Audio Math in QA40xPlot

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Fundamental Signal RMS Voltage

The RMS voltage at the fundamental frequency is simply the height of the fundamental frequency's bin in the FFT.

V @ Frequency

Signal RMS Voltage

The RMS voltage is calculated one of two ways. For time domain data it's just the RMS definition

$$\boldsymbol{V_{RMS}} = \frac{\sqrt{\left(\sum_{t=0}^{T} V_t^2\right)}}{n}$$

For bandlimited frequency data it's a bit different since frequencies values are like densities.

$$\boldsymbol{V_{RMS}} = \frac{\sqrt{\left(\sum_{fmin}^{fmax} V_f^2\right)}}{ENBW(windowing)}$$

ENBW is the equivalent noise bandwidth of the fft windowing method. In some ways it's a measure of how much the signal is smeared into adjacent channels. It can be calculated by

$$\boldsymbol{ENBW} = \frac{\sqrt{\left(\sum_{t=0}^{T} W_t^2\right)}}{\sum_{t=0}^{T} W_t}$$

Where W_t is the fft weight at time *t*. Note this is scale-independent of the weights.

Harmonic Distortion

Using the frequency domain results the distortion voltage, in VRMS, of each harmonic is simply the height of the harmonic bin, V @ Harmonic Frequency.

The total harmonic distortion (THD) is the RMS total distortion divided by fundamental.

$$\boldsymbol{THD} = \frac{\sqrt{\left(\sum_{t=2}^{N} V_{Ft}^2 \ \right)}}{V_F}$$

Currently QA40xPlot uses N=7. Note that distortion spectra are clearly in-phase but the above approximation is good enough.

Intermodulation Distortion

CCIF style math for IMD

When close together fundamentals (**f_H/f_L < 2**) use the 2nd order CCIF2 or 3rd order CCIF3. QA40xPlot uses CCIF3.

CCIF2 uses a single value

CCIF2 IMD = $\frac{V_{f_H - f_L}}{V_{f_H} + V_{f_L}}$

CCIF3 uses a different single value

CCIF3 IMD = $\frac{\sqrt{V_{f_H-f_L}^2 + \left(V_{2f_L-f_H} + V_{2f_H-f_L}\right)^2}}{V_{f_H} + V_{f_L}}$

SMPTE/DIN IMD (or MOD IMD)

When the fundamentals are far apart (**f_H/f_L > 7**) use SMPTE/DIN math

SMPTE/DIN IMD = $\frac{\sqrt{\left(V_{fH-} \quad + V_{fH+fL}\right)^2 + \left(V_{fH-2fL} + V_{fH+2f} \quad\right)^2}}{V_{fH}}$

RMS Power IMD

Finally, when **2 < f_H/f_L < 7** use IMD RMS power methods using RMS addition

POWER IMD = $\frac{\sqrt{V_{fH-}^2 \quad + V_{fH+fL}^2 + V_{fL-}^2 \quad + V_{fL+2fH}^2 + V_{fH-2f}^2 \ + V_{fH+2f}^2}}{\sqrt{V_{fH}^2 + V_{fL}^2}}$

Noise Weighting Curves

From A-weighting - Wikipedia

The function(f) for C weighting is:

W_C(f)=$\frac{12194^2 * f^2}{(f^2+20.6^2)(f^2+1219\ \ ^2)}$

Adding two real-axis poles to the C-weighting transfer function gives us A-weighting:

W_A(f)= $\frac{12194^2 * f^4}{(f^2+20.6^2)(f^2+1219\ \ ^2)\sqrt{(f^2+107.7^2)(f^2+737.9^2)}}$

And unweighted effectively is

W_Z(f)=1.0

The weights are normalized to 1.0 at 1KHz.

Calculating Time Delay of a Signal

Use autocorrelation in the frequency domain by the following technique.

