9004	9005	9006	9007	9008	9009	9010
8001	8002	8003	8004	8005	8006	8007	8008	8009	8010
7001	7002	7003	7004	7005	7006	7007	7008	7009	7010
6001	6002	6003	6004	6005	6006	6007	6008	6009	6010
5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1001	1002	1003	1004	1005	1006	1007	1008	1009	1010

Figure 5
AQ3R2S010d – Plate with random grid disorder coefficient = 0.4

LI											
		10001	10002	10003	10004	10005	10006	10007	10008	10009	10010
		9001	9002	9003	9004	9005	9006	9007	9008	9009	9010
		8001	8002	8003	8004	8005	8006	8007	8008	8009	8010
		7001	7002	7003	7004	7005	7006	7007	7008	7009	7010
		6001	6002	6003	6004	6005	6006	6007	6008	6009	6010
		5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
		4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
		3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
		2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
	Υ	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010
		<u></u>									

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Figure 6
AQ3R2S010e – Plate with random grid disorder coefficient = 0.5

_					_				_	
	10001	10002	10003	10004	10005	10006	10007	10008	10009	10010
	9001	9002	9003	9004	9005	9006	9007	9008	9009	9010
	8001	8002	8003	8004	8005	8006	8007	8008	8009	8010
	7001	7002 \	7003	7004	7005	7006	7007	7008	7009	7010
	6001	6002	6003	6004	6005	6006	6007	6008	6009	6010
	5001	5002	5003	5004	5005	5006	5007	5008	5009	5010
	4001	4002	4003	4004	4005	4006	4007	4008	4009	4010
	3001	3002	3003	3004	3005	3006	3007	3008	3009	3010
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Y	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010
ļ			/							

V1 L2 C1

Figure 7
AQ3R2S010f – Plate with random grid disorder coefficient = 0.6

	10001 10002 10003 10004 10005 10006 10007 10008 10009 10010
	9001 9002 9003 9004 9005 9006 9007 9008 9009 9010
	8001 8002 8003 8004 8005 8006 8007 8008 8009 8010
	7001 7002 7003 7004 7005 7006 7007 7008 7009 7010
	6001 6002 6003 6004 6005 6006 6007 6008 6009 6010
	5001 5002 5003 5004 5005 5006 5007 5008 5009 5010
	4001 4002 4003 4004 4005 4006 4007 4008 4009 4010
	3001 3002 3003 3004 3005 3006 3007 3008 3009 3010
	2001 2002 2003 2004 2005 2006 2007 2008 2009 2010
Y	1001 /1002 /1003 /1004 /1005 /1006 /1007 /1008 /1009 /1010

Figure 8
AQ3R2S010g – Plate with random grid disorder coefficient = 0.7

	10001 10002 10003 10004 10005 10006 10007 10008 10009 10010
	9001 9002 9003 9004 9005 9006 9007 9008 9009 9010
	8001 8002 8003 8004 8005 8006 8007 8008 8009 8010
	7001 7002 7003 7004 7005 7006 7007 7008 7009 7010
	6001 6002 6003 6004 6005 6006 6007 6008 6009 6010
	5001 5002 5003 5004 5005 5006 5007 5008 5009 5010
	4001 4002 4003 4004 4005 4006 4007 4008 4009 4010
	3001 3002 3003 3004 3005 3006 3007 3008 3009 3010
	2001 2002 2003 2004 2005 2006 2007 2008 2009 2010
Y	1001 1002 1003 1004 1005 1006 1007 1008 1009 1010

Figure 9
AQ3R2S010h – Plate with random grid disorder coefficient = 0.8

10007 10008 10009 10010 \ 10003 \ \9010| , 8002\ B010 /7010<sup>)</sup> 4009 4010 3009 3010 7<sub>2009</sub> \ 1008, /1010 

## Appendix A-1

Simply supported square plate. 1/4 sym. Plate overall dimensions 2" X 2" QUAD4 is MIN4 element

Plate thickness 0.02" (thin)

Table A1-1

## Convergence study - increasing mesh size Thin plate: 0.02" using MIN4 quad elements Uniform pressure load 0.05 psi

#### Central point lateral displacement (in)

Mesh	QUAD4 (MIM4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)	QUAD4K (AQ2U1Sxxx)	TRIA3K (AT2U1Sxxx)
2	4.188881E-04	3.882019E-04	4.053825E-04	4.064101E-04
5	4.092078E-04	4.044364E-04	4.069221E-04	4.072906E-04
10	4.077716E-04	4.066298E-04	4.070296E-04	4.071537E-04
20	4.074049E-04	4.071399E-04	4.070529E-04	4.070879E-04
50	4.072946E-04	4.072582E-04	4.070591E-04	4.070651E-04
100	4.072773E-04	4.072688E-04	4.070600E-04	4.070615E-04
200	4.072728E-04	4.072707E-04	4.070602E-04	4.070605E-04

### Max M<sub>x</sub> or M<sub>y</sub> bending moment (lb-in/in)

Mesh	QUAD4 (MIN4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)	QUAD4K (AQ2U1Sxxx)	TRIA3K (AT2U1Sxxx)
2	-8.298327E-03	-8.251984E-03	-7.932362E-03	-7.980317E-03
5	-9.376329E-03	-9.402860E-03	-9.316309E-03	-9.326607E-03
10	-9.528890E-03	-9.538238E-03	-9.513549E-03	-9.514432E-03
20	-9.566863E-03	-9.568613E-03	-9.562834E-03	-9.562405E-03
50	-9.577319E-03	-9.577013E-03	-9.576632E-03	-9.576411E-03
100	-9.578778E-03	-9.578567E-03	-9.578603E-03	-9.578518E-03
200	-9.579139E-03	-9.579062E-03	-9.579095E-03	-9.579065E-03

## Max M<sub>xy</sub> twisting moment (lb-in/in)

Mesh	QUAD4 (MIN4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)	QUAD4K (AQ2U1Sxxx)	TRIA3K (AT2U1Sxxx)
		(	,	· · · · · · · · · · · · · · · · · · ·
2	4.791267E-03	4.423924E-03	4.823450E-03	4.763773E-03
5	6.074485E-03	6.069230E-03	6.075395E-03	6.067006E-03
10	6.361276E-03	6.381598E-03	6.360069E-03	6.360019E-03
20	6.454034E-03	6.466028E-03	6.453200E-03	6.454149E-03
50	6.486215E-03	6.489589E-03	6.485967E-03	6.486422E-03
100	6.491816E-03	6.492906E-03	6.491745E-03	6.491934E-03
200	6.493429E-03	6.493751E-03	6.493411E-03	6.493480E-03

