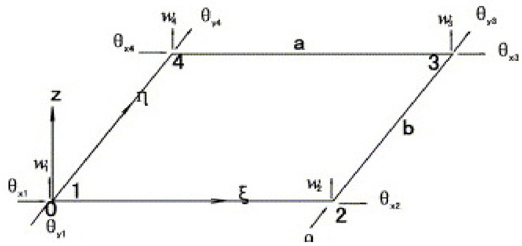
QUAD4 Reissner-Mindlin Plate Finite element Formulation



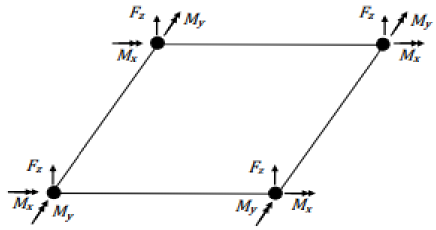


plate element nodal displacement matrix (degrees of freedom)

$$[d]=\begin{bmatrix} w_1 \\ \theta_{x1} \\ \theta_{y1} \\ w_2 \\ \theta_{x1} \\ \theta_{y1} \\ w_3 \\ \theta_{x1} \\ \theta_{y1} \\ w_4 \\ \theta_{x1} \\ \theta_{y1} \end{bmatrix}$$

$$u = u_b + u_s, \qquad u_b = -z \cdot \theta_x, \quad u_s = -z \cdot \phi_x$$
$$u = -z \cdot \theta_x - z \cdot \phi_x$$
$$u = -z \cdot (\theta_x + \phi_x), \qquad \beta_x = \theta_x + \phi_x$$
$$u = -z \cdot \beta_x$$

plate element displacements and rotations

$$\begin{bmatrix} u \\ v \\ w \\ \theta_x \\ \theta_y \\ \theta_z = 0 \end{bmatrix} = \begin{bmatrix} -z \cdot \theta_x \\ -z \cdot \theta_y \\ w \\ \theta_x \\ \theta_y \\ 0 \end{bmatrix}$$

plate element deformations due bending and shear

$$\begin{bmatrix} \varepsilon_b \\ \gamma_s \end{bmatrix} = [D] \cdot [d_b]$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} = 0 \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{bmatrix}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(-z \cdot \theta_x - z \cdot \phi_x) = -z \cdot \frac{\partial \theta_x}{\partial x}$$
$$\frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(-z \cdot \theta_y - z \cdot \phi_y) = -z \cdot \frac{\partial \theta_y}{\partial y}$$
$$\frac{\partial w}{\partial z} = 0$$
$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial}{\partial y}(-z \cdot \theta_x - z \cdot \phi_x) + \frac{\partial}{\partial x}(-z \cdot \theta_y - z \cdot \phi_y) = -z \cdot \frac{\partial \theta_x}{\partial y} - z \cdot \frac{\partial \theta_y}{\partial x} = -z \cdot \left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right)$$
$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}(-z \cdot \theta_x - z \cdot \phi_x) + \frac{\partial w}{\partial x} = -\theta_x + \frac{\partial w}{\partial x}$$
$$\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial}{\partial z}(-z \cdot \theta_y - z \cdot \phi_y) + \frac{\partial w}{\partial y} = -\theta_y + \frac{\partial w}{\partial y}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} -z \cdot \dfrac{\partial \theta_x}{\partial x} \\ -z \cdot \dfrac{\partial \theta_y}{\partial y} \\ 0 \\ -z \cdot \left(\dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \right) \\ -\theta_x + \dfrac{\partial w}{\partial x} \\ -\theta_y + \dfrac{\partial w}{\partial y} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \begin{bmatrix} z \cdot k_x \\ z \cdot k_y \\ 0 \\ z \cdot k_{xy} \\ -\phi_x \\ -\phi_y \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}_{PSEUDO} = \begin{bmatrix} k_x \\ k_y \\ 0 \\ k_{xy} \\ -\phi_x \\ -\phi_y \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}_{PSEUDO} = \begin{bmatrix} -\dfrac{\partial \theta_x}{\partial x} \\ -\dfrac{\partial \theta_y}{\partial y} \\ 0 \\ -\left(\dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \right) \\ -\theta_x + \dfrac{\partial w}{\partial x} \\ -\theta_y + \dfrac{\partial w}{\partial y} \end{bmatrix}}$$

$$N_1(\xi,\eta) = \frac{1}{4} \cdot (1-\xi) \cdot (1-\eta)$$

$$N_2(\xi,\eta) = \frac{1}{4} \cdot (1+\xi) \cdot (1-\eta)$$

$$N_3(\xi,\eta) = \frac{1}{4} \cdot (1+\xi) \cdot (1+\eta)$$

$$N_4(\xi,\eta) = \frac{1}{4} \cdot (1-\xi) \cdot (1+\eta)$$

$$w = \sum_{i=1}^{4} N_i \cdot w_i$$

$$\theta_x = \sum_{i=1}^{4} N_i \cdot \theta_{xi}$$

$$\theta_{yx} = \sum_{i=1}^{4} N_i \cdot \theta_{yi}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}_{PSEUDO} = \begin{bmatrix} -\sum_{i=1}^{4} \frac{\partial}{\partial x}\left(N_i \cdot \theta_{xi}\right) \\ -\sum_{i=1}^{4} \frac{\partial}{\partial y}\left(N_i \cdot \theta_{yi}\right) \\ 0 \\ -\sum_{i=1}^{4}\left(\frac{\partial}{\partial y}\left(N_i \cdot \theta_{xi}\right) + \frac{\partial}{\partial x}\left(N_i \cdot \theta_{yi}\right)\right) \\ \sum_{i=1}^{4}\left(-N_i \cdot \theta_{xi} + \frac{\partial}{\partial x}\left(N_i \cdot w_i\right)\right) \\ \sum_{i=1}^{4}\left(-N_i \cdot \theta_{yi} + \frac{\partial}{\partial y}\left(N_i \cdot w_i\right)\right) \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_y = 0 \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}_{PSEUDO} = \sum_{i=1}^{4} \begin{bmatrix} 0 & -\frac{\partial N_i}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_i}{\partial y} \\ 0 & -\frac{\partial N_i}{\partial y} & -\frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial x} & -N_i & 0 \\ \frac{\partial N_i}{\partial y} & 0 & -N_i \end{bmatrix} \cdot \begin{bmatrix} w_i \\ \theta_{xi} \\ \theta_{yi} \end{bmatrix}$$

$$[\varepsilon]_{PSEUDO} = \sum_{i=1}^{4} \begin{bmatrix} B_{Bi} \\ B_{Si} \end{bmatrix} \cdot [d]_i$$

