Comparison of MYSTRAN plate element results with theory (MYSTRAN results of 12/2005 using new QUAD4)

Introduction

In order to check the various MYSTRAN plate elements (QUAD4 and TRIA3 Mindlin elements, QUAD4K and TRIA3K Kirchoff elements) several test runs were made. Appendices B-1 through B-4 show the results. All of the runs are for a 2"x2" square plate with only ¼ of the plate modeled due to symmetry. Two plate thicknesses were analyzed (0.02" thin plate and 0.20" thick plate). The plates were subjected to two sets of loadings (with boundary conditions indicated):

- Simply supported plate subjected to:
    - uniform pressure load and

- Stress-free plate (supported in rigid body mode only) subjected to:
    - constant bulk temperature change and
    - temperature gradient through the plate thickness

The plate physical and material properties were:

- Length and width = 2.0"
- E = 10.9×10^6
- G = 4.19×10^6
- Poissons ratio = 0.3
- Pressure load = 0.05 lb/in^2

The following is a key to the tables in the appendices:

- Appendices B-1 through B-4 are results when the QUAD4 element is the new element (MIN4T) with:
    - App B-1: Simply supported 0.02" thick plate with pressure load
    - App B-2: Simply supported 0.20"thick plate with pressure load
    - App B-3: Stress-free 0.02" thick plate with bulk and gradient thermal load
    - App B-4: Stress-free 0.20" thick plate with bulk and gradient thermal load

Each appendix has results for a convergence study in which the plate mesh was increased from a 2x2 mesh (for the ¼ plate) to 5x5, 10x10, 20x20, 50x50, 100x100 and finally a 200x200 mesh. In these, the elements are all square. In addition to this convergence study, the appendices also have the results of runs wherein the model with a 10x10 mesh had elements distorted (no longer square). This was done by applying a random perturbation to the grids which are interior to the plate with a successfully more severe random coefficient. Figures 1 – 9 show the meshes for the 10x10 plate with the various distortions analyzed. As the random coefficient increased there came a time where the mesh distortion was so great that some elements had fatal errors (e.g. an interior angle of a quad element exceeded 180^o).

Table 1 below gives the theoretical results for the displacements and forces to compare with the tables in the appendices for the pressure load results. Equations for the theoretical results were taken from reference 1 for thin plates and from references 2 and 3 for moderately thick plates. The equations were solved in fortran programs written by the author. For the thin plate a comparison could be made for the max values of displacements and forces based on tables in reference 1 that give nondimensional coefficients for simply supported plates under a uniform pressure load. For the thick plate there were no comparison values available in reference 1 other than to use the thick plate equations for the thin plate. Columns 1 and 2 in Table 1 are the solutions from reference 1 for both the thin and thick plate. The column 2 values do not account for a plate thickness that is finite relative to its in-plane dimensions. Thus, for the thick plate, the author programmed a solution available from the literature for the theoretical displacements and element forces using Mindlin plate theory that does account for the finite thickness relative to the in-plane dimensions.

Table 1: Theoretical results for simply supported plate with uniform pressure load

	1	2	3	4
Max displacement or force	Kirchoff thin plate eqn solution (ref 1)		Mindlin thick plate eqn solution (programmed from ref 3)	
	t = 0.02"	t = 0.20"	t = 0.02"	t = 0.20"
displacement (in)	$+4.070 \times 10^{-4}$	$+4.070 \times 10^{-7}$	$+4.072 \times 10^{-4}$	$+4.249 \times 10^{-7}$
bending moment (lb-in/in)	$+9.577 \times 10^{-3}$	$+9.577 \times 10^{-3}$	$+9.578 \times 10^{-3}$	$+9.607 \times 10^{-3}$
twisting moment (lb-in/in)	-6.496×10^{-3}	-6.496×10^{-3}	-6.493×10^{-3}	-6.225×10^{-3}
transverse shear (lb/in)	-3.375×10^{-2}	-3.375×10^{-2}	-3.376×10^{-2}	-3.374×10^{-2}

For a plate whose thickness to length (t/a) is negligible, columns 1 and 3 should be the same. The fact that they are close would seem to validate the thick plate solution programmed. Columns 1 (or 3) and 4 then would be the values to compare with the MYSTRAN QUAD4 and TRIA3 results for the 0.02" and 0.20" plates, respectively. Columns 1 and 2 would be the expected values from MYSTRAN for the QUAD4K and TRIA3K Kirchoff elements for the 0.02" and 0.20" plates, respectively.

Table 2 gives the results for the stress free thermal loadings. Note that the displacements are the same for the thin or thick plate which is expected.

Table 2: Theoretical results for simply supported thick or thin plate with stress free thermal expansion

	1	2
Max displacement or force	1.0 deg bulk temp change (all plate thicknesses)	1.0 deg temp gradient (all plate thicknesses)
displacement (in)	2.0×10^{-5}	-2.0×10^{-5}
bending moment (lb-in/in)	0	0
twisting moment (lb-in/in)	0	0
transverse shear (lb/in)	0	0

Conclusions

The convergence study shows excellent agreement with theory for the thick and thin plates modeled with the QUAD4 or TRIA3 elements for even moderately fine meshes. The Kirchoff QUAD4K and TRIA3K give excellent results when the plate is thin (as the theory for Kirchoff plates assumes) but, as expected, give poor results when the plate is thick. The QUAD4K and TRIA3K should only be used for thin plates and only then as a backup to the more general QUAD4 and TRIA3 elements which are good for thin or thick plates.

The study of the effects of mesh distortion also show good agreement until the mesh distortion gets quite large. Keep in mind that the results are for a 10 by 10 mesh has some deviation from theory since the mesh is not very large. The results should really be compared to the 10 by 10 mesh for no element distortion to discern the effects of distortion from a square shape.

References

1. “Theory of plates and shells”, S. Timoshenko

2. “On bending of elastic plates”, Eric Reissner, Quarterly of Applied Mathematics, Vol V, 1947, pp 55-68

3. “Effect of shear deformations on the bending of elastic plates”, V. Salerno and M. Goldberg, J of Applied Mathematics, Transactions of the ASME, March 1960, pp 54-58

