Group 3 Lab report
Experiment 2: Noise Measurements

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1 Introduction

The objective of this experiment was to measure the noise spectra emitted by a resistor, followed by analysis of different noise types, temperature dependent noise levels, and noise mitigation[1]. This was to be accomplished via a spectrum analyser and analysing the noise levels at room temperature (298 K) and liquid nitrogen (77 K). Specifically, we sought to observe the Johnson-Nyquist noise[2] and see how we could minimise it.

1.1 Johnson-Nyquist Noise

Johnson-Nyquist noise occurs due to thermal agitation of the electric charges in a conductor coupled to a thermal environment[1, 2]. This noise only depends on the temperature in Kelvin, T, and the bandwidth Δf. By probing at a constant bandwidth, we observe that $N \propto T$.

$$N = k_B T \Delta f \tag{1}$$

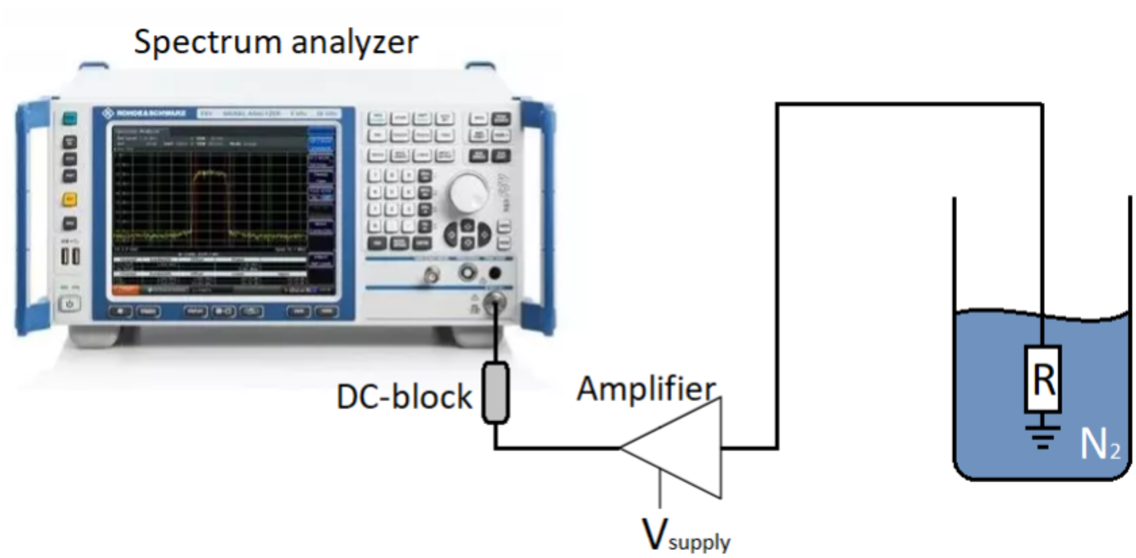
2 Method

For this experiment, we conducted 6 different measurements using a spectrum analyser. Specifically, we used the FSV40-N spectrum analyser, which can observe frequencies from 9 kHz to 40 GHz. Then, we measure the Johnson-Nyquist noise via a spectrum analyser. Our settings for the spectrum analyser are shown here:

Parameters	Set values
Mode	Spectrum
Reference Level	173 pW
Attenuation	0 dB
Sweep time	100 ms
Resolution Bandwidth	2 MHz
Video Bandwidth	10 kHz
Frequency	6 GHz
Span	0 Hz

Here, the resolution bandwidth is the minimum bandwidth over which two signals are still separable, whereas the video bandwidth can be thought of as a low pass filter which filters noise. Only the resolution bandwidth reduces the noise floor; the video bandwidth only reduces the noise on the trace.

First, we measure over different frequencies, sweeping from 0 to 12 GHz. This was achieved with slightly different settings from the one in the table, where we set the span to 12 GHz. Next, we measure the noise level over a 100 ms time interval at 6 GHz. This measurement is then repeated with a 50 Ω resistor, which is then replaced by a 40 dB amplifier. Finally, we measure with both the resistor and the amplifier attached. So far, all measurements have happened at room temperature, which we take to be 298 K. However, for the final experiment, we repeat the measurement with both the resistor and the amplifier in a liquid nitrogen bath. This gives us a second measurement at approximately 298 K, with which we can compare equation 1. The final setup is shown here:



2.1 Amplifiers

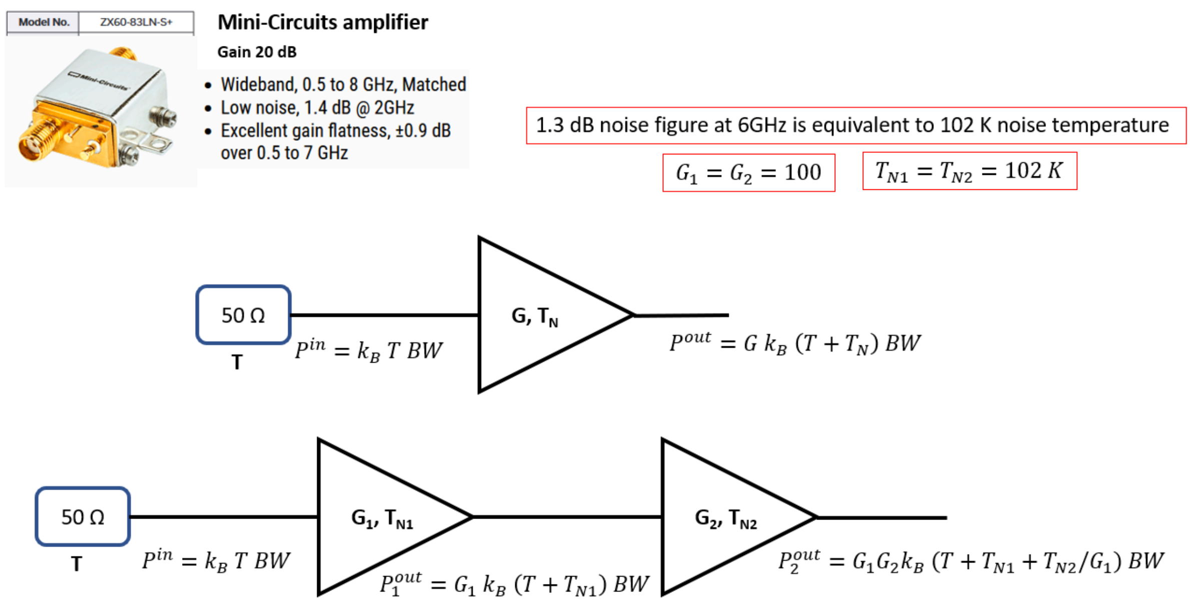
For our experiment, we use 2 20 dB amplifiers connected one after the other. These amplifiers are identical, hence the order does not matter, however, it is quite essential for multi-amplifier setups. For a 2 amplifier setup, the power output is

$$P_{\text{out}} = G_1 G_2 k_B \left(T + T_{N_1} + \frac{T_{N_2}}{G_1} \right) \Delta f \tag{2}$$

We can rewrite this equation to get the temperature instead:

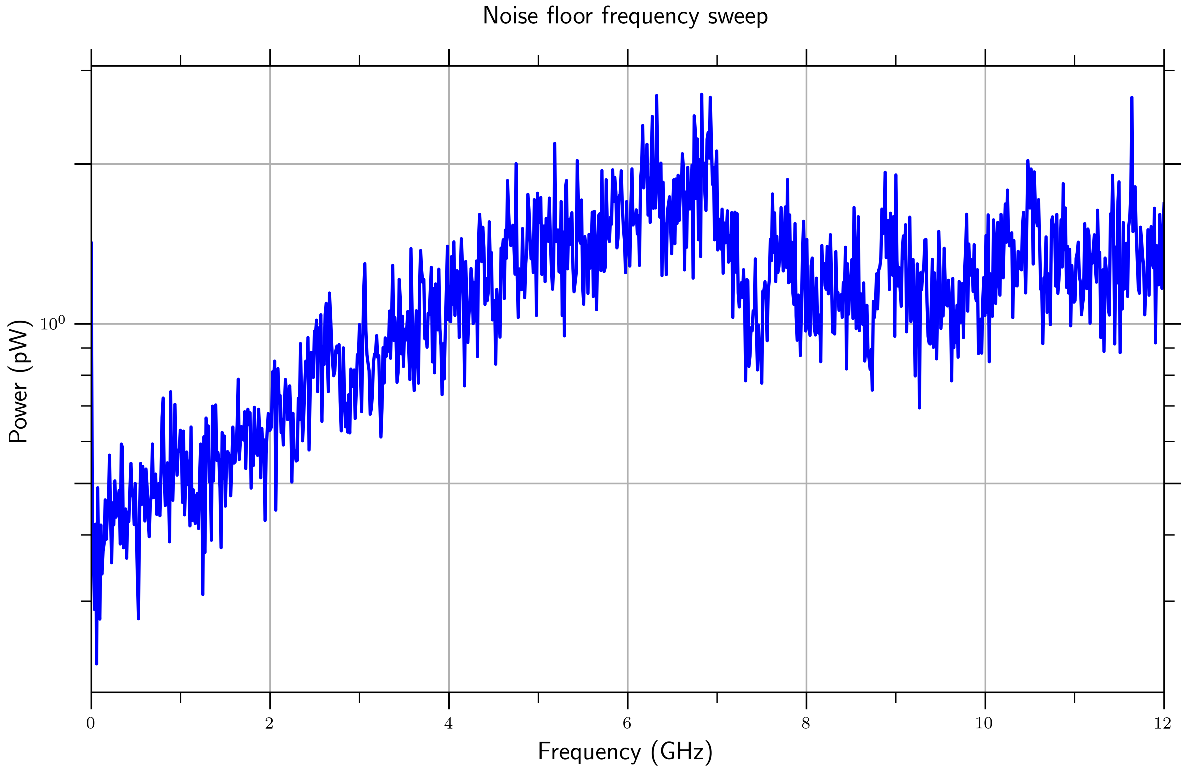
$$T = \frac{P_{\text{out}}}{G_1 G_2 k_B \Delta f} - T_{N_1} - \frac{T_{N_2}}{G_1} \tag{3}$$