## $\text{MaxV}_{\text{x}}$ or $\text{V}_{\text{y}}$ transverse shear force (lb/in)

Mesh	QUAD4 (MIN4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)
2	-2.197407E-02	-2.355221E-02
5	-2.886149E-02	-2.980993E-02
10	-3.128842E-02	-3.210377E-02
20	-3.252104E-02	-3.310807E-02
50	-3.326685E-02	-3.343383E-02
100	-3.351638E-02	-3.356282E-02
200	-3.364128E-02	-3.366052E-02

#### Table A1-2

## Random disorder of element shape - mesh size is 10 x 10 Thin plate: 0.02" using MIN4 quad elements Uniform pressure load 0.05 psi

Central point lateral displacement (in)

Random Grid Disorder	QUAD4 (MIN4) (AQ3R1S010)	TRIA3 (AT3R1S010)	QUAD4K (AQ2R1S010)	TRIA3K (AT2R1S010)
2.00.00.	` ,	,	` '	,
.0	4.077716E-04	4.066298E-04	4.070296E-04	4.071537E-04
.1	4.077502E-04	4.065895E-04	4.069936E-04	4.070827E-04
.2	4.076948E-04	4.065019E-04	4.069458E-04	4.069921E-04
.3	4.076048E-04	4.063725E-04	4.068937E-04	4.069003E-04
.4	4.073618E-04	4.062242E-04	4.068626E-04	4.067432E-04
.5	4.069259E-04	4.059689E-04	4.067344E-04	4.066651E-04
.6	4.065474E-04	4.053971E-04	4.066827E-04	4.066200E-04
.7	See Note 1	4.052982E-04	See Note 1	4.063957E-04
.8	See Note 1	4.046979E-04	See Note 1	4.062411E-04

Max M<sub>x</sub> or M<sub>y</sub> bending moment (lb-in/in)

	- A			
Random Grid Disorder	QUAD4 (MIN4) (AQ3R1S010)	TRIA3 (AT3R1S010)	QUAD4K (AQ2R1S010)	TRIA3K (AT2R1S010)
District	(AQ31110010)	(A131(13010)	(AQZIT10010)	(A121(10010)
.0	-9.528890E-03	-9.538238E-03	-9.513549E-03	-9.514432E-03
.1	-9.518489E-03	-9.531171E-03	-9.511662E-03	-9.508362E-03
.2	-9.526729E-03	-9.540563E-03	-9.497620E-03	-9.497151E-03
.3	-9.515448E-03	-9.528532E-03	-9.509454E-03	-9.505900E-03
.4	-9.526196E-03	-9.518584E-03	-9.488904E-03	-9.480915E-03
.5	-9.499431E-03	-9.503319E-03	-9.471925E-03	-9.481681E-03
.6	-9.503035E-03	-9.467728E-03	-9.462858E-03	-9.453775E-03
.7	See Note 1	-9.503531E-03	See Note 1	-9.486368E-03
.8	See Note 1	-9.454101E-03	See Note 1	-9.485915E-03

Max M<sub>xv</sub> twisting moment (lb-in/in)

max may twisting moment (is min)					
Random Grid Disorder	QUAD4 (MIN4) (AQ3R1S010)	TRIA3 (AT3R1S010)	QUAD4K (AQ2R1S010)	TRIA3K (AT2R1S010)	
.0	6.361276E-03	6.381598E-03	6.360069E-03	6.360019E-03	
.1	6.368621E-03	6.378455E-03	6.361073E-03	6.361410E-03	
.2	6.364122E-03	6.390043E-03	6.370030E-03	6.377284E-03	
.3	6.321933E-03	6.420099E-03	6.359695E-03	6.375967E-03	
.4	6.360112E-03	6.463871E-03	6.340293E-03	6.381823E-03	
.5	6.374931E-03	6.409916E-03	6.366337E-03	6.401488E-03	
.6	6.211167E-03	6.496164E-03	6.357278E-03	6.390720E-03	
.7	See Note 1	6.462622E-03	See Note 1	6.371801E-03	
.8	See Note 1	6.719344E-03	See Note 1	6.332287E-03	

MaxV<sub>x</sub> or V<sub>y</sub> transverse shear force (lb/in)

, , , , , , , , , , , , , , , , , , ,				
Random Grid Disorder	QUAD4 (MIN4) (AQ2R1S010)	TRIA3 (AT3R1S010)		
.0	-3.128842E-02	-3.210377E-02		
.1	-3.137333E-02	-3.249074E-02		
.2	-3.126812E-02	-3.328399E-02		
.3	-3.152239E-02	-3.352161E-02		
.4	-3.190008E-02	-3.592427E-02		
.5	-3.187843E-02	-3.431050E-02		
.6	-3.305208E-02	-3.296402E-02		
.7	See Note 1	-3.407969E-02		
.8	See Note 1	-3.609102E-02		

## Appendix A-2

Simply supported square plate. 1/4 sym. Plate overall dimensions 2" X 2" QUAD4 is MIN4 element

Plate thickness 0.20" (thick)

Table A2-1

## Convergence study - increasing mesh size Thick plate: 0.20" using MIN4 quad elements Uniform pressure load 0.05 psi

## Central point lateral displacement (in)

Mesh	QUAD4 (MIM4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)	QUAD4K (AQ2US2xxx)	TRIA3K (AT2US2xxx)
2	4.425585E-07	4.146006E-07	4.053825E-04	4.064101E-07
5	4.306603E-07	4.270713E-07	4.069221E-04	4.072906E-07
10	4.287975E-07	4.280076E-07	4.070296E-04	4.071537E-07
20	4.283199E-07	4.281360E-07	4.070529E-04	4.070879E-07
50	4.281852E-07	4.281575E-07	4.070591E-04	4.070651E-07
100	4.281659E-07	4.281593E-07	4.070600E-04	4.070615E-07
200	4.281611E-07	4.281595E-07	4.070602E-07	4.070606E-07

## ${\bf Max}\ {\bf M_x}\ {\bf or}\ {\bf M_y}\ {\bf bending}\ {\bf moment}\ ({\bf lb\text{-}in/in})$

Mesh	QUAD4 (MIN4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)	QUAD4K (AQ2US2xxx)	TRIA3K (AT2US2xxx)
2	-8.327611E-03	-8.286672E-03	-7.932362E-03	-7.980317E-03
5	-9.385025E-03	-9.384681E-03	-9.316309E-03	-9.326607E-03
10	-9.531068E-03	-9.522622E-03	-9.513549E-03	-9.514432E-03
20	-9.567238E-03	-9.562965E-03	-9.562834E-03	-9.562405E-03
50	-9.577337E-03	-9.576334E-03	-9.576632E-03	-9.576411E-03
100	-9.578779E-03	-9.578481E-03	-9.578603E-03	-9.578518E-03
200	-9.579140E-03	-9.579054E-03	-9.579094E-03	-9.579056E-03

