Day: M T W T F S Date: / /

$f(x,y) = x^3 - y^3 - 2xy + 6$

taking partial derivative w.r.t x,

$f_x = \frac{\partial}{\partial x} x^3 - \frac{\partial}{\partial x} y^3 - \frac{\partial}{\partial x} 2xy + \frac{\partial}{\partial x} 6$

$= 3x^2 - 2x \quad - (i)$

taking partial derivative w.r.t y,

$f_y = \frac{\partial}{\partial y} x^3 - \frac{\partial}{\partial y} y^3 - \frac{\partial}{\partial y} 2xy + \frac{\partial}{\partial y} 6$

$f_y = -3y^2 - 2x - (ii)$

$3x^2 - 2y = 0 \Rightarrow x = -\frac{2}{3} ; y = \frac{2}{3}$
$-3y^2 - 2x = 0$

$x = 0$
$y = 0$

$f_{xx} = 6x|(0,0), (-2/3, 2/3) = 0, -4$

$f_{yy} = -6y|(0,0), (-2/3, 2/3) = 0, -4$

$f_{xy} = 0|(0,0), (-2/3, 2/3) = 0, 0$

Apply criterion for maximum for $(0,0)$,

$D \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{xx} \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$

for $(-2/3, 2/3)$:

$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{xx} \end{vmatrix} = \begin{vmatrix} -4 & 0 \\ 0 & -4 \end{vmatrix} = 16$

So, $(-2/3, 2/3)$ is maxima.