As such, the noise temperature for the first amplifier determines most of the noise. The details for our amplifiers are given below:



3 Results

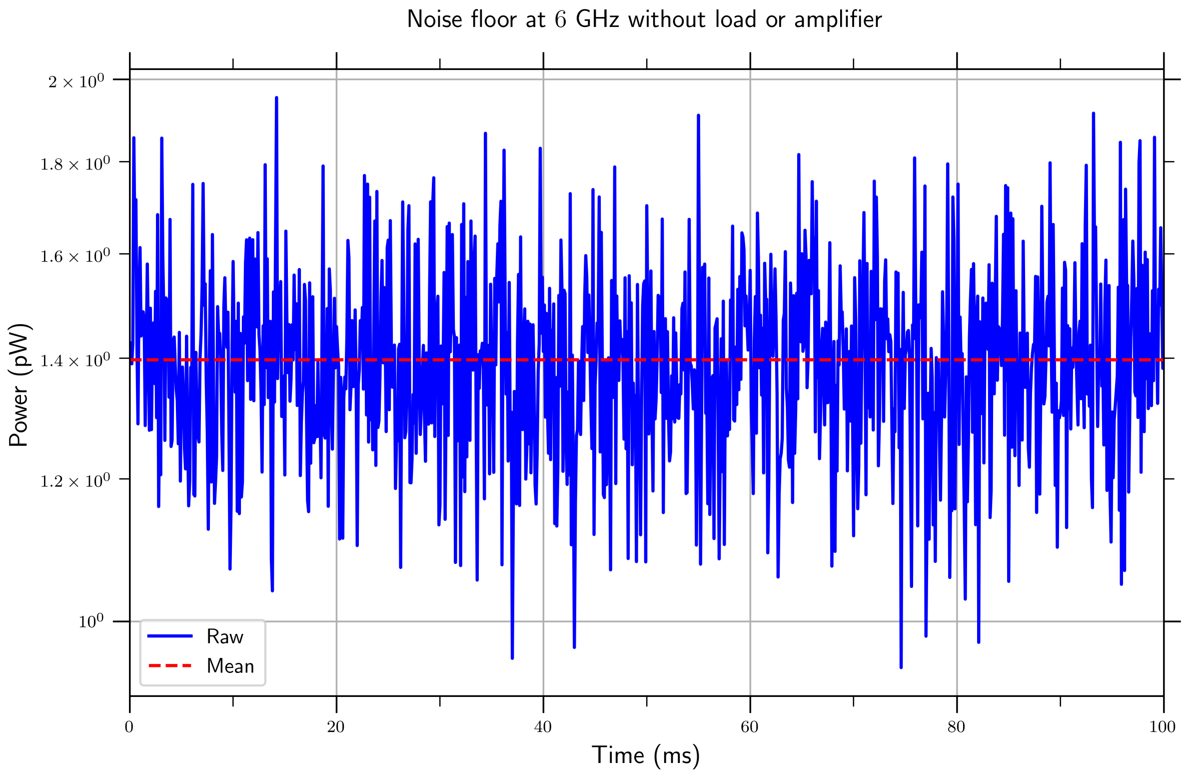
3.1 Noise floor sweep from 0 to 12 GHz without load or amplifier



As we can see, the noise is very stable around 1 pW, with the noise peeking around 6.5 GHz at around 1.5 pW. This also follows the noise specifications quite well, however, the gain flatness is not quite ±0.9 dB between 0.5 GHz and 7 GHz, as it is around 13 dB.

