The BAR Element

I. Stiffness Matrix

The equations for the BAR element are

$$K U = P + P_T \qquad \begin{cases} P = \text{applied loads + constraints} \\ P_T = \text{equiv. thermal loads} \end{cases} \qquad \text{I-(1)}$$

Define:

$$k_a = \frac{AE}{l}, \quad k_t = \frac{GJ}{l}, \quad \beta = \frac{12EI_{12}}{l^3}$$

$$\hat{k}_1 = \frac{l^2}{4}R_1 + \frac{EI_1}{l}, \quad \hat{k}_2 = \frac{l^2}{4}R_2 + \frac{EI_2}{l}, \quad \hat{k}_3 = \frac{l^2}{4}R_1 - \frac{EI_1}{l}, \quad \hat{k}_4 = \frac{l^2}{4}R_2 - \frac{EI_2}{l} \qquad \text{I-(2)}$$

where

$$R_1 = \frac{12EI_1}{l^3}\left(1+\gamma_1\right)^{-1}$$

$$R_2 = \frac{12EI_2}{l^3}\left(1+\gamma_2\right)^{-1} \qquad \text{I-(3)}$$

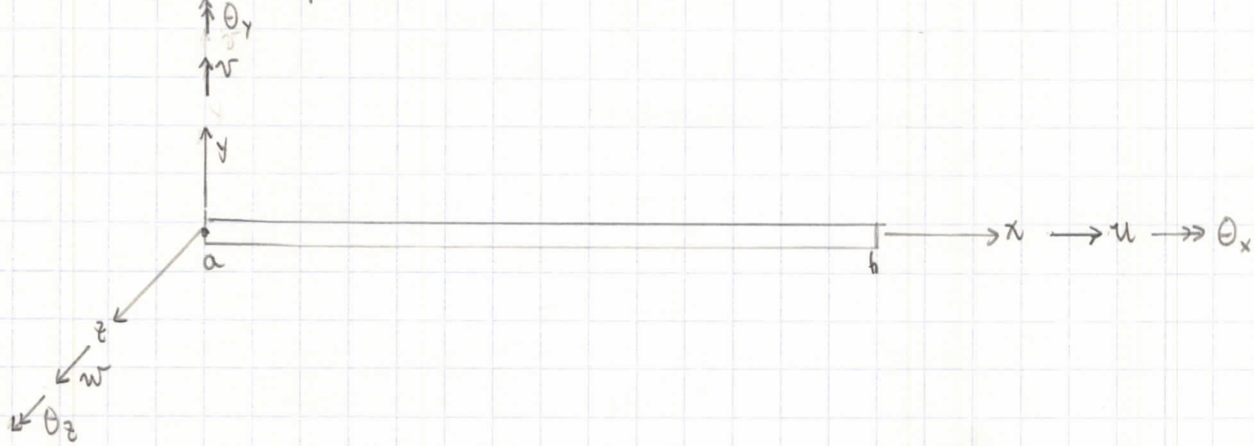
and

$$\gamma_1 = \begin{cases} \dfrac{12EI_1}{K_1 A G l^2} & \text{if } I_{12} = 0 \text{ and } K_1 \neq 0 \\ 0 & \text{if } I_{12} \neq 0 \text{ or } K_1 = 0 \end{cases} \qquad \text{I-(4)}$$

$$\gamma_2 = \begin{cases} \dfrac{12EI_2}{K_2 A G l^2} & \text{if } I_{12} = 0 \text{ and } K_2 \neq 0 \\ 0 & \text{if } I_{12} \neq 0 \text{ or } K_2 = 0 \end{cases}$$

and K_1, K_2 are the area factors for shear (a zero value is assigned to γ_1, γ_2 when K_1 and K_2 are zero which is interpreted to be the case of no shear flexibility (which is actually $K_1, K_2 \to \infty$).