$$[B]_{4x(5x3)=5x12} = \left[\begin{vmatrix} 0 & -\frac{\partial N_1}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_1}{\partial y} \\ 0 & -\frac{\partial N_1}{\partial y} & -\frac{\partial N_1}{\partial x} \\ \frac{\partial N_1}{\partial x} & -N_1 & 0 \\ \frac{\partial N_1}{\partial y} & 0 & -N_1 \end{vmatrix} \begin{vmatrix} 0 & -\frac{\partial N_2}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_2}{\partial y} \\ 0 & -\frac{\partial N_2}{\partial y} & -\frac{\partial N_2}{\partial x} \\ \frac{\partial N_2}{\partial x} & -N_2 & 0 \\ \frac{\partial N_2}{\partial y} & 0 & -N_2 \end{vmatrix} \begin{vmatrix} 0 & -\frac{\partial N_3}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_3}{\partial y} \\ 0 & -\frac{\partial N_3}{\partial y} & -\frac{\partial N_3}{\partial x} \\ \frac{\partial N_3}{\partial x} & -N_i & 0 \\ \frac{\partial N_3}{\partial y} & 0 & -N_3 \end{vmatrix} \begin{vmatrix} 0 & -\frac{\partial N_4}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_4}{\partial y} \\ 0 & -\frac{\partial N_4}{\partial y} & -\frac{\partial N_4}{\partial x} \\ \frac{\partial N_4}{\partial x} & -N_4 & 0 \\ \frac{\partial N_4}{\partial y} & 0 & -N_4 \end{vmatrix} \right]$$

In natural coordinate system

$$[B]_{4x(5x3)=5x12} = \left[\begin{vmatrix} 0 & -\dfrac{\partial N_1}{\partial x} & 0 \\ 0 & 0 & -\dfrac{\partial N_1}{\partial y} \\ 0 & -\dfrac{\partial N_1}{\partial y} & -\dfrac{\partial N_1}{\partial x} \\ \dfrac{\partial N_1}{\partial x} & -N_1 & 0 \\ \dfrac{\partial N_1}{\partial y} & 0 & -N_1 \end{vmatrix} \quad |2| \quad |3| \quad |4| \right]$$

using chain rule:

$$[B]_{4x(5x3)=5x12} = \left[\begin{vmatrix} 0 & -\left(\dfrac{\partial N_1}{\partial \xi}\cdot\dfrac{\partial \xi}{\partial x}+\dfrac{\partial N_1}{\partial \eta}\cdot\dfrac{\partial \eta}{\partial x}\right) & 0 \\ 0 & 0 & -\left(\dfrac{\partial N_1}{\partial \xi}\cdot\dfrac{\partial \xi}{\partial y}+\dfrac{\partial N_1}{\partial \eta}\cdot\dfrac{\partial \eta}{\partial y}\right) \\ 0 & -\left(\dfrac{\partial N_1}{\partial \xi}\cdot\dfrac{\partial \xi}{\partial y}+\dfrac{\partial N_1}{\partial \eta}\cdot\dfrac{\partial \eta}{\partial y}\right) & -\left(\dfrac{\partial N_1}{\partial \xi}\cdot\dfrac{\partial \xi}{\partial x}+\dfrac{\partial N_1}{\partial \eta}\cdot\dfrac{\partial \eta}{\partial x}\right) \\ \left(\dfrac{\partial N_1}{\partial \xi}\cdot\dfrac{\partial \xi}{\partial x}+\dfrac{\partial N_1}{\partial \eta}\cdot\dfrac{\partial \eta}{\partial x}\right) & -N_1 & 0 \\ \left(\dfrac{\partial N_1}{\partial \xi}\cdot\dfrac{\partial \xi}{\partial y}+\dfrac{\partial N_1}{\partial \eta}\cdot\dfrac{\partial \eta}{\partial y}\right) & 0 & -N_1 \end{vmatrix} \quad |2| \quad |3| \quad |4| \right]$$

Jacobian matrix

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial X}{\partial \xi} & \dfrac{\partial Y}{\partial \xi} \\ \dfrac{\partial X}{\partial \eta} & \dfrac{\partial Y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}\left(\dfrac{\partial N_i}{\partial \xi}\right)\cdot x_i & \sum_{i=1}^{n}\left(\dfrac{\partial N_i}{\partial \xi}\right)\cdot y_i \\ \sum_{i=1}^{n}\left(\dfrac{\partial N_i}{\partial \eta}\right)\cdot x_i & \sum_{i=1}^{n}\left(\dfrac{\partial N_i}{\partial \eta}\right)\cdot y_i \end{bmatrix}, \qquad \det|J| = J_{11}\cdot J_{22} - J_{12}\cdot J_{21}$$

$$[J]^{-1} = \begin{bmatrix} \dfrac{\partial \xi}{\partial X} & \dfrac{\partial \eta}{\partial X} \\ \dfrac{\partial \xi}{\partial Y} & \dfrac{\partial \eta}{\partial Y} \end{bmatrix} = \frac{1}{\det|J|}\cdot\begin{bmatrix} \dfrac{\partial y}{\partial \eta} & -\dfrac{\partial y}{\partial \xi} \\ -\dfrac{\partial x}{\partial \eta} & \dfrac{\partial x}{\partial \xi} \end{bmatrix} = \frac{1}{\det|J|}\cdot\begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix}$$

$$\frac{\partial \xi}{\partial X} = \frac{1}{\det|J|}\cdot\frac{\partial y}{\partial \eta} = \frac{1}{\det|J|}\cdot J_{22} = J_{11}^{-1}$$

$$\frac{\partial \eta}{\partial X} = -\frac{1}{\det|J|}\cdot\frac{\partial y}{\partial \xi} = -\frac{1}{\det|J|}\cdot J_{12} = J_{12}^{-1}$$

$$\frac{\partial \xi}{\partial Y} = -\frac{1}{\det|J|}\cdot\frac{\partial x}{\partial \eta} = -\frac{1}{\det|J|}\cdot J_{21} = J_{21}^{-1}$$

$$\frac{\partial \eta}{\partial Y} = \frac{1}{\det|J|}\cdot\frac{\partial x}{\partial \xi} = \frac{1}{\det|J|}\cdot J_{11} = J_{22}^{-1}$$

$$
[B]_{4x(5x3)=5x12} = \left[\begin{array}{c|c|c|c} \left| \begin{matrix} 0 & -\left(\frac{\partial N_1}{\partial \xi}\cdot J_{11}^{-1} + \frac{\partial N_1}{\partial \eta}\cdot J_{12}^{-1}\right) & 0 \\ 0 & 0 & -\left(\frac{\partial N_1}{\partial \xi}\cdot J_{21}^{-1} + \frac{\partial N_1}{\partial \eta}\cdot J_{22}^{-1}\right) \\ 0 & -\left(\frac{\partial N_1}{\partial \xi}\cdot J_{21}^{-1} + \frac{\partial N_1}{\partial \eta}\cdot J_{22}^{-1}\right) & -\left(\frac{\partial N_1}{\partial \xi}\cdot J_{11}^{-1} + \frac{\partial N_1}{\partial \eta}\cdot J_{12}^{-1}\right) \\ \left(\frac{\partial N_1}{\partial \xi}\cdot J_{11}^{-1} + \frac{\partial N_1}{\partial \eta}\cdot J_{12}^{-1}\right) & -N_1 & 0 \\ \left(\frac{\partial N_1}{\partial \xi}\cdot J_{21}^{-1} + \frac{\partial N_1}{\partial \eta}\cdot J_{22}^{-1}\right) & 0 & -N_1 \end{matrix} \right| & 2 & 3 & 4 \end{array} \right]
$$