## Max M<sub>xy</sub> twisting moment (lb-in/in)

Mesh	QUAD4 (MIN4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)	QUAD4K (AQ2US2xxx)	TRIA3K (AT2US2xxx)
2	4.813814E-03	4.500427E-03	4.823450E-03	4.763773E-03
5	6.080656E-03	6.089201E-03	6.075395E-03	6.067006E-03
10	6.361956E-03	6.379003E-03	6.360069E-03	6.360019E-03
20	6.453592E-03	6.461321E-03	6.453200E-03	6.454149E-03
50	6.486000E-03	6.487857E-03	6.485967E-03	6.486422E-03
100	6.491767E-03	6.492338E-03	6.491745E-03	6.491934E-03
200	6.493441E-03	6.493610E-03	6.493411E-03	6.493481E-03

## $MaxV_x$ or $V_y$ transverse shear force (lb/in)

Mesh	QUAD4 (MIN4)	TRIA3
WCSH	(AQ3US2xxx)	(AT3US2xxx)
2	-2.182056E-02	-2.338886E-02
5	-2.882820E-02	-2.916144E-02
10	-3.128044E-02	-3.133337E-02
20	-3.251968E-02	-3.252939E-02
50	-3.326676E-02	-3.327051E-02
100	-3.351635E-02	-3.351810E-02
200	-3.364124E-02	-3.364134E-02

Table A2-2
Random disorder of element shape - mesh size is 10 x 10
Thick plate: 0.20" using MIN4 quad elements
Uniform pressure load 0.05 psi

Central point lateral displacement (in)

Random Grid	QUAD4 (MIN4)	TRIA3	QUAD4K	TRIA3K
Disorder	(AQ3R2S010)	(AT3R2S010)	(AQ2R2S010)	(AR2R2S010)
.0	4.287975E-07	4.280076E-07	4.070296E-04	4.071537E-07
.1	4.287897E-07	4.279977E-07	4.069889E-07	4.070835E-07
.2	4.287657E-07	4.279538E-07	4.069567E-07	4.069980E-07
.3	4.287153E-07	4.278728E-07	4.069297E-07	4.069015E-07
.4	4.286709E-07	4.277729E-07	4.068571E-07	4.067849E-07
.5	4.285825E-07	4.277427E-07	4.066887E-07	4.067714E-07
.6	4.284095E-07	4.274602E-07	See Note 1	4.066252E-07
.7	4.282386E-07	4.272412E-07	4.063414E-07	4.063621E-07
.8	See Note 1	4.273250E-07	See Note 1	4.062954E-07

Max M<sub>x</sub> or M<sub>y</sub> bending moment (lb-in/in)

Random Grid Disorder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)	QUAD4K (AQ2R2S010)	TRIA3K (AR2R2S010)
.0	-9.531068E-03	-9.522622E-03	-9.513549E-03	-9.514432E-03
.1	-9.526352E-03	-9.516944E-03	-9.508573E-03	-9.509657E-03
.2	-9.525470E-03	-9.513496E-03	-9.500159E-03	-9.497733E-03
.3	-9.515931E-03	-9.510518E-03	-9.497190E-03	-9.498315E-03
.4	-9.517207E-03	-9.502445E-03	-9.492558E-03	-9.484133E-03
.5	-9.500221E-03	-9.499946E-03	-9.468693E-03	-9.478471E-03
.6	-9.473250E-03	-9.482645E-03	See Note 1	-9.475569E-03
.7	-9.525772E-03	-9.482962E-03	-9.479518E-03	-9.450705E-03
.8	See Note 1	-9.488444E-03	See Note 1	-9.472364E-03

Max M<sub>xy</sub> twisting moment (lb-in/in)

Random Grid Disorder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)	QUAD4K (AQ2R2S010)	TRIA3K (AR2R2S010)
.0	6.361956E-03	6.379003E-03	6.360069E-03	6.360019E-03
.1	6.368551E-03	6.386316E-03	6.363487E-03	6.360086E-03
.2	6.392715E-03	6.398629E-03	6.367205E-03	6.367530E-03
.3	6.346974E-03	6.390431E-03	6.347635E-03	6.389979E-03
.4	6.365845E-03	6.433151E-03	6.361833E-03	6.356812E-03
.5	6.350245E-03	6.424353E-03	6.375497E-03	6.376842E-03
.6	6.330921E-03	6.421069E-03	See Note 1	6.386909E-03
.7	6.379596E-03	6.419936E-03	6.329188E-03	6.416808E-03
.8	See Note 1	6.458207E-03	See Note 1	6.414755E-03

MaxV<sub>x</sub> or V<sub>y</sub> transverse shear force (lb/in)

Random Grid Disorder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)
.0	-3.128044E-02	-3.133337E-02
.1	-3.133322E-02	-3.132976E-02
.2	-3.135833E-02	-3.136309E-02
.3	-3.123436E-02	-3.139836E-02
.4	-3.131467E-02	-3.125302E-02
.5	-3.119922E-02	-3.130338E-02
.6	-3.099358E-02	-3.098793E-02
.7	-3.094443E-02	-3.154259E-02
.8	See Note 1	-3.127497E-02

## Appendix A-3

Square plate supported only at the central point. 1/4 sym.
Plate overall dimensions 2" X 2"
QUAD4 is MIN4 element

Plate thickness 0.02" (thin)

Table A3-1

# Convergence study - increasing mesh size Thin plate: 0.02" using MIN4 quad elements Stress free thermal expansion – 1.0 deg bulk temperature change

### Central point lateral displacement (in)

Mesh	QUAD4 (MIM4) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
10	2.000000E-05	2.000000E-05	2.000000E-05	2.00000E-05
20	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
50	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
100	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
200	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05

## Max stress (lb/in²)

Mesh	QUAD4 (MIN4) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	E-13	E-13	E-13	E-13
5	E-13	E-12	E-12	E-12
10	E-12	E-12	E-12	E-12
20	E-12	E-12	E-12	E-12
50	E-11	E-11	E-11	E-11
100	E-10	E-10	E-10	E-10
200	E-09	E-10	E-09	E-10

Table A3-2

## Random disorder of element shape - mesh size is 10 x 10 Thin plate: 0.02" using MIN4 quad elements Stress free thermal expansion – 1.0 deg bulk temperature change

#### Central point lateral displacement (in)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.1	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.3	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.4	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.6	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.7	2.000000E-05	2.000000E-05	See Note 1	2.000000E-05
.8	See Note 1	2.000000E-05	See Note 1	2.000000E-05