To find the time difference between two receipts of similar signals we use this->

L_f = FFT(L_t)

R_f = FFT(R_t)

Values = Abs(IFFT(L_f * Conj(R_f)).Real)

T_{max} = index of largest Value. If T_{max} > L/2 unwrap the value as negative T_{max}->T_{max}-L/2 where L = length of arrays

Time delay = T_{delta} * T_{max}

Biquad Filters

A Biquad filter is the way to generate an RIAA equalization curve. It's a ratio of quadratic polynomials.

$$H(s) = \frac{(1 + Z_1 S) * (1 + Z_2 S\ )}{(1 + P_1 S) * (1 + P_2 S)}$$

Where Z1 and Z2 are the zeros and P1 and P2 are the poles expressed as time points. So, RIAA uses 3 [or 4] time points where: T == 1 / (2 π F). The fourth point (the 50KHz pole) is to avoid overflows at high frequencies where there's no audio anyway.

RIAA Preemphasis

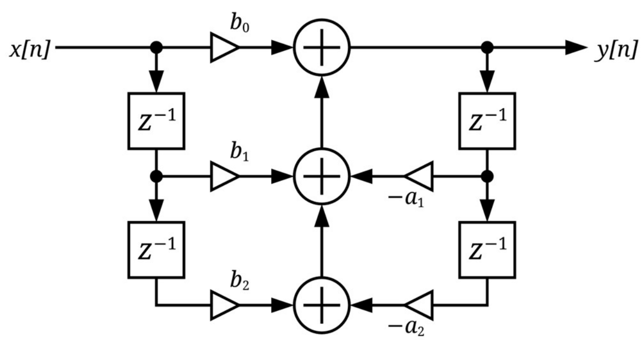
Variable	Value	Frequency
Z1	3180 µS	50.05 Hz
Z2	75 µS	2123 Hz
P1	318 µS	500.5 Hz
P2	3.18 µS	50050 Hz

Filtering in the frequency (FFT) domain use S=jω in the complex domain. Where ω=2 π F, so

$$H(\mathrm{f}) = \frac{(1 + jZ_1 * 2\pi F) * (1 + jZ_2 * 2\pi\mathrm{F}\ )}{(1 + jP_1 * 2\pi\mathrm{F}) * (1 + jP_2 * 2\pi\mathrm{F})}$$

Time Domain Filtering

With the above biquad equation, the digital Z domain filter (where Z-1 is last step, Z-2 is 2 steps ago…) becomes



Or Y[t] = $b_0X_t + b_1X_{t-1}+b_2X_{t-2} – a_1Y_{t-1} – a_2Y_{t-2}$

The filter coefficients are calculated from the transform $T' = e^{-\frac{1}{f_sT}}$

From https://linearaudio.net/sites/linearaudio.net/files/v10 sw app1 table a-1.xlsx

Biquad IIR Coefficients for Popular Sampling Frequencies	poles / zeros	a coefficients / b coefficients	Gain at 1kHz (dB)
48 kHz	[52.5257285337, 2287.22026941]	[1. -1.73273489, 0.73451925]	13.16179286
	[21833.6972506, 536.805364274]	[1. -0.75549731, -0.16463057]	
With the added 50 kHz zero	[52.3147934774, 2274.69993092]	[1. -1.7340031, 0.73577185]	12.87643103
	[20053.9676045, 533.822795553]	[1. -0.79733573, -0.1260212]	
96 kHz	[50.1482157346, 2125.92919847]	[1. -1.866632, 0.86705829]	18.62517791
	[38433.3251281, 501.903676141]	[1. -0.85352776, -0.11046426]	
With the added 50 kHz zero	[50.1411966368, 2125.49371853]	[1. -1.86665737, 0.86708352]	17.67778023
	[30478.7992823, 501.800206324]	[1. -0.96898135, 0.00125169]	
192 kHz	[50.0537159311, 2121.63154226]	[1. -1.93126252, 0.93137234]	24.35100343
	[75313.5235611, 500.558587601]	[1. -0.87968338, -0.10237808]	
With the added 50 kHz zero	[50.0537545743, 2121.60617514]	[1. -1.93126329, 0.93137311]	21.66968409
	[41656.8730375, 500.552807373]	[1. -1.17308964, 0.18626087]	