constitutive matrices

$$
[E]_b = \frac{E\cdot t^3}{12\cdot\left(1-\nu^2\right)}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}
$$

$$
[E]_s = \frac{\left(K_s = 0.833\right)\cdot E\cdot t}{2\cdot\left(1+\nu\right)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = K_S\cdot G\cdot t\cdot\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

$$
[E]_{5x5} = \begin{bmatrix} \frac{E\cdot t^3}{12\cdot\left(1-\nu^2\right)}\cdot\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \frac{K_s\cdot E\cdot t}{2\cdot\left(1+\nu\right)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}
$$

stiffness matrix

$$[K]_{12x12} = \int_V [B]^T_{12x5} \cdot [E]_{5x5} \cdot [B]_{5x12} \cdot dV$$

$$[K]_{12x12} = \int_A [B]^T_{12x5} \cdot [E]_{5x5} \cdot [B]_{5x12} \cdot t \cdot dA$$

$$[K]_{12x12} = \int_A [B]^T_{12x5} \cdot [E]_{5x5} \cdot [B]_{5x12} \cdot t \cdot dx \cdot dy$$

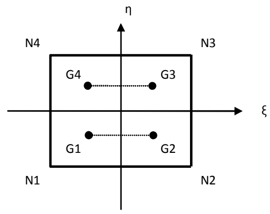
$$[K]_{12x12} = \int_{-1}^{1}\int_{-1}^{1} [B]^T_{12x5} \cdot [E]_{5x5} \cdot [B]_{5x12} \cdot \det[J] \cdot d\xi \cdot d\eta$$

$$[K]_{12x12} = [B]^T_{12x5} \cdot [E]_{5x5} \cdot [B]_{5x12} \cdot \det[J] \cdot w_i \cdot w_j$$

ξ=±0.5773502691896257
η=±0.5773502691896257
wg1=1.0
wg2=1.0

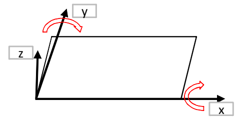
ξ=0.0
η=0.0
wg1=2.0

QUAD4 Reissner-Mindlin Plate Bending



x1= -0.5
y1= -0.5
x2= 0.5
y2= -0.5
x3= 0.5
y3= 0.5
x4= -0.5
y4= 0.5

coordinates of element nodes in <u>element coordinate system</u>



det|J|=(x2-x1)*(y4-y1)-(y2-y1)*(x4-x1)= 1.00

*perfect element is when det|J|=1

det|J|max= 0.250

det|J|min= 0.250

J.R.=det|J|max/det|J|min= 1.00 Jacobian Ratio

*perfect element is when J.R.=1

t= 0.01 thickness

E= 70,000 modulus of elasticity

v= 0.3 poisson's coefficient

Gauss Point 1

ξ= -0.577 gaus point coordinates in natural coordinate system

η= -0.577

wi= 1.000 gaus weights

wj= 1.000

A=0.5*(x1*y2-y1*x2)+(x2*y3-y2*x3)+(x3*y4-y3*x4)+(x4*y1-y4*x1)= 1 area of element

N1=0.25*(1-ξ)*(1-η)= 0.622
N2=0.25*(1+ξ)*(1-η)= 0.167
N3=0.25*(1+ξ)*(1+η)= 0.045
N4=0.25*(1-ξ)*(1+η)= 0.167

dN1/dξ=-0.25*(1-η)= -0.394 derivatives of element shape functions
dN2/dξ=0.25*(1-η)= 0.394
dN3/dξ=0.25*(1+η)= 0.106
dN4/dξ=-0.25*(1-η)= -0.106

dN1/dη=-0.25*(1-ξ)= -0.394
dN2/dη=-0.25*(1+ξ)= -0.106
dN3/dη=0.25*(1+ξ)= 0.106
dN4/dη=0.25*(1-ξ)= 0.394

J11=ΣdNi/dξi*xi= 0.500 coefficients of jacobian matrix
J12=ΣdNi/dξi*yi= 0.000
J21=ΣdNi/dηi*xi= 0.000
J22=ΣdNi/dηi*yi= 0.500

det|J|=J11*J22-J12*J21= 0.250 jacobian matrix determinant

J11-1=1/det|J|*J22= 2.000 coefficients of inverted jacobian matrix
J12-1=-1/det|J|*J12= 0.000
J21-1=-1/det|J|*J21= 0.000
J22-1=1/det|J|*J11= 2.000

[J]=	0.500	0.000	check using excel minverse()
	0.000	0.500	
[J]-1=	2.000	0.000	
	0.000	2.000	

[B]=

0	0.789	0	0	-0.789	0	0	-0.211	0	0	0.211	0
0	0	0.789	0	0	0.211	0	0	-0.211	0	0	0.000
0	0.789	0.789	0	0.000	0.000	0	-0.211	-0.211	0	0.000	0.211
-0.789	-0.622	0	0.789	-0.167	0	0.211	-0.045	0	-0.211	-0.167	0
-0.789	0	-0.622	-0.211	0	-0.167	0.211	0	-0.045	0.789	0	-0.167

[E]b=

0.006	0.002	0	0	0
0.002	0.006	0	0	0
0	0	0.000	0	0
0	0	0	0	0
0	0	0	0	0

Bending

[B]t=

0.000	0.000	0.000	-0.789	-0.789
0.789	0.000	0.789	-0.622	0.000
0.000	0.789	0.789	0.000	-0.622
0.000	0.000	0.000	0.789	-0.211
-0.789	0.000	0.000	-0.167	0.000
0.000	0.211	0.000	0.000	-0.167
0.000	0.000	0.000	0.211	0.211
-0.211	0.000	-0.211	-0.045	0.000
0.000	-0.211	-0.211	0.000	-0.045
0.000	0.000	0.000	-0.211	0.789
0.211	0.000	0.000	-0.167	0.000
0.000	0.000	0.211	0.000	-0.167

det[J]= 0.250

[E]*[B]=

0	0.0050556	0.0015167	0	-0.005056	0.0004064	0	-0.001355	-0.000406	0	0.0013546	0
0	0.0015167	0.0050556	0	-0.001517	0.0013546	0	-0.000406	-0.001355	0	0.0004064	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