Figure 1
AQ3U2S010 – Plate with no element random distortion

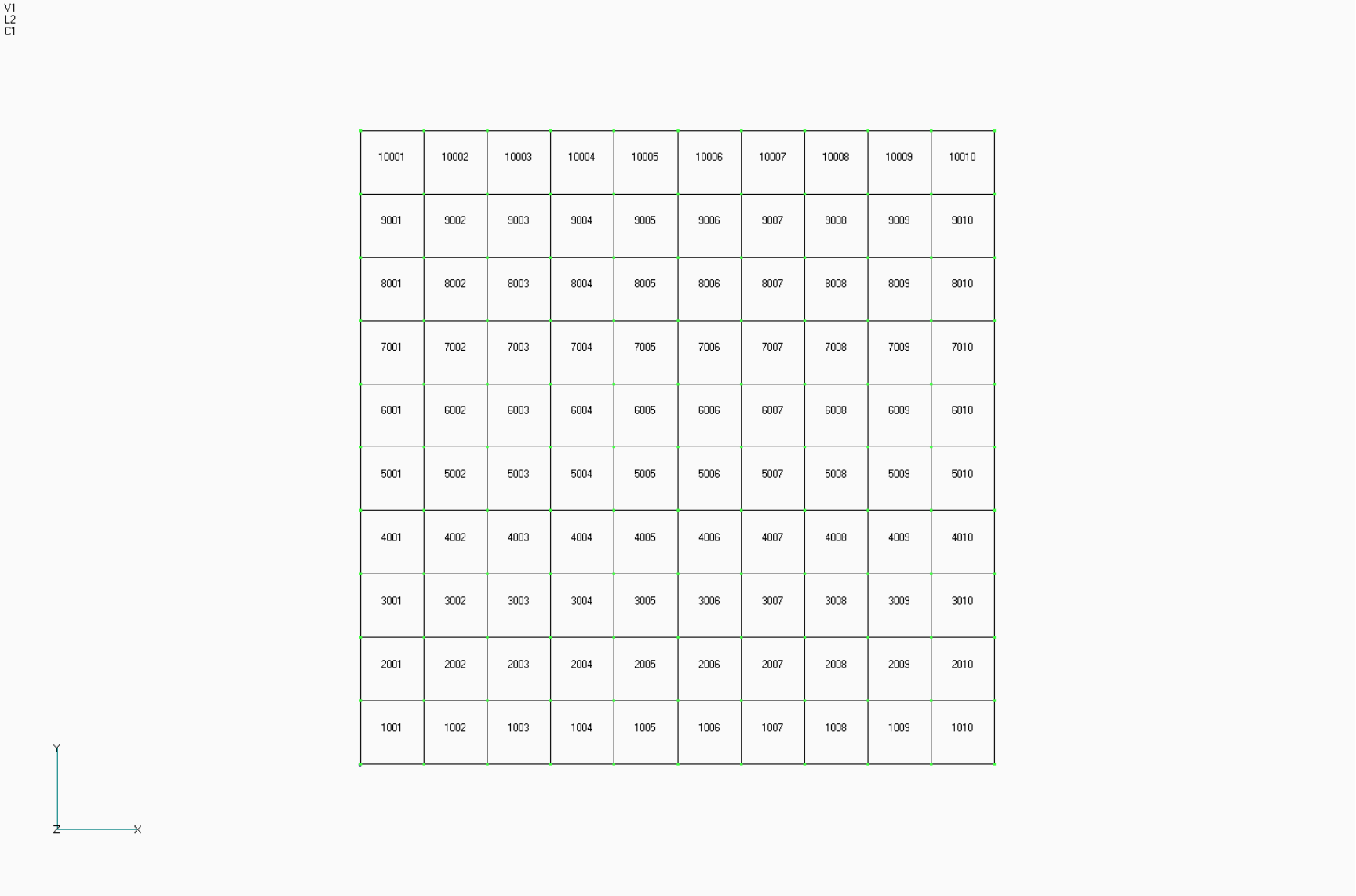


Figure 2
AQ3R2S010a – Plate with random grid disorder coefficient = 0.1

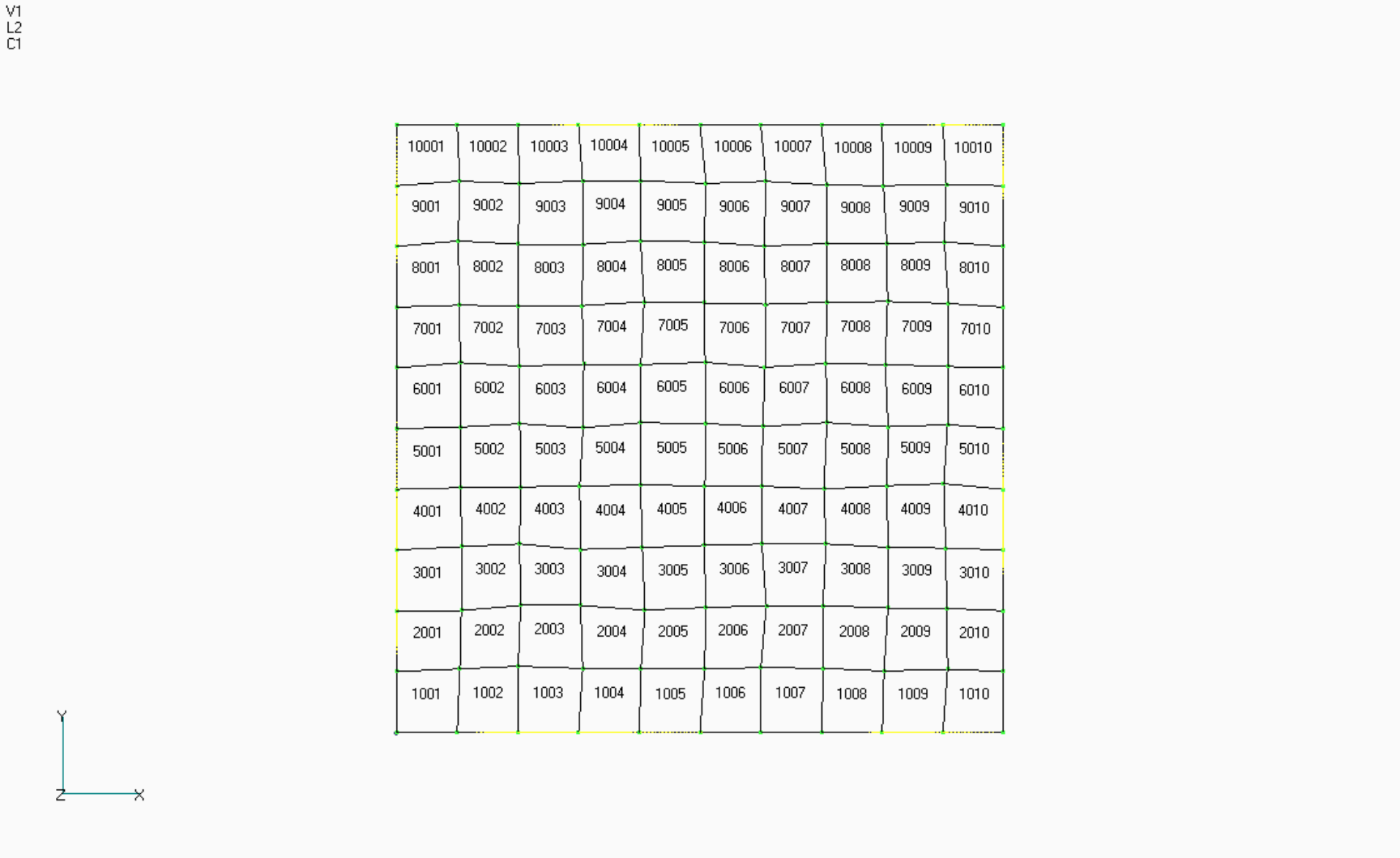


Figure 3