## Max stress (lb/in<sup>2</sup>)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	E-12	E-12	E-12	E-12
.1	E-11	E-12	E-12	E-12
.2	E-11	E-12	E-12	E-12
.3	E-11	E-12	E-12	E-12
.4	E-11	E-12	E-12	E-12
.5	E-11	E-12	E-12	E-12
.6	E-11	E-12	E-11	E-12
.7	E-11	E-12	See Note 1	E-12
.8	See Note 1	E-12	See Note 1	E-11

## Table A3-3

# Convergence study - increasing mesh size Thin plate: 0.02" using MIN4 quad elements Stress free thermal expansion – 1.0 deg temperature gradient through thickness

## Central point lateral displacement (in)

Mesh	QUAD4 (MIM4) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
5	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
10	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
20	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
50	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
100	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
200	-2.000000E-05	-2.000000E-05	-2.000000E-05	-1.999998E-05

## Max stress (lb/in²)

Mesh	QUAD4 (MIN4) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	E-13	E-13	E-14	E-14
5	E-12	E-12	E-12	E-13
10	E-11	E-11	E-11	E-12
20	E-11	E-11	E-09	E-09
50	E-09	E-08	E-08	E-08
100	E-08	E-09	E-08	E-06
200	E-08	E-08	E-06	E-05

#### Table A3-4

## Random disorder of element shape - mesh size is 10 x 10 Thin plate: 0.02" using MIN4 quad elements Stress free thermal expansion – 1.0 deg temperature gradient through thickness

#### Central point lateral displacement (in)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.1	-1.999996E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.2	-1.999907E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.3	-1.999685E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.4	-1.999206E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.5	-1.998105E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.6	-1.997207E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.7	-1.993467E-05	-2.000000E-05	See Note 1	-2.000000E-05
.8	See Note 1	-2.000000E-05	See Note 1	-1.998677E-05

## Max stress (lb/in²)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	E-11	E-11	E-11	E-12
.1	E-04	E-11	E-11	E-11
.2	E-03	E-11	E-11	E-11
.3	E-02	E-11	E-11	E-11
.4	E-02	E-11	E-11	E-12
.5	E-02	E-11	E-11	E-11
.6	E-02	E-11	E-11	E-11
.7	E-02	E-11	See Note 1	E-11
.8	See Note 1	E-11	See Note 1	E-02

## Appendix A-4

Square plate supported only at the central point. 1/4 sym.
Plate overall dimensions 2" X 2"
QUAD4 is MIN4 element

Plate thickness 0.20" (thick)

Table A4-1

# Convergence study - increasing mesh size Thick plate: 0.20" using MIN4 quad elements Stress free thermal expansion – 1.0 deg bulk temperature change

### Central point lateral displacement (in)

Mesh	QUAD4 (MIM4) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
10	2.000000E-05	2.000000E-05	2.000000E-05	2.00000E-05
20	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
50	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
100	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
200	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05

## Max stress (lb/in²)

Mesh	QUAD4 (MIN4) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	E-13	E-13	E-13	E-13
5	E-12	E-12	E-12	E-12
10	E-12	E-12	E-12	E-12
20	E-12	E-11	E-12	E-11
50	E-10	E-10	E-10	E-10
100	E-10	E-10	E-10	E-10
200	E-09	E-09	E-09	E-09

Table A4-2

## Random disorder of element shape - mesh size is 10 x 10 Thick plate: 0.20" using MIN4 quad elements Stress free thermal expansion – 1.0 deg bulk temperature change

#### Central point lateral displacement (in)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.1	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.3	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.4	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.6	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.7	2.000000E-05	2.000000E-05	See Note 1	2.000000E-05
.8	See Note 1	2.000000E-05	See Note 1	2.000000E-05

## Max stress (lb/in<sup>2</sup>)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	E-12	E-12	E-12	E-12
.1	E-12	E-12	E-12	E-12
.2	E-12	E-12	E-12	E-12
.3	E-12	E-12	E-12	E-12
.4	E-12	E-12	E-12	E-12
.5	E-12	E-12	E-12	E-12
.6	E-12	E-12	E-12	E-12
.7	E-12	E-12	See Note 1	E-12
.8	See Note 1	E-12	See Note 1	E-11

## Table A4-3

# Convergence study - increasing mesh size Thick plate: 0.20" using MIN4 quad elements Stress free thermal expansion – 1.0 deg temperature gradient through thickness

## Central point lateral displacement (in)

Mesh	QUAD4 (MIM4) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
5	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
10	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
20	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
50	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
100	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
200	-2.000000E-05	-2.000000E-05	-2.000000E-05	-1.999998E-05

## Max stress (lb/in²)

Mesh	QUAD4 (MIN4) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	E-13	E-13	E-13	E-13
5	E-13	E-12	E-11	E-11
10	E-11	E-11	E-10	E-11
20	E-11	E-10	E-09	E-09
50	E-10	E-10	E-07	E-07
100	E-10	E-10	E-06	E-06
200	E-09	E-09	E-04	E-05

#### Table A4-4

## Random disorder of element shape - mesh size is 10 x 10 Thick plate: 0.20" using MIN4 quad elements Stress free thermal expansion – 1.0 deg temperature gradient through thickness

#### Central point lateral displacement (in)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.1	-1.999998E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.2	-1.999987E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.3	-1.999959E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.4	-1.999953E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.5	-1.999741E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.6	-1.999633E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.7	-1.998706E-05	-2.000000E-05	See Note 1	-2.000000E-05
.8	See Note 1	-2.000000E-05	See Note 1	-2.000000E-05

#### Max stress (lb/in²)

Random Grid Disorder	QUAD4 (MIN4) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	E-11	E-11	E-10	E-11
.1	E-04	E-11	E-10	E-10
.2	E-03	E-11	E-10	E-10
.3	E-03	E-12	E-10	E-11
.4	E-03	E-12	E-10	E-10
.5	E-02	E-11	E-10	E-10
.6	E-02	E-11	E-10	E-10
.7	E-01	E-11	See Note 1	E-10
.8	See Note 1	E-11	See Note 1	E-10

## **Appendix B-1**

Simply supported square plate. 1/4 sym. Plate overall dimensions 2" X 2" QUAD4 is MIN4T element

Plate thickness 0.02" (thin)

Table B1-1

## Convergence study - increasing mesh size Thin plate: 0.02", Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.070E-04 inches