[B]t*[E]*[B]=

0	0	0	0	0	0	0	0	0	0	0	0
0	0.0039872	0.0011962	0	-0.003987	0.0003205	0	-0.001068	-0.000321	0	0.0010684	0
0	0.0011962	0.0039872	0	-0.001196	0.0010684	0	-0.000321	-0.001068	0	0.0003205	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.003987	-0.001196	0	0.0039872	-0.000321	0	0.0010684	0.0003205	0	-0.001068	0
0	0.0003205	0.0010684	0	-0.000321	0.0002863	0	-8.59E-05	-0.000286	0	8.588E-05	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.001068	-0.000321	0	0.0010684	-8.59E-05	0	0.0002863	8.588E-05	0	-0.000286	0
0	-0.000321	-0.001068	0	0.0003205	-0.000286	0	8.588E-05	0.0002863	0	-8.59E-05	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0.0010684	0.0003205	0	-0.001068	8.588E-05	0	-0.000286	-8.59E-05	0	0.0002863	0
0	0	0	0	0	0	0	0	0	0	0	0

stiffness matrix in gaus point 1

[K]Gi=w1*w2*[B]t*[E]*[B]*det|J|=

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00100	0.00030	0.00000	-0.00100	0.00008	0.00000	-0.00027	-0.00008	0.00000	0.00027	0.00000
0.00000	0.00030	0.00100	0.00000	-0.00030	0.00027	0.00000	-0.00008	-0.00027	0.00000	0.00008	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00100	-0.00030	0.00000	0.00100	-0.00008	0.00000	0.00027	0.00008	0.00000	-0.00027	0.00000
0.00000	0.00008	0.00027	0.00000	-0.00008	0.00007	0.00000	-0.00002	-0.00007	0.00000	0.00002	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00027	-0.00008	0.00000	0.00027	-0.00002	0.00000	0.00007	0.00002	0.00000	-0.00007	0.00000
0.00000	-0.00008	-0.00027	0.00000	0.00008	-0.00007	0.00000	0.00002	0.00007	0.00000	-0.00002	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00027	0.00008	0.00000	-0.00027	0.00002	0.00000	-0.00007	-0.00002	0.00000	0.00007	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

x1=	-0.5	coordinates of element nodes in element coordinate system
y1=	-0.5	
x2=	0.5	
y2=	-0.5	
x3=	0.5	
y3=	0.5	
x4=	-0.5	
y4=	0.5	
t=	0.01	thickness
E=	70,000	modulus of elasticity
v=	0.3	poisson's coefficient

Gauss Point 2

ξ=	0.577	gaus point coordinates in natural coordinate system
η=	-0.577	
wi=	1.000	gaus weights
wj=	1.000	
A=0.5*(x1*y2-y1*x2)+(x2*y3-y2*x3)+(x3*y4-y3*x4)+(x4*y1-y4*x1)=	1	area of element

N1=0.25*(1-ξ)*(1-η)=	0.167	dN1/dξ=-0.25*(1-η)=	-0.394	derivatives of element shape functions
N2=0.25*(1+ξ)*(1-η)=	0.622	dN2/dξ=0.25*(1-η)=	0.394	
N3=0.25*(1+ξ)*(1+η)=	0.167	dN3/dξ=0.25*(1+η)=	0.106	
N4=0.25*(1-ξ)*(1+η)=	0.045	dN4/dξ=-0.25*(1-η)=	-0.106	
		dN1/dη=-0.25*(1-ξ)=	-0.106	
		dN2/dη=-0.25*(1+ξ)=	-0.394	
		dN3/dη=0.25*(1+ξ)=	0.394	
		dN4/dη=0.25*(1-ξ)=	0.106	

J11=ΣdNi/dξi*xi=	0.500	coefficients of jacobian matrix
J12=ΣdNi/dξi*yi=	0.000	
J21=ΣdNi/dηi*xi=	0.000	
J22=ΣdNi/dηi*yi=	0.500	
det|J|=J11*J22-J12*J21=	0.250	jacobian matrix determinant
J11-1=1/det|J|*J22=	2.000	coefficients of inverted jacobian matrix
J12-1=-1/det|J|*J12=	0.000	
J21-1=-1/det|J|*J21=	0.000	
J22-1=1/det|J|*J11=	2.000	

[J]=	0.500	0.000	check using excel minverse()
	0.000	0.500	
[J]-1=	2.000	0.000	
	0.000	2.000	

[B]=

0	0.789	0	0	-0.789	0	0	-0.211	0	0	0.211	0
0	0	0.211	0	0	0.789	0	0	-0.789	0	0	0.000
0	0.211	0.789	0	0.000	0.000	0	-0.789	-0.211	0	0.000	0.211
-0.789	-0.167	0	0.789	-0.622	0	0.211	-0.167	0	-0.211	-0.045	0
-0.211	0	-0.167	-0.789	0	-0.622	0.789	0	-0.167	0.211	0	-0.045

[E]b=

0.006	0.002	0	0	0
0.002	0.006	0	0	0
0	0	0.000	0	0
0	0	0	0	0
0	0	0	0	0

[B]t=

0	0	0	-0.788675	-0.211325
0.7886751	0	0.2113249	-0.166667	0
0	0.2113249	0.7886751	0	-0.166667
0	0	0	0.7886751	-0.788675
-0.788675	0	0	-0.622008	0
0	0.7886751	0	0	-0.622008
0	0	0	0.2113249	0.7886751
-0.211325	0	-0.788675	-0.166667	0
0	-0.788675	-0.211325	0	-0.166667
0	0	0	-0.211325	0.2113249
0.2113249	0	0	-0.044658	0
0	0	0.2113249	0	-0.044658

det[J]= 0.250

[E]*[B]=

0	0.0050556	0.0004064	0	-0.005056	0.0015167	0	-0.001355	-0.001517	0	0.0013546	0
0	0.0015167	0.0013546	0	-0.001517	0.0050556	0	-0.000406	-0.005056	0	0.0004064	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

[B]t*[E]*[B]=

0	0	0	0	0	0	0	0	0	0	0	0
0	0.0039872	0.0003205	0	-0.003987	0.0011962	0	-0.001068	-0.001196	0	0.0010684	0
0	0.0003205	0.0002863	0	-0.000321	0.0010684	0	-8.59E-05	-0.001068	0	8.588E-05	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.003987	-0.000321	0	0.0039872	-0.001196	0	0.0010684	0.0011962	0	-0.001068	0
0	0.0011962	0.0010684	0	-0.001196	0.0039872	0	-0.000321	-0.003987	0	0.0003205	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.001068	-8.59E-05	0	0.0010684	-0.000321	0	0.0002863	0.0003205	0	-0.000286	0
0	-0.001196	-0.001068	0	0.0011962	-0.003987	0	0.0003205	0.0039872	0	-0.000321	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0.0010684	8.588E-05	0	-0.001068	0.0003205	0	-0.000286	-0.000321	0	0.0002863	0
0	0	0	0	0	0	0	0	0	0	0	0

