MID TERM PAPER-DE (ANSWER SHEET)

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Question # 01

Q① $\frac{dy}{dx} = \frac{2-4y-8x}{4+4y+8x}$

Solution:-

$$\frac{dy}{dx} = \frac{\cancel{2}(1-2y-4x)}{{}_{2}\cancel{4}(1+y+2x)}$$

$$2\frac{dy}{dx} = \frac{1-2y-4x}{1+y+2x} \longrightarrow ①$$

let $u = y+2x \rightarrow du = dy + 2$

$$\frac{du}{dx} = \frac{dy}{dx} + 2$$

$$\frac{du}{dx} - 2 = \frac{dy}{dx}$$

Now substituting in ①.

$$2\left(\frac{du}{dx} - 2\right) = \frac{1-2u}{1+u}$$

$$2\frac{du}{dx} = \frac{1-2u}{1+u} + 2$$

$$2\frac{du}{dx} = \frac{1-\cancel{2u}+2+\cancel{2u}}{1+u}$$

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$$\frac{2\,du}{dx} = \frac{1}{1+u}$$

$$\int (1+u)\,du = \int \frac{1}{2}\,dx$$

$$u + \frac{u^2}{2} = \frac{1}{2}x \longrightarrow (2).$$

putting

$$u = y + 2x$$

$$y + 2x + \frac{(y+2x)^2}{2} = \frac{1}{2}x$$

$$2y + 4x + y^2 + 4yx + 4x^2 = x$$

$$\cancel{2}\ 2y + 4x + (y+2x)^2 - x = C$$

$$2y + 3x + (y+2x)^2 = C.$$

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Question # 02

State & solve Bernoulli's Equation.

Statement:-

Any Differential Equation of Type

$$\frac{dy}{dx} + P(x)y = Q(x)\,y^n \quad \text{where}$$

$n \in \mathbb{R}$ & $n \neq 0, 1$ is called

Bernoulli's Equation.

Solution:-

Step # 01

$$y^{-n}\frac{dy}{dx} + p(x)\,y^{1-n} = Q(x) \longrightarrow ①$$

Step # 02

Set $y^{1-n} = z$

$$\Rightarrow \frac{d}{dx}\left(y^{1-n}\right) = \frac{d}{dx}(z)$$

$$\Rightarrow 1-n\, y^{1-n-1} \cdot \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow y^{n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dy}{dx} \longrightarrow ②$$

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Step #03 Now substitituting (2) in (1) we get.

$$\frac{1}{1-n}\frac{dz}{dx} + P(x)\cdot z = Q(x).$$

$$\frac{dz}{dx} + (1-n)\,P(x)\,z = (-n)\,Q(x)$$

where $p(x) = (1-n)\,P(x)$ &
$q(x) = (1-n)\,Q(x)$

which is linear Differential Equation. i-e $\frac{dz}{dx} + p(x)z = q(x)$. → (3)

Step #04 Find Integrating factor.

$$I\cdot F = e^{\int p(x)\,dx}$$

step# 05
Multiplying I-F with (3).

$$e^{\int p(x)\,dx}\cdot\frac{dz}{dx} + e^{\int p(x)\,dx}\,p(x)\cdot z = e^{\int p(x)\,dx}\,q(x)$$

i-e $$\frac{d}{dx}\left[e^{\int p(x)\,dx}\cdot z\right] = m(x)$$

where
$$m(x) = e^{\int p(x)\,dx}\cdot q(x).$$

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Step # 06

Now by Variable Separable.

$$\int d\left[e^{\int P(x)\,dx} \cdot z\right] = \int m(x)\,dx.$$

i.e $e^{\int P(x)\,dx} \cdot z = M(x) + C$

i.e $z = e^{-\int P(x)\,dx}\left[M(x) + C\right].$

i.e $\boxed{y^{1-n} = e^{-\int P(x)\,dx}\left[M(x) + C\right]}$

is the solution of Bernoulli's Equation.