AQ3R2S010b – Plate with random grid disorder coefficient = 0.2

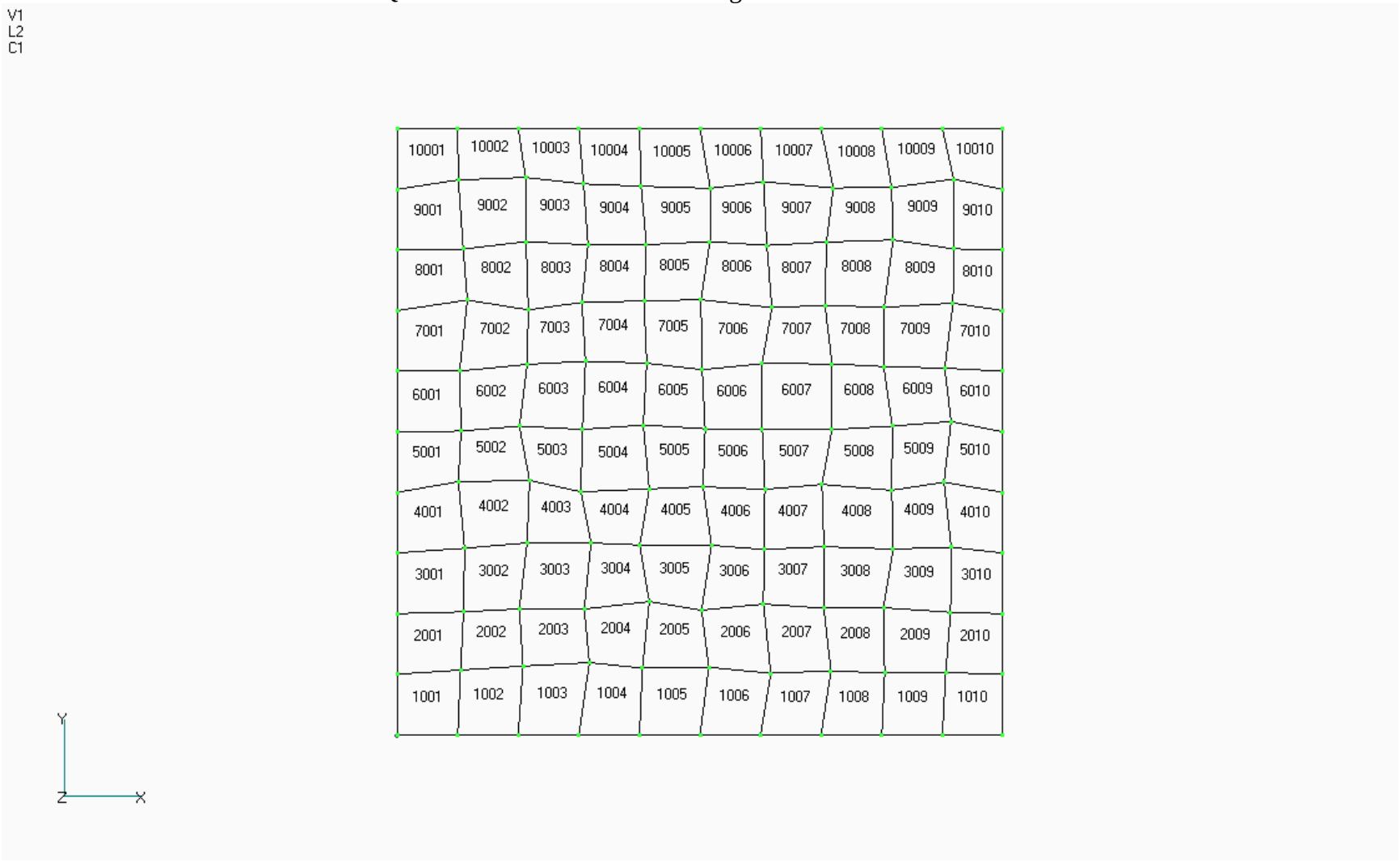


Figure 4
AQ3R2S010c – Plate with random grid disorder coefficient = 0.3

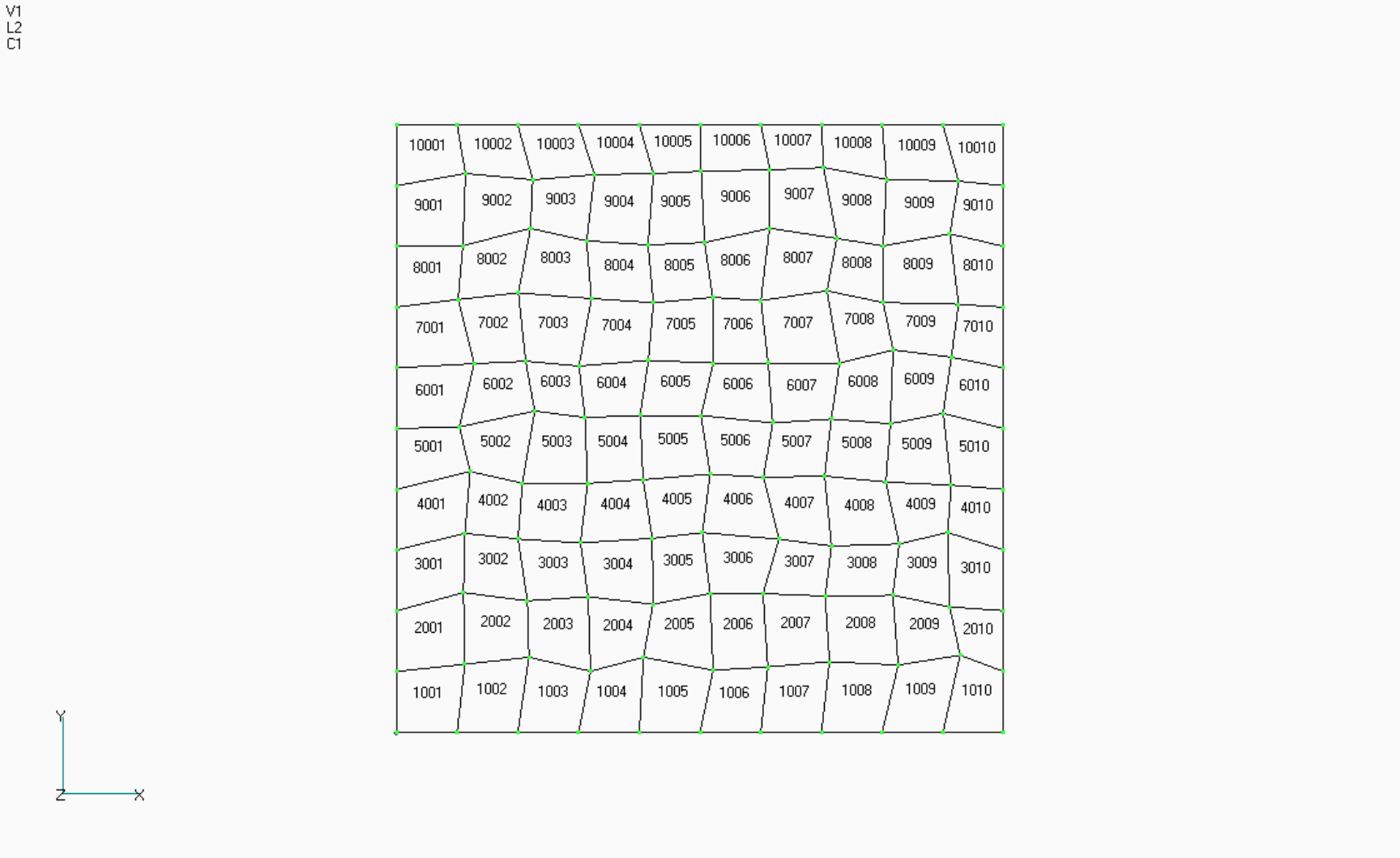


Figure 5
