Notes on estimating BLP demand and supply side jointly.

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11/28/2017

Note: This is a supplement to my notes on BLP demand estimation using the random-coefficients logit model.

Motivation

In some cases we will want to fully specify a supply relationship and estimate it jointly with the demand-side equations (for example, see BLP (1995)). This fits into the regulat BLP demand model easily by adding moment conditions to the GMM objective function. The increase in computational and programming complexity is small for standard static supply-side models. As usual, estimating demand and supply jointly has the advantage of increasing the efficiency of the estimates, at the cost of requiring more structure. The cost and benefits are specific to each application and data set.

The Model: Pricing problem of the multiproduct firm

Suppress market/time subscript for simplicity of notation. There are F firms, each of which produce some subset, say J_f, of the J products. Given the demand system $s(x, p; \theta)$, the profits of firm f are

$$\Pi_f = \sum_{jinJ_f} (p_j - mc_j) M s_j(x, p; \theta)$$

Each firm is assumed to choose prices that maximize its profit. Thus any product produced by firm f must have a price, p_j, that satisfies the first order conditions

$$s_j + \sum_{r \in J_f} (p_r - mc_r) \frac{\partial s_r}{\partial p_j} = 0$$

The J first order conditions imply price-cost markups $(p_j - mc_j)$ for each good. Using matrix notation, this set of first order conditions can be rewritten as

$$s - \Omega(p - mc) = 0$$

where Ω is a $J \times J$ matrix, whose (j, r) element is equal to $-\frac{\partial s_r}{\partial p_j}$ if r and j are produced by the same firm, and 0 otherwise. Solving for price-cost markup gives

$$p - mc = \Omega^{-1} s$$

Define markup as

$$m = \Omega^{-1} s$$

And let marginal cost to be linear in cost characteristics

$$mc_j = w_j \gamma + \omega_j$$

Estimation Algorithm

As coefficient on price α appears in price elasticities, and thus in first order conditions, it enters optimization problem in a non-linear fashion when supply side equations are added.

Step 0:

- Draw ν_i (and D_i if needed) for a set of consumers (say NS=50).
- Initialize mean utility δ_{jt} based on homogenous logit.
- Choose initial values for θ_2, α.

Step 1:

- For given θ_2 (and initial draws of ν_i and D_i) compute household deviations from mean utility $\mu_{ijt}(x_{jt}, p_{jt}, \nu_i, D_i; \theta_2)$.

- For given mean utility δ_{jt} and θ_2, compute predicted shares. The integral in (1) can be approximated by simulation with

$$s_{jt}(\delta_t, \theta_2) = \frac{1}{NS} \sum_{i=0}^{NS} s_{ijt}(\delta_t, x_{jt}, p_{jt}, D_i, \nu_i; \theta_2) = \frac{1}{NS} \sum_{i=0}^{NS} \frac{\exp\{\delta_{jt} + \mu_{ijt}\}}{\sum_{k=0}^{J} \exp\{\delta_{kt} + \mu_{ikt}\}}$$

- For given α, θ_2 and individual shares, s_{ijt}, compute cross-price derivatives

$$\frac{\partial s_{jt}}{\partial p_{jt}} = \frac{1}{NS} \sum_{i=0}^{NS} \alpha_i s_{ijt}(1 - s_{ijt})$$

$$\frac{\partial s_{jt}}{\partial p_{kt}} = \frac{1}{NS} \sum_{i=0}^{NS} \alpha_i s_{ijt} s_{ikt}$$

- Compute markup $m_{jt}(\theta_2, \alpha)$ from simulated shares and cross-price derivatives.

Step 2:

- Contraction Mapping : given θ_2, search for δ_t such that market shares computed in *Step 1* are equal to the observed market shares, $s_{jt} = s_{jt}(\delta_t, \theta_2)$. This is a non-linear system of equations that is solved numerically using contraction mapping proposed by BLP (1995)

$$\delta_t^{h+1} = \delta_t^h + \ln s_{jt} - \ln s_{jt}(\delta_t^h, \theta_2)$$

  Iteration continues until $||\delta_t^{h+1} - \delta_t^h||$ is below a specified tolerance level.

Step 3:

- Estimate the linear parameters θ_1 and γ using the fact that

$$\delta_{jt}(s_{jt}, \theta_2) + \alpha p_{jt} = x_{jt}\beta + \xi_{jt}$$

$$(p_{jt} - m_{jt}(s_{jt}, \alpha, \theta_2)) = w_{jt}\gamma + \omega_{jt}$$

Stack IV moment conditions

$$E \begin{bmatrix} Z_d{}'\xi \\ Z_c{}'\omega \end{bmatrix} = 0$$

- Compute GMM objective function $Q(\alpha, \theta_2)$

Step 4:

- Minimize $Q(\alpha, \theta_2)$ over α and θ_2 with *Steps 1-3* nested for every (α, θ_2) trial.

References

Berry, S., Levinsohn, J., & Pakes, A. (1995). Automobile prices in market equilibrium. *Econometrica: Journal of the Econometric Society*, 841-890.