Audio Math

Fundamental Signal RMS Voltage

The RMS voltage at the fundamental frequency is simply the height of the fundamental frequency's bin in the FFT.

V @ Frequency

Signal RMS Voltage

The RMS voltage is calculated one of two ways. For time domain data it's just the RMS definition

$$V_{RMS} = \frac{\sqrt{\left(\sum_{t=0}^{T} V_t^2\right)}}{n}$$

For bandlimited frequency data it's a bit different since frequencies values are like densities.

$$V_{RMS} = \frac{\sqrt{\left(\sum_{fmin}^{fmax} V_f^2\right)}}{ENBW(windowing)}$$

ENBW is the equivalent noise bandwidth of the fft windowing method. In some ways it's a measure of how much the signal is smeared into adjacent channels. It can be calculated by

$$ENBW = \frac{\sqrt{\left(\sum_{t=0}^{T} W_t^2\right)}}{\sum_{t=0}^{T} W_t}$$

Where W_t is the fft weight at time t. Note this is scale-independent of the weights.

Harmonic Distortion

Using the frequency domain results the distortion voltage, in VRMS, of each harmonic is simply the height of the harmonic bin, V @ Harmonic Frequency.

The total harmonic distortion (THD) is the RMS total distortion divided by fundamental.

$$THD = \frac{\sqrt{\left(\sum_{t=2}^{N} V_{Ft}^{2}\right)}}{V_{F}}$$

Currently QA40xPlot uses N=7. Note that distortion spectra are clearly in-phase but the above approximation is good enough.

Intermodulation Distortion

CCIF style math for IMD

When close together fundamentals ($f_H/f_L < 2$) use the 2nd order CCIF2 or 3rd order CCIF3. QA40xPlot uses CCIF3.

CCIF2 uses a single value

CCIF2 IMD =
$$\frac{V_{f_H - f_L}}{V_{f_H} + V_{f_L}}$$

CCIF3 uses a different single value

CCIF3 IMD =
$$\frac{\sqrt{V_{f_H-f_L}^2 + (V_{2f_L-f_H} + V_{2f_H-f_L})^2}}{V_{f_H} + V_{f_I}}$$

SMPTE/DIN IMD (or MOD IMD)

When the fundamentals are far apart $(f_H/f_L > 7)$ use SMPTE/DIN math

$$\mathbf{SMPTE/DIN}\;\mathsf{IMD} = \frac{\sqrt{\left(V_{fH-fL} + V_{fH+fL}\right)^2 + \left(V_{fH-2fL} + V_{fH+2f}\right)^2}}{V_{fH}}$$

RMS Power IMD

Finally, when $2 < f_H/f_L < 7$ use IMD RMS power methods using RMS addition

$$\begin{aligned} \textbf{POWER} \text{ IMD} &= \frac{\sqrt{V_{fH-f}^2 + V_{fH+f}^2 + V_{fL-2f}^2 + V_{fL+2f}^2 + V_{fH-2fL}^2 + V_{fH+2f}^2}}{\sqrt{V_{fH}^2 + V_{fL}^2}} \end{aligned}$$

Noise Weighting Curves

From A-weighting - Wikipedia

The function(f) for C weighting is:

$$\mathbf{W}_{\mathbf{C}}(\mathbf{f}) = \frac{12194^2 * f^2}{(f^2 + 20.6^2)(f^2 + 12194^2)}$$

Adding two real-axis poles to the C-weighting transfer function gives us A-weighting:

$$\mathbf{W}_{A}(\mathbf{f}) = \frac{12194^{2} * f^{4}}{(f^{2} + 20.6^{2})(f^{2} + 1219^{2})\sqrt{(f^{2} + 107.7^{2})(f^{2} + 73^{2})^{2}}}$$

And unweighted effectively is

$$W_z(f)=1.0$$

The weights are normalized to 1.0 at 1KHz.