AQ3R2S010d – Plate with random grid disorder coefficient = 0.4

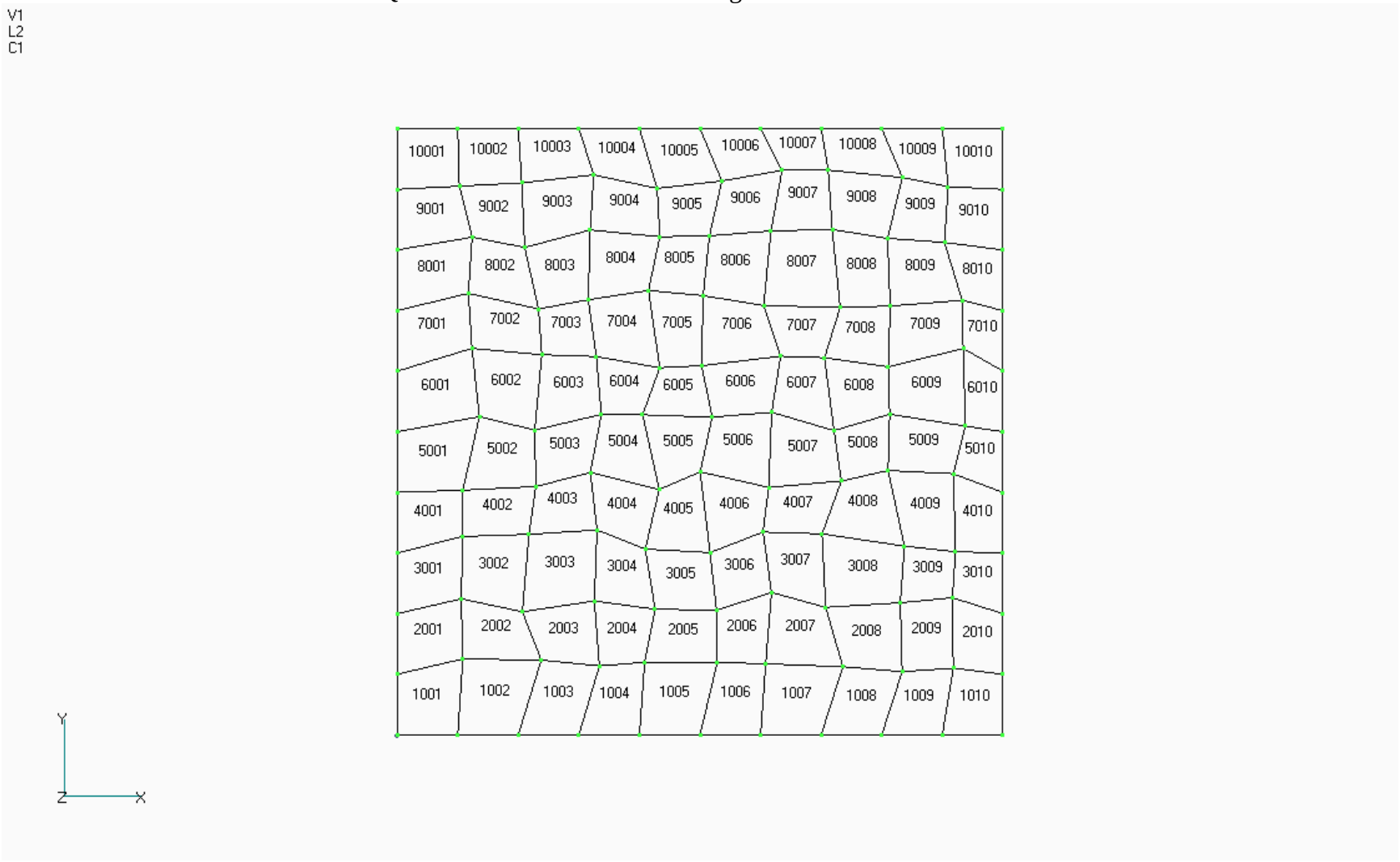


Figure 6
AQ3R2S010e – Plate with random grid disorder coefficient = 0.5

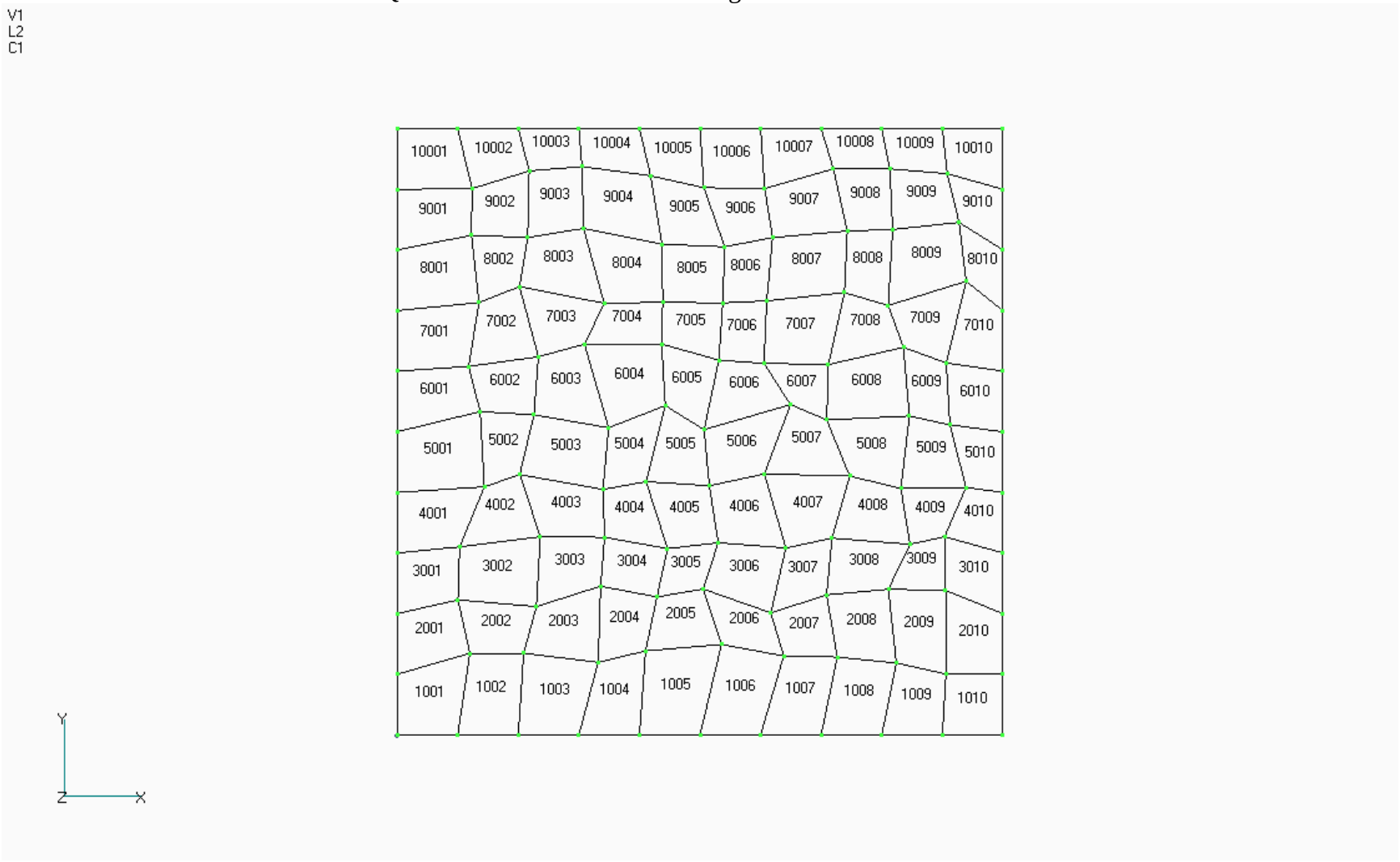


Figure 7