stiffness matrix in gaus point 2

[K]Gi=w1*w2*[B]t*[E]*[B]*det|J|=

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00100	0.00008	0.00000	-0.00100	0.00030	0.00000	-0.00027	-0.00030	0.00000	0.00027	0.00000
0.00000	0.00008	0.00007	0.00000	-0.00008	0.00027	0.00000	-0.00002	-0.00027	0.00000	0.00002	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00100	-0.00008	0.00000	0.00100	-0.00030	0.00000	0.00027	0.00030	0.00000	-0.00027	0.00000
0.00000	0.00030	0.00027	0.00000	-0.00030	0.00100	0.00000	-0.00008	-0.00100	0.00000	0.00008	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00027	-0.00002	0.00000	0.00027	-0.00008	0.00000	0.00007	0.00008	0.00000	-0.00007	0.00000
0.00000	-0.00030	-0.00027	0.00000	0.00030	-0.00100	0.00000	0.00008	0.00100	0.00000	-0.00008	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00027	0.00002	0.00000	-0.00027	0.00008	0.00000	-0.00007	-0.00008	0.00000	0.00007	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

x1=	-0.5	coordinates of element nodes in <u>element coordinate system</u>
y1=	-0.5	
x2=	0.5	
y2=	-0.5	
x3=	0.5	
y3=	0.5	
x4=	-0.5	
y4=	0.5	
t=	0.01	thickness
E=	70,000	modulus of elasticity
v=	0.3	poisson's coefficient

Gauss Point 3

ξ=	0.577	gaus point coordinates in natural coordinate system
η=	0.577	
wi=	1.000	gaus weights
wj=	1.000	
A=0.5*(x1*y2-y1*x2)+(x2*y3-y2*x3)+(x3*y4-y3*x4)+(x4*y1-y4*x1)=	1	area of element

N1=0.25*(1-ξ)*(1-η)=	0.045	dN1/dξ=-0.25*(1-η)=	-0.106	derivatives of element shape functions
N2=0.25*(1+ξ)*(1-η)=	0.167	dN2/dξ=0.25*(1-η)=	0.106	
N3=0.25*(1+ξ)*(1+η)=	0.622	dN3/dξ=0.25*(1+η)=	0.394	
N4=0.25*(1-ξ)*(1+η)=	0.167	dN4/dξ=-0.25*(1-η)=	-0.394	

dN1/dη=-0.25*(1-ξ)=	-0.106	
dN2/dη=-0.25*(1+ξ)=	-0.394	
dN3/dη=0.25*(1+ξ)=	0.394	
dN4/dη=0.25*(1-ξ)=	0.106	
J11=ΣdNi/dξi*xi=	0.500	coefficients of jacobian matrix
J12=ΣdNi/dξi*yi=	0.000	
J21=ΣdNi/dηi*xi=	0.000	
J22=ΣdNi/dηi*yi=	0.500	
det|J|=J11*J22-J12*J21=	0.250	jacobian matrix determinant
J11-1=1/det|J|*J22=	2.000	coefficients of inverted jacobian matrix
J12-1=-1/det|J|*J12=	0.000	
J21-1=-1/det|J|*J21=	0.000	
J22-1=1/det|J|*J11=	2.000	

[J]=	0.500	0.000	check using excel minverse()
	0.000	0.500	
[J]-1=	2.000	0.000	
	0.000	2.000	

[B]=

0	0.211	0	0	-0.211	0	0	-0.789	0	0	0.789	0
0	0	0.211	0	0	0.789	0	0	-0.789	0	0	0.000
0	0.211	0.211	0	0.000	0.000	0	-0.789	-0.789	0	0.000	0.789
-0.211	-0.045	0	0.211	-0.167	0	0.789	-0.622	0	-0.789	-0.167	0
-0.211	0	-0.045	-0.789	0	-0.167	0.789	0	-0.622	0.211	0	-0.167

[E]b=

0.006	0.002	0	0	0
0.002	0.006	0	0	0
0	0	0.000	0	0
0	0	0	0	0
0	0	0	0	0

[B]t=

0	0	0	-0.211325	-0.211325
0.2113249	0	0.2113249	-0.044658	0
0	0.2113249	0.2113249	0	-0.044658
0	0	0	0.2113249	-0.788675
-0.211325	0	0	-0.166667	0
0	0.7886751	0	0	-0.166667
0	0	0	0.7886751	0.7886751
-0.788675	0	-0.788675	-0.622008	0
0	-0.788675	-0.788675	0	-0.622008
0	0	0	-0.788675	0.2113249
0.7886751	0	0	-0.166667	0
0	0	0.7886751	0	-0.166667

det[J]= 0.250

[E]*[B]=

0	0.0013546	0.0004064	0	-0.001355	0.0015167	0	-0.005056	-0.001517	0	0.0050556	0
0	0.0004064	0.0013546	0	-0.000406	0.0050556	0	-0.001517	-0.005056	0	0.0015167	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

[B]t*[E]*[B]=

0	0	0	0	0	0	0	0	0	0	0	0
0	0.0002863	8.588E-05	0	-0.000286	0.0003205	0	-0.001068	-0.000321	0	0.0010684	0
0	8.588E-05	0.0002863	0	-8.59E-05	0.0010684	0	-0.000321	-0.001068	0	0.0003205	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.000286	-8.59E-05	0	0.0002863	-0.000321	0	0.0010684	0.0003205	0	-0.001068	0
0	0.0003205	0.0010684	0	-0.000321	0.0039872	0	-0.001196	-0.003987	0	0.0011962	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.001068	-0.000321	0	0.0010684	-0.001196	0	0.0039872	0.0011962	0	-0.003987	0
0	-0.000321	-0.001068	0	0.0003205	-0.003987	0	0.0011962	0.0039872	0	-0.001196	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0.0010684	0.0003205	0	-0.001068	0.0011962	0	-0.003987	-0.001196	0	0.0039872	0
0	0	0	0	0	0	0	0	0	0	0	0

stiffness matrix in gaus point 3

[K]Gi=w1*w2*[B]t*[E]*[B]*det|J|=

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00007	0.00002	0.00000	-0.00007	0.00008	0.00000	-0.00027	-0.00008	0.00000	0.00027	0.00000
0.00000	0.00002	0.00007	0.00000	-0.00002	0.00027	0.00000	-0.00008	-0.00027	0.00000	0.00008	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00007	-0.00002	0.00000	0.00007	-0.00008	0.00000	0.00027	0.00008	0.00000	-0.00027	0.00000
0.00000	0.00008	0.00027	0.00000	-0.00008	0.00100	0.00000	-0.00030	-0.00100	0.00000	0.00030	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00027	-0.00008	0.00000	0.00027	-0.00030	0.00000	0.00100	0.00030	0.00000	-0.00100	0.00000
0.00000	-0.00008	-0.00027	0.00000	0.00008	-0.00100	0.00000	0.00030	0.00100	0.00000	-0.00030	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00027	0.00008	0.00000	-0.00027	0.00030	0.00000	-0.00100	-0.00030	0.00000	0.00100	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

x1=	-0.5	coordinates of element nodes in element coordinate system
y1=	-0.5	
x2=	0.5	
y2=	-0.5	
x3=	0.5	
y3=	0.5	
x4=	-0.5	
y4=	0.5	
t=	0.01	thickness
E=	70,000	modulus of elasticity
v=	0.3	poisson's coefficient