The degrees of freedom for the BAR element are:



$$U = \begin{Bmatrix} u_a \\ v_a \\ w_a \\ \theta_{xa} \\ \theta_{ya} \\ \theta_{za} \\ u_b \\ v_b \\ w_b \\ \theta_{xb} \\ \theta_{yb} \\ \theta_{zb} \end{Bmatrix} , \quad P = \begin{Bmatrix} P_{xa} \\ P_{ya} \\ P_{za} \\ M_{xa} \\ M_{ya} \\ M_{za} \\ P_{xb} \\ P_{yb} \\ P_{zb} \\ M_{xb} \\ M_{yb} \\ M_{zb} \end{Bmatrix} \qquad \text{I-(5)}$$

Eqn (1) can be partitioned as

$$K = \begin{bmatrix} K_{aa} & K_{ab} \\ K_{ab}^{T} & K_{bb} \end{bmatrix}$$

The partitions of K are shown on the following page

$$
K = \begin{array}{cccccccccccc|c}
u_a & v_a & w_a & \theta_{xa} & \theta_{ya} & \theta_{za} & u_b & v_b & w_b & \theta_{xb} & \theta_{yb} & \theta_{zb} & \\
k_a & 0 & 0 & 0 & 0 & 0 & -k_a & 0 & 0 & 0 & 0 & 0 & u_a \\
0 & R_1 & \beta & 0 & -\frac{l}{2}\beta & \frac{l}{2}R_1 & 0 & -R_1 & -\beta & 0 & -\frac{l}{2}\beta & \frac{l}{2}R_1 & v_a \\
0 & \beta & R_2 & 0 & -\frac{l}{2}R_2 & \frac{l}{2}\beta & 0 & -\beta & -R_2 & 0 & -\frac{l}{2}R_2 & \frac{l}{2}\beta & w_a \\
0 & 0 & 0 & k_t & 0 & 0 & 0 & 0 & 0 & -k_t & 0 & 0 & \theta_{xa} \\
0 & -\frac{l}{2}\beta & -\frac{l}{2}R_2 & 0 & \hat{k}_2 & -\frac{l^2}{3}\beta & 0 & \frac{l}{2}\beta & \frac{l}{2}R_2 & 0 & \hat{k}_4 & -\frac{l^2}{6}\beta & \theta_{ya} \\
0 & \frac{l}{2}R_1 & \frac{l}{2}\beta & 0 & -\frac{l^2}{3}\beta & \hat{k}_1 & 0 & -\frac{l}{2}R_1 & -\frac{l}{2}\beta & 0 & -\frac{l^2}{6}\beta & \hat{k}_3 & \theta_{za} \\
\hline
-k_a & 0 & 0 & 0 & 0 & 0 & k_a & 0 & 0 & 0 & 0 & 0 & u_b \\
0 & -R_1 & -\beta & 0 & \frac{l}{2}\beta & -\frac{l}{2}R_1 & 0 & R_1 & \beta & 0 & \frac{l}{2}\beta & -\frac{l}{2}R_1 & v_b \\
0 & -\beta & -R_2 & 0 & \frac{l}{2}R_2 & -\frac{l}{2}\beta & 0 & \beta & R_2 & 0 & \frac{l}{2}R_2 & -\frac{l}{2}\beta & w_b \\
0 & 0 & 0 & -k_t & 0 & 0 & 0 & 0 & 0 & k_t & 0 & 0 & \theta_{xb} \\
0 & -\frac{l}{2}\beta & -\frac{l}{2}R_2 & 0 & \hat{k}_4 & -\frac{l^2}{6}\beta & 0 & \frac{l}{2}\beta & \frac{l}{2}R_2 & 0 & \hat{k}_2 & -\frac{l^2}{3}\beta & \theta_{yb} \\
0 & \frac{l}{2}R_1 & \frac{l}{2}\beta & 0 & -\frac{l^2}{6}\beta & \hat{k}_3 & 0 & -\frac{l}{2}R_1 & -\frac{l}{2}\beta & 0 & -\frac{l^2}{3}\beta & \hat{k}_1 & \theta_{zb}
\end{array}
= \left[\begin{array}{c|c} K_{aa} & K_{ab} \\ \hline K_{ab}^T & K_{bb} \end{array}\right] \qquad \text{I-(6)}
$$

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