AQ3R2S010f – Plate with random grid disorder coefficient = 0.6

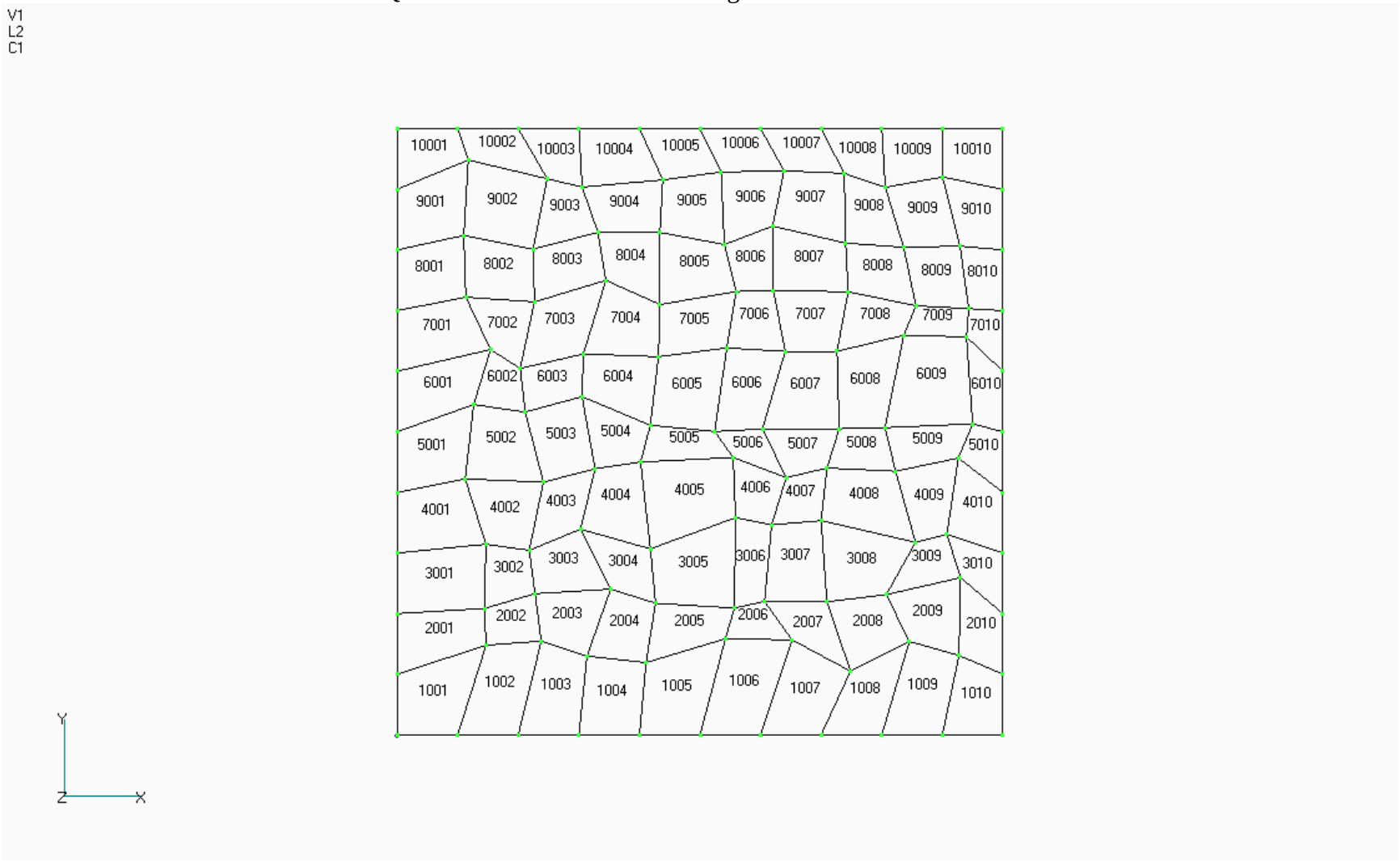


Figure 8
AQ3R2S010g – Plate with random grid disorder coefficient = 0.7

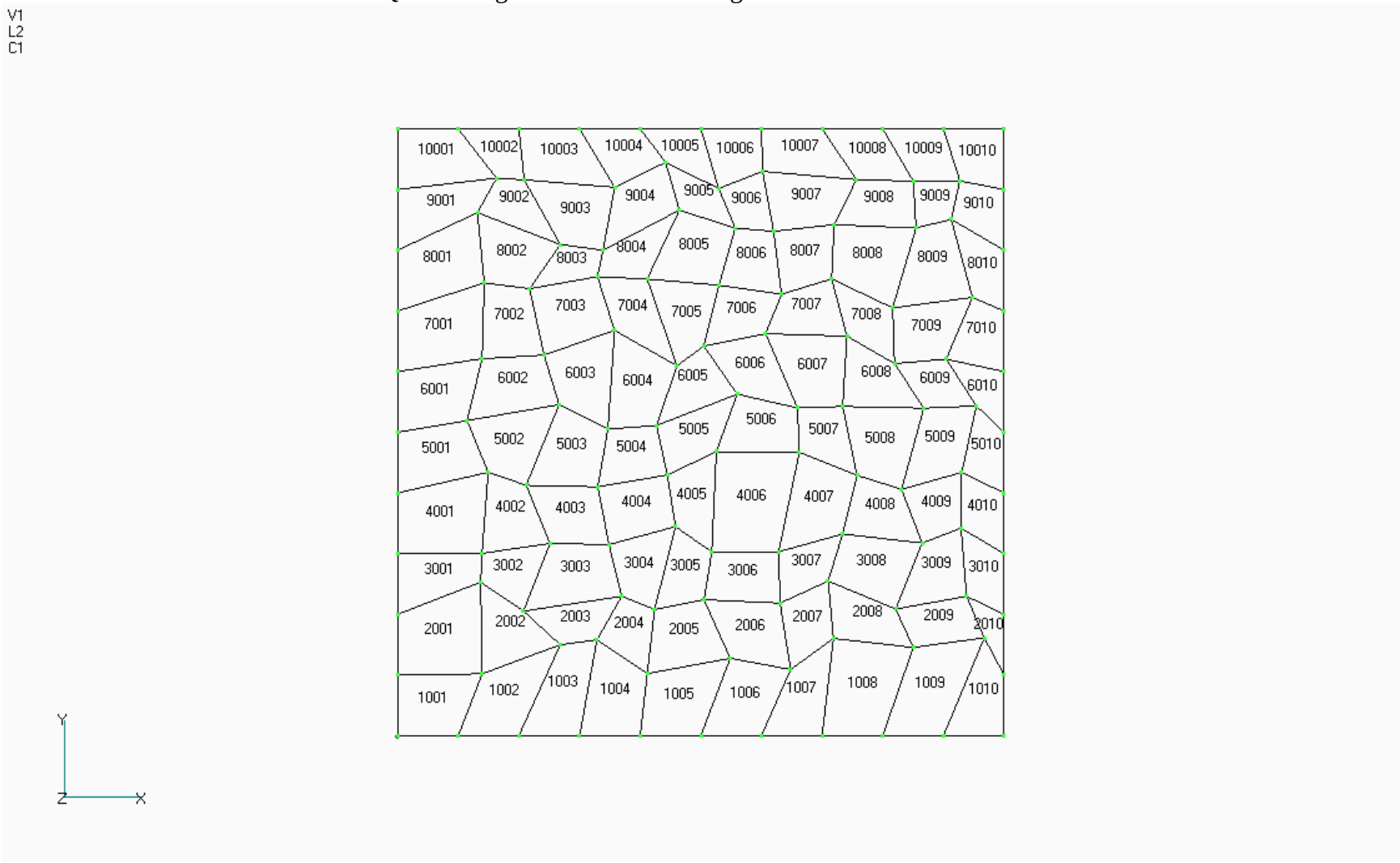
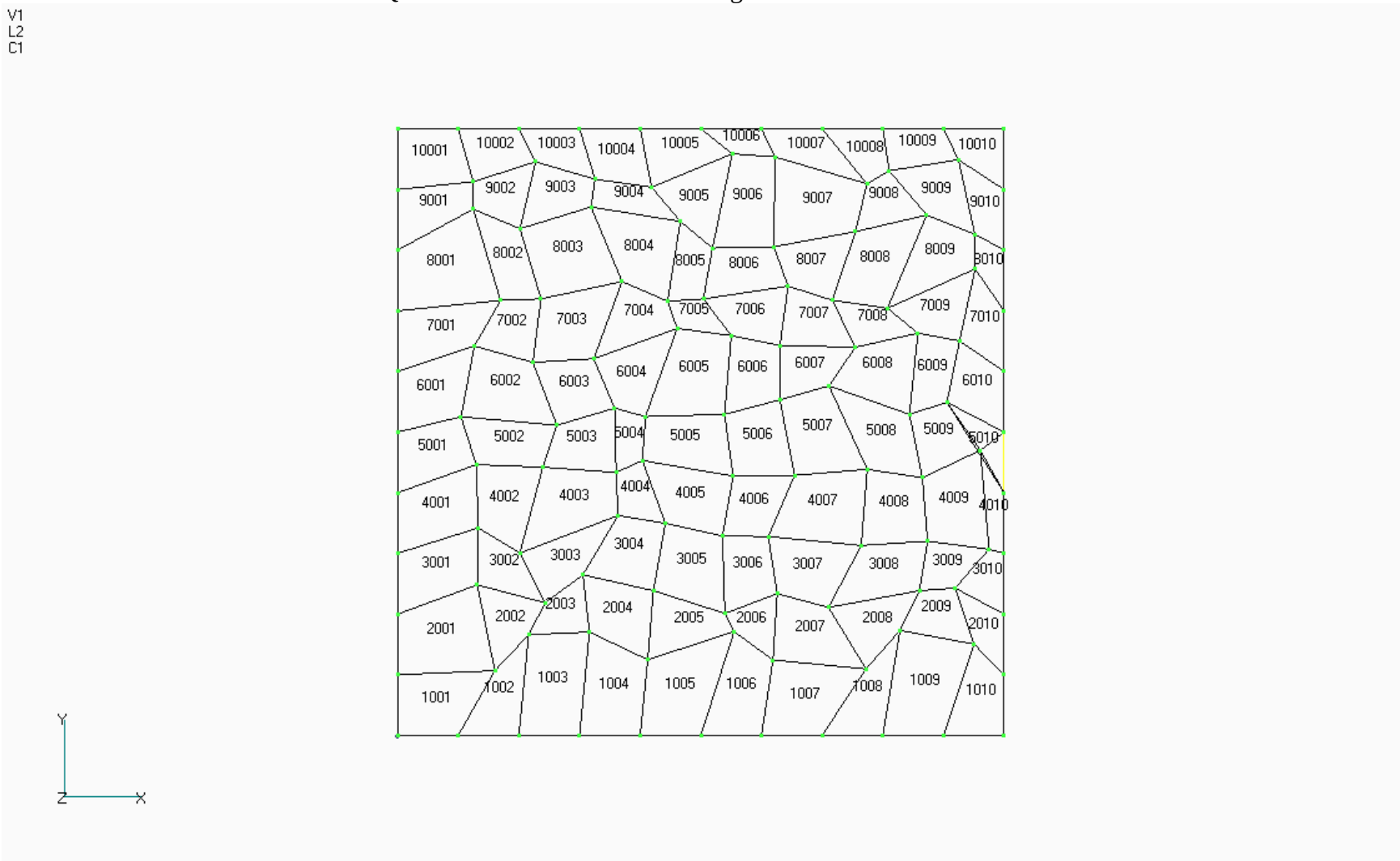