Random	QUAD4 (MIN4T)		TRIA3		QUAD4K		TRIA3K	
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	4.177474E-04	2.6%	3.882019E-04	-4.6%	4.053825E-04	-0.4%	4.064101E-04	-0.1%
5	4.089617E-04	0.5%	4.044364E-04	-0.6%	4.069221E-04	0.0%	4.072906E-04	0.1%
10	4.076979E-04	0.2%	4.066298E-04	-0.1%	4.070296E-04	0.0%	4.071537E-04	0.0%
20	4.073804E-04	0.1%	4.071399E-04	0.0%	4.070529E-04	0.0%	4.070879E-04	0.0%
50	4.072894E-04	0.1%	4.072582E-04	0.1%	4.070591E-04	0.0%	4.070651E-04	0.0%
100	4.072759E-04	0.1%	4.072688E-04	0.1%	4.070600E-04	0.0%	4.070615E-04	0.0%
200			4.072707E-04	0.1%	4.070602E-04	0.0%	4.070605E-04	0.0%

Max x or y bending moment: theoretical = -9.577E-03 lb-in/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	-8.041848E-03	-16.0%	-8.251984E-03	-13.8%	-7.932362E-03	-17.2%	-7.980317E-03	-16.7%
5	-9.329311E-03	-2.6%	-9.402860E-03	-1.8%	-9.316309E-03	-2.7%	-9.326607E-03	-2.6%
10	-9.516958E-03	-0.6%	-9.538238E-03	-0.4%	-9.513549E-03	-0.7%	-9.514432E-03	-0.7%
20	-9.563905E-03	-0.1%	-9.568613E-03	-0.1%	-9.562834E-03	-0.1%	-9.562405E-03	-0.2%
50	-9.576865E-03	0.0%	-9.577013E-03	0.0%	-9.576632E-03	0.0%	-9.576411E-03	0.0%
100	-9.578667E-03	0.0%	-9.578567E-03	0.0%	-9.578603E-03	0.0%	-9.578518E-03	0.0%
200			-9.579062E-03	0.0%	-9.579095E-03	0.0%	-9.579065E-03	0.0%

Max twisting moment: theoretical = -6.496E-03 lb-in/in

	max thiomig memoria area or local to the figure									
Random	QUAD4 (MIN4T)		TRIA3		QUAD4K		TRIA3K			
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error		
2	-4.642756E-03	-28.5%	-4.423924E-03	-31.9%	-4.823450E-03	-25.7%	-4.763773E-03	-26.7%		
5	-6.039231E-03	-7.0%	-6.069230E-03	-6.6%	-6.075395E-03	-6.5%	-6.067006E-03	-6.6%		
10	-6.350917E-03	-2.2%	-6.381598E-03	-1.8%	-6.360069E-03	-2.1%	-6.360019E-03	-2.1%		
20	-6.451331E-03	-0.7%	-6.466028E-03	-0.5%	-6.453200E-03	-0.7%	-6.454149E-03	-0.6%		
50	-6.485836E-03	-0.2%	-6.489589E-03	-0.1%	-6.485967E-03	-0.2%	-6.486422E-03	-0.1%		
100	-6.491724E-03	-0.1%	-6.492906E-03	0.0%	-6.491745E-03	-0.1%	-6.491934E-03	-0.1%		
200			-6.493751E-03	0.0%	-6.493411E-03	0.0%	-6.493480E-03	0.0%		

Max x or y transverse shear: theoretical = -3.375E-02 lb/in

Random	QUAD4 (MIN4T)		) TRIA3		QUAD4K		TRIA3K	
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	-2.120397E-02	-37.2%	-2.355221E-02	-30.2%				
5	-2.873378E-02	-14.9%	-2.980993E-02	-11.7%				
10	-3.125650E-02	-7.4%	-3.210377E-02	-4.9%				
20	-3.251324E-02	-3.7%	-3.310807E-02	-1.9%				
50	-3.326562E-02	-1.4%	-3.343383E-02	-0.9%				
100	-3.351608E-02	-0.7%	-3.356282E-02	-0.6%				
200			-3.366052E-02	-0.3%				

Table B1-2

Random disorder of element shape - mesh size is 10 x 10 Thin plate: 0.02", Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.070E-04 inches

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	4.076979E-04	0.2%	4.066298E-04	-0.1%	4.070296E-04	0.0%	4.071537E-04	0.0%
0.1	4.076845E-04	0.2%	4.065895E-04	-0.1%	4.069936E-04	0.0%	4.070827E-04	0.0%
0.2	4.076601E-04	0.2%	4.065019E-04	-0.1%	4.069458E-04	0.0%	4.069921E-04	0.0%
0.3	4.076150E-04	0.2%	4.063725E-04	-0.2%	4.068937E-04	0.0%	4.069003E-04	0.0%
0.4	4.075636E-04	0.1%	4.062242E-04	-0.2%	4.068626E-04	0.0%	4.067432E-04	-0.1%
0.5	4.075042E-04	0.1%	4.059689E-04	-0.3%	4.067344E-04	-0.1%	4.066651E-04	-0.1%
0.6	4.074408E-04	0.1%	4.053971E-04	-0.4%	4.066827E-04	-0.1%	4.066200E-04	-0.1%
0.7			4.052982E-04	-0.4%			4.063957E-04	-0.1%
8.0			4.046979E-04	-0.6%			4.062411E-04	-0.2%

Max x or y bending moment: theoretical = -9.577E-03 lb-in/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-9.516958E-03	-0.6%	-9.538238E-03	-0.4%	-9.513549E-03	-0.7%	-9.514432E-03	-0.7%
0.1	-9.508684E-03	-0.7%	-9.531171E-03	-0.5%	-9.511662E-03	-0.7%	-9.508362E-03	-0.7%
0.2	-9.513518E-03	-0.7%	-9.540563E-03	-0.4%	-9.497620E-03	-0.8%	-9.497151E-03	-0.8%
0.3	-9.498255E-03	-0.8%	-9.528532E-03	-0.5%	-9.509454E-03	-0.7%	-9.505900E-03	-0.7%
0.4	-9.508094E-03	-0.7%	-9.518584E-03	-0.6%	-9.488904E-03	-0.9%	-9.480915E-03	-1.0%
0.5	-9.475692E-03	-1.1%	-9.503319E-03	-0.8%	-9.471925E-03	-1.1%	-9.481681E-03	-1.0%
0.6	-9.510552E-03	-0.7%	-9.467728E-03	-1.1%	-9.462858E-03	-1.2%	-9.453775E-03	-1.3%
0.7			-9.503531E-03	-0.8%			-9.486368E-03	-0.9%
۷.8			-9.454101E-03	-1.3%			-9.485915E-03	-1.0%