3.2 Noise floor measurements at 6 GHz without load or amplifier

3.2.1 Raw data with the mean highlighted



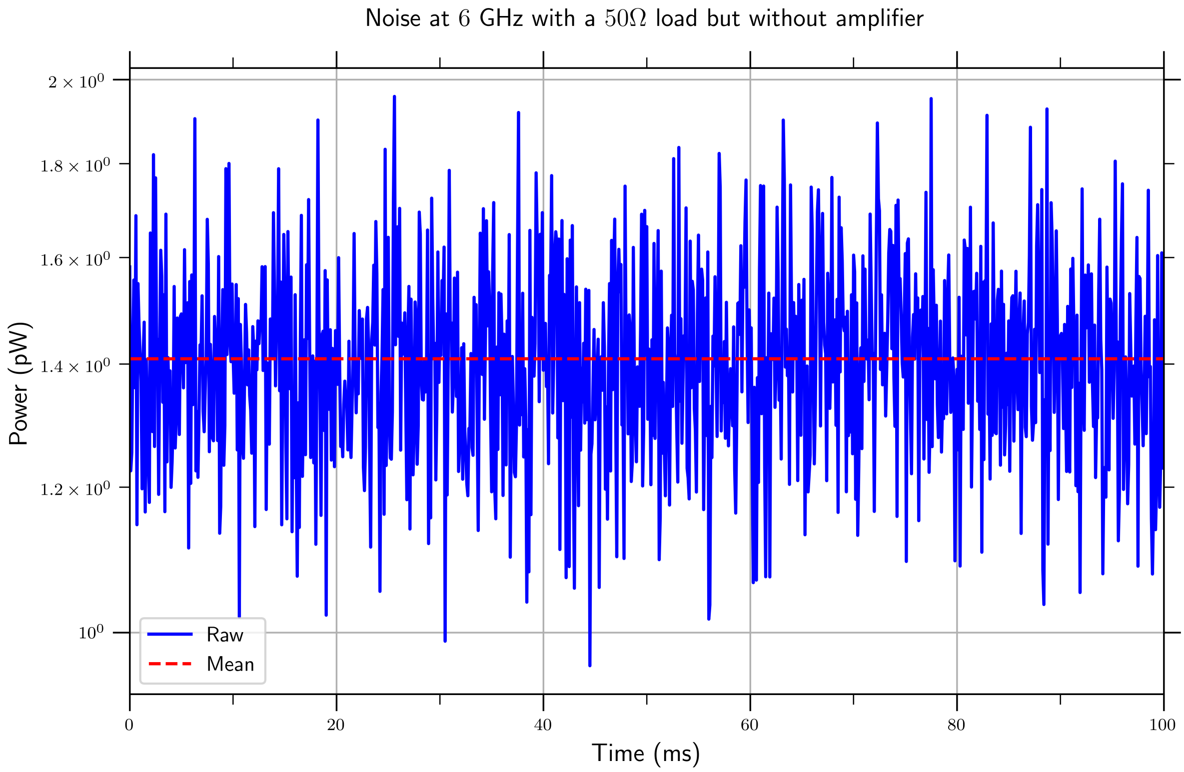
3.2.2 Noise analysis

The actual average power is 1.397 pW. The noise level calculated via root mean square is 1.407 pW. The noise level calculated via peak-to-peak is 1.011 pW.

The signal-to-noise ratio calculated via root mean square is 0.9930. The signal-to-noise ratio calculated via peak-to-peak is 1.382.