Figure 9
AQ3R2S010h – Plate with random grid disorder coefficient = 0.8



Appendix B-1

Simply supported square plate. 1/4 sym. Plate overall dimensions 2" X 2"
QUAD4 is MIN4T element

Plate thickness 0.02” (thin)

Table B1-1

Convergence study - increasing mesh size
Thin plate: 0.02” using MIN4T quad elements
Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.070E-04 inches

Mesh	QUAD4 (MIM4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)	QUAD4K (AQ2U1Sxxx)	TRIA3K (AT2U1Sxxx)
2	4.177474E-04	3.882019E-04	4.053825E-04	4.064101E-04
5	4.089617E-04	4.044364E-04	4.069221E-04	4.072906E-04
10	4.076979E-04	4.066298E-04	4.070296E-04	4.071537E-04
20	4.073804E-04	4.071399E-04	4.070529E-04	4.070879E-04
50	4.072894E-04	4.072582E-04	4.070591E-04	4.070651E-04
100	4.072759E-04	4.072688E-04	4.070600E-04	4.070615E-04
200		4.072707E-04	4.070602E-04	4.070605E-04

Max M_x or M_y bending moment: theoretical = 9.577E-03 lb-in/in

Mesh	QUAD4 (MIN4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)	QUAD4K (AQ2U1Sxxx)	TRIA3K (AT2U1Sxxx)
2	-8.041848E-03	-8.251984E-03	-7.932362E-03	-7.980317E-03
5	-9.329311E-03	-9.402860E-03	-9.316309E-03	-9.326607E-03
10	-9.516958E-03	-9.538238E-03	-9.513549E-03	-9.514432E-03
20	-9.563905E-03	-9.568613E-03	-9.562834E-03	-9.562405E-03
50	-9.576865E-03	-9.577013E-03	-9.576632E-03	-9.576411E-03
100	-9.578667E-03	-9.578567E-03	-9.578603E-03	-9.578518E-03
200		-9.579062E-03	-9.579095E-03	-9.579065E-03

Max M_{xy} twisting moment: theoretical = -6.496E-03 lb-in/in

Mesh	QUAD4 (MIN4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)	QUAD4K (AQ2U1Sxxx)	TRIA3K (AT2U1Sxxx)
2	-4.642756E-03	-4.423924E-03	-4.823450E-03	-4.763773E-03
5	-6.039231E-03	-6.069230E-03	-6.075395E-03	-6.067006E-03
10	-6.350917E-03	-6.381598E-03	-6.360069E-03	-6.360019E-03
20	-6.451331E-03	-6.466028E-03	-6.453200E-03	-6.454149E-03
50	-6.485836E-03	-6.489589E-03	-6.485967E-03	-6.486422E-03
100	-6.491724E-03	-6.492906E-03	-6.491745E-03	-6.491934E-03
200		-6.493751E-03	-6.493411E-03	-6.493480E-03

MaxV_x or V_y transverse shear force: theoretical = -3.375E-02 lb/in

Mesh	QUAD4 (MIN4) (AQ3U1Sxxx)	TRIA3 (AT3U1Sxxx)
2	-2.120397E-02	-2.355221E-02
5	-2.873378E-02	-2.980993E-02
10	-3.125650E-02	-3.210377E-02
20	-3.251324E-02	-3.310807E-02
50	-3.326562E-02	-3.343383E-02
100	-3.351608E-02	-3.356282E-02
200		-3.366052E-02

Table B1-2

Random disorder of element shape - mesh size is 10 x 10
Thin plate: 0.02” using MIN4T quad elements
Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.070E-04 inches

Random Grid Disprder	QUAD4 (MIN4) (AQ3R1S010)	TRIA3 (AT3R1S010)	QUAD4K (AQ2R1S010)	TRIA3K (AT2R1S010)
.0	4.076979E-04	4.066298E-04	4.070296E-04	4.071537E-04
.1	4.076845E-04	4.065895E-04	4.069936E-04	4.070827E-04
.2	4.076601E-04	4.065019E-04	4.069458E-04	4.069921E-04
.3	4.076150E-04	4.063725E-04	4.068937E-04	4.069003E-04
.4	4.075636E-04	4.062242E-04	4.068626E-04	4.067432E-04
.5	4.075042E-04	4.059689E-04	4.067344E-04	4.066651E-04
.6	4.074408E-04	4.053971E-04	4.066827E-04	4.066200E-04
.7	See Note 1	4.052982E-04	See Note 1	4.063957E-04
.8	See Note 1	4.046979E-04	See Note 1	4.062411E-04

Max M_x or M_y bending moment: theoretical = 9.577E-03 lb-in/in

Random Grid Disprder	QUAD4 (MIN4) (AQ3R1S010)	TRIA3 (AT3R1S010)	QUAD4K (AQ2R1S010)	TRIA3K (AT2R1S010)
.0	-9.516958E-03	-9.538238E-03	-9.513549E-03	-9.514432E-03
.1	-9.508684E-03	-9.531171E-03	-9.511662E-03	-9.508362E-03
.2	-9.513518E-03	-9.540563E-03	-9.497620E-03	-9.497151E-03
.3	-9.498255E-03	-9.528532E-03	-9.509454E-03	-9.505900E-03
.4	-9.508094E-03	-9.518584E-03	-9.488904E-03	-9.480915E-03
.5	-9.475692E-03	-9.503319E-03	-9.471925E-03	-9.481681E-03
.6	-9.510552E-03	-9.467728E-03	-9.462858E-03	-9.453775E-03
.7	See Note 1	-9.503531E-03	See Note 1	-9.486368E-03
.8	See Note 1	-9.454101E-03	See Note 1	-9.485915E-03

Max M_{xy} twisting moment: theoretical = -6.496E-03 lb-in/in

Random Grid Disprder	QUAD4 (MIN4) (AQ3R1S010)	TRIA3 (AT3R1S010)	QUAD4K (AQ2R1S010)	TRIA3K (AT2R1S010)
.0	-6.350917E-03	-6.381598E-03	-6.360069E-03	-6.360019E-03
.1	-6.355576E-03	-6.378455E-03	-6.361073E-03	-6.361410E-03
.2	-6.350873E-03	-6.390043E-03	-6.370030E-03	-6.377284E-03
.3	-6.308518E-03	-6.420099E-03	-6.359695E-03	-6.375967E-03
.4	-6.350845E-03	-6.463871E-03	-6.340293E-03	-6.381823E-03
.5	-6.370382E-03	-6.409916E-03	-6.366337E-03	-6.401488E-03
.6	-6.220156E-03	-6.496164E-03	-6.357278E-03	-6.390720E-03
.7	See Note 1	-6.462622E-03	See Note 1	-6.371801E-03
.8	See Note 1	-6.719344E-03	See Note 1	-6.332287E-03

MaxV_x or V_y transverse shear force: theoretical = -3.375E-02 lb/in

Random Grid Disprder	QUAD4 (MIN4) (AQ2R1S010)	TRIA3 (AT3R1S010)
.0	-3.125650E-02	-3.230785E-02
.1	-3.137727E-02	-3.249074E-02
.2	-3.131193E-02	-3.328399E-02
.3	-3.227968E-02	-3.352161E-02
.4	-3.253596E-02	-3.592427E-02
.5	-3.573537E-02	-3.431050E-02
.6	-3.739516E-02	-3.296402E-02
.7	See Note 1	-3.407969E-02
.8	See Note 1	-3.609102E-02