Max twisting moment: theoretical = -6.496E-03 lb-in/in

			9		000	,		
Random	QUAD4 (MI	N4T)	TRIA3		QUAD4K TRI		TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-6.350917E-03	-2.2%	-6.381598E-03	-1.8%	-6.360069E-03	-2.1%	-6.360019E-03	-2.1%
0.1	-6.355576E-03	-2.2%	-6.378455E-03	-1.8%	-6.361073E-03	-2.1%	-6.361410E-03	-2.1%
0.2	-6.350873E-03	-2.2%	-6.390043E-03	-1.6%	-6.370030E-03	-1.9%	-6.377284E-03	-1.8%
0.3	-6.308518E-03	-2.9%	-6.420099E-03	-1.2%	-6.359695E-03	-2.1%	-6.375967E-03	-1.8%
0.4	-6.350845E-03	-2.2%	-6.463871E-03	-0.5%	-6.340293E-03	-2.4%	-6.381823E-03	-1.8%
0.5	-6.370382E-03	-1.9%	-6.409916E-03	-1.3%	-6.366337E-03	-2.0%	-6.401488E-03	-1.5%
0.6	-6.220156E-03	-4.2%	-6.496164E-03	0.0%	-6.357278E-03	-2.1%	-6.390720E-03	-1.6%
0.7			-6.462622E-03	-0.5%			-6.371801E-03	-1.9%
8.0			-6./19344 <b>Ľ</b> -03	3.4%			-6.332287E-03	-2.5%

Max x or y transverse shear: theoretical = -3.375E-02 lb/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ŀ	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-3.125650E-02	-7.4%	-3.230785E-02	-4.3%				
0.1	-3.137727E-02	-7.0%	-3.249074E-02	-3.7%				
0.2	-3.131193E-02	-7.2%	-3.328399E-02	-1.4%				
0.3	-3.227968E-02	-4.4%	-3.352161E-02	-0.7%				
0.4	-3.253596E-02	-3.6%	-3.592427E-02	6.4%				
0.5	-3.573537E-02	5.9%	-3.431050E-02	1.7%				
0.6	-3.739516E-02	10.8%	-3.296402E-02	-2.3%				
0.7			-3.407969E-02	1.0%				
۵.۷			-3.609102E-02	6.9%				

## **Appendix B-2**

Simply supported square plate. 1/4 sym. Plate overall dimensions 2" X 2" QUAD4 is MIN4T element

Plate thickness 0.20" (thick)

## Convergence study - increasing mesh size Thick plate: 0.20", Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.294E-07 inches

Random	QUAD4 (M	IN4T)	TRIA3	TRIA3		QUAD4K		<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	4.395387E-07	2.4%	4.146006E-07	-3.4%	4.053825E-07	-5.6%	4.064101E-07	-5.4%
5	4.300534E-07	0.2%	4.270713E-07	-0.5%	4.069221E-07	-5.2%	4.072906E-07	-5.1%
10	4.286389E-07	-0.2%	4.280076E-07	-0.3%	4.070296E-07	-5.2%	4.071537E-07	-5.2%
20	4.282798E-07	-0.3%	4.281360E-07	-0.3%	4.070529E-07	-5.2%	4.070879E-07	-5.2%
50	4.281788E-07	-0.3%	4.281575E-07	-0.3%	4.070591E-07	-5.2%	4.070651E-07	-5.2%
100	4.281643E-07	-0.3%	4.281593E-07	-0.3%	4.070600E-07	-5.2%	4.070615E-07	-5.2%
200			4.281595E-07	-0.3%	4.070602E-07	-5.2%	4.070606E-07	-5.2%

Max x or y bending moment: theoretical = -9.607E-03 lb-in/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ŀ	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	-8.074155E-03	-16.0%	-8.286672E-03	-13.7%	-7.932362E-03	-17.4%	-7.980317E-03	-16.9%
5	-9.340636E-03	-2.8%	-9.384681E-03	-2.3%	-9.316309E-03	-3.0%	-9.326607E-03	-2.9%
10	-9.520004E-03	-0.9%	-9.522622E-03	-0.9%	-9.513549E-03	-1.0%	-9.514432E-03	-1.0%
20	-9.564479E-03	-0.4%	-9.562965E-03	-0.5%	-9.562834E-03	-0.5%	-9.562405E-03	-0.5%
50	-9.576896E-03	-0.3%	-9.576334E-03	-0.3%	-9.576632E-03	-0.3%	-9.576411E-03	-0.3%
100	-9.578669E-03	-0.3%	-9.578481E-03	-0.3%	-9.578603E-03	-0.3%	-9.578518E-03	-0.3%
200			-9.579054E-03	-0.3%	-9.579094E-03	-0.3%	-9.579056E-03	-0.3%

Max twisting moment: theoretical = -6.225E-03 lb-in/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	-4.668448E-03	-25.0%	-4.500427E-03	-27.7%	-4.823450E-03	-22.5%	-4.763773E-03	-23.5%
5	-6.052187E-03	-2.8%	-6.089201E-03	-2.2%	-6.075395E-03	-2.4%	-6.067006E-03	-2.5%
10	-6.354222E-03	2.1%	-6.379003E-03	2.5%	-6.360069E-03	2.2%	-6.360019E-03	2.2%
20	-6.451367E-03	3.6%	-6.461321E-03	3.8%	-6.453200E-03	3.7%	-6.454149E-03	3.7%
50	-6.485563E-03	4.2%	-6.487857E-03	4.2%	-6.485967E-03	4.2%	-6.486422E-03	4.2%
100	-6.491641E-03	4.3%	-6.492338E-03	4.3%	-6.491745E-03	4.3%	-6.491934E-03	4.3%
200			-6.493610E-03	4.3%	-6.493411E-03	4.3%	-6.493481E-03	4.3%

Max x or y transverse shear: theoretical = -3.374E-02 lb/in

Random	( )		TRIA3		QUAD4K TRIA3I		<	
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	-2.112100E-02	-37.4%	-2.338886E-02	-30.7%				
5	-2.870941E-02	-14.9%	-2.916144E-02	-13.6%				
10	-3.125004E-02	-7.4%	-3.133337E-02	-7.1%				
20	-3.251203E-02	-3.6%	-3.252939E-02	-3.6%				
50	-3.326553E-02	-1.4%	-3.327051E-02	-1.4%				
100	-3.351604E-02	-0.7%	-3.351810E-02	-0.7%				
200			-3.364134E-02	-0.3%				

Random disorder of element shape - mesh size is 10 x 10 Thick plate: 0.20", Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.294E-07 inches