3.3 Noise measurements at 6 GHz with a 50Ω load but no amplifier

3.3.1 Raw data with the mean highlighted



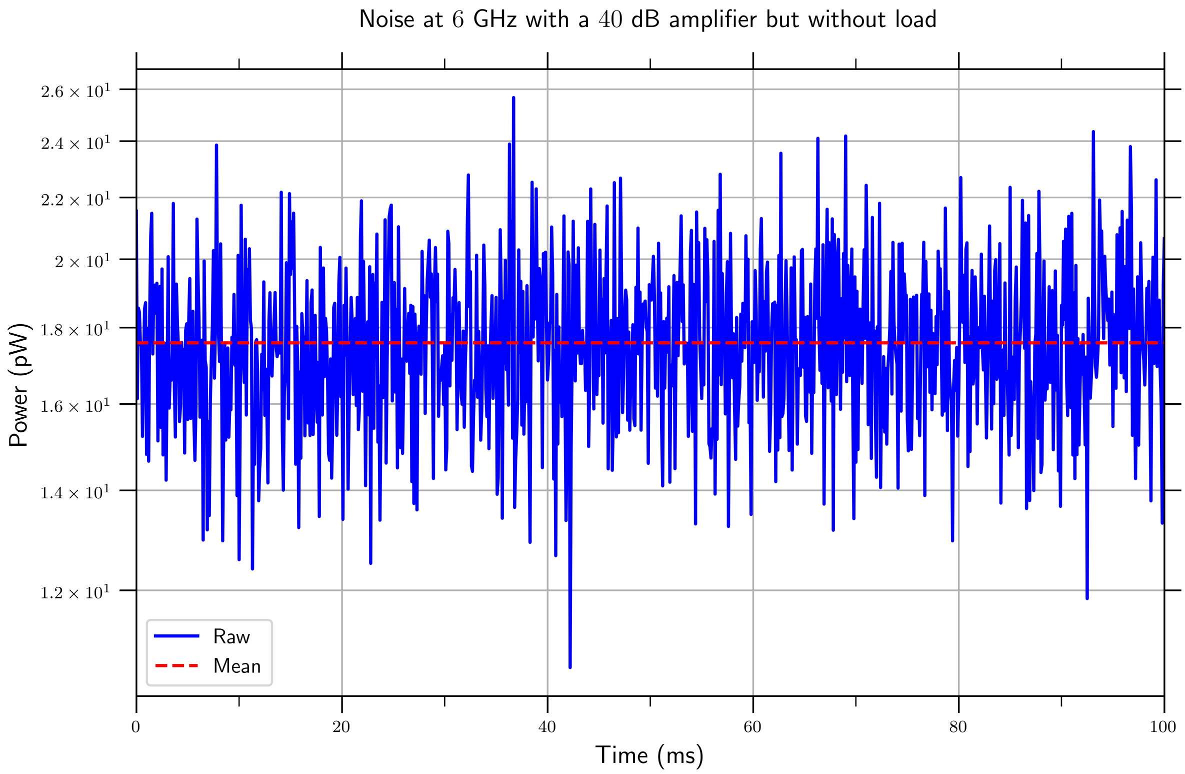
3.3.2 Noise analysis

The average power is 1.410 pW. The noise level calculated via root mean square is 1.420 pW. The noise level calculated via peak-to-peak is 0.9987 pW.

The signal-to-noise ratio calculated via root mean square is 0.9929. The signal-to-noise ratio calculated via peak-to-peak is 1.411.