Note 1: Random disorder of internal grids is so large as to cause one or more elements to have a fatal error (e.g. interior angle > 180 deg, element not numbered CW or CCW, or negative area)

Appendix B-2

Simply supported square plate. 1/4 sym. Plate overall dimensions 2" X 2"
QUAD4 is MIN4T element

Plate thickness 0.20” (thick)

Table B2-1

Convergence study - increasing mesh size
Thick plate: 0.20” using MIN4T quad elements
Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.294-07 in

Mesh	QUAD4 (MIM4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)	QUAD4K (AQ2US2xxx)	TRIA3K (AT2US2xxx)
2	4.395387E-07	4.146006E-07	4.053825E-04	4.064101E-07
5	4.300534E-07	4.270713E-07	4.069221E-04	4.072906E-07
10	4.286389E-07	4.280076E-07	4.070296E-04	4.071537E-07
20	4.282798E-07	4.281360E-07	4.070529E-04	4.070879E-07
50	4.281788E-07	4.281575E-07	4.070591E-04	4.070651E-07
100	4.281643E-07	4.281593E-07	4.070600E-04	4.070615E-07
200		4.281595E-07	4.070602E-07	4.070606E-07

Max M_x or M_y bending moment: theoretical = 9.607E-03 lb-in/in

Mesh	QUAD4 (MIN4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)	QUAD4K (AQ2US2xxx)	TRIA3K (AT2US2xxx)
2	-8.074155E-03	-8.286672E-03	-7.932362E-03	-7.980317E-03
5	-9.340636E-03	-9.384681E-03	-9.316309E-03	-9.326607E-03
10	-9.520004E-03	-9.522622E-03	-9.513549E-03	-9.514432E-03
20	-9.564479E-03	-9.562965E-03	-9.562834E-03	-9.562405E-03
50	-9.576896E-03	-9.576334E-03	-9.576632E-03	-9.576411E-03
100	-9.578669E-03	-9.578481E-03	-9.578603E-03	-9.578518E-03
200		-9.579054E-03	-9.579094E-03	-9.579056E-03

Max M_{xy} twisting moment: theoretical = -6.225E-03 lb-in/in

Mesh	QUAD4 (MIN4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)	QUAD4K (AQ2US2xxx)	TRIA3K (AT2US2xxx)
2	-4.668448E-03	-4.500427E-03	-4.823450E-03	-4.763773E-03
5	-6.052187E-03	-6.089201E-03	-6.075395E-03	-6.067006E-03
10	-6.354222E-03	-6.379003E-03	-6.360069E-03	-6.360019E-03
20	-6.451367E-03	-6.461321E-03	-6.453200E-03	-6.454149E-03
50	-6.485563E-03	-6.487857E-03	-6.485967E-03	-6.486422E-03
100	-6.491641E-03	-6.492338E-03	-6.491745E-03	-6.491934E-03
200		-6.493610E-03	-6.493411E-03	-6.493481E-03

MaxV_x or V_y transverse shear force: theoretical = -3.374E-02 lb/in

Mesh	QUAD4 (MIN4) (AQ3US2xxx)	TRIA3 (AT3US2xxx)
2	-2.112100E-02	-2.338886E-02
5	-2.870941E-02	-2.916144E-02
10	-3.125004E-02	-3.133337E-02
20	-3.251203E-02	-3.252939E-02
50	-3.326553E-02	-3.327051E-02
100	-3.351604E-02	-3.351810E-02
200		-3.364134E-02

Table B2-2

Random disorder of element shape - mesh size is 10 x 10
Thick plate: 0.20" using MIN4T quad elements
Uniform pressure load 0.05 psi

Central point lateral displacement: theoretical = 4.294E-07 in

Random Grid Disprder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)	QUAD4K (AQ2R2S010)	TRIA3K (AR2R2S010)
.0	4.286389E-07	4.280076E-07	4.070296E-04	4.071537E-07
.1	4.286363E-07	4.279977E-07	4.069889E-07	4.070835E-07
.2	4.286255E-07	4.279538E-07	4.069567E-07	4.069980E-07
.3	4.286026E-07	4.278728E-07	4.069297E-07	4.069015E-07
.4	4.285964E-07	4.277729E-07	4.068571E-07	4.067849E-07
.5	4.285648E-07	4.277427E-07	4.066887E-07	4.067714E-07
.6	4.285039E-07	4.274602E-07	See Note 1	4.066252E-07
.7	4.284351E-07	4.272412E-07	4.063414E-07	4.063621E-07
.8	See Note 1	4.273250E-07	See Note 1	4.062954E-07

Max M_x or M_y bending moment: theoretical = 9.607E-03 lb-in/in

Random Grid Disprder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)	QUAD4K (AQ2R2S010)	TRIA3K (AT2R2S010)
.0	-9.520004E-03	-9.522622E-03	-9.513549E-03	-9.514432E-03
.1	-9.516887E-03	-9.516944E-03	-9.508573E-03	-9.509657E-03
.2	-9.515005E-03	-9.513496E-03	-9.500159E-03	-9.497733E-03
.3	-9.510936E-03	-9.510518E-03	-9.497190E-03	-9.498315E-03
.4	-9.503080E-03	-9.502445E-03	-9.492558E-03	-9.484133E-03
.5	-9.495133E-03	-9.499946E-03	-9.468693E-03	-9.478471E-03
.6	-9.485838E-03	-9.482645E-03	See Note 1	-9.475569E-03
.7	-9.523367E-03	-9.482962E-03	-9.479518E-03	-9.450705E-03
.8	See Note 1	-9.488444E-03	See Note 1	-9.472364E-03

Max M_{xy} twisting moment: theoretical = -6.225E-03 lb-in/in

Random Grid Disprder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)	QUAD4K (AQ2R2S010)	TRIA3K (AT2R2S010)
.0	-6.354222E-03	-6.379003E-03	-6.360069E-03	-6.360019E-03
.1	-6.358936E-03	-6.386316E-03	-6.363487E-03	-6.360086E-03
.2	-6.377200E-03	-6.398629E-03	-6.367205E-03	-6.367530E-03
.3	-6.335701E-03	-6.390431E-03	-6.347635E-03	-6.389979E-03
.4	-6.359404E-03	-6.433151E-03	-6.361833E-03	-6.356812E-03
.5	-6.337010E-03	-6.424353E-03	-6.375497E-03	-6.376842E-03
.6	-6.323262E-03	-6.421069E-03	See Note 1	-6.386909E-03
.7	-6.361620E-03	-6.419936E-03	-6.329188E-03	-6.416808E-03
.8	See Note 1	-6.458207E-03	See Note 1	-6.414755E-03

MaxV_x or V_y transverse shear force: theoretical = -3.374E-02 lb/in

Random Grid Disprder	QUAD4 (MIN4) (AQ3R2S010)	TRIA3 (AT3R2S010)
.0	-3.125004E-02	-3.133337E-02
.1	-3.131184E-02	-3.132976E-02
.2	-3.129734E-02	-3.136309E-02
.3	-3.129752E-02	-3.139836E-02
.4	-3.133671E-02	-3.125302E-02
.5	-3.128780E-02	-3.130338E-02
.6	-3.120514E-02	-3.098793E-02
.7	-3.094243E-02	-3.154259E-02
.8	See Note 1	-3.127497E-02

Note 1: Random disorder of internal grids is so large as to cause one or more elements to have a fatal error (e.g. interior angle > 180 deg, element not numbered CW or CCW, or negative area)

Appendix B-3

Square plate supported only at the central point. 1/4 sym.
Plate overall dimensions 2" X 2"
QUAD4 is MIN4T element

Plate thickness 0.02” (thin)

Table B3-1

Convergence study - increasing mesh size
Thin plate: 0.02” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.E-05 in

Mesh	QUAD4 (MIM4T) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
10	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
20	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
50	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
100	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
200		2.000000E-05	2.000000E-05	2.000000E-05