Random	QUAD4 (M	IN4T)	TRIA3		QUAD4	K	TRIA3ŀ	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	4.286389E-07	-0.2%	4.280076E-07	-0.3%	4.070296E-07	-5.2%	4.071537E-07	-5.2%
0.1	4.286363E-07	-0.2%	4.279977E-07	-0.3%	4.069889E-07	-5.2%	4.070835E-07	-5.2%
0.2	4.286255E-07	-0.2%	4.279538E-07	-0.3%	4.069567E-07	-5.2%	4.069980E-07	-5.2%
0.3	4.286026E-07	-0.2%	4.278728E-07	-0.4%	4.069297E-07	-5.2%	4.069015E-07	-5.2%
0.4	4.285964E-07	-0.2%	4.277729E-07	-0.4%	4.068571E-07	-5.2%	4.067849E-07	-5.3%
0.5	4.285648E-07	-0.2%	4.277427E-07	-0.4%	4.066887E-07	-5.3%	4.067714E-07	-5.3%
0.6	4.285039E-07	-0.2%	4.274602E-07	-0.5%	See Note 1		4.066252E-07	-5.3%
0.7	4.284351E-07	-0.2%	4.272412E-07	-0.5%	4.063414E-07	-5.4%	4.063621E-07	-5.4%
۷.8	See Note 1		4.2/3250E-0/	-0.5%	See Note 1		4.062954E-07	-5.4%

Max x or y bending moment: theoretical = -9.607E-03 lb-in/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-9.520004E-03	-0.9%	-9.522622E-03	-0.9%	-9.513549E-03	-1.0%	-9.514432E-03	-1.0%
0.1	-9.516887E-03	-0.9%	-9.516944E-03	-0.9%	-9.508573E-03	-1.0%	-9.509657E-03	-1.0%
0.2	-9.515005E-03	-1.0%	-9.513496E-03	-1.0%	-9.500159E-03	-1.1%	-9.497733E-03	-1.1%
0.3	-9.510936E-03	-1.0%	-9.510518E-03	-1.0%	-9.497190E-03	-1.1%	-9.498315E-03	-1.1%
0.4	-9.503080E-03	-1.1%	-9.502445E-03	-1.1%	-9.492558E-03	-1.2%	-9.484133E-03	-1.3%
0.5	-9.495133E-03	-1.2%	-9.499946E-03	-1.1%	-9.468693E-03	-1.4%	-9.478471E-03	-1.3%
0.6	-9.485838E-03	-1.3%	-9.482645E-03	-1.3%	See Note 1		-9.475569E-03	-1.4%
0.7	-9.523367E-03	-0.9%	-9.482962E-03	-1.3%	-9.479518E-03	-1.3%	-9.450705E-03	-1.6%
۷.8	See Note 1		-9.488444E-U3	-1.2%	See Note 1		-9.472364E-03	-1.4%

Max twisting moment: theoretical = -6.225E-03 lb-in/in

			<u> </u>					
Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-6.354222E-03	2.1%	-6.379003E-03	2.5%	-6.360069E-03	2.2%	-6.360019E-03	2.2%
0.1	-6.358936E-03	2.2%	-6.386316E-03	2.6%	-6.363487E-03	2.2%	-6.360086E-03	2.2%
0.2	-6.377200E-03	2.4%	-6.398629E-03	2.8%	-6.367205E-03	2.3%	-6.367530E-03	2.3%
0.3	-6.335701E-03	1.8%	-6.390431E-03	2.7%	-6.347635E-03	2.0%	-6.389979E-03	2.7%
0.4	-6.359404E-03	2.2%	-6.433151E-03	3.3%	-6.361833E-03	2.2%	-6.356812E-03	2.1%
0.5	-6.337010E-03	1.8%	-6.424353E-03	3.2%	-6.375497E-03	2.4%	-6.376842E-03	2.4%
0.6	-6.323262E-03	1.6%	-6.421069E-03	3.1%	See Note 1		-6.386909E-03	2.6%
0.7	-6.361620E-03	2.2%	-6.419936E-03	3.1%	-6.329188E-03	1.7%	-6.416808E-03	3.1%
8.0	See Note 1		-6.458207E-03	3.7%	See Note 1		-6.414755E-03	3.0%

Max x or y transverse shear: theoretical = -3.375E-02 lb/in

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3I	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-3.125004E-02	-7.4%	-3.133337E-02	-7.2%				
0.1	-3.131184E-02	-7.2%	-3.132976E-02	-7.2%				
0.2	-3.129734E-02	-7.3%	-3.136309E-02	-7.1%				
0.3	-3.129752E-02	-7.3%	-3.139836E-02	-7.0%				
0.4	-3.133671E-02	-7.2%	-3.125302E-02	-7.4%				
0.5	-3.128780E-02	-7.3%	-3.130338E-02	-7.2%				
0.6	-3.120514E-02	-7.5%	-3.098793E-02	-8.2%				
0.7	-3.094243E-02	-8.3%	-3.154259E-02	-6.5%				
۵.۷	See Note 1		-3.12/49/E-02	-7.3%				

## **Appendix B-3**

Square plate supported only at the central point. 1/4 sym.
Plate overall dimensions 2" X 2"
QUAD4 is MIN4T element

Plate thickness 0.02" (thin)

Table B3-1

## Convergence study - increasing mesh size Thin plate: 0.02", stress free thermal expansion - 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.000E-05 inches

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4K TRIA3K			(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
5	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
10	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
20	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
50	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
100	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
200			2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (M	IN4T)	TRIA3	1	QUAD4	·K	TRIA3ŀ	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	E-13		E-13		E-13		E-13	
5	E-13		E-12		E-12		E-12	
10	E-12		E-12		E-12		E-12	
20	E-12		E-12		E-12		E-12	
50	E-11		E-11		E-11		E-11	
100	E-10		E-10		E-10		E-10	
200			E-10		E-09		E-10	

Table B3-2

Random disorder of element shape - mesh size is 10 x 10 Thin plate: 0.02", stress free thermal expansion - 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.000E-05 inches

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.1	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.2	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.3	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.4	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.5	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.6	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.7	2.000000E-05	0.0%	2.000000E-05	0.0%	See Note 1		2.000000E-05	0.0%
8.0	See Note 1		2.00000E-05	0.0%	See Note 1		2.000000E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (M	IN4T)	TRIA3		QUAD4	K	TRIA3ŀ	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	E-12		E-12		E-12		E-12	
0.1	E-12		E-12		E-12		E-12	
0.2	E-12		E-12		E-12		E-12	
0.3	E-12		E-12		E-12		E-12	
0.4	E-12		E-12		E-12		E-12	
0.5	E-12		E-12		E-12		E-12	
0.6	E-12		E-12		E-11		E-12	
0.7	E-12		E-12		See Note 1		E-12	
0.8	See Note 1		<b>Ŀ-1</b> 2		See Note 1		E-11	

Table B3-3

## Convergence study - increasing mesh size

Thin plate: 0.02", stress free thermal expansion - 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.000E-05 inches