3.4 Noise measurements at 6 GHz with an amplifier but no load

3.4.1 Raw data with the mean highlighted



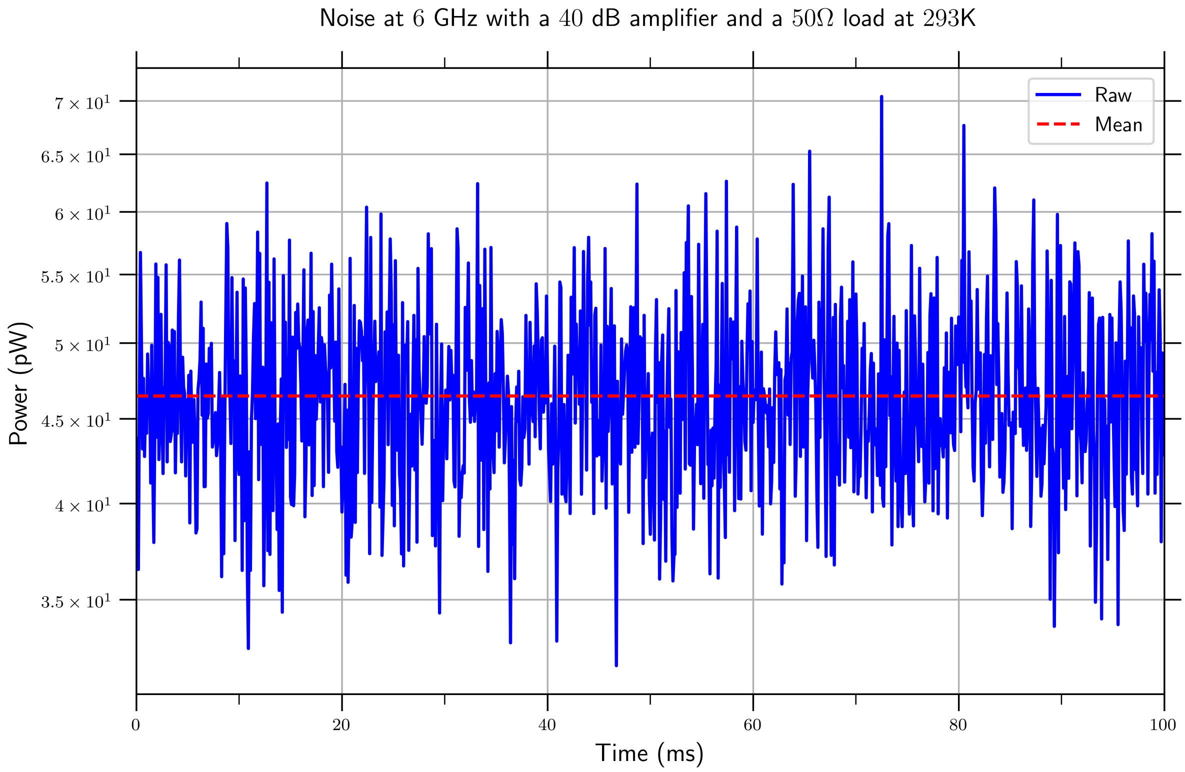
3.4.2 Noise analysis

The average power is 17.58 pW. The noise level calculated via root mean square is 17.71 pW. The noise level calculated via peak-to-peak is 15.01 pW.

The signal-to-noise ratio calculated via root mean square is 0.9927. The signal-to-noise ratio calculated via peak-to-peak is 1.171.

3.5 Noise measurements at $6\,\mathrm{GHz}$ with a $40\,\mathrm{dB}$ amplifier and a $50\,\Omega$ load at $298\,\mathrm{K}$