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Question # 03 Solve

$$y'[\sinh(3y) - 2xy) = y^2$$

$$y(1) = 1$$

Solution:-

$$y'[\sinh(3y) - 2xy) = y^2$$

$$y' = \frac{y^2}{(\sinh 3y) - 2xy}$$

$$\frac{dx}{dy} = \frac{\sinh 3y - 2xy}{y^2}$$

$$\frac{dx}{dy} = \frac{\sinh 3y}{y^2} - \frac{2xy}{y^2}$$

$$\frac{dx}{dy} - \frac{2\cancel{y}}{y^{\cancel{2}}} \cdot x = \frac{\sinh 3y}{y^2}$$

$$x' + \frac{2}{y} \cdot x = \frac{\sinh 3y}{y^2}$$

$$P(y) = +2/y \quad \neq \quad h = \int P(y)\,dy = \cancel{\int} \int 2/y\,dy$$

$$= \ln y^2$$

$$y = e^{-h}\left[\int e^{h} \cdot r(y) + c\right].$$

$$= e^{-\ln y^2}\left[\int e^{\ln y^2} \cdot \frac{\sinh 3y}{y^2} + c\right.$$

$$= -y^2\left(\int \cancel{y^2} \frac{\sinh 3y}{\cancel{y^2}}\right] + c$$

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$$x = y^{-2}\left[\frac{1}{3}\cosh(3y) + C\right]$$

$$x = \frac{1}{3y^2}\cosh 3y + cy^{-2}$$

At $y(1) = 1$

$$1 = \frac{1}{3}\cosh(3) + C$$

$$1 = \frac{1}{\cancel{3}}\,\frac{\cancel{9}\,3}{10} + C$$

$$1 = 0.3 + C$$

$$C = 1 - 0.3 = 0.7$$

Hence

$$x = \frac{1}{3y^2}\cosh 3y + 0.7y^{-2}.$$

Question # 04 :-

~~Solve~~ $\cancel{y'' + y' + 9.25y = e^{-x}}$.

$\cancel{y(0) = 7, \quad y'(0) =}$

Solve $3y'' + 10y' + 3y = e^{x} + 5\cos x$

Solution:-

$$y = y_h + y_p$$

$\cancel{y_h =}$ $$3y'' + 10y' + 3y = 0$$

$$3m^2 + 10m + 3 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$m = \frac{-10 \pm \sqrt{100 - 4(3)(3)}}{2(3)}$$

$$= \frac{-10 \pm \sqrt{100-36}}{6}$$

$$= \frac{-10 \pm 8}{6}$$

$m_1 = \frac{-2}{6}$ or $\frac{-18}{6}$

$m_1 = -1/3$ or $m_2 = -3$.

Roots are real & different hence case (I)

$$y_h = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$= c_1 e^{-\frac{1}{3}x} + c_2 e^{-3x}$$

$$y_p = e^x + 5\cos x$$

Hence

$$y = c_1 e^{-\frac{1}{3}x} + c_2 e^{-3x} + e^x + 5\cos x$$

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Question #05

Solve $y'' + y' + 9.25y = e^{-x}$

$y(0) = 7 \quad y'(0) = 2.$

Solution

homogenous equation will be

$$y'' + y' + 9.25y = 0$$

$$m^2 + m + 9.25 = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1 - 4(1)(9.25)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 - 37}}{2}$$

$$= \frac{-1 \pm \sqrt{-36}}{2}$$

Case (III)

$$= \frac{-1 \pm 6i}{2} \Rightarrow \boxed{\frac{-1}{2} \pm 3i}$$

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~~$y_n = C_1 \cos$~~

$y_n = (C_1 \cos bx + C_2 \sin bx) e^{ax}$

where $m = a \pm bi$

$y_n = (C_1 \cos 3x + C_2 \sin 3x) e^{-1/2 x}$

$y_p = e^{-x}$

$y = y_n + y_p$

$y = (C_1 \cos 3x + C_2 \sin 3x) e^{-1/2 x} + e^{-x} \rightarrow (1)$

$7 = (C_1 \cos(0) + C_2 \sin(0)) e^{0} + e^{0}$

$\boxed{7 = C_1}$

$y' = (-3C_1 \sin 3x + 3C_2 \cos 3x) e^{-1/2 x} - \frac{1}{2}(C_1 \cos 3x + C_2 \sin 3x) e^{-1/2 x} - e^{-x}$

$2 = ((-3C_1 \sin(0) + 3C_2 \cos(0)) e^{0} - \frac{1}{2} C_1 \cos(0) - 1.$

$2 = +3C_2 - \frac{1}{2} C_1 - 1$

$3 = 3C_2 - \frac{1}{2} C_1$

$3 = 3C_2 - \frac{1}{2}(7)$

$3 - \frac{7}{2} = 3C_2$

$-\frac{1}{2} = 3C_2$

$C_2 = \boxed{-1/6}$

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putting this in ① we get -

$$y = \left(7\cos 3x - \frac{1}{6}\sin 3x\right)e^{-\frac{1}{2}x} + e^{-x}$$

↳ Answer.