Random	QUAD4 (MIN4T)		TRIA3		QUAD4	QUAD4K TRIA3K		
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
5	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
10	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
20	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
50	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
100	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
200			-2.000000E-05	0.0%	-2.000000E-05	0.0%	-1.999998E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (M	IN4T)	TRIA3		QUAD4	·K	TRIA3I	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	E-13		E-13		E-14		E-14	
5	E-12		E-12		E-12		E-13	
10	E-11		E-11		E-11		E-12	
20	E-10		E-11		E-09		E-09	
50	E-09		E-08		E-08		E-08	
100	E-07		E-09		E-08		E-06	
200			E-08		E-06		E-05	

Table B3-4

## Random disorder of element shape - mesh size is 10 x 10

Thin plate: 0.02", stress free thermal expansion - 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.000E-05 inches

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.1	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.2	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.3	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.4	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.5	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.6	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.7	-2.000000E-05	0.0%	-2.000000E-05	0.0%	See Note 1		-2.000000E-05	0.0%
۷.8	See Note 1		-2.000000E-05	0.0%	See Note 1		-2.000000E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (M	IN4T)	TRIA3		QUAD4	K	TRIA3ŀ	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	E-11		E-11		E-11		E-12	
0.1	E-12		E-11		E-11		E-11	
0.2	E-12		E-11		E-11		E-11	
0.3	E-11		E-11		E-11		E-11	
0.4	E-11		E-11		E-11		E-12	
0.5	E-11		E-11		E-11		E-11	
0.6	E-10		E-11		E-11		E-11	
0.7	E-11		E-11		See Note 1		E-11	
۵.۷	See Note 1		<b>Ŀ-</b> 11		See Note 1		E-02	

## **Appendix B-4**

Square plate supported only at the central point. 1/4 sym.
Plate overall dimensions 2" X 2"
QUAD4 is MIN4T element

Plate thickness 0.20" (thick)

Table B4-1

## Convergence study - increasing mesh size Thick plate: 0.20", stress free thermal expansion - 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.000E-05 inches

Random	QUAD4 (M	IN4T)	TRIA3		QUAD4	K	TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
5	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
10	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
20	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
50	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
100	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
200			2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (M	IN4T)	TRIA3	3	QUAD4	·K	TRIA3ŀ	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	E-13		E-13		E-13		E-13	
5	E-12		E-12		E-12		E-12	
10	E-11		E-12		E-12		E-12	
20	E-11		E-11		E-12		E-11	
50	E-10		E-10		E-10		E-10	
100	E-10		E-10		E-10		E-10	
200			E-09		E-09		E-09	

Table B4-2

Random disorder of element shape - mesh size is 10 x 10 Thick plate: 0.20", stress free thermal expansion - 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.000E-05 inches

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	4K TRIA3K		(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.1	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.2	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.3	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.4	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.5	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.6	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%	2.000000E-05	0.0%
0.7	2.000000E-05	0.0%	2.000000E-05	0.0%	See Note 1		2.000000E-05	0.0%
۷.8	See Note 1		2.000000E-05	0.0%	See Note 1		2.000000E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (M	IN4T)	TRIA3		QUAD4	K	TRIA3ŀ	<
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	E-11		E-12		E-12		E-12	
0.1	E-12		E-12		E-12		E-12	
0.2	E-12		E-12		E-12		E-12	
0.3	E-12		E-12		E-12		E-12	
0.4	E-12		E-12		E-12		E-12	
0.5	E-12		E-12		E-12		E-12	
0.6	E-12		E-12		E-12		E-12	
0.7	E-12		E-12		See Note 1		E-12	
۷.8	See Note 1		<b>Ŀ-1</b> 2		See Note 1		E-11	

Table B4-3

## Convergence study - increasing mesh size

Thick plate: 0.20", stress free thermal expansion - 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.000E-05 inches

Random	QUAD4 (MI	N4T)	TRIA3		QUAD4	K	TRIA3ł	(
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2 5 10 20	-2.000000E-05 -2.000000E-05 -2.000000E-05 -2.000000E-05	0.0% 0.0% 0.0% 0.0%	-2.000000E-05 -2.000000E-05 -2.000000E-05 -2.000000E-05	0.0% 0.0% 0.0% 0.0%	-2.000000E-05 -2.000000E-05 -2.000000E-05 -2.000000E-05	0.0% 0.0% 0.0% 0.0%	-2.000000E-05 -2.000000E-05 -2.000000E-05 -2.000000E-05	0.0% 0.0% 0.0% 0.0%
50 100 200	-2.000000E-05 -2.000000E-05	0.0% 0.0%	-2.000000E-05 -2.000000E-05 -2.000000E-05	0.0% 0.0% 0.0%	-2.00000E-05 -2.00000E-05 -2.00000E-05	0.0% 0.0% 0.0%	-2.00000E-05 -2.00000E-05 -1.999998E-05	0.0% 0.0% 0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (MIN4T)		TRIA3		QUAD4K		TRIA3K	
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
2	E-12		E-13		E-13		E-13	
5	E-12		E-12		E-11		E-11	
10	E-11		E-11		E-10		E-11	
20	E-11		E-10		E-09		E-09	
50	E-10		E-10		E-07		E-07	
100	E-10		E-10		E-06		E-06	
200			E-09		E-04		E-05	

Table B4-4

## Random disorder of element shape - mesh size is 10 x 10

Thick plate: 0.20", stress free thermal expansion - 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.000E-05 inches

Random	QUAD4 (MIN4T)		TRIA3		QUAD4K		TRIA3K	
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.1	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.2	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.3	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.4	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.5	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.6	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%	-2.000000E-05	0.0%
0.7	-2.000000E-05	0.0%	-2.000000E-05	0.0%	See Note 1		-2.000000E-05	0.0%
8.0	See Note 1		-2.000000E-05	0.0%	See Note 1		-2.00000E-05	0.0%

Max stress: theoretical = 0 lb/in^2

Random	QUAD4 (MIN4T)		TRIA3		QUAD4K		TRIA3K	
Grid Disorder	Displ	Error	Displ	Error	Displ	Error	Displ	Error
0	E-11		E-11		E-10		E-11	
0.1	E-11		E-11		E-10		E-10	
0.2	E-11		E-11		E-10		E-10	
0.3	E-11		E-12		E-10		E-11	
0.4	E-11		E-12		E-10		E-10	
0.5	E-11		E-11		E-10		E-10	
0.6	E-11		E-11		E-10		E-10	
0.7	E-09		E-11		See Note 1		E-10	
8.0	See Note 1		<b>Ŀ-</b> 11		See Note 1		E-10	