Gauss Point 4

ξ=	-0.577	gaus point coordinates in natural coordinate system
η=	0.577	
wi=	1.000	gaus weights
wj=	1.000	
A=0.5*(x1*y2-y1*x2)+(x2*y3-y2*x3)+(x3*y4-y3*x4)+(x4*y1-y4*x1)=	1	area of element

N1=0.25*(1-ξ)*(1-η)=	0.167	dN1/dξ=-0.25*(1-η)=	-0.106	derivatives of element shape functions
N2=0.25*(1+ξ)*(1-η)=	0.045	dN2/dξ=0.25*(1-η)=	0.106	
N3=0.25*(1+ξ)*(1+η)=	0.167	dN3/dξ=0.25*(1+η)=	0.394	
N4=0.25*(1-ξ)*(1+η)=	0.622	dN4/dξ=-0.25*(1-η)=	-0.394	

dN1/dη=-0.25*(1-ξ)=	-0.394
dN2/dη=-0.25*(1+ξ)=	-0.106
dN3/dη=0.25*(1+ξ)=	0.106
dN4/dη=0.25*(1-ξ)=	0.394

J11=ΣdNi/dξi*xi=	0.500	coefficients of jacobian matrix
J12=ΣdNi/dξi*yi=	0.000	
J21=ΣdNi/dηi*xi=	0.000	
J22=ΣdNi/dηi*yi=	0.500	
det|J|=J11*J22-J12*J21=	0.250	jacobian matrix determinant
J11-1=1/det|J|*J22=	2.000	coefficients of inverted jacobian matrix
J12-1=-1/det|J|*J12=	0.000	
J21-1=-1/det|J|*J21=	0.000	
J22-1=1/det|J|*J11=	2.000	

[J]=	0.500	0.000	check using excel minverse()
	0.000	0.500	
[J]-1=	2.000	0.000	
	0.000	2.000	

[B]=

0	0.211	0	0	-0.211	0	0	-0.789	0	0	0.789	0
0	0	0.789	0	0	0.211	0	0	-0.211	0	0	0.000
0	0.789	0.211	0	0.000	0.000	0	-0.211	-0.789	0	0.000	0.789
-0.211	-0.167	0	0.211	-0.045	0	0.789	-0.167	0	-0.789	-0.622	0
-0.789	0	-0.167	-0.211	0	-0.045	0.211	0	-0.167	0.789	0	-0.622

[E]b=

0.006	0.002	0	0	0
0.002	0.006	0	0	0
0	0	0.000	0	0
0	0	0	0	0
0	0	0	0	0

[B]t=

0	0	0	-0.211325	-0.788675
0.2113249	0	0.7886751	-0.166667	0
0	0.7886751	0.2113249	0	-0.166667
0	0	0	0.2113249	-0.211325
-0.211325	0	0	-0.044658	0
0	0.2113249	0	0	-0.044658
0	0	0	0.7886751	0.2113249
-0.788675	0	-0.211325	-0.166667	0
0	-0.211325	-0.788675	0	-0.166667
0	0	0	-0.788675	0.7886751
0.7886751	0	0	-0.622008	0
0	0	0.7886751	0	-0.622008

det[J]= 0.250

[E]*[B]=

0	0.0013546	0.0015167	0	-0.001355	0.0004064	0	-0.005056	-0.000406	0	0.0050556	0
0	0.0004064	0.0050556	0	-0.000406	0.0013546	0	-0.001517	-0.001355	0	0.0015167	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

[B]t*[E]*[B]=

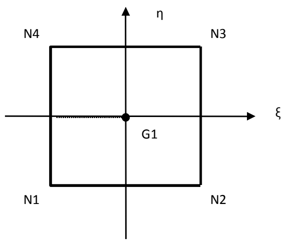
0	0	0	0	0	0	0	0	0	0	0	0
0	0.0002863	0.0003205	0	-0.000286	8.588E-05	0	-0.001068	-8.59E-05	0	0.0010684	0
0	0.0003205	0.0039872	0	-0.000321	0.0010684	0	-0.001196	-0.001068	0	0.0011962	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.000286	-0.000321	0	0.0002863	-8.59E-05	0	0.0010684	8.588E-05	0	-0.001068	0
0	8.588E-05	0.0010684	0	-8.59E-05	0.0002863	0	-0.000321	-0.000286	0	0.0003205	0
0	0	0	0	0	0	0	0	0	0	0	0
0	-0.001068	-0.001196	0	0.0010684	-0.000321	0	0.0039872	0.0003205	0	-0.003987	0
0	-8.59E-05	-0.001068	0	8.588E-05	-0.000286	0	0.0003205	0.0002863	0	-0.000321	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0.0010684	0.0011962	0	-0.001068	0.0003205	0	-0.003987	-0.000321	0	0.0039872	0
0	0	0	0	0	0	0	0	0	0	0	0

stiffness matrix in gaus point 4

[K]Gi=w1*w2*[B]t*[E]*[B]*det|J|=

0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00007	0.00008	0.00000	-0.00007	0.00002	0.00000	-0.00027	-0.00002	0.00000	0.00027	0.00000
0.00000	0.00008	0.00100	0.00000	-0.00008	0.00027	0.00000	-0.00030	-0.00027	0.00000	0.00030	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00007	-0.00008	0.00000	0.00007	-0.00002	0.00000	0.00027	0.00002	0.00000	-0.00027	0.00000
0.00000	0.00002	0.00027	0.00000	-0.00002	0.00007	0.00000	-0.00008	-0.00007	0.00000	0.00008	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	-0.00027	-0.00030	0.00000	0.00027	-0.00008	0.00000	0.00100	0.00008	0.00000	-0.00100	0.00000
0.00000	-0.00002	-0.00027	0.00000	0.00002	-0.00007	0.00000	0.00008	0.00007	0.00000	-0.00008	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00027	0.00030	0.00000	-0.00027	0.00008	0.00000	-0.00100	-0.00008	0.00000	0.00100	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

QUAD4 Reissner-Mindlin Plate Out of Plane Shear



x1=	-0.5	coordinates of element nodes in <u>element coordinate system</u>
y1=	-0.5	
x2=	0.5	
y2=	-0.5	
x3=	0.5	
y3=	0.5	
x4=	-0.5	
y4=	0.5	
t=	0.01	thickness
E=	70,000	modulus of elasticity G=E/(2*(1+v))= 26,923 shear modulus of elasticity
v=	0.3	poisson's coefficient
Ks=	0.833	plate shear corrtorection factor

Gauss Point 1

ξ=	0.000	gaus point coordinates in natural coordinate system
η=	0.000	
wi=	2.000	gaus weights
wj=	2.000	
A=0.5*(x1*y2-y1*x2)+(x2*y3-y2*x3)+(x3*y4-y3*x4)+(x4*y1-y4*x1)=	1	area of element