3.5.1 Raw data with the mean highlighted



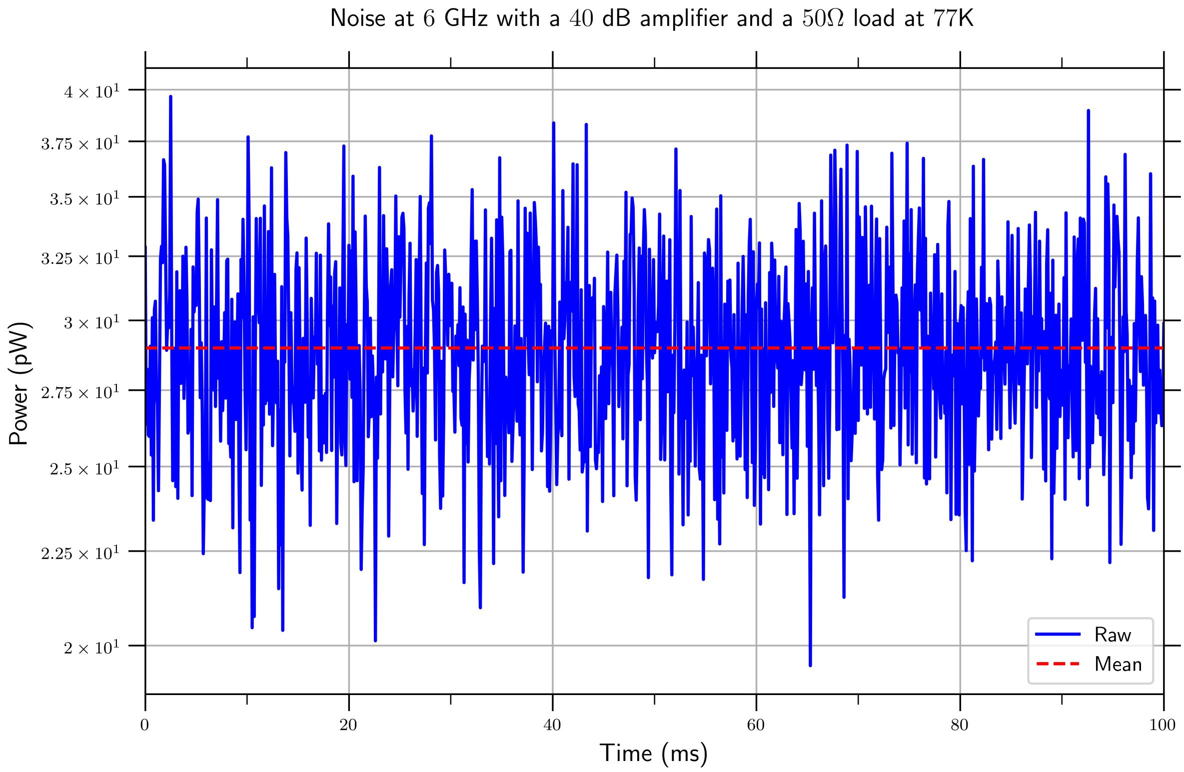
3.5.2 Noise analysis

The average power is $46.45\,\mathrm{pW}$. The noise level calculated via root mean square is $46.79\,\mathrm{pW}$. The noise level calculated via peak-to-peak is $38.51\,\mathrm{pW}$.

The signal-to-noise ratio calculated via root mean square is 0.9927. The signal-to-noise ratio calculated via peak-to-peak is 1.206.

3.6 Noise measurements at 6 GHz with a 40 dB amplifier and a 50 Ω load at 77 K

3.6.1 Raw data with the mean highlighted



3.6.2 Noise analysis

The average power is 28.99 pW. The noise level calculated via root mean square is 29.18 pW. The noise level calculated via peak-to-peak is 20.15 pW.

The signal-to-noise ratio calculated via root mean square is 0.9932. The signal-to-noise ratio calculated via peak-to-peak is 1.438.

4 Conclusion

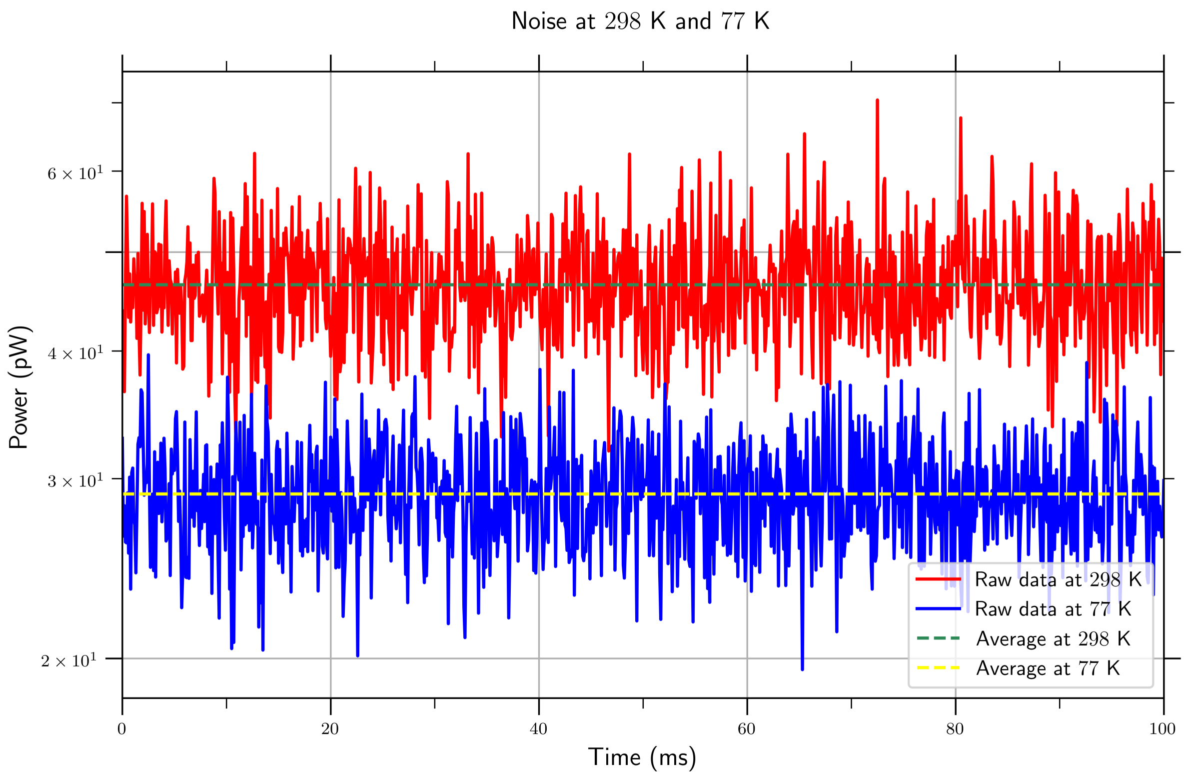
As a short reminder, for the 5 time sweeps, we get an average power output of 1.397 pW, 1.410 pW, 17.58 pW, 46.45 pW and 28.99 pW, corresponding to nothing, only load, only amplifier, both load and amplifier at room temperature and both load and amplifier in a liquid nitrogen bath.