Max stress: theoretical = 0 lb/in^2

Mesh	QUAD4 (MIN4T) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	E-13	E-13	E-13	E-13
5	E-13	E-12	E-12	E-12
10	E-12	E-12	E-12	E-12
20	E-12	E-12	E-12	E-12
50	E-11	E-11	E-11	E-11
100	E-10	E-10	E-10	E-10
200		E-10	E-09	E-10

Table B3-2

Random disorder of element shape - mesh size is 10 x 10
Thin plate: 0.02” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.E-05 in

Random Grid Disprder	QUAD4 (MIN4) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.1	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.3	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.4	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.6	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.7	2.000000E-05	2.000000E-05	See Note 1	2.000000E-05
.8	See Note 1	2.000000E-05	See Note 1	2.000000E-05

Max stress: theoretical = 0 lb/in^2

Random Grid Disprder	QUAD4 (MIN4) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	E-12	E-12	E-12	E-12
.1	E-12	E-12	E-12	E-12
.2	E-12	E-12	E-12	E-12
.3	E-12	E-12	E-12	E-12
.4	E-12	E-12	E-12	E-12
.5	E-12	E-12	E-12	E-12
.6	E-12	E-12	E-11	E-12
.7	E-12	E-12	See Note 1	E-12
.8	See Note 1	E-12	See Note 1	E-11

Note 1: Random disorder of internal grids is so large as to cause one or more elements to have a fatal error (e.g. interior angle > 180 deg, element not numbered CW or CCW, or negative area)

Table B3-3

Convergence study - increasing mesh size
Thin plate: 0.02” using MIN4T quad elements
Stress free thermal expansion – 1.0- deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.E-05 in

Mesh	QUAD4 (MIM4T) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
5	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
10	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
20	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
50	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
100	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
200		-2.000000E-05	-2.000000E-05	-1.999998E-05

Max stress: theoretical = 0 lb/in^2

Mesh	QUAD4 (MIN4T) (BQ3U1Fxxx)	TRIA3 (BT3U1Fxxx)	QUAD4K (BQ2U1Fxxx)	TRIA3K (BT2U1Fxxx)
2	E-13	E-13	E-14	E-14
5	E-12	E-12	E-12	E-13
10	E-11	E-11	E-11	E-12
20	E-10	E-11	E-09	E-09
50	E-09	E-08	E-08	E-08
100	E-07	E-09	E-08	E-06
200		E-08	E-06	E-05

Table B3-4

Random disorder of element shape - mesh size is 10 x 10
Thin plate: 0.02” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.E-05 in

Random Grid Disprder	QUAD4 (MIN4T) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.1	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.2	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.3	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.4	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.5	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.6	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.7	-2.000000E-05	-2.000000E-05	See Note 1	-2.000000E-05
.8	See Note 1	-2.000000E-05	See Note 1	-1.998677E-05

Max stress: theoretical = 0 lb/in^2

Random Grid Disprder	QUAD4 (MIN4T) (BQ3R1F010)	TRIA3 (BT3R1F010)	QUAD4K (BQ2R1F010)	TRIA3K (BT2R1F010)
.0	E-11	E-11	E-11	E-12
.1	E-12	E-11	E-11	E-11
.2	E-12	E-11	E-11	E-11
.3	E-11	E-11	E-11	E-11
.4	E-11	E-11	E-11	E-12
.5	E-11	E-11	E-11	E-11
.6	E-10	E-11	E-11	E-11
.7	E-11	E-11	See Note 1	E-11
.8	See Note 1	E-11	See Note 1	E-02

Note 1: Random disorder of internal grids is so large as to cause one or more elements to have a fatal error (e.g. interior angle > 180 deg, element not numbered CW or CCW, or negative area)

Appendix B-4

Square plate supported only at the central point. 1/4 sym.
Plate overall dimensions 2" X 2"
QUAD4 is MIN4T element

Plate thickness 0.20” (thick)

Table B4-1

Convergence study - increasing mesh size
Thick plate: 0.20” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.E-05 in

Mesh	QUAD4 (MIM4T) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
10	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
20	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
50	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
100	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
200		2.000000E-05	2.000000E-05	2.000000E-05

Max stress: theoretical = 0 lb/in^2

Mesh	QUAD4 (MIN4T) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	E-13	E-13	E-13	E-13
5	E-12	E-12	E-12	E-12
10	E-11	E-12	E-12	E-12
20	E-11	E-11	E-12	E-11
50	E-10	E-10	E-10	E-10
100	E-10	E-10	E-10	E-10
200		E-09	E-09	E-09

Table B4-2

Random disorder of element shape - mesh size is 10 x 10
Thick plate: 0.20” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg bulk temperature change

Central point lateral displacement: theoretical = 2.E-05 in

Random Grid Disprder	QUAD4 (MIN4T) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.1	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.2	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.3	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.4	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.5	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.6	2.000000E-05	2.000000E-05	2.000000E-05	2.000000E-05
.7	2.000000E-05	2.000000E-05	See Note 1	2.000000E-05
.8	See Note 1	2.000000E-05	See Note 1	2.000000E-05

Max stress: theoretical = 0 lb/in^2

Random Grid Disprder	QUAD4 (MIN4T) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	E-11	E-12	E-12	E-12
.1	E-12	E-12	E-12	E-12
.2	E-12	E-12	E-12	E-12
.3	E-12	E-12	E-12	E-12
.4	E-12	E-12	E-12	E-12
.5	E-12	E-12	E-12	E-12
.6	E-12	E-12	E-12	E-12
.7	E-12	E-12	See Note 1	E-12
.8	See Note 1	E-12	See Note 1	E-11

Note 1: Random disorder of internal grids is so large as to cause one or more elements to have a fatal error (e.g. interior angle > 180 deg, element not numbered CW or CCW, or negative area)

Table B4-3

Convergence study - increasing mesh size
Thick plate: 0.20” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.E-05 in

Mesh	QUAD4 (MIM4T) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
5	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
10	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
20	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
50	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
100	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
200		-2.000000E-05	-2.000000E-05	-1.999998E-05

Max stress: theoretical = 0 lb/in^2

Mesh	QUAD4 (MIN4T) (BQ3U2Fxxx)	TRIA3 (BT3U2Fxxx)	QUAD4K (BQ2U2Fxxx)	TRIA3K (BT2U2Fxxx)
2	E-12	E-13	E-13	E-13
5	E-12	E-12	E-11	E-11
10	E-11	E-11	E-10	E-11
20	E-11	E-10	E-09	E-09
50	E-10	E-10	E-07	E-07
100	E-10	E-10	E-06	E-06
200		E-09	E-04	E-05

Table B4-4

Random disorder of element shape - mesh size is 10 x 10
Thick plate: 0.20” using MIN4T quad elements
Stress free thermal expansion – 1.0 deg temperature gradient through thickness

Central point lateral displacement: theoretical = -2.E-05 in

Random Grid Disprder	QUAD4 (MIN4T) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.1	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.2	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.3	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.4	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.5	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.6	-2.000000E-05	-2.000000E-05	-2.000000E-05	-2.000000E-05
.7	-2.000000E-05	-2.000000E-05	See Note 1	-2.000000E-05
.8	See Note 1	-2.000000E-05	See Note 1	-2.000000E-05

Max stress: theoretical = 0 lb/in^2

Random Grid Disprder	QUAD4 (MIN4T) (BQ3R2F010)	TRIA3 (BT3R2F010)	QUAD4K (BQ2R2F010)	TRIA3K (BT2R2F010)
.0	E-11	E-11	E-10	E-11
.1	E-11	E-11	E-10	E-10
.2	E-11	E-11	E-10	E-10
.3	E-11	E-12	E-10	E-11
.4	E-11	E-12	E-10	E-10
.5	E-11	E-11	E-10	E-10
.6	E-11	E-11	E-10	E-10
.7	E-09	E-11	See Note 1	E-10
.8	See Note 1	E-11	See Note 1	E-10

Note 1: Random disorder of internal grids is so large as to cause one or more elements to have a fatal error (e.g. interior angle > 180 deg, element not numbered CW or CCW, or negative area)