N1=0.25*(1-ξ)*(1-η)=	0.250	dN1/dξ=-0.25*(1-η)=	-0.250	derivatives of element shape functions
N2=0.25*(1+ξ)*(1-η)=	0.250	dN2/dξ=0.25*(1-η)=	0.250	
N3=0.25*(1+ξ)*(1+η)=	0.250	dN3/dξ=0.25*(1+η)=	0.250	
N4=0.25*(1-ξ)*(1+η)=	0.250	dN4/dξ=-0.25*(1-η)=	-0.250	

dN1/dη=-0.25*(1-ξ)=	-0.250	
dN2/dη=-0.25*(1+ξ)=	-0.250	
dN3/dη=0.25*(1+ξ)=	0.250	
dN4/dη=0.25*(1-ξ)=	0.250	
J11=ΣdNi/dξi*xi=	0.500	coefficients of jacobian matrix
J12=ΣdNi/dξi*yi=	0.000	
J21=ΣdNi/dηi*xi=	0.000	
J22=ΣdNi/dηi*yi=	0.500	
det|J|=J11*J22-J12*J21=	0.250	jacobian matrix determinant
J11-1=1/det|J|*J22=	2.000	coefficients of inverted jacobian matrix
J12-1=-1/det|J|*J12=	0.000	
J21-1=-1/det|J|*J21=	0.000	
J22-1=1/det|J|*J11=	2.000	

[J]=	0.500	0.000	check using excel minverse()
	0.000	0.500	
[J]-1=	2.000	0.000	
	0.000	2.000	

[B]=

0	0.500	0	0	-0.500	0	0	-0.500	0	0	0.500	0
0	0	0.500	0	0	0.500	0	0	-0.500	0	0	0.000
0	0.500	0.500	0	0.000	0.000	0	-0.500	-0.500	0	0.000	0.500
-0.500	-0.250	0	0.500	-0.250	0	0.500	-0.250	0	-0.500	-0.250	0
-0.500	0	-0.250	-0.500	0	-0.250	0.500	0	-0.250	0.500	0	-0.250

[E]b=

0.000	0.000	0	0	0
0.000	0.000	0	0	0
0	0	0.002	0	0
0	0	0	224.269	0
0	0	0	0	224.269

In plane shear

Out of plane shear

[B]t=

0.000	0.000	0.000	-0.500	-0.500
0.500	0.000	0.500	-0.250	0.000
0.000	0.500	0.500	0.000	-0.250
0.000	0.000	0.000	0.500	-0.500
-0.500	0.000	0.000	-0.250	0.000
0.000	0.500	0.000	0.000	-0.250
0.000	0.000	0.000	0.500	0.500
-0.500	0.000	-0.500	-0.250	0.000
0.000	-0.500	-0.500	0.000	-0.250
0.000	0.000	0.000	-0.500	0.500
0.500	0.000	0.000	-0.250	0.000
0.000	0.000	0.500	0.000	-0.250

det[J]= 0.250

[E]*[B]=

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0.0011218	0.0011218	0	0	0	0	-0.001122	-0.001122	0	0	0.0011218
-112.13462	-56.06731	0	112.13462	-56.06731	0	112.13462	-56.06731	0	-112.1346	-56.06731	0
-112.13462	0	-56.06731	-112.1346	0	-56.06731	112.13462	0	-56.06731	112.13462	0	-56.06731

[B]t*[E]*[B]=

112.13462	28.033654	28.033654	0	28.033654	28.033654	-112.1346	28.033654	28.033654	0	28.033654	28.033654
28.033654	14.017388	0.0005609	-28.03365	14.016827	0	-28.03365	14.016266	-0.000561	28.033654	14.016827	0.0005609
28.033654	0.0005609	14.017388	28.033654	0	14.016827	-28.03365	-0.000561	14.016266	-28.03365	0	14.017388
0	-28.03365	28.033654	112.13462	-28.03365	28.033654	0	-28.03365	28.033654	-112.1346	-28.03365	28.033654
28.033654	14.016827	0	-28.03365	14.016827	0	-28.03365	14.016827	0	28.033654	14.016827	0
28.033654	0	14.016827	28.033654	0	14.016827	-28.03365	0	14.016827	-28.03365	0	14.016827
-112.13462	-28.03365	-28.03365	0	-28.03365	-28.03365	112.13462	-28.03365	-28.03365	0	-28.03365	-28.03365
28.033654	14.016266	-0.000561	-28.03365	14.016827	0	-28.03365	14.017388	0.0005609	28.033654	14.016827	-0.000561
28.033654	-0.000561	14.016266	28.033654	0	14.016827	-28.03365	0.0005609	14.017388	-28.03365	0	14.016266
0	28.033654	-28.03365	-112.1346	28.033654	-28.03365	0	28.033654	-28.03365	112.13462	28.033654	-28.03365
28.033654	14.016827	0	-28.03365	14.016827	0	-28.03365	14.016827	0	28.033654	14.016827	0
28.033654	0.0005609	14.017388	28.033654	0	14.016827	-28.03365	-0.000561	14.016266	-28.03365	0	14.017388

stiffness matrix in gaus point i   **[K]Gi=w1*w2*[B]t*[E]*[B]*det|J|=**

112.13	28.03	28.03	0.00	28.03	28.03	-112.13	28.03	28.03	0.00	28.03	28.03
28.03	14.02	0.00	-28.03	14.02	0.00	-28.03	14.02	0.00	28.03	14.02	0.00
28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.02	-28.03	0.00	14.02
0.00	-28.03	28.03	112.13	-28.03	28.03	0.00	-28.03	28.03	-112.13	-28.03	28.03
28.03	14.02	0.00	-28.03	14.02	0.00	-28.03	14.02	0.00	28.03	14.02	0.00
28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.02	-28.03	0.00	14.02
-112.13	-28.03	-28.03	0.00	-28.03	-28.03	112.13	-28.03	-28.03	0.00	-28.03	-28.03
28.03	14.02	0.00	-28.03	14.02	0.00	-28.03	14.02	0.00	28.03	14.02	0.00
28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.02	-28.03	0.00	14.02
0.00	28.03	-28.03	-112.13	28.03	-28.03	0.00	28.03	-28.03	112.13	28.03	-28.03
28.03	14.02	0.00	-28.03	14.02	0.00	-28.03	14.02	0.00	28.03	14.02	0.00
28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.02	-28.03	0.00	14.02

Total stiffness matrix of plate element:

	T3	R1	R2	T3	R1	R2	T3	R1	R2	T3	R1	R2
	112.13	28.03	28.03	0.00	28.03	28.03	-112.13	28.03	28.03	0.00	28.03	28.03
	28.03	14.02	0.00	-28.03	14.01	0.00	-28.03	14.02	0.00	28.03	14.02	0.00
	28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.02	-28.03	0.00	14.02
	0.00	-28.03	28.03	112.13	-28.03	28.03	0.00	-28.03	28.03	-112.13	-28.03	28.03
	28.03	14.01	0.00	-28.03	14.02	0.00	-28.03	14.02	0.00	28.03	14.02	0.00
[K]=	28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.01	-28.03	0.00	14.02
	-112.13	-28.03	-28.03	0.00	-28.03	-28.03	112.13	-28.03	-28.03	0.00	-28.03	-28.03
	28.03	14.02	0.00	-28.03	14.02	0.00	-28.03	14.02	0.00	28.03	14.01	0.00
	28.03	0.00	14.02	28.03	0.00	14.01	-28.03	0.00	14.02	-28.03	0.00	14.02
	0.00	28.03	-28.03	-112.13	28.03	-28.03	0.00	28.03	-28.03	112.13	28.03	-28.03
	28.03	14.02	0.00	-28.03	14.02	0.00	-28.03	14.01	0.00	28.03	14.02	0.00
	28.03	0.00	14.02	28.03	0.00	14.02	-28.03	0.00	14.02	-28.03	0.00	14.02

Nastran file for element stiffness matrix generation in punch file:

```
$
NASTRAN SYSTEM(173)=2
NASTRAN SYSTEM(310)=1
$
SOL 101
CEND
$
SEALL = ALL
SUPER = ALL
$
TITLE = QUAD4_Plate_Shear
$
ECHO = NONE
MAXLINES = 999999999
K2GG = KAAX
$
BEGIN BULK
$
$ Parameters
PARAM,POST,0
PARAM,NOCOMPS,-1
PARAM,PRTMAXIM,YES
PARAM,K6ROT,100.
PARAM,PRGPST,NO
PARAM,AUTOSPC,YES
PARAM,SNORM,20.
PARAM,EXTOUT,DMIGPCH
$
$       GID   RCID     x     y     z
$======|=======|=======|=======|=======|=======|=======|=======|=======|=======|
GRID       1          -0.5   -0.5    0.
GRID       2           0.5   -0.5    0.
GRID       3           0.5    0.5    0.
GRID       4          -0.5    0.5    0.
$
$
$
$       MID     E      G      v
$======|=======|=======|=======|=======|=======|=======|=======|=======|=======|
MAT1       1  70000.          .3
$
$
$
$        ID   MID1     tm   MID2 12I/t^3   MID3   TS/T   NSM
$======|=======|=======|=======|=======|=======|=======|=======|=======|=======|
PSHELL     10         0.01     1           1
$
$
$       EID    PID    G1     G2     G3     G4   THETA  ZOFFS
$======|=======|=======|=======|=======|=======|=======|=======|=======|=======|
CQUAD4     1     10     1      2      3      4
$
$
$
$       DOF    GID1   GID2   GID3   GID4
$======|=======|=======|=======|=======|=======|=======|=======|=======|=======|
ASET1   123456     1      2      3      4
ENDDATA
```

Plate element stiffness matrix output in Nastran punch file:

```
DMIG    KAAX       0    6    2    0              24
DMIG*   KAAX                   1           3
*              1           3 5.443310210D-01
DMIG*   KAAX                   1           4
*              1           3 1.360827553D-01
*              1           4 7.073902708D-02
DMIG*   KAAX                   1           5
*              1           3-1.360827553D-01
*              1           4-1.041666638D-03
*              1           5 7.073902708D-02
DMIG*   KAAX                   2           3
*              1           3-5.058826669D-01
*              1           4-1.168585782D-01
*              1           5 1.360827553D-01
*              2           3 5.443310210D-01
DMIG*   KAAX                   2           4
*              1           3-1.168585782D-01
*              1           4-5.792181049D-02
*              1           5 8.012820915D-05
*              2           3 1.360827553D-01
*              2           4 7.073902708D-02
DMIG*   KAAX                   2           5
*              1           3-1.360827553D-01
*              1           4-8.012820915D-05
*              1           5 6.646552303D-02
*              2           3 1.360827553D-01
*              2           4 1.041666638D-03
*              2           5 7.073902708D-02
DMIG*   KAAX                   3           3
*              1           3 4.674343127D-01
*              1           4 1.168585782D-01
*              1           5-1.168585782D-01
*              2           3-5.058826669D-01
*              2           4-1.360827553D-01
*              2           5-1.168585782D-01
*              3           3 5.443310210D-01
DMIG*   KAAX                   3           4
*              1           3-1.168585782D-01
*              1           4-6.005856252D-02
*              1           5 1.041666638D-03
*              2           3 1.360827553D-01
*              2           4 6.646552303D-02
*              2           5 8.012820915D-05
*              3           3-1.360827553D-01
*              3           4 7.073902708D-02
DMIG*   KAAX                   3           5
*              1           3 1.168585782D-01
*              1           4 1.041666638D-03
*              1           5-6.005856252D-02
*              2           3-1.168585782D-01
*              2           4-8.012820915D-05
*              2           5-5.792181049D-02
*              3           3 1.360827553D-01
*              3           4-1.041666638D-03
*              3           5 7.073902708D-02
DMIG*   KAAX                   4           3
*              1           3-5.058826669D-01
*              1           4-1.360827553D-01
*              1           5 1.168585782D-01
*              2           3 4.674343127D-01
*              2           4 1.168585782D-01
*              2           5 1.168585782D-01
*              3           3-5.058826669D-01
*              3           4 1.168585782D-01
*              3           5-1.360827553D-01
*              4           3 5.443310210D-01
DMIG*   KAAX                   4           4
*              1           3 1.360827553D-01
*              1           4 6.646552303D-02
*              1           5-8.012820915D-05
*              2           3-1.168585782D-01
*              2           4-6.005856252D-02
*              2           5-1.041666638D-03
*              3           3 1.168585782D-01
*              3           4-5.792181049D-02
*              3           5 8.012820915D-05
*              4           3-1.360827553D-01
*              4           4 7.073902708D-02
DMIG*   KAAX                   4           5
```

```
*               1           3 1.168585782D-01
*               1           4 8.012820915D-05
*               1           5-5.792181049D-02
*               2           3-1.168585782D-01
*               2           4-1.041666638D-03
*               2           5-6.005856252D-02
*               3           3 1.360827553D-01
*               3           4-8.012820915D-05
*               3           5 6.646552303D-02
*               4           3-1.360827553D-01
*               4           4 1.041666638D-03
*               4           5 7.073902708D-02
DMIG    VAX          0      9      2      0                      1
DMIG*   VAX                        1             0
*               1           3 1.000000000D+00
*               1           4 1.000000000D+00
*               1           5 1.000000000D+00
*               2           3 1.000000000D+00
*               2           4 1.000000000D+00
*               2           5 1.000000000D+00
*               3           3 1.000000000D+00
*               3           4 1.000000000D+00
*               3           5 1.000000000D+00
*               4           3 1.000000000D+00
*               4           4 1.000000000D+00
*               4           5 1.000000000D+00
```