At room temperature, we observe that simply adding the load does not showcase a large change, however, with the 40 dB amplifier, we observe that adding the load causes the noise to almost double.

Additionally, by going over all the noise analyses, we observe that the root mean square noise level calculation is more accurate as it is less prone to being affected by outliers. Additionally, it was very close to the average power, as it resulted in a consistent signal-to-noise ratio of around 0.99.

4.1 Noise dependence on temperature

By comparing the complete setup at 298 K and 77 K, we can check how the noise dependence changes. Here, we have the plot with both results overlaid:



As we can see, the noise drops by almost a factor of 2 from 46.45 pW to 28.99 pW.

4.2 Temperature analysis

We can calculate the theoretical noise we expect from a few of the setups and compare them to our actual results. We kept the resolution bandwidth fixed at 2 MHz, and the gain and noise temperature of the amplifiers are fixed at 20 dB, or 100, and 102 K respectively.

First, we can use equation 1 to calculate the expected noise with just a $50\,\Omega$ load at $298\,\mathrm{K}$.

The expected power is $k_B T \Delta f = k_B \times 298\,\mathrm{K} \times 2\,\mathrm{MHz} \approx 8.229\,\mathrm{fW}$.

However, the actual average power is $1.410\,\mathrm{pW}$, which is about 2 orders of magnitude larger.

Similarly, we can use equation 2 to calculate the expected noise with the $40\,\mathrm{dB}$ amplifier and a $50\,\Omega$ load at $298\,\mathrm{K}$ and $77\,\mathrm{K}$.

At $298\,\mathrm{K}$, the expected power is $G^2 k_B \Delta f \left(T + T_N + \frac{T_N}{G}\right) = 100^2 \times k_B \times 2\,\mathrm{MHz} \times \left(298\,\mathrm{K} + 102\,\mathrm{K} + \frac{102\,\mathrm{K}}{100}\right) \approx 110.7\,\mathrm{pW}$.

However, the actual average power is $46.45\,\mathrm{pW}$, which is about half of the theoretical power.

At $77\,\mathrm{K}$, the expected power is $G^2 k_B \Delta f \left(T + T_N + \frac{T_N}{G}\right) = 100^2 \times k_B \times 2\,\mathrm{MHz} \times \left(77\,\mathrm{K} + 102\,\mathrm{K} + \frac{102\,\mathrm{K}}{100}\right) \approx 49.71\,\mathrm{pW}$.

However, the actual average power is $28.99\,\mathrm{pW}$, which is about half of the theoretical power once again.

In fact, estimating the actual temperatures based on the Johnson-Nyquist noise using equations 1 and 3 respectively gives us $51\,060\,\mathrm{K}, 65.20\,\mathrm{K}$ and $1.967\,\mathrm{K}$ respectively, which are wildly different when compared to the actual values of $298\,\mathrm{K}, 298\,\mathrm{K}$ and $77\,\mathrm{K}$ respectively. This strongly suggests that there were additional noise sources which were causing this discrepancy. However, the liquid nitrogen noise can potentially be explained by considering that the actual temperature is smaller than the noise temperature of the amplifiers.

In conclusion, while we get wildly different noises from the expected values, we do observe the rapid drop in noise as we cool from room temperature to liquid nitrogen. However, Johnson-Nyquist noise alone is not sufficient to explain the exact values. As such, Johnson-Nyquist noise is not a good tool to use as a primary thermometer, even though it has a linear dependence on time.

5 References

References

[1] Timm Fabian Mörstedt and Suman Kundu. *Experiment 2: Noise Measurements*. 2023.

[2] H. Nyquist. "Thermal Agitation of Electric Charge in Conductors". In: *Physical Review* 32.1 (July 1, 1928), pp. 110–113. DOI: 10.1103/PhysRev.32.110. URL: https://link.aps.org/doi/10.1103/PhysRev.32.110 (visited on 09/26/2023).