Multidisciplinary Design Optimization Techniques Applied to Diving Cylinder

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Following a brief introduction in which background information regarding diving cylinders is presented, a formulation for a multidisciplinary optimization problem considering material selection, mechanics of materials and economics is proposed with accompanying objective functions, design variables and constraint functions, along with their implementation in MATLAB. A high-fidelity 3D finite element simulation is used to obtain maximum von Mises stress values for a combination of internal pressures, wall thickness and material selections. This high-fidelity model is represented by a surface surrogate model, which is used in the optimization code to dramatically reduce computational time. Optimization under uncertainty methods are considered, results are analyzed and suggestions for potential future work are discussed.

I. Nomenclature

t	=	wall thickness
P_i	=	internal pressure
r_m	=	mean radius
r_o	=	outside radius
r_i	=	inside radius
h	=	cylinder height
S_y	=	yield strength
S_{max}	=	maximum stress
ρ	=	density
C	=	cost of cylinder
c	=	cost of material per kilogram
V	=	volume of material in cylinder
p	=	surface equation coefficient
R	=	reliability

II. Introduction

Diving cylinders are cylindrical tanks used to store high-pressure gas, and are often used to provide oxygen to divers while under the water's surface. Typically, they are manufactured out of steel or aluminum alloys, and can be designed to accommodate a range of internal pressures, although 3,000 psi is usually the standard working pressure [1]. The design of diving cylinders is a good candidate for optimization problem design, as a high-pressure tank that is too weak to hold gas is subject to bursting, while a tank that is too strong or uses an unnecessarily expensive material can be significantly more costly to produce than is needed. In addition, the design equations for cylinders are relatively simple and so can be fairly easily implemented in optimization codes.

In order to design a multidisciplinary optimization problem, the first step is to formulate the problem. This is expanded upon in greater detail in the following section, but the primary objective will be to minimize the cost of manufacturing the cylinder, while ensuring that the internal pressure acting on the walls does not cause the material to deform plastically.

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III. Problem Formulation

As noted previously, the objective of the optimization problem will be to minimize the cost of producing the cylinder. The factors contributing to the manufacturing process can be relatively complex and variable, but for the sake of simplicity, the manufacturing cost is assumed to be simply a product of the volume of material used to produce the cylinder, the density of the material, and the cost of material per kilogram.

$$\min_t C = c\rho\left[h\pi((r_i + t)^2 - r_i{}^2) + \tfrac{4}{3}\pi((r_i + t)^3 - r_i{}^3)\right] \tag{1}$$

The volume portion of Eq. (1) above is derived from the volume of the outside cylinder subtracted from the volume of the inside cylinder, in addition to the hemispherical ends, which share the same wall thickness as the cylinder. The height and internal radius values are fixed, while the wall thickness, t, serves as the single design variable.

In order to obtain the constraint functions, there were primarily two possible options. One would be to approximate the maximum stress experienced by the cylinder by using hoop stress equations intended for thin-walled pressure vessels. Another option would be to develop a finite element model (FEM) to model the geometry of the cylinder and the magnitude of the internal pressure, and from that obtain the maximum stress experienced by the cylinder. The latter option was chosen as a high-fidelity model is a project requirement, and it would likely be more accurate, as the thin-wall assumption may not apply to all possible cases considered. Therefore, a FEM can better account for the particular geometry of the cylinder used in this analysis.

Some problems with using a FEM in this case are that, for one, it can be difficult to integrate into MATLAB scripts, which will be used for this optimization problem, and, being a high-fidelity code, FEM analyses take too much time to run. In testing, each FEM problem can be solved within about 5 seconds, but in an optimization problem that can take hundreds or thousands of iterations, this can very dramatically slow down the process and be enormously computationally expensive. As an alternative, a handful of FEM simulations were run and a representative sampling of initial conditions and results were obtained, and by using surface fitting techniques, a surrogate model was developed to use in its place.

The FEM was developed using the same model dimensions specified in the MATLAB code. After developing the cross-section of the cylinder in SolidWorks, it was revolved 360° about the axis, forming a solid 3D shape with a hollow cavity. The bottom surface was kept flat in order to provide a convenient surface to fix the geometry.

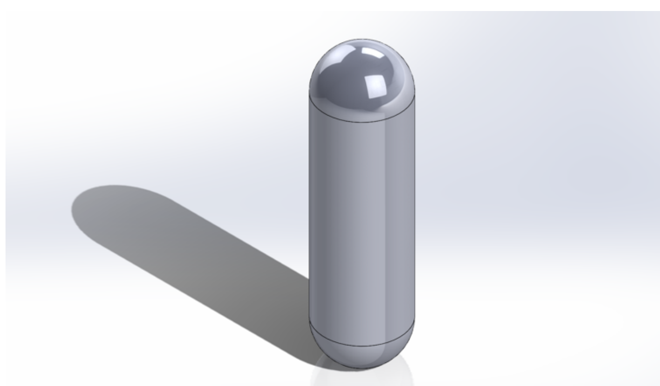

Fig. 1 Solid Model of Cylinder.

An internal pressure load was applied on the internal surface of the cylinder. To reduce the number of independent trials conducted, a time-dependent simulation was performed in SolidWorks with the internal pressure programmed to start at 0 Pa, then increase linearly to twice the standard working pressure, approximately 41.4 MPa, at the end of 6 seconds. This way, one trial could be run, and maximum stress data could be extracted from 7 different pressures in a quick succession at a single wall thickness value.

In all, the FEA (finite element analysis) simulation was run for combinations of 7 different internal pressures, 7 different wall thicknesses, and two different materials, which means there were a total of ninety-eight different maximum stress measurements taken, forming a fairly clear picture of how varying these three parameters affects the maximum stress[2]. The relevant material properties used in SolidWorks appear in the table below.

Table 1 Relevant Material Properties

	AISI 4130 Steel	Aluminum 6061
Elastic Modulus	205 GPa	69 GPa
Poisson's Ratio	0.285	0.33
Shear Modulus	80 GPa	26 GPa
Mass Density	7,850 kg/m^3	2,700 kg/m^3
Yield Strength	460 MPa	55.1485 MPa
Cost	$1,101.06/kg	$33.35/kg

Following the extraction of the data from the FEM, the data was imported into MATLAB, and the surrogate model was developed using the surface fitting command to generate the coefficients for the following surface equation[3].

$$S_{max}(P_i, t) = p_{00} + p_{10}P_i + p_{01}t + p_{20}{P_i}^2 + p_{11}P_i t + p_{02}t^2 + p_{21}{P_i}^2 t + p_{12}P_i t^2 + p_{03}t^3 \quad (2)$$

Since the two materials have different stress profiles, the coefficients of Eq. (2) vary based on the material properties, although the general shape of the surface is comparable. See the two figures below for a direct comparison.

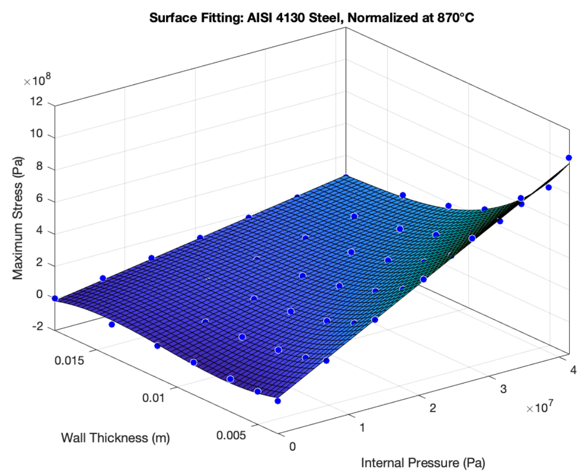

Fig. 2 Surrogate Model Surface of AISI 4130 Steel, Normalized at 870°C.

[2] The raw data for wall thickness, internal pressure, material and maximum stress can be found in the supplemental Microsoft Excel spreadsheet, "3D Curve Data.xlsx".

[3] MATLAB files used to provide surface equations for each material are provided in the supplemental files folder.

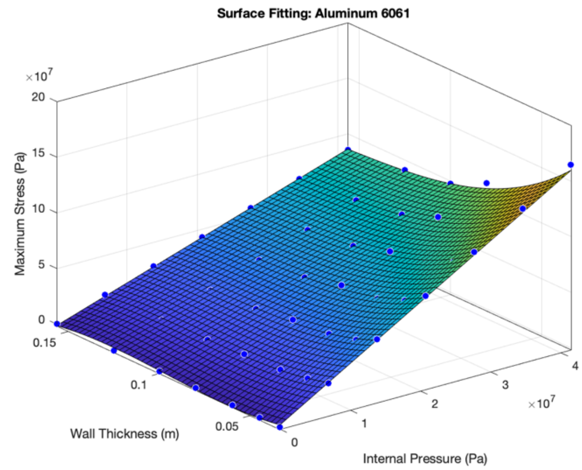

Fig. 3 Surrogate Model Surface of Aluminum 6061.

Since the goal is to avoid plastic deformation, the maximum stress acting on the cylinder must not exceed the yield strength of each material. Therefore, the nonlinear constraint can be specified as follows.

$$p_{00} + p_{10}P_i + p_{01}t + p_{20}{P_i}^2 + p_{11}P_i t + p_{02}t^2 + p_{21}{P_i}^2 t + p_{12}P_i t^2 + p_{03}t^3 - S_y \leq 0 \tag{3}$$

The lower bound on the wall thickness is 0, since this parameter cannot take a negative value, and while the upper limit is not as crucial to specify, it is set at an extremely generous value of 1 m. Setting a low upper bound on the wall thickness would likely have the effect of causing weaker materials to become unfeasible. Assuming the target operating internal pressure is known, the cylinder's wall thickness can be optimized.

In MATLAB, all of the constants are defined, such as the cylinder height, internal radius, target internal pressure, and others. Several material-dependent properties are also specified, such as yield strength, density and cost per kilogram. The surface function coefficients are entered in a series of arrays, and the optimization itself can begin.

As there are no linear equality or inequality constraints, those matrices are left blank. The lower and upper bounds of the wall thickness design variable are specified, along with a small initial guess, which is important for this particular optimization problem. The algorithm used is a combination of a discrete problem, which substitutes properties specific to each material used, into fmincon, which is a gradient-based solver found in MATLAB's Optimization Toolbox. This algorithm was chosen because the surrogate models themselves are gradient-based, therefore accurate results can be expected that are obtained less computationally expensively and more quickly than competing non-gradient-based algorithms, such as the genetic algorithm or the particle swarm algorithm. This step also calls the nonlinear constraint specified in Eq. (3). Fmincon then finds the minimum viable volume for the material, multiplies this by the material's density and cost, and compares it to previous values, only keeping the lowest cost material. This process is repeated for each material, and at the end, the code will present the final optimized wall thickness, the lowest cost, and the material number, which specifies which material to use.

IV. Results and Discussion

When using the configurations specified in the code as it is, the program finds that it is more cost-effective to create the cylinder out of 6061 aluminum, as the final cost would be approximately $4,593.51, compared to $16,927.51 if the cylinder were made of 4130 steel. This is despite the fact that, due to the superior mechanical properties of steel, a steel cylinder would require far less material volume than aluminum, at 0.00196 m^3 compared to 0.0510 m^3, respectively. This is caused by the more expensive material cost of steel vs aluminum, at least as specified in this problem. Consequently, the optimal thickness of the optimal steel vs aluminum cylinder also varies, at 4.331 mm vs 75.380 mm, respectively.

The above results produce an optimized cost, material volume and wall thickness under the assumption that there is no uncertainty associated with the design parameters, or that the models are completely deterministic. It assumes that, in the real world, the working pressure will be *exactly* 3,000 psi, and that the cylinder can be manufactured such that the wall thickness can be an extraordinarily precise valve. In the real world, these sorts of assumptions often simply are not valid. This sort of model does not account for the possibility, for instance, that maybe a technician may fill a tank a bit over capacity, or that a manufacturing process may result in a tank wall that is a bit too thin in some places, or even that the materials themselves may have some impurities causing unanticipated weaknesses. Due to these possibilities, it is important design the tank with the intention of going beyond the bare minimum performance specifications to account for any uncertainties and reduce the failure rate.

There are multiple ways to consider OUU (Optimization Under Uncertainty), but the method used here will be the deterministic design approach, which consists of safety factors applied to the design parameters. To give just one example, if a factor of safety of 1.14 is applied to the target internal pressure, this means the optimization problem is being solved again, but with a higher working pressure than before. This necessitates a stronger cylinder, thus a larger wall thickness, a larger material volume and higher costs. Therefore, if a cylinder is made which stores gas at a pressure slightly higher than the standard working pressure, or the walls are manufactured slightly below the optimal wall thickness in practice, this should not end in failure.

The reliability value is determined by assigning random values, which fall along a scaled normal distribution, to the wall thickness, with the optimal value serving as the mean, and simply evaluating the constraint functions to determine if they are still satisfied. For each case in which any of the constraint functions are not met, i.e., when the maximum stress exceeds the yield strength, that case is counted as a failure. For each case in which all of the constraints are met, that case is counted as a success. The reliability value, R, is simply the ratio of the successes to the total number of trials. So, for instance, an R value of 0.5 would mean that the design would fail exactly 50% of the time, and succeed exactly 50% of the time, probabilistically.

When the design is considered prior to OUU techniques, the reliability value can vary somewhat, as it is a probabilistic measure, but typically only barely exceeds 0.5 in repeated executions of the script. After applying OUU methods, with a factor of safety of 1.14 applied to the working pressure, and considering the actual wall thickness can vary 0.01 m multiplied by a random number selected from a normal distribution from the optimal value, the reliability value dramatically increases, typically over 0.99. Even seemingly slight increases to the factor of safety, such as 1.2, result in a perfect reliability value of 1. This means that if the cylinder were manufactured to withstand a working pressure 1.2 times the standard working pressure, there would be zero failures in 10,000 cases. The table below shows the results of varying the factor of safety and how it affects the reliability value, as well as some of the optimal values, assuming 10,000 trials.

Table 2 Factor of Safety vs Reliability Values

Factor of Safety	Reliability	Optimal Material	Minimum Wall Thickness (m)	Minimum Volume (m^3)	Minimum Cost (USD)
none	0.5153	Al 6061	0.0754	0.0510	$4,593.51
1.1	0.9750	Al 6061	0.0949	0.0708	$6,372.22
1.15	0.9997	Al 6061	0.1087	0.0866	$7,798.35
1.2	1.0000	Al 6061	0.1269	0.1098	$9,890.73

V. Conclusions and Future Work

The methods proposed in this paper can be used to illustrate the relationship between the internal working pressure, the wall thickness, material selection and cost of manufacturing a diving cylinder. They are an attempt to formulate and solve a multidisciplinary design optimization problem that integrates the disciplines of material science, mechanics of materials and economics. A surrogate model was used in place of a computationally expensive FEA code to obtain maximum stresses acting on the cylinder, and the effects of incorporating OUU methods was also explored. However, a number of limitations became apparent in the design process, and future work could focus on overcoming these shortcomings. Some of them will be very briefly discussed.

First, this model, for demonstrative purposes, only incorporated material properties and surrogate models for two materials. Any useful code intended for practical use ought to have a far greater variety of materials available for selection. Second, regarding the 3D model of the cylinder, as the wall thickness increased, the model developed a few sharper edges, particularly on the internal surfaces. These sharp edges can create localized regions of stress concentrations when loads are applied. This can result in FEM models exceeding their yield strength sooner than a real-life product would, resulting in optimal wall thicknesses that are likely far thicker than actually needed. Future work should seek to better understand their effect and, if necessary, eliminate these sorts of design flaws. Third, the price of the materials used above may not be precise, as it is difficult to find pricing data on materials that is a direct, 1:1 comparison. Materials are often sold in very large sheets, rolls or rods, which may or may not be consistent across material types, can vary considerably in price depending on what form they are sold in, and even based on which vendor is used. As the code depends on the price of material to obtain an optimal cost, obtaining accurate 1:1 relative material pricing is essential. Finally, this particular optimization algorithm only considers the cylinder's basic structural properties in a single nonlinear constraint. There may be many more sorts of design specifications and standards for diving cylinders that could be formulated as design variables and constraints, but which were neglected in this analysis for the sake of simplicity. This is a fairly short list of possible areas of future work, illustrating that there is still much more which can be done to create a more robust and practically useful code.

References

[1] Dive Gear Express LLC, "How to Select a SCUBA Tank," [Online]. Available: https://www.divegearexpress.com/library/articles/how-to-select-a-scuba-tank. [Accessed